Econometrica, Vol. 81, No. 2 (March, 2013), 489–534

IDENTIFICATION AND INFERENCE IN ASCENDING AUCTIONS WITH CORRELATED PRIVATE VALUES

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We introduce and apply a new nonparametric approach to identification and inference on data from ascending auctions. We exploit variation in the number of bidders across auctions to nonparametrically identify useful bounds on seller profit and bidder surplus using a general model of correlated private values that nests the standard independent private values (IPV) model. We also translate our identified bounds into closed form and asymptotically valid confidence intervals for several economic measures of interest. Applying our methods to much studied U.S. Forest Service timber auctions, we find evidence of correlation among values after controlling for a rich vector of relevant auction covariates; this correlation causes expected profit, the profit-maximizing reserve price, and bidder surplus to be substantially lower than conventional (IPV) analysis of the data would suggest.

KEYWORDS: Ascending auction, correlated values, nonparametric identification.

1. INTRODUCTION

APPLYING INSIGHTS FROM AUCTION THEORY to real world settings requires knowledge of the primitives of the game being played by bidders. In the case of a private value ascending (English) auction, the main primitive of interest is the latent distribution of bidder valuations. Knowledge of this distribution allows one to forecast expected revenue, bidder surplus, and the effects of a change in auction design.

Ascending auctions, however, present a unique empirical challenge. Since each auction ends when every bidder but one is unwilling to raise his bid, bidding behavior never reveals how high the winner would have been willing to go. Even in the idealized environment of a "button auction," where each losing bidder's private valuation is exactly revealed from his bidding behavior, the joint distribution of valuations among *all* bidders cannot be nonparametrically identified, even when bidders are assumed to be symmetric.[2]

The standard approach in the literature has, therefore, been to impose stronger structural assumptions to achieve identification. This is most commonly done by assuming that bidders' valuations, in addition to being symmetric, are independent. Nonparametric identification of the distribution of

[1]We are very thankful to a co-editor and three anonymous referees for their insightful feedback. We also thank Steven Durlauf, Elena Krasnokutskaya, Jeremy Fox, Phil Haile, Bruce Hansen, Ken Hendricks, Han Hong, Ali Hortaçsu, Harry Paarsch, Jack Porter, Rob Porter, Art Shneyerov, Chris Taber, seminar participants at Chicago, Chicago Booth, Concordia, Duke, Harvard/MIT, Michigan, Northwestern, NYU, Olin Business School, Stanford, UCL, Wisconsin and Yale, and conference attendees at IIOC 2009 and 2010, SITE 2009, Cowles Summer Conference 2010, the 2011 ASSA Winter Meetings, and CAPCP 2011 for helpful comments. Aradillas-López acknowledges the support of the NSF through Grant SES-0922062.

[2]Athey and Haile (2002, Theorem 4).

 DOI: 10.3982/ECTA9431

valuations from bid data in this setting was established by Athey and Haile (2002) and Haile and Tamer (2003). Independent private values (IPV) is also a standard identifying restriction used in the empirical literature on first-price auctions (see, for example, Guerre, Perrigne, and Vuong (2000, 2009)).

Of course, the IPV model is misspecified if there is dependence among bidder valuations. Whether values are independent and, if not, whether this misspecification is innocuous or relevant for policy analysis are ultimately empirical questions. This perspective motivates the goal of our paper, which is to put forth a framework for the empirical analysis of English auctions that allows for private values to be correlated. We can thus examine whether the key policy conclusions reached under IPV are robust to possible correlation among bidder values in the data.

Our approach to nonparametric identification in English auctions is to focus attention on seller profits and bidder surplus as the relevant primitives of interest (rather than the full joint distribution of valuations). Seller profits and bidder surplus are the basic measures used to evaluate policies that impact auction design. We exploit the fact that both of these measures depend only on the marginal distributions of the two highest valuations and not on any other features of the joint distribution. We then show how bounds on these marginal distributions are identified from the data under a general model of correlated private values, which allows for nonparametric identification of bounds on these payoff functions. These are the first positive results on identification in English auctions that allow for private values to be correlated even conditional on observables.

Our identification results naturally give rise to an inference strategy for empirical work. We show that our bounds on seller and bidder surplus can be expressed as known transformations of expectations that are identified from the data. This allows us to construct asymptotically valid closed-form confidence intervals from data on transaction price and observed characteristics of a sample of auctions. A key feature of this strategy is that we allow for full nonparametric conditioning on continuous covariates; this enables us to empirically distinguish correlation of valuations among bidders from observable heterogeneity in the bidding environment across auctions.

To illustrate the policy usefulness of our approach, we study data from U.S. Forest Service (USFS) timber auctions. The ascending auctions in these data have been exclusively analyzed in the past under the IPV assumption. Even conditional on the rich vector of available covariates (the presence of which is often used to defend the IPV assumption), we find evidence of positive dependence among valuations. Comparing our bounds side-by-side with the traditional IPV estimates, we find substantially lower expected seller profit and bidder surplus, as well as substantially lower profit-maximizing reserve prices. These findings may help to reconcile the seemingly conservative reserve prices found in practice in comparison to the more aggressive policies that IPV often suggests.[3]

We also analyze a major change in reserve price policy that took place within the USFS during the 1980s and 1990s. In response to widespread concerns that reserve prices were being set too low, the USFS switched from an accounting-based approach to timber appraisal to a transaction-based approach. Our analysis suggests that, given the technology available at the time, the level of reserve prices chosen by the USFS in implementing the transaction-based approach was close to optimal and did indeed increase expected profits. However, our methodology also suggests an alternative policy, which would have increased profits nearly twice as much without further raising the average level of reserve prices.

Our paper proceeds as follows. In Section 2, we offer a brief review of the closely related literature. In Section 3, we present our model and identification results. In Section 4, we apply our results to timber auction data and examine the main policy questions. Section 5 concludes. Appendix A contains proofs, as well as supporting results and extensions referenced in the text. A separate Appendix B in the Supplemental Material (Aradillas-López, Gandhi, and Quint (2013)) presents the details of the econometric approach used in our application.

2. RELATED LITERATURE

Despite the widespread use of ascending auctions in practice, the literature on identification in ascending auctions remains fairly sparse. Paarsch (1997) modeled ascending auctions as button auctions—bidders hold down a button to remain active as the price rises continuously, releasing the button to drop out, and thus each losing bidder's willingness to pay is learned exactly from his bid—and estimated a parametric model of independent private values. Donald and Paarsch (1996) discussed identification and estimation of parametric IPV models. Hong and Shum (2003) estimated a parametric, affiliated values model. Athey and Haile (2002) showed that in button auctions, an IPV model is nonparametrically identified from transaction prices alone. Haile and Tamer (2003) departed from the button auction model and introduced an "incomplete" model of bidding in English auctions, replacing a full model of the game and the assumption of equilibrium play with two "behavioral" assumptions[4]; within the IPV setting, they showed that this allows them to nonparametrically identify bounds on the distribution of valuations.

[3]See Paarsch (1997) for a comparison between the reserve prices suggested by an IPV model and those used in practice using Canadian timber auctions. He concluded that reserve prices should be three times higher than those that were actually being used at the time. A different but related comparison was also performed by Haile and Tamer (2003) on USFS data.

[4]These assumptions are (i) bidders do not bid more than they are willing to pay and (ii) bidders do not allow an opponent to win at a price they are willing to beat.

Athey and Haile (2002) also presented a fundamental nonidentification result: a symmetric private-values model cannot be nonparametrically identified from ascending-auction data even in the ideal case of a button auction. One consequence of this finding is there has thus been virtually no empirical work on ascending auctions with correlated private values.[5] We are the first investigators to provide positive identification results for a nonparametric model of correlated valuations, and our progress is achieved by narrowing the focus of identification to seller and bidder payoff functions (rather than the full joint distribution of valuations).

The literature on first-price auctions offers several approaches to identification of models with correlated values, but these approaches do not solve the empirical problem posed by ascending auctions. For example, Li, Perrigne, and Vuong (2002) extended the nonparametric identification technique introduced by Guerre, Perrigne, and Vuong (2000) to affiliated private values; Li, Perrigne, and Vuong (2000), Krasnokutskaya (2011), and Hu, McAdams, and Shum (2011) adapted measurement error techniques from the econometrics literature to identify models of first-price auctions with unobserved heterogeneity. These approaches, however, cannot be applied to ascending auctions.[6]

A number of empirical papers have also studied Forest Service auctions. Paarsch (1997) estimated his model on data from British Columbian timber auctions. Baldwin, Marshall, and Richard (1997) provided much institutional background on U.S. timber auctions; their focus is to test for collusion. Haile (2001) considered the effects of resale on valuations. Haile, Hong, and Shum (2003) tested for common values against private values, using the fact that the Forest Service runs both first-price and ascending auctions. Lu and Perrigne (2008) used this same variation to estimate risk aversion among bidders. Athey and Levin (2001), Athey, Levin, and Seira (2011), and Haile and Tamer (2003) analyzed USFS data to study mechanism design issues. Of these papers, only Athey, Levin, and Seira (2011) considered the possibility of correlated private values, but they were forced to limit their empirical analysis to first-price auctions precisely because of the nonidentification problem discussed above.[7]

[5] Roberts (2009) estimated a model of IPV with unobserved heterogeneity, but assumed that sellers set reserve prices in a way that is strictly monotonic in the unobserved characteristic, allowing it to be backed out from the data.

[6] These approaches model valuations as a function of a common unobservable and an error term that is independent across bidders. However, ascending auctions reveal at best a *selected* sample of the order statistics of valuations, since the highest valuation is never revealed. Because a given pair of order statistics will always be stochastically dependent even if the underlying random variables are independent, this selection problem causes the independence assumption on the errors that underlies the measurement error approach to fail; see Athey and Haile (2002, Section 3.2) for further discussion.

[7] Referring to an earlier version of Athey, Levin, and Seira (2011), Athey and Haile (2006, p. 33) write "ALS show that conditional on observable characteristics of an auction, bids are positively correlated within a first-price auction. To account for this correlation, ALS select a model

They found correlation in values to be present and economically significant, and thus their paper is a complement to our own.

3. THEORY

3.1. *Environment and Bidding Behavior*

Here, we introduce our theoretical framework, and, in particular, three assumptions about the environment and bidding behavior that we will maintain throughout the paper. Let N (a random variable) denote the number of bidders in an auction and let n denote a value in the support of N.[8]

ASSUMPTION 1: *Bidders have symmetric private values*.

For an n-bidder auction, let $(V_1, V_2, \dots, V_n)$ denote the private values of the bidders and let $\mathbf{F}^n$ denote their probability distribution.[9] Assumption 1 is very standard in the literature and is well suited to a variety of applications.

ASSUMPTION 2: *The transaction price in an auction is the greater of the reserve price and the second-highest bidder's willingness to pay*.

Assumption 2 is an assumption on equilibrium play, but it can be motivated in a number of ways. It is the exact outcome that occurs in the dominant-strategy equilibrium of a button auction (discussed above), a stylized version of an ascending auction. It would also hold approximately (to within one bid increment) in the incomplete model of open-outcry ascending auctions of Haile and Tamer (2003) if bidders do not use "jump bids" at the end of an auction, a condition whose appropriateness can be checked directly in the data.[10] This assumption can also be motivated pragmatically: in some data sets, only the transaction price is recorded, so relying on multiple bids is impossible.[11]

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of independent private values with unobserved heterogeneity. ... This model is not identified in data from ascending auctions; thus, ALS focus their structural estimation on first-price auctions."

[8]To be proper, N should refer to the number of *potential* bidders—the number of buyers who learn their valuations and would win the auction given a high enough private value—so that we can interpret the winning bidder as the "highest of N." Observability of this number is problematic in some settings, although (as we discuss below) not in our application. If N is observed with error but there are multiple noisy measures of it for each auction, we could adapt the ideas of An, Hu, and Shum (2010) to identify the distribution of transaction price corresponding to each n, and then to proceed with our theory below.

[9]Symmetry is then the requirement that $\mathbf{F}^n(v_1, \dots, v_n) = \mathbf{F}^n(v_{\sigma(1)}, \dots, v_{\sigma(n)})$ for $\sigma : \{1, \dots, n\} \to \{1, \dots, n\}$ any permutation. This is equivalent to valuations being exchangeable random variables.

[10]In particular, if the top two bids in an auction are relatively close to one another, the condition is approximately satisfied.

[11]For example, the data used by Asker (2010) to study the behavior of a collusive bidding ring contained detailed data about activities within the ring, but only the transaction prices of the auction-house auctions themselves.

Slightly modified versions of our main results would also hold if Assumption 2 were replaced by the bidding assumptions of Haile and Tamer (2003), and the top two bids in each auction were observed; we present these modified results in Appendix A.1.

Finally, we impose a restriction that the dependence among bidder values is nonnegative.

ASSUMPTION 3: *For each n, the joint distribution $\mathbf{F}^n$ is such that for any v and i, the probability* $\Pr(V_i < v | N = n, \|\{j \neq i : V_j < v\}\| = k)$ *is nondecreasing in k.*

This assumption is new to the literature; its appeal is that it is sufficiently general to nest all the standard models of correlated private values:

- *Symmetric, affiliated private values*, where the joint distribution of valuations $\mathbf{F}^n$ has a density function $\mathbf{f}^n$ that is log-supermodular[12].
- *Symmetric, conditionally independent private values*, where bidder valuations are independent and identically distribution (i.i.d.) draws from a distribution $F_V(\cdot|\theta)$ that depends on an unobserved random variable θ, so that $\mathbf{F}^n(v_1, \dots, v_n) = E_\theta\{\prod_{i=1}^n F_V(v_i|\theta)\}$.
- *Independent private values with unobserved heterogeneity*, where valuations are again i.i.d. draws from $F_V(\cdot|\theta)$, but now θ is observed by bidders and only unobserved to the analyst.

LEMMA 1: *Assumption 3 is satisfied by any model of* (i) *symmetric*, *affiliated private values*, (ii) *symmetric*, *conditionally independent private values*, *or* (iii) *symmetric*, *independent private values with unobserved heterogeneity*.

Affiliation is the standard formulation of positive dependence in the theoretical auction literature, going back to Milgrom and Weber (1982). The latter two models are more common in the empirical literature; they differ in whether the bidders themselves observe θ or only their own valuation.[13] This informational difference has consequences for first-price auctions because equilibrium play depends on a bidder's beliefs about his opponents' valuations. In our setting, however, the auction outcome depends only on bidders' valuations, not their beliefs about opponents' valuations, so the distinction is immaterial; we need not even specify what, beyond his own private value, each bidder knows. Note also that our model nests both of these without any restriction on the dimensionality of the unobserved variables or the way in which they effect the distribution of valuations.

[12]See the Appendix of Milgrom and Weber (1982) for the definition of affiliation when $\mathbf{F}^n$ does not admit a density function.

[13]In the conditionally-independent-values model, θ is unobserved, so bidders themselves perceive values as being correlated; in the IPV model with unobserved heterogeneity, bidders perceive values as independent (conditional on observables), while the analyst perceives them as correlated.

