1. PCVRP formulation

1.1. Introduction

This formulation is a simple PCVRP (Periodic Capacitated Vehicle Routing Problem) were the vehicles have a capacity according to the demand they can attend and must be scheduled different time periods. The objective is simple: assign a route to every vehicle everyday to attend the demand of all the clients.

1.2. Sets

The PCVRP is designed upon a connected and directed Graph $G = (N, A)$ where N is the set of nodes and A is the set of arcs. The set of clients C and a depot 0, $(N = C \cup \{0\})$ are also given. The cost of using the arc (i, j) is represented by c_{ij} (because the fleet is homogeneous, the cost is the same for all the vehicles). The set of days is Δ. The index set of available vehicles to serve the demand is K and the demand of every client is $d_{c\delta}$. Lastly, as it is supposed a homogeneous fleet, the capacity, q, is the same for all vehicles. A largely enough value ($\mathbf{M}$) is used. It can be bounded by the number of clients.

1.3. Variables

It uses the following sets of variables:

- $x_{ij}^{k\delta} \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by the k^{th} vehicle in the day δ, and zero otherwise. The number of variables of this family is $|K| \cdot |\Delta| \cdot |N^2|$.

- $u_{i\delta} \geq 0$: continuous auxiliary variable which represents the position in which node i is visited on its route in day δ. The objective of this variable is to ensure that the routes of each vehicle are well-defined and avoid cycling. The number of variables of this family is $|N| \cdot |\Delta|$.

1.4. Formulation

$$\text{(VRP)} \min \quad Z = \sum_{\delta\in\Delta}\sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij} \cdot x_{ij}^{k\delta} \tag{1}$$

$$\text{subject to} \quad \sum_{k\in K}\sum_{j\in N} x_{cj}^{k\delta} = 1 \qquad \forall\, c \in C, \forall\, \delta \in \Delta \tag{2}$$

$$\sum_{c\in C}\sum_{j\in N} d_{c\delta} \cdot x_{cj}^{k\delta} \leq q \qquad \forall\, k \in K, \forall\, \delta \in \Delta \tag{3}$$

$$\sum_{j\in N} x_{0j}^{k\delta} = 1 \qquad \forall\, k \in K, \forall\, \delta \in \Delta \tag{4}$$

$$\sum_{i\in N} x_{i0}^{k\delta} = 1 \qquad \forall\, k \in K, \forall\, \delta \in \Delta \tag{5}$$

$$\sum_{i\in N} x_{ic}^{k\delta} - \sum_{j\in N} x_{cj}^{k\delta} = 0 \qquad \forall\, c \in C,\; k \in K, \forall\, \delta \in \Delta \tag{6}$$

$$u_{0\delta} = 1 \qquad \forall\, \delta \in \Delta \tag{7}$$

$$u_{c\delta} \geq (u_{i\delta} + 1) - M\left(1 - \sum_{k\in K} x_{ic}^{k\delta}\right) \quad \forall\, i \in N,\; c \in C, \forall\, \delta \in \Delta \tag{8}$$

$$x_{ij}^{k\delta} \in \{0,1\} \qquad \forall\, i,\; j \in N,\; k \in K,\; \delta \in \Delta \tag{9}$$

$$2 \leq u_{c\delta} \leq |N| \qquad \forall\, c \in C, \forall\, \delta \in \Delta \tag{10}$$

Constraints (2) ensure that every client is visited at least once everyday. Inequalities (3) limit the amount of clients served by a vehicle according to their capacity everyday. Constraints (4) and (5) guarantee that the vehicles depart and finish the route at the depot 0 always. Equalities (6) assure that the flow continues through the network. Equation (7) is used to secure that the first node to be visited according to auxiliary variables u is the depot everyday. Inequalities (8) determine that the value of $u_{j\delta}$ must be higher than $u_{i\delta}$ everyday, when i is the previously visited node. These two last sets of constraints avoid the possibility of having subcircuits in the routes. The last two expressions define the domain of the decision variables.