1\. A

$$X_t + 0.2X_{t-1} - 0.48X_{t-2} = W_t$$

$$(1 + 0.2B - 0.48B^2)X_t = W_t$$

$$(1 + 0.8B)(1 - 0.6B) = 0$$

$$B = \frac{5}{3} \text{ or } -\frac{5}{4} \Rightarrow |B| > 1 \Rightarrow \text{causal}$$

$$\theta(z) = 1 \Rightarrow \text{Invertible}$$

B.

$$X_t + 1.9X_{t-1} - 0.88X_{t-2} = W_t + 0.2W_{t-1} + 0.7W_{t-2}$$

$$(1 + 1.9B - 0.88B^2)X_t = (1 + 0.2B + 0.7B^2)W_t$$

$$\frac{-1.9 \pm \sqrt{1.9^2 + 4 \cdot 0.88}}{2 \times -0.88} = \overset{z_1}{-0.438}, \overset{z_2}{2.597}$$

$$|z_1| < 1 \quad |z_2| > 1 \Rightarrow \text{Not causal}$$

$$\frac{-0.2 \pm \sqrt{0.04 - 2.8}}{1.4} \Rightarrow z_1 = \frac{1}{7} + \frac{\sqrt{69}}{7}i, \quad z_2 = \frac{1}{7} -$$

$$|z_1, z_2| > 1 \Rightarrow \text{Invertible}$$

C.

$$X_t + 0.6X_{t-1} = W_t + 1.2W_{t-1}$$

$$(1 + 0.6B)X_t = (1 + 1.2B)W_t$$

$$z = -\frac{1}{0.6} = -\frac{5}{3} \Rightarrow |z_1| > 1 \Rightarrow \text{Causal}$$

$$z_1 = -\frac{1}{1.2} = -\frac{5}{6} \Rightarrow |z_1| < 1 \Rightarrow \text{Not invertible}$$

D. $X_t + 1.8X_{t-1} + 0.81X_{t-2} = W_t - 0.4W_{t-1} + 0.04W_{t-2}$

$(1+1.8B+0.81B^2)X_t = (1-0.4B+0.04B^2)W_t$

$\hookrightarrow (0.9z_1+1)^2 \Rightarrow z_1 = -\frac{1}{0.9} = -\frac{10}{9} \Rightarrow |z| > 1 \Rightarrow$ Caus

$\rightarrow (0.2z_2-1)^2 \Rightarrow z_2 = \frac{1}{0.2} = \frac{10}{2} = 5 \Rightarrow |z_2| > 1 \Rightarrow$ Inver

2. A.C.D causal

A. $X_t + 0.2X_{t-1} - 0.48X_{t-2} = W_t$ $\qquad \rho(0) + 0.2\rho(1) - 0.48\gamma(2) =$

$$\begin{cases} \gamma(0) + 0.2\gamma(-1) - 0.48\gamma(-2) = \sigma^2, \\ \gamma(1) + 0.2\gamma(0) - 0.48\gamma(-1) = 0 \end{cases}$$

$\rho(0) = 1,\ \rho(1) = -\frac{0.2}{0.52} = -\frac{5}{13}$ #

$\gamma(t) + 0.2\gamma(t-1) - 0.48\gamma(t-2) = 0$

$\rho(t) + 0.2\rho(t-1) - 0.48\rho(t-2) = 0$

$\rho(t) = c_1 z_1^{-t} + c_2 z_2^{-t}$, $z_1 = -\frac{5}{4}$, $z_2 = \frac{5}{3}$ by 1.

$$\begin{cases} t=0 \rightarrow c_1 + c_2 = 1 \\ t=1 \Rightarrow c_1 \cdot \left(-\frac{5}{4}\right)^{-1} + c_2\left(\frac{5}{3}\right)^{-1} = -\frac{5}{13} \end{cases} \Rightarrow \begin{cases} c_1 = \frac{64}{91} \\ c_2 = \frac{27}{91} \end{cases}$$

$\Rightarrow \rho(t) = \frac{64}{91}\left(-\frac{5}{4}\right)^{-t} + \frac{27}{91}\left(\frac{5}{3}\right)^{-t}$, $t \geq 2$

