Energy Infrastructure Planning: Forecasting

Carlos Abad

November 7, 2014

Who am I?

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- PhD student in the IEOR department

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- Advisors: Prof. Vijay Modi and Prof. Garud Iyengar

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- PhD student in the IEOR department
- Advisors: Prof. Vijay Modi and Prof. Garud Iyengar
- Research:
  - Robust control algorithms for solar micro-grids
  - Control, signal detection, and forecasting methods for managing DR programs

References

- Hyndman, R. J. & Athanasopoulos, G.(2013) Forecasting: principles and practice.
- www.otexts.org/fpp/
- R package fpp

Outline

Time series data

- Time series consists of sequences of observations collected over time.

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- Hourly electricity demand

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- Daily maximum temperature
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Time series in R

Main package used in this course

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> library(fpp)
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This loads:

- some data for use in examples and exercises
- **forecast** package (for forecasting functions)
- **tseries** package (for a few time series functions)
- **fma** package (for lots of time series data)
- expsmooth package (for more time series data)
- lmtest package (for some regression functions)

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- Easier time aggregation

Outline

Notation

- y_t: observed value at time t

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- $\hat{y}_{T+h|T}$: forecast for time $T + h$ made at time T with historical information up to time T

Some simple forecasting methods

Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

Naïve method (for time series only)

- Forecasts equal to last observed value.

Seasonal naïve method

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Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

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Some simple forecasting methods

- Mean: `meanf(x, h=20)`
- Naive: `naive(x, h=20)` or `rwf(x, h=20)`
- Seasonal naive: `snaive(x, h=20)`
- Drift: `rwf(x, drift=TRUE, h=20)`

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Outline

Forecasting residuals

Residuals in forecasting: difference between observed value and its forecast based on all previous observations: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

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Measures of forecast accuracy

Let y_t denote the tth observation and $\hat{y}_{t|t-1}$ denote its forecast based on all previous data, where $t = 1, \dots, T$. Then the following measures are useful.

$$\text{MAE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|$$

$$\text{MSE} = T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \qquad \text{RMSE} = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2}$$

$$\text{MAPE} = 100T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/|y_t|$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

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works well. Then MASE is equivalent to MAE relative to a naive method.

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For seasonal time series,

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works well. Then MASE is equivalent to MAE relative to a seasonal naive method.

Training and test sets

Available data

Training set (e.g., 80%)	Test set (e.g., 20%)

- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

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Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into “training” set and “test” set. Training set used to estimate parameters. Forecasts are made for test set.
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Outline

Time series graphics

- **Time plots**
  R command: `plot` or `plot.ts`
- **Seasonal plots**
  R command: `seasonplot`
- **Seasonal subseries plots**
  R command: `monthplot`
- **Lag plots**
  R command: `lag.plot`
- **ACF plots**
  R command: `Acf`

Seasonal plots

- Data plotted against the individual "seasons" in which the data were observed. (In this case a "season" is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
- In R: `seasonplot`

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Seasonal subseries plots

- Data for each season collected together in time plot as separate time series.
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Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

