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ARMA vs NLARMA: Comparative Analysis for Time Series Forecasting

Abstract

This report investigates the performance differences between traditional linear ARMA models and nonlinear ARMA (NLARMA) models in time series forecasting. Using a simulated stock price dataset, we compare an ARMA(5,0) model with a neural-network-based NLARMA model (and its optimized variant). We present the mathematical formulations of each model, analyze their fit to the data via visual plots, and benchmark their accuracy using Root Mean Squared Error (RMSE). The findings show that the NLARMA model substantially outperforms ARMA on this nonlinear data, though it requires more data and tuning. The report also discusses practical applications (e.g. in finance, energy, healthcare) and the relative strengths and limitations of linear vs. nonlinear approaches.

Executive Summary

Traditional ARMA models assume linear relationships in time series data, which can limit their accuracy on real-world phenomena that exhibit nonlinear dynamics [1] [2]. NLARMA models extend ARMA by allowing a nonlinear function (such as a neural network) to capture complex patterns [3]. In our comparative study, a simulated stock price series (created by a nonlinear rule) was fitted by both methods. The NLARMA model (a multi-layer perceptron with lagged inputs) achieved far lower RMSE (≈3.84) than the ARMA benchmark (≈11.70) on test data [4]; after hyperparameter tuning the NLARMA RMSE fell to ≈1.66. Visual inspection of the series and forecasts (Fig.1–2) confirms that the NLARMA predictions closely track the actual price movements, whereas the linear model would lag. We conclude that NLARMA offers significant gains in forecasting accuracy for nonlinear time series, at the expense of higher data demands and reduced interpretability [5] [6]. The models and findings have relevance to stock market forecasting, energy load prediction, biomedical signal analysis, and more [7]. Full details, data plots, and references are given below.

Background on ARMA and NLARMA

ARMA Models

Autoregressive Moving Average (ARMA) models are a classical approach for time series forecasting. An ARMA(p,q) model assumes that the current value y_t depends linearly on its past p values and on q past noise terms. Specifically, the AR component is

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + e_t,$$

and the MA component is

$$y_t = \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_q e_{t-q} + e_t,$$

so the full ARMA model is:

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} \ + \ \sum_{j=1}^{q} \theta_j e_{t-j} \ + \ e_t,$$

where e_t is white noise [1]. ARMA assumes *linear* dependence on past values. This makes the model relatively simple and interpretable, but it cannot capture nonlinear patterns [2]. In practice, ARMA is often used for economic or financial data when linear relationships are adequate.

NLARMA (Nonlinear ARMA) Models

Nonlinear ARMA (NLARMA) generalizes ARMA by replacing the linear terms with a nonlinear function. In an NLARMA model, the forecast equation is written as:

$$y_t = f(y_{t-1}, y_{t-2}, \dots, y_{t-p}, e_{t-1}, e_{t-2}, \dots, e_{t-q}) + e_t,$$

where $f(\cdot)$ is a nonlinear function (for example, represented by a neural network) [3]. The idea is that f can learn complex relationships between past values and errors, capturing dynamics that a linear ARMA would miss. In our study, the NLARMA model is implemented with a multi-layer neural network (an MLP) using lagged values of the series as inputs. Because f is nonlinear, NLARMA can flexibly model curved or chaotic trends [3]. The trade-off is that NLARMA typically requires more data and tuning to train the nonlinear function, and its predictions are less interpretable than the coefficients in a linear ARMA model [6] [5].

Data and Visualization

Figure 1: Simulated stock price time series (synthetic data). This series was generated by a nonlinear rule (roughly $x_t = \sin(0.2t) + 0.5x_{t-1}^2 + \eta_t$) [8]. The blue line (left) shows the price over 100 days, exhibiting an overall upward trend with fluctuations. The nonlinearity (e.g. the sinusoidal component) and volatility in the series suggest that a linear ARMA model may struggle to capture its pattern [8] [2]. The dataset was split into training and test segments (blue range, then beyond). From this plot, one can see that the series is not a simple linear trend, motivating the use of a nonlinear model like NLARMA.

The simulated series (Fig.1) has moderate noise and oscillatory behavior. It was constructed to mimic a stock price with nonlinear feedback (prior day's squared effect plus noise). The plot’s smooth but curved trajectory illustrates that predicting future values requires modeling nonlinear dependencies. In particular, ARMA’s assumption of linearity (“fails to capture nonlinear patterns” [2]) may lead to systematic bias on data like this.

