

1. PCVRP formulation

1.1. Sets

The PCVRP is designed upon a connected and directed Graph $G = (N, A)$ where N is the set of nodes and A is the set of arcs. The set of clients C and a depot 0, ($N = C \cup \{0\}$) are also given. The cost of using the arc (i, j) is represented by c_{ij} (because the fleet is homogeneous, the cost is the same for all the vehicles). The set of days is Δ . The index set of available vehicles to serve the demand is K and the demand of every client is $d_{i\delta}$. Lastly, as it is supposed a homogeneous fleet, the capacity, q , is the same for all vehicles.

1.2. Variables

It uses the following sets of variables:

- $x_{ij}^{k\delta} \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by the k^{th} vehicle in the day δ , and zero otherwise. The number of variables of this family is $|K||\Delta| \cdot |N^2|$.
- $u_{i\delta} \geq 0$: continuous auxiliary variable which represents the position in which node i is visited on its route in day δ . The objective of this variable is to ensure that the routes of each vehicle are well-defined and avoid cycling. The number of variables of this family is $|N||\Delta|$.

1.3. Formulation

$$(\text{VRP}) \min \quad Z = \sum_{\delta \in \Delta} \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} \cdot x_{ij}^{k\delta} \quad (1)$$

$$\text{subject to} \quad \sum_{k \in K} \sum_{j \in N} x_{ij}^{k\delta} = 1 \quad \forall i \in C, \forall \delta \in \Delta \quad (2)$$

$$\sum_{i \in C} \sum_{j \in N} d_{i\delta} \cdot x_{ij}^{k\delta} \leq q \quad \forall k \in K, \forall \delta \in \Delta \quad (3)$$

$$\sum_{j \in N} x_{0j}^{k\delta} = 1 \quad \forall k \in K, \forall \delta \in \Delta \quad (4)$$

$$\sum_{i \in N} x_{i0}^{k\delta} = 1 \quad \forall k \in K, \forall \delta \in \Delta \quad (5)$$

$$\sum_{i \in N} x_{ih}^{k\delta} - \sum_{j \in N} x_{hj}^{k\delta} = 0 \quad \forall h \in C, k \in K, \forall \delta \in \Delta \quad (6)$$

$$u_{0\delta} = 1 \quad \forall \delta \in \Delta \quad (7)$$

$$u_{j\delta} \geq (u_{i\delta} + 1) - M \left(1 - \sum_{k \in K} x_{ij}^{k\delta} \right) \quad \forall i \in N, j \in C, \forall \delta \in \Delta \quad (8)$$

$$x_{ij}^{k\delta} \in \{0, 1\} \quad \forall i, j \in N, k \in K \quad (9)$$

$$2 \leq u_{i\delta} \leq |N| \quad \forall i \in C, \forall \delta \in \Delta S \quad (10)$$

Constraints (2) ensure that every client is visited at least once everyday. Inequalities (3) limit the amount of clients served by a vehicle according to their capacity everyday. Constraints (4) and (5) guarantee that the vehicles depart and finish the route at the depot 0 always. Equalities (6) assure that the flow continues through the network. Equation (7) is used to secure that

the first node to be visited according to auxiliary variables u is the depot everyday. Inequalities (8) determine that the value of $u_{j\delta}$ must be higher than $u_{i\delta}$ everyday, when i is the previously visited node. These two last sets of constraints avoid the possibility of having subcircuits in the routes. The last two expressions define the domain of the decision variables.