## 1. HFMDCVRP formulation

### *1.1. Sets*

Similarly to the CVRP, the HFMDVRP is designed upon a connected and directed Graph $G = (N, A)$. The set of nodes $N$ is composed by a set of clients $C$ and a set of depots $D$ ($N = C \cup D$). The set of arcs is represented by $A$. The set $K$ corresponds to the set of available vehicles to attend the demand. As the fleet is supposed to be heterogeneous, the capacity of each vehicle $k$ is given by $q_k$. The cost of using the arc $(i, j)$ is represented with $c_{ij}^k$ if it is traversed by the vehicle $k$. Note that now the costs are different according to the type of vehicle. The demand of every client is $d_i$. Lastly, as the depots have a finite capacity to attend the clients, a maximum depot capacity $R_i$ is added to each depot $i$ (this capacity is given in terms of amount of demand a depot can serve).

### *1.2. Variables*

It uses the following sets of variables:

- $x_{ij}^k \in \{0, 1\}$: binary variable which takes the value 1 if the arc $(i, j)$ is used by the $k^{th}$ vehicle, and zero otherwise. The number of variables of this family is $|K| \cdot |N^2|$.

- $f_{ij}^k \geq 0$: continuous auxiliary variable which represents the demand already attended when vehicle $k$ use arc $(i, j)$. The objective of this variable is to ensure that the routes of each vehicle are well-defined and avoid cycling. It is also important to evaluate the capacity of the depots. The number of variables of this family is $|K| \cdot |N^2|$.

- $w_d^k \in \{0, 1\}$: Binary variable which takes the value 1 if vehicle $k$ is served from depot $d$ and zero otherwise. The number of variables of this family is $|K| \cdot |D|$.

- $z_i^k \in \{0, 1\}$: Binary variable which takes the value 1 if vehicle $k$ serves client $i$ and zero otherwise. The number of variables of this family is $|K| \cdot |C|$.

*1.3. Formulation*

$$\text{(HFMDVRP) min} \quad Z = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij}^k \cdot x_{ij}^k \tag{1}$$

$$\text{subject to} \quad \sum_{k \in K} z_i^k = 1 \qquad \forall\, i \in C \tag{2}$$

$$\sum_{i \in C} d_i \cdot z_i^k \leq q_k \qquad \forall\, k \in K \tag{3}$$

$$\sum_{j \in N} x_{hj}^k = w_h^k \qquad \forall\, h \in D,\ k \in K \tag{4}$$

$$\sum_{i \in N} x_{ih}^k = w_h^k \qquad \forall\, h \in D,\ k \in K \tag{5}$$

$$\sum_{i \in N} x_{ih}^k - \sum_{j \in N} x_{hj}^k = 0 \qquad \forall\, h \in C,\ k \in K \tag{6}$$

$$\sum_{h \in D} \sum_{j \in C} f_{hj}^k = 0 \qquad \forall\, k \in K \tag{7}$$

$$\sum_{i \in N} f_{ih}^k - \sum_{j \in N} f_{hj}^k + d_h \cdot z_h^k = 0 \quad \forall\, h \in C,\ k \in K \tag{8}$$

$$\sum_{i \in N} \sum_{k \in K} f_{ih}^k \leq R_h \qquad \forall\, h \in D \tag{9}$$

$$w_h^k \in \{0,1\} \qquad \forall\, h \in D,\ k \in K \tag{10}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall\, i,j \in N,\ k \in K \tag{11}$$

$$0 \leq f_{ij}^k \leq q_k \cdot x_{ij}^k \qquad \forall\, i,j \in N,\ k \in K \tag{12}$$

$$z_i^k \in \{0,1\} \qquad \forall\, i \in N,\ k \in K \tag{13}$$

Constraints (2) ensure that every client is visited at least once. Inequalities (3) limit the amount of clients served by a vehicle according to their capacity. Constraints (4) and (5) guarantee that the vehicles depart and finish the route at the depot assigned to them. Equalities (6) assure that the flow continues through the network. Equation (7) determines that all the initial arcs have not attended any demand. Equations (8) adds the demand attended in every client to the next arc used. Inequalities (9) guarantee that the capacity of every depot is not exceeded. The last four expressions define the domain of the decision variables.