#

(c)

$$X_t + 0.6X_{t-1} = W_t + 1.2W_{t-1}$$

$$\gamma(t) + 0.6\gamma(t-1) = 0$$

$$\begin{cases} \gamma(0) + 0.6\gamma(1) = \sigma^2(1 + 0.6 \times 1.2) \\ \gamma(1) + 0.6\gamma(0) = \sigma^2(1.2) \end{cases}$$

$$\Rightarrow \begin{cases} \gamma(1) = 0.2625\sigma^2 \\ \gamma(0) = 1.5625\sigma^2 \end{cases} \Rightarrow \gamma(t) = (-0.6)^{t-1}\gamma(1)$$

$$\Rightarrow \rho(t) = \underbrace{\frac{0.2625}{1.5625}}_{\rho(1)}(-0.6)^{t-1}, \quad t \geq 1$$

d)

$$X_t + 1.8X_{t-1} + 0.81X_{t-2} = W_t - 0.4W_{t-1} + 0.04W_{t-2}$$

$$\begin{cases} \gamma(0) + 1.8\gamma(1) + 0.81\gamma(2) = \sigma^2(1 + 2.2 \times 0.4 + 0.04 \times 3.19) \\ \gamma(1) + 1.8\gamma(0) + 0.81\gamma(1) = \sigma^2(-0.4 + 0.04 \times (-2.2)) \\ \gamma(2) + 1.8\gamma(1) + 0.81\gamma(0) = \sigma^2(0.04) \end{cases}$$

$$\gamma(0) + 1.8\gamma(1) + 0.81\gamma(2) = 2.0076\sigma^2$$

$$1.8\gamma(0) + 1.81\gamma(1) = -0.488\sigma^2$$

$$0.81\gamma(0) + 1.8\gamma(1) + \gamma(2) = 0.04\sigma^2$$

$$\begin{cases} \gamma(0) = 545.403\sigma^2 \\ \gamma(1) = -542.1517\sigma^2 \\ \gamma(2) = 535.05077\sigma^2 \end{cases}$$

$$\rho(1) = \frac{\gamma(1)}{\gamma(0)} = -0.994969$$

$$\rho(2) = \frac{\gamma(2)}{\gamma(0)} = 0.9810183396$$

$$\rho(t) = -1.8\rho(t-1) - 0.81\rho(t-2)$$

For PACF, please refer to codes.

3.

A. $X_t + 0.2 X_{t-1} - 0.48 X_{t-2} = W_t$

Using Taylor Expansion on

$$\frac{1}{1+0.2x-0.48x^2}$$

$$1 - 0.2x + 0.52x^2 - 0.2x^3 + 0.2896x^4 - 0.15392x^5 + 0.168792x^6$$

c. $X_t + 0.6 X_{t-1} = W_t + 1.2 W_{t-1}$

Taylor expand $\frac{1+1.2z}{1+0.6z}$

$$1 + 0.6x - 0.36x^2 + 0.216x^3 - 0.1296x^4 + 0.07776x^5 - 0.046656x^6$$

d $X_t + 1.8 X_{t-1} + 0.81 X_{t-2} = W_t - 0.4 W_{t-1} + 0.04 W_{t-2}$

Taylor expand $\frac{1-0.4x+0.04x^2}{1+1.8x+0.81x^2}$

$$1 - 2.2x + 3.19x^2 - 3.96x^3 + 4.5441x^4 - 4.97178x^5 + 5.26848x^6$$

4. $Y_t = W_t + a_1 W_{t-1} + a_{12} W_{t-12}$

$\gamma(Y_{t+h}, Y_t) = \mathbb{E}[Y_{t+h} \cdot Y_t]$

$= \mathbb{E}[(W_{t+h} + a_1 W_{t+h-1} + a_{12} W_{t+h-12}) \cdot (W_t + a_1 W_{t-1} + a_{12} W_{t-12})]$

$$= \begin{cases} h=0 & \sigma^2 + a_1^2 \sigma^2 + a_{12}^2 \sigma^2 \\ h=\pm 1 & a_1 \sigma^2 \\ h=\pm 11 & a_{12} a_1 \sigma^2 \\ h=\pm 12 & a_{12} \sigma^2 \end{cases}$$