3.2. *Identification*

Next, we show how auction data on the transaction price and number of bidders can be used to identify primitives of interest despite the fact the joint distribution of valuations $\mathbf{F}^n$ cannot be identified. For a given realization n of N, let $V_{n:n} \geq V_{n-1:n} \geq \cdots \geq V_{1:n}$ denote the order statistics of bidder values and let $F_{k:n}(\cdot)$ denote the cumulative distribution function of $V_{k:n}$. Assumption 2 implies that in the absence of binding reserve prices, the distribution $F_{n-1:n}$ is identified from the distribution of transaction prices over n-bidder auctions; these distributions $F_{n-1:n}$ for each n will be the basic building blocks of our identification strategy.[14]

Let v_0 denote the value of the unsold good to the seller (i.e., the opportunity cost of selling the object). Let $\pi_n(r)$ denote the seller's expected profit in an n-bidder auction with reserve price r, and let $\mathrm{BS}_n(r)$ denote the expected surplus achieved by the winning bidder, that is, the expected difference between the winner's valuation and the price paid. (By symmetry, each bidder's ex ante expected payoff is $\frac{1}{n}\mathrm{BS}_n(r)$.) These payoff functions can be expressed as

$$\pi_n(r) = \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v) - v_0 - F_{n:n}(r)(r - v_0), \tag{1}$$

$$\mathrm{BS}_n(r) = \int_0^\infty \max\{r, v\}\, dF_{n:n}(v) - \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v). \tag{2}$$

These expressions are functions only of the two *marginal* distributions $F_{n-1:n}$ and $F_{n:n}$, a fact that has not been previously recognized in the empirical literature but has a powerful empirical consequence: identification of seller and bidder payoffs in ascending auctions can be boiled down to the identification of just these two marginal distributions.[15] Under Assumption 2, $F_{n-1:n}$ is directly revealed by transaction price, so what remains to be identified is $F_{n:n}$. However, because nothing directly linked to $V_{n:n}$ is observed in ascending auctions—the auction ends as soon as the second-to-last bidder drops out—it will not be possible to point-identify $F_{n:n}$ in general. Our strategy is instead to seek useful

[14]In our application, these distributions are identified conditional on a vector of covariates X, which is implicitly being held fixed in the theoretical exposition.

[15]In a different but analogous setting, Athey and Haile (2007) pointed out that identification of the *joint* distribution of $V_{n-1:n}$ and $V_{n:n}$ is sufficient for "evaluation of rent extraction by the seller, the effects of introducing a reserve price, and the outcomes under a number of alternative selling mechanisms." The key difference here is that the representation of payoffs (1) and (2) depends only on the two marginal distributions of these order statistics, not the joint distribution. The cost, however, of focusing on just the marginals is that we cannot evaluate the effect of changing to a different auction format such as a first-price auction; although if bidders are risk-neutral and values are symmetric and affiliated, an ascending auction revenue-dominates a first-price auction with the same reserve price (Milgrom and Weber (1982)), so the question we can answer (the optimal ascending auction) is in some sense the most relevant one.

bounds on $F_{n:n}$. Bounds on $F_{n:n}$ will suffice to identify bounds on bidder and seller surplus, as well as on the optimal reserve price, which we now show.

LEMMA 2: *If $F^L_{n:n}(v)$ and $F^U_{n:n}(v)$ are two distribution functions such that $F_{n:n}(v) \in [F^L_{n:n}(v), F^U_{n:n}(v)]$ for all v, then the following bounds hold*[16]:

(i) $\pi_n(r) \in [\pi^L_n(r), \pi^U_n(r)]$ *for every $r \geq v_0$, where*

$$\pi^L_n(r) \equiv \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v) - v_0 - F^U_{n:n}(r)\cdot(r - v_0),$$

$$\pi^U_n(r) \equiv \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v) - v_0 - F^L_{n:n}(r)\cdot(r - v_0).$$

(ii) $\mathrm{BS}_n(r) \in [\mathrm{BS}^L_n(r), \mathrm{BS}^U_n(r)]$ *for every $r \geq v_0$, where*

$$\mathrm{BS}^L_n(r) \equiv \int_0^\infty \max\{r, v\}\, dF^U_{n:n}(v) - \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v),$$

$$\mathrm{BS}^U_n(r) \equiv \int_0^\infty \max\{r, v\}\, dF^L_{n:n}(v) - \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v).$$

(iii) $\max_r \pi_n(r) \in [\max_r \pi^L_n(r), \max_r \pi^U_n(r)]$.

(iv) $\arg\max_r \pi_n(r) \in [r^L_n, r^U_n]$, *where r^L_n and r^U_n are the min and the max, respectively, of the set $\{r \geq v_0 : \pi^U_n(r) \geq \max_{r'} \pi^L_n(r')\}$.*

The main empirical problem now becomes identifying bounds on $F_{n:n}$. Our strategy will be to appeal to variation in the number of bidders to identify such bounds. To motivate why we do so, we first observe that if we only use data from auctions of a single size n, the range of values of $F_{n:n}$ consistent with the data is wide.

To see this, for $n \geq 2$, define a function $\phi_n : [0, 1] \to [0, 1]$ implicitly by

$$v = \int_0^{\phi_n(v)} n(n-1)s^{n-2}(1-s)\, ds. \tag{3}$$

Note that ϕ_n is the inverse of the function mapping x to $nx^{n-1} - (n-1)x^n$, which takes any distribution function to the distribution of the second-highest of n independent draws from it. Thus if the valuations $(V_1, \dots, V_n)$ are independent, then $F_{n:n} = (\phi_n(F_{n-1:n}))^n$. Our next result, however, shows that for the general correlated values case, there is a wide range of possibilities for $F_{n:n}$.

[16]The bounds on π_n, BS_n, and $\max_r \pi_n(r)$ are sharp in the sense that if $F_{n:n} = F^L_{n:n}$ and $F_{n:n} = F^U_{n:n}$ are both admissible, then none of these bounds can be tightened. If it is also the case that every distribution function $F_{n:n}$ satisfying the pointwise bounds is admissible, the bounds on $\arg\max_r \pi_n(r)$ are sharp as well. However, we do not expect this to typically be the case given Assumption 3 or a more restrictive model such as affiliated private values.

LEMMA 3: *For any n and v, under Assumptions 1 and 3, $F_{n:n}(v) \in [(\phi_n(F_{n-1:n}(v)))^n, F_{n-1:n}(v)]$ and both bounds are sharp.*[17]

The lower bound on $F_{n:n}$ is consistent with a model of independent private values, and the upper bound is consistent with perfectly correlated values. Thus, Lemma 3 implies that using only the information in transaction price from n-bidder auctions, there is no way to tell the degree of correlation among bidder values, which could range from independence (the best-case scenario for seller profit) to perfect correlation (the worst-case scenario).

Lemma 3 sheds some light on the effect of correlated values on seller profits. Recall that for $r > v_0$, $\pi_n(r)$ is decreasing in $F_{n:n}$; Lemma 3, therefore, says that relative to the IPV benchmark, correlation in values can only decrease seller profits. In fact, in an informal sense, expected profits will be lower when values are "more highly correlated." To see why, we can write $F_{n:n}(r)$ as $F_{n-1:n}(r)\Pr(V_{n:n} < r|V_{n-1:n} < r)$. The function $F_{n-1:n}(r)$ is observed; when values are more highly correlated, $\Pr(V_{n:n} < r|V_{n-1:n} < r)$ will be higher, making profits lower.

Unfortunately, Lemma 3 implies no restriction on the degree of correlation among valuations, so applying Lemma 2 with $F^L_{n:n} = \phi_n(F_{n-1:n})^n$ and $F^U_{n:n} = F_{n-1:n}$ will give bounds on π_n and BS_n that may be too wide to give meaningful policy conclusions. We now show how to exploit variation in the number of bidders across auctions to potentially tighten these bounds.

3.3. *Using Variation in the Number of Bidders*

To see how variation in N provides information about $F_{n:n}$ for a particular n, we begin with the following observation. If n bidders are chosen at random out of $n+1$, the highest valuation of the sample will be either the highest of the original group (with probability $\frac{n}{n+1}$) or the second highest (with probability $\frac{1}{n+1}$). Thus, if we let $F^n_{k:m}$ denote the distribution of the kth order statistic out of m bidder valuations taken at random from an auction with n bidders ($n \geq m \geq k$), then[18]

$$F^{n+1}_{n:n}(v) = \frac{1}{n+1}F_{n:n+1}(v) + \frac{n}{n+1}F_{n+1:n+1}(v). \tag{4}$$

What we learn from this relation, however, depends on the relationship between $F_{n:n}$ and $F^{n+1}_{n:n}$. That is, it depends on whether, and how, bidders in an n-bidder auction differ from bidders in an $n+1$-bidder auction. One possible assumption we can make about this relationship is the following.

[17]This is proven in Quint (2008) for symmetric, affiliated values, but not for the more general case given here.

[18]In the event that $F^{n+1}_{n:n} = F_{n:n}$, (4) is a case of Eq. (9) from Athey and Haile (2002).

DEFINITION 1: Let $\mathbf{F}_m^n$ be the joint distribution of m randomly chosen bidders in an n-bidder auction. Valuations are *independent of* N if $\mathbf{F}_m^n = \mathbf{F}_m^{n'}$ for any $m \leq n, n'$.

Athey and Haile (2002) defined "exogenous participation" to mean this same property.[19] Independence between valuations and N has been assumed in a number of empiricals papers on first-price auctions. Guerre, Perrigne, and Vuong (2000) made this assumption as a way to improve the efficiency of their nonparametric estimator of the distribution of private values (assuming IPV), effectively allowing them to pool information from auctions of different sizes. Haile, Hong, and Shum (2003) made this same assumption to test between common and private values.[20] Guerre, Perrigne, and Vuong (2009) used this assumption to identify the coefficient of risk aversion in a model with risk-averse bidders; Gillen (2009) used it to identify the distribution of bidders' level of strategic sophistication in a "level-k" behavioral model. We can use this assumption to obtain potentially tighter bounds on $F_{n:n}$, and, consequently (by Lemma 2), on bidder and seller surplus.

If valuations are independent of N, then $F_{n:n}^{n+1} = F_{n:n}$; successive application of (4) then allows us to express the unobserved distribution $F_{n:n}$ in the following way.

LEMMA 4: *Fix n and $\bar{n} > n$. If valuations are independent of N, then for any v,*

$$F_{n:n}(v) = \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{\bar{n}} F_{\bar{n}:\bar{n}}(v).$$

Using Lemma 3 to bound the trailing term $F_{\bar{n}:\bar{n}}$ then gives

$$F_{n:n}(v) \leq \overline{F}_{n:n}(v) \equiv \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{\bar{n}} F_{\bar{n}-1:\bar{n}}(v), \tag{5}$$

$$F_{n:n}(v) \geq \underline{F}_{n:n}(v) \equiv \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{\bar{n}} \big(\phi_{\bar{n}}\big(F_{\bar{n}-1:\bar{n}}(v)\big)\big)^{\bar{n}}.$$

Since the bounds $\overline{F}_{n:n}$ and $\underline{F}_{n:n}$ are functions only of $\{F_{m-1:m}\}_{m=n+1}^{\bar{n}}$, they are *point-identified* from data on transaction price across auctions with varying numbers of bidders. Using these bounds in place of $F_{n:n}^U$ and $F_{n:n}^L$, respectively, in Lemma 2 gives the following point-identified upper and lower bounds:

[19]This is also equivalent to a model where there is a single distribution of potential bidder values $(V_1, \ldots, V_{\bar{n}})$, and a random subset of those potential bidders participate in a given auction.

[20]They extend their empirical strategy to allow for endogenous participation based on standard models of entry or to use an instrument for the number of bidders.

THEOREM 1: *Fix n and $\bar{n} > n$. If valuations are independent of N, then for any $r \geq v_0$,*

$$\pi_n(r) \geq \underline{\pi}_n(r) \equiv \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v) - v_0 - \overline{F}_{n:n}(r) \cdot (r - v_0),$$

$$\pi_n(r) \leq \overline{\pi}_n(r) \equiv \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v) - v_0 - \underline{F}_{n:n}(r) \cdot (r - v_0),$$

$$\mathrm{BS}_n(r) \geq \underline{\mathrm{BS}}_n(r) \equiv \int_0^\infty \max\{r, v\}\, d\overline{F}_{n:n}(v) - \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v),$$

$$\mathrm{BS}_n(r) \leq \overline{\mathrm{BS}}_n(r) \equiv \int_0^\infty \max\{r, v\}\, d\underline{F}_{n:n}(v) - \int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v).$$

Likewise, $\max_r \pi_n(r) \in [\max_r \underline{\pi}_n(r), \max_r \overline{\pi}_n(r)]$ *and* $\arg\max_r \pi_n(r) \in \{r : \overline{\pi}_n(r) \geq \max_{r'} \underline{\pi}_n(r')\}$.

Observe that if valuations are independent of N, then for each r, the intervals $[\underline{\pi}_n(r), \overline{\pi}_n(r)]$ and $[\underline{\mathrm{BS}}_n(r), \overline{\mathrm{BS}}_n(r)]$ shrink as $\bar{n}$ increases, and collapse to the true values of $\pi_n(r)$ and $\mathrm{BS}_n(r)$ as $\bar{n} \to \infty$.

To see what these bounds are capable of accomplishing, we now present an example.

EXAMPLE 1: Consider a symmetric IPV model with unobserved heterogeneity. There are three bidders. An unobserved "demand shifter" θ takes two values, $\theta \in \{H, L\}$, with equal probability. Conditional on a realization of θ, bidder valuations are i.i.d. log normal: $\ln(V_i) \sim N(2.5, 0.5)$ when $\theta = H$ and $\ln(V_i) \sim N(2.0, 0.5)$ when $\theta = L$. The seller's reservation value is $v_0 = 5$.

Figure 1 shows actual expected profit $\pi_3(\cdot)$ and expected bidder surplus $\mathrm{BS}_3(\cdot)$ (the dashed lines), as well as the tightest bounds that can be put on them using only $F_{2:3}$ (the solid lines).

Actual expected profit is maximized at a reserve price of $r = 10.1$, but the bounds on optimal reserve price implied by Lemmas 2 and 3 are [5.0, 17.8],

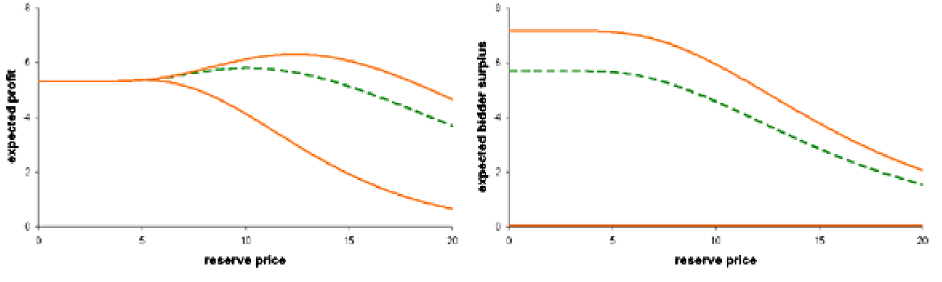

FIGURE 1.—Bounds on $\pi_3(r)$ and $\mathrm{BS}_3(r)$ using only $F_{2:3}$ (from Lemmas 2 and 3).

