QUANTIFYING

A Story of Mathematics and Me

Amogh Arakali

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by

Amogh Arakali
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***Note 1**: There are two ways to read this rather long essay of nearly 10,000 words. One, if you have enough time, start from the beginning and work your way to the end. Two, if you're pressed for time, jump to Section 5 "Reflections" for the heart of my essay and then read through the other sections in your own preferred order.*

***Note 2**: The Prologue may be distressing for some people. I promise you that things get better, but if you continue to feel uncomfortable, please feel free to skip to Part 1.*

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Prologue

On a quiet evening in 2003, I thought I was going to die. My breath held itself in my chest, refusing to come out. My throat squeezed, compressing the space inside, willing the air in my lungs to emerge. My eyes shut tight. My body heaved up and down in quick, short movements, trying to claw back some state of normalcy. Eventually, I began to gasp, my breath coming out in sporadic bursts, each burst accompanied by a deep, guttural sound as air stuttered through my vocal cords. Tears burst from my eyes. Finally, I collapsed into a foetal position on the floor, sobbing without control.

Dramatic as this incident may sound, the cause was quite mundane — routine, really. Earlier that day, I had received yet another Progress Report from my school. With her usual caustic wit, my teacher had remarked "Shall I call it a Lack of Progress Report?" as she handed the document to me. I had failed to improve my Mathematics scores for the umpteenth time. There were only a few months to go before I sat for the Class 10 CBSE Board Examinations, the results of which were crucial for my career prospects. My future, at age fourteen, felt shaky.

Now I was sitting at home, alone, waiting for my parents to return. I knew we'd repeat the same dull process that took place after every examination. I would start the evening with heated exchanges, move on to a night of shame and self-flagellation, and end with a morning of apologies topped by a promise to do better next time. This was a promise I always intended to keep, but had no idea how. And in yet another turning of the same wheel, I'd experiment with new methods, fail, panic, hastily adopt someone else's practices, fail again, and write another examination knowing fully well what the results were going to be. As I sat there that evening, I had a sudden vision of this process repeating itself for years, no matter where I went or what I did. I saw myself in the future, losing job after job, opportunity after opportunity, before being driven into penury and worse. This was when I had my panic attack.

By the time my parents returned home that evening, I had begun to pull myself together. However, I was still shaken. I couldn't understand what had happened to me. "Surely..." I thought to myself. "...surely, it's not normal to react like this to a set of test scores." In my experience, few of my fellow-students took test results this badly. Bad scores were usually met in the classroom with blank faces and silent perusals of the scored papers. After a few minutes, most people would resume their normal selves. Some would even crack jokes or flash idiotic grins at the exasperated teacher. Even those who cried did so quietly, almost stoically. As far as I knew, no one went to pieces and descended to hysterics like I had that evening. I began to wonder if there was something wrong with me.

As my parents began to settle in, I knew what I had to say. I wanted to tell them I was broken. I did not know what was wrong with me and I did not know how to fix myself. I needed help and not just with exams. As I went into my parents' room to hand them my Report, I wanted to ask for counselling. Not just for my poor performance in Mathematics —

I was convinced my problem lay somewhere much deeper. Perhaps a part of me was askew. Perhaps I was born with a genetic defect and required medical help. At the root of it all was a single sentence laced with frustration “Why can’t I solve mathematical problems?”

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Part 1: “Why Can’t I See What You’re Seeing?”

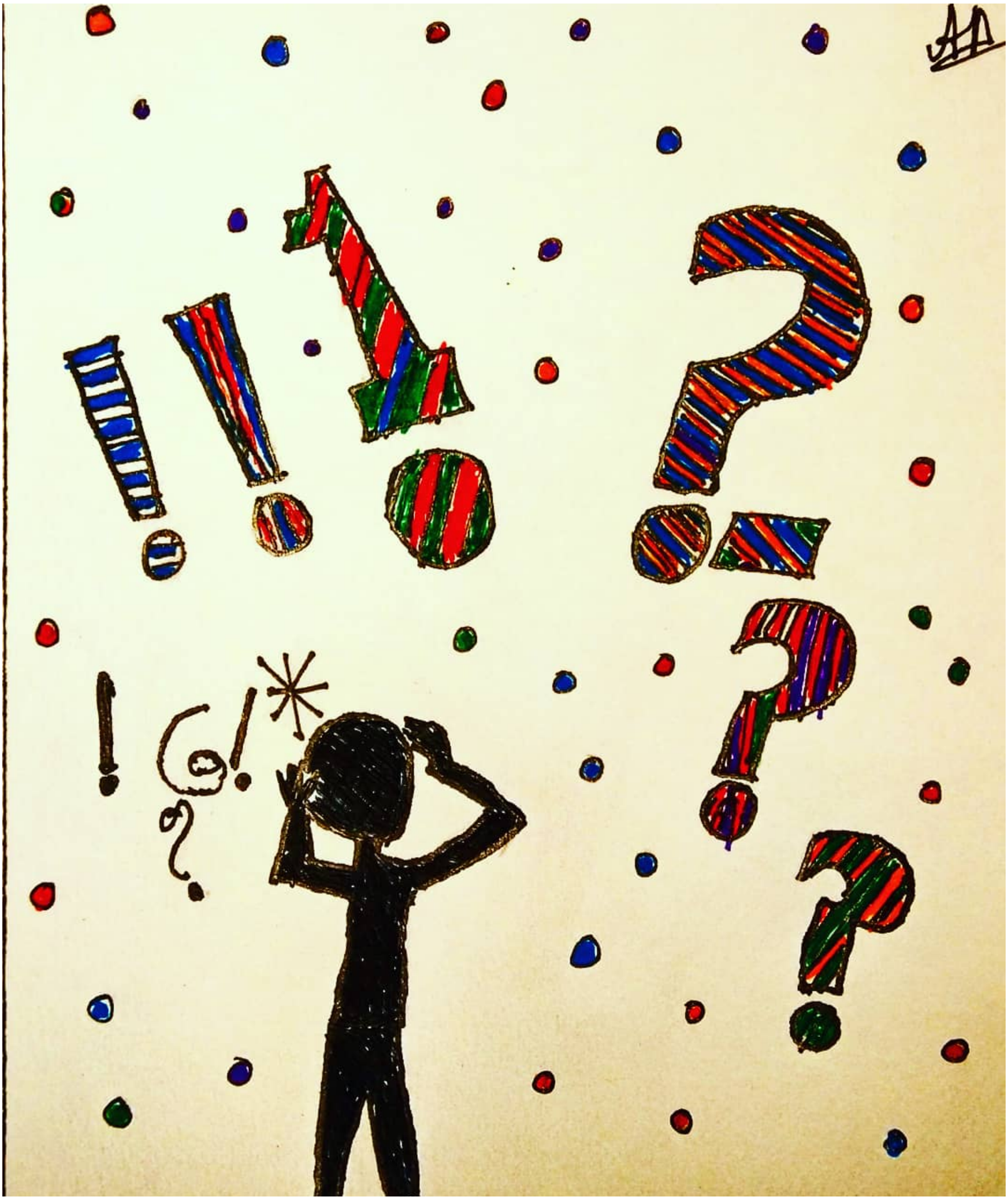
In my experience, people with an affinity for Mathematics see the world differently from those without. Time after time, I’d catch myself staring blankly at a set of equations while someone next to me would have already worked out three or four different approaches to solving them.

Yet, when I would ask them to explain their process to me, they’d struggle. How can one explain the process by which two plus two equals four? As far as they’re concerned, the solution is almost obvious. There are just a couple of tweaks or manipulations required, and the challenge (for them) is to figure out the correct tweaks. Most accounts of famous problems being solved speak of “flashes of insight”, of pieces fitting together like a jigsaw puzzle, of moments when everything falls into place. Even though tough problems require effort, the process by which that work takes place is not easy to explain. Mathematics, for all its modern-day links to science and method, often gives the impression of an art, where the best artists are unable to explain the source of their inspiration.

