

CS 5300 Advanced Algorithms
HW # 3

1. Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 2 & \text{if } n = 1 \\ T(n-1) + 2 & \text{otherwise} \end{cases}$$

Show, by induction, that $T(n) = 2n$

2. Use Induction to show that $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
3. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = T\left(\frac{n}{2}\right) + \Theta(1)$. Use the substitution method to verify your answer.
4. Solve the following recurrence: $T(n) = 4T\left(\frac{n}{2}\right) + \Theta(n^2 \lg n)$
5. Solve the following recurrence: $T(n) = 5T\left(\frac{n}{2}\right) + \Theta(n^3)$
6. Solve the following recurrence: $T(n) = 8T\left(\frac{n}{2}\right) + \Theta(n^2)$