1. CVRP formulation integrating in one matrix all the vehicles

1.1. Sets

The CVRP is designed upon a connected and directed Graph $G = (N, A)$ where N is the set of nodes and A is the set of arcs. The set of clients C and a depot 0, ($N = C \cup \{0\}$) are also given. The cost of using the arc (i, j) is represented by c_{ij} (because the fleet is homogeneous, the cost is the same for all the vehicles). The number of vehicles available is $Vehic$ and the demand of every client is d_i. Lastly, as it is supposed a homogeneous fleet, the capacity, q, is the same for all vehicles.

1.2. Variables

It uses the following sets of variables:

- $x_{ij} \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by some vehicle.
- f_{ij}: represents the cumulative packages delivered of the vehicle in every node. Because a matrix will carry all the information, this variable is needed to control the capacity of the vehicles. This variable also guarantees that there will not be any subcircuit.

1.3. Formulation

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\begin{aligned}
\text{(VRP) min} \quad & Z = \sum_{i \in N} \sum_{j \in N} c_{ij} \cdot x_{ij} && && (1)\\
\text{subject to} \quad & \sum_{j \in N} x_{ij} = 1 && \forall\, i \in C && (2)\\
& \sum_{j \in N} x_{0j} \leq Vehic && && (3)\\
& \sum_{i \in N} x_{i0} = \sum_{j \in N} x_{0j} && && (4)\\
& \sum_{i \in N} x_{ih} - \sum_{j \in N} x_{hj} = 0 && \forall\, h \in C && (5)\\
& \sum_{j \in N} f_{0j} = 0 && && (6)\\
& \sum_{j \in N} f_{ij} - \sum_{j \in N} f_{ji} - d_i = 0 && \forall\, i \in C && (7)\\
& x_{ii} = 0 && \forall\, i \in N && (8)\\
& x_{ij} \in \{0, 1\} && \forall\, i,\ j \in N, && (9)\\
& 0 \leq f_{ij} \leq q * x_{ij} && \forall\, i \in N, j \in N && (10)
\end{aligned}
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Constraints (2) ensure that every client is visited at least once. Constraint (3) guarantees that the maximum number of vehicles used is the number of vehicles available. Equality (4) guarantee that the same number of vehicles that departed from the depot return to it. Equalities (5) assure that the flow continues through the network. Equation (6) makes the initial flow of every vehicle equal to zero and (7) adds the demand of every node to the flow. The last two expressions define the domain of the decision variables.