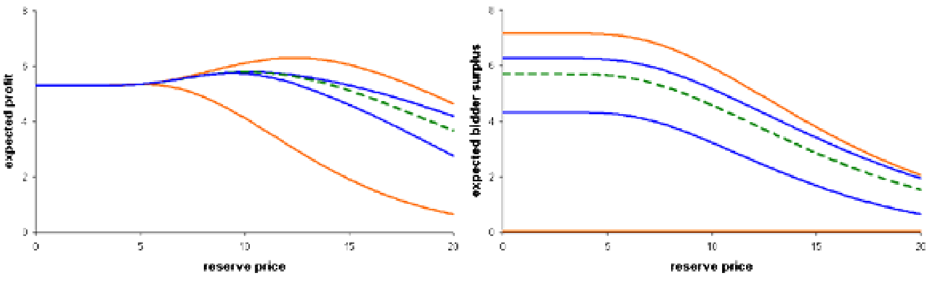

FIGURE 2.—Bounds on $\pi_3(r)$ and $\mathrm{BS}_3(r)$ from Theorem 1, using $\{F_{n-1:n}\}_{n=3}^{12}$.

and over much of that range, the upper bound on expected profit is more than twice the lower bound.[21] At the profit-maximizing reserve price, actual expected bidder surplus is \$4.57, but the range consistent with the distribution $F_{2:3}$ is [\$0.00, \$5.90]. Notice that the lower bound on bidder surplus is simply the x-axis because we cannot rule out the perfectly correlated case.

Now suppose that the number of bidders varies "exogenously" (independently of θ and valuations) between 3 and 12. Figure 2 shows the bounds on π_3 and BS_3 from Theorem 1. Again, the true values of $\pi_3(\cdot)$ and $\mathrm{BS}_3(\cdot)$ are the dashed lines. The lighter solid lines are the bounds calculated only from auctions with three bidders. The heavier solid lines are the bounds given in Theorem 1, with $\bar{n} = 12$.

Note that the bounds from Theorem 1 are much tighter. With $\bar{n} = 12$, the bounds on optimal reserve price are [8.9, 11.6], and over that range, the upper and lower bounds on expected profit differ by no more than 4%. At the true optimal reserve price $r = 10.1$, Theorem 1 bounds expected bidder surplus to lie within the interval [\$3.20, \$5.14] versus the bounds [\$0.00, \$5.90] using only the data from $n = 3$.

Of course, in some applications, the assumption that valuations are independent of N may be unrealistic. If, even conditional on observable covariates, the objects for sale are heterogeneous, we might expect auctions for more valuable objects to attract more bidders, leading to a positive relationship between bidder valuations and the number of bidders in an auction. Next, we show that one-sided bounds still hold in this case. We will rely on a weaker identifying assumption, essentially stochastic monotonicity—higher N must be associated with higher valuations in a particular sense.

DEFINITION 2: Valuations are *stochastically increasing in N* if $n > n'$ implies $F^n_{m:m} \succsim_{\mathrm{FOSD}} F^{n'}_{m:m}$ for any $m \leq n'$.

[21]A more restrictive model of valuations such as affiliation could shrink this interval to [5.0, 12.4], with both bounds being sharp.

That is, valuations are stochastically increasing in N if a group of bidders sampled from a larger auction tend to have higher valuations than a group of bidders sampled from a smaller auction, where "higher" is measured as a first-order stochastic dominance (FOSD) ranking of the highest order statistic. In Appendix A.2, we discuss three standard models of endogenous participation in auctions—those of Levin and Smith (1996), Samuelson (1985), and Marmer, Shneyerov, and Xu (2010)—and show conditions under which equilibrium play in each model would lead to valuations stochastically increasing in N. Thus, our definition of stochastically increasing valuations accommodates many standard models of endogenous entry.

When valuations are not independent of N but rather stochastically increasing, one-sided analogs of Lemma 4 and Theorem 1 hold:

LEMMA 5: *Fix n and $\bar{n} > n$. If valuations are stochastically increasing in N, then for any v, $F_{n:n}(v) \geq \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{\bar{n}} F_{\bar{n}:\bar{n}}(v)$.*

THEOREM 2: *Fix n and $\bar{n} > n$. If valuations are stochastically increasing in N, then for any $r \geq v_0$, $\pi_n(r) \leq \overline{\pi}_n(r)$ and $\mathrm{BS}_n(r) \leq \overline{\mathrm{BS}}_n(r)$, where $\overline{\pi}_n$ and $\overline{\mathrm{BS}}_n$ are as defined in Theorem 1.*

Theorem 2 can be interpreted as a robustness check on the bounds in Theorem 1. Theorem 1 gives two-sided bounds on $\pi_n(r)$ and $\mathrm{BS}_n(r)$ when valuations are independent of N; Theorem 2 says that both *upper* bounds are still valid even if valuations are, instead, only stochastically increasing in N.[22]

3.4. *Further Tightening the Bounds*

Since, by assumption, the distributions $\{F_{n-1:n}\}$ are point-identified, the only potential source of slackness in the bounds $\underline{\pi}_n$, $\overline{\pi}_n$, $\underline{\mathrm{BS}}_n$, and $\overline{\mathrm{BS}}_n$ is the trailing term $\frac{n}{\bar{n}} F_{\bar{n}:\bar{n}}(v)$ in the expression for $F_{n:n}$ in Lemma 4. Per Lemma 3, the bounds we place on $\frac{n}{\bar{n}} F_{\bar{n}:\bar{n}}$ are the tightest available given *only* the information contained in $F_{\bar{n}-1:\bar{n}}$. However, additional information could potentially be used to tighten the bounds on $F_{\bar{n}:\bar{n}}$, and, therefore, the bounds on π_n and BS_n.

One source of additional information is losing bids. While Assumption 2 says nothing about the interpretation of losing bids, we could apply the first assumption of Haile and Tamer (2003)—that no bidder bids higher than his private value—and interpret a losing bid as a lower bound on the bidder's valuation. Each losing bid would give a different lower bound on $F_{\bar{n}:\bar{n}}$, and we could use the pointwise max of these bounds to potentially tighten the upper

[22]The lower bounds on π_n and BS_n implied by Lemma 3 (and illustrated in Figure 1) still hold without any assumption about variation in N, but are likely to be too "loose" to be practically useful: for example, the lower bound on π_n is strictly decreasing on $r > v_0$ and the lower bound on BS_n is 0.

bounds on π_n and BS_n. In Appendix A.3, we show how this would be done and discuss why we do not pursue this strategy in our empirical application.

4. AN APPLICATION TO TIMBER AUCTIONS

We have shown how data from ascending auctions can be used to point-identify bounds on seller profits, profit-maximizing reserve price, and expected bidder surplus in auctions where bidder valuations may be correlated. We now seek to examine the empirical usefulness of our bounds. In particular, we apply our identification strategy to study United States Forest Service (USFS) timber auctions, which are auctions that have received significant scrutiny in the literature because of the many different policy problems they pose. However, no existing work has studied the English auctions in these data in a way that allows for correlated values, which makes them an ideal application for our bounds approach. The key questions we address are whether correlation among bidders' values exists in timber auctions, and if so, what economic consequences it has for welfare and auction design.

4.1. *Institution and Data*

USFS timber auctions are used to allocate the right to harvest timber from a tract of public land. Prior to the auction, the government conducts a cruise of the auction tract and publishes a report containing the characteristics of the tract uncovered by the cruise. The government also announces a reserve price for the auction, which is equal to its appraisal value for the tract. The auctions themselves are conducted in two rounds. In the first round, bidders submit sealed bids, which must exceed the appraisal value for the bidder to qualify for the auction. In the second round, those bidders who qualified in the first round compete in an ascending auction, which begins at the highest of the sealed bids from the first round. We use publicly available data covering all USFS timber auctions held between 1978 and 1996.[23]

One major feature of timber auctions is that they are highly heterogeneous, both in terms of the types of sales contracts they employ as well as the underlying timber being sold. This heterogeneity plays an important role for the applicability of our identification strategy to the data, which we now discuss.

Contract Type

Our Assumption 1 states that the bidders in an auction have private values. We focus on a subset of the auctions that are most likely to satisfy this assumption. As suggested by Baldwin, Marshall, and Richard (1997), we consider only scaled sales, where bids are per unit of timber actually harvested,

[23]These data are available in the original form collected by the timber service on Phil Haile's website, which is the raw source we use.

and, therefore, common-value uncertainty about the total amount of timber on the tract should not affect valuations. In addition, like Haile and Tamer (2003), we focus on sales in Region 6 (encompassing mostly Oregon), where bidders typically do not conduct their own pre-auction cruises for scaled-sales auctions (see, e.g., Natural Resources Management Corp (1997)), thus removing any private signals about common value uncertainty concerning aspects of the timber not captured by the cruise report.[24] Finally, we consider only sales whose contracts expire within a year, to minimize the effect of resale on valuations as suggested by Haile (2001). All of these elements combine to make the private values assumption a reasonable approximation.

Appraisal Method

Our Assumption 2 on equilibrium bidding behavior allows us to identify the distributions of the order statistic $\{F_{n-1:n}\}$ from the transaction price data (which in turn are the basic building blocks of our identification strategy) as long as the reserve price is nonbinding. We thus focus on auctions held between 1982 and 1990, which is a period when the appraisal value for Region 6 was set according to the so-called *residual method* and generally recognized to be well below actual bidder values (and therefore nonbinding in practice).[25] Indeed, a major policy concern of the timber service is how best to appraise timber, and we will revisit the debate that took place concerning the residual method in our empirical analysis below.

A further advantage of restricting the data to this period is that if reserve prices were not binding, the announced reserve would not prevent any potential bidders from participating in the first round or advancing to the second (open-outcry) round. A key feature of our data is that we observe the number of bidders participating in this second round and thus have a fairly direct measure of N. In our analysis, we use only those auctions where the number of bidders N is between 2 and 11.[26]

[24]Baldwin, Marshall, and Richard (1997) investigated collusion in auctions in Region 6 prior to 1982, and discussed major changes made by the timber service in 1982; collusion appears to have been less of a concern after that. See also Haile and Tamer (2003).

[25]In the 2003 working paper version of Campo, Guerre, Perrigne, and Vuong (2011), the authors write, "It is well known that this reserve price does not act as a screening device to participating," and perform analysis confirming that "the possible screening effect of the reserve price is negligible" (p. 33). (In the published version, they simply write, "As in the previous empirical studies, we consider a nonbinding reserve price.") See also Haile (2001), Froeb and McAfee (1988), and Haile and Tamer (2003).

[26]Like Haile and Tamer (2003), we drop auctions with one bidder from the analysis, because without binding reserve prices, they give us no information about the distribution of valuations. Bounds on $F_{1:1}$ are still identified from $\{F_{m-1:m}\}_{m\geq 2}$ via Lemma 4 or 5, and $\max_r (r - v_0)(1 - F_{1:1}(r))$ can then be solved to find the optimal reserve price when $N = 1$. We also drop observations with $N = 12$, as this appears to be top-coding for "12 or more" and our methods depend on accurate measurement of N.

Cruise Report

The timber being sold in USFS auctions is highly heterogeneous from tract to tract. Our Assumption 3 allows for very general patterns of correlated values, including correlation induced by unobserved heterogeneity, and thus it is not essential for our identification approach that we control for tract heterogeneity (which would be necessary with an IPV model). Nevertheless, failing to control for tract information that the timber service in fact observes would limit the policy relevance of our application; any correlation in values we find empirically might simply be due to the effects of heterogeneity among tracts that the timber service can take into account but the empirical analysis does not.

A key feature of the USFS data is that it contains detailed information about each auction from the government's cruise report, which summarizes the USFS's information set concerning the quality of timber in the tract. To control for the information contained in the cruise reports, we combine two different sets of covariates emphasized in the previous literature. Following Haile and Tamer (2003), we control for the estimated sales value of the timber (per unit of timber), the estimated harvesting cost (per unit of timber), the estimated manufacturing cost (per unit of timber), the species concentration index (the HHI (Herfindahl index) computed based on the relative volumes of the different species present), and the total volume of timber sold in the 6 months prior to each auction (as a measure of the bidding firms' existing inventory), and we control for the geographic zone of the tract being sold by including only sales in Zone 2 (which contains fairly homogeneous timber and accounts for the majority of the data). A different approach was used by Lu and Perrigne (2008), who treated the appraisal value itself as a sufficient statistic for the auction's characteristics. This was motivated by the institutional fact that appraisal value was set as a function of all the information in the government cruise report and by the empirical fact that appraisal value on its own explained most of the variation in transaction price relative to the other covariates. In our data, appraisal value still contains information about transaction price when included alongside of the controls from Haile and Tamer (2003). Rather than choosing between the two approaches, we combine them, using all the covariates from Haile and Tamer (2003) as well as the appraisal value.

Table I gives some summary statistics of the data. Note that the highest bid (transaction price) and second-highest bid have very similar distributions. In fact, the median ratio of second-highest bid to transaction price is 0.994. Also note the large gap between appraisal value and transaction price, evidence that appraisal values were conservative estimates and did not serve as binding reserve prices. Together, these give us further confidence in the applicability of Assumption 2 to the data.

Another feature of the data is that the number of bidders participating in each auction bears only a weak relationship to the observable covariates that

TABLE I

SUMMARY STATISTICS OF AUCTIONS USED IN THE ANALYSIS

Variable[a]	Mean	Std. Dev.	Min.	Max.
Transaction price	143.92	166.30	5.34	4,263.78
Second-highest bid	142.18	166.08	5.34	4,263.07
Number of bidders	5.3	2.5	2	11
Sales value	418.98	66.88	190.85	1,354.69
Manufacturing cost	184.63	44.70	39.93	282.52
Harvesting cost	135.45	36.22	55.25	358.43
Density	23.1	25.0	0.003	240
Species concentration	0.655	0.225	0.258	1.000
6-month inventory	2,160,817	453,273	906,916	3,529,826
Appraisal value	73.70	63.22	4.08	825.00
Volume	845	1,015	15	11,800
Number of observations: 1,113				

[a]Transaction price, second-highest bid, sales value, manufacturing cost, harvesting cost, and appraisal value are in dollars per thousand board-feet; 6-month inventory and timber volume are in thousands of board-feet.

characterize the auction.[27] If bidders are not systematically selecting to participate based on observables, it may suggest they are also not selecting based on unobservables, and, therefore, independence between valuations and N may be a plausible assumption. In a separate paper (Aradillas-López, Gandhi, and Quint (2011)), we introduce and apply a formal test of independence between valuations and N, and fail to reject it in these auctions.[28] Thus in the analysis that follows, we estimate two-sided bounds on the measures of interest; as noted earlier, if valuations were instead stochastically increasing in N, the upper bounds on expected profit and bidder surplus would still be valid, and one-sided inference could be conducted based only on those upper bounds.

4.2. *Estimation*

Given Theorem 1, estimation of our bounds becomes a question of estimating $F_{n-1:n}(\cdot|X)$, the distribution of transaction price conditional on X, for $n = 2, \ldots, \bar{n}$. While these distributions are nonparametrically identified, the best approach to estimation will depend on the data set: if the sample size is small and/or if X contains a rich collection of covariates, the "curse of dimensionality" could impact the precision of nonparametric estimators.

One natural approach—similar to the estimation strategy of Athey, Levin, and Seira (2011)—would be to parametrize the distributions $F_{n-1:n}(\cdot|X)$ by as-

[27]The R^2 of regressing N on the entire vector of covariates is 0.06.

[28]Roberts and Sweeting (2012) found evidence of selective entry in California auctions. The apparent reason for this difference is that bidders in California conduct their own cruises, so the cost of participation is much greater than in Region 6, where they typically do not.

suming they belong to a known family (e.g., mixtures of normals) indexed by a finite-dimensional parameter. This would allow for parametric estimation without imposing any parametric assumptions on the *joint* distribution of values. Another approach would be to impose a single index restriction, where the distribution of $V_{n-1:n}|X$ is estimated nonparametrically but assumed to depend on X only through a scalar index. An estimation procedure such as the one in Ai (1997), which extends the least-squares-based semiparametric estimators in Robinson (1988) and Ichimura and Lee (1991) to a maximum likelihood estimator (MLE) setting, could be employed. Standard specification tests for likelihood models or semiparametric index restrictions (see Fan and Li (1996)) could be applied to evaluate the validity of these assumptions.

