1) a) $p + p \rightarrow n + \bar{n}$

Reactions allowed by the strong interaction conserve:

- energy
- * baryon #
- * charge Q
- * strangeness
- * parity
- * isospin and $\vec{J}$

$(uud) + (uud) \longrightarrow (ddu) + (\bar{d}\bar{d}\bar{u})$ $\boxed{\text{not allowed}}$

Baryon # – $1 + 1 \rightarrow 1 - 1$ X

b) $K^+ + n \rightarrow \Sigma^+ + \pi^0$

$(u\bar{s}) + (ddu) \rightarrow (uus) + (u\bar{d})$ ✓ $\boxed{\text{allowed}}$

Strangeness ✓

Baryon #: $0 + 1 \rightarrow 1 + 0$ ✓

Parity: $- \ + \longrightarrow + \ -$ ✓

$\vec{J}$: $0 + \frac{1}{2} \longrightarrow \frac{1}{2} + 0$ ✓

Q: $1 + 0 \longrightarrow 1 + 0$ ✓

c) $d + d \rightarrow He + \pi^0$

$d = p + n$, $He = 2p + 2n$

Baryon #: $2 + 2 \rightarrow 4 + 0$ ✓ $\boxed{\text{allowed}}$

Parity: $+ \ + \longrightarrow + \ +$ ✓

Strangeness: ✓ (no s quarks)

$\vec{J}$: $0 + 0 \rightarrow 0 + 0$ ✓

Q: $1 + 1 \rightarrow 2 + 0$ ✓

$\pi^- + p \to \Lambda + K^0$

$(u\bar{d}) + (uud) \to (uds) + (d\bar{s})$ $\boxed{\text{Not allowed}}$

Strangeness: $0_s \to 2_s$ X

$\Xi^- \to \Sigma^0 + K^-$

$(dss) \to (uds) + (u\bar{s})$

Strangeness: $2s \to 2s$ ✓ $\boxed{\text{not allowed}}$

Baryon #: $1 \to 1 + 0$ ✓

Parity: $+ \to + \; -$ X

$\omega \to \eta \, \pi^0$

$\boxed{\text{not allowed}}$

Parity: $- \to - \; -$ ✓

Charge Conjugation: $- \to + \; +$ X (C must be conserved)

$\rho^0 \to \eta \, \pi^0$

$\boxed{\text{not allowed}}$

Parity: $- \to - \; -$ ✓

Charge Conjugation: $- \to + \; +$ X

② a) $\pi^- + p \to K^0 \Sigma^0$

$\pi^- p : |1,-1\rangle |\tfrac{1}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$K^0 \Sigma^0 : |\tfrac{1}{2},-\tfrac{1}{2}\rangle |1,0\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$I_3 = Q - \tfrac{1}{2}(A+S)$

$\Sigma^0 I_3 = 0 - \tfrac{1}{2}(A-1)$

$\Sigma^0 I_3 = 0 - 0 = 0$

$\pi^- + p \to K^0 + \Sigma^0 : \sqrt{\tfrac{1}{3}}\sqrt{\tfrac{2}{3}}\,\langle \tfrac{3}{2},-\tfrac{1}{2}|H|\tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\sqrt{\tfrac{1}{3}}\,\langle \tfrac{1}{2},-\tfrac{1}{2}|H|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$\Rightarrow \tfrac{\sqrt{2}}{3} M_{3/2} - \tfrac{\sqrt{2}}{3} M_{1/2}$

$\sigma_{\pi^- p \to K^0 \Sigma^0} = |\langle \pi^- p|H|K^0 \Sigma^0\rangle|^2$

b) $\pi^- + p \to K^+ \Sigma^-$

$K^+ \Sigma^- : |\tfrac{1}{2},\tfrac{1}{2}\rangle |1,-1\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$\pi^- + p \to K^+ + \Sigma^- : \sqrt{\tfrac{1}{3}}\sqrt{\tfrac{1}{3}}\,\langle \tfrac{3}{2},-\tfrac{1}{2}|H|\tfrac{3}{2},-\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\sqrt{\tfrac{2}{3}}\,\langle \tfrac{1}{2},-\tfrac{1}{2}|H|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$\Rightarrow \tfrac{1}{3} M_{3/2} + \tfrac{2}{3} M_{1/2}$

$\sigma_{\pi^- p \to K^+ \Sigma^-} = |\langle \pi^- p|H|K^+ \Sigma^-\rangle|^2$

c) $\pi^+ + p \to K^+ \Sigma^+$

$K^+ \Sigma^+ : |\tfrac{1}{2},\tfrac{1}{2}\rangle |1,1\rangle = |\tfrac{3}{2},\tfrac{3}{2}\rangle$

$\pi^+ p : |1,1\rangle |\tfrac{1}{2},\tfrac{1}{2}\rangle = |\tfrac{3}{2},\tfrac{3}{2}\rangle$

$\pi^+ p \to K^+ + \Sigma^+ : \langle \tfrac{3}{2},\tfrac{3}{2}|H|\tfrac{3}{2},\tfrac{3}{2}\rangle \Rightarrow M_{3/2}$

$\sigma_{\pi^+ p \to K^+ \Sigma^+} = |\langle \pi^+ p|H|K^+ \Sigma^+\rangle|^2$

Ratios: $\sigma_c : \sigma_b : \sigma_a = 9|M_{3/2}|^2 : |M_{3/2} + 2M_{1/2}|^2 : 2|M_{3/2} - M_{1/2}|^2$

$\Rightarrow \boxed{9 : 2 : 1}$

3) $\Xi^- + p \to \Lambda^0 + \Lambda^0$, $\Lambda^0 \to \pi^- p$

Ξ^- spin $s_1 = \frac{1}{2} \pm$
p spin $s_2 = \frac{1}{2} \pm$
$\Rightarrow S_{\Xi^- p} = \frac{1}{2} \pm \frac{1}{2} = 1$ or 0

We know that $L_{\Xi^- p} = 0$, so $\vec{J}_{\Xi^- p} = 0+1 = 1$ or $0+0=0$

So $\vec{J}_{\Lambda\Lambda}$ must also be 0/1 by conservation laws

π^- spin $s_1 = 0$, so Λ spin $S_{\Lambda_1} = \frac{1}{2} \pm$
$S_{\Lambda_2} = \frac{1}{2} \pm$
$\Rightarrow S_{\Lambda\Lambda} = 0$ or 1

So our allowed combinations are

$L=0$ $S=0$ $J=0$
$S=1$ $J=1$

$L=1$ $S=0$ $J=1$
$S=1$ $J=1$

$L=2$ $S=1$ $J=1$

$\Lambda\Lambda$ can be anti symmetric in orientation : $(-1)^{L+S+1} \Rightarrow -1$
so $L+S$ must be even

$\Rightarrow$ $L=0, S=0 \Rightarrow J=0,$ $S=0$ $\boxed{P_{\Xi} = (-1)^0 = +}$
(anti-symmetric spin orientation)

Alternatively, if the spin orientation is symmetric: $L=1, S=0 \Rightarrow J=0$ $J=1$ $\rightarrow \boxed{P_{\Xi} = (-1)^1 = -}$