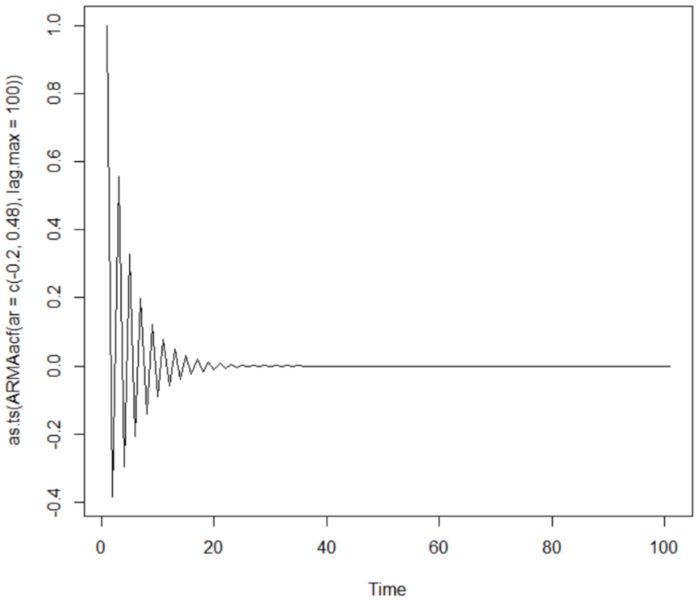
$\rho(1) = \frac{a_1}{1 + a_1^2 + a_{12}^2}$

$\rho(11) = \frac{a_{12} a_1}{1 + a_1^2 + a_{12}^2}$

$\rho(12) = \frac{a_{12}}{1 + a_1^2 + a_{12}^2}$

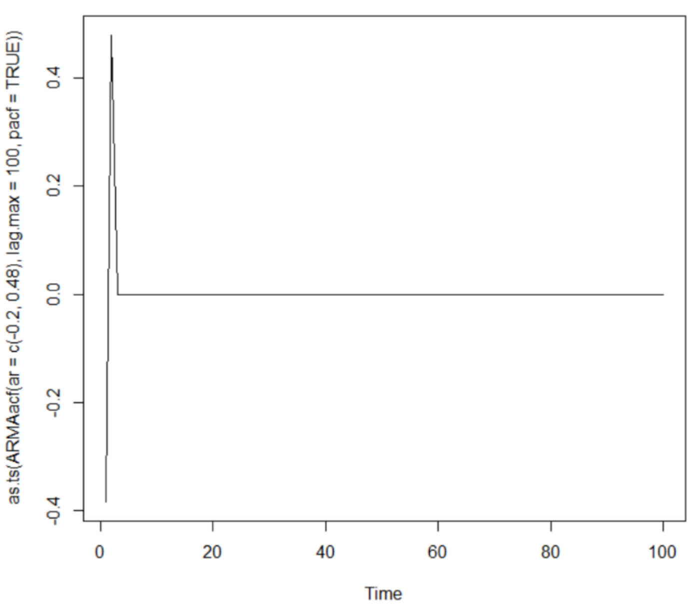
5.A.Theoretical ACF

```
> #theoretical ACF values
> ARMAacf(ar=c(-0.2,0.48),lag.max=8)
        0          1          2          3          4          5          6          7          8 
1.0000000 -0.3846154  0.5569231 -0.2960000  0.3265231 -0.2073846  0.1982080 -0.1391862  0.1229771 
```