A misspecified parametric or semiparametric model, however, would likely have effects that are observationally equivalent to those of residual correlation among values. Since one of our main empirical questions is whether bidder values are correlated once observables are controlled for, we have, therefore, taken a fully nonparametric approach to estimation. Inference is simplified by the fact that while expected profit and bidder surplus are nonlinear functions of $\{F_{n-1:n}\}$, they can be written as expectations of random variables identified in the data. Thus, they can be estimated using sample analogs, allowing for analytically closed-form confidence intervals.

Given our sample size and the number of continuous covariates in our data, the curse of dimensionality and overall goodness of fit are a concern. To address these issues, we first estimate a parametric "reference model" and then select our bandwidths to minimize the estimation "error" of our nonparametric estimators relative to this reference model. Choosing a bandwidth through the use of a parametric reference model is a well known approach in nonparametric estimation (see Silverman (1986) and Ichimura and Todd (2007)). Our reference model appears to fit the data reasonably well, but this is not essential, as the nonparametric estimates are asymptotically valid even if the reference model is misspecified. The Supplemental Material gives a fully self-contained presentation of our econometric strategy: we define our nonparametric estimators, derive their asymptotic properties, and use them as a basis for analytically constructing pointwise confidence intervals for both seller profit and bidder surplus.

4.3. *Seller Profit and Bidder Surplus*

We now present estimates of our fundamental objects of interest: seller profit and bidder surplus. We extend our earlier notation to include observable covariates X. If an auction in the data is characterized by $N = n$ bidders and tract covariates $X = x$, then the private values $(V_1, \dots, V_n)$ are assumed to be drawn from a joint distribution $\mathbf{F}^n(\cdot|x)$ satisfying Assumption 3. This distribution of valuations determines the seller's profit $\pi_n(r|x)$ and bidder surplus $\text{BS}_n(r|x)$ for any reserve price $r \geq 0$.

Seller Profit for a Given Number of Bidders

We begin by estimating the bounds on seller profit $\pi_n(r|x)$ for various values of n. As a benchmark case of a representative auction, we consider the "median auction," that is, the pointwise median of the vector of covariates X.[29] We use $\bar{n} = 11$, the upper bound of the support of N in our data. For the benchmark case, we assume that v_0, the seller's valuation, is equal to the appraisal value, which is \$60 (per thousand board-feet) for the median auction. (We consider the effect of alternative assumptions about v_0 below).

Figure 3 shows our estimates of the bounds on expected profit (the shaded region), as well as the pointwise 95% confidence interval (the darker dashed lines), plotted against reserve price for different values of n. For comparison, the point estimate and 95% confidence interval for expected profit under IPV are shown as lighter lines (marked "Assuming IPV").[30] The following pattern is apparent from these graphs:

- For smaller values of n—auctions with two, three, four, or five bidders—our bounds on π_n are fairly tight in the relevant range of reserve prices (those close to the optimal reserve price), and the IPV estimate clearly overstates both the profit-maximizing reserve price and the increase in profits from using it.
- For larger values of n—auctions with six or more bidders—competition is sufficiently strong to eliminate most of the benefit of using a strategic reserve price; even the IPV model suggests the increase in profits from setting reserve price optimally is quite small. For $n \geq 6$, the confidence intervals for IPV and our bounds begin to overlap, and the implications of the two models for profits become harder to disentangle. Note that the main reason our bounds appear wider in this case is because as n gets large, $\frac{n}{\bar{n}}$—the coefficient on the term in π_n that is not point-identified—grows as well, so the "unknown" term becomes a larger part of the estimate of π_n. The confidence intervals are also growing wider as n grows because the data are thinner at the top: two-thirds of our data are from auctions with $N \leq 6$.

The overall qualitative pattern revealed by the profit curves is consistent with the presence of positive dependence in bidder valuations, since as noted above, correlation in values will reduce the benefits of a strategic reserve. Observe that for a wide range of reserve prices, the confidence interval for the IPV model does not overlap with the confidence interval for our bounds. Thus our bounds are sufficiently informative to reveal a significant departure of the data from IPV.

We can further quantify the policy significance of this departure. Table II presents the level of the optimal reserve implied by each model, as well as the effects of these reserve prices on profits. We focus on smaller values of n, where

[29]The first column of Figure 9 in Appendix A compares estimates of expected profit at this "median" auction and other percentiles of X; the qualitative results are similar.

[30]IPV benchmarks for π_n and BS_n are calculated by plugging $F_{n:n}(v|x) = (\phi_n(F_{n-1:n}(v|x)))^n$ into (1) and (2); estimation and inference for the IPV case are presented in the Supplemental Material alongside estimation and inference for our bounds.

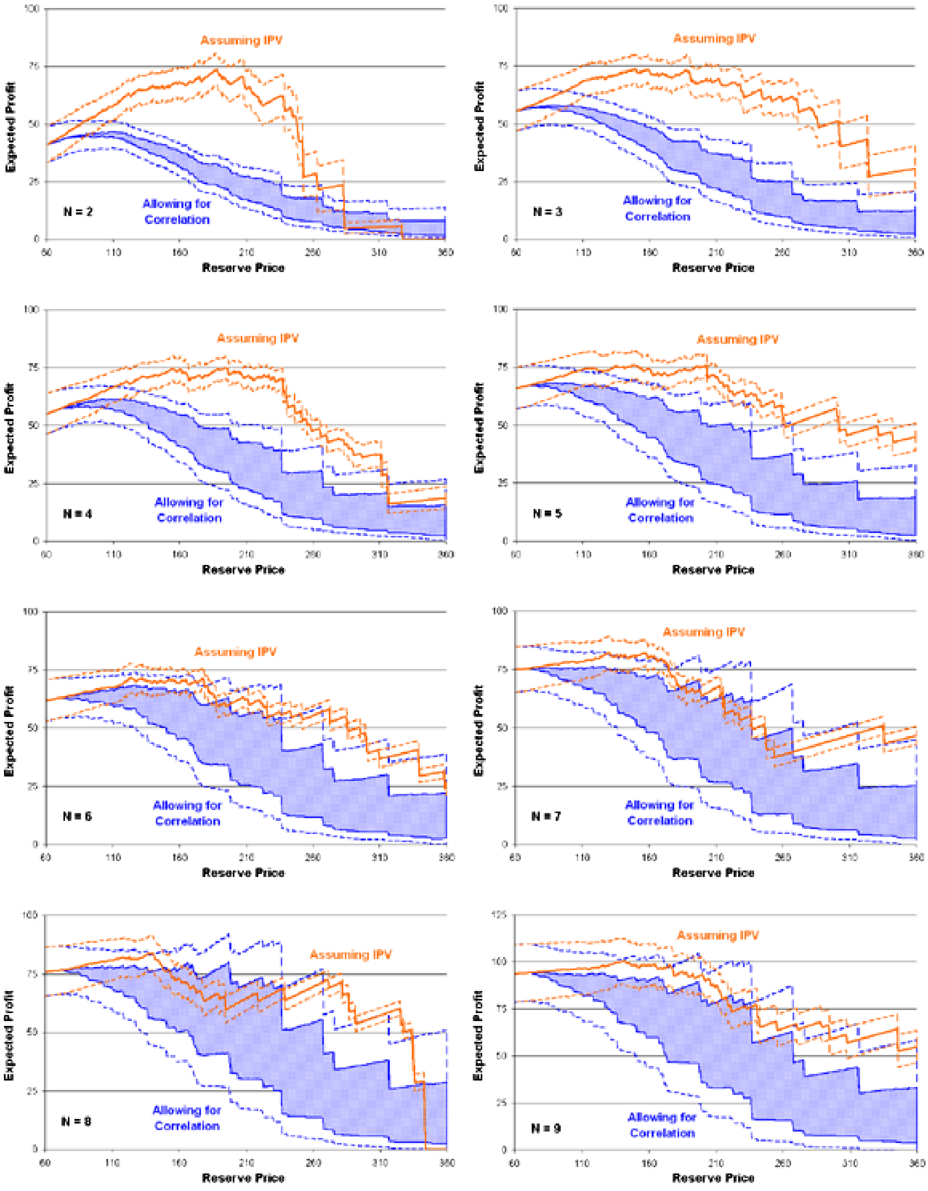

FIGURE 3.—Expected profit for benchmark auction, conditional on N, with v_0 = appraisal value.

TABLE II

BOUNDS ON PROFIT AND OPTIMAL RESERVE, BENCHMARK AUCTION, CONDITIONAL ON N

	Number of Bidders N			
	2	3	4	5
Expected profits with $r = v_0$	41.03	55.51	54.92	65.95
Bounds on optimal reserve price	[82, 123]	[73, 110]	[80, 141]	[72, 109]
Bounds on $\max_r \pi_n(r\|x)$	[44.71, 46.57]	[56.99, 57.88]	[58.33, 61.43]	[67.09, 68.13]
Optimal reserve price under IPV	188	149	195	149
Predicted profits under IPV	73.78	73.70	74.86	76.00
Profits with correlated values	[23.50, 32.53]	[42.54, 50.95]	[29.56, 48.59]	[49.51, 63.54]

reserve price matters most. Consider the case of $N = 2$ (the first column of the table) as an example and notice the following facts:

- The first row shows that setting reserve price r equal to the seller's valuation v_0 (here, 60) would yield expected profit of 41.03.
- The second row shows that our point estimates on $\overline{\pi}_2$ and $\underline{\pi}_2$ bound the optimal reserve price to lie between 82 and 123. Although this interval may be seen as somewhat wide, it excludes the optimal reserve of 188 that is implied by the IPV model (seen in the fourth row).
- The contrast is even more apparent if we consider the profits attained at the optimal reserve: whereas our bound estimates imply expected profits at the optimal reserve of at least 44.71 and at most 46.57 (the third row), the expected profit under IPV is estimated to be as high as 73.78 (the fifth row).
- Finally observe that if we were to implement the optimal reserve suggested by IPV, then the actual profits predicted by our bounds would be between 23.50 and 32.53 (the last row), which is a loss for the seller compared to setting the nonstrategic reserve $r = v_0$.

The remaining columns show the same results for $n = 3$, 4, and 5, and the conclusions are similar. In all four cases, our upper bound on the optimal reserve price is at least 25% lower than the level of the reserve price that would appear optimal under IPV. And in all four cases, using the reserve that appears optimal under IPV would actually lead to lower profits than simply setting $r = v_0$.

Seller Profit Unconditional on the Number of Bidders

These results suggest that in auctions with few bidders—auctions where the choice of reserve price matters most—the reserve prices recommended by an IPV analysis of the data are substantially too high in actuality and would lead to lower profits than simply setting reserve equal to v_0. However, the problem facing the seller in timber auctions is somewhat more subtle. In principle, given the two-round structure of the auction, the seller could potentially set a reserve price for the second round after the first round concludes, with full information

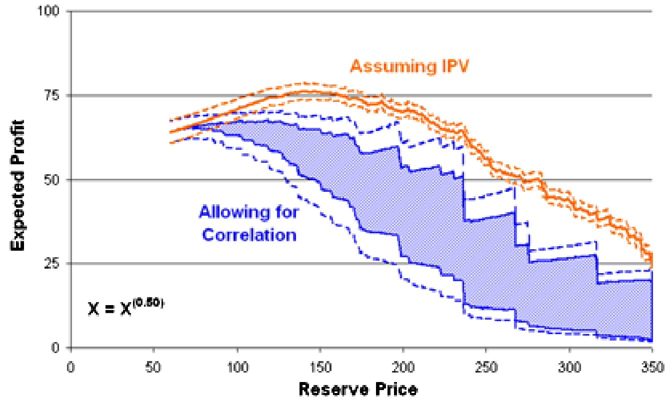

FIGURE 4.—Expected profit for benchmark auction, unconditional on N.

about N. In practice, however, the forest service announces the reserve price before the first round and, therefore, without knowing the realization of the number of bidders N. Therefore, next we consider the expected profit of the seller *unconditional* on the number of bidders, which is

$$\pi(r|x) = \sum_n p_N(n|x)\pi_n(r|x),$$

where $p_N(\cdot|x)$ is the distribution of N among auctions with covariates $X = x$. The question is whether our insights above are robust to averaging profits across the different numbers of potential bidders.

Figure 4 shows our bounds estimates and pointwise 95% confidence interval for $\pi(\cdot|x)$ at the median auction, along with the corresponding IPV estimates. The qualitative takeaway is similar to the conclusions above on $\pi_n(r|x)$ for $n \leq 5$: IPV substantially overstates both the optimal reserve price and the resulting expected profit. The first column of Table III replicates the analysis of Table II for *unconditional* profits. As can be seen, our point estimates bound the optimal reserve price (unconditional on N) to lie between 73 and 136, whereas the IPV model finds an optimal reserve price of 141. Use of the reserve price that appears optimal under IPV would lead to expected profits between 79% and 101% of those earned by setting $r = v_0$.

Effect of Seller's Valuation

The seller's actual profit depends critically on the seller's valuation v_0 for an unsold tract. We have assumed that this value is equal to the seller's own appraisal value of the tract. Because the appraisal method during the period of our sample was known to largely underestimate bidders' willingness to pay, we view this choice as a conservative estimate of the seller's outside option (i.e., selling the tract to an outside party or selling it in a future auction). However,

TABLE III

BOUNDS ON PROFIT AND OPTIMAL RESERVE, BENCHMARK AUCTION, *UNCONDITIONAL* ON N

	Seller's Valuation v_0			
	60	30	90	120
Expected profits with $r = v_0$	64.15	91.50	41.51	25.02
Bounds on optimal reserve price	[73, 136]	[65, 76]	[96, 236]	[125, 267]
Bounds on $\max_r \pi(r\|x)$	[65.47, 67.53]	[92.93, 93.23]	[42.46, 48.51]	[26.15, 36.30]
Optimal reserve price under IPV	141	119	187	211
Predicted profits under IPV	76.25	99.16	57.52	43.80
Profits with correlated values	[50.98, 65.03]	[75.79, 88.11]	[27.77, 46.64]	[16.80, 33.75]

if exercising this outside option would entail additional costs, it is possible that the seller's value v_0 would be even lower than this. On the other hand, scaled sales require the timber service to measure the timber actually harvested so as to calculate payment, and it is possible that exercising the outside option (through a lump-sum contract) would avoid these costs; in this case, v_0 could be higher than appraisal value. It is thus useful to examine the effect of changing our assumption on v_0 on our above conclusions.

