t-tests

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```
[62]: import scipy.stats as stats
      import pandas as pd
      import numpy as np
      import statsmodels.api as sm
      import pylab
 [6]: df = pd.read_csv('/content/titanic.csv')
 [7]: df.head()
 [7]:
         PassengerId Survived
                                 Pclass
                    1
                    2
      1
                              1
                                       1
                    3
      2
                              1
                                       3
      3
                    4
                                       1
                              1
                    5
                              0
                                       3
                                                         Name
                                                                  Sex
                                                                         Age
                                                                              SibSp \
      0
                                     Braund, Mr. Owen Harris
                                                                 male
                                                                        22.0
         Cumings, Mrs. John Bradley (Florence Briggs Th... female 38.0
      1
      2
                                      Heikkinen, Miss. Laina
                                                               female
                                                                        26.0
                                                                                  0
      3
              Futrelle, Mrs. Jacques Heath (Lily May Peel)
                                                               female 35.0
                                                                                  1
      4
                                    Allen, Mr. William Henry
                                                                 male 35.0
                                                                                  0
         Parch
                           Ticket
                                       Fare Cabin Embarked
      0
             0
                        A/5 21171
                                    7.2500
                                              {\tt NaN}
                                                          S
                                                          С
      1
                         PC 17599
                                   71.2833
                                              C85
             0
      2
                STON/02. 3101282
                                    7.9250
                                                          S
                                              NaN
      3
             0
                           113803 53.1000
                                             C123
                                                          S
             0
                           373450
                                    8.0500
                                                          S
                                              {\tt NaN}
 [8]: df.shape
```

[8]: (891, 12)

1 Single Sample t-test

• $H0: Population_mean = 35$

```
• H1 : Population_mean <=35
        • Alpha: 0.05
[11]: population_age = df['Age']
[13]: population_age.dropna()
[13]: 0
              22.0
      1
             38.0
      2
              26.0
      3
             35.0
      4
              35.0
      885
             39.0
      886
             27.0
      887
             19.0
      889
             26.0
      890
             32.0
      Name: Age, Length: 714, dtype: float64
[23]: sample_age = population_age.dropna()
      sample_age1 = sample_age.sample(30)
[24]: sample_age1
[24]: 182
              9.0
      510
              29.0
      265
              36.0
      211
              35.0
      79
             30.0
      735
             28.5
      323
             22.0
      373
             22.0
      848
             28.0
      398
             23.0
      110
             47.0
      407
              3.0
      551
             27.0
      746
             16.0
      615
             24.0
             21.0
      56
      782
             29.0
      267
             25.0
      119
              2.0
      688
              18.0
```

```
429
              32.0
      272
              41.0
               4.0
      63
              26.0
      315
      289
              22.0
              20.0
      131
      663
              36.0
              21.0
      115
              31.0
      439
      400
              39.0
      Name: Age, dtype: float64
[25]: sample_age1.shape
```

[25]: (30,)

Assumptions to perform Single-Sample-t-test

- Normality: Since we have a sample size of 30 a/c to central limit theorem we can assume it to be normal
- Independence of sample
- Random Sampling
- Unknown Population Std. deviation

1.1.1 Performing Shapiro-Wilk test

• This test is used to check normality

```
[26]: from scipy.stats import shapiro
[31]: pop_mean = 35
      shapiro(sample_age1)
[31]: ShapiroResult(statistic=0.9610083699226379, pvalue=0.3286316990852356)
```

- Since the p-value is greater than 0.05, therefore the sample data is normally distributed
- 1.1.3 Basically it was not necessary to perform this test, but still I did it.

```
[32]: t_statistic , p_value = stats.ttest_1samp(sample_age1,pop_mean)
[33]: print('t-statistic: ',t_statistic)
     t-statistic: -5.136536349467585
[36]: print('p_value: ',p_value/2)
```

p_value: 8.677479419803319e-06

1.1.4 Since it is one tailed, we divide p-value by 2

```
[37]: alpha = 0.05
if p_value < alpha:
    print("Reject Null Hypothesis")
else:
    print("Accept Null Hypothesis")</pre>
```

Reject Null Hypothesis

1.1.5 Therefore our null hypothesis of mean age being 35 is wrong

• i.e. the population mean age is less than 35

1.1.6 Checking whether our process is right or wrong

```
[40]: df['Age'].mean()
```

[40]: 29.69911764705882

1.1.7 There mean age is actually less than 35 i.e 29.70, which proves our test is right

2 Independent two-sample test

2.1 Assumptions for the test

- Independence of obeservations
- Normality
- Equal Variances: The variances of two independent groups should be equal
- Random Sampling
- H0: The average age of male and female has no difference
- H1: The average age of male is significantly higher than average age of female.
- Alpha = 0.05

```
[46]: df[['Age','Sex']].dropna()
```

```
[46]:
            Age
                     Sex
      0
           22.0
                    male
      1
           38.0 female
      2
           26.0
                  female
      3
           35.0
                  female
      4
           35.0
                    male
      885
           39.0
                  female
      886
           27.0
                    male
      887
           19.0 female
```

