

# Recommender Systems

Statistical Data Mining II

Spring 2016

Rachael Hageman Blair

# Outline

- Introduction
- Nearest Neighbor Methods
- Terminology
- Collaborative Filtering
- Conclusions
  
- Thinking points

# What is Recommender System (Rec Sys)

- Take a guess?

Yes! Rec Sys is a method designed to make recommendations!

- There are many forms of recommendation system.
- Example: Amazon, News, Blogs, Friends.

# Non-personalized recommendations

Instant Video   Digital Music Store   Cloud Drive   **Fire HD 7**   Appstore for Android   Digital Games & Software   Audible Audiobooks

Fire tablets ranked  
"Highest in Customer Satisfaction  
with Tablets"

—according to J.D. Power 2014  
U.S. Tablet Satisfaction Study, Vol. 2



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**fire HD7**  
~~\$139~~ **\$119**

> [Shop now](#)

Gifts with Stories to Tell   **Black Friday Deals Deals Week**   Holiday Toy List



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# Personalized recommendations

## Frequently Bought Together



Price for all three: **\$187.49**

[Add all three to Cart](#) [Add all three to Wish List](#)

[Show availability and shipping details](#)

- ☒ **This item:** The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition (Springer ... by Trevor Hastie Hardcover **\$66.46**
- ☒ An Introduction to Statistical Learning: with Applications in R (Springer Texts in Statistics) by Gareth James Hardcover **\$59.10**
- ☒ Applied Predictive Modeling by Max Kuhn Hardcover **\$61.93**

## Customers Who Bought This Item Also Bought

Page 1 of 17



An Introduction to Statistical Learning: ...  
› Gareth James  
★★★★★ 38  
**#1 Best Seller** in Artificial Intelligence  
Hardcover  
\$59.10 ✓Prime



Applied Predictive Modeling  
› Max Kuhn  
★★★★★ 27  
**#1 Best Seller** in Biostatistics  
Hardcover  
\$61.93 ✓Prime



Pattern Recognition and Machine Learning...  
› Christopher M. Bishop  
★★★★☆ 101  
**#1 Best Seller** in Artificial Intelligence...  
Hardcover  
\$62.60 ✓Prime



Machine Learning: A Probabilistic...  
› Kevin P. Murphy  
★★★★☆ 35  
Hardcover  
\$82.53 ✓Prime



Convex Optimization  
Stephen Boyd  
★★★★★ 19  
Hardcover  
\$76.00 ✓Prime



All of Statistics: A Concise Course in...  
› Larry Wasserman  
★★★★★ 23  
Hardcover  
\$81.56 ✓Prime

# Power of Recommendation Systems

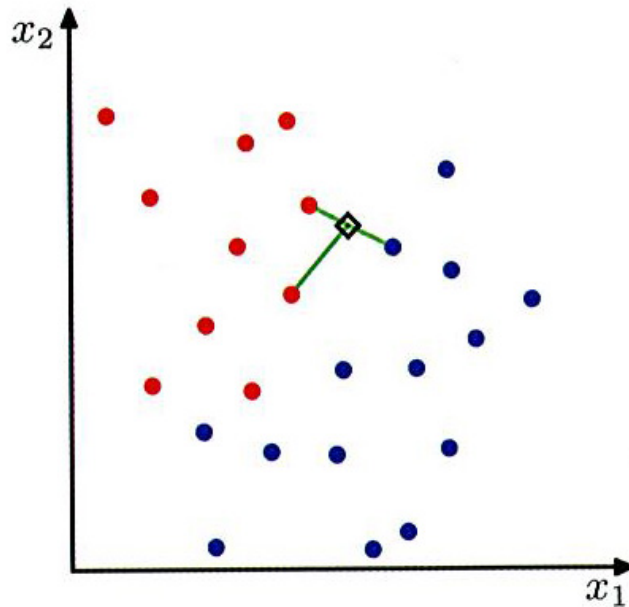
- Netflix: 2/3 of the movies watched are recommended
- Google News: recommendations generate 38% more clickthrough
- Amazon: 35% sales from recommendations

\*new wave of competitions.....