Continuing to focus on the median auction, Figure 5 shows expected profit, unconditional on N, under different assumptions about v_0. Naturally, an in-

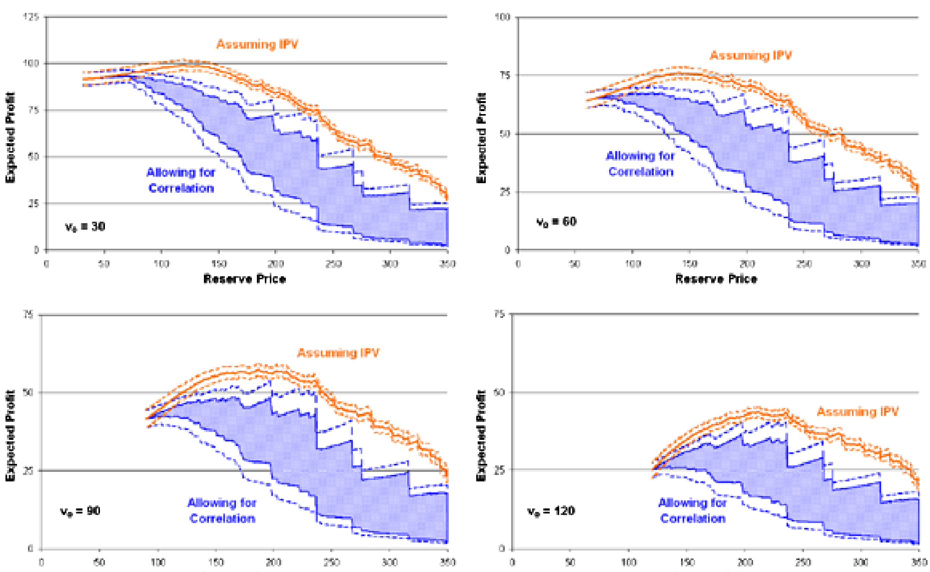

FIGURE 5.—Expected profit for benchmark auction, unconditional on N, at various v_0.

crease in the seller's opportunity cost reduces the level of profits and increases the optimal reserve price. However, the qualitative patterns discussed above—in particular, how our bounds compare to the point estimates from IPV—remain intact. The last three columns of Table III summarize the changes in expected profit and optimal reserve price at different values of v_0. One main conclusion reached earlier—that the IPV model substantially overstates the increase in profits from using an aggressive reserve price—is extremely robust to these alternative assumptions about v_0.

Bidder Surplus

We now move our focus to the second half of the welfare question: bidder surplus. Figure 6 shows expected bidder surplus for the benchmark auction unconditional on N. The main pattern that visually emerges is that IPV substantially overstates bidder surplus. Consider the range of reserve prices from 60 to 200, which is the range of reserve prices that are optimal or near optimal under any of the models. Over this range, the upper bound on bidder surplus is between 60% and 66% of the IPV point estimate, and the lower bound is between 6% and 20% of the IPV estimate. The second column of Figure 9 in Appendix A shows expected bidder surplus at other values of X; the qualitative takeaway is similar to Figure 6.

Figure 10 in Appendix A shows expected bidder surplus $\mathrm{BS}_n(r|x)$ for various values of n at the benchmark auction. For every n, the IPV estimate of bidder surplus is above our bounds. As with seller profit, for $n \leq 5$, the difference in bidder surplus between our model and IPV is dramatic: on the relevant range of reserve prices $r \in [60, 200]$, our upper bound on bidder surplus is typically below half the IPV estimate, our lower bound is typically below one-quarter, and the pointwise confidence intervals under the two models do not overlap. For $n \geq 6$, on the other hand, our upper bound is consistently above

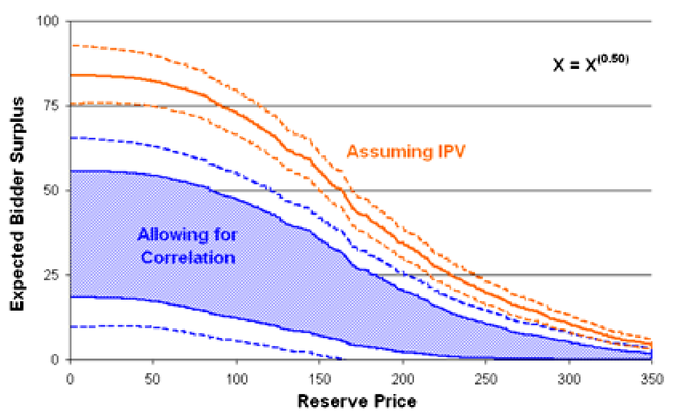

FIGURE 6.—Expected bidder surplus for benchmark auction, unconditional on N.

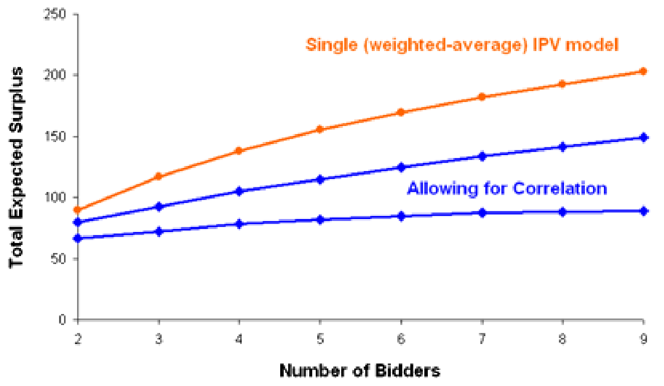

FIGURE 7.—Expected total surplus for median auction, $r = v_0$, conditional on N.

one-half of, and sometimes fairly close to, the IPV estimate, and the confidence intervals often overlap.

One interesting feature of bidder surplus that is reflected in the graphs is that theory does not constrain bidder surplus to be either increasing or decreasing with the number of bidders. Although the highest willingness to pay increases with the number of bidders, so does the expected price that must be paid, so the net effect on bidder surplus is ambiguous.[31] On the other hand, *total* surplus—bidder surplus plus seller profit—must be weakly increasing in the number of bidders. One consequence of correlated values, however, is that the incremental contribution of an additional bidder to total surplus will be smaller than under an IPV model. Figure 7 illustrates this effect in our data, comparing total surplus in the benchmark auction with a reserve price of $r = v_0$ under our model and IPV as N changes.[32] Figure 7 shows the actual incremental social value of each additional bidder to be at most about two-thirds that suggested by an IPV model and possibly much less. This can have important policy implications, particularly for assessing the efficiency of entry.[33]

[31]Putting aside reserve price, bidder surplus is the expected difference between the top two order statistics; even under IPV, this can be either increasing or decreasing in n, depending on whether the parent distribution from which valuations are drawn has an increasing or decreasing hazard rate.

[32]The IPV model used here is a weighted average of the IPV models estimated separately for each N. If total surplus is compared across IPV models estimated separately for each N, the result is nonmonotonic in N, suggesting again that the IPV model does not fit our data particularly well.

[33]As noted above, in the Region 6 auctions, bidders typically do not conduct their own cruises for scaled sales. However, in other regions (particularly California), it appears bidders do conduct their own cruises so as to learn their value, which introduces a fixed cost of entry (see Roberts and Sweeting (2012) and Athey, Levin, and Seira (2011)). Athey, Levin, and Seira (2011) estimated the fixed cost of a cruise to be roughly \$5,000, which also accords with common institutional wisdom. If the environment changed so that bidders chose to, or were forced to, conduct their own cruises in Region 6 and costs were of the same magnitude, and using the fact that the median tract

4.4. *Evaluation of Reserve Price Policies*

We now shift our attention away from evaluating the effects of correlation on outcomes in a single (representative) auction and consider the effects of a *policy* change applied across all auctions. A central policy question facing the USFS is the choice of reserve price it sets at timber auctions. The residual method used by the USFS to determine appraisal values during the period of our data was based on estimates of accounting profits; in practice, these accounting estimates were widely seen as being too conservative, yielding appraisal values that were artificially deflated.[34] This prompted a policy change toward a market-based approach to appraising timber that was implemented using a method known as transaction evidence appraisal (TEA).[35] The TEA method, which is described in detail in Athey, Crampton, and Ingraham (2003), sets the appraisal value (and hence reserve price) for a tract to be the predicted value of the winning bid in a given auction based on historical data, discounted by some "rollback factor" to encourage competition. USFS policy requires rollback rates to be no more than 30% of the predicted transaction price; Athey, Crampton, and Ingraham (2003) reported rollback rates actually employed in various locations ranging from 5% to 30%. The TEA was introduced into different regions at different times, but for Region 6 (the region we study) the introduction did not happen until after 1990. We can thus study what would have been the effects of introducing the TEA into Region 6 during our pre-1990 sample period as a measure of its performance.

We can think of a reserve price policy r_i as a mapping from X (the details of a particular tract) to a reserve price $r_i(X) \in \mathbb{R}$. An effect we would obviously like to study is the expected profits from a given policy, $E_{X,N}[\pi_N(r_i(X)|X)]$. However, the USFS also operates under a mandate to sell at least 85% of the timber it offers at auction (see, e.g., Haile and Tamer (2003)). Thus another relevant measure of performance is the expected fraction of tracts that go unsold, that is, the fraction of auctions at which the reserve price will not be met, which is $E_{X,N}[F_{N:N}(r_i(X)|X)]$. This expectation should not exceed 15% if the mandate is being respected. The USFS, therefore, faces an interesting trade-off in setting reserve price: a more aggressive reserve price (a smaller rollback factor) might extract more rents from bidders, but potentially at the cost of violating the mandate.

To study this trade-off and the effects of the TEA policy, we replicate such a policy by regressing the transaction price on the auction covariates X and

has 550 thousand board-feet of timber, the IPV model would suggest that the efficient number of bidders (the number of bidders that maximizes total surplus net of fixed costs) on the median tract is 9. Our more general correlated values analysis would instead conclude that the efficient auction size would be at most six bidders, and possibly fewer.

[34]See Combes (1980) for a thorough description of how the residual method of appraisal was carried out in practice.

[35]For a discussion of the actual policy debate between the residual method and the TEA as it pertains to Region 6, see USFS National Appraisals Working Group (1987).

TABLE IV

AVERAGE "NO-SALE" PROBABILITY FOR TEA POLICY WITH VARIOUS ROLLBACK FACTORS[a]

	Results Assuming IPV		Our Results	
Rollback Factor	Point Estimates	95% Confidence Interval	Bound Estimates	95% Confidence Interval
0%	19.5%	[17.9%, 21.0%]	[27.1%, 37.7%]	[25.9%, 42.2%]
10%	14.0%	[12.7%, 15.3%]	[21.2%, 29.6%]	[20.1%, 33.5%]
20%	9.4%	[8.3%, 10.5%]	[15.8%, 21.9%]	[14.9%, 25.2%]
30%	5.8%	[3.6%, 8.0%]	[11.0%, 15.1%]	[10.3%, 17.5%]

[a] Reserve price given X = (expected transaction price given X) × (1 − rollback factor). Median values of $\frac{\text{reserve price}}{\text{appraisal value}}$ are 2.09, 1.89, 1.68, and 1.47, respectively.

then applying various rollback factors to that predicted value. Tables IV and V presents our results for this policy simulation.

Here we see a rather stark result. Allowing for the correlation in bidder values in our data, it appears that only at a 30% rollback factor does the no sale probability fall close to the mandated 15% level. In contrast, IPV analysis would suggest the mandate could be satisfied by as little as a 10% rollback policy. Moreover, under an IPV model, profits would appear to be increasing as the rollback factor falls all the way to 0; our bounds suggest that this is not the case. The conclusions of our correlated values analysis seem to agree with the Forest Service's approach to actually implementing TEA in Region 6. According to a U.S. Government Accountability Office analysis of timber sales under TEA between 1992 and 1996 (Meissner (1997)), the average rollback rate used in Region 6 was 31.1%, while the average rollback rates used in the six other

TABLE V

AVERAGE PER-AUCTION (UNSCALED) PROFITS FOR TEA POLICY WITH VARIOUS ROLLBACK FACTORS[a]

	Results Assuming IPV		Our Results	
Rollback Factor	Point Estimates	95% Confidence Interval	Bound Estimates	95% Confidence Interval
0%	72.75	[70.28, 75.21]	[61.15, 67.96]	[57.47, 70.26]
10%	71.82	[69.24, 74.41]	[63.99, 68.18]	[60.89, 70.53]
20%	70.04	[67.35, 72.73]	[65.58, 67.74]	[62.90, 70.11]
30%	67.90	[65.11, 70.69]	[66.05, 66.85]	[63.58, 69.25]
Baseline	65.10	[62.15, 68.05]	65.10	[62.15, 68.05]

[a] Profits are in dollars per thousand board-feet (mbf). For reference, the average auction in our data had a volume of 845 mbf and our sample comprised about 1 million mbf.

regions where TEA was implemented were between 7% and 20%. This is consistent with the prediction of our bounds: even at a rollback rate of 20%, the fraction of successful sales in Region 6 would fall below 85%.

Table V shows that the lower bound on profits increases with the rollback factor, at least up to 30%; in fact, the USFS appears to have seen the 30% cap on rollback factors as binding.[36] Thus, we might wonder whether even higher rollback rates—lower reserve prices than those implemented under TEA—would have been preferable, at least under our "pessimistic" (lower-bound) profits.

We can address this by explicitly calculating, for each auction X, the reserve price that maximizes the estimated lower bound on profit $\underline{\pi}(r|x)$, and compare this lower-bound profit maximization policy to the 30% TEA policy. We find that the reserve prices that maximize the lower bound on profit are on average very close to those implied by the TEA policy: the median ratio of reserve price to appraisal value under the former is 1.48, while under TEA with a 30% rollback, it is 1.47. Thus, given the technology available at the time, the Forest Service seems to have chosen the optimal level of "aggressiveness" for reserve prices.

However, using the same data, our policy of maximizing the lower bound on expected profit would allow the Forest Service to achieve better performance. Like the TEA policy, our policy would likely satisfy the 85% sales mandate: the probability of no sale would be between 13.5% and 16.9%. But our policy would give higher profits: the bounds on expected profit are [\$67.08, \$68.10], higher than the bounds of [\$66.05, \$66.85] under the TEA policy. Comparing these numbers to the benchmark policy of setting reserve price equal to appraisal value, our policy would have offered between a 3.0% and 4.6% increase in profits, while the TEA policy would have increased profits by 1.5% to 2.7%. Thus, while the two policies set similar levels of reserve prices on average, our methodology allows reserve prices to be more closely tailored to each individual auction, leading to higher expected profits.

5. CONCLUSION

Our paper presents techniques for identification and estimation of a general model of ascending auctions with correlated private values. We empirically analyze USFS timber data and find that even after controlling for a rich set of auction covariates, residual correlation persists among bidder valuations and has significant implications for seller and bidder payoffs as well as optimal behavior.

[36]The USFS rollback rate in Region 6 averaged 31.1%, even though rates above 30% were generally not allowed.

A key innovation in our analysis was to restrict attention to identifying only those primitives that were necessary to answer the relevant economic question of interest (rather than trying to identify all the primitives of the model). By doing so, we were able to nonparametrically identify bounds on bidder and seller payoffs in ascending auctions with correlated values even though the joint distribution of valuations is not identified. While the techniques presented here are tightly tailored to ascending auctions, we believe this general approach of tying the identification problem to the relevant economic policy question may have more widespread applicability.

One possible fruitful area to extend the existing results is bidder asymmetry. Our existing approach is not incompatible with asymmetry. For example, suppose, as described in Athey, Levin, and Seira (2011), that each bidder is either a large sawmill or a small logging firm, with mills having private values drawn from a different distribution than loggers. Ex ante, view each bidder as having the same, independent probability of being a mill, so that the marginal distribution of each bidder's valuation is a weighted average of the mill distribution and the logger distribution. The fact that each bidder's identity is realized and known to his opponents by the time bidding occurs would be important in a first-price auction, since equilibrium bids depend on beliefs about opponent valuations, but it is irrelevant in an ascending auction, where (by assumption) the outcome does not depend on bidders' beliefs about other bidders' valuations. Thus, under the assumption that the "type" of each bidder is an independent random event (and independent of N), our model extends to asymmetric bidders. If instead, say, the propensity of bidders to be mills rather than loggers were positively associated with N, and the mill distribution yielded stochastically higher valuations than the logger distribution, then valuations would be stochastically increasing in N and our weaker results (Theorem 2) would hold. Nevertheless, it may be a relevant empirical question to recover the type specific value distributions, and we leave this as a subject for future research.

