Critical Path Method and Program Evaluation and Review Technique

Project Networks

A network used to represent a project is called a **project network**

1. Activity information: Break down the project into its individual *activities* (at the desired level of detail).

2. Precedence relationships: Identify the *immediate predecessor(s)* for each activity.

3. Time information: Estimate the *duration* of each activity.

A**ctivity-on-arc (AOA)** project network, where each activity is represented by an *arc.* A node is used to separate an activity (an outgoing arc) from each of its immediate predecessors (an incoming arc).

The sequencing of the arcs thereby shows the precedence relationships between the activities.

A**ctivity-on-node (AON)** project network, where each activity is represented by a *node.*

Network Representation

Each activity of the project is represented by an arc pointing in the direction of progress in the project. The nodes of the network establish the precedence relationships among the different activities.

Three rules are available for constructing the network.

Rule 1. *Each activity is represented by one, and only one, arc.*

Rule 2. *Each activity must be identified by two distinct end nodes.*

Rule 3. *To maintain the correct precedence relationships, the following questions mus be answered as each activity is added to the network:*

(a) *What activities must immediately precede the current activity?*

(b) *What activities must follow the current activity?*

(c) *What activities must occur concurrently with the current activity?*

Activity	Precedence	Duration
A	-	2
B	-	5
C	-	4
D	B	5
E	A	7
F	A	3
G	B	3
H	C,D	6
I	C,D	2
J	E	5
K	F,G,H	4
L	F,G,H	3
M	I	12
N	J.K	8

Critical Path Method:

A **path** through a project network is one of the routes following the arcs from the START node to the FINISH node.

The **length** of a path is the *sum* of the (estimated) *durations* of the activities on the path.

It is the longest path in the network.

1. Total duration needed to complete the project
2. Classification of the activities o the project critical and noncritical

We define the following notations

ES_i = Earlier start time event i

LC_i = Latest completion time of event j

D_{ij} = Duration of activity (i,j)

