

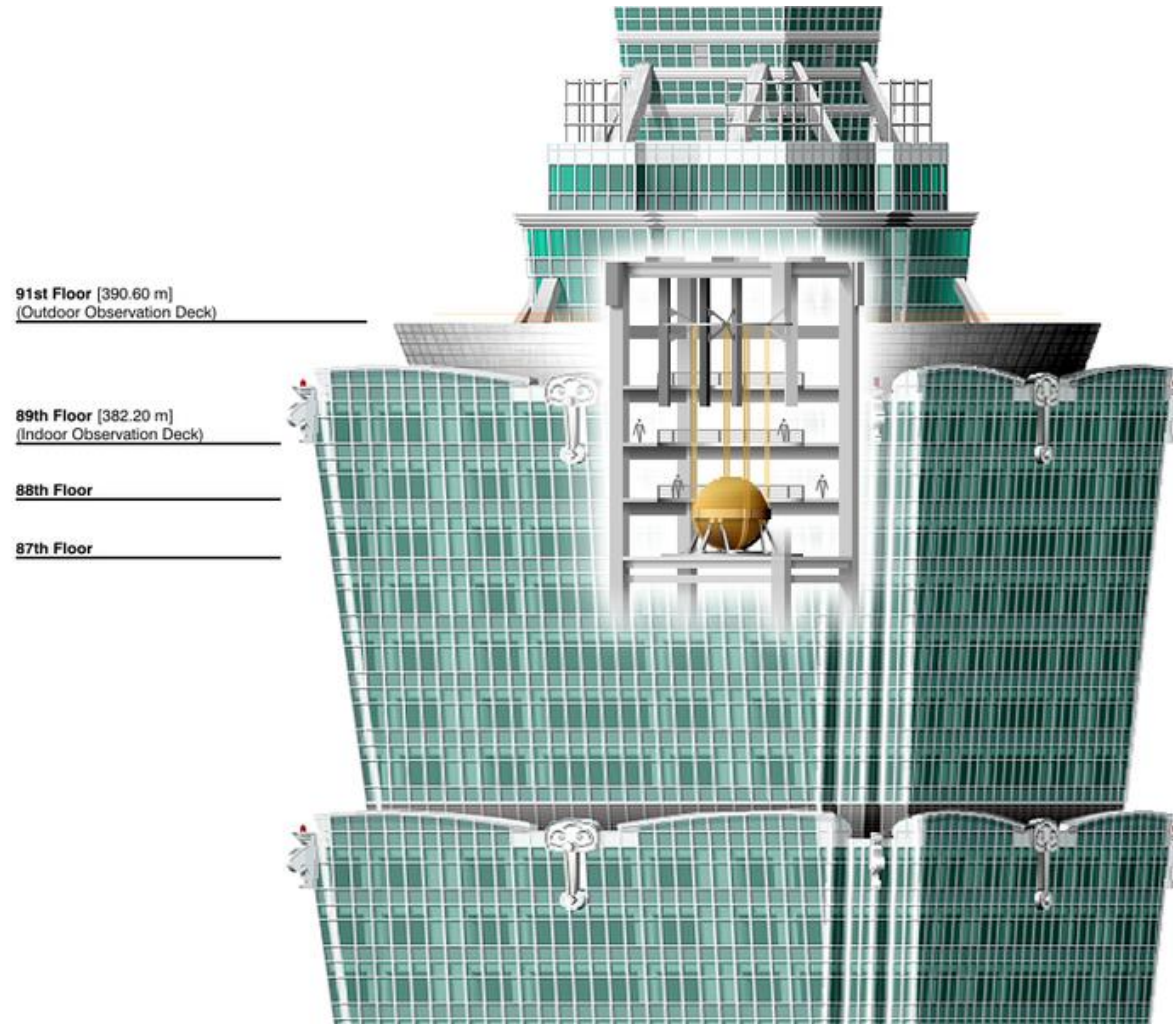
# **Oscillation and fundamental of wave optics**

# **Oscillation and fundamental of wave optics**

Periodic motion-simple harmonic motion-characteristics of simple harmonic motion-vibration of simple springs mass system. Resonance-definition, damped harmonic oscillator – heavy, critical and light damping, energy decay in a damped harmonic oscillator, quality factor, forced mechanical and electrical oscillators.