APPENDIX A: EXTENSIONS AND PROOFS

A.1. *Extending Our Bounds to the Incomplete Model of Haile and Tamer*

Consider each bidder's bid to be the highest bid he makes during an auction. Let $B_{n:n} \geq B_{n-1:n} \geq \cdots \geq B_{1:n}$ be the order statistics of bids in an n-bidder auction, and let $G_{k:n}$ be the distribution of $B_{k:n}$. The bidding assumptions of Haile and Tamer (2003) are that bidders never bid more than their valuations and that no bidder allows an opponent to win at a price he would have been willing to outbid. This means that $V_{k:n} \geq B_{k:n}$ for every k, and $V_{n-1:n} \leq B_{n:n} + \Delta$, where Δ is the minimum bid increment at the end of the auction. Together, these imply that $F_{n-1:n}(v) \in [G^{\Delta}_{n:n}(v), G_{n-1:n}(v)]$, where $G^{\Delta}_{n:n}$ is the distribution of $B_{n:n} + \Delta$, and that $F_{n:n}(v) \leq G_{n:n}(v)$. Lemma 3 gives $F_{n:n}(v) \geq (\phi_n(F_{n-1:n}(v)))^n \geq (\phi_n(G^{\Delta}_{n:n}(v)))^n$.

As long as the top two bids and the minimum bid increment are recorded in the data, the distributions $G_{n-1:n}$, $G_{n:n}$, and $G^{\Delta}_{n:n}$ are identified. If we continue to assume that counterfactual auctions satisfy Assumption 2, then our bounds become

$$\underline{\pi}_n(r) \equiv \int_0^\infty \max\{r, v\}\, dG_{n-1:n}(v) - v_0 - \left(\sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} G_{m-1:m}(r) + \frac{n}{\bar{n}} G_{\bar{n}:\bar{n}}(r)\right)(r - v_0),$$

$$\overline{\pi}_n(r) \equiv \int_0^\infty \max\{r, v\}\, dG^{\Delta}_{n:n}(v) - v_0 - \left(\sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} G^{\Delta}_{m:m}(r) + \frac{n}{\bar{n}} \left(\phi_{\bar{n}}\left(G^{\Delta}_{\bar{n}:\bar{n}}(r)\right)\right)^{\bar{n}}\right)(r - v_0),$$

$$\underline{\mathrm{BS}}_n(r) \equiv \int_0^\infty \max\{r, v\}\, d\left(\sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} G_{m-1:m}(v) + \frac{n}{\bar{n}} G_{\bar{n}:\bar{n}}(v)\right) - \int_0^\infty \max\{r, v\}\, dG^{\Delta}_{n:n}(v),$$

$$\overline{\mathrm{BS}}_n(r) \equiv \int_0^\infty \max\{r, v\}\, d\left(\sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} G^{\Delta}_{m:m}(v) + \frac{n}{\bar{n}} \left(\phi_{\bar{n}}\left(G^{\Delta}_{\bar{n}:\bar{n}}(v)\right)\right)^{\bar{n}}\right) - \int_0^\infty \max\{r, v\}\, dG_{n-1:n}(v).$$

These come from plugging bounds on both $F_{n-1:n}$ and $F_{n:n}$ into (1) and (2). Since $\max\{r, v\}$ is increasing in v, its expected value increases with a first-order stochastic shift in the distribution of v, so the upper bound on $F_{n-1:n}$ gives the minimal possible value of $\int_0^\infty \max\{r, v\}\, dF_{n-1:n}(v)$, which is, therefore, used in $\underline{\pi}_n(r)$ and $\overline{\mathrm{BS}}_n(r)$; likewise the lower bound on $F_{n:n}$ gives the maximal value of $\int_0^\infty \max\{r, v\}\, dF_{n:n}(v)$, which is used in $\overline{\mathrm{BS}}_n(r)$. (Also note that it is because $\pi_n(r)$ and $\mathrm{BS}_n(r)$ are both additively separable into a term depending only on $F_{n-1:n}$ and another depending only on $F_{n:n}$ that we are able to plug in bounds on each marginal without considering their *joint* distribution.)

A.2. *Sufficient Conditions for V Stochastically Increasing in N*

Consider an environment of independent private values with one-dimensional unobserved heterogeneity. $\theta \in \Re$ varies across auctions (and is not observed by the analyst), valuations are i.i.d. $\sim F(\cdot|\theta)$, and $\theta > \theta'$ implies $F(\cdot|\theta) \succsim_{\mathrm{FOSD}} F(\cdot|\theta')$.

Within this environment, consider first the entry model of Levin and Smith (1996): $\bar{n}$ potential bidders observe θ but not their own valuations, then decide simultaneously whether to pay a cost c to participate in the auction. (Levin and Smith interpreted c as the cost of investigating the object and learning one's valuation.) For each realization of θ, bidders play a symmetric, mixed-strategy equilibrium in the entry game.

THEOREM A1: *If the equilibrium probability of entry is increasing in θ, then the entry game of Levin and Smith (1994) generates valuations stochastically increasing in N. A sufficient condition is for expected bidder surplus in an n-bidder auction to be weakly increasing in θ for each n.*

Second, consider the same environment, but under the model of Samuelson (1985): $\bar{n}$ potential bidders each observe both θ and their own valuation before deciding whether to enter at cost c. For each realization of θ, bidders play a symmetric, cutoff-strategy equilibrium in the entry game.

THEOREM A2: *If valuations and θ are related via the monotone likelihood ratio property (MLRP), that is, if $\frac{f(v|\theta)}{f(v|\theta')}$ is increasing in v for $\theta > \theta'$, then the entry game of Samuelson (1985) generates valuations stochastically increasing in N.*

Finally, consider a different environment, the "affiliated model of entry" studied by Marmer, Shneyerov, and Xu (2010). Potential bidders each get a signal S_i related to their (unobserved) valuation V_i before deciding whether to enter. Valuations and signals $(V_1, V_2, \dots, V_{\bar{n}}, S_1, S_2, \dots, S_{\bar{n}})$ are affiliated, and their distribution is symmetric in the obvious way.[37]

THEOREM A3: *If a symmetric equilibrium exists in cutoff strategies, then the entry game of Marmer, Shneyerov, and Xu (2010) generates valuations stochastically increasing in N.*[38]

PROOF OF THEOREMS A1–A3: For the Levin–Smith result, suppose θ is continuous; let $P(\cdot)$ denote its prior distribution, let p denote its density, and let $P(\cdot|N = n)$ denote its posterior distribution conditional on exactly n of the $\bar{n}$ potential bidders choosing to participate. For a given realization of θ, let q_θ be the equilibrium entry probability of each bidder (given the symmetric

[37] $f(V_1, V_2, \dots, V_{\bar{n}}, S_1, S_2, \dots, S_{\bar{n}}) = f(V_{\sigma(1)}, V_{\sigma(2)}, \dots, V_{\sigma(\bar{n})}, S_{\sigma(1)}, S_{\sigma(2)}, \dots, S_{\sigma(\bar{n})})$, where f is the joint distribution of values and signals, and $\sigma : \{1, 2, \dots, \bar{n}\} \to \{1, 2, \dots, \bar{n}\}$ is any permutation.

[38] Marmer, Shneyerov, and Xu assumed V_i and S_i are affiliated, but (V_i, S_i) is independent of (V_j, S_j) for $i \neq j$, which guarantees existence of a unique symmetric cutoff-strategy equilibrium. Here, we allow general affiliation among $(V_1, V_2, \dots, V_{\bar{n}}, S_1, S_2, \dots, S_{\bar{n}})$; our result holds for any symmetric cutoff equilibrium, provided one exists (which is not guaranteed).

mixed-strategy equilibrium of the entry game). We will show that q_θ increasing in θ implies $P(\cdot|N=n)$ decreasing in n. Bayes' law gives

$$P(\tilde{\theta}|n) = \frac{\displaystyle\int_{-\infty}^{\tilde{\theta}} p(\theta)\binom{\bar{n}}{n} q_\theta^n (1-q_\theta)^{\bar{n}-n}\,d\theta}{\displaystyle\int_{-\infty}^{+\infty} p(\theta)\binom{\bar{n}}{n} q_\theta^n (1-q_\theta)^{\bar{n}-n}\,d\theta} = R\left(\frac{\displaystyle\int_{\tilde{\theta}}^{+\infty} p(\theta)\binom{\bar{n}}{n} q_\theta^n (1-q_\theta)^{\bar{n}-n}\,d\theta}{\displaystyle\int_{-\infty}^{\tilde{\theta}} p(\theta)\binom{\bar{n}}{n} q_\theta^n (1-q_\theta)^{\bar{n}-n}\,d\theta}\right),$$

where $R(x) \equiv \frac{1}{1+x}$. Consider the change to the argument of R as n increases: the integrand in the numerator gets multiplied by $\frac{q_\theta}{1-q_\theta} \geq \frac{q_{\tilde{\theta}}}{1-q_{\tilde{\theta}}}$, while the integrand in the denominator gets multiplied by $\frac{q_\theta}{1-q_\theta} \leq \frac{q_{\tilde{\theta}}}{1-q_{\tilde{\theta}}}$, so the argument of R increases; since R is decreasing, this means $P(\tilde{\theta}|N=n)$ is decreasing in n, so $n > n'$ implies $P(\cdot|N=n) \succsim_{\text{FOSD}} P(\cdot|N=n')$. (With discrete θ, the same proof applies with sums replacing integrals.) Since $F^n_{m:m}(v) = E_{\theta|N=n}\{F^m(v|\theta)\}$ and (by assumption) $F^m(v|\theta)$ is decreasing in θ, this then implies $F^n_{m:m}(v) \leq F^{n'}_{m:m}(v)$, so valuations are stochastically increasing in N.

For the sufficient condition, let $u_n(\theta)$ denote each bidder's expected surplus in an n-bidder auction given a realization θ. q_θ is the value of q that solves $c = \sum_{n=0}^{\bar{n}-1}[\binom{\bar{n}-1}{n} q^n (1-q)^{\bar{n}-1-n}] u_{n+1}(\theta)$. Since $u_n(\theta)$ is decreasing in n, if it is increasing in θ for each n, then the right-hand side is increasing in θ and decreasing in q, which would make the solution q_θ increasing in θ.

For the Samuelson result, the same logic from above—higher θ means more entry, therefore higher N means higher θ, which gives the result—still holds, but with two changes. Let $v^*(\theta)$ denote the equilibrium entry threshold at a realized value of θ; then $1 - F(v^*(\theta)|\theta)$ replaces q_θ as each potential bidder's entry probability, and $F(v|\theta, v > v^*(\theta)) = \frac{F(v|\theta) - F(v^*(\theta)|\theta)}{1-F(v^*(\theta)|\theta)}$ replaces $F(v|\theta)$ as the distribution of valuations among those bidders who enter. $v^*(\theta)$ is the solution to $c = (v-r)(F(v|\theta))^{\bar{n}-1}$; since $F(v|\theta)$ is decreasing in θ, $v^*(\theta)$ must be increasing in θ and $F(v^*(\theta)|\theta)$ must be decreasing in θ, so the entry probability is increasing in θ and so (as above) $P(\theta|N=n)$ is decreasing in n. What remains is to show that for $\theta > \theta'$, $\frac{F(\cdot|\theta) - F(v^*(\theta)|\theta)}{1-F(v^*(\theta)|\theta)} \succsim_{\text{FOSD}} \frac{F(\cdot|\theta') - F(v^*(\theta')|\theta')}{1-F(v^*(\theta')|\theta')}$.

The MLRP implies that $\frac{f(\cdot|\theta)}{1-F(\cdot|\theta)}$ is decreasing in θ and, therefore, that $\log(1 - F(v|\theta))$ has increasing differences in v and θ. This means that for $\theta > \theta'$, $\log(1 - F(v|\theta)) - \log(1 - F(v|\theta'))$ is increasing in v or $\log(\frac{1-F(v|\theta)}{1-F(v|\theta')})$ is increasing in v or $\frac{1-F(v|\theta)}{1-F(v|\theta')}$ is increasing in v. Since $v^*(\theta)$ is increasing in θ, then, for any $v > v^*(\theta)$,

$$\frac{1-F(v|\theta)}{1-F(v|\theta')} \geq \frac{1-F(v^*(\theta)|\theta)}{1-F(v^*(\theta)|\theta')} \geq \frac{1-F(v^*(\theta)|\theta)}{1-F(v^*(\theta')|\theta')},$$

from which we get

$$\begin{aligned}\frac{F(v|\theta)-F(v^*(\theta)|\theta)}{1-F(v^*(\theta)|\theta)} &= 1-\frac{1-F(v|\theta)}{1-F(v^*(\theta)|\theta)}\\ &\leq 1-\frac{1-F(v|\theta')}{1-F(v^*(\theta')|\theta')}\\ &= \frac{F(v|\theta')-F(v^*(\theta')|\theta')}{1-F(v^*(\theta')|\theta')}\end{aligned}$$

and, therefore, $F(v|\theta, v > v^*(\theta)) \succsim_{\text{FOSD}} F(v|\theta', v > v^*(\theta'))$, giving the result.

Finally, for the model of Marmer Shneyerov, and Xu, let e_i be an indicator function taking value 1 if bidder i participates in the auction and 0 otherwise. If a symmetric cutoff-strategy equilibrium exists with cutoff $\bar{s}$, then $e_i = \mathbf{1}\{S_i \geq \bar{s}\}$ is a nondecreasing transformation of S_i, so $(V_1, \ldots, V_{\bar{n}}, e_1, \ldots, e_{\bar{n}})$ are affiliated. Given symmetry, we can write $F^n_{m:m}(v)$ as

$$\Pr(V_1, V_2, \ldots, V_m \leq v | e_1, e_2, \ldots, e_n = 1, e_{n+1}, \ldots, e_{\bar{n}} = 0).$$

Since $\Pr(V_1, V_2, \ldots, V_m \leq v)$ is the expected value of a function that is decreasing in every V_i, under affiliation, it is decreasing in each e_i, and so $F^n_{m:m}(v) \leq F^{n'}_{m:m}(v)$ for $n > n'$. *Q.E.D.*

A.3. *Using More Bids to Tighten the Bounds*

The bounds presented in the text rely only on observation of the distribution $F_{n-1:n}$ for various values of n; the bounds in Appendix A.1 rely only on observation of $G_{n-1:n}$, $G_{n:n}$, and $G^{\Delta}_{n:n}$. However, if other losing bids are observed, their distributions $G_{k:n}$ could be used to tighten the lower bound on $F_{n:n}$, and, therefore, the upper bounds on π_n and BS_n. For example, under the assumption (from Haile and Tamer (2003)) that each bidder's bid is no higher than his valuation, implying that $V_{k:n} \leq B_{k:n}$ for each k, here is how the losing bid distributions $G_{n-2:n}$ and $G_{n-3:n}$ would yield new, possibly tighter lower bounds on $F_{n:n}$:

THEOREM A4: *Under Assumptions 1 and 3, and the bidding assumptions of Haile and Tamer (2003), $F_{n:n}(v) \geq z_1$ and $F_{n:n}(v) \geq z_2$, where z_1 and z_2 are the*

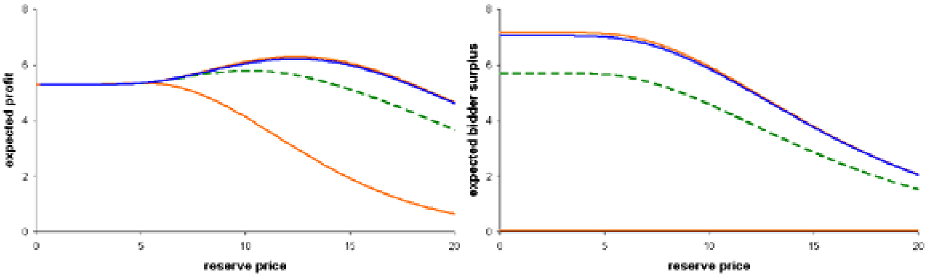

FIGURE 8.—Tightening the upper bounds on $\pi_3(r)$ and $\mathrm{BS}_3(r)$ using all bids.

unique solutions to

$$\frac{z_1}{(G^{\Delta}_{n:n}(v)-z_1)^2}=\frac{n-1}{2n}\frac{1}{G_{n-2:n}(v)-G^{\Delta}_{n:n}(v)},$$

$$\frac{(z_2)^2}{(G^{\Delta}_{n:n}(v)-z_2)^3}=\frac{(n-1)(n-2)}{6n^2}\times\frac{1}{G_{n-3:n}(v)-G^{\Delta}_{n:n}(v)-\frac{n-1}{2n}\frac{1}{z_2}(G^{\Delta}_{n:n}(v)-z_2)^2}.$$

Since both left-hand sides are strictly increasing in the variable z_1 or z_2, and both right-hand sides are either constant or decreasing, they define unique values for z_1 and z_2. Each additional losing bid would similarly provide a new lower bound on $F_{n:n}$.

