1. HFMDCVRP formulation

1.1. Introduction

This formulation is for a HFMDCVRP (Heterogeneous Fleet Multi Depot Capacitated Vehicle Routing Problem) were the fleet is composed by different vehicles and more than one depot can dispatch the vehicles. The objective is simple: assign a route and a depot to every vehicle to attend the demand of all the clients.

1.2. Sets

The HFMDCVRP is designed upon a connected and directed Graph $G = (N, A)$. The set of nodes N is composed by a set of clients C and a set of depots D ($N = C \cup D$). The set of arcs is represented by A. The set K corresponds to the set of available vehicles to attend the demand. As the fleet is supposed to be heterogeneous, the capacity of each vehicle k is given by q_k. The cost of using the arc (i, j) is represented with c_{ij}^k for vehicle k. The demand of every client is dem_c. Lastly, as the depots have a finite capacity to attend the vehicles, a maximum depot capacity R_d is added to each depot d (this capacity is given in terms of number of vehicles the depot can attend). A largely enough value ($\mathbf{M}$) is used. It can be bounded by the number of clients.

1.3. Variables

It uses the following sets of variables:

- $x_{ij}^k \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by the k^{th} vehicle, and zero otherwise. The number of variables of this family is $|K| \cdot |N^2|$.

- $u_i \geq 0$: continuous auxiliary variable which represents the position in which node i is visited on its route. The objective of this variable is to ensure that the routes of each vehicle are well-defined and avoid cycling. The number of variables of this family is $|N|$.

- $w_d^k \in \{0, 1\}$: Binary variable which takes the value 1 if vehicle k is served from depot d and zero otherwise. The number of variables of this family is $|K| \cdot |D|$.

1.4. Formulation

$$\text{(HFMDCVRP)} \min \quad Z = \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij}^k \cdot x_{ij}^k \tag{1}$$

$$\text{subject to} \quad \sum_{k\in K}\sum_{j\in N} x_{cj}^k = 1 \qquad \forall\ c \in C \tag{2}$$

$$\sum_{c\in C}\sum_{j\in N} dem_c \cdot x_{cj}^k \le q_k \qquad \forall\ k \in K \tag{3}$$

$$\sum_{j\in N} x_{dj}^k = w_d^k \qquad \forall\ d \in D,\ k \in K \tag{4}$$

$$\sum_{i\in N} x_{id}^k = w_d^k \qquad \forall\ d \in D,\ k \in K \tag{5}$$

$$\sum_{i\in N} x_{ic}^k - \sum_{j\in N} x_{cj}^k = 0 \qquad \forall\ c \in C,\ k \in K \tag{6}$$

$$\sum_{k\in K} w_d^k \le R_d \qquad \forall\ d \in D \tag{7}$$

$$u_d = 1 \qquad \forall\ d \in D \tag{8}$$

$$u_c \ge (u_i + 1) - M\left(1 - \sum_{k\in K} x_{ic}^k\right) \qquad \forall\ i \in N,\ c \in C \tag{9}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall\ i,\ j \in N,\ k \in K \tag{10}$$

$$2 \le u_c \le |N| \qquad \forall\ c \in C \tag{11}$$

$$w_d^k \in \{0,1\} \qquad \forall\ d \in D,\ k \in K \tag{12}$$

Constraints (2) ensure that every client is visited at least once. Inequalities (3) limit the amount of clients served by a vehicle according to their capacity. Constraints (4) and (5) guarantee that the vehicles depart and finish the route at the depot assigned to them. Equalities (6) assure that the flow continues through the network. Inequality (6) limits the number of vehicles that a depot can attend. Equation (8) is used to secure that the first node to be visited according to auxiliary variables u is the depot. Inequalities (9) determine that the value of u_j must be higher than u_i, when i is the previously visited node. These two last sets of constraints avoid the possibility of having subcircuits in the routes. The last three expressions define the domain of the decision variables.