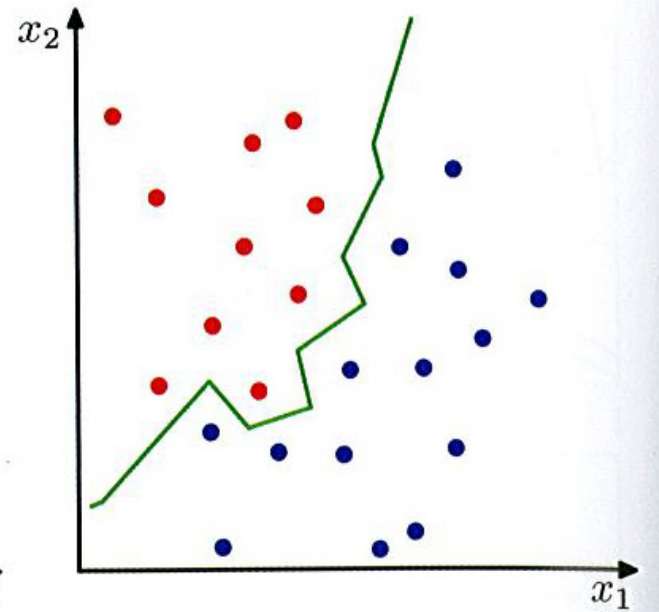
Netflix – 1 million dollar prize to beat algorithm by 10%.



# Nearest-neighbor methods



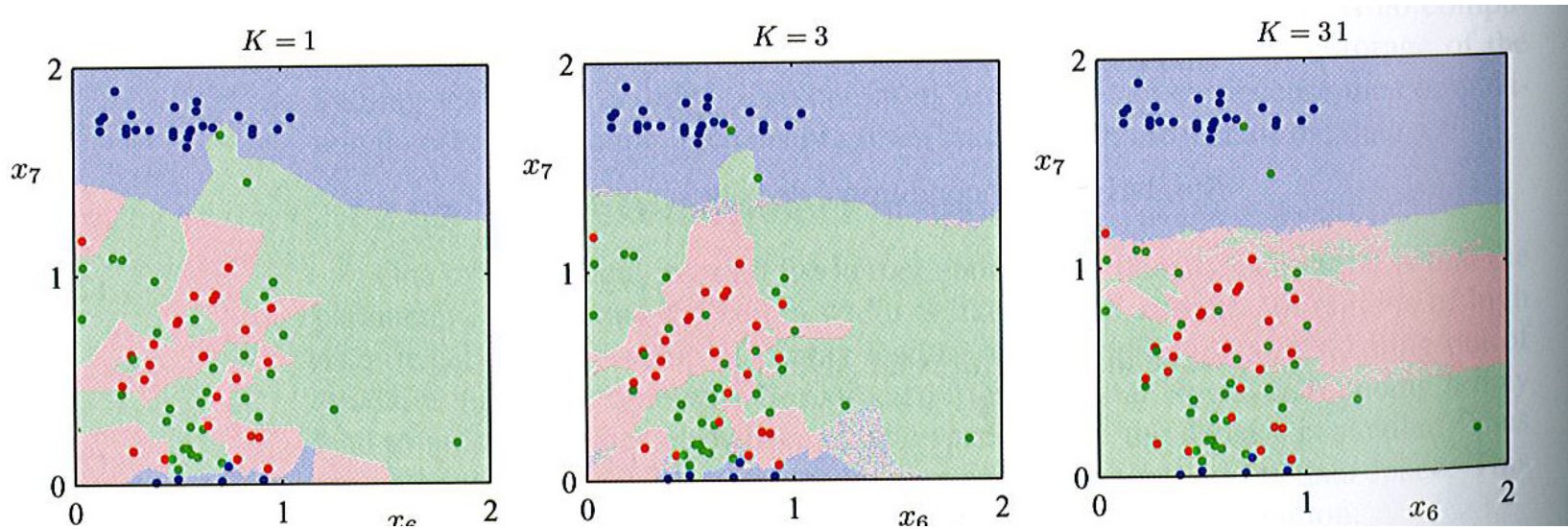
A new point arrives, it is classified according to the majority class membership of its  $K$  closest neighbors.