To see the effect such bounds would have, we return to the example discussed in the text. Figure 8 shows actual expected profit and bidder surplus (dashed line), the upper and lower bounds that come from exact knowledge of the distribution $F_{2:3}$ (light solid lines), and the upper bound calculated using the bound z_1 from Theorem A4 (dark solid line), if the lowest bidder in a three-bidder auction bid all the way up to his valuation ($B_{1:3}=V_{1:3}$ so $G_{1:3}=F_{1:3}$).

As we can see, the incorporation of the additional losing bid gives only a very slight improvement to the upper bounds on π_3 and BS_3 calculated only from $F_{2:3}$. And this is under the best-case assumption that the third-highest bidder bids his true valuation; if the third-highest bidder routinely bid strictly lower than his value, the upper bound would be even higher and would not improve even on the existing bound. (The improvement from using $F_{2:4}$ when $N=4$ is similarly small.) In addition, while losing bids could potentially tighten the upper bound, they cannot improve the lower bounds on π_n or BS_n at all.[39] Thus,

[39]Both lower bounds depend on the upper bound on $F_{n:n}$. Regardless of the distribution of losing bids in the data, the bidding assumptions of Haile and Tamer (2003) are consistent with

in our application, we choose to ignore the additional information contained in losing bids, and use only the transaction prices of past auctions.[40]

PROOF OF THEOREM A4: The proof of Lemma 3 below will show that under Assumption 3, $\frac{\binom{n}{k+1}^{-1}(F_{n-(k+1):n}(v)-F_{n-k:n}(v))}{\binom{n}{k}^{-1}(F_{n-k:n}(v)-F_{n-(k-1):n}(v))}$ is increasing in k. This means

$$\frac{\binom{n}{2}^{-1}(F_{n-2:n}(v)-F_{n-1:n}(v))}{\frac{1}{n}(F_{n-1:n}(v)-F_{n:n}(v))} \geq \frac{\frac{1}{n}(F_{n-1:n}(v)-F_{n:n}(v))}{F_{n:n}(v)}.$$

Under the assumptions of Haile and Tamer (2003), $F_{n-1:n}(v) \geq G^{\Delta}_{n:n}(v)$ and $F_{n-2:n}(v) \leq G_{n-2:n}(v)$, so

$$\begin{aligned} \frac{F_{n:n}(v)}{\frac{1}{n^2}(G^{\Delta}_{n:n}(v)-F_{n:n}(v))^2} &\geq \frac{F_{n:n}(v)}{\frac{1}{n^2}(F_{n-1:n}(v)-F_{n:n}(v))^2} \\ &\geq \frac{\binom{n}{2}}{F_{n-2:n}(v)-F_{n-1:n}(v)} \geq \frac{\binom{n}{2}}{G_{n-2:n}(v)-G^{\Delta}_{n:n}(v)}. \end{aligned}$$

Since $\frac{x}{(1/n^2)(G^{\Delta}_{n:n}(v)-x)^2}$ is strictly increasing in x, the outer inequality gives a lower bound on $F_{n:n}(v)$.

For the second result,

$$\begin{aligned} \frac{\binom{n}{3}^{-1}(F_{n-3:n}(v)-F_{n-2:n}(v))}{\binom{n}{2}^{-1}(F_{n-2:n}(v)-F_{n-1:n}(v))} &\geq \frac{\binom{n}{2}^{-1}(F_{n-2:n}(v)-F_{n-1:n}(v))}{\binom{n}{1}^{-1}(F_{n-1:n}(v)-F_{n:n}(v))} \\ &\geq \frac{\binom{n}{1}^{-1}(F_{n-1:n}(v)-F_{n:n}(v))}{\binom{n}{0}^{-1}F_{n:n}(v)} \end{aligned}$$

---

every bidder's valuation in each auction being equal to the transaction price in that auction, in which case $F_{n:n} = G_{n:n}$.

[40]There are two further arguments against incorporating losing bids in the analysis. First, if we did, then the lower bound on $F_{n:n}$ would be the pointwise maximum of multiple distinct bounds, leading to a bias in estimation. Haile and Tamer (2003) confronted and dealt with this issue, but it complicates the analysis. And second, due to the difference terms in the denominators of the expressions in Theorem A4, we worry these bounds could be unstable on moderately sized samples.

implies $\frac{\binom{n}{3}^{-1}(F_{n-3:n}(v)-F_{n-2:n}(v))}{\binom{n}{1}^{-1}(F_{n-1:n}(v)-F_{n:n}(v))} \geq (\frac{\binom{n}{1}^{-1}(F_{n-1:n}(v)-F_{n:n}(v))}{\binom{n}{0}^{-1}F_{n:n}(v)})^2$. Letting $x = F_{n:n}(v)$, this is

$$\frac{x^2}{\left(\frac{1}{n}(F_{n-1:n}(v)-x)\right)^3} \geq \frac{\binom{n}{3}}{F_{n-3:n}(v)-F_{n-2:n}(v)}.$$

We get a lower bound for $F_{n-2:n}(v)$ from $\frac{\binom{n}{2}^{-1}(F_{n-2:n}(v)-F_{n-1:n}(v))}{(1/n)(F_{n-1:n}(v)-x)} \geq \frac{(1/n)(F_{n-1:n}(v)-x)}{x}$, which gives

$$\begin{aligned} F_{n-2:n}(v) &\geq F_{n-1:n}(v) + \binom{n}{2}\frac{\left(\frac{1}{n}(F_{n-1:n}(v)-x)\right)^2}{x} \\ &\geq G^{\Delta}_{n:n}(v) + \binom{n}{2}\frac{\left(\frac{1}{n}(G^{\Delta}_{n:n}(v)-x)\right)^2}{x}. \end{aligned}$$

Combining $\frac{x^2}{((1/n)(F_{n-1:n}(v)-x))^3} \geq \frac{\binom{n}{3}}{F_{n-3:n}(v)-F_{n-2:n}(v)}$ with $F_{n-1:n}(v) \geq G^{\Delta}_{n:n}(v)$, $F_{n-3:n}(v) \leq G_{n-3:n}(v)$, and $F_{n-2:n}(v) \geq G^{\Delta}_{n:n}(v) + \binom{n}{2}\frac{((1/n)(G^{\Delta}_{n:n}(v)-x))^2}{x}$ gives

$$\begin{aligned} &\frac{x^2}{\left(\frac{1}{n}(G^{\Delta}_{n:n}(v)-x)\right)^3} \\ &\quad \geq \frac{\binom{n}{3}}{G_{n-3:n}(v) - \left(G^{\Delta}_{n:n}(v) + \binom{n}{2}\frac{\left(\frac{1}{n}(G^{\Delta}_{n:n}(v)-x)\right)^2}{x}\right)}. \end{aligned}$$

The left-hand side is again strictly increasing in x, and the right-hand side is strictly decreasing in x, giving the result. *Q.E.D.*

A.4. *Proof of Lemma 1*

Let $V_{-i} = (V_1, \dots, V_{i-1}, V_{i+1}, \dots, V_n)$. If values are affiliated, then $\Pr(V_i < v|V_{-i})$ is decreasing in V_{-i}, since it is the expected value of $\mathbf{1}\{V_i < v\}$, a decreasing function of V_i; Assumption 3 follows.

For the other two models, suppose values are i.i.d. $\sim F(\cdot|\theta)$.[41] Let P be the prior distribution of θ, let p be its density, and let all integrals below be over the support of p. Fix v; conditional on exactly m out of $m+k$ bidders having valuations weakly less than v, Bayes' law puts posterior density

$$p(\theta|m,k)=\frac{p(\theta)\binom{m+k}{m}(F(v|\theta))^m(1-F(v|\theta))^k}{\int p(\theta')\binom{m+k}{m}(F(v|\theta'))^m(1-F(v|\theta'))^k\,d\theta'}$$

on the value of θ, so conditional on m bidders having valuations below v and k having values above, the probability that the next bidder has value below v is

$$\begin{aligned}\Pr(V_i<v|m,k)&=\int p(\theta|m,k)F(v|\theta)\,d\theta\\&=\frac{\int p(\theta)(F(v|\theta))^{m+1}(1-F(v|\theta))^k\,d\theta}{\int p(\theta')(F(v|\theta'))^m(1-F(v|\theta'))^k\,d\theta'}.\end{aligned}$$

Next, let $q(\theta)=p(\theta)(F(v|\theta))^{m-1}(1-F(v|\theta))^k\,d\theta$, $\bar{q}=\int q(\theta)\,d\theta$, and $\tilde{q}(\theta)=\frac{q(\theta)}{\bar{q}}$. Since $q(\theta)$ is everywhere nonnegative, so is $\tilde{q}(\theta)$; since $\tilde{q}(\cdot)$ (by construction) integrates to 1, it is a density function. Letting $\tilde{\theta}$ be a random variable with density $\tilde{q}$, we can write

$$\begin{aligned}0&\le \operatorname{Var}\big(F(v|\tilde{\theta})\big)\\&=\int \big(F(v|\theta)\big)^2\tilde{q}(\theta)\,d\theta-\left(\int F(v|\theta)\tilde{q}(\theta)\,d\theta\right)^2\\&=\frac{1}{\bar{q}^2}\left(\int \big(F(v|\theta)\big)^2 q(\theta)\,d\theta\int q\big(\theta'\big)\,d\theta'-\left(\int F(v|\theta)q(\theta)\,d\theta\right)^2\right)\\&=\frac{\int q(\theta')\,d\theta'\int F(v|\theta)q(\theta)\,d\theta}{\bar{q}^2}\\&\quad\times\left(\frac{\int (F(v|\theta))^2q(\theta)\,d\theta}{\int F(v|\theta)q(\theta)\,d\theta}-\frac{\int F(v|\theta)q(\theta)\,d\theta}{\int q(\theta)\,d\theta}\right)\end{aligned}$$

[41]The two models differ in whether bidders observe only V_i or both θ and V_i, but this does not affect the joint distribution of $\{V_i\}$.

$$= \frac{\int F(v|\theta)q(\theta)\,d\theta}{\bar{q}} \times \left(\frac{\int p(\theta)(F(v|\theta))^{m+1}(1-F(v|\theta))^k\,d\theta}{\int p(\theta')(F(v|\theta'))^{m}(1-F(v|\theta'))^k\,d\theta'} - \frac{\int p(\theta)(F(v|\theta))^{m}(1-F(v|\theta))^k\,d\theta}{\int p(\theta')(F(v|\theta'))^{m-1}(1-F(v|\theta'))^k\,d\theta'} \right).$$

This means that $\frac{\int p(\theta)(F(v|\theta))^{m+1}(1-F(v|\theta))^k\,d\theta}{\int p(\theta')(F(v|\theta'))^{m}(1-F(v|\theta'))^k\,d\theta'} \geq \frac{\int p(\theta)(F(v|\theta))^{m}(1-F(v|\theta))^k\,d\theta}{\int p(\theta')(F(v|\theta'))^{m-1}(1-F(v|\theta'))^k\,d\theta'}$, meaning that $\Pr(V_i < v|m, k)$ (the probability $V_i < v$ given m bidders have values below v and k have values above v) is increasing in m. By exactly analogous steps, we can show it decreasing in k, and so $\Pr(V_i < v|m, n-1-m) \geq \Pr(V_i < v|m-1, n-m)$, which is exactly Assumption 3.

A.5. *Proof of Lemma 2*

Given equation (1) in the text, if $F_{n:n}(r) \geq F^L_{n:n}(r)$, then

$$\begin{aligned}\pi_n(r) &= \int_0^\infty \max\{r, v\}\,dF_{n-1:n}(v) - v_0 - F_{n:n}(r)(r - v_0)\\ &\leq \int_0^\infty \max\{r, v\}\,dF_{n-1:n}(v) - v_0 - F^L_{n:n}(r)(r - v_0) = \pi^U_n(r).\end{aligned}$$

If $F_{n:n}(v) \geq F^L_{n:n}(v)$ for every v, then $F^L_{n:n} \succsim_{\text{FOSD}} F_{n:n}$, so $\int g(v)\,dF^L_{n:n}(v) \geq \int g(v)\,dF_{n:n}(v)$ for any nondecreasing function g. Then by (2),

$$\begin{aligned}\text{BS}_n(r) &= \int_0^\infty \max\{r, v\}\,dF_{n:n}(v) - \int_0^\infty \max\{r, v\}\,dF_{n-1:n}(v)\\ &\leq \int_0^\infty \max\{r, v\}\,dF^L_{n:n}(v) - \int_0^\infty \max\{r, v\}\,dF_{n-1:n}(v) = \text{BS}^U_n(r).\end{aligned}$$

Analogous arguments show that if $F_{n:n}(v) \leq F^U_{n:n}(v)$ for every v, then $\pi_n(r) \geq \pi^L_n(r)$ and $\text{BS}_n(r) \geq \text{BS}^L_n(r)$.

Let $r^* = \arg\max \pi_n(r)$ and $r_* = \arg\max \pi^L_n(r)$. If $\pi^L_n(r) \leq \pi_n(r) \leq \pi^U_n(r)$ for every r, then $\max_r \pi^U_n(r) \geq \pi^U_n(r^*) \geq \pi_n(r^*) = \max_r \pi_n(r)$ and $\max_r \pi^L_n(r) = \pi^L_n(r_*) \leq \pi_n(r_*) \leq \max_r \pi_n(r)$. Finally, similar to the proof of Theorem 4 in

Haile and Tamer (2003),

$$\pi_n^U\bigl(r^*\bigr) \geq \pi_n\bigl(r^*\bigr) \geq \pi_n(r_*) \geq \pi_n^L(r_*) = \max_{r'} \pi_n^L\bigl(r'\bigr),$$

so $r^* \in \{r : \pi_n^U(r) \geq \max_{r'} \pi_n^L(r')\}$.

