1D_lineraly_Streteched_rod_with_introduced_body_force_(E=1)

```
import numpy as np  # For numerical operations
import tensorflow as tf  # For building and training the neural network
import matplotlib.pyplot as plt  # For plotting results

# Define the Neural Network for solving the PDE
class SimplePINN(tf.keras.Model):
    def __init__(self, layer_sizes):
        super(SimplePINN, self).__init__()
        # Create hidden layers with 'tanh' activation
        self.hidden_layers = [tf.keras.layers.Dense(size, activation='tanh') for size in layer_sizes[:-1]]
        # Output layer (no activation function since this is a regression task)
        self.output_layer = tf.keras.layers.Dense(layer_sizes[-1])

    def call(self, x):
        # Pass data through each hidden layer
        for layer in self.hidden_layers:
            x = layer(x)
        # Return the result from the output layer
        return self.output_layer(x)

# Function to compute PDE residual (difference from satisfying the equation)
def compute_pde_residual(model, x_vals, young_modulus):
    # Use nested gradient tapes to calculate second derivatives (needed for PDE residual)
    with tf.GradientTape(persistent=True) as outer_tape:
        with tf.GradientTape() as inner_tape:
            inner_tape.watch(x_vals)
            outer_tape.watch(x_vals)
            predicted_u = model(x_vals)  # Predict displacement using model

        # First derivative of displacement (strain)
        u_x = inner_tape.gradient(predicted_u, x_vals)
        # Compute stress using Hooke's Law (stress = modulus * strain)
        stress = young_modulus * u_x

    # Second derivative of stress (needed for the PDE residual)
    stress_x = outer_tape.gradient(stress, x_vals)
    del inner_tape, outer_tape  # Clean up tapes after use

    return stress_x  # Return the computed PDE residual (difference from zero)

# Loss function to enforce the PDE residual and boundary conditions
def compute_total_loss(model, x_points, x_left, x_right, young_modulus, force):
    # Calculate PDE residual loss
    pde_loss = tf.reduce_mean(tf.square(compute_pde_residual(model, x_points, young_modulus)))
    # Enforce boundary condition at the left end (displacement u = 0 at x = 0)
    left_boundary_loss = tf.reduce_mean(tf.square(model(x_left)))
    # Enforce boundary condition at the right end (displacement u = t / E at x = 1)
    right_boundary_loss = tf.reduce_mean(tf.square(model(x_right) - force / young_modulus))
    # Total loss combines PDE residual and boundary conditions
    return pde_loss + left_boundary_loss + right_boundary_loss

# Create points to check the PDE (collocation points) and boundary points
num_collocation_points = 100
x_points = tf.convert_to_tensor(np.linspace(0, 1, num_collocation_points).reshape(-1, 1), dtype=tf.float32)
x_left = tf.convert_to_tensor([[0.0]], dtype=tf.float32)
x_right = tf.convert_to_tensor([[1.0]], dtype=tf.float32)

# Model parameters
young_modulus = 10  # Young's modulus, E
force = 1  # Force applied at the end of the rod
layer_sizes = [1, 20, 20, 20, 1]  # Neural network structure: 1 input, 3 hidden layers, 1 output