For a long time, I looked at such artists with a mixture of awe, envy, and frustration. There is something remarkable about being able to mentally multiply fourteen by nineteen or independently prove a property of triangles using little more than axioms. Frustration stemmed from the fact that I usually failed at such endeavours. I’ve lost count of the times I’d tentatively try, before someone would stare at my first step with a puzzled frown and ask “Why are you doing that?”. Math teacher after math teacher would approach me with the words “See, this is so simple...”. They would then rapidly work out a solution while I would dig my fingernails into my desk wondering “What’s going on in your head? How did you know **that** was the correct step to take and not the six others I’m wrestling with right now? Why can’t I see what you’re seeing?”

This issue with mathematics would eventually come to define a significant portion of my life. As I grew older, I would break my head figuring out explanations for my inability to ‘get’ mathematics. It didn’t help that I lived in an environment that prized mathematical talent. I grew up in Bengaluru, a city known for scientists, aerospace engineers, and software programmers. My own family had an astrophysicist, a mathematical economist, a chartered accountant, and half-a-dozen engineers. Later, when I moved to Mumbai, accounts of my poor mathematical abilities would raise eyebrows since “south Indians are supposed to be good at math”. Meanwhile, as India acquired a global reputation for technology and software services, Indians were stereotypically associated with strong mathematical talent across the globe. A mathematically deficient Indian student was an oxymoron — at best, an object of curiosity and at worst, a secret to be hushed up.

I had always scored poorly in math exams, but until I completed Primary School, I did fairly well in other subjects. I would top my class in Science, and score high in English and Social Studies. I’d perform well in extra-curricular assignments, usually scoring high grades for science projects or interactive tasks. All of this went downhill in High School and my scores on most subjects slipped. Opportunities for extra-curricular work diminished as we were told to focus on our Class 10 Board Exams. Worst of all, I began failing in math tests — my first experiences of exam failure. Most of those early years of failure were filled with bewilderment. I had no idea where I was going wrong, my teachers had no idea what was wrong with their methods, and my parents (academic achievers themselves) struggled to understand my confusion.



Looking back, I realise that there was more than one challenge at the time. The first challenge was my obvious lack of intuition for Mathematics. However, I now know that this was hardly the biggest challenge. Instead, several other factors compounded the problems caused by this deficit — the constraints faced by the mathematically skilled while communicating with people like me; an examination system that prioritised speed and memory over understanding (this issue requires its own book); a learning environment with few alternatives to such forms of assessment; an emphasis on rote-learning over diverse approaches to problem-solving; and my own reluctance to practice problem-solving. The last was a vicious cycle — I couldn't enthuse myself to work on problems I couldn't understand. In turn, this meant I had lesser experience in successful problem-solving, which led to lesser enthusiasm. This was compounded by the emphasis on rote-learning; even if I did occasionally stumble onto a solution, it would often be dismissed, since I had not follow the 'approved' steps that would grant me scores in the Board Exams.

These multiple challenges came together at a bad time. As the Board Examinations loomed, everyone around me talked of future careers and moving on. Due to an old interest in science, I had ambitions of building a career in Astrophysics. However, it was clear that Pre-University Colleges wouldn't accept a student with low math scores into their Science programmes. My parents began a long process of trying to change my mind about Astrophysics, while my teachers continued to emphasise my poor Mathematics scores. As I continued to worry, my anxiety slowly became chronic. It was around this time that I had my panic attack.

Unfortunately, as much as I tried, I could not bring myself to openly ask for help that day. Those were days when mental health still conjured images of asylums and electroshock therapies (at least in my limited schoolchild experience) and I was embarrassed to bring it up. At best, I could manage a few hints about how I was feeling. My mother listened patiently. However, her first concern was getting me to focus on my exams again. Given that my most visible challenge was insufficient practice, she concentrated on calming me down and putting me to practice sums one more time. I could hardly blame her. Over several years, I had unsuccessfully used every gimmick, strategy, and moonshot I could think of to better my math scores. With exams around the corner, there was no time for new strategies that threatened to become diversions.

Thankfully, I began to improve in other subjects as we approached the exam dates. In the final set of preparatory tests before the board exams, I showed the highest rate of improvement in my class, jumping from an average of 55-60% to about 70%. In the board exams themselves, I managed to wrangle an average of 73% across five subjects. However, my Mathematics score was an unsurprising 55%. Everyone was relieved at my overall average, but the math score effectively ended my hopes of building a career as an Astrophysicist . After another long set of debates and arguments, I agreed to avoid a Science track in Pre-University College. Both my family and my teachers felt that the further I stayed away from the math-heavy Science track, the more likely I was to succeed in life.

While my parents and teachers insisted I avoid taking up Science programmes after Class 10, they still debated on where I should go instead. Some, like my father, believed it would be best if I avoided Mathematics completely and spare myself future distress. He was (and is) a strong critic of societal pressures on students to take up subjects they could not handle. Others, like my mother, had more mixed responses. While she appreciated my father's points, my mother was also a staunch believer in facing one's fears and learning to deal with unpleasant work. As far as she was concerned, moving me away from Science was to give me more time to tackle my mathematical weaknesses. While she wasn't opposed to me entering the humanities, she urged me to also consider careers in law, accountancy, or finance. She argued that my choices shouldn't be about avoiding discomforts, but about giving myself the space to work through them. It was through her that I first learned about the discipline that I would eventually pick — economics.

Economics is a curious choice for someone who had just been told to stay away from Mathematics. At first glance, the discipline seems to be nothing but math. Textbooks on the subject are liberally sprinkled with cartesian diagrams and linear equations. Advanced papers are littered with Greek symbols raised to curious powers. Importantly, some of the most crucial work done by Economists — testing the applicability of economic models to social phenomena — requires a complex combination of mathematical, statistical, and social analysis tools that even seasoned scholars find a nightmare to inculcate. And yet, this weird subject that refused to box itself as either natural or social science would eventually become more than the base for my future career. It would also serve as my lens to review my relationship with mathematics in a more favourable light.

My real journey with economics would commence two years later, after I joined undergraduate college. However, I look back to pre-university college as the point where my choice to study economics started to turn around my relationship with mathematics.

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Part 2: Of Butterflies and Uncomfortable Silence

"If they change their criteria in two years, will you redo pre-university?" My Principal looked at me exasperatedly. I swallowed uncomfortably. I then adopted an innocent expression that I hoped would say "I know you think this is a bad choice, but please humour me". This was a classic strategy I'd observed quiet, obedient students adopt when they faced their teacher's ire for making a bad choice. Unimpressed, my principal stared back at me, waiting for me to answer his question. We proceeded to stare blankly at each other for half-a-minute. He then looked away and began shuffling through some documents on his desk with a nonchalant air, waiting to see if I would give up. He didn't know it but if I could have moved, I would have stepped out, assuming that he wasn't interested in my request. However, I had never walked out on a teacher without their permission before. My feet were rooted to the ground and my arms refused to move. A few minutes later, my Principal sighed. "Fine." he said. "Get me a letter from your parents giving you permission to switch subjects and I'll approve the shift."

Years later, while reading about Complex Systems, I would learn about the Butterfly Effect — how a butterfly flapping its wings in South America could be the starting point of a complex set of events leading up to a cyclone on the other side of the world. While this story is largely apocryphal, such types of emergent phenomena exist all around us. A single wet-market in Wuhan allegedly triggered a global pandemic. A series of unusual rainstorms over the Arabian Empty Quarter would lead to locust swarms devouring food crops all the way from Ethiopia to western India two years later. Butterfly effects can be found in our individual lives as well. In November 2020, a BuzzFeed article listed over a dozen stories of ordinary people who ended up meeting life-partners, getting fantastic jobs, or inheriting wealth thanks to completely random events such as tripping on a sidewalk or catching a cold.

As far as my own life was concerned, those few seconds of silence in my Principal's office was arguably the biggest butterfly effect I've experienced to date (that I am aware of). Had I gathered the courage to walk out early, or had my Principal directly told me to leave, my life would have gone down a path incredibly different from my present one. Even speculating about where this alternative path would have taken me is difficult.