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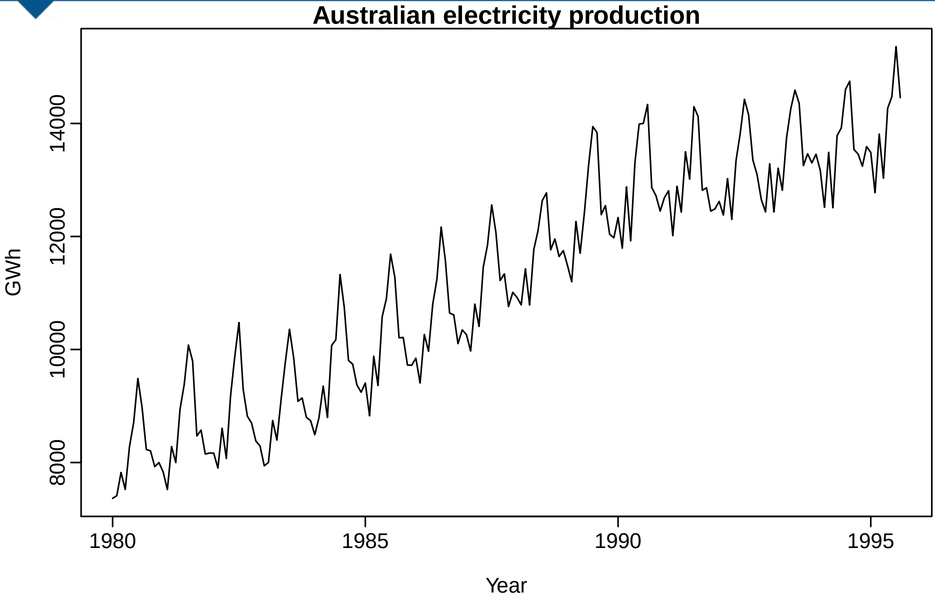

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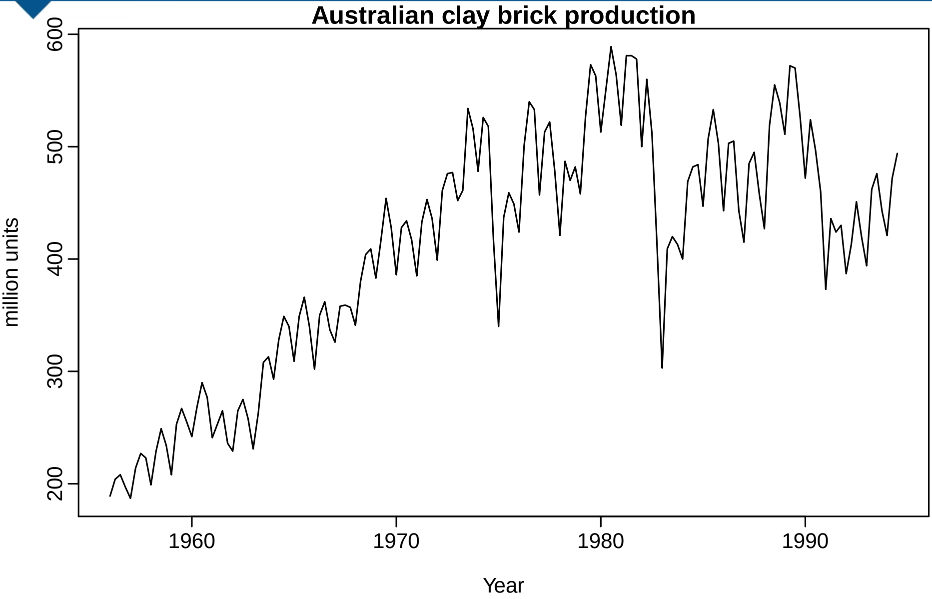

Stationarity

Definition

If $\{y_t\}$ is a stationary time series, then for all s, the distribution of $(y_t, \ldots, y_{t+s})$ does not depend on t.

A **stationary series** is:

- roughly horizontal
- constant variance
- no patterns predictable in the long-term