# Initialize the neural network model
model = SimplePINN(layer_sizes)

# Set up the optimizer for training
optimizer = tf.keras.optimizers.Adam()

# Training loop
num_epochs = 2000
for epoch in range(num_epochs):
    # Record gradients with respect to model parameters
    with tf.GradientTape() as tape:
        # Compute total loss (PDE residual + boundary conditions)
        loss = compute_total_loss(model, x_points, x_left, x_right, young_modulus, force)
    # Compute gradients of loss with respect to model parameters
    gradients = tape.gradient(loss, model.trainable_variables)
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    # Update model weights using gradients
    optimizer.apply_gradients(zip(gradients, model.trainable_variables))

    # Print the loss every 100 epochs for monitoring
    if epoch % 100 == 0:
        print(f"Epoch {epoch}, Loss: {loss.numpy()}")

# Evaluate the model by generating predictions for test points
x_test = tf.convert_to_tensor(np.linspace(0, 1, 100).reshape(-1, 1), dtype=tf.float32)
predicted_u = model(x_test)  # Predicted displacement values

# Calculate the analytical solution for comparison
x_true = np.linspace(0, 1, 100)
true_u = (force / young_modulus) * x_true  # u(x) = (t / E) * x

# Plot the model's predictions and the analytical solution
plt.figure(figsize=(10, 6))
plt.plot(x_test, predicted_u, label="PINN Prediction", linestyle='-', marker='o')
plt.plot(x_true, true_u, label="Analytical Solution", linestyle='--')
plt.xlabel("x")
plt.ylabel("Displacement u(x)")
plt.title("Displacement Field u(x) for 1D Stretching Rod")
plt.legend()
plt.grid()
plt.show()
```
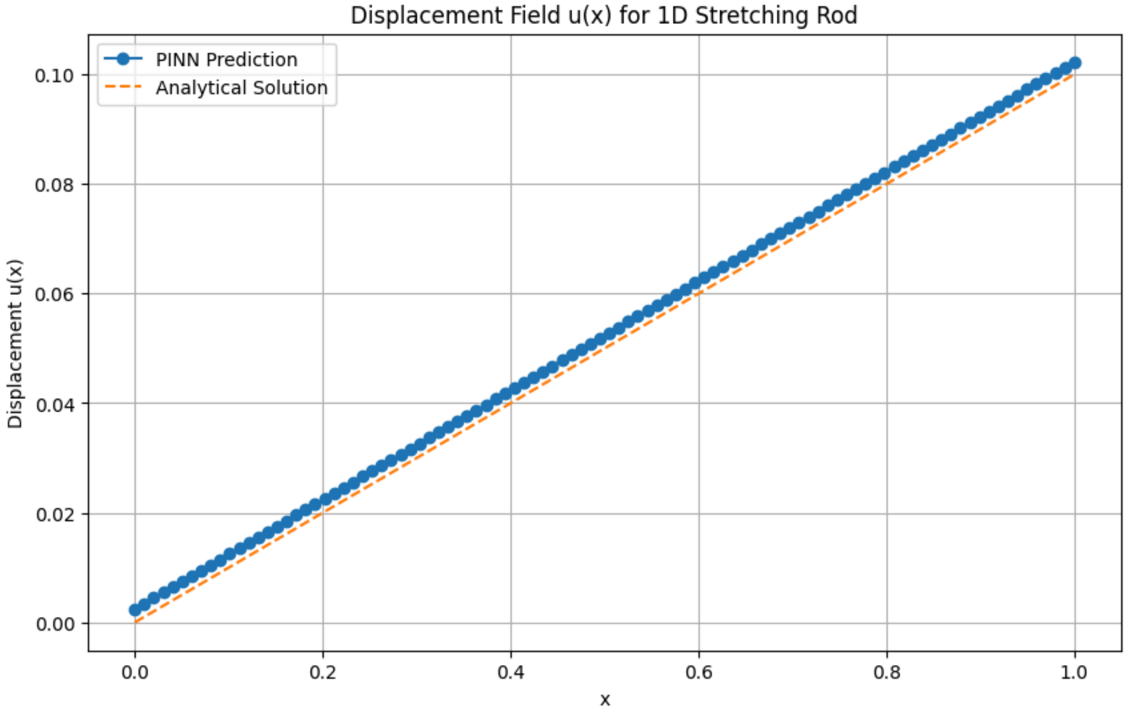

```
Epoch 0, Loss: 0.032336484640836716
Epoch 100, Loss: 0.00031464529456570745
Epoch 200, Loss: 0.00012176908057881519
Epoch 300, Loss: 7.53565618651919e-05
Epoch 400, Loss: 5.21550391567871e-05
Epoch 500, Loss: 3.782523708650842e-05
Epoch 600, Loss: 2.8365375328576192e-05
Epoch 700, Loss: 2.186592246289365e-05
Epoch 800, Loss: 1.7258462321478873e-05
Epoch 900, Loss: 1.3903932995162904e-05
Epoch 1000, Loss: 1.1403702046663966e-05
Epoch 1100, Loss: 9.50087451201398e-06
Epoch 1200, Loss: 8.025343049666844e-06
Epoch 1300, Loss: 6.861677775304997e-06
Epoch 1400, Loss: 5.9299277381796855e-06
Epoch 1500, Loss: 5.173565114091616e-06
Epoch 1600, Loss: 1.733864155539777e-05
Epoch 1700, Loss: 4.104262643522816e-06
Epoch 1800, Loss: 3.88304306397913e-06
Epoch 1900, Loss: 3.371555294506834e-06
```




```
import tensorflow as tf
import numpy as np
import matplotlib.pyplot as plt

# Constants
EA = 1  # Stiffness coefficient (assume EA=1 for simplicity)
pi = np.pi
```

```python
# Analytical Solution
def analytical_solution(x):
    return (1 / (pi**2 * EA)) * np.sin(pi * x) + (1 + 1 / pi) * x

# Define the governing equation as a loss function
def pinn_loss(model, x):
    with tf.GradientTape(persistent=True) as tape:
        tape.watch(x)
        u = model(x)
        u_x = tape.gradient(u, x)
    u_xx = tape.gradient(u_x, x)
    del tape

    f = EA * u_xx + tf.sin(pi * x)  # Governing equation