When I joined pre-university college, I had, in deference to my parents' wishes, avoided taking up mathematics. However, keeping my mother's advice, I'd chosen to pursue subjects related to Business and Commerce, hoping to shift fully to Economics during under-graduation. My Pre-University college offered a combination of Accountancy, Business Studies, Economics, and Statistics. Accountancy and Statistics were both quantitative subjects, but my parents felt they would not be as tough to deal with as Mathematics itself. Therefore, I began my final years of schooling with the idea that my time with Mathematics had ended.

Barely two weeks later, I would change my mind. While researching future career paths in economics, I suddenly faced an uncomfortable fact. Most undergraduate programmes in Economics refused to accept applicants who hadn't taken up mathematics at pre-university

level. I was particularly chagrined by the absurdly high requirements of the colleges at Delhi University, generally considered back then as the best place to study Economics in India. Hesitantly, I wrote an email to one of the colleges in Delhi. Does Statistics suffice? I asked. The reply was terse and direct — "**In order to be considered for admission, all applicants must have studied mathematics at the +2 (pre-university) level. Statistics is not considered a suitable alternative**."

Within a few minutes of receiving their reply, I had made up my mind. I did not want to go through another round of uncertainty about my career in two years where I might have to rethink choices again. A few hours later, I was tapping on my Principal's door. Having studied in schools where Principals were mostly seen as distant figures to be respected and not disturbed, this was a new experience for me. I entered, gulped a couple of times, gave a respectful nod, whispered "Good Afternoon, sir" and gingerly eased into the chair he offered me. As soon as I sat down however, a sense of calm washed over me. I had a feeling it wouldn't last, so I began speaking. I wanted to become an economist. I was hoping to do this by getting into a university like the one at Delhi. The Delhi folks had informed me that I needed to do mathematics to be considered for admission. Would it be possible for me to switch my statistics course for a mathematics one?

As I mentioned before, my Principal wasn't very pleased but agreed to honour my request if I could get my parents' consent. Surprisingly, this proved easier than I hoped. After months of arguments, I had expected at least another round of heated exchanges. As it was, I called my mother at her office. Within a few hours, I had come around to her office, picked the consent letter up, and had returned to the college to hand it over to my Principal. By the end of the day, I was handed a new time-table and told to join a different classroom the following morning. My father was rather vexed at the whole affair, but I suspect the endless arguments had gotten to him too. That would be the last time he'd try pushing me in particular directions in my career. In the future, any career discussions with him would resemble conversations between friends or equals rather than parent and child.

Over the following two years, I would commence my first steps in repairing my relationship with mathematics. It was in pre-university college that I began to suspect how my difficulties with math were actually a combination of multiple challenges rather than a single weakness. What helped me significantly was that I was dealing with not one, but two quantitative subjects — accountancy and math — and I was dealing with different difficulties in each. For instance, I was pleasantly surprised when I had few problems with Calculus, largely considered one of the toughest parts of High School math. On the other hand, I struggled to apply the simple book-keeping rule of "Debit all expenses, Credit all incomes". I'd trip and fall over basic Probability Theory but could reasonably solve Business Optimisation problems. Meanwhile, I was getting an average of 90-95% in Business Studies (unusually high for a non-quantitative subject) and around 80-85% in Economics, suggesting that it wasn't unfamiliarity with concepts that was affecting my Accountancy scores. By the time I completed pre-university to enter undergraduate college, I had begun to suspect that my "math weakness" was not impossible to surmount. I had gathered enough courage to continue with the subject, even in undergraduate college.

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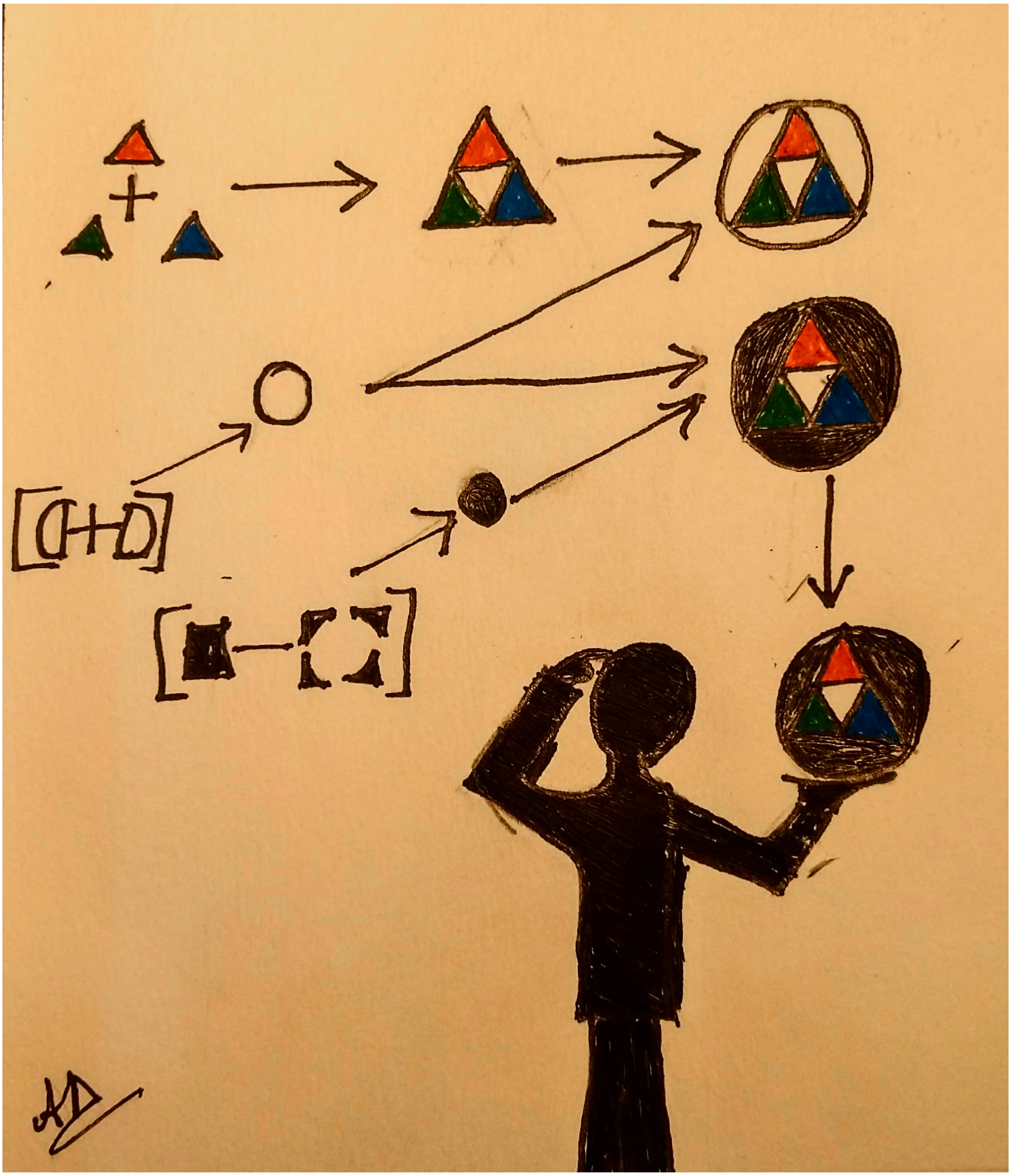
Part 3: What They Always Had To Do

We tend to look at an obstacle as something to be fought and defeated. Our languages wash themselves with metaphors of struggles, battles, duels, defeats, and victories to describe the relationship between humans and their challenges. A mountain climber is a soldier in a fight against the mountain, even when the mountain itself has no say in the affair. Depression or melancholy are demons to be battled in order to be happy. Myths around the world speak of heroes whose roads were blocked by trolls, devils, rakshasas, sphinxes, and dragons, to be beaten before forging ahead. Once these demons are vanquished, we tend to believe, our lives will be forever free, to be shaped as per our wills.