Model Predictions vs Actual

Figure 2: NLARMA (neural network) predictions vs actual stock prices over the test period. In this plot, the solid blue curve is the historical price series (up to the test point), and the dashed red curve shows the NLARMA model’s forecast for the following days, while the dashed green curve shows the actual future prices. We observe that the NLARMA predictions (red) closely follow the actual prices (green) in both level and direction. The trained model successfully captures the ups and downs of the test data. This visual fit indicates the model’s effectiveness: the forecasting line tracks the real price, with only small deviations. Such close alignment reflects the low RMSE achieved by NLARMA (as shown below) and supports the numerical findings that NLARMA significantly outperforms ARMA on this nonlinear series.

Performance Benchmarking

The models' accuracy was evaluated using Root Mean Squared Error (RMSE) on a held-out test set. Table 1 reports the RMSE for the linear ARMA benchmark (implemented as ARIMA(5,0,0)) and the NLARMA neural models (with and without hyperparameter optimization). The ARMA model, constrained by its linear form, gives a high RMSE, whereas the NLARMA model yields much lower errors.

Model	RMSE	Remarks
ARMA (ARIMA(5,0,0)) (Linear)	11.6988 [4]	High error due to linearity; performs poorly on this nonlinear data.
NLARMA (Neural Network)	3.8417 [4]	Captures nonlinear dependencies, substantially better than ARMA.
NLARMA (Optimized NN)	1.6559 [4]	Best result after hyperparameter tuning (tanh, one hidden layer, etc.) [4].

Table 1: RMSE comparison of models (lower is better). Values are from the study's results [4]. All models were trained on the same data. The optimized NLARMA (with grid-searched architecture and learning rate) achieves the lowest error (~1.66), improving dramatically over the unoptimized NLARMA (~3.84) and far outperforming the linear ARMA (~11.70) [4]. In practice, this means the nonlinear model's forecasts are much closer to the true values.

Applications Across Domains

Nonlinear time series models have broad applications. In **finance**, NLARMA can improve stock market forecasts by capturing volatility and complex market patterns [7]. In **energy systems**, it can model nonlinear consumption or load patterns (e.g. daily or seasonal cycles with non-additive effects). Similarly, in **weather and environmental modeling**, nonlinear models can adapt to chaotic dynamics in meteorological data. In **healthcare and biomedical engineering**, nonlinear ARMA methods can enhance predictions of physiological signals, such as ECG or EEG data, where relationships are not purely linear [7]. These domains benefit from the extra flexibility of NLARMA, as noted in the study's list of real-world applications [7]. In contrast, when data is well-described by linear trends (or data is very limited), traditional ARMA models may suffice due to their simplicity.

Strengths and Limitations

ARMA (Linear): Strengths include simplicity, ease of interpretation, and low computational cost. Linear models require less data to fit and their parameters (the φ and θ coefficients) are transparent. However, their major limitation is the assumption of linearity – they "fail to capture nonlinear patterns" in the data [2]. On truly nonlinear time series, ARMA may systematically underfit, leading to large forecast errors as seen above.

NLARMA (Neural Network): The nonlinear model's strengths lie in its flexibility. It "captures complex dynamics" and often "outperforms linear models on nonlinear time series" [5], as our results demonstrate. Using machine learning frameworks, NLARMA can learn arbitrarily complicated relationships given enough data. The trade-offs are that NLARMA typically *requires more training data* and careful tuning of hyperparameters [6]. It is also more prone to overfitting (hence the need for cross-validation) and its learned function f is harder to interpret than ARMA's linear coefficients [6]. Thus, while powerful, NLARMA models come with higher complexity. In summary, ARMA is preferable for simpler or linear datasets, whereas NLARMA is advantageous when the series is nonlinear and ample data/resources are available.

Conclusion

In this comparative study, the nonlinear ARMA approach (implemented via neural networks) showed clear advantages for forecasting a nonlinear stock price series. The NLARMA model achieved significantly lower RMSE than the linear ARMA model, confirming that allowing for nonlinearity can greatly improve accuracy [4]. We reviewed the mathematical formulations of both models and demonstrated, through plots and error metrics, how NLARMA better tracks the actual data. However, NLARMA's benefits come with practical costs (more data, tuning, less transparency) [6]. The choice of model ultimately depends on the application domain: for complex domains like finance or biomedical signals, NLARMA's power can outweigh its drawbacks, whereas simpler ARMA may suffice for well-behaved linear processes. Overall, the study highlights that extending linear models with nonlinear components is a promising direction for enhancing time series forecasts.

References

- Yinka-Banjo, C. O., & Akinyemi, M. I. (2023). *Stock Market Prediction Using a Hybrid of Deep Learning Models*. International Journal of Financial Studies, Economics and Management, 2(2), 1–16.
- Stojkoska, B., Taskovska, K., & Utkovski, Z. (2022). *Time Series Prediction with Neural Networks: A Review*. In 2022 57th International Scientific Conference on Information, Communication and Energy Systems and Technologies (ICEST) (pp. 1–4). IEEE.

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