    return tf.reduce_mean(tf.square(f))

# Boundary condition loss
def boundary_loss(model):
    # Boundary at x = 0
    x0 = tf.convert_to_tensor([[0.0]])
    u0 = model(x0)

    # Boundary at x = 1 (force boundary condition)
    x1 = tf.convert_to_tensor([[1.0]])
    with tf.GradientTape() as tape:
        tape.watch(x1)
        u1 = model(x1)
    u1_x = tape.gradient(u1, x1)

    force_condition = EA * u1_x - 1  # F = EA * du/dx
    return tf.reduce_mean(tf.square(u0)) + tf.reduce_mean(tf.square(force_condition))

# Total loss function
def total_loss(model, x):
    return pinn_loss(model, x) + boundary_loss(model)

# Define the neural network
class PINN(tf.keras.Model):
    def __init__(self):
        super(PINN, self).__init__()
        self.hidden_layers = [tf.keras.layers.Dense(30, activation='tanh')] #for _ in range(3)
        self.hidden_layers = [tf.keras.layers.Dense(30, activation='tanh')]
        self.hidden_layers = [tf.keras.layers.Dense(30, activation='tanh')]
        self.output_layer = tf.keras.layers.Dense(1, activation=None)

    def call(self, x):
        for layer in self.hidden_layers:
            x = layer(x)
        return self.output_layer(x)

# Generate training data
x_train = tf.convert_to_tensor(np.linspace(0, 1, 100).reshape(-1, 1), dtype=tf.float32)

# Initialize the model
model = PINN()

# Optimizer
optimizer = tf.keras.optimizers.Adam(learning_rate=0.001)

# Training loop
epochs = 6000
for epoch in range(epochs):
    with tf.GradientTape() as tape:
        loss = total_loss(model, x_train)
    gradients = tape.gradient(loss, model.trainable_variables)
    optimizer.apply_gradients(zip(gradients, model.trainable_variables))

    if epoch % 500 == 0:
        print(f"Epoch {epoch}, Loss: {loss.numpy()}")

# Evaluate the model
x_test = np.linspace(0, 1, 100).reshape(-1, 1)
x_test_tensor = tf.convert_to_tensor(x_test, dtype=tf.float32)
u_pred = model(x_test_tensor).numpy()

# Analytical solution
u_analytical = analytical_solution(x_test)

# Plot results
plt.figure(figsize=(8, 6))
plt.plot(x_test, u_pred, label='PINN Prediction', linewidth=2)
plt.plot(x_test, u_analytical, label='Analytical Solution', linestyle='dashed')
```

```
plt.xlabel('x')
plt.ylabel('u(x)')
plt.legend()
plt.title('Comparison of PINN and Analytical Solution')
plt.show()
```

```
/usr/local/lib/python3.10/dist-packages/keras/src/layers/layer.py:391: UserWarning: `build()` was called on layer 'pinn', however th
  warnings.warn(
WARNING:tensorflow:Calling GradientTape.gradient on a persistent tape inside its context is significantly less efficient than callir
Epoch 0, Loss: 2.1505513191223145
Epoch 500, Loss: 0.0775962620973587
Epoch 1000, Loss: 0.06997445225715637
Epoch 1500, Loss: 0.030236031860113144
Epoch 2000, Loss: 0.0069174617528915405
Epoch 2500, Loss: 0.0027424923609942198
Epoch 3000, Loss: 0.0006284067640081048
Epoch 3500, Loss: 0.0002152338420273736
Epoch 4000, Loss: 0.00015700410585850477
Epoch 4500, Loss: 0.00012040235742460936
Epoch 5000, Loss: 8.601861918577924e-05
Epoch 5500, Loss: 5.94029770581983e-05
```