# Taipei Financial Center



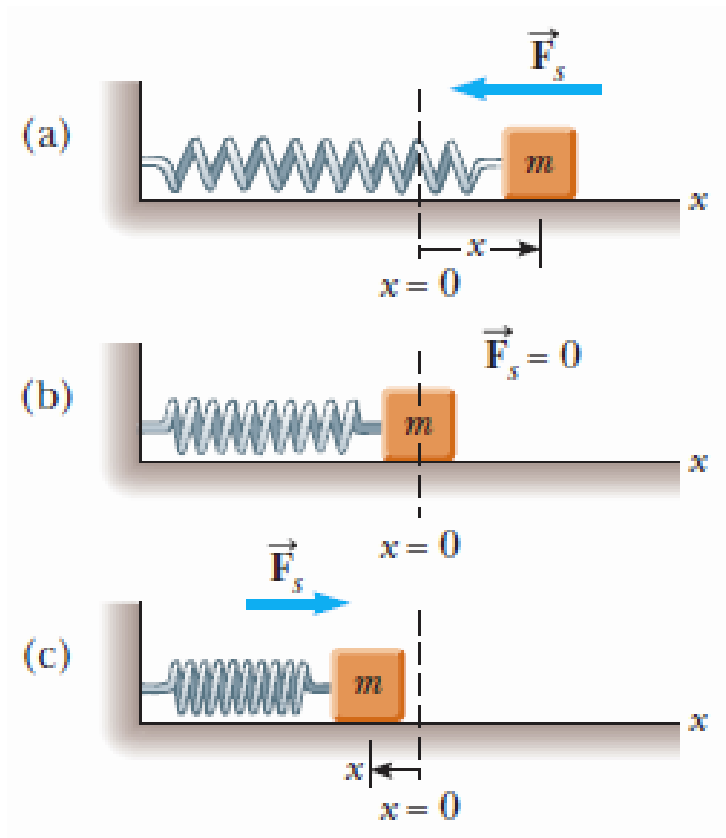
| Height        |                       |
|---------------|-----------------------|
| Architectural | 509.2 m<br>(1,671 ft) |
| Tip           | 509.2 m<br>(1,671 ft) |



To reduce swaying in tall buildings because of wind, tuned dampers are placed near the top of the building. These mechanisms include an object of large mass that oscillates under computer control at the same frequency as the building, reducing the swaying. The large sphere in the photograph on the left is part of the tuned damper system of the building in the photograph on the right, called Taipei 101, in Taiwan. The building, also called the Taipei Financial Center, was completed in 2004, at which time it held the record for the world's tallest building.

# Periodic Motion ?

A special kind of (periodic) motion occurs when the force that acts on a particle is always directed toward an equilibrium position and is proportional to the position of the particle relative to the equilibrium position.



$$F_s = -kx$$

When the particle attached to an idealized massless spring is located at a position  $x$ , the spring exerts a force  $F_s$  on it given by Hooke's law.

**Three condition for the Occurrence of Simple Harmonic oscillations. In case of mechanical oscillators, three condition must be satisfied for the occurrence of simple harmonic oscillation.**

**(1) There must be a position of stable equilibrium.**

**(2) There must be no dissipation of energy. **

**(3) The acceleration should be proportional to the displacement and opposite in direction.**



## Basic Concepts and Formulae

### Simple Harmonic Motion (SHM)

In SHM the restoring force ( $F$ ) is proportional to the displacement but is oppositely directed.

$$F = -kx$$

where  $k$  is a constant, known as force constant or spring constant. The negative sign in  $F = -kx$  implies that the force is opposite to the displacement.

