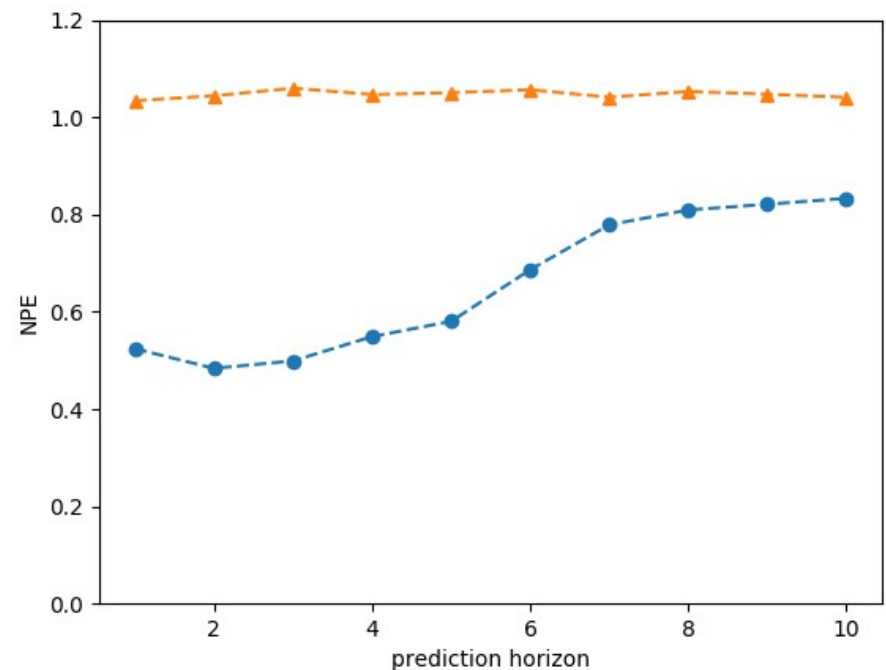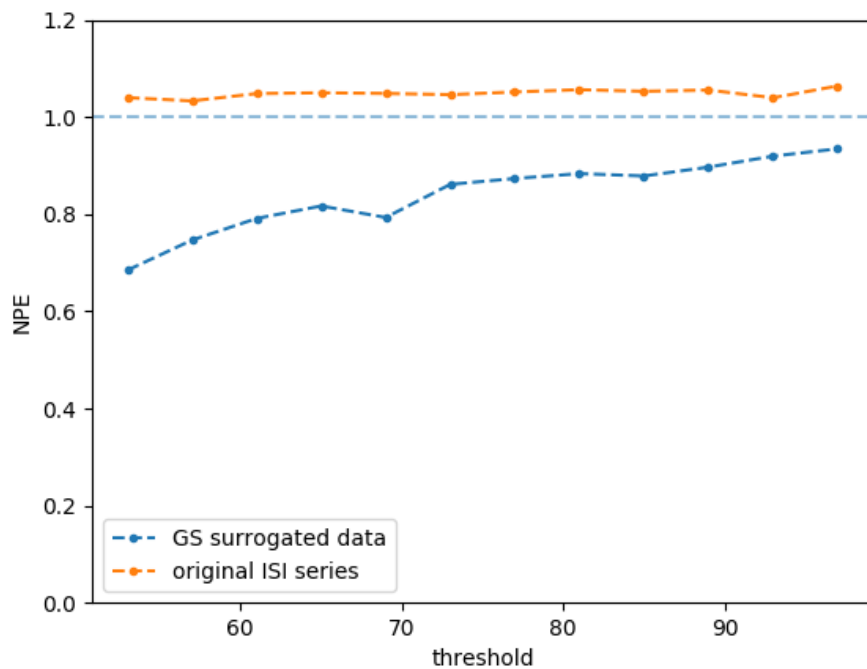# Results Ⅱ : Predictability

- Predictability assumption for NPE.

- Calcul of NPE : Gaussian shuffle & ISIs series

- Calcul of predictability limit: Gaussian shuffle & ISIs series

# Reconstruction of strange attractors via inter-spike intervals

Àlex Arcas Cuerda &
Ramón Marc García Seuma

# Index

- Geometry from a Time Series

- Reconstruction of Dynamical Systems from Interspike Intervals (ISI) I: Introduction

- Reconstruction of Dynamical Systems from Interspike Intervals (ISI) II: Concepts

- Results I : Topology

- Results II : Predictability

- Conclusions

# Geometry from a time series

- Give insight into a dynamical system from a time series.

- Reconstruction of the topology.

- Characterization of the Liapunov exponent.

- Procedure to find the dimensionality.