When  $K=1$ , the decision boundary is a hyper-plane that form perpendicular bisectors for pairs of points from different classes.



# Nearest-neighbor methods



$K$  - pertains to the fit. The  $k$ - nearest neighbor fit for  $\hat{Y}(x)$  is defined as follows:

$$\hat{Y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i.$$

Neighborhood of  $x$  defined by the  $k$  closest points  $x_i$ .

Small  $K$  - many small regions.  
Larger  $K$  - fewer large regions.



# Terminology

- Content-based systems: examine properties of the items recommended. For instance, if a netflix user watches a lot of westerns, then a movie recommended related to “westerns” may be suggested.
- Collaborative filtering schemes: recommend items based on similarity between users and/or items. The items recommended to a user are those that are preferred by similar users.

# Terminology

- Utility Matrix – the data is represented as user-item pairs. Entries of the matrix reflect a given “rating” of the user on the item. Values come from an ordinal set.

	Harry Potter			Twilight	Star Wars		
	HP1	HP2	HP3	TW	SW1	SW2	SW3
<i>A</i>	4			5	1		
<i>B</i>	5	5	4				
<i>C</i>				2	4	5	
<i>D</i>		3					3

- Data is large and sparse.
- Goal: Predict the empty spaces that would be highly rated for a person.

# Terminology

	Harry Potter			Twilight	Star Wars		
	HP1	HP2	HP3	TW	SW1	SW2	SW3
<i>A</i>	4			5	1		
<i>B</i>	5	5	4				
<i>C</i>				2	4	5	
<i>D</i>		3					3

## How can we populate the utility matrix?

1. Ask people to rate items they buy/watch/read.
2. Make inferences based on users' behavior – includes purchasing, watching, adding to cart, or even clicking. They are watching your every click!

# Terminology

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	?	4.0	4.0	2.0	1.0	2.0	?	?
$u_2$	3.0	?	?	?	5.0	1.0	?	?
$u_3$	3.0	?	?	3.0	2.0	2.0	?	3.0
$u_4$	4.0	?	?	2.0	1.0	1.0	2.0	4.0
$u_5$	1.0	1.0	?	?	?	?	?	1.0
$u_6$	?	1.0	?	?	1.0	1.0	?	1.0
$u_a$	?	?	4.0	3.0	?	1.0	?	5.0
$r_a$	3.5	4.0		1.3		2.0		

(a)

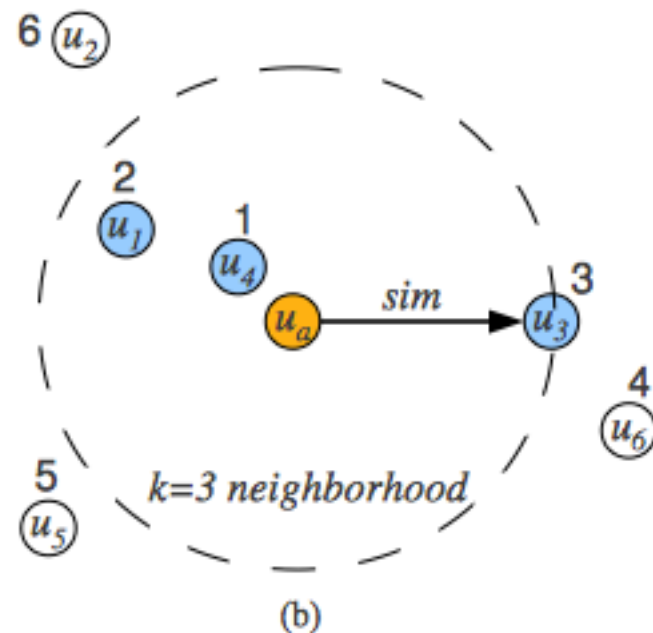


Figure 1: User-based collaborative filtering example with (a) rating matrix and estimated ratings for the active user, and (b) user neighborhood formation.