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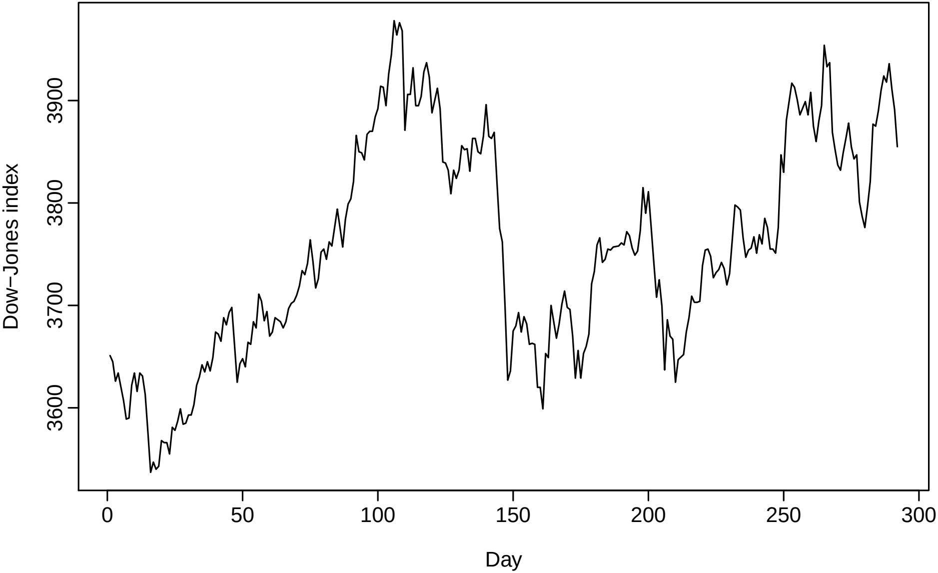
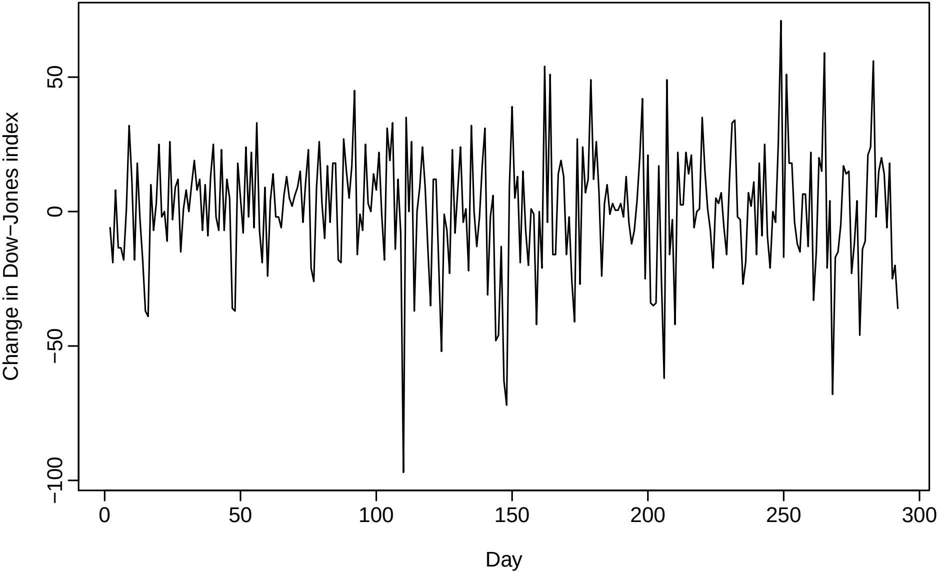

Dow−Jones index
3600
3700
3800
3900
0
50
100
150
200
250
300
Day

Stationary?



Change in Dow−Jones index
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0
−50
−100
0
50
100
150
200
250
300
Day

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Transformations help to **stabilize the variance**.

For ARIMA modelling, we also need to **stabilize the mean**.

Non-stationarity in the mean

Identifying non-stationary series

- time plot.
- The ACF of stationary data drops to zero relatively quickly
- The ACF of non-stationary data decreases slowly.
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Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

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We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k / c_0$$

- r_1 indicates how successive values of y relate to each other
- r_2 indicates how y values two periods apart relate to each other
- r_k is *almost* the same as the sample correlation between y_t and y_{t-k}.

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Recognizing seasonality in a time series

If there is seasonality, the ACF at the seasonal lag (e.g., 12 for monthly data) will be **large and positive**.

- For seasonal monthly data, a large ACF value will be seen at lag 12 and possibly also at lags 24, 36, . . .
- For seasonal quarterly data, a large ACF value will be seen at lag 4 and possibly also at lags 8, 12, . . .