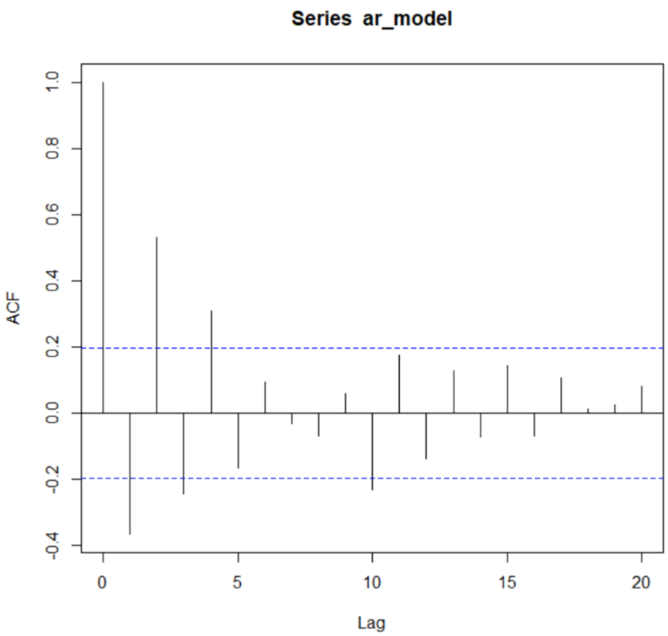
Theoretical PACF

```
> ARMAacf(ar=c(-0.2,0.48),lag.max=8,pacf=TRUE)
[1] -3.846154e-01  4.800000e-01  0.000000e+00  8.465242e-17  7.759805e-17 -4.873234e-17 -4.147969e-17 -4.232621e-18
```



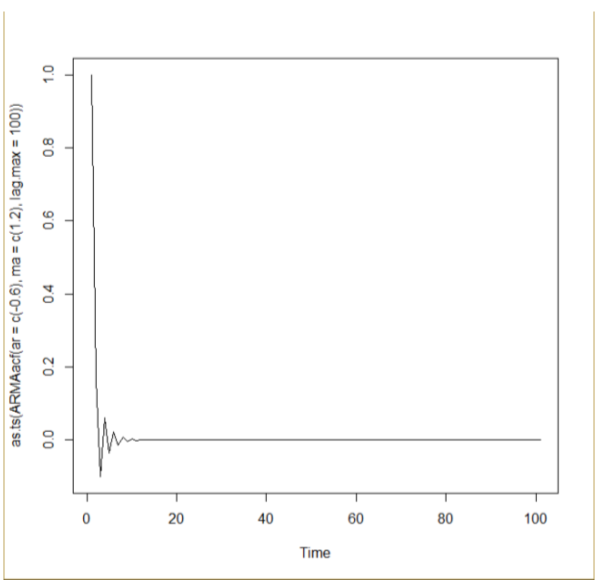
Self generated Observed acf

```
Autocorrelations of series 'ar_model', by lag

     0      1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17 
 1.000 -0.365  0.531 -0.244  0.308 -0.166  0.095 -0.030 -0.068  0.060 -0.230  0.175 -0.138  0.127 -0.073  0.145 -0.068  0.106 
    18     19     20 
 0.014  0.024  0.081 
```



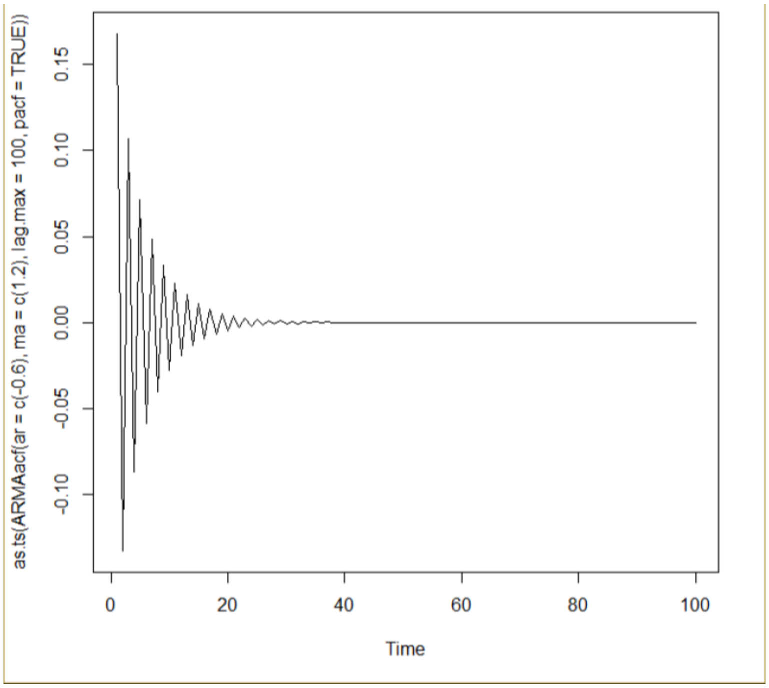
C.Theoretical ACF

```
> #C
> #theoretical ACF values
> ARMAacf(ar=c(-0.6),ma=c(1.2),lag.max=8)
           0            1            2            3            4            5            6            7            8 
 1.000000000  0.168000000 -0.100800000  0.060480000 -0.036288000  0.021772800 -0.013063680  0.007838208 -0.004702925 
```



