

1. HFMDPCVRPTW formulation

1.1. Introduction

This formulation is a PCVRP (Periodic Capacitated Vehicle Routing Problem) where the fleet of vehicles is heterogeneous and can be served from different depots. Additionally the clients must be served within a time-window. The clients have a defined frequency of visits in the time span:

- T6: client must be visited everyday.
- T3: clients must be visited three times per week.
- T2: clients must be visited two times per week.
- T1: clients must be visited once per week.
- Q1: clients must be visited once every two weeks.

1.2. Sets and parameters

- Directed Graph $G = (N, A)$ where N is the set of nodes and A is the set of arcs.
- Set of clients C .
- Set of depots D .
- Set of days Δ .
- Set of vehicles K .
- Set of patterns of visits available P .

- $F_{p\delta}$ is 1 if the pattern p requires the client to be visited in the day δ and 0 otherwise.
- H_{cp} is 1 if client c is eligible for pattern p and 0 otherwise.
- The capacity of every vehicle is q_k .
- The demand of every client for each visit is dem_c .
- The time to serve every client is s_c .
- The window of time in which every client must be attended is given by $[a_c, b_c]$.
- The time to traverse every arc with every vehicle is given by t_{ij}^k .
- The capacity to attend vehicles in every depot is R_d .
- A largely enough value (\mathbf{M}) is used. It can be bounded by maximum time of a route.

1.3. Variables

It uses the following sets of variables:

- $x_{ij}^{k\delta} \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by the k^{th} vehicle in the day δ , and 0 otherwise.
- $T_{i\delta} \geq 0$: time at which service starts in client i in the day δ .
- $y_{cp} \in \{0, 1\}$: binary variable which takes the value 1 if client c is visited according to pattern p and 0 otherwise.
- $w_d^k \in \{0, 1\}$: Binary variable which takes the value 1 if vehicle k is served from depot d and zero otherwise.

1.4. Formulation

$$(\text{VRP}) \min \quad Z = \sum_{\delta \in \Delta} \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} \cdot x_{ij}^{k\delta} \quad (1)$$

$$\text{subject to} \quad \sum_{k \in K} \sum_{j \in N} x_{cj}^{k\delta} = \sum_{p \in P} y_{cp} \cdot H_{cp} \cdot F_{p\delta} \quad \forall c \in C, \forall \delta \in \Delta \quad (2)$$

$$\sum_{p \in P} y_{cp} = 1 \quad \forall c \in C \quad (3)$$

$$\sum_{c \in C} \sum_{j \in N} dem_c \cdot x_{cj}^{k\delta} \leq q_k \quad \forall k \in K, \forall \delta \in \Delta \quad (4)$$

$$\sum_{j \in N} x_{dj}^{k\delta} \leq w_d^k \quad \forall k \in K, \forall \delta \in \Delta, \forall d \in D \quad (5)$$

$$\sum_{i \in N} x_{id}^{k\delta} = \sum_{j \in N} x_{dj}^{k\delta} \quad \forall k \in K, \forall \delta \in \Delta, \forall d \in D \quad (6)$$

$$\sum_{i \in N} x_{ic}^{k\delta} - \sum_{j \in N} x_{cj}^{k\delta} = 0 \quad \forall c \in C, \forall k \in K, \forall \delta \in \Delta \quad (7)$$

$$\sum_{k \in K} w_d^k \leq R_d \quad \forall d \in D \quad (8)$$

$$T_{d\delta} = 0 \quad \forall \delta \in \Delta, \forall d \in D \quad (9)$$

$$T_{c\delta} \geq a_c \quad \forall \delta \in \Delta, \forall c \in C \quad (10)$$

$$T_{c\delta} \leq b_c - t_c \quad \forall \delta \in \Delta, \forall c \in C \quad (11)$$

$$T_{c\delta} \geq (T_{i\delta} + \sum_{k \in K} x_{ic}^{k\delta} \cdot t_{ic}^k + s_i) - M \left(1 - \sum_{k \in K} x_{ic}^{k\delta} \right) \quad \forall i \in N, \forall c \in C, \forall \delta \in \Delta \quad (12)$$

$$x_{ij}^{k\delta} \in \{0, 1\} \quad \forall i, j \in N, k \in K, \delta \in \Delta \quad (13)$$

$$0 \leq T_{c\delta} \leq M \quad \forall c \in C, \forall \delta \in \Delta \quad (14)$$

$$y_{cp} \in \{0, 1\} \quad \forall c \in C, \forall p \in P \quad (15)$$

$$w_d^k \in \{0, 1\} \quad \forall d \in D, \forall k \in K \quad (16)$$

Constraints (2) ensure that every client is visited according to the frequency selected for that client. Equalities (3) guarantee that a pattern is selected for every client. Inequalities (4) limit the amount of clients served by a vehicle according to their capacity everyday. Constraints (5) and (6) guarantee that the vehicles can only depart and finish the route at the depot it is assigned. Equalities (7) assure that the flow continues through the network. Inequalities (8) keep the capacity of the depots at check. Equation (9) is used to secure that the time of departure from the depot is 0. Constraints (10) and (11) forces the time of arrival of the routes to be inside the time windows. Inequalities (12) determine that, when i is the previously visited node, the time of arrival to the new node is greater than the time needed to get from c to i plus the time expended in i . The last two expressions define the domain of the decision variables.