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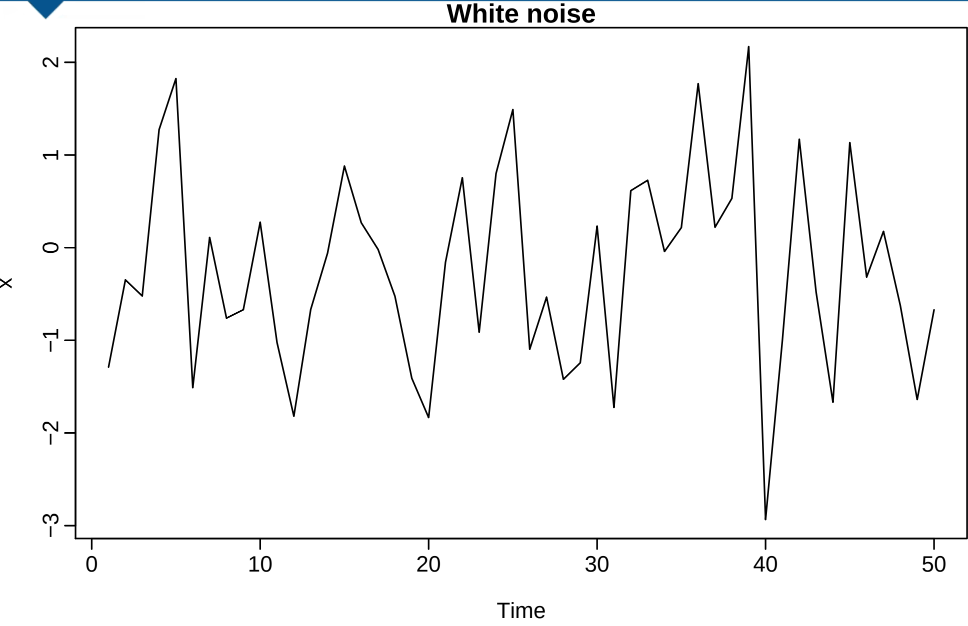
Example: White noise



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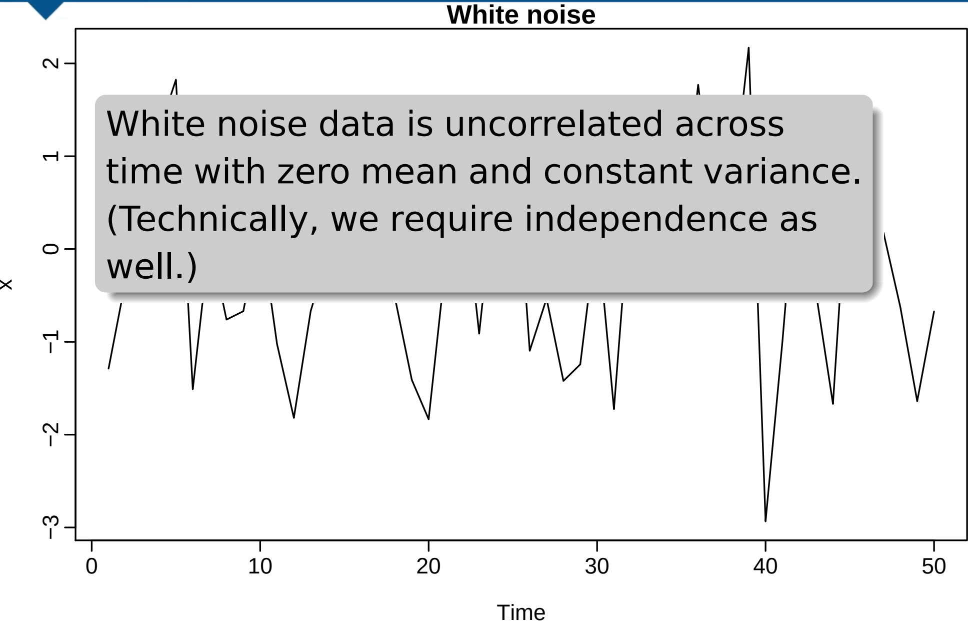