When the mass is released, the force produces acceleration  $a$  given by

$$a = F/m = (-k/m)x = -\omega^2 x$$

$$\text{where } \omega^2 = k/m$$

$$\text{and } \omega = 2\pi f$$

$\omega$  is the angular frequency.



is the angular frequency.

Differential equation for SHM:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Most general solution for is

$$x = A \sin(\omega t + \varepsilon)$$

where  $A$  is the amplitude,  $(\omega t + \varepsilon)$  is called the phase and  $\varepsilon$  is called the phase difference.

## **Characteristics of Simple Harmonic Motion:**

Similarly, the solution of differential equation can be given as

$$x = A \sin (\omega t + \phi)$$

Here  $A$  denotes amplitude of oscillatory system,  $(\omega t + \phi)$  is called phase and is called  $\phi$  is initial phase/phase constant/phase angle.

### **Velocity in SHM:-**

$$x = A \sin (\omega t + \phi)$$

$$\frac{dx}{dt} = A \cos (\omega t + \phi)$$

$$v = A \cos (\omega t + \phi)$$

The minimum value of  $v$  is 0 (as minimum value of  $A \sin (\omega t + \phi) = 0$  & maximum value is  $A$ ). The maximum value of  $v$  is called velocity amplitude.

### **Acceleration in SHM:-**

$$\frac{d^2x}{dt^2} = -A^2 \sin(\omega t + \phi)$$

$$a = -A^2 \sin(\omega t + \phi)$$

The minimum value of 'a' is 0 & maximum value is  $A\omega^2$ . The maximum value of 'a' is called acceleration amplitude.

Also,  $a = \omega^2 x$  (from equation (5))

$$a \propto -y$$

It is also the condition for SHM.

### **Time period in SHM:-**

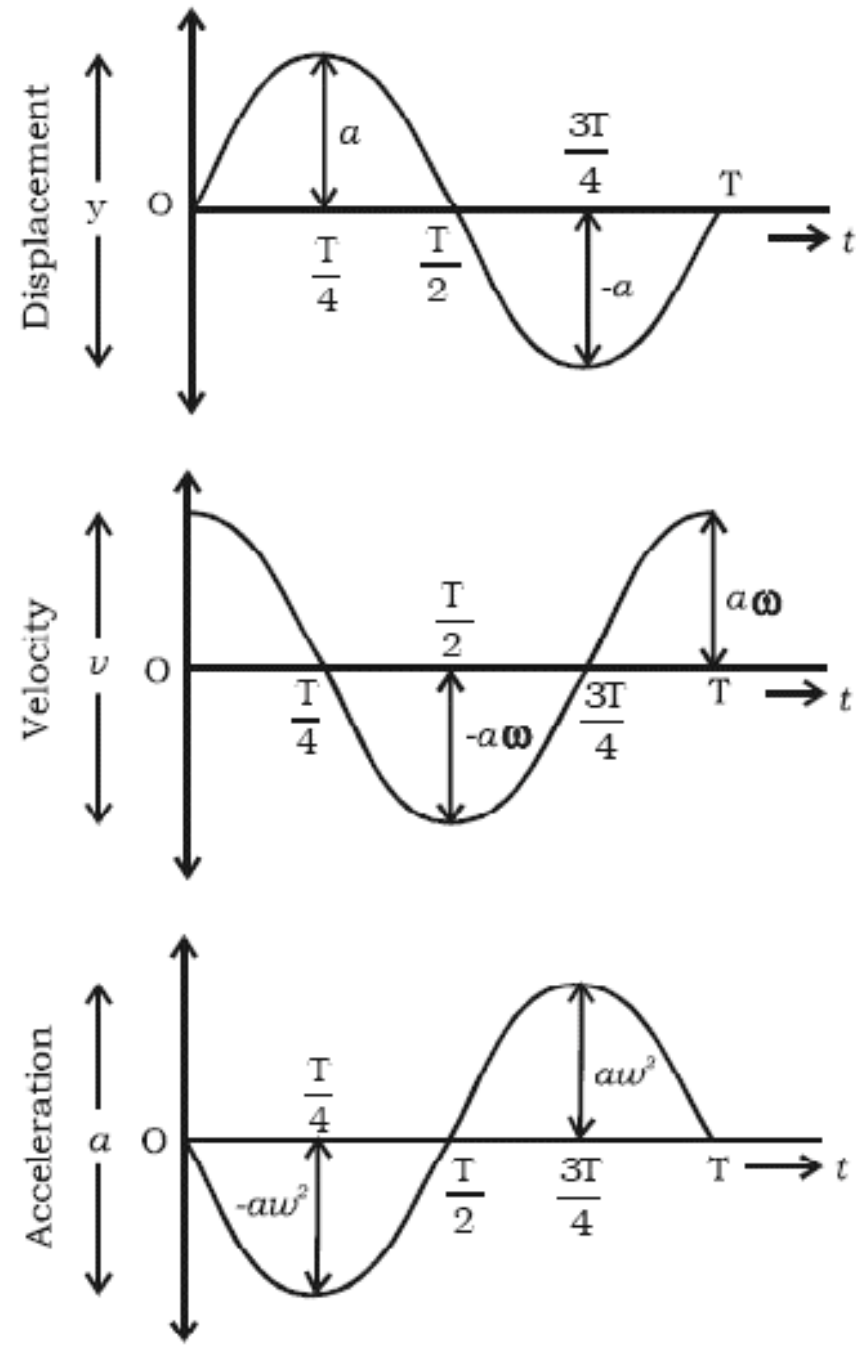
The time required for one complete oscillation is called the time period (T). It is related to the angular frequency ( $\omega$ ) by.