# Terminology

Long tail – can explain in part the advantages of online retailers compared to traditional “brick and mortar stores”.

Large bookstore at the corner -> thousands of books

Bookstore online -> millions of books

Physical newspaper -> limited articles

Online news -> unlimited articles

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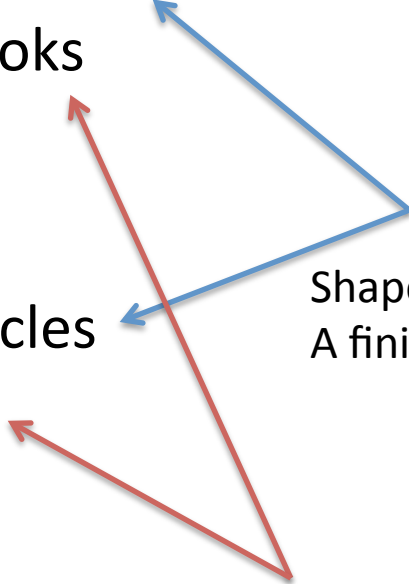
Bookstore online -> millions of books

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Shaped by aggregate numbers  
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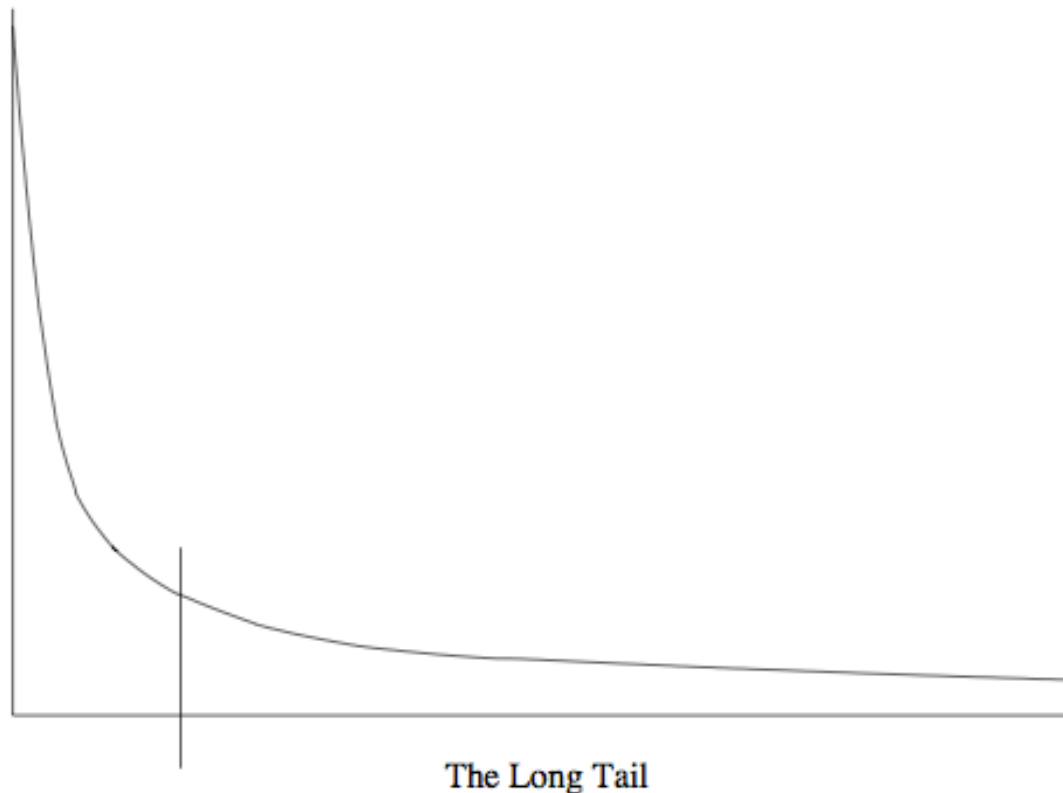
Forced to make recommendations  
Can't show everything.



```
graph LR; A[Large bookstore at the corner -> thousands of books] -- blue arrow --> B[Bookstore online -> millions of books]; C[Physical newspaper -> limited articles] -- blue arrow --> D[Online news -> unlimited articles]; B -- red arrow --> A; D -- red arrow --> C;
```

