2.17 (S&S)

Let Mt represent the cardivascular mortality series discussed in chapter 1, example 1.27. Fit an AR(2) model to the data using linear regression and using Yule-Walker

a. Compare the parameter estimates obtained by the two methods.

Using the ar function in R, we can get the ϕ_1 and ϕ_2

```
> ar_yw$ar
[1] 0.4339481 0.4375768
> as.vector(ar_ols$ar)
[1] 0.4285906 0.4417874
```

<- using Yule Walker

<-using linear regression

We can see that the results we obtain from both methods seems to be similar.

b. Compare the estimated standard errors of the coefficients obtained by linear regression with the corresponding asymptotic approximations as given in property p2.9

The coefficients obtained by linear regression with the corresponding asymptotic approximations

```
> ar_ols$asy.se.coef$ar
[1] 0.03979433 0.03976163
```

To obtain the estimated standard errors, we find the gamma0 to 4 of the data, where

$$\Gamma(0) = 99.77697$$

$$\Gamma(1) = 76.98478$$

$$\Gamma(2) = 77.06748$$

$$\Gamma(3) = 67.95442$$

And $\begin{bmatrix}\gamma_1\\\gamma_2\end{bmatrix} = \begin{bmatrix}76.98478\\77.06748\end{bmatrix}$

Then we can get

$$\hat{\phi} = \begin{bmatrix}\Gamma(0) & \Gamma(1)\\\Gamma(1) & \Gamma(0)\end{bmatrix}^{-1} \begin{bmatrix}\gamma_1\\\gamma_2\end{bmatrix}$$

Using these two, we can obtain $\hat{\sigma}^2 = \hat{\gamma}(0) - \hat{\phi}_1\hat{\gamma}(1) - \hat{\phi}_2\hat{\gamma}(2) = 32.64662$

Finally, we can get the estimated standard error by plugging above numbers into

$$\sqrt{\frac{\hat{\sigma}^2\Gamma_2^{-1}}{n}} = 0.03989471$$

We can see that the standard error we calculated is somewhat similar to the one we obtained by using the linear regression with corresponding asymptotic approximations.

5.1 (B&D)

The sunspot numbers filed as SUNSPOTS.DAT have a smaple autocovariances of

gamma hat (0) = 1382.2

gamma hat (1) = 1114.4

gamma hat (2) = 591.73

gamma hat (3) = 96.216

Using these values find the Yule-Walker estimates of psi 1, psi 2, sigma in the model

Xt = psi1 Xt-1 + psi 2 Xt-2 + Wt

Using Yule Walker, we get:

$$\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \begin{bmatrix}\Gamma(0) & \Gamma(1)\\ \Gamma(1) & \Gamma(0)\end{bmatrix}^{-1} \begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix} = \begin{bmatrix}1.3175495\\ -0.6341682\end{bmatrix}$$

Then use

$$\hat{\sigma}^2 = \hat{\gamma}(0) - \hat{\phi}_1\hat{\gamma}(1) - \hat{\phi}_2\hat{\gamma}(2) = 289.1791$$

3. With Burg fit an AR(2) model to the lake data. This is example 5.1.4 in the Davis Book.

Simply by plugging the lake data into the ar model in R, we can get

```
ar(la_ke,method="burg")

Call:
ar(x = la_ke, method = "burg")

Coefficients:
      1        2
 1.0449  -0.2456

Order selected 2  sigma^2 estimated as  0.4789
```

4. Innovations fit an ARMA(1,1) model to the lake data. This is example 5.1.6 in the Davis Book.

Since R only has innovations fit a MA model, we use that and transform MA into ARMA(1,1)

First we fit the data into the MA model, getting

```
> ia(as.ts(la_ke),2,17)
$phi
[1] 0

$theta
[1] 1.0830783 0.7835384

$sigma2
[1] 0.7148892

$aicc
[1] 253.8393

$se.phi
[1] 0

$se.theta
[1] 0.1010153 0.1489096
```

Since we are trying to transform it into ARMA(1,1), if we see MA as ARMA(0,2) we can get

$$\psi_1 = \theta_1,\ \psi_2 = \theta_2 (Since\ \phi\ of\ the\ MA\ model\ doesn't\ exist)$$

Then we calculate the $\theta\ and\ \phi$ of the ARMA(1,1)

$$\phi = \frac{\psi_2}{\psi_1} = 0.7234365$$

$$\theta = \psi_1 - \phi = \psi_1 - \frac{\psi_2}{\psi_1} = 0.3596418$$

Which gives us the same result as shown in example in the textbook.

Codes used: (Written in R)

```
#1 load data
card_io <- read.table("Cardio.txt")
car_dio <- as.ts(card_io[,1])
#1.fit yule walker and ols
ar_yw <- ar(car_dio,method="yule-walker")
ar_ols <- ar(car_dio,method="ols")
#print results
ar_yw$ar
as.vector(ar_ols$ar)

#2.estimated std error obtained by liner regression
ar_ols$asy.se.coef$ar
#using calculations to get the estimated standard errors
temp <- acf(car_dio,lag.max=4,plot=FALSE,"covariance")
gammatrix <- toeplitz(as.vector(c(temp$acf[1],temp$acf[2])))
r_sq <- matrix(c(temp$acf[2],temp$acf[3]),2,1)
car_phi <- solve(gammatrix)%*%r_sq
car_sigma2 <- temp$acf[1]-car_phi[1]*temp$acf[2]-car_phi[2]*temp$acf[3]
est_std_err <- sqrt(car_sigma2/length(car_dio)*solve(gammatrix))[1,1]
#compare results
est_std_err
ar_ols$asy.se.coef$ar[1]

#2
#import data
sun_spot <- read.table("SUNSPOT.txt")
#creating the gamma2 matrix and the r matrix
gamma <- c(1382.2,1114.4,591.73,96.216)
gam_ma <- matrix(c(gamma[1],gamma[2],gamma[2],gamma[1]),2,2)
r_2 <- matrix(c(gamma[2],gamma[3]),2,1)
#solving for phi and sigma squared
phi <- solve(gam_ma)%*%r_2
sigma_2 <- gamma[1]-phi[1]*gamma[2]-phi[2]*gamma[3]

#3
#import and fit burg model
la_ke <- read.table("lake.txt")
ar(la_ke,method="burg")

#4
#using library itsmr
library(itsmr)
#estimate MA coefficients using innovations algorithm, using m=17
ia(as.ts(la_ke),2,17)
#transforming MA to ARMA(1,1)
arma_phi <- temp$theta[2]/temp$theta[1]
arma_theta <- temp$theta[1]-temp$theta[2]/temp$theta[1]
```