For Forward computation

$$ES_j = \max_i(ES_i + D_{ij})$$

For Backward computation

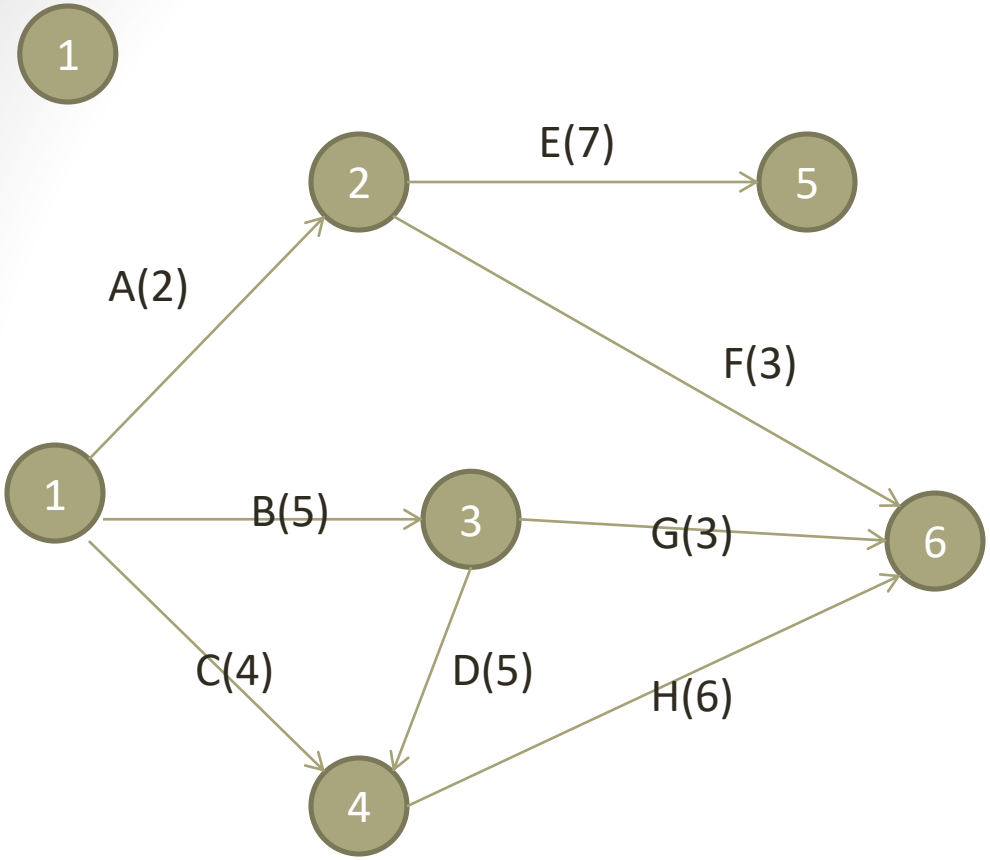
$$LC_i = \min_j(LC_j - D_{ij})$$

Optimization Critical Path

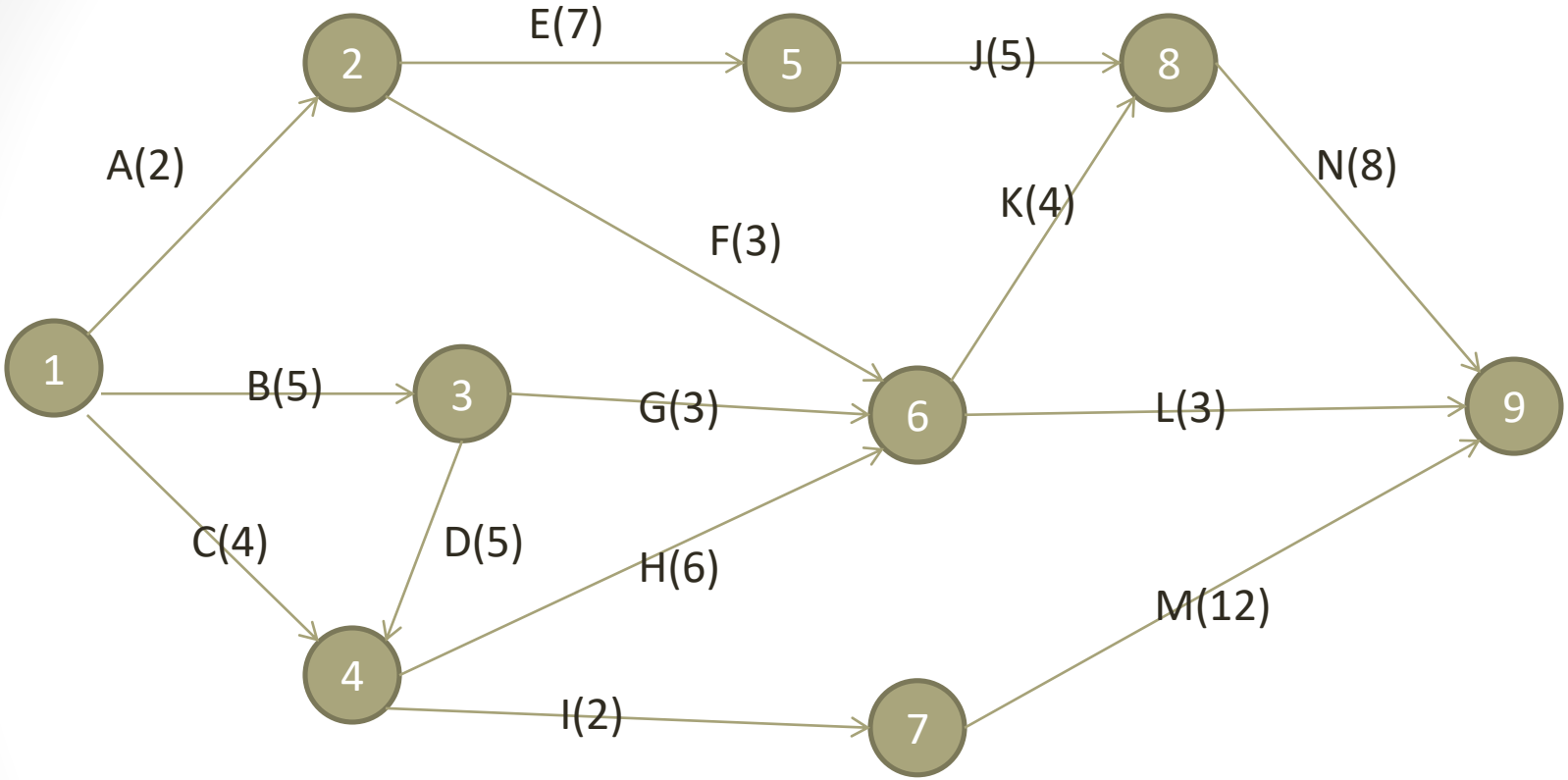
$$ES_i = ES_j$$

$$LC_i = LC_j$$

$$ES_j - ES_i = LC_j - LC_i = D_{ij}$$