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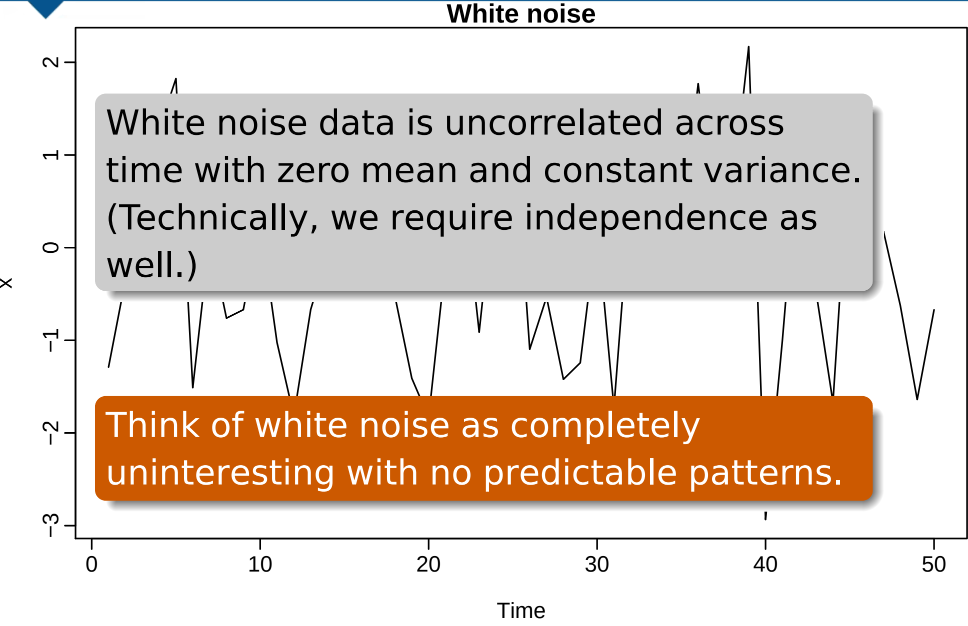

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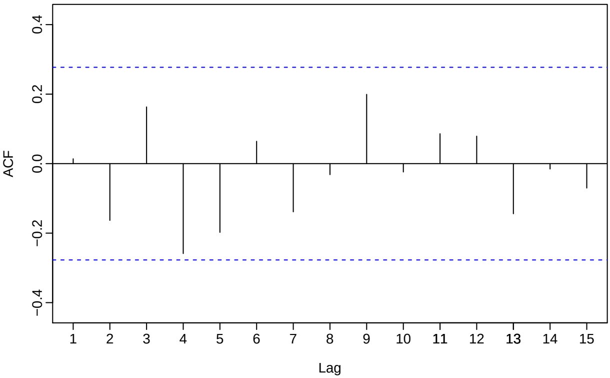
Think of white noise as completely uninteresting with no predictable patterns.

Example: White noise

$$
\begin{aligned}
r_1 &= 0.013 \\
r_2 &= -0.163 \\
r_3 &= 0.163 \\
r_4 &= -0.259 \\
r_5 &= -0.198 \\
r_6 &= 0.064 \\
r_7 &= -0.139 \\
r_8 &= -0.032 \\
r_9 &= 0.199 \\
r_{10} &= -0.240
\end{aligned}
$$



Sample autocorrelations for white noise series.
For uncorrelated data, we would expect each autocorrelation to be close to zero.

Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically $\text{N}(0,1/T)$.

- 95% of all r_k for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the *critical values*.