When I began undergraduate college, I defined my relationship with mathematics on these terms. I was in a race to success, and math was a hurdle to be surmounted. I was hardly alone in this. Most people around me had very particular ideas of what college students were supposed to do — get high-income jobs, move up the ladder in their industries, shift to a rich country, and "settle down" (marry and have children). These were all goals to be achieved. Anything that came in the way of these goals were hurdles that the world threw at us, to be fought and overcome. In High School, my grandmother would give me dramatic inspirational speeches by coming into my room, raising her fist, glaring at me, and talking of **shistu** (Kannada for 'effort') in order to **gellu** ('win'). In college, others would fill that role and the metaphors they used would almost always contain some allusion to fighting.

I don’t want to mock this too much. Like all metaphors, the ones of battle and struggle have their moments and I’ve used them repeatedly over the years to revive myself after failure. That said, the overwhelming prevalence of these metaphors (at the cost of other, more benign framings) ended up blinkering the way I perceived my challenges. A mathematical problem was something to be fought and won over. Once beaten, it was under your control. Understanding math had a simple objective — beating the exam and taking my place among those for whom math was never an issue. In the end, I hoped to be free of self-doubt and confusion, leaving me free to surge onward to better things. Instead, undergraduate college turned out to be a somewhat different experience. For that, I have Economics to thank. In order to understand why, it’s important to know a little bit more about the discipline.



There are many debates and arguments about what Economics actually is. I prefer to think of it as studying the relationships between human values and our material world. As human beings, we hold multiple ideas of what is 'valuable' to us. When we begin to act upon these ideas, we shape the material universe around us to adhere to these values in our heads. For instance, when daily nourishment was the greatest value in peoples' heads, more pieces of wood were shaped into plates, and more streams were diverted to irrigate crops. Similarly, the material world can shape our values as well, such as when the cycle of seasons taught us to worship rain gods at specific times of the year. Even today, the **materiality** of our world shapes and is shaped by human values — the value we place on comfort shapes our cities into structures of brick and concrete, while our increasing engagement with computers incentivises us to value Logic. Economics is essentially the study of all these processes and relationships. Admittedly, this is not how most students are taught to view the subject. However, a simple overview of the domains Economists work in can substantiate my claim. Economists tend to work at junctions where abstract values in our minds interact with the material world.

This results in Economics placing quite a heavy focus on the material, the non-sentient, and quite often, the non-living. This is unusual for a social science. Some branches of Anthropology, Geography, and History have similar focus, but it's rare to find a social science where the fundamentals of the discipline intertwine so deeply with the material. Of five major economic phenomena — Demand, Supply, Production, Consumption and Exchange — all describe fundamental links between human values and the physical and material contexts within which these phenomena take place. For instance, Demand combines human desire with a material capability to satisfy that desire, while Supply and Production cannot ignore the physicality of the processes that allow products to be made and distributed. Building food security requires understanding how crops are grown and distributed. Cellphone production is constrained by technological limits of extracting rare earth minerals. Simply put, a significant part of Economics involves exploring our knowledge of the material. As a result, in addition to studying society and value, Economics needs the physical and material sciences, as well as their frameworks and methods of understanding the world.

Therefore, it shouldn't be surprising that Economists have long attempted to inject the Scientific Method into their practice (albeit often clumsily). Both the deductive and empirical components of the Scientific Method have been assimilated into Economic Thought, the former to build models of how economic phenomena occur, and the latter to develop methods of testing these models. From here, it was only a short step towards intensifying the role of mathematics, the universal language of the physical sciences, within Economics. At various points in history, there have been equally varied rationalisations for these incorporations. Leon Walras chased the idea of general economic equilibrium after being inspired by developments in 19th Century physics, Vassily Leontief's Input-Output Analysis was founded on dreams of a materialistic planned economy. Daly and Georgescu-Rogen linked economic activity to thermodynamics and entropy. Each development deepened and justified the increased links between mathematics and economics. By the beginning of the 21st Century, we reached a point where universities preferred admitting undergraduate Mathematics students rather than undergraduate Economists into their graduate programmes.

There are critics, of course. Mainstream economists themselves bemoaned the increased 'mathification' of their field, while continuing to publish papers littered with Greek symbols. Libertarian and certain leftist schools openly rejected the use of Mathematics, claiming math to be unnecessary at best and oppressive at worst. With the rise of the internet in 1990s and 2000s, blogs began to fill with conspiracy theories about how Mathematics was a smokescreen to cloak Deep State control of the economy. At the other end of the spectrum, universities, corporations, and think-tanks embraced mathification with a vengeance, trying to quantify everything from the costs of changing the shape of milk cartons to designing carbon credit markets that transacted in 'Rights to Pollute'. In turn, their actions enraged those who saw math as a way to paper over social and political concerns. This led to Economics becoming a battlefield of ideas on the use of Mathematics to solve social problems.

Navigating this quagmire in the late 2000s required me to rethink my relationship with math multiple times. Often, I was tempted to follow other peoples' lead and embrace ideologies which rejected mathematics. Yet, a part of me couldn't help feeling uncomfortable. For example, certain libertarians argued that quantification of the economy was unnecessary when free markets would determine prices more efficiently than any mathematical model. And yet, my experiments with free market models showed them to be sensitive to either elite capture or eventually reverting to political institutions for economic justice. In either case, rejecting mathematical models had little relevance as a methodological choice (although I agreed that math alone was insufficient). Similar arguments could be made against claims by other schools of thought.

These experiences pushed me unpack my ideas of mathematics. It would take several more years, but undergraduate college is where I began to change my belief about mathematics as a hurdle to be beaten and left behind. Instead, I began to see the utility of a longer engagement with the field, unconstrained by the immediate burdens of exams and job market signalling. Those were still early days, but I started appreciating the role mathematics played in strengthening good economic analysis. I paid attention to differences between various branches of mathematics and how the use of different methods and tools could lead different economic perspectives on similar issues. I still did not fully understand the difference between deduction and empiricism, but I started to notice how Mathematics was sometimes used as a 'language' to build arguments while at other times, it was used as a tool to measure and quantify. Looking back, it's a bit shocking that no one pulled me aside and pointed out all this, but that is an unfortunate constraint faced by an overburden education system. As long as you're passing exams, even well-intentioned teachers don't have much bandwidth to pay attention to what you're missing. There are always a dozen students weaker than you who need more urgent attention.

This is not to suggest that all was well on the examinations front. On the contrary, I was struggling with exams quite badly, although I was managing to avoid failure. However, the challenges were different this time. Unlike High School, where I struggled to understand the approaches to solving problems, here I was more often faced with trade-offs. I was finding ways to solve problems, but my lack of intuition made me slow, implying that exam preparations would require considerable time and practice. However, this cut into my time available to actually dive into issues I cared about. If I had to work on mathematical problems, I had to choose between the problem that I was interested in and the problem that was relevant for an upcoming exam. I did not have the capacity for both.

What made things worse was that most topics I was interested in — monetary policy, financial instruments, international trade — were all math-heavy, while 'safe bets' for passing exams were not. This implied that I either risk taking up math-heavy subjects to pursue my interests, or allocate many more hours learning these topics on my own. Additionally, I had to think about jobs. There were few employment opportunities for Economics students who did not have some quantitative experience. As a result, I took up a range of quantitative subjects to give myself a shot at employment. If I did well in these subjects, I had a chance at a job interview. If I dropped these subjects however, I'd have no chance. As a net result, I stretched myself too thin. I was permanently juggling between interesting problems and relevant problems, job-oriented courses and theoretical courses, studying for pleasure and studying to pass exams — and not doing well in anything.

The stress began to take its toll. In my second year, I was so nervous during a Statistics exam that I forgot to turn a page over and missed half the listed questions. I broke up with my girlfriend because I couldn't handle a relationship as well as my upcoming exams. At the beginning of my final year, I was asked to leave my college hostel due to my poor exam performance. I spent a couple of months shifting from place to place in Mumbai. There were nights I would study on railway benches because the place I was staying at was too noisy. I finally got a respite when a family friend was kind enough to take me in for a month. After this, my good disciplinary record allowed me to return to the college hostel when one of the other students dropped out. I'd never been more thankful for being a 'good boy' at college, a boring reputation that finally paid some dividends. I somehow put in a final burst of effort and scraped through a First Class to get my Bachelor's Degree.