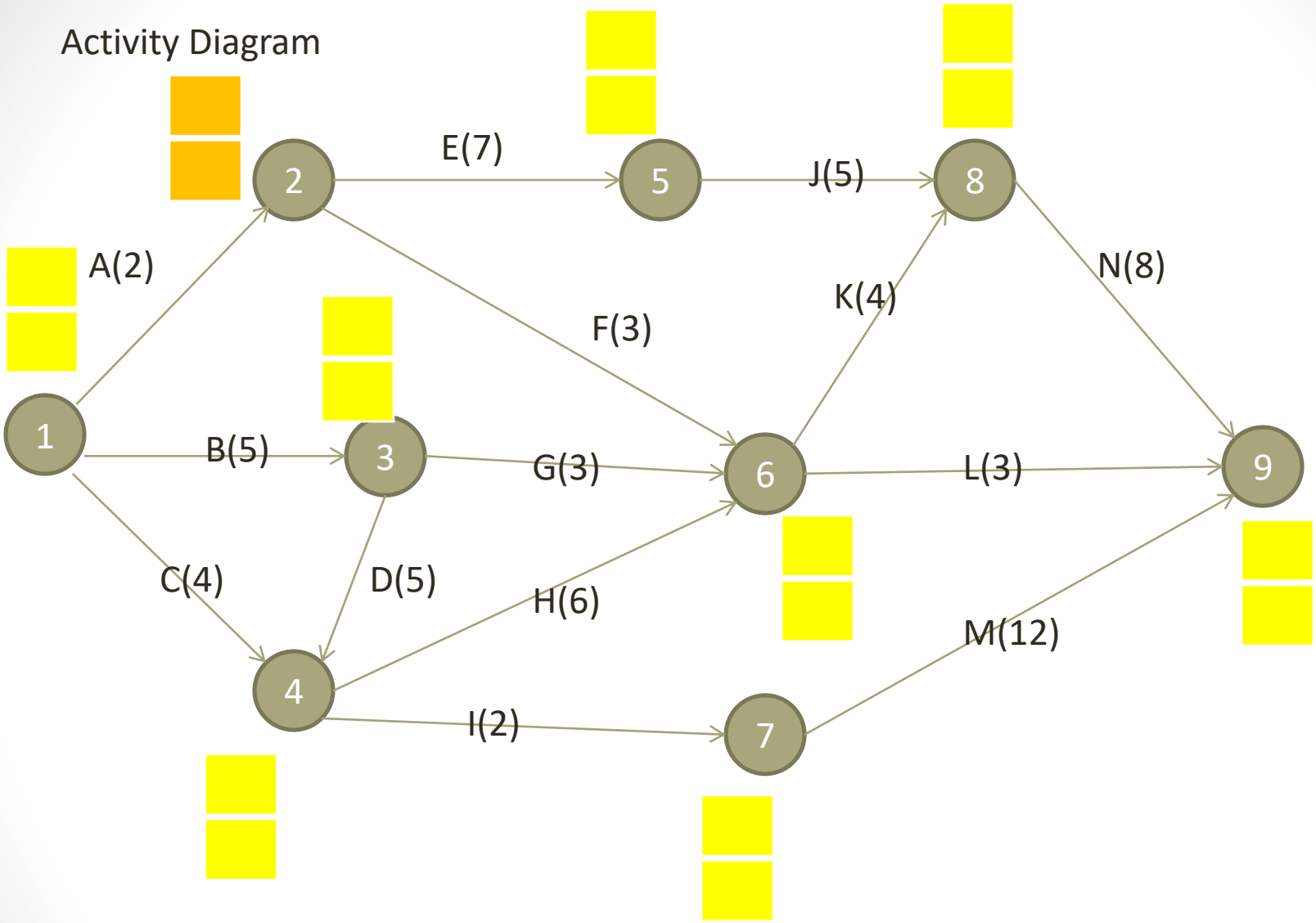

1
2
E(7)
5
A(2)
F(3)
1
B(5)
3
G(3)
6
C(4)
D(5)
H(6)
4

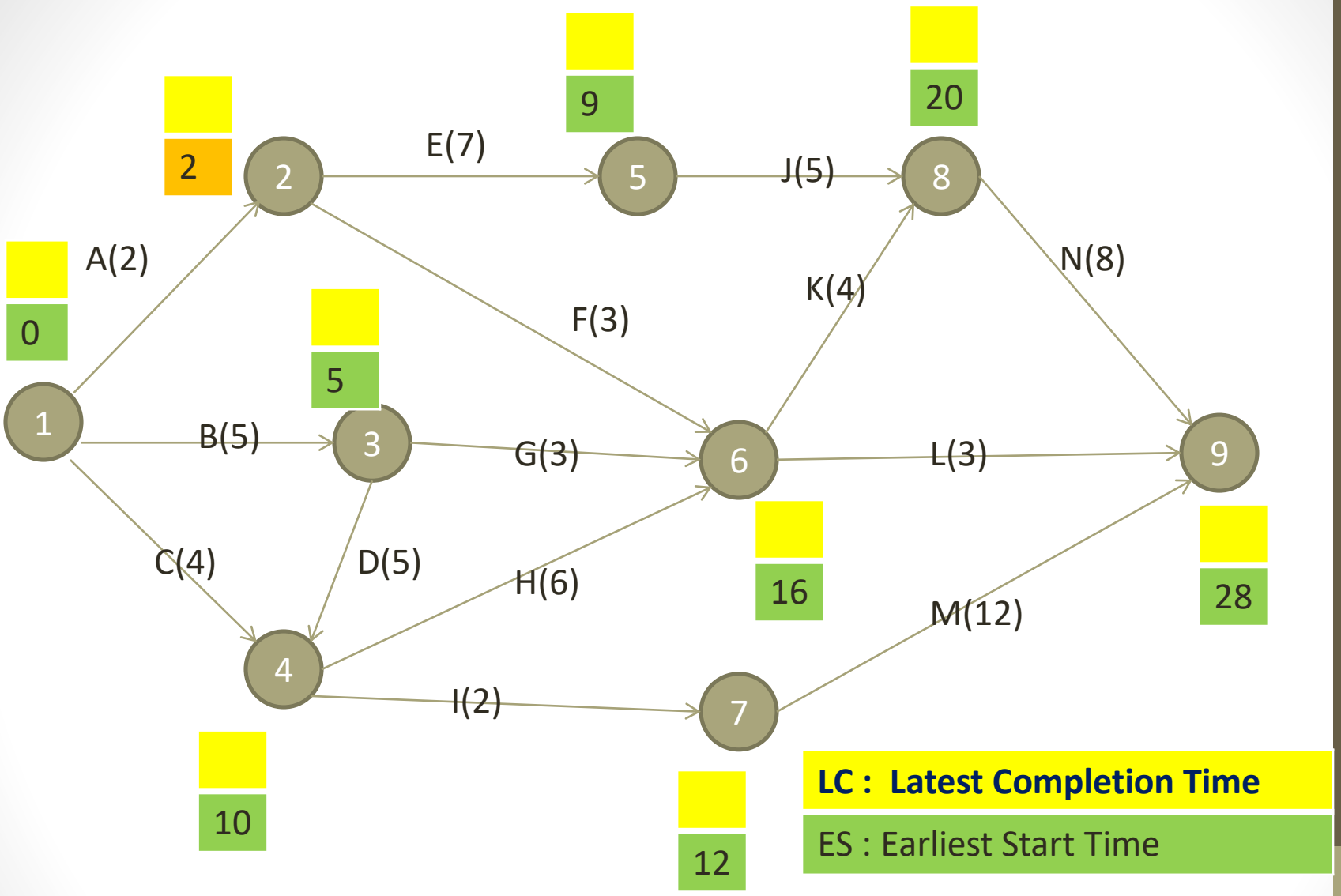
Activity Diagram



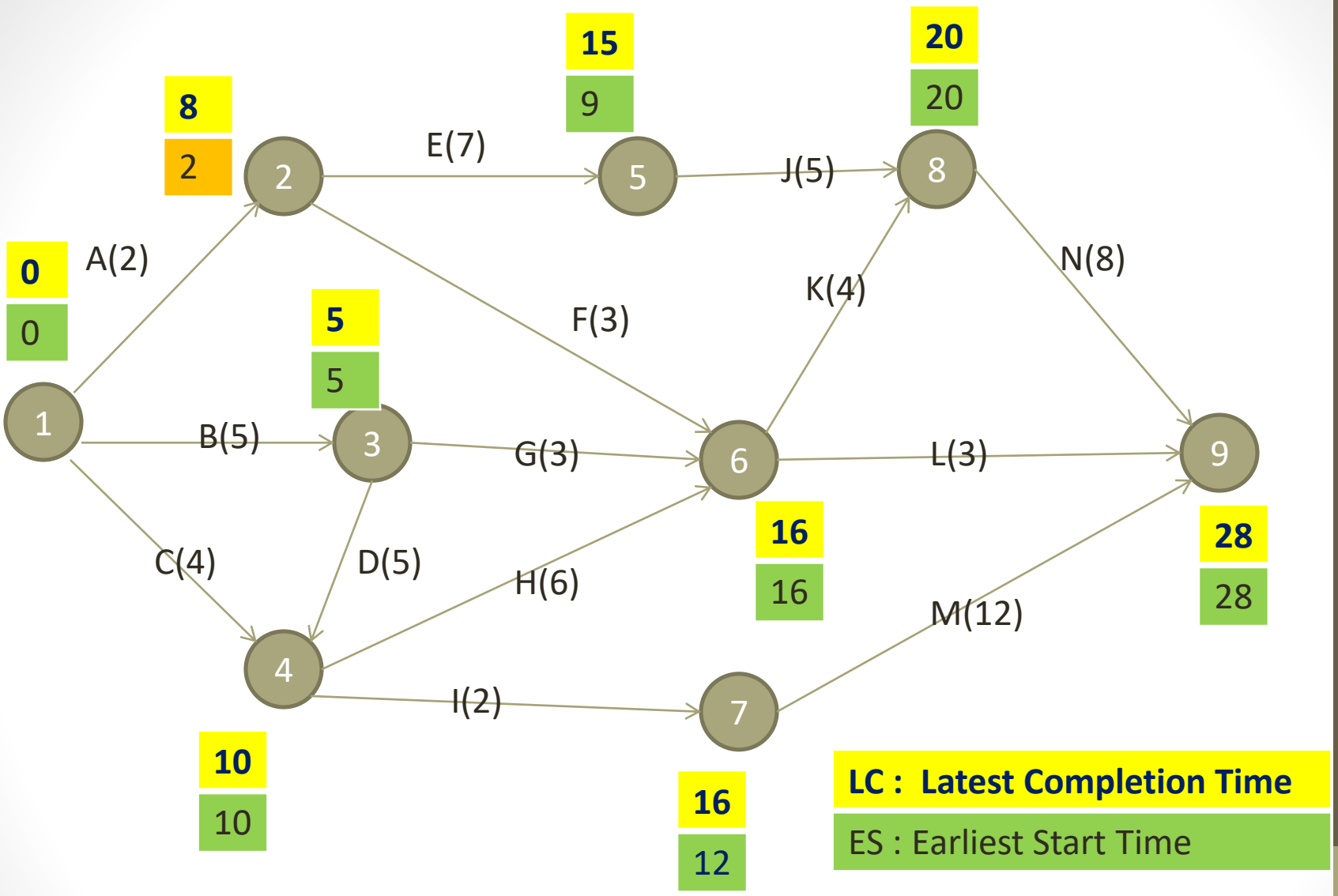
1 Node

Arrow

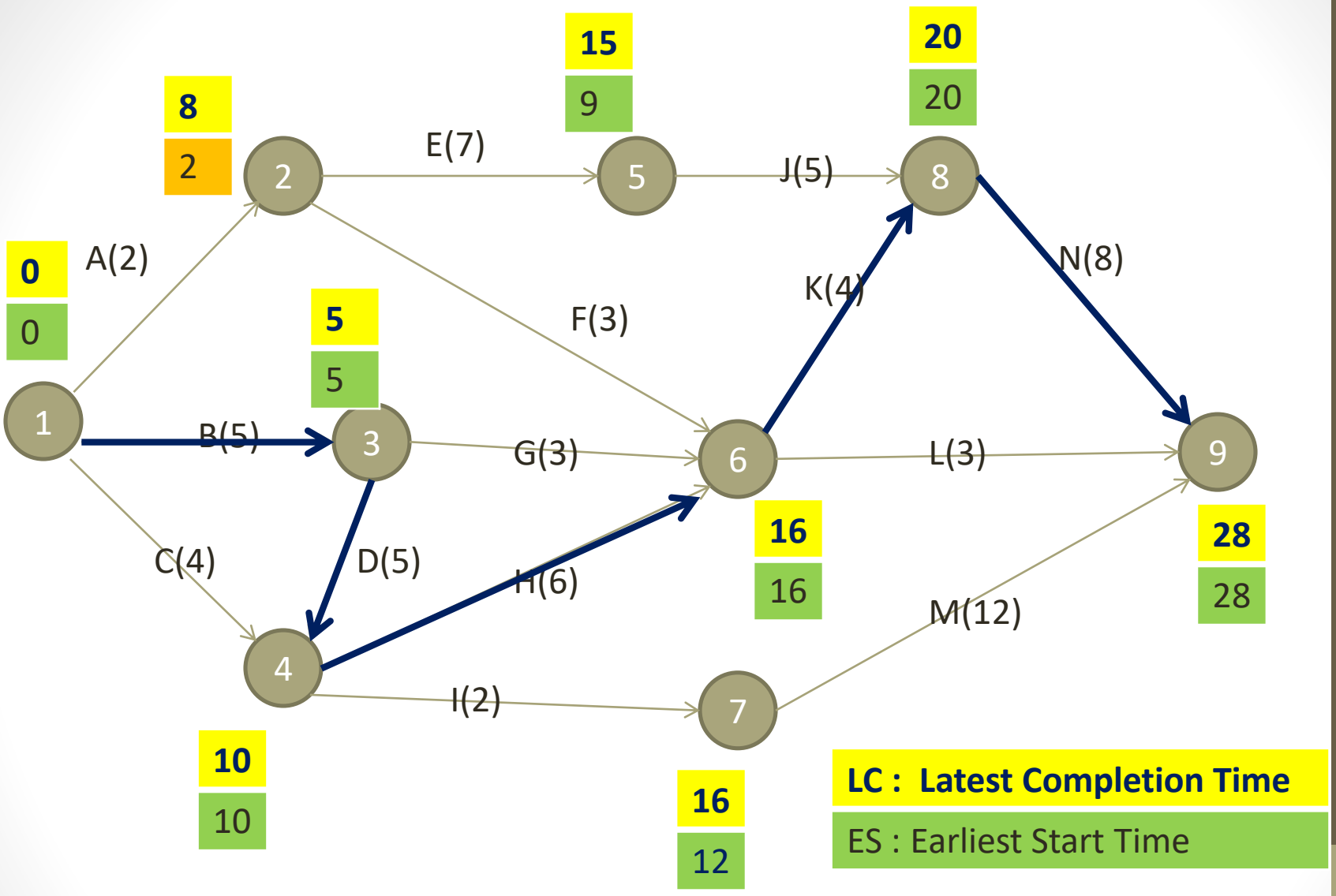

Activity Diagram
A(2)
B(5)
C(4)
D(5)
E(7)
F(3)
G(3)
H(6)
I(2)
J(5)
K(4)
L(3)
M(12)
N(8)
1
2
3
4
5
6
7
8
9



Forward Pass computes the earliest start times from the start node and moves through the network to the end node.



Backward Pass computes latest completion time that starts at end node and moves through the network to the start node node.