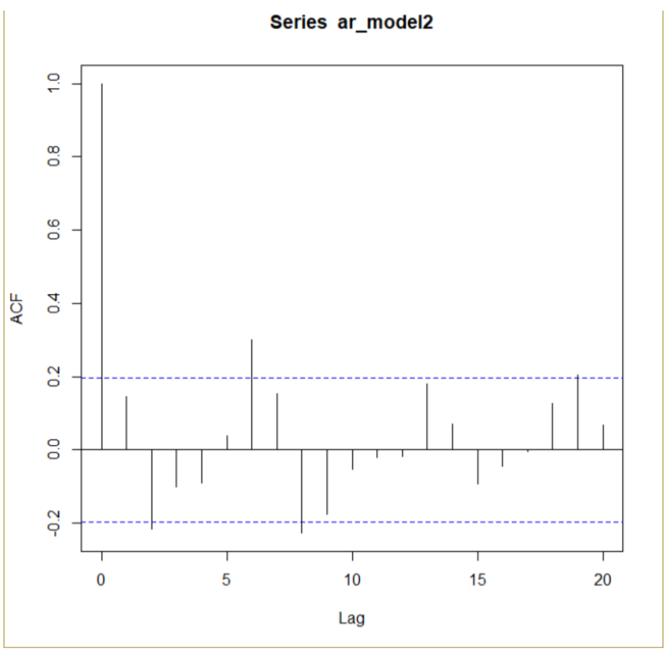
Theoretical PACF

```
> ARMAacf(ar=c(-0.6),ma=c(1.2),lag.max=8,pacf=TRUE)
[1]  0.16800000 -0.13277134  0.10681285 -0.08692126  0.07127254 -0.05873949  0.04857795 -0.04026931
```



Self generated Observed acf

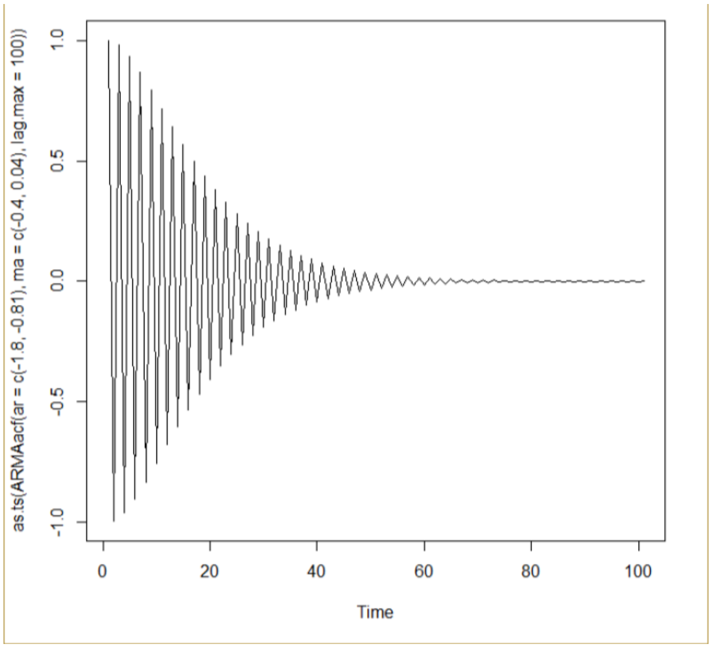
```
Autocorrelations of series 'ar_model2', by lag

     0      1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17 
 1.000  0.147 -0.217 -0.100 -0.090  0.039  0.300  0.154 -0.227 -0.176 -0.053 -0.022 -0.017  0.180  0.069 -0.092 -0.044 -0.005 
    18     19     20 
 0.127  0.205  0.067 
```



