1. HFMDPCVRPTW formulation

1.1. Introduction

This formulation is a PCVRP (Periodic Capacitated Vehicle Routing Problem) where the fleet of vehicles is heterogeneous and it can be served from different depots. The vehicles are allowed to start the route everyday from different depots but must finish in the same depot were the route started. Additionally, clients have a time window in which they must be attended (is not possible to violate this time window). Clients also have possible patterns of attention. These pattern of visits are: daily, thrice per week, twice per week, once per week, once every two weeks. As Sundays are days-off, the time-span of the problem is of 12 days.

1.2. Sets

- N is the set of all the nodes (including the depots). Subindexes i, j are used to indicate an element of N.
- C is the set of clients. Subindex c is used to indicate an element of C.
- D is the set of depots. Subindex d is used to indicate an element of D.
- Δ is the set of days. Subindex δ is used to indicate an element of Δ. As referenced before, the scheduling span is of 12 days.
- K is the set of vehicles. Subindex k is used to indicate an element of K.
- P is the set of patterns of visits available. Subindex p is used to indicate an element of P.

1.3. Parameters

- $F_{p\delta}$ is 1 if the pattern p requires the client to be visited in the day δ and 0 otherwise.
- H_{cp} is 1 if client c is eligible for pattern p and 0 otherwise.
- q_k is the capacity of vehicle k un units of demand.
- dem_c is the demand of client c for each visit.
- s_c is the time to serve client c.
- $[a_c, b_c]$ is the time window in which client c must be attended.
- t_{ij}^k is the time to traverse arc (i, j) with vehicle k.
- R_d is the capacity (in number of vehicles) of depot d.
- A largely enough value (**M**) is used. It can be bounded by maximum time of a route.

1.4. Variables

It uses the following sets of variables:

- $x_{ij}^{k\delta} \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by vehicle k in the day δ, and 0 otherwise.
- $T_{ij}^{k\delta} \geq 0$: time at which service starts in node j from node i in the vehicle k on day δ.
- $y_{cp} \in \{0, 1\}$: binary variable which takes the value 1 if client c is visited according to pattern p and 0 otherwise.

- $w_d^{k\delta} \in \{0,1\}$: Binary variable which takes the value 1 if vehicle k is served from depot d on day δ and zero otherwise.

- $f_{ij}^{k\delta}$ demand already attended when vehicle k arrives to j from i on day δ.

1.5. Formulation

$$\text{(VRP) min} \quad Z = \sum_{\delta \in \Delta} \sum_{d \in D} \sum_{j \in N} \sum_{k \in K} T_{dj}^{k\delta} - T_{jd}^{k\delta} + t_{dj}^{k} \cdot x_{dj}^{k\delta} \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in N, j \neq c} \sum_{k \in K} x_{cj}^{k\delta} = \sum_{p \in P} F_{p\delta} \cdot y_{cp} \qquad \forall \delta \in \Delta, \forall c \in C \tag{2}$$

$$\sum_{p \in P} H_{cp} \cdot y_{cp} \geq 1 \qquad \forall c \in C \tag{3}$$

$$\sum_{j \in N, j \neq d} x_{dj}^{k\delta} \leq w_{d}^{k\delta} \qquad \forall \delta \in \Delta, \forall d \in D, \forall k \in K \tag{4}$$

$$\sum_{d \in D} w_{d}^{k\delta} \leq 1 \qquad \forall \delta \in \Delta, \forall k \in K \tag{5}$$

$$\sum_{i \in N, i \neq d} x_{id}^{k\delta} = \sum_{j \in N, j \neq d} x_{dj}^{k\delta} \qquad \forall \delta \in \Delta, \forall d \in D, \forall k \in K \tag{6}$$

$$\sum_{i \in N, i \neq c} x_{ic}^{k\delta} - \sum_{j \in N, j \neq c} x_{cj}^{k\delta} = 0 \qquad \forall \delta \in \Delta, \forall c \in C, \forall k \in K \tag{7}$$

$$\sum_{k \in K} w_{d}^{k\delta} \leq R_{d} \qquad \forall \delta \in \Delta, \forall d \in D \tag{8}$$

$$\sum_{i \in N, i \neq c} \sum_{k \in K} T_{ic}^{k\delta} \geq a_{c} \qquad \forall \delta \in \Delta, \forall c \in C \tag{9}$$

$$\sum_{i \in N, i \neq c} \sum_{k \in K} T_{ic}^{k\delta} \leq b_{c} - s_{c} \qquad \forall \delta \in \Delta, \forall c \in C \tag{10}$$

$$\sum_{i \in N, i \neq c} T_{ic}^{k\delta} + s_c + t_{cj}^{k} \cdot x_{cj}^{k\delta} - M \cdot (1 - x_{cj}^{k\delta}) \leq T_{cj}^{k\delta} \quad \forall \delta \in \Delta, \forall c \in C, \forall j \in N (c \neq j), \forall k \in K \tag{11}$$

$$\sum_{\delta \in \Delta} \sum_{c \in C} \sum_{d \in D} f_{dc}^{k\delta} = 0 \qquad \forall k \in K \tag{12}$$

$$\sum_{i \in N, i \neq c} (f_{ic}^{k\delta} + dem_c \cdot x_{ci}^{k\delta}) - \sum_{j \in N, j \neq c} f_{cj}^{k\delta} = 0 \quad \forall \delta \in \Delta, \forall c \in C, \forall k \in K \tag{13}$$

$$f_{ij}^{k\delta} \leq q_k \cdot x_{ij}^{k\delta} \qquad \forall \delta \in \Delta, \forall i \in N, \forall j \in N, \forall k \in K \tag{14}$$

$$x_{ij}^{k\delta} \in \{0, 1\} \qquad \forall \delta \in \Delta, \forall i \in N, \forall j \in N, \forall k \in K \tag{15}$$

$$0 \leq T_{ij}^{k\delta} \leq M \cdot x_{ij}^{k\delta} \qquad \forall \delta \in \Delta, \forall i \in N, \forall j \in N, \forall k \in K \tag{16}$$

$$y_{cp} \in \{0, 1\} \qquad \forall c \in C, \forall p \in P \tag{17}$$

$$w_d^k \in \{0, 1\} \qquad \forall d \in D, \forall k \in K \tag{18}$$

The objective function (1) minimizes the total time of all routes. Constraints (2) to (3) define that every client is visited using at least one of their allowed patterns. Constraints (4) to (7) ensure that flow continues through the network: (4) define the start of the routes, (5) allow only one route per day per vehicle, (6) force the route to finish at the same depot it started and (7) ensure that every node is left after every visit. Constraints (8) are the physical capacity constraints of every depot. Constraint (9) to (11) condition the arrival time of the vehicles in the time window of every client: (9) make the vehicles arrive after the time window opens, (10) make the vehicles arrive before they can not service the client without violating the time window and (11) actualize the values of variable T. Constraints (12) to (13) are the subtour elimination constraints and actualize the load of demand attended in every visit. Constraints (14) to (18) are the domain of the decision variables.