Critical Path: 1 --- 3 ---- 4 ---- 6 ---- 8 --- 9

5 5 6 4 8 = 28 weeks

Activity	Precedence	Duration	Total Float time	Free Float time
A: 1 - 2	-	2	6	0
B: 1 - 3	-	5	0	0
C: 1 – 4	-	4	6	6
D: 3 – 4	B	5	0	0
E: 2 – 5	A	7	6	0
F: 2 – 6	A	3	11	11
G: 3 – 6	B	3	8	8
H: 4- 6	C,D	6	0	0
I: 4 – 7	C,D	2	4	0
J: 5 - 8	E	5	6	6
K: 6 - 8	F,G,H	4	0	0
L: 6 - 9	F,G,H	3	9	9
M: 7 - 9	I	12	4	4
N: 8 - 9	J.K	8	0	0

$$TF_{ij} = LC_j - ES_i - D_{ij}$$

$$FF_{ij} = ES_j - ES_i - D_{ij}$$

PERT Networks

PERT differs from CPM in that it bases the duration of an activity on three estimates:

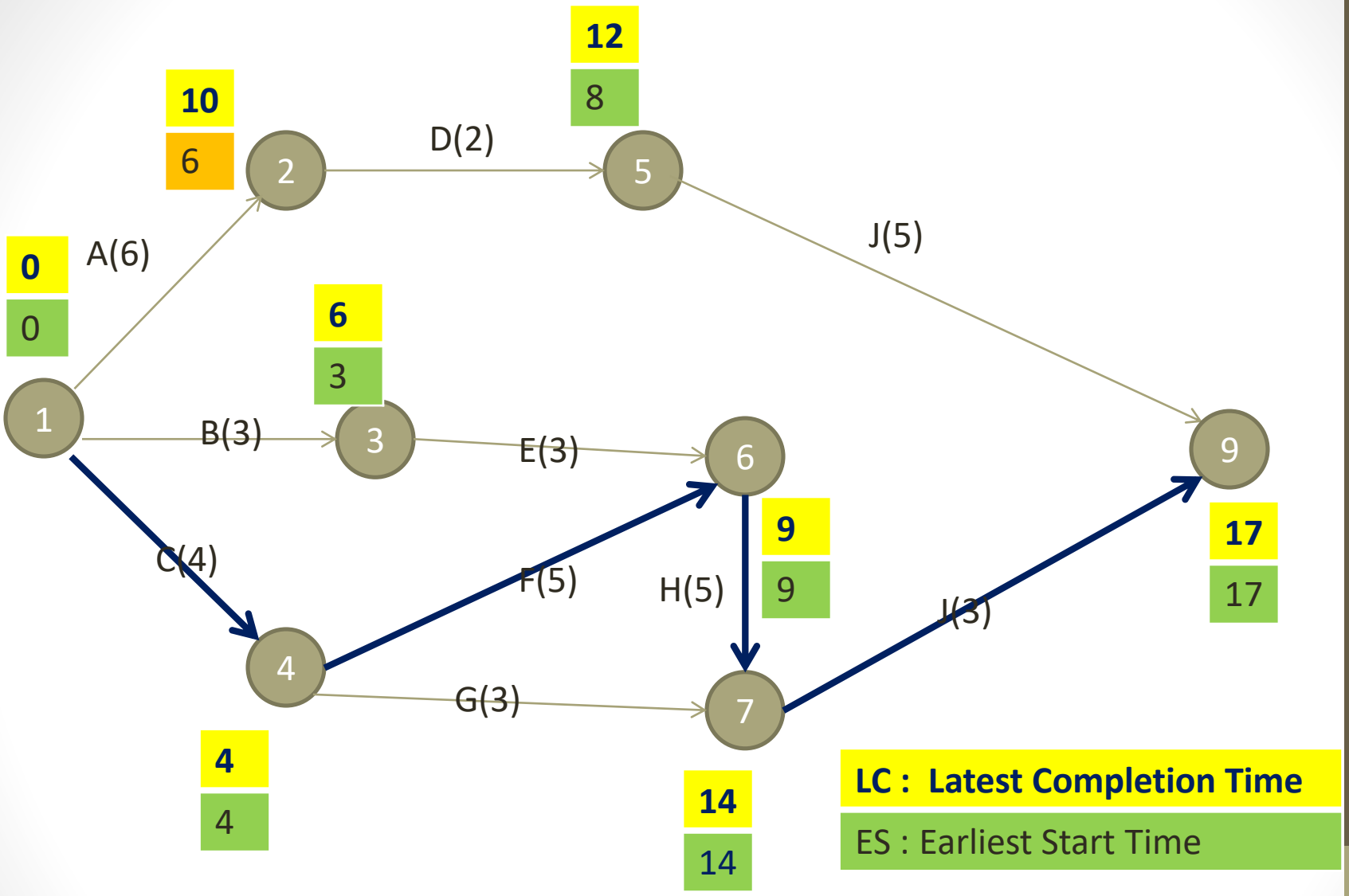
1. **Optimistic time,** a, which occurs when execution goes extremely well.
2. **Most likely time,** m, which occurs when execution is done under normal conditions.
3. **Pessimistic time,** b, which occurs when execution goes extremely poorly.

The range (a, b) encloses all possible estimates of the duration of an activity. The estimate m lies somewhere in the range (a, b). Based on the estimates, the average duration time, $\overline{D}$, and variance, v, are approximated as:

$$\overline{D} = \frac{a + 4m + b}{6}$$

$$v = \left(\frac{b - a}{6}\right)^2$$

Activity	Precedence	Duration			Mean	Variance
		Optimistic	Most Likely	Pessimistic		
A	-	5	6	7	6	0.11
B	-	1	3	5	3	0.44
C	-	1	4	7	4	1.0
D	A	1	2	3	2	0.11
E	B	1	2	9	3	1.78
F	C	1	5	9	5	1.78
G	C	2	2	8	3	1.0
H	E,F	4	4	10	5	1.0
I	D	2	5	8	5	1.0
J	H,G	2	2	8	3	1.0



Critical Path: 1 --- 4 ---- 6 ---- 7 --- 8

4 5 5 3 = 17

Activity	Mean	Variance
C	4	1.00
F	5	1.78
H	5	1.00
J	3	1.00
	$\sigma^2 =$	4.78

What will be the probability of completion time of the project on or before 22 weeks.