D)

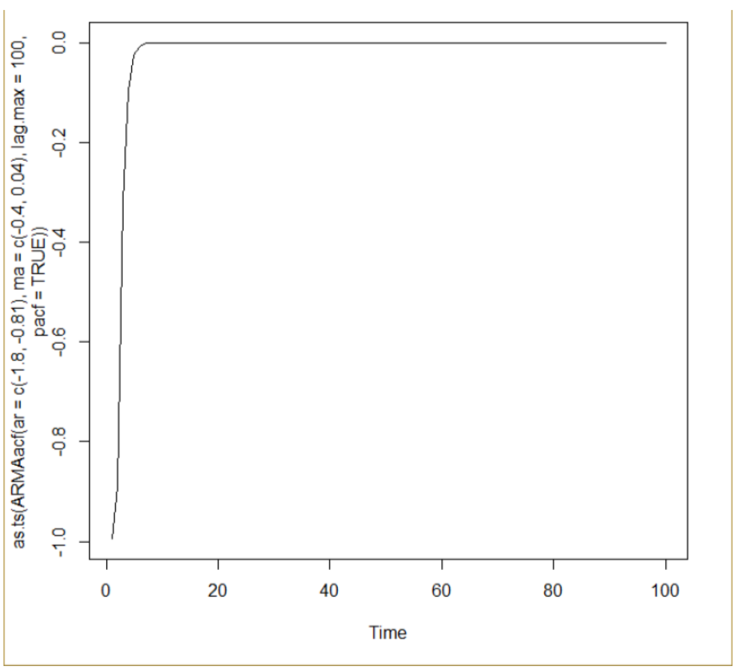
Theoretical ACF

```
· #D
· #theoretical ACF values
· ARMAacf(ar=c(-1.8,-0.81),ma=c(-0.4,0.04),lag.max=8)
        0          1          2          3          4          5          6          7          8
1.0000000 -0.9949693  0.9810181 -0.9599075  0.9332087 -0.9022507  0.8681522 -0.8318508  0.7941283
```



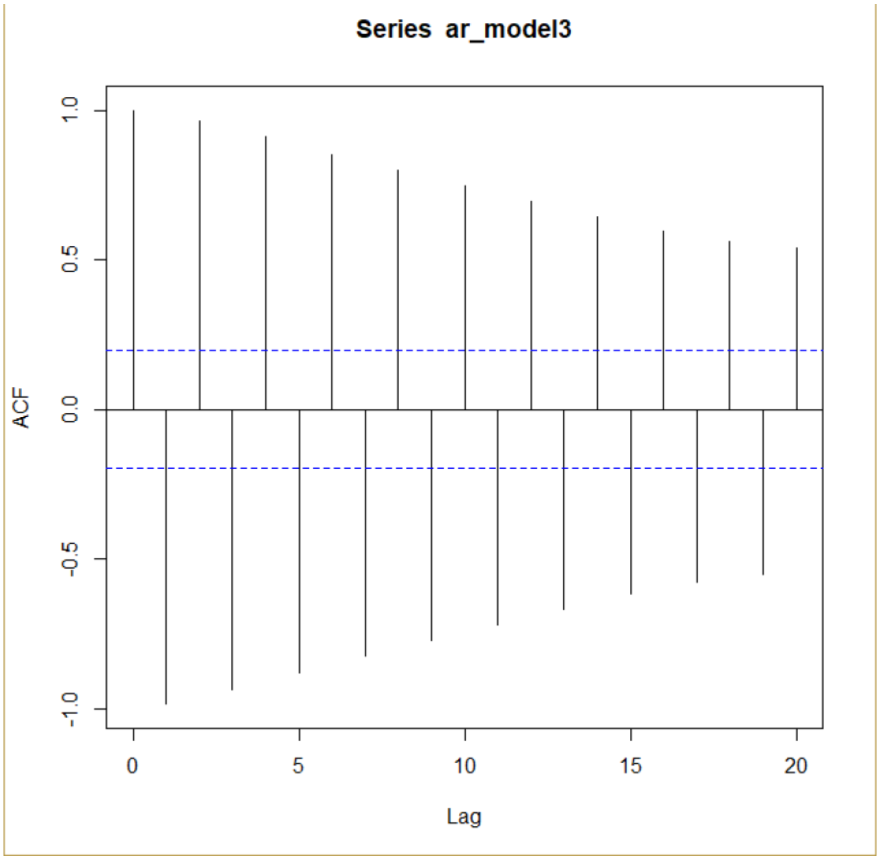
Theoretical PACF

```
> ARMAacf(ar=c(-1.8,-0.81),ma=c(-0.4,0.04),lag.max=8,pacf=TRUE)
[1] -0.994969332 -0.891374028 -0.319773935 -0.095381584 -0.025470338 -0.006375248 -0.001531326 -0.000357521
```