```
889 26.0
                   male
      890 32.0
                   male
      [714 rows x 2 columns]
[47]: male_pop = df[df['Sex'] == 'male']['Age'].dropna()
[48]: female_pop = df[df['Sex'] == 'female']['Age'].dropna()
[51]: male_pop.head()
[51]: 0
            22.0
            35.0
            54.0
      6
      7
             2.0
      12
            20.0
      Name: Age, dtype: float64
[52]: female_pop.head()
[52]: 1
           38.0
           26.0
      2
      3
           35.0
           27.0
      8
      9
           14.0
      Name: Age, dtype: float64
[55]: print(female_pop.shape)
      print(male_pop.shape)
     (261,)
     (453,)
[77]: sample_male = male_pop.sample(30)
      sample_female = female_pop.sample(30)
[78]: sample_male.shape
[78]: (30,)
[79]: sample_female.shape
[79]: (30,)
```

2.1.1 Shapiro test to check for normality

```
[80]: print(shapiro(sample_male))
print(shapiro(sample_female))
```

ShapiroResult(statistic=0.9784983992576599, pvalue=0.7843736410140991) ShapiroResult(statistic=0.9559767842292786, pvalue=0.24363601207733154)

- 2.1.2 since p_value of both sample is greater than 0.05, they are normal
- 2.1.3 Levene test to check the whether both groups have equal variances

```
[81]: from scipy.stats import levene
[82]: levene(sample_male , sample_female)
```

- [82]: LeveneResult(statistic=2.9567637031766094, pvalue=0.09085181613089263)
 - 2.1.4 Since p-value > 0.05, we do not reject the null hypothesis of the variances of both group being same.
 - 3 F-test to check equality of variances

```
[83]: variance1 = np.var(sample_male, ddof=1)
variance2 = np.var(sample_female, ddof=1)
```

```
[84]: f_value = variance1 / variance2
```

```
[85]: df1 = len(sample_male) - 1
df2 = len(sample_female) - 1
```

```
[86]: p_value = stats.f.cdf(f_value, df1, df2)
```

```
[87]: print('Degree of freedom 1:',df1)
print('Degree of freedom 2:',df2)
print("F-statistic:", f_value)
print("p-value:", p_value)
```

Degree of freedom 1: 29 Degree of freedom 2: 29 F-statistic: 1.7173672839111591 p-value: 0.9243640868083862

3.0.1 Since p-value is > 0.05 we accept the null hypothesis

```
[88]: t_statistic , p_value = stats.ttest_ind(sample_male,sample_female)
```

```
[89]: print('t-statistic: ',t_statistic)
    print('p_value: ',p_value/2)

    t-statistic: 2.44583001377923
    p_value: 0.008753503445615367

[90]: alpha = 0.05
    if p_value < alpha:
        print("Reject Null Hypothesis")
    else:
        print("Accept Null Hypothesis")</pre>
```

Reject Null Hypothesis

- 3.0.2 Therefore we have to accept the null hypothesis.
- 3.0.3 Welch t-test (if we do not have equal varinaces)
 - Same as two-sample t test , only we have to set the parameter, equal_var=False

4 Paired two-sample t-tests

4.0.1 Assumptions

- Paired Observations
- Normality: The difference between paired observations should be normal
- Independence of pairs
- H0: There is no difference of wieghts before and after treatment
- H1: There is a significant difference
- Alpha: 0.05

```
[96]: weight_before = np.array([80, 92, 75, 68, 85, 78, 73, 90, 70, 88, 76, 84, 82, \( \times \) 77, 91])
weight_after = np.array([78, 93, 81, 67, 88, 76, 74, 91, 69, 88, 77, 81, 80, \( \times \) 79, 88])
```

```
[97]: weight_differences = weight_after - weight_before
```

```
[99]: shapiro_test = stats.shapiro(weight_differences)
print("Shapiro-Wilk test:", shapiro_test)
```

Shapiro-Wilk test: ShapiroResult(statistic=0.9220570921897888, pvalue=0.20704729855060577)

4.0.2 We accept the null hypothesis of difference of paired obeseravtions is normal

```
[101]: mean_diff = np.mean(weight_differences)
       std_diff = np.std(weight_differences, ddof=1)
[102]: n = len(weight_differences)
       t_statistic = mean_diff / (std_diff / np.sqrt(n))
       df = n - 1
[103]: t_statistic
[103]: 0.10482848367219182
[104]: alpha = 0.05
       p_value = stats.t.cdf(t_statistic, df)
[105]: p_value
[105]: 0.5410005146857456
      4.0.3 Since p > alpha we accept the null hypothesis
      4.0.4 Verification
[107]: weight_before.mean()
[107]: 80.6
[109]: weight_after.mean()
[109]: 80.6666666666667
  []:
```