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

### **Frequency in SHM:-**

The number of oscillation per time is called frequency or it is the reciprocal of time period.

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{m}{k}}$$



*Graphical representation*

## Energy of Mechanical oscillator

A body executing simple harmonic oscillations is called a simple harmonic oscillator. simple harmonic oscillator possesses potential energy and kinetic energy.

So the Kinetic Equation is given as

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \varphi) \\ &= \frac{1}{2}m\omega^2 A^2 [1 - \sin^2(\omega t + \varphi)] \\ &= \frac{1}{2}m\omega^2 (A^2 - x^2) \\ &= \frac{1}{2}k(A^2 - x^2) \end{aligned}$$

The potential energy,  $U$ , is given by mass is moved against the restoring force. So the work is done which is restored as potential energy

$$= U = - \int_0^x F dx = \int_0^x kx dx = \frac{1}{2} kx^2$$

= The total energy of the simple harmonic oscillator is

$$E = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2} k(A^2 - x^2) + \frac{1}{2} kx^2$$

$$= \frac{1}{2} kA^2 = \textit{constant (Energy is conserved)}$$

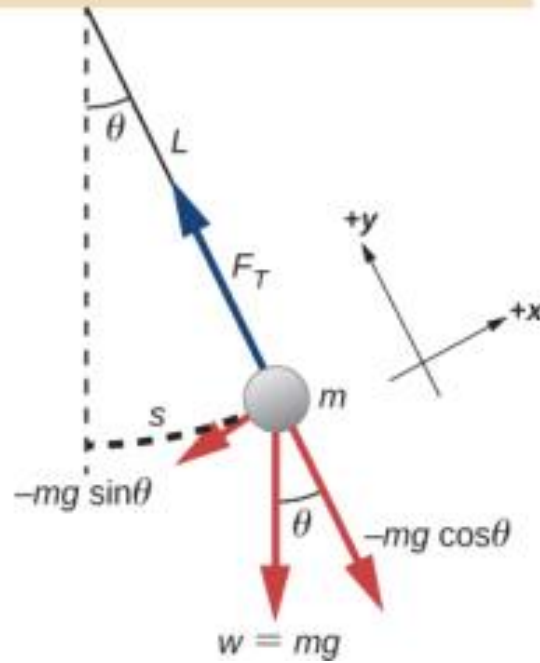
The, the total energy of a simple harmonic oscillator does not depend on time and is a constant of the motion.

