Enhancing Time Series Forecasting with Nonlinear ARMA Models (NLARMA)

A Comparative Study with ARMA on Simulated Stock Price Data

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Introduction

- Time series forecasting is essential in finance, energy, and healthcare.
- Traditional models like ARMA assume linearity.
- Real-world phenomena often exhibit nonlinear behavior.
- **Goal:** Demonstrate the power of Nonlinear ARMA models (NLARMA) in forecasting.

ARMA in Traditional Models

- **ARMA(p,q)** = Autoregressive (AR) + Moving Average (MA)
- **AR:** $y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + e_t$
- **MA:** $y_t = \sum_{j=1}^{q} \theta_j e_{t-j} + e_t$
- **Full Model:**

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + e_t$$

- Assumes linear dependence on past values and noise.
- **Limitation:** Fails to capture nonlinear patterns.

What is NLARMA?

- **NLARMA:** Nonlinear Autoregressive Moving Average
- Generalization of ARMA using nonlinear functions:

$$y_t = f(y_{t-1}, ..., y_{t-p}, e_{t-1}, ..., e_{t-q}) + e_t$$

- f is a nonlinear function (e.g., neural networks, kernels)
- Better suited for capturing complex dynamics in real data.

NLARMA Mathematical Formulation

$$y_t = f(y_{t-1}, y_{t-2}, \ldots, y_{t-p}, e_{t-1}, \ldots, e_{t-q}) + e_t$$

- y_t: Current value
- y_{t-i}: Lagged values
- e_t: White noise
- f: Nonlinear function

Simulated Dataset

- Equation used:

$$x_t = \sin(0.2t) + 0.5 \cdot x_{t-1}^2 + \eta_t$$

- **Columns:**
  - Simulated_Stock_Price
  - Previous_Close
  - Two_Days_Ago_Close
  - Price_Volatility
- Dataset: 100 rows (daily frequency)

NARMA-like Model (Neural Network)

- Tool: `MLPRegressor` (2 layers: 100 and 50 neurons)
- Inputs: lagged stock prices (5 days)
- Output: next day's price
- Normalization: MinMaxScaler
- Optimizer: Adam, Activation: ReLU

Hyperparameter Optimization

- To optimize the NLARMA model, we applied **Grid Search** with cross-validation.
- Evaluated combinations of:
  - Hidden layer sizes: (50), (100), (50, 50)
  - Activation functions: ReLU, Tanh
  - Regularization (`alpha`): 0.0001, 0.001
  - Learning rates: 0.001, 0.01
- Objective: Minimize Root Mean Squared Error (RMSE) on validation set

Best Parameters Found:

- `activation`: **tanh**
- `alpha`: **0.0001**
- `hidden_layer_sizes`: **(100,)**
- `learning_rate_init`: **0.001**

ARMA Benchmark Model

- Tool: `statsmodels.ARIMA`
- Parameters: ARIMA(5, 0, 0)
- Trained on same training set
- Forecasts made for comparison with NARMA

Model Performance Comparison

Model	RMSE	Remarks
ARIMA (Linear)	11.6988	-Limited by linearity assumption; performs poorly on nonlinear sequences
NARMA (Neural Network)	3.8417	-Captures nonlinear dependencies; better performance than ARIMA
NARMA (Optimized)	**1.6559**	-Best performance after hyperparameter tuning using GridSearchCV (activation: tanh, hidden layers: (100,), learning rate: 0.001, alpha: 0.0001)

Table: Comparison of models based on Root Mean Squared Error (RMSE)

Real-World Applications

- **Stock Market Forecasting:** Captures volatility
- **Energy Load Prediction:** Reacts to nonlinear consumption patterns
- **Weather Modeling:** Handles chaotic trends
- **Biomedical Signals:** ECG, EEG analysis

Why Use NLARMA?

- Captures complex dynamics
- Outperforms linear models on nonlinear time series
- Works well with machine learning frameworks

Limitations

- Requires more training data
- Prone to overfitting without regularization
- Less interpretable than ARMA

Conclusion

- NLARMA is a powerful alternative to ARMA for nonlinear forecasting
- Demonstrated better performance on simulated data
- Widely applicable in many real-world domains

References

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Thank You!