1. Common CVRP formulation

1.1. Introduction

This formulation is a simple CVRP (Capacitated Vehicle Routing Problem) where the vehicles have a capacity according to the demand they can attend. The objective is simple: assign a route to every vehicle to attend the demand of all the clients.

1.2. Sets

This CVRP is designed upon a connected and directed Graph $G = (N, A)$ where N is the set of nodes and A is the set of arcs. The set of clients C and a depot 0, ($N = C \cup \{0\}$) are also given. The cost of using the arc (i, j) is represented by c_{ij} (because the fleet is homogeneous, the cost is the same for all the vehicles). The index set of available vehicles to serve the demand is K and the demand of every client is d_c. Lastly, as it is supposed a homogeneous fleet, the capacity, q, is the same for all vehicles. A largely enough value ($\mathbf{M}$) is used. It can be bounded by the number of clients.

1.3. Variables

It uses the following sets of variables:

- $x_{ij}^k \in \{0, 1\}$: binary variable which takes the value 1 if the arc (i, j) is used by the k^{th} vehicle, and zero otherwise. The number of variables of this family is $|K| \cdot |N^2|$.
- $u_i \geq 0$: continuous auxiliary variable which represents the position in which node i is visited on its route. The objective of this variable is

to ensure that the routes of each vehicle are well-defined and avoid cycling. The number of variables of this family is $|N|$.

1.4. Formulation

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\begin{aligned}
\text{(VRP)}\ \min \quad & Z = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} \cdot x_{ij}^k && && (1)\\
\text{subject to} \quad & \sum_{k \in K} \sum_{j \in N} x_{cj}^k = 1 && \forall\, c \in C && (2)\\
& \sum_{c \in C} \sum_{j \in N} d_c \cdot x_{cj}^k \le q && \forall\, k \in K && (3)\\
& \sum_{j \in N} x_{0j}^k = 1 && \forall\, k \in K && (4)\\
& \sum_{i \in N} x_{i0}^k = 1 && \forall\, k \in K && (5)\\
& \sum_{i \in N} x_{ic}^k - \sum_{j \in N} x_{cj}^k = 0 && \forall\, c \in C,\ k \in K && (6)\\
& u_0 = 1 && && (7)\\
& u_c \ge (u_i + 1) - M \left(1 - \sum_{k \in K} x_{ic}^k\right) && \forall\, i \in N,\ c \in C && (8)\\
& x_{ij}^k \in \{0, 1\} && \forall\, i,\ j \in N,\ k \in K && (9)\\
& 2 \le u_c \le |N| && \forall\, c \in C && (10)
\end{aligned}
$$

Constraints (2) ensure that every client is visited at least once. Inequalities (3) limit the amount of clients served by a vehicle according to their capacity. Constraints (4) and (5) guarantee that the vehicles depart and finish the route at the depot 0. Equalities (6) assure that the flow continues through the network. Equation (7) is used to secure that the first node to be visited according to auxiliary variables u is the depot. Inequalities (8) determine that the value of u_j must be higher than u_i, when i is the previously visited node. These two last sets of constraints avoid the possibility of having subcircuits in the routes. The last two expressions define the domain of the decision variables.