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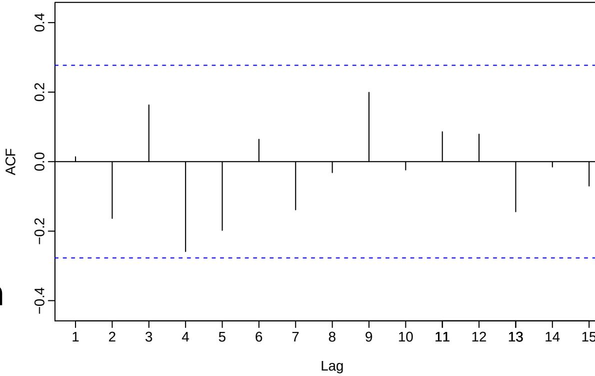
Autocorrelation

Example:

$T = 50$ and so critical values at $\pm 1.96/\sqrt{50} = \pm 0.28$.

All autocorrelation coefficients lie within these limits, confirming that the data are white noise.



(More precisely, the data cannot be distinguished from white noise.)

ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Dow-Jones naive forecasts revisited

$$\hat{y}_{t|t-1} = y_{t-1}$$
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Non-stationarity in the mean

Identifying non-stationary series

- time plot.
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Differencing

- Differencing helps to **stabilize the mean**.
- The differenced series is the *change* between each observation in the original series: $y'_t = y_t - y_{t-1}$.
- The differenced series will have only $T - 1$ values since it is not possible to calculate a difference y'_1 for the first observation.

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Second-order differencing

Occasionally the differenced data will not appear stationary and it may be necessary to difference the data a second time:

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\begin{aligned}
y''_t &= y'_t - y'_{t-1} \\
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A seasonal difference is the difference between an observation and the corresponding observation from the previous year.

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where $m =$ number of seasons.

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When both seasonal and first differences are applied. . .

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- first differences are the change between **one observation and the next**;
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Outline

Autoregressive models

Autoregressive (AR) models:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + e_t,$$

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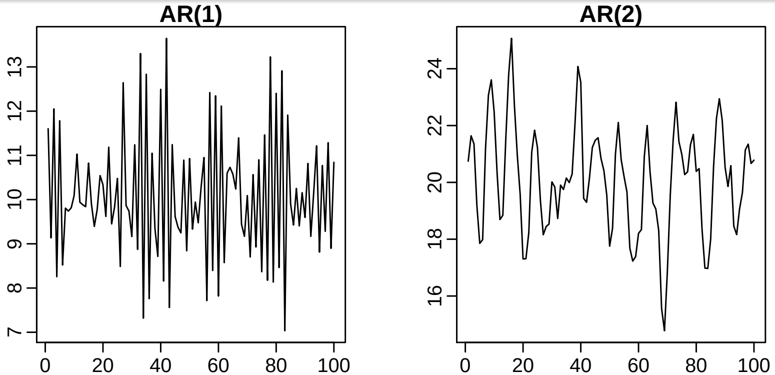

AR(1) model

$$y_t = c + \phi_1 y_{t-1} + e_t$$

- When $\phi_1 = 0$, y_t is **equivalent to WN**
- When $\phi_1 = 1$ and $c = 0$, y_t is **equivalent to a RW**
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- When $\phi_1 < 0$, y_t tends to **oscillate between positive and negative values**.

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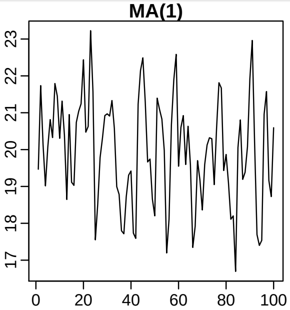

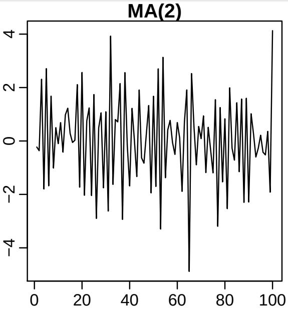

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- Predictors include both **lagged values of y_t and lagged errors.**
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Autoregressive Integrated Moving Average models

ARIMA(p, d, q) model

AR: $p =$ order of the autoregressive part

I: $d =$ degree of first differencing involved

MA: $q =$ order of the moving average part.