A.6. *Proof of Lemma 3*

The proof is as in Quint (2008). By definition, $V_{n:n} \geq V_{n-1:n}$, so $F_{n:n}(v) \leq F_{n-1:n}(v)$. For the lower bound, fix n and v, and let $P_k^n = F_{n-k:n}(v) - F_{n-k+1:n}(v)$ be the probability that exactly k of n bidders have valuations above v ($P_0^n = F_{n:n}(v)$). Given symmetry,

$$\begin{aligned}
\frac{\binom{n}{k+1}^{-1} P_{k+1}^n}{\binom{n}{k}^{-1} P_k^n} &= \Pr(V_1, \ldots, V_k > v; V_{k+1}, \ldots, V_{n-1} \leq v) \\
&\quad \times \Pr(V_n > v | V_1, \ldots, V_k > v; V_{k+1}, \ldots, V_{n-1} \leq v) \\
&\quad /\bigl(\Pr(V_1, \ldots, V_k > v; V_{k+1}, \ldots, V_{n-1} \leq v) \\
&\quad \times \Pr(V_n \leq v | V_1, \ldots, V_k > v; V_{k+1}, \cdots, V_{n-1} \leq v)\bigr) \\
&= \frac{\Pr(V_n > v | V_1, \ldots, V_k > v; V_{k+1}, \ldots, V_{n-1} \leq v)}{\Pr(V_n \leq v | V_1, \ldots, V_k > v; V_{k+1}, \cdots, V_{n-1} \leq v)},
\end{aligned}$$

which by Assumption 3 is weakly increasing in k.

Next, let $p = \phi_n(F_{n-1:n}(v))$, so that $F_{n-1:n}(v) = \phi_n^{-1}(p) = np^{n-1} - (n-1)p^n$. We will show that $P_0^n \geq p^n$. Suppose not.

Note that $F_{n-1:n}(v) = P_0^n + P_1^n$. So if $P_0^n < p^n$, then

$$\begin{aligned}
P_1^n &= \bigl(np^{n-1} - (n-1)p^n\bigr) - P_0^n \\
&> \bigl(np^{n-1} - (n-1)p^n\bigr) - p^n = np^{n-1}(1-p)
\end{aligned}$$

and so

$$\frac{(1/n)P_1^n}{P_0^n} > \frac{p^{n-1}(1-p)}{p^n} = \frac{1-p}{p}.$$

Since

$$\frac{\binom{n}{2}^{-1} P_2^n}{\binom{n}{1}^{-1} P_1^n} \geq \frac{\binom{n}{1}^{-1} P_1^n}{\binom{n}{0}^{-1} P_0^n} > \frac{1-p}{p}$$

and $P_1^n > np^{n-1}(1-p)$, we get $\binom{n}{2}^{-1} P_2^n > p^{n-2}(1-p)^2$.

Likewise

$$\frac{\binom{n}{3}^{-1} P_3^n}{\binom{n}{2}^{-1} P_2^n} \geq \frac{\binom{n}{2}^{-1} P_2^n}{\binom{n}{1}^{-1} P_1^n} > \frac{1-p}{p},$$

so $\binom{n}{3}^{-1} P_3^n > p^{n-3}(1-p)^3$, and so on.

This gives us $P_1^n + P_0^n = np^{n-1} - (n-1)p^n = \binom{n}{1} p^{n-1}(1-p) + \binom{n}{0} p^n$ and $P_k^n > \binom{n}{k} p^{n-k}(1-p)^k$ for $k > 1$; together, these imply $\sum_{k=0}^n P_k^n > \sum_{k=0}^n \binom{n}{k} p^{n-k}(1-p)^k = 1$, a contradiction, proving $P_0^n \geq p^n$.

For sharpness of the lower bound, note that an IPV model satisfies Assumptions 1 and 3, and gives $(F_{n:n}(v))^{1/n} = F_V(v) = \phi_n(F_{n-1:n}(v))$ (where F_V is the marginal distribution of a V_i) and, therefore, $F_{n:n}(v) = (\phi_n(F_{n-1:n}(v)))^n$. For sharpness of the upper bound, consider a conditionally independent private values model with $\theta \sim F_{n-1:n}$ and $V_i = \theta$ for all i; this too satisfies Assumptions 1 and 3, and gives $F_{n:n} = F_{n-1:n}$.

A.7. *Proof of Lemmas 4 and 5*

We prove Lemma 5 by induction on $\bar{n}$. The base case ($\bar{n} = n+1$) is simply

$$F_{n:n}(v) \geq \frac{1}{n+1} F_{n:n+1}(v) + \frac{n}{n+1} F_{n+1:n+1}(v),$$

which is implied by $F_{n:n}^{n+1} \succsim_{\mathrm{FOSD}} F_{n:n}$ and (4). For the inductive step, if $F_{n:n}(v) \geq \sum_{m=n+1}^{K} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{K} F_{K:K}(v)$, then

$$\begin{aligned}
&\sum_{m=n+1}^{K+1} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{K+1} F_{K+1:K+1}(v) \\
&\quad = \sum_{m=n+1}^{K} \frac{n}{(m-1)m} F_{m-1:m}(v) \\
&\qquad + \frac{n}{K(K+1)} F_{K:K+1}(v) + \frac{n}{K+1} F_{K+1:K+1}(v) \\
&\quad = \sum_{m=n+1}^{K} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{K} F_{K:K}(v) - \frac{n}{K} F_{K:K}(v) \\
&\qquad + \frac{n}{K(K+1)} F_{K:K+1}(v) + \frac{n}{K+1} F_{K+1:K+1}(v)
\end{aligned}$$

$$
\begin{aligned}
&\le F_{n:n}(v) - \frac{n}{K} F_{K:K}(v) + \frac{n}{K(K+1)} F_{K:K+1}(v) + \frac{n}{K+1} F_{K+1:K+1}(v) \\
&= F_{n:n}(v) + \frac{n}{K}\left(-F_{K:K}(v) + \frac{1}{K+1} F_{K:K+1}(v) + \frac{K}{K+1} F_{K+1:K+1}(v)\right) \\
&\le F_{n:n}(v),
\end{aligned}
$$

with the last inequality coming again from $F_{K:K}(v) \ge F^{K+1}_{K:K}(v) = \frac{1}{K+1} F_{K:K+1}(v) + \frac{K}{K+1} F_{K+1:K+1}(v)$. Lemma 4 is the same proof, with all inequalities replaced by equalities.

A.8. *Proof of Theorems 1 and 2*

As noted in the text, if valuations are independent of N, Lemmas 3 and 4 together give

$$
\begin{aligned}
F_{n:n}(v) &= \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{\bar{n}} F_{\bar{n}:\bar{n}}(v) \\
&\ge \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m} F_{m-1:m}(v) + \frac{n}{\bar{n}} \big(\phi_{\bar{n}}\big(F_{\bar{n}-1:\bar{n}}(v)\big)\big)^{\bar{n}} \\
&= \underline{F}_{n:n}(v)
\end{aligned}
$$

and likewise $F_{n:n}(v) \le \overline{F}_{n:n}(v)$. Applying Lemma 2 with $F^L_{n:n} = \underline{F}_{n:n}$ and $F^U_{n:n} = \overline{F}_{n:n}$ gives Theorem 1. If valuations are stochastically increasing in N, then Lemmas 3 and 5 together give $F_{n:n}(v) \ge \underline{F}_{n:n}(v)$ for all v; applying Lemma 2 with $F^L_{n:n} = \underline{F}_{n:n}$ (and $F^U_{n:n} = 1$) gives $\pi_n(r) \le \overline{\pi}_n(r)$ and $\mathrm{BS}_n(r) \le \overline{\mathrm{BS}}_n(r)$.

A.9. *Graphs Omitted From Text*

Figure 9 shows our estimated bounds, IPV point estimates, and confidence intervals for $\pi(\cdot|X)$ and $\mathrm{BS}(\cdot|X)$ for various values of X. The third row reproduces Figures 4 and 6, the results for the auction characterized by the pointwise median of X in the data. The other rows give other percentiles of X. Those covariates that are negatively correlated with transaction price are taken in the opposite order as those that are positively correlated with transaction price: so the auction $X^{(90)}$ is defined by the 90th percentile values of SALEVAL, CONCENTR, and APPRAISAL, and the 10th percentile of MFGCOST, HARVCOST, and INVENTORY, representing a "high value" auction. Figure 10 shows $\mathrm{BS}_n(\cdot|x)$ for the pointwise median of X and various values of n.

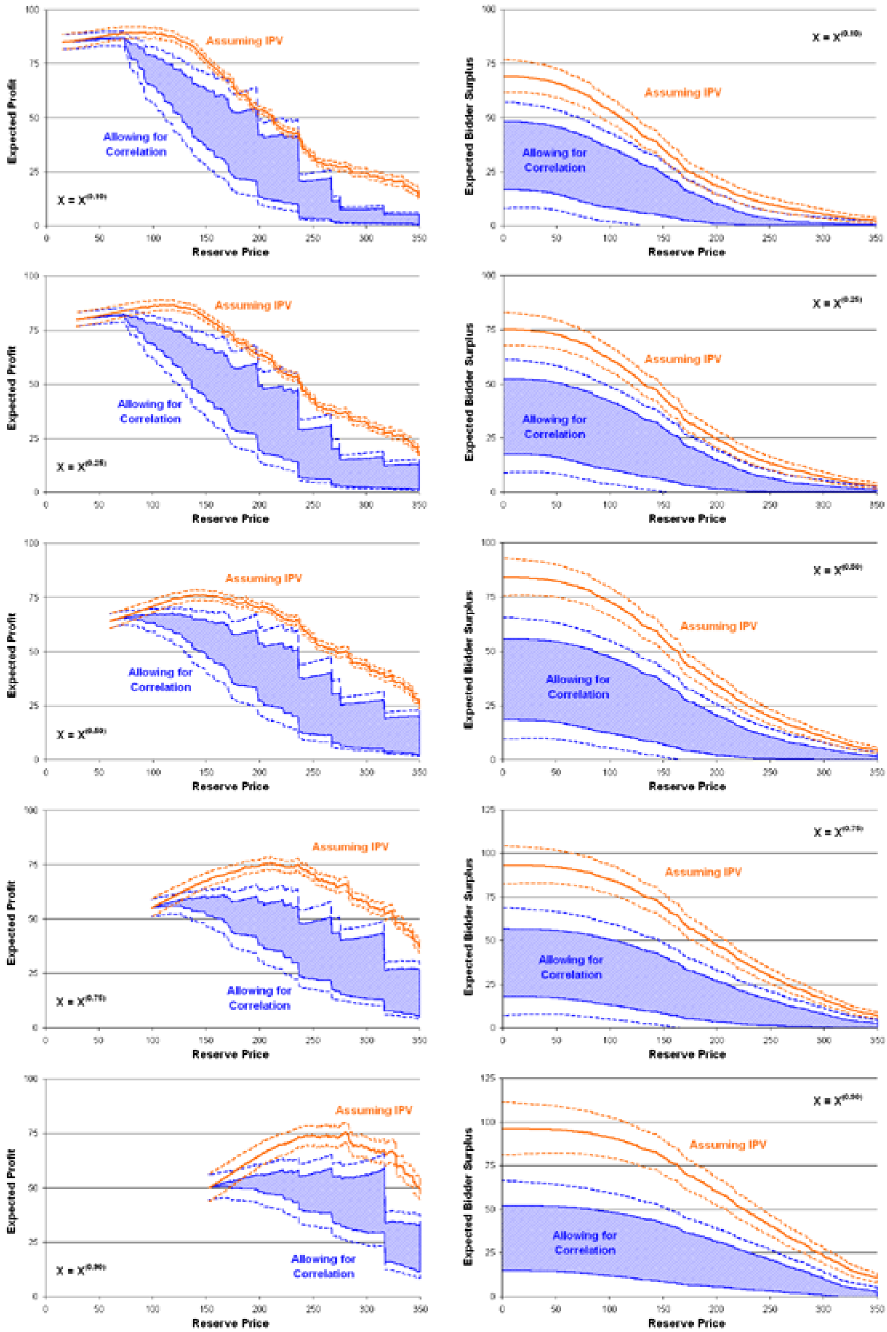

FIGURE 9.—Expected profit and bidder surplus, unconditional on N, for various percentiles of X.

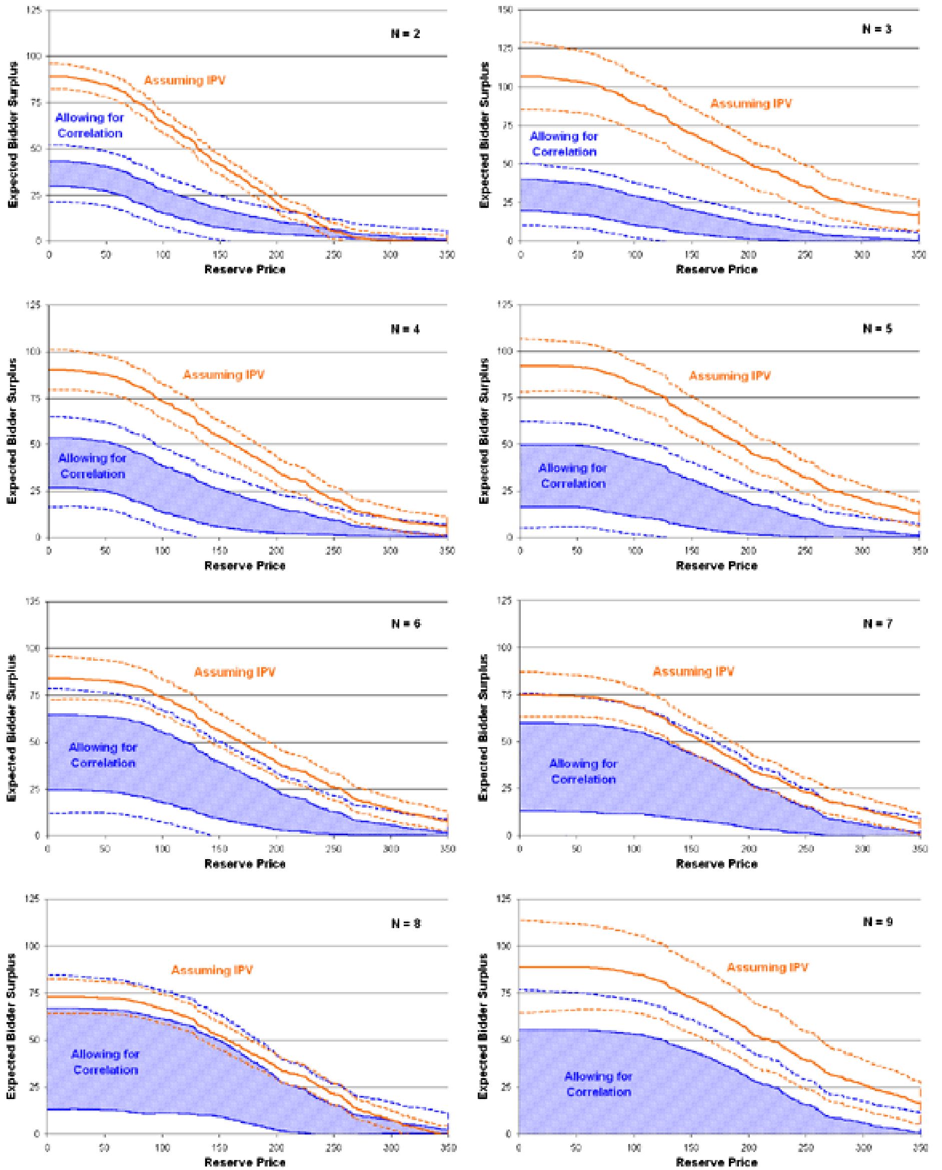

FIGURE 10.—Expected winning bidder's surplus for median X, various N.

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Manuscript received July, 2010; final revision received July, 2012.