$\sigma = \sqrt{4.78} = 2.186$

$P(Z \leq 22) = P\left(\frac{x-\mu}{\sigma} \leq \frac{22-17}{2.19}\right) = P(Z \leq 2.28) = 0.9887$

Crashing of Project Network

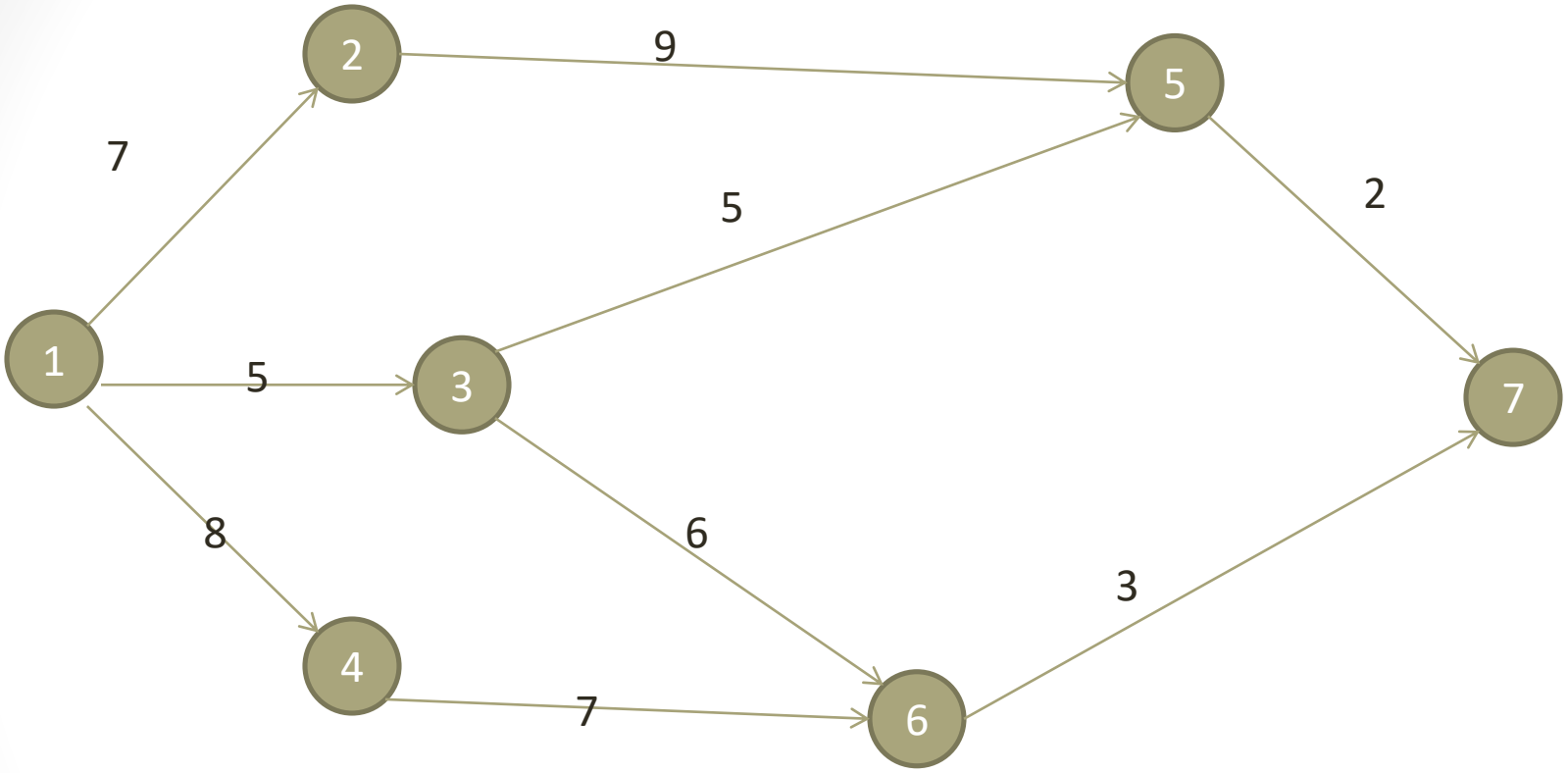
Activity	Normal Time	Normal Cost	Crash Time	Crash Cost	Slope
1-2	7	700	4	850	50
1-3	5	500	3	700	100
1-4	8	600	5	1200	200
2-5	9	800	7	1250	125
3-5	5	700	3	1000	150
3-6	6	1100	5	1300	200
4-6	7	1200	5	1450	125
5-7	2	400	1	500	100
6-7	3	500	2	850	350

Indirect cost per week is Rs. 200.

Find the optimal crash project to completion time.

$$\text{Slope} = \frac{Crash\ cost - Normal\ cost}{Normal\ Time - Crash\ Time} = (850 - 700)/(7\text{-}4) = 50$$