I didn't do well enough to be considered for many job interviews though and I decided that continuing my studies would be a good option. Hopefully, a graduate degree would give me more freedom to choose subjects I was interested in and there would be less emphasis on scoring in exams. Unfortunately, this was not the case. The university I got into had a good programme and enthusiastic teachers. However, it had an extremely intensive examination system, where a set of six to eight examinations would be held every nine weeks. There was no final dissertation or capstone project. Suddenly, it looked as if the trade-offs from my undergraduate days were back and I was going to repeat my mistakes all over again. To an extent, I did. But my Master's Degree was also where I finally set my relationship with mathematics right.

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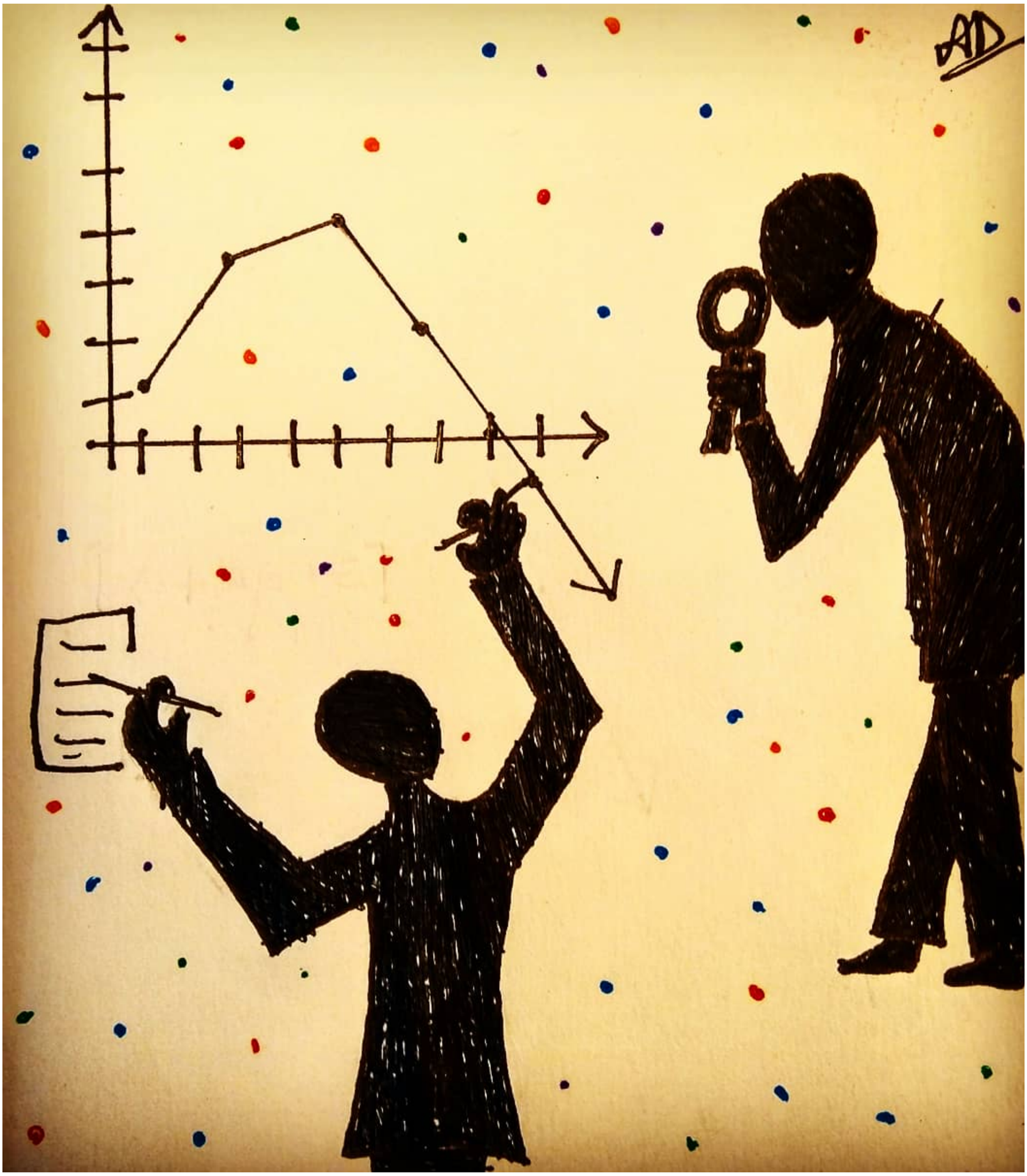
Part 4: Trade-Offs

My undergraduate college was in southern Mumbai and early on, I discovered that my campus was a short walk away from the Bombay Stock Exchange. A few times every month, I would walk down to the Exchange and observe everything going on around it. It was mostly for fun, but I also picked up a lot. I noticed how street vendors around the exchange didn't sell snacks or produce, but specialised in IPO application forms and financial newspapers instead. I observed how the busiest hours for restaurants and cafes in the area coincided with the opening and closing of trading hours. I understood the meaning of the word 'mania' when I saw limousines drive up to purchase stacks of forms on the day of the Exchange's biggest IPO in history. A year later, I understood the meaning of 'recession', when the restaurants slashed their menus and fired staff after a massive crash led to an exodus of stock traders to their native Gujarati villages. Stressful and harrowing as 2008 was, it was also a year I could observe economic forces in action — not just at the Stock Exchange, but all around the business centre that was south Mumbai.

I was assisted in this endeavour by a small bookshop on the way to the Stock Exchange, that had an excellent economics section. Whenever I could afford it, I would pop down to make a purchase from the shop. It was here that I read Gurucharan Das' **India Unbound**, one of the few books that defended India's liberalisation of its economy by using personal narrative. I picked up Muhammad Yunus' **Banker to the Poor** at a time when microfinance was a hot new topic in development economics. It was also where I first began to explore mathematics and statistics outside my prescribed textbooks and papers. Out of curiosity, I picked up Nassim Nicholas Taleb's **Fooled by Randomness**, Satyajit Das' **Traders, Guns and Money** and several obscure books on the mathematics and statistics of Finance.

It was an interesting time to be studying this. Lehman Brothers and Bear Sterns declared bankruptcy around this time, triggering chain reactions culminating in a global collapse of financial systems. Overnight, popular ideas of what drove financial markets were turned upside down. Investment Bankers, Fund Managers, Financial Analysts, and Wall Street traders who were celebrated just a few months earlier were now castigated as villains who destroyed lives. In these contexts, the mathematical and statistical aspects of Finance were greeted with skepticism. This wasn't without justification. Mathematics had been frequently used to mask inept or unethical financial practices, skillfully wielded to portray investment strategies as rational and scientific, while exploiting the gullibility of investors who preferred to skim over the details.



While learning about these practices pushed many to be skeptical of mathematics and statistics as a whole, I went in a different direction. Around the same time, I was learning about Terms & Conditions in software contracts that most users never bothered to read. I argued with myself that if lawyers have to deal with the fine print of written contracts, economists cannot ignore the mathematical and statistical fine print of financial models. It is the responsibility of a professional economist to be able to critique the nuances of a model that can confuse the layperson.

It was (and is) easier said than done. The fundamentals of financial mathematics, such as rationalisations for spreading risk across a diverse portfolio, were fairly easy to grasp. However, once one dives into specific models, the going gets tougher. Behind each equation lies a plethora of assumptions made by the modeller, reflecting not just rational logic but also a complex combination of individual biases, industry practices, availability of data, marketing requirements, and social incentives. A modeller may favour one practice, such as Monte Carlo sampling, in order to appease bosses or clients excited by stochastic models. Alternatively, a model may be tweaked by the final investors and traders to speak to the dynamics of asset markets being influenced by events around the world. Mapping the elements of a model to these multiple factors is a task that never ceases to be easy to an outsider.