# Terminology

Long tail – can explain in part the advantages of online retailers compared to traditional “brick and mortar stores”. They can use the entire distribution.





## *Into Thin Air and Touching the Void*

An extreme example of how the long tail, together with a well designed recommendation system can influence events is the story told by Chris Anderson about a book called *Touching the Void*. This mountain-climbing book was not a big seller in its day, but many years after it was published, another book on the same topic, called *Into Thin Air* was published. Amazon's recommendation system noticed a few people who bought both books, and started recommending *Touching the Void* to people who bought, or were considering, *Into Thin Air*. Had there been no on-line bookseller, *Touching the Void* might never have been seen by potential buyers, but in the on-line world, *Touching the Void* eventually became very popular in its own right, in fact, more so than *Into Thin Air*.

# Data Acquisition

Two Basic Approaches:

1. Ask users' to rate items.

Generally, people don't want to waste time for a stranger or companies gain. Responses may be bias.

2. Can make inferences from users' behavior.

If you click "it", you like it. Binary, but 0 is an NA.

# Popular approaches

Two Basic Approaches to recommender systems:

1. Content-based

Examine similarity of items.

2. Collaborative Filtering

Focus on the relationship between users and items.



# Popular approaches

Two Basic Approaches to recommender systems:

1. Content-based

Examine similarity of items.

2. Collaborative Filtering

Focus on the relationship between users and items.



- memory-based ~ work in real memory
- model-based ~ work on clusters of preference (scalability)



# Collaborative Filtering

Formally, we have a set of users:  $U = \{u_1, u_2, \dots, u_m\}$

and a set of items:  $I = \{i_1, i_2, \dots, i_n\}$ .

Ratings are stored in a  $(m \times n)$  user-item rating matrix  $R = (r_{jk})$ , where each row represents a user  $u_j$  and columns represent items  $i_k$ .

\*The matrix is sparse!

\*Item ratings are on a fixed scale --- > regression.



# Collaborative Filtering

**Objective:** Create recommendations for an active user,  $u_a \in U$ .

**Unknown items:** We define the set of items unknown to the user  $u_a$  as:  $I_a = I \setminus \{i_l \in I \mid r_{al} = 1\}$ .

## Classical Tasks:

- Predict ratings for all items  $I_a$ .
- Create top-rated lists for user  $u_a$ .



# Collaborative Filtering

## User-based collaborative filtering:

- Memory based
- Mimics “word of mouth”.
- Users with similar preferences rate items similarly.

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	?	4.0	4.0	2.0	1.0	2.0	?	?
$u_2$	3.0	?	?	?	5.0	1.0	?	?
$u_3$	3.0	?	?	3.0	2.0	2.0	?	3.0
$u_4$	4.0	?	?	2.0	1.0	1.0	2.0	4.0
$u_5$	1.0	1.0	?	?	?	?	?	1.0
$u_6$	?	1.0	?	?	1.0	1.0	?	1.0
$u_a$	?	?	4.0	3.0	?	1.0	?	5.0
$r_a$	3.5	4.0			1.3		2.0	

(a)

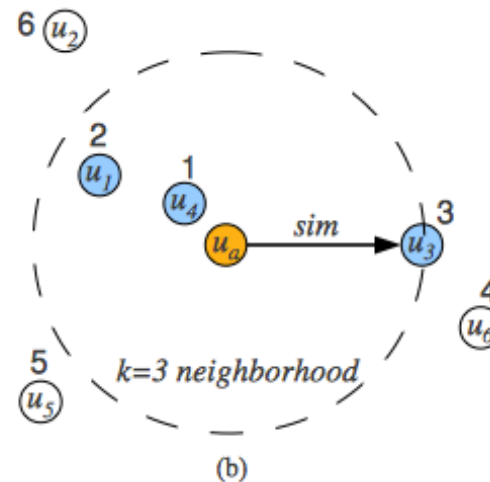


Figure 1: User-based collaborative filtering example with (a) rating matrix and estimated ratings for the active user, and (b) user neighborhood formation.