Activity Diagram in Normal condition



Critical Paths

Path 1: 7 + 9 + 2 = **18**

Path 2: 5 + 5 +2 = 12

Path 3: 5 + 6 +3 = 14

Path4: 8 + 7 + 3 = **18**

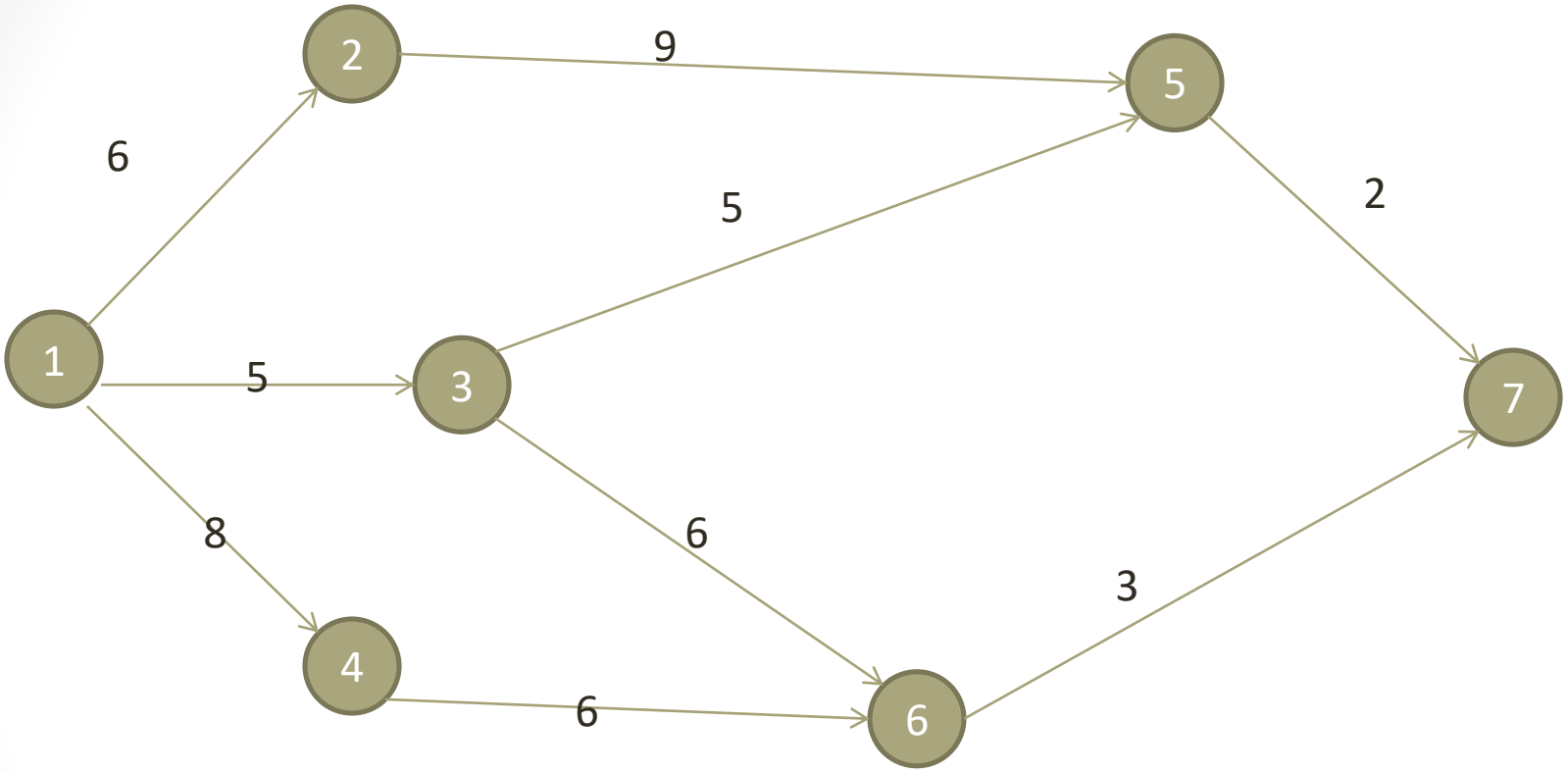
Total cost = 6500 + 200*18 = 10100

Indirect cost per week is 200.

Completion time is 18 weeks

Critical path	Critical Activity	Crash Limit	Slope
1-2-5-7	1-2	**3**	50 (Low)
	2-5	2	225
	5-7	1	100
1-4-6-7	1-2	3	200
	4-6	**2**	125 (Low)
	6-7	1	350
Total Cost = 10100 + (50 +125) – 200 = 10075			

Activity Diagram in Normal condition



Critical Paths

Path 1: 6 + 9 + 2 = **17**

Path 2: 5 + 5 +2 = 12

Path 3: 5 + 6 +3 = 14

Path4: 8 + 6 + 3 = **17**

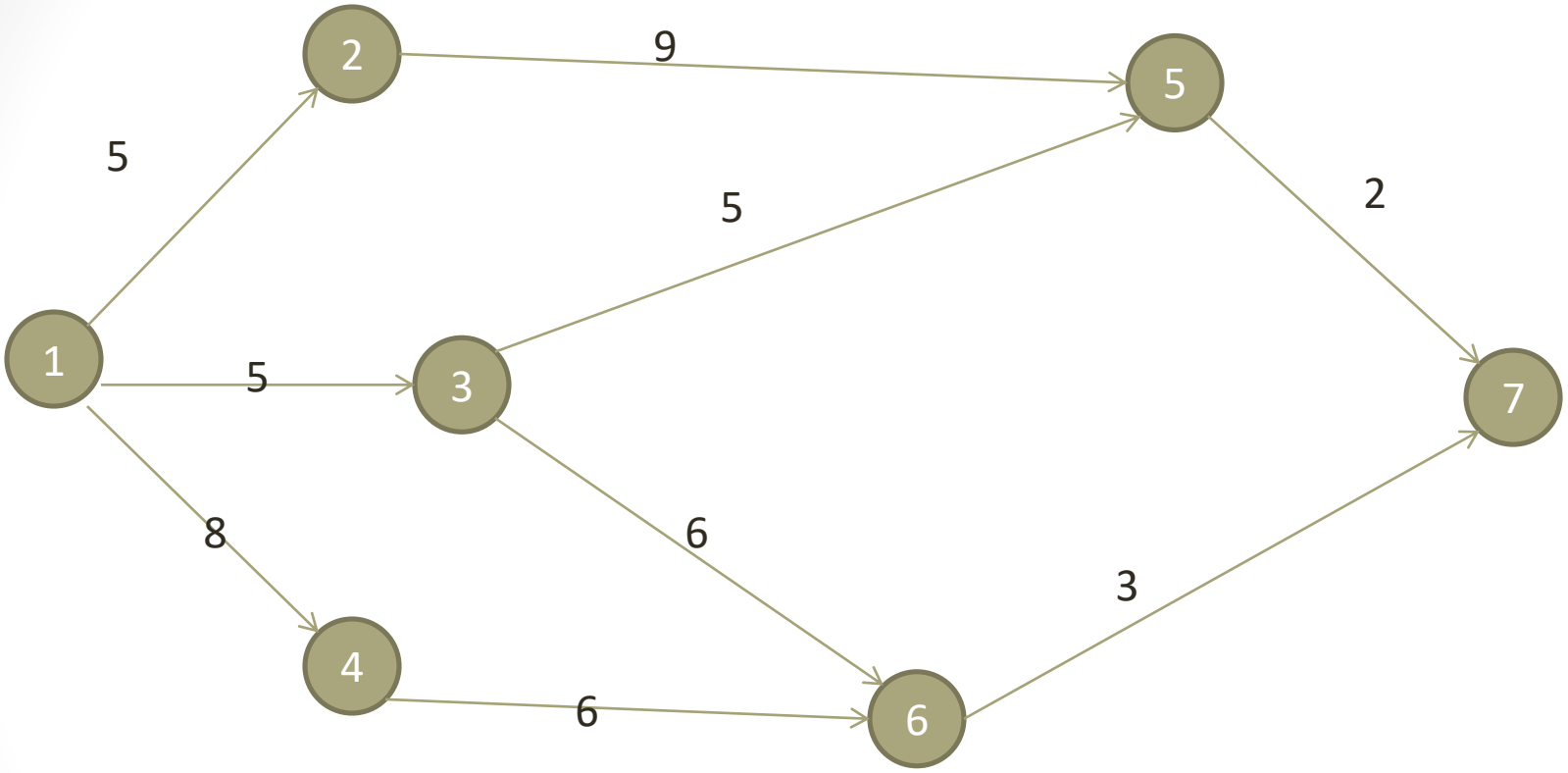
Total cost = 10100 = (50 +125) - 200 = 10075

Indirect cost per week is 200.