# Condition for the occurrence of SHM

- There must be a position of stable equilibrium.
- There should not be dissipation of energy.
- Acceleration is proportional to the displacement  $x$  and opposite in direction.



# SIMPLE PENDULUM



A simple pendulum consists of a point mass  $m$  suspended by a light string of Length  $L$ . The restoring force  $F$  is the tangential component of the net force.

$$F = - mg \sin \theta$$

For SHM if  $\theta \ll \text{small}$  then  $\sin \theta \approx \theta$

i.e  $F = - mg\theta$

$$\Rightarrow F = - mg \frac{x}{L}$$

$$\Rightarrow m \frac{d^2 x}{dt^2} = - mg \frac{x}{L}$$

$$\Rightarrow \frac{d^2 x}{dt^2} = -g \frac{x}{L}$$

$$\Rightarrow \frac{d^2 x}{dt^2} + \omega^2 x = 0$$

$$; \omega = \sqrt{\frac{g}{L}}$$

## FREE OSCILLATION:

are oscillations that appears in a system as a result of a single initial deviation from its state of equilibrium. Simple Pendulum when displaced from equilibrium position and left to oscillate it undergo oscillation with frequency

$$\nu = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad ; \quad \omega = \sqrt{\frac{g}{L}}$$

$$\text{Time period } T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

So time period and frequency of simple pendulum depends only on length of string and gravity. It is independent of mass.

## Natural frequency

is the frequency at which a system tends to oscillate in the absence of any driving or damping force.

For Simple pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

For simple harmonic motion (spring mass system)

$$\omega = \sqrt{\frac{k}{m}}$$

The motion pattern of a system oscillating at its natural frequency is called the **normal mode**.

A **simple pendulum** of length  $L$  exhibits simple harmonic motion for small angular displacements from the vertical, with a period of

$$T = 2\pi \sqrt{\frac{L}{g}}$$

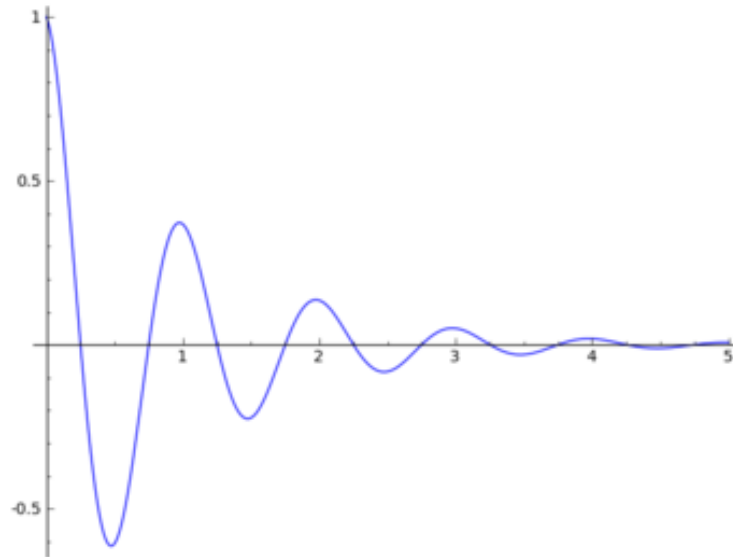
The period of a simple pendulum is independent of the mass of the suspended object.