- White noise model: ARIMA(0,0,0)
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Understanding ARIMA models

- If $c = 0$ and $d = 0$, the long-term forecasts will go to zero.
- If $c = 0$ and $d = 1$, the long-term forecasts will go to a non-zero constant.
- If $c = 0$ and $d = 2$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 0$, the long-term forecasts will go to the mean of the data.
- If $c \neq 0$ and $d = 1$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 2$, the long-term forecasts will follow a quadratic trend.

ACF and PACF plots

- Recall that k-th autocorrelation r_k measures the linear relationship between y_t and y_{t-k}

ACF and PACF plots

- Recall that k-th autocorrelation r_k measures the linear relationship between y_t and y_{t-k}
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ACF and PACF plots

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Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{Likelihood}) + 2p$$

where p is the number of estimated parameters in the model.

- *Minimizing* the AIC gives the best model for prediction.

AIC corrected (for small sample bias)

$$\text{AIC}_\text{C} = \text{AIC} + \frac{2(p+1)(p+2)}{n-p}$$

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Backshift notation

A very useful notational device is the backward shift operator, B, which is used as follows:

$$By_t = y_{t-1} \, .$$

In other words, B, operating on y_t, has the effect of **shifting the data back one period**. Two applications of B to y_t **shifts the data back two periods**:

$$B(By_t) = B^2 y_t = y_{t-2} \, .$$

For monthly data, if we wish to shift attention to "the same month last year," then B^{12} is used, and the notation is $B^{12} y_t = y_{t-12}$.

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Backshift notation for ARIMA

- ARMA model:

$$\begin{aligned} y_t &= c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + e_t + \theta_1 e_{t-1} + \cdots + \theta_q e_{t-q} \\ &= c + \phi_1 B y_t + \cdots + \phi_p B^p y_t + e_t + \theta_1 B e_t + \cdots + \theta_q B^q e_t \end{aligned}$$

$$\phi(B) y_t = c + \theta(B) e_t$$

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Seasonal ARIMA models

$$\text{ARIMA} \;\; (p, d, q) \;\; (P, D, Q)_m$$

where $m =$ number of periods per season.

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Regression with ARIMA errors

Regression models

$$y_t = b_0 + b_1 x_{1,t} + \cdots + b_k x_{k,t} + n_t$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \dots, x_{k,t}$.
- Usually, we assume that n_t is WN.
- Now we want to allow n_t to be autocorrelated.

Example: n_t = ARIMA(1,1,1)

$$y_t = b_0 + b_1 x_{1,t} + \cdots + b_k x_{k,t} + n_t$$

where $\quad (1 - \phi_1 B)(1 - B)n_t = (1 - \theta_1 B)e_t$

and e_t is white noise .

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where $\quad (1 - \phi_1 B)(1 - B) n_t = (1 - \theta_1 B) e_t$

and e_t is white noise .

Regression with ARIMA errors

Regression models

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- y_t modeled as function of k explanatory variables $x_{1,t}, \dots, x_{k,t}$.
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- Be careful in distinguishing n_t from e_t.
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After differencing all variables

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Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Original data

$$y_t = b_0 + b_1 x_{1,t} + \cdots + b_k x_{k,t} + n_t$$

$$\text{where} \quad \phi(B)(1-B)^d n_t = \theta(B) e_t$$

After differencing all variables

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$$\text{where} \quad \phi(B) n_t = \theta(B) e_t$$

$$\text{and} \quad y'_t = (1-B)^d y_t, \quad \text{etc.}$$

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Modeling procedure

Problems with OLS and autocorrelated errors

1. OLS no longer the best way to compute coefficients as it does not take account of time-relationships in data.
2. Standard errors of coefficients are incorrect — most likely too small. This invalidates tests and prediction intervals.