Self generated Observed acf

```
Autocorrelations of series ‘ar_model3’, by lag

     0      1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17 
 1.000 -0.985  0.964 -0.938  0.910 -0.881  0.852 -0.824  0.798 -0.773  0.748 -0.721  0.694 -0.667  0.642 -0.617  0.594 -0.576 
    18     19     20 
 0.561 -0.550  0.540 
```



Codes used(written in R):

```
#open up new tab for plot
x11()

#theoretical ACF values
ARMAacf(ar=c(-0.2,0.48),lag.max=8)
ARMAacf(ar=c(-0.2,0.48),lag.max=8,pacf=TRUE)
plot(as.ts(ARMAacf(ar=c(-0.2,0.48),lag.max=100)))
plot(as.ts(ARMAacf(ar=c(-0.2,0.48),lag.max=100,pacf=TRUE)))
#generated ACF values
rand_om <- rnorm(100,0,1)
ar_model <- rand_om
ar_model[1] <- 1
ar_model[2] <- 1
for(i in c(3:100)){
  ar_model[i] <- -0.2*ar_model[i-1]+0.48*ar_model[i-2]+rand_om[i]
}
#print(ar_model)
ar_acf <- acf(ar_model,type="correlation",plot=T)
ar_acf
ar_pacf <- acf(ar_model,type="partial")


#C
#theoretical ACF values
ARMAacf(ar=c(-0.6),ma=c(1.2),lag.max=8)
ARMAacf(ar=c(-0.6),ma=c(1.2),lag.max=8,pacf=TRUE)
plot(as.ts(ARMAacf(ar=c(-0.6),ma=c(1.2),lag.max=100)))
plot(as.ts(ARMAacf(ar=c(-0.6),ma=c(1.2),lag.max=100,pacf=TRUE)))
#generated ACF values
rand_om2 <- rnorm(100,0,1)
ar_model2 <- rand_om2
ar_model2[1] <- 1
for(i in c(2:100)){
  ar_model2[i] <- -0.6*ar_model2[i-1]+rand_om2[i]+1.2*rand_om2[i-1]
}
#print(ar_model)
ar_acf <- acf(ar_model2,type="correlation",plot=T)
ar_acf


#D
#theoretical ACF values
ARMAacf(ar=c(-1.8,-0.81),ma=c(-0.4,0.04),lag.max=8)
ARMAacf(ar=c(-1.8,-0.81),ma=c(-0.4,0.04),lag.max=8,pacf=TRUE)
plot(as.ts(ARMAacf(ar=c(-1.8,-0.81),ma=c(-0.4,0.04),lag.max=100)))
plot(as.ts(ARMAacf(ar=c(-1.8,-0.81),ma=c(-0.4,0.04),lag.max=100,pacf=TRUE)))
```

```
#generated ACF values
rand_om3 <- rnorm(100,0,1)
ar_model3 <- rand_om3
ar_model3[1] <- 1
ar_model3[2] <- 1
for(i in c(3:100)){
  ar_model3[i] <- -1.8*ar_model3[i-1]-0.81*ar_model3[i-2]+rand_om3[i]-0.4*rand_om3[i-
1]+0.04*rand_om3[i-2]
}
#print(ar_model)
ar_acf <- acf(ar_model3,type="correlation",plot=T)
ar_acf
```