A **physical pendulum** exhibits simple harmonic motion for small angular displacements from equilibrium about a pivot that does not go through the center of mass. The period of this motion is

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

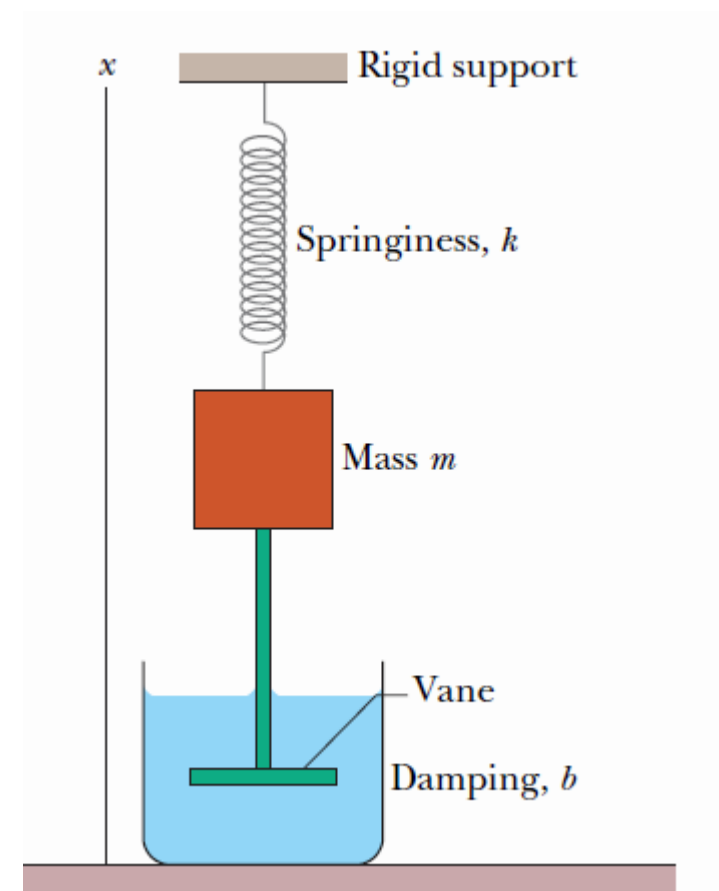
# **DAMPED MECHANICAL OSCILLATOR**

**Damped oscillations:** An ideal pendulum once set into motion, continues to oscillate between two spatial positions forever without a decrease in actual practice. No body can oscillate for an indefinite time. If we watch an oscillating pendulum, we shall find that its **amplitude of oscillation goes on decreasing due to resistance** offered both at the supports and by the surrounding air; and ultimately it stops. The **phenomenon of decay in the amplitude of oscillation is known as damping. Damped oscillation are not sinusoidal, but are much more complex.**



**Damping force is resistive : it opposes motion (i.e. is always in opposite direction to motion). A damped system is subjected to the following two force :**

- (1) A restoring force proportional to displacement but oppositely directed and**
- (2) A frictional force proportional to the velocity but oppositely directed.**





We write the damping force as  $F' = -bv$

Where  $b$  is a constant that depends on the medium and the shape of the body. The resultant force on the body is

$$F + F' = -kx - bv$$

Therefore the equation of motion of the body is

$$\begin{array}{c} \text{ma} = -kx - bv \\ \swarrow \quad \searrow \quad \searrow \\ \text{(inertial force)} = \quad \text{(Restoring)} \quad \text{(damping force)} \end{array}$$

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt}$$

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt}$$

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

**Solution of eqn**

$$x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \varphi)$$

**This angular frequency is given by**

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

The solution of this equation is

$$x = Ae^{-\frac{bt}{2m}} \cos(\omega t + \phi) \quad (\text{a})$$

where  $x_m$  is the amplitude and  $\omega'$  is the angular frequency of the damped oscillator. This angular frequency is given by