And yet, by the time I began my Master's Degree, I began to better understand the role played by mathematics and other quantitative subjects in my discipline. I started to become more comfortable with the notion of abstract mathematics as a language, a mirror that reflected the emotions and aspirations that lay behind very human actors. At the same time, this led me to explore and understand the use of mathematics in other fields (particularly physics, a discipline I retained an interest in from my school days). During my Master's Programme, I picked up several popular books dealing with mathematics in other fields. I began to read about the history of mathematics from Stephen Hawking's **God Created The Integers**. I read Roger Penrose's controversial arguments favouring quantum mechanics in human thought via his book, **The Shadows of the Mind**. This in turn led me to his magnum opus, **The Road to Reality**, that traces the construction of space-time. I explored philosophical ideas like Searle's Chinese Room Experiment, from where I was led to the idea of the Turing Test (this was a few years before Artificial Intelligence returned as a popular buzzword in the 2010s). I was introduced to Claude Shannon and models of information transfer. I skimmed through Douglas Hofstadter's **Godel, Escher, and Bach** before my interest was diverted by Godel's original text on his Impossibility Theorem.

With every new idea, I obtained a new perspective on mathematics and I would compare every new perspective with the mathematics and econometrics I was (still) struggling with in my Master's Degree classes. During a break at home, I mentioned my newfound interest in abstract math to my father. He responded by gifting me two books — Terrence Tao's guide to abstract mathematics, **Solving Mathematical Problems** and G. Polya's popular book on mathematical method, **How to Solve It**. I used these to work on problems both within my taught courses as well as my out-of-syllabus explorations. I continued to find examinations tough, but something had fundamentally changed. I no longer saw mathematics as this incomprehensible field that was to be either avoided or fought. I realised that there were situations where, despite the continuing difficulties, I actually enjoyed working with math. I was beginning to see how the subject connected to real world issues and this made the field much more understandable.

A key shift in my thinking that facilitated this was my approach to equations. As I began to see abstract mathematics as a language, I asked myself if translating equations into formal logical statements in English would help me understand the concepts better. I began this practice in the final years of my undergraduate programme, but really picked it up at Master's. To my pleasant surprise, this worked for a majority of the models I was studying. More complicated operations remained difficult to translate (translating these would result in dozens of pages of written notes for a single equation — a time-consuming process), but the attempt was largely helpful as translations allowed me to engage with the underlying concepts on a terrain I was more familiar with. More importantly, this pushed me to explore the links between language, thinking, and mathematics. Two major books that helped me in this endeavour were Steven Pinker's **The Language Instinct** and **The Stuff of**

Thought. Exploring linguistics helped me understand my own thinking better and pushed me to rationalise my approaches to problem-solving.

I'd be remiss if I did not mention the helpful people in my Master's Programme. While exams continued to drive me mad, my teachers and classmates created an environment where exploration and independent thinking were encouraged despite the evaluation system. I had a group of friends who were more than happy to dive into long conversations about economics, mathematics, philosophy, computers and linguistics. PhD scholars in the department would chat with us about new things they'd come across. My Microeconomics teacher pushed us to explain complicated microeconomic models written on the blackboard and I used these to build confidence in English translations. My Econometrics professor openly exhibited his own interest in multiple fields and my first introduction to Computational Social Sciences was via his work. My Gender Economics professor pushed back against exam-based evaluation by setting us a paper-writing assignment and I topped the class with an attempt to model monetary policy effects upon a gender-segregated economy. This came as a great relief at a time I was struggling with other subjects. It showed me that a good environment could facilitate better performance and not all my failures were my own fault.

My greatest leaps in confidence came in my Urban Economics classes. In my second year, A friend introduced me to a book by Eric Beinhocker, **The Origin of Wealth**, on a branch of mathematics called Complex Systems. By their very nature, Complex Systems are hard to define, but they can be roughly imagined as systems where the whole system exhibits properties greater than the sum of its parts. Both Cities and Economies could arguably be framed as Complex Systems and I spent many hours reading about Complex Systems Theory and its applications to both. Complex Systems wasn't part of our syllabus, but in later Urban Economics classes, my Professor began flipping the classroom and made us students teach each other about urban topics. I grabbed an empty slot that no one else wanted and ran three lectures using Beinhocker's book as a reference. In my enthusiasm, I overshot and inundated my classmates with too much information. But preparing for those lectures sent me strong signals that I was getting better, both at working with difficult mathematical ideas, as well as applying them to other fields.

By the end of my Master's Degree however, I was faced with a difficult decision. Leaps and bounds in confidence aside, the examination system was still a major hurdle. By now, I had realised that my weaknesses in math were separate from my weaknesses in passing exams, each requiring a unique skill set. If Mathematics required understanding concepts and practicing problems, passing exams required speed, memory, timing, and replication. Combined, these two weaknesses were large enough to fail me, but resolving them required working on them separately. With examinations every nine weeks, this left me no time to actually learn anything new. I would have to give up my explorations, my self-study, and my connecting everything I've learned into a common body of knowledge.

This led me to one of the most difficult decisions I've ever made — difficult, because it's a terrifying move in an education system like India's. In my final semester, I chose to prioritise my learning over passing exams. I would continue to give time to examinations, especially the ones I was weak in, but not at the cost of learning. In many ways, this was an obviously foolish move. In other ways, it wasn't. I had realised that no matter how much effort I put into exams, several constraints would prevent me from obtaining more than a marginal improvement in my scores. Therefore, if I wanted to take something away from my Master's Degree, I might as well choose to improve my knowledge. In a sense, I chose to favour a longer term engagement with mathematics over immediate requirements.

The final semester nearly broke me, both physically and mentally. I not only had the five subjects I was studying that semester, I had three others whose exams I'd failed in the previous semesters and needed to retake. The result was a timetable that consisted of eight back-to-back exams. Unfortunately, the quantitative subjects were smack in the middle. I spent the final weeks of my Master's in a feverish dream, trying to balance my studies with my exam preparation. When the exams commenced, I'd walk into the examination hall every morning, write for three hours, walk straight back to my hostel room, and begin preparing for the next day.

When I stepped out of the hall on the final day, I collapsed on the ground outside and burst into tears of relief. A friend had a car and a group of us went for a long drive to the edge of Mumbai to cool down. I spent most of the trip staring out of the window. A few months later, I would learn that my efforts just about allowed me to obtain my Master's Degree. However, on that final day, I had stopped thinking about this. Instead, as we moved to the edge of the city, I was wondering how urban marketplaces would expand into the countryside and whether some of these expansions could be framed as mathematical models. I suddenly felt grateful for the learning I'd obtained the previous two years to think in such ways.

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Part 5: Reflections

My journey with mathematics did not finish there, but I think the end of my Master's Degree is a good point to stop. I would spend many more years building, refining, and correcting my knowledge of mathematics, but that tale is a sequel that builds upon this story, to be told another day.

Right now, it would suffice to say that I've been engaging with math on my own for nearly a decade. I completed my first, largely quantitative research paper in 2019, a study of Indian government spending on climate change adaptation. A year before, I began to run a small session as part of a larger Data Skills Lab at my workplace, focusing on the analysis of employment data. Successfully completing my first data-heavy session was a moment of immense pleasure, as I realised that I had built enough capabilities to teach the applications of basic mathematics and statistics in economic issues.

In a way, I have come full circle. As a school student, I struggled with understanding how mathematically talented people thought and approached problems. As a researcher, I now draw upon those very same experiences to understand where others struggle and what obstacles they may encounter. I'll probably never reach the speed or flexibility of someone with greater mathematical aptitude, but that's okay. Most people I encounter will probably face challenges similar to mine. Therefore, in some ways, my experiences allow me to communicate mathematics better than those who have greater skill or aptitude.