- Some review.
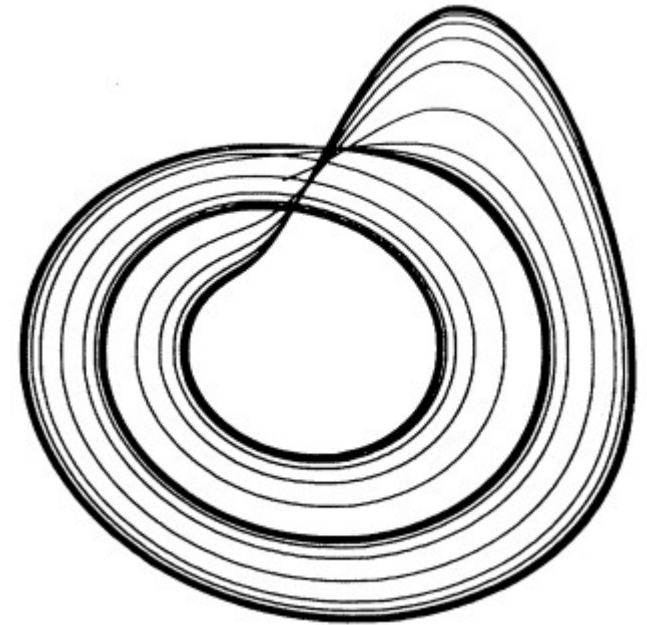


| | Characteristic exponent value |
| --- | --- |
| $(x,y,z)$ system [Eq. (1)] | $0.0677 \pm 0.0005$ |
| $(y,\dot{y},\ddot{y})$ system [Eq. (2)] | $0.0680 \pm 0.0005$ |
| $(x,\dot{x})$ return map reconstruction | $0.0677 \pm 0.0001$ |

# Reconstruction of Dynamical Systems from ISI Ⅰ: Introduction

- Takens's theorem for attractor reconstruction.

- Integrate-and-fire model: point process.

- Applications: noise filtering, prediction of chaotic series and control of unstable periodic orbits.
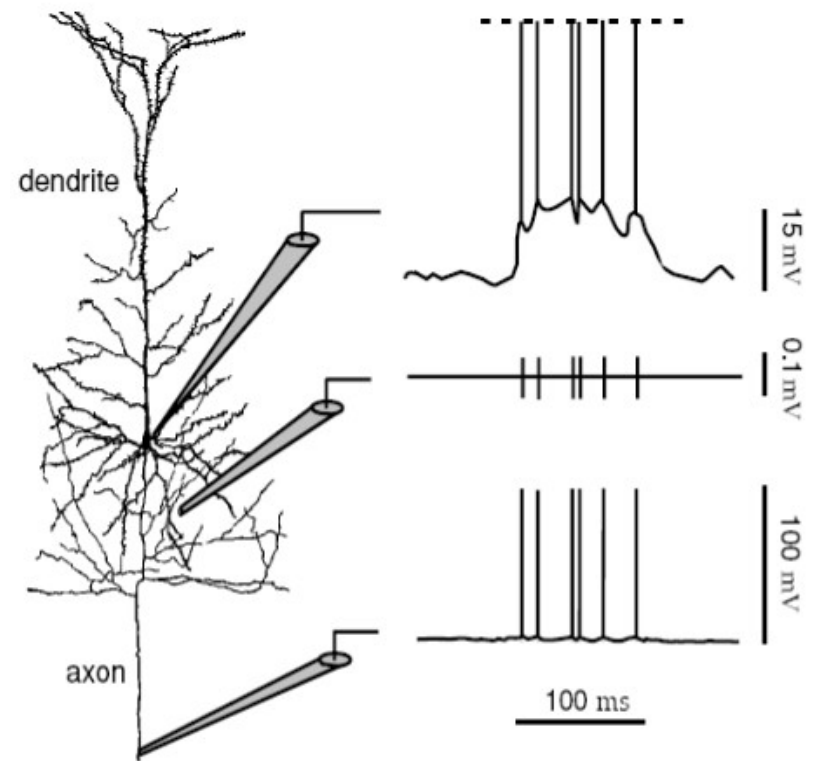
# Reconstruction of Dynamical Systems from ISI II:  Concepts

- Create trajectories of a certain dynamics

- Select a signal S(t) and a threshold Θ so to extract the firing times $T_i$ with:

$$\int_{T_i}^{T_{i+1}} S(t)\, dt = \Theta.$$

- Define the Interspike Intervals as:
$t_i = T_i - T_{i-1}$

**Lorenz attractor**

$$\frac{dx}{dt} = a(y - x)$$

$$\frac{dy}{dt} = x(b - z) - y$$

$$\frac{dz}{dt} = xy - cz$$

**Rössler attractor**

$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + ay$$

$$\frac{dz}{dt} = b + z(x - c)$$

# Reconstruction of Dynamical Systems from ISI II: Concepts

- Vectors $(t_i, t_{i-1}, \ldots, t_{i-m+1})$ can be used to reconstruct the attractor.