# Collaborative Filtering

## User-based collaborative filtering:

$$\text{sim}_{\text{Pearson}}(\vec{x}, \vec{y}) = \frac{\sum_{i \in I} (\vec{x}_i - \bar{\vec{x}})(\vec{y}_i - \bar{\vec{y}})}{(|I| - 1) \text{sd}(\vec{x}) \text{sd}(\vec{y})} \quad (1)$$

and

$$\text{sim}_{\text{Cosine}}(\vec{x}, \vec{y}) = \frac{\vec{x} \cdot \vec{y}}{\|\vec{x}\| \|\vec{y}\|}, \quad (2)$$

where  $\vec{x} = r_x$  and  $\vec{y} = r_y$  represent the row vectors in  $\mathbf{R}$  with the two users' profile vectors.  $\text{sd}(\cdot)$  is the standard deviation and  $\|\cdot\|$  is the  $l^2$ -norm of a vector. For calculating similarity using rating data only the dimensions (items) are used which were rated by both users.





# Collaborative Filtering

## User-based collaborative filtering:

Once the neighborhood is identified, the user ratings are aggregated to form a prediction:

$$\hat{r}_{aj} = \frac{1}{|\mathcal{N}(a)|} \sum_{i \in \mathcal{N}(a)} r_{ij}$$

Can also think about a weighted average:

$$\hat{r}_{aj} = \frac{1}{\sum_{i \in \mathcal{N}(a)} s_{ai}} \sum_{i \in \mathcal{N}(a)} s_{ai} r_{ij}$$

Where  $s_{ai}$  is the similarity between the active and the  $i^{\text{th}}$  user.

# Collaborative Filtering

## User-based collaborative filtering:

Elimination of rating bias, can be done via normalization, or z-score, etc. For example:

$$h(r_{ui}) = r_{ui} - \bar{r}_u,$$

\*Note: heavy need for memory: users have to all be compared via similarity, and the user database has to be in memory.



# Collaborative Filtering

## Item-based collaborative filtering:

- model-based – looks at similarities of items.
- $n \times n$  similarity matrix, items with low relationships are removed from the matrix (dimension reduction/filtering).  
 $n \times k$  --- trimmed

Weighted sum of user rating's on similar items.

$$\hat{r}_{ui} = \frac{1}{\sum_{j \in S(i)} s_{ij}} \sum_{j \in S(i)} s_{ij} r_{uj}$$



# Collaborative Filtering

## Item-based collaborative filtering:

S	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$	$u_a$
$i_1$	-	0.1	0	<b>0.3</b>	<b>0.2</b>	<b>0.4</b>	0	0.1	2
$i_2$	0.1	-	<b>0.8</b>	<b>0.9</b>	0	<b>0.2</b>	0.1	0	?
$i_3$	0	<b>0.8</b>	-	0	<b>0.4</b>	0.1	0.3	<b>0.5</b>	?
$i_4$	<b>0.3</b>	<b>0.9</b>	0	-	0	<b>0.3</b>	0	0.1	?
$i_5$	<b>0.2</b>	0	<b>0.7</b>	0	-	<b>0.2</b>	0.1	0	4
$i_6$	<b>0.4</b>	<b>0.2</b>	0.1	<b>0.3</b>	0.1	-	0	0.1	?
$i_7$	0	<b>0.1</b>	<b>0.3</b>	0	0	0	-	0	?
$i_8$	<b>0.1</b>	0	<b>0.9</b>	<b>0.1</b>	0	0.1	0	-	5

$r_a$    -   0.0   4.6   2.8   -   2.7   0.0   -

We know these.