Second problem more serious because it can lead to misleading results.

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Outline

Time series decomposition

$$Y_t = f(S_t, T_t, E_t)$$

where
$$\begin{aligned} Y_t &= \text{data at period } t \\ S_t &= \text{seasonal component at period } t \\ T_t &= \text{trend-cycle component at period } t \\ E_t &= \text{remainder (or irregular or error) component at period } t \end{aligned}$$

Additive decomposition: $Y_t = S_t + T_t + E_t$.

Multiplicative decomposition: $Y_t = S_t \times T_t \times E_t$.

Time series decomposition

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Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Logs turn multiplicative relationship into an additive relationship:

$$Y_t = S_t \times T_t \times E_t \quad \Rightarrow \quad \log Y_t = \log S_t + \log T_t + \log E_t.$$

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Seasonal adjustment

- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by

$$Y_t - S_t = T_t + E_t$$

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Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method. E.g.,
  - Holt's method — next topic
  - Random walk with drift model
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.

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Simple methods

Random walk forecasts

$$\hat{y}_{T+1|T} = y_T$$

Average forecasts

$$\hat{y}_{T+1|T} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

- Want something in between that weights most recent data more highly.
- Simple exponential smoothing uses a weighted moving average with weights that decrease exponentially.

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Simple Exponential Smoothing

Forecast equation

$$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1-\alpha)y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + \cdots ,$$

where $0 \leq \alpha \leq 1$.

	Weights assigned to observations for:			
Observation	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$
y_T	0.2	0.4	0.6	0.8
y_{T-1}	0.16	0.24	0.24	0.16
y_{T-2}	0.128	0.144	0.096	0.032
y_{T-3}	0.1024	0.0864	0.0384	0.0064
y_{T-4}	$(0.2)(0.8)^4$	$(0.4)(0.6)^4$	$(0.6)(0.4)^4$	$(0.8)(0.2)^4$
y_{T-5}	$(0.2)(0.8)^5$	$(0.4)(0.6)^5$	$(0.6)(0.4)^5$	$(0.8)(0.2)^5$

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Simple Exponential Smoothing

Weighted average form

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

for $t = 1, \ldots, T$, where $0 \leq \alpha \leq 1$ is the smoothing parameter.

The process has to start somewhere, so we let the first forecast of y_1 be denoted by ℓ_0. Then

$$\begin{aligned}
\hat{y}_{2|1} &= \alpha y_1 + (1-\alpha)\ell_0 \\
\hat{y}_{3|2} &= \alpha y_2 + (1-\alpha)\hat{y}_{2|1} \\
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Substituting each equation into the following equation:

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Exponentially weighted average

$$\hat{y}_{T+1|T} = \sum_{j=0}^{T-1} \alpha(1-\alpha)^j y_{T-j} + (1-\alpha)^T\ell_0$$

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Simple exponential smoothing

Initialization

- Last term in weighted moving average is $(1-\alpha)^T\hat{\ell}_0$.
- So value of ℓ_0 plays a role in *all* subsequent forecasts.
- Weight is small unless α close to zero or T small.
- Common to set $\ell_0 = y_1$. Better to treat it as a parameter, along with α.

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- We can choose α and ℓ_0 by minimizing MSE:

$$\text{MSE} = \frac{1}{T-1}\sum_{t=2}^{T}(y_t - y_{t|t-1})^2$$

- Unlike regression there is no closed form solution — use numerical optimization.

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Multi-step forecasts

$$\hat{y}_{T+h|T} = \hat{y}_{T+1|T}, \qquad h = 2, 3, \dots$$

- A "flat" forecast function.
- Remember, a forecast is an estimated mean of a future value.
- So with no trend, no seasonality, and no other patterns, the forecasts are constant.

Simple exponential smoothing

Multi-step forecasts

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