Completion time is 17 weeks

Critical path	Critical Activity	Crash Limit	Slope
1-2-5-7	1-2	**2**	50 (Low)
	2-5	2	225
	5-7	1	100
1-4-6-7	1-2	3	200
	4-6	**1**	125 (Low)
	6-7	1	350
Total Cost = 10075 + (50 +125) – 200 = 10050			

Activity Diagram in Normal condition



Critical Paths

Path 1: 5 + 9 + 2 = **16**

Path 2: 5 + 5 +2 = 12

Path 3: 5 + 6 +3 = 14

Path4: 8 + 5 + 3 = **16**

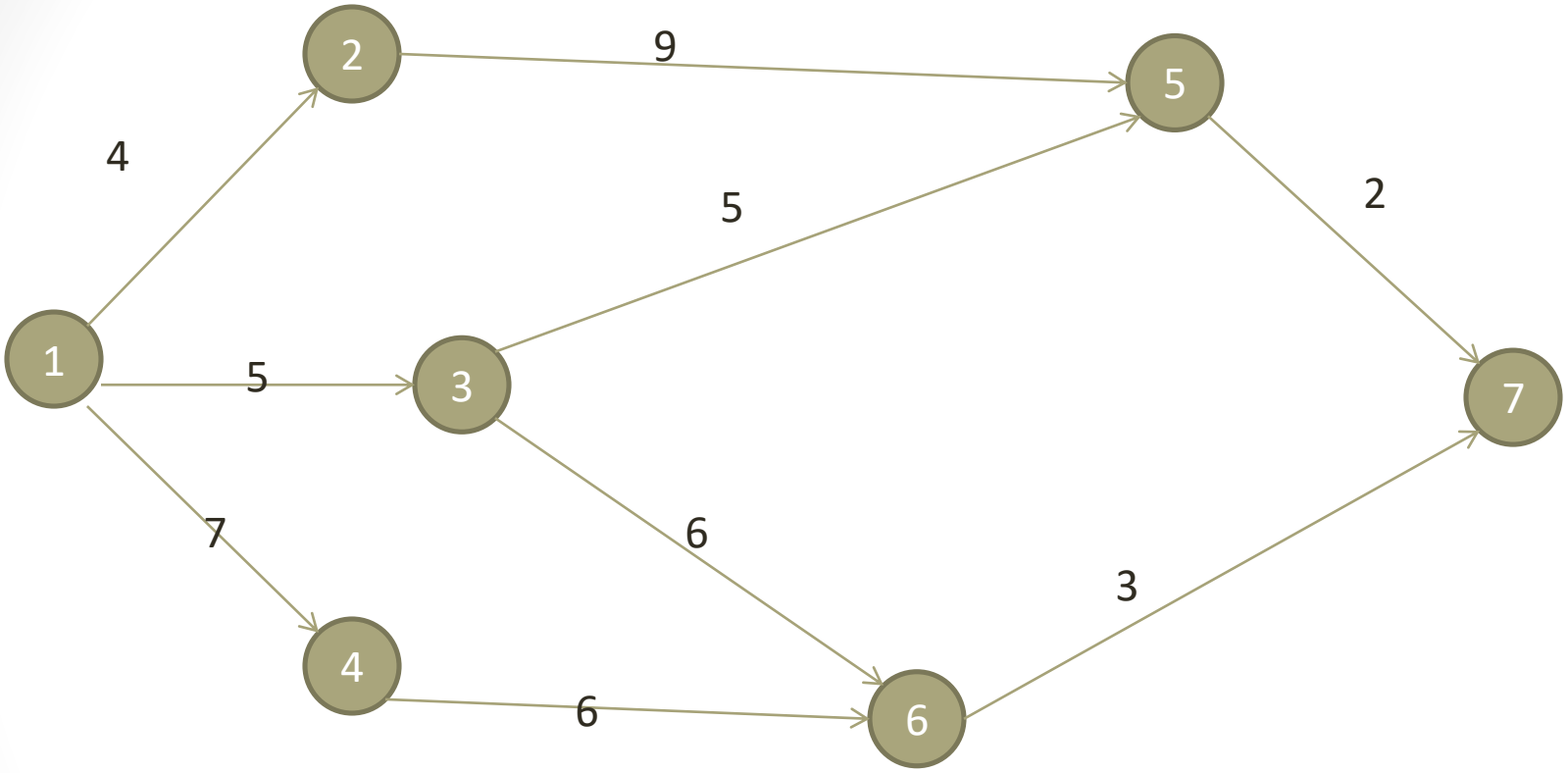
Total cost = 10075 = (50 +125) - 200 = 10050

Indirect cost per week is 200.

Completion time is 16 weeks

Critical path	Critical Activity	Crash Limit	Slope
1-2-5-7	1-2	**1**	50 (Low)
	2-5	2	225
	5-7	1	100
1-4-6-7	1-2	3	200 (Low)
	4-6	**0**	125
	6-7	1	350
Total Cost = 10050 + (50 +200) – 200 = 10100			

Activity Diagram in Normal condition



Critical Paths

Path 1: 4 + 9 + 2 = **15**

Path 2: 5 + 5 +2 = 12

Path 3: 5 + 6 +3 = 14

Path4: 7 + 5 + 3 = **15**

Total cost = 10050 + (50 + 200) - 200 = 10100

Indirect cost per week is 200.

The cost increases so we can stop.

Completion time is 16 weeks and cost will be 10050