$k=3$

Everything else  
Is discarded

# Collaborative Filtering

Item-based collaborative filtering:

S	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$	$u_a$
$i_1$	-	0.1	0	0.3	0.2	0.4	0	0.1	2
$i_2$	0.1	-	0.8	0.9	0	0.2	0.1	0	?
$i_3$	0	0.8	-	0	0.4	0.1	0.3	0.5	?
$i_4$	0.3	0.9	0	-	0	0.3	0	0.1	?
$i_5$	0.2	0	0.7	0	-	0.2	0.1	0	4
$i_6$	0.4	0.2	0.1	0.3	0.1	-	0	0.1	?
$i_7$	0	0.1	0.3	0	0	0	-	0	?
$i_8$	0.1	0	0.9	0.1	0	0.1	0	-	5
$r_a$	-	0.0	4.6	2.8	-	2.7	0.0	-	

$k=3$

Everything else  
Is disregarded

$$> 1/(.7+.9)*(2*0 + 4*.7 + 5*0.9)$$

[1] 4.5625

# Collaborative Filtering 0-1

## **Less information available on 0-1:**

- People don't want to do ratings.
- Preference has to be inferred by analyzing usage behavior.

In this setting, the 1's are easy to understand!

# Collaborative Filtering 0-1

## **Less information available on 0-1:**

- People don't want to do ratings.
- Preference has to be inferred by analyzing usage behavior.

In this setting, the 1's are easy to understand!

The zeros are more mysterious:

S1) The customer did not need the product right now.

S2) The customer did not know about the product.

S3) The customer did not like the product.

Same issues hold in “click-stream” modeling.

# Collaborative Filtering 0-1

In the 0-1 case with  $r_{jk} \in 0, 1$  where we define:

$$r_{jk} = \begin{cases} 1 & \text{user } u_j \text{ is known to have a preference for item } i_k \\ 0 & \text{otherwise.} \end{cases}$$

Two possibilities:

- Assume “missing”.
- Assume “negative”.
- Some trade-off's have been proposed.

$$\text{sim}_{\text{Jaccard}}(\mathcal{X}, \mathcal{Y}) = \frac{|\mathcal{X} \cap \mathcal{Y}|}{|\mathcal{X} \cup \mathcal{Y}|}, \quad (7)$$

where  $\mathcal{X}$  and  $\mathcal{Y}$  are the sets of the items with a 1 in user profiles  $u_a$  and  $u_b$ , respectively.



# Conclusions

- Recommender systems are not going away.
- Collaborative Filtering via user-based effective but expensive.
- Collaborative Filtering via item-based more realistic in terms of computation, subtly less accurate.
- Really a “many” supervised learning problems.
- The 0-1 case less understood, but more relevant.

# Thinking points.....

- The 0-1 case less understood, but more relevant.
- Conceptually compare and draw parallels the ideas of association rules, to the 0-1 case for recommender systems (using Jaccard Similarity).
- Is there a useful analogy?
- How do the “outputs” and “overall objectives” compare?

Discussion Board open until next Friday.

Up to +5 points extra credit on Homework Grade.

# Medical Applications: Why Recommender System

- Handles data of huge size well
- Handles sparse data well

## From Netflix to Heart Attacks: Collaborative Filtering in Medical Datasets

Shahzaib Hassan  
University of Michigan  
2260 Hayward St.  
Ann Arbor, MI 48109  
shahzaib@eecs.umich.edu

Zeeshan Syed  
University of Michigan  
2260 Hayward St.  
Ann Arbor, MI 48109  
zhs@eecs.umich.edu

Users = Patients  
Items = patient characteristics  
& outcome of interest  
Ratings = "Ratings"

	Sex	Age	Blood pressure	...	Disease outcome
A					
B					
C					
D					
...					

- <http://cran.r-project.org/web/packages/recommenderlab/vignettes/recommenderlab.pdf>