This essay might make it seem as if the majority of my life was a struggle with mathematics. Truthfully, albeit mathematics played an important part in my life, it was hardly the only thing I worked on. In fact, a greater portion of my life and thinking has been shaped by economics and urban studies (perhaps one day, I'll write about these too). Over the years, I've also dived into studies of anthropology, geography, sociology, information science, and history. As an adult, I rediscovered a love for Kannada, my native language, and through it, scholarship on Kannada epigraphy and classical literature. A study on water body conservation pushed me to learn more about hydrology, geology, and ecology. I draw upon my old interest in physics to write science fiction and fantasy for fun in my spare time. I have just begun learning digital art. I've volunteered for civic campaigns. I have developed a reputation as a long-distance walker, often covering 20-25 KM a day to explore my city. I've learned (sometimes painfully) that solving the big problems shouldn't come at the cost of other pleasures that life has to offer.

So why have I written this essay? Is it to inspire people to take up mathematics? Is it to comfort other students who may be experiencing similar challenges? To feel good about myself? To express gratitude to those who helped me along the way? To simply record a major part of my life? There have been times when I wanted to write this essay for precisely these reasons. But none of these feel particularly appropriate today. If I have struggled, so have many others, some much more than me. Our bookstores and media portals are filled to the brim with stories to inspire generations. Mine is just one more struggle among the millions who compete fiercely for the limited space in our education system.

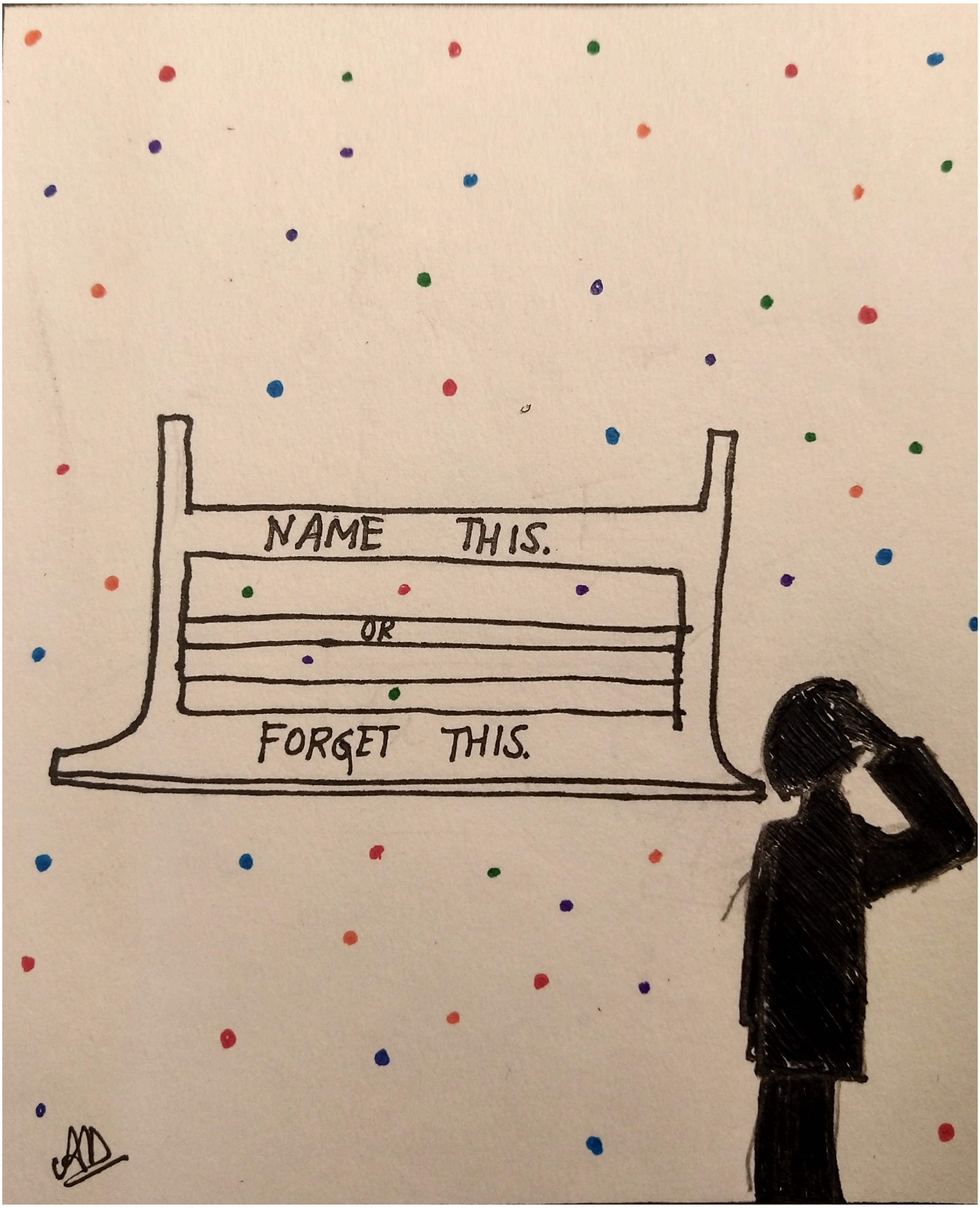
If I have pin one reason down for writing this, I'd go back to the part where I talked about the fallacy of seeing mathematics as an obstacle. As I'd mentioned, I suspect this framing comes from social notions of 'success' being framed in particular ways. For most, a successful mathematician is one who does well in tough exams, is hired by an organisation with stringent entry barriers (like an investment bank or an Ivy League university) and is able to earn either money or fame (or both) from their mathematical work. In these framings, success is the child of aspiration and exclusivity. The rarer and more desirable a particular position is, the more likely the person holding it is labelled as 'successful' by society. It's easy to associate mathematics with aspiration and exclusivity, and from there as something to be mastered to enter a club of successful people.

This is compounded by the fact that most of us don't like using math in our daily lives. For the vast majority of us, mathematics is not intuitive. It is the tongue of a Platonic world, built step by step from axioms and postulates to arrive at rational conclusions. Most people see equations as incomprehensible hieroglyphs, while statements like "fail to reject a null hypothesis" make us pause before we can grasp its meaning. An intuition for mathematics is available only to a few. In such a scenario, there is little quotidian use of advanced math to counter its image as being restricted to experts. This opens up troublesome situations. Portraying Mathematics as a discrete hurdle, crossed by a select few, allows a discipline or profession to be garbed in respectability when it is 'mathified'. An ordinary person can be intimidated enough by a display of math to not scrutinise claims and "leave it to the experts" rather than commit a mistake themselves.

The more cynical of us have long recognised this. This is why sometimes mainstream economic research can feel like a mathematical arms race, where the more obscure the math, the more likely you are to be taken seriously. Economists are not alone here. Chartered accountants can be notoriously exclusive about the workings of their profession, while much of finance or physics or ecology remains out of reach for outsiders, with discourse taking place behind closed conferences and paywalled journals. Until recently, the relatively open discipline of computing was an exception, but even here, open-source movements are being overtaken by copyrighted algorithms and subscription software which hide the underlying math and logic from ordinary scrutiny.

Now, it is true that certain problems are best left to experts. Laypeople cannot be expected to conduct cardiac surgeries, so why expect them to analyse complex algorithms? However, there is a difference between learning to perform a surgery and learning how the heart works. Even if one never wishes to become a cardiac surgeon, learning about the workings of the heart can still be useful in many situations, starting with improving one's basic health. I argue that it's similar with mathematics. Learning more Mathematics, while tough, should not be seen as an initiation into some arcane cult. As I've mentioned before, it's a language to view and describe the world around us. It's always useful to keep in touch with a language, even if one can never become as good as a native speaker.

My own engagement stands as proof of this. By standard measures of 'success', I've neither topped exams nor held jobs in heavily quantitative fields. And yet, a constant engagement with mathematics not only proves useful in my daily life, but acts as a reasonable guard against falling for claims like the ones mentioned above. For example, econometric models often force us to add an "error term", just a regular reminder to modellers that no model can simulate reality perfectly. Whenever I face a problem in my daily life as well, through force of habit, I've begun including error terms of a sort, reminders to (a) ask myself what I've overlooked and (b) make concessions for the fact that I may have missed something. This may sound rather obvious, but readers should carefully analyse their own lives. It's astonishing how often we overlook these simple points in our daily decision-making. Engaging in a continuous, never-ending process of improving myself in math keeps me on my toes and has saved me from crisis on more than one occasion.



