1. HFMDPCVRPTW formulation

1.1. Introduction

This formulation is a PCVRP (Periodic Capacitated Vehicle Routing Problem) where the fleet of vehicles is heterogeneous and can be served from different depots. Additionally the clients must be served within a time-window. The clients have a defined frequency of visits in the time span:

- T6: client must be visited everyday.
- T3: clients must be visited three times per week.
- T2: clients must be visited two times per week.
- T1: clients must be visited once per week.
- Q1: clients must be visited once every two weeks.

1.2. Sets and parameters

- Set of all the nodes N. Subindexes i, j will be used to indicate a element of N.
- A are the arcs formed between all nodes.
- Directed Graph $G = (N, A)$.
- Set of clients C. Subindex c will be used to indicate a element of C.
- Set of depots D. Subindex d will be used to indicate a element of D.
- Set of days Δ. Subindex δ will be used to indicate a element of Δ.

- Set of types of vehicles K. Subindex k will be used to indicate a element of K.
- Set of patterns of visits available P. Subindex p will be used to indicate a element of P.
- $F_{p\delta}$ is 1 if the pattern p requires the client to be visited in the day δ and 0 otherwise.
- H_{cp} is 1 if client c is eligible for pattern p and 0 otherwise.
- The capacity of type of vehicle k is q_k and the number of vehicles available is veh_k.
- The demand of client c for each visit is dem_c.
- The time to serve client c is s_c.
- The window of time in which client c must be attended is given by $[a_c, b_c]$.
- The time to traverse arc (i, j) with type of vehicle k is given by t_{ij}^k.
- The capacity to attend vehicles in depot d is R_d.
- A largely enough value ($\mathbf{M}$) is used. It can be bounded by maximum time of a route.

1.3. Variables

It uses the following sets of variables:

- $x_{ij}^{k\delta} \in \{0,1\}$: binary variable which takes the value 1 if the arc (i,j) is used by the k^{th} type of vehicle in the day δ, and 0 otherwise.

- $T_{i\delta} \geq 0$: time at which service starts in node i in the day δ.

- $y_{cp} \in \{0,1\}$: binary variable which takes the value 1 if client c is visited according to pattern p and 0 otherwise.

- $w_{cd}^{\delta} \in \{0,1\}$: Binary variable which takes the value 1 if client c is attended from depot d on day δ.

- $f_{ij}^{k\delta}$ demand already attended when type of vehicle k arrives to j from i on day δ.

1.4. Formulation

$$\text{(VRP) min} \quad Z = \sum_{\delta \in \Delta} \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij}^{k} \cdot x_{ij}^{k\delta} \tag{1}$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{j \in N} x_{cj}^{k\delta} = \sum_{p \in P} F_{p\delta} \cdot y_{cp} \qquad \forall c \in C, \forall \delta \in \Delta \tag{2}$$

$$\sum_{p \in P} H_{cp} \cdot y_{cp} = 1 \qquad \forall c \in C \tag{3}$$

$$x_{dc}^{k\delta} \leq w_{cd}^{\delta} \qquad \forall c \in C, \forall d \in D, \forall k \in K, \forall \delta \in \Delta \tag{4}$$

$$x_{cd}^{k\delta} \leq w_{cd}^{\delta} \qquad \forall c \in C, \forall d \in D, \forall k \in K, \forall \delta \in \Delta \tag{5}$$

$$\sum_{i \in N} x_{ic}^{k\delta} - \sum_{j \in N} x_{cj}^{k\delta} = 0 \qquad \forall c \in C, \forall k \in K, \forall \delta \in \Delta \tag{6}$$

$$\sum_{c \in C} \sum_{k \in K} x_{cd}^{k\delta} \leq R_d \qquad \forall d \in D, \forall \delta \in \Delta \tag{7}$$

$$T_{d\delta} = 0 \qquad \forall \delta \in \Delta, \forall d \in D \tag{8}$$

$$T_{c\delta} \geq a_c \qquad \forall \delta \in \Delta, \forall c \in C \tag{9}$$

$$T_{c\delta} \leq b_c - s_c \qquad \forall \delta \in \Delta, \forall c \in C \tag{10}$$

$$T_{c\delta} \geq (T_{i\delta} + \sum_{k \in K} t_{ic}^{k} \cdot x_{ic}^{k\delta} + s_i) - M \left(1 - \sum_{k \in K} x_{ic}^{k\delta}\right) \quad \forall i \in N, \forall c \in C, \forall \delta \in \Delta \tag{11}$$

$$\sum_{c \in C} \sum_{d \in D} x_{cd}^{k\delta} \leq veh_k \qquad \forall k \in K, \forall \delta \in \Delta \tag{12}$$

$$\sum_{c \in C} \sum_{d \in D} \sum_{\delta \in \Delta} f_{dc}^{k\delta} = 0 \qquad \forall k \in K \tag{13}$$

$$\sum_{i \in N} (f_{ic}^{k\delta} + dem_c \cdot x_{ci}^{k\delta}) - \sum_{j \in N} f_{cj}^{k\delta} = 0 \qquad \forall k \in K, \forall \delta \in \Delta \tag{14}$$

$$f_{ij}^{k\delta} \leq q_k \cdot x_{ij}^{k\delta} \qquad \forall i \in N, \forall j \in N, \forall k \in K, \forall \delta \in \Delta \tag{15}$$

$$x_{ij}^{k\delta} \in \{0, 1\} \qquad \forall i \in N, \forall j \in N, \forall k \in K, \forall \delta \in \Delta \tag{16}$$

$$0 \leq T_{c\delta} \leq M \qquad \forall c \in C, \forall \delta \in \Delta \tag{17}$$

$$y_{cp} \in \{0, 1\} \qquad \forall c \in C, \forall p \in P \tag{18}$$

$$w_d^k \in \{0, 1\} \qquad \forall d \in D, \forall k \in K \tag{19}$$

Constraints (2) ensure that every client is visited according to the frequency selected for that client. Equalities (3) guarantee that a pattern is selected for every client. Constraints (4) and (5) guarantee that the type of vehicles can only depart and finish the route at the depot were they started. Equalities (6) assure that the flow continues through the network. Inequalities (7) keep the capacity of the depots at check. Equation (8) is used to secure that the time of departure from the depot is 0. Constraints (9) and (10) forces the time of arrival of the routes to be inside the time windows. Inequalities (11) determine that, when i is the previously visited node, the time of arrival to the new node is greater than te time needed to get from c to i plus the time expended in i. (12) guarantee that the maximum number of vehicles for each type is not exceeded. Equalities (13) assign the load from the depot to be zero. (14) assure that every time a client is attended the demand is added to the load. The last five expressions define the domain of the decision variables.