linear long-wave speed over the topography is critical,

$$c_l = \frac{U_o}{1-h} + (U_{1c} - U_{2c})\frac{1-h-2d_{1c}}{1-h} - \left[\left(1 - \frac{(U_{1c}-U_{2c})^2}{1-h}\right)\frac{d_{1c}(1-h-d_{1c})}{1-h}\right]^{1/2} = 0, \tag{2.10}$$

in the frame translating with the gravity current. Here $U_{1c,2c}$ and $d_{1c,2c}$ are the speeds and thicknesses of the respective layers over the obstacle crest. The bounding curves in (h, U_o) parameter space for critical flow can be found explicitly by solving

$$d_o(1-d_o)\left(\frac{U_o}{c_o}\right)^2\left[\frac{d_o^2}{d_{1c}^3} + \frac{(1-d_o)^2}{(1-d_{1c}-d_oh)^3}\right] - 1 = 0, \tag{2.11}$$

$$\frac{1}{2}d_o(1-d_o)\left(\frac{U_o}{c_o}\right)^2\left[\frac{d_o^2}{d_{1c}^2} + \frac{(1-d_o)^2}{(1-d_{1c}-d_oh)^2}\right] + d_{1c} + d_o(h-1) = 0, \tag{2.12}$$

where

$$c_o = \sqrt{d_o(1-d_o)} \tag{2.13}$$

is the linear long-wave speed in the two-layer ambient; see Baines (1995), in which (3.6.7) corresponds to (2.10) and (3.6.5) corresponds to (2.11)–(2.12). In figure 2(*c,d*), the boundary curves corresponding to the onset of critical flow over topography of height h are shown for $d_o = 0.1$ and 0.3. It can be seen that the slower Holyer–Huppert solution branch always falls below the lower boundary of the critical regime from Baines' theory. The Holyer–Huppert upper branch is for the most part above the bounding supercritical curve, except for $S \rightarrow 1$ where these solutions are found below Baines' limit. Note that Baines' supercritical curve is, like the subcritical curve, parabolic for small h, but becomes flat when h is beyond a critical value. Baines (1984) explains that the parabolic branch terminates at a specific h (which depends on d_o) and is met by a spurious branch, so that the supercritical boundary to the right is constant and equal to the value of U_o at the termination point. This value of U_o is very close but not equal to C_{cs} (and also varies slightly with d_o). However, since C_{cs} is the fastest wave supported by the ambient waveguide, and consistent with Stastna & Peltier (2005), we argue that C_{cs} is the appropriate upper limit for critical flow in the flat region and we terminate the upper parabolic curve at $U_o = C_{cs}$. The subtle distinction between C_{cs} and the Baines upper limit is not explored further.

3. Incorporation of an upstream bore into conjugate flow theory

For combinations of (h, U_o) between the subcritical and supercritical boundaries, Baines (1984) showed that the flow over the obstacle crest must be critical, and disturbances will propagate upstream in the form of internal bores, which have been observed in experiments (Baines 1984; Melville & Helfrich 1987). These upstream bores may be undular trains of large-amplitude internal waves due to non-hydrostatic dispersion (Grimshaw & Smyth 1986; Melville & Helfrich 1987) and in the extended KdV model the upstream disturbance can be a monotonic, or conjugate, bore (Melville & Helfrich 1987). Recall that non-hydrostatic effects are included in these models, but are not present in Baines' hydrostatic theory. In the critical flow regime, Baines (1984) introduced an upstream bore connected to the flow near the region of topography. Rottman & Simpson (1989) extended this framework to gravity currents in the