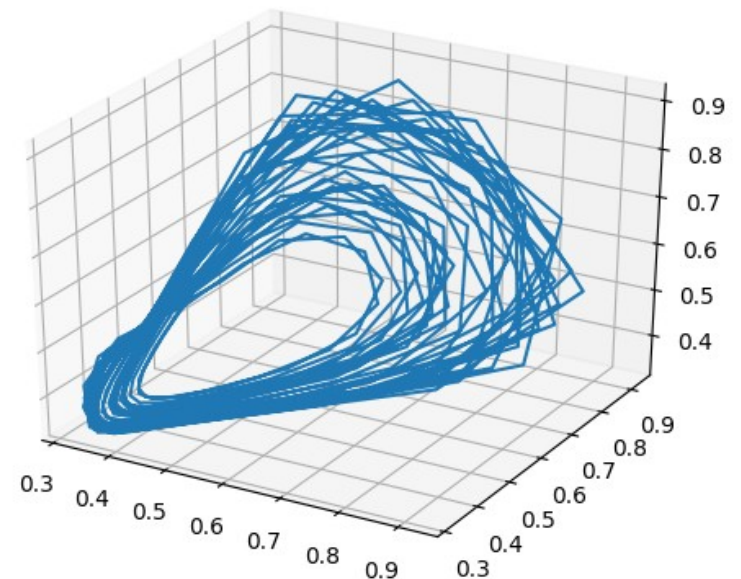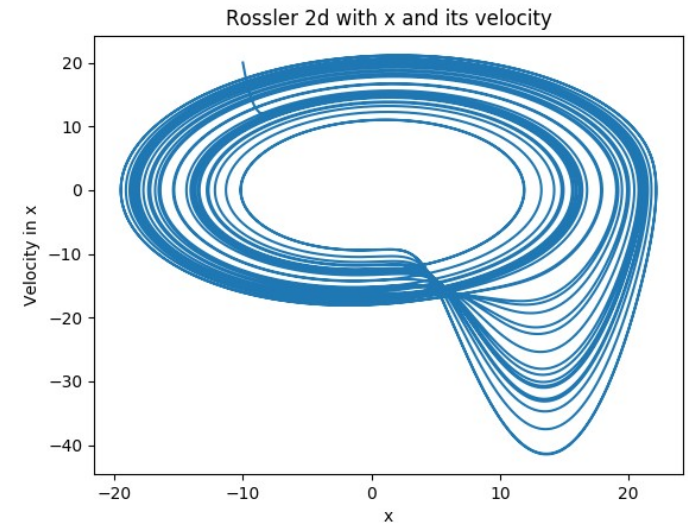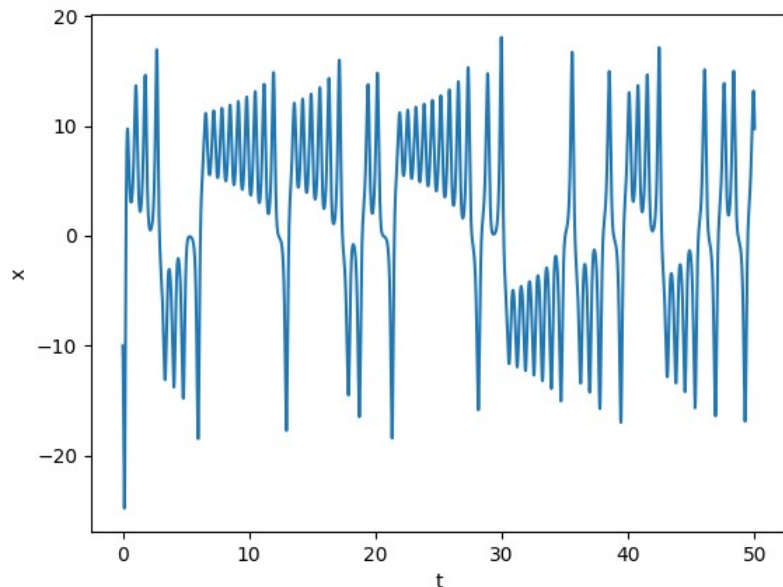  *Takens's theorem: $m > 2D_0$ is sufficient, $m=3$*

- Prediction algorithm: Determinism can be recognized in front of surrogate data using:

$$\mathrm{NPE} = \frac{\langle (p_{i_0} - t_{i_0+h})^2 \rangle^{1/2}}{\langle (m - t_{i_0+h})^2 \rangle^{1/2}},$$

# Results Ⅰ： Topology

- Reconstruction of trajectories : Rossler & Lorentz

- Reconstruction of phase space : 2d & 3d



Rossler 2d with x and its velocity

# Conclusions

- Really counter intuitive and interesting new ideas of studying chaos learned.

- Succeeded while reproducing the paper results, although not all.

- Glad to see how our lecture during the curse helped us to understand the papers even to notice some mistakes.