This has also forced me to re-evaluate my own notions of success over the years. I still ascribe importance to position, privilege, and money — in our world, it's impossible to survive without doing so. However, my engagement with mathematics and the way this engagement slowly changed my life has forced to rethink what my obstacles actually are. For me, success is never a goal to be reached, but a process of continuous improvement at a pace I can handle. Meanwhile position, privilege, and money are more akin to obstacles that must be handled, as creatively as possible, if you want the space to focus on your own flourishing.

All well and good, but does reframing one's relationship with a challenge allow us to overcome it? Perhaps not, but it can teach us to see how overcoming may not be our greatest concern. It took a while, but I finally understood that mathematics is not an obstacle or challenge, but just another component of human knowledge. And human knowledge has multiple paths to comprehension, even if some of these paths are long and circuitous. My own long route has taught me to appreciate the scenery much much more. By writing this essay, I hope I have prompted readers to think about challenges in their own lives (not necessarily math!) and find new paths to deal with them. Your journey may not be like mine, but it will certainly be unique. That is usually better than falling back on received ideas.

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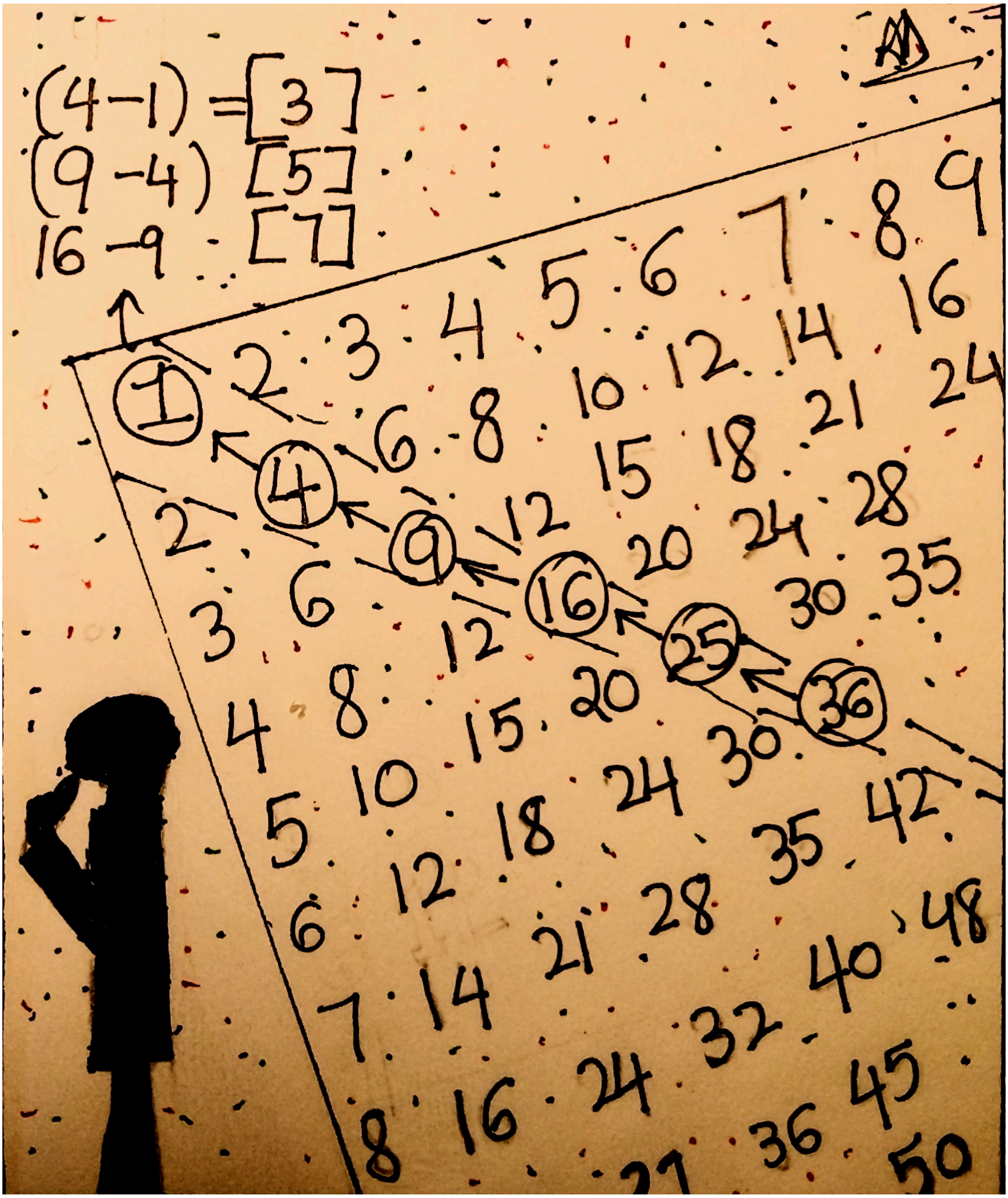
Epilogue

It happened on a quiet Sunday afternoon a few years ago. I had just finished a bout of reading and wasn't in the mood for another book. I was avoiding the computer and cellphone due to strained eyes from a week of staring at screens. Out of boredom, I pulled out a notebook and began doodling on one of its pages.

Gradually, I began shaping a pattern in the ink. I listed down the numbers from 1 to 10 horizontally at the top of the page. Using the same number 1, I listed a column of numbers from 1 to 10, from top to bottom, thereby forming the top and left edges of a number grid. I moved to the next column. Starting with the number 2, I began listing the multiples of 2 until 20. I moved to the next column. Starting with the number 3, I listed the multiples of 3 until 30. I moved to the next column. In this manner, I finally built a matrix of 10 rows and 10 columns, where each row and each column represented the multiples of the numbers from 1 to 10. Every schoolchild has seen this matrix, in a slightly different form, as multiplication tables. I had just removed text denoting the operations and worked only with the multiples.

I was still playing. I wasn't solving any particularly deep problem and was certainly not looking for deep insights. I now began moving diagonally. Starting from the top-left corner and moving to the bottom-right, I began subtracting every number in the diagonal from the bigger number that followed. And that's when it hit me. The differences were all odd numbers. Curious, I moved to the next diagonal. The differences were even. I quickly moved through every diagonal and realised that the differences alternated between even and odd for each diagonal. Every diagonal yielded one of two sets of unique numbers (3, 5, 7, 9, 11...) or (2, 4, 6, 8, 10...). I had just discovered a method for formally defining and separating even and odd numbers from each other. In other words, there was a moment when all the pieces fell into place and a mathematical property was revealed to me in a flash of insight.

It was probably nothing great. I suspected that if I went online and looked it up, I would probably find a million others who had been discovering and rediscovering this property since the beginning of history. Bramhagupta or Avicenna would have probably written



treatises on it. Some precocious whiz-kid may have talked about it in an interview. It may even be one of the first things mathematics majors are taught in number theory. I told myself that perhaps I should look it up, and see if anyone else had figured this out before me.

But I didn’t. I realised that I did not care how many others had discovered this property and how unique my discovery was. It did not matter. What mattered was that, for the first major time in my life, — after decades of frustration, doubt, resolve, and engagement — I wasn’t wringing answers out of a mathematical problem. Instead, mathematics was speaking to me.

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ABOUT ME

My name is Amogh Arakali. I live in Bengaluru, India and I've previously lived in Mumbai. While I work as as a researcher on economies and urbanisation, I also spend my free time reading, blogging, writing science fiction and fantasy, and taking long walks.

This essay is a personal project, not affiliated with any organisation that I work or study at. My other amateur projects ('amateur' here meaning 'not associated with paid professional work') can be found on arakaliamogh.wordpress.com.

My professional work can be found at amogharakaliwork.wordpress.com.