$$\omega = \sqrt{\omega_o^2 - \frac{b^2}{4m^2}} \quad (\text{b})$$

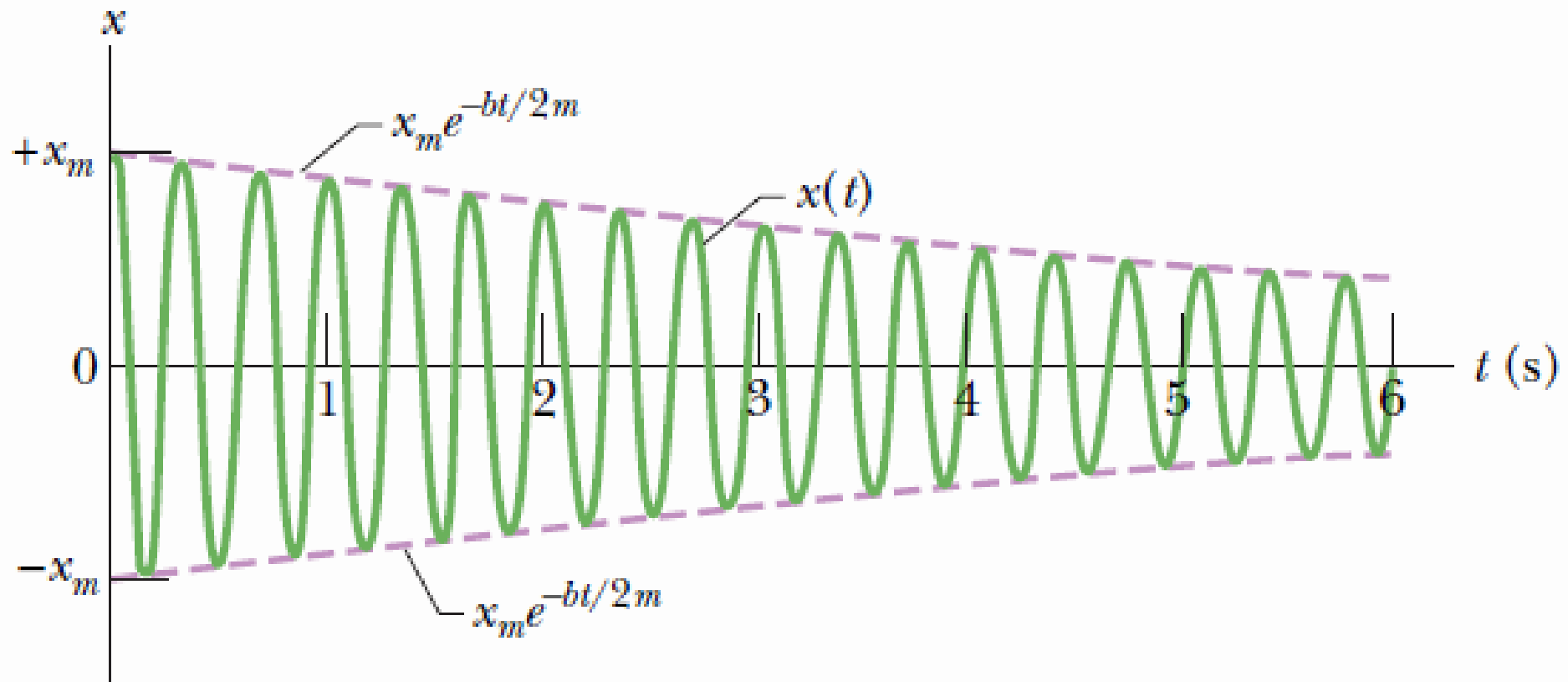
If  $b = 0$  (there is no damping), then equation (b) reduces to SHM equation of an undamped oscillator and the displacement equation (a) becomes an undamped oscillator.

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi), \quad (\mathbf{A})$$

We can regard Eq. A as a cosine function whose amplitude, which is  $x_m e^{-bt/2m}$ , gradually decreases with time. For an undamped oscillator, the mechanical energy is constant and is given by ( $E = \frac{1}{2} k x_m^2$ ). If the oscillator is damped, the mechanical energy is not constant but decreases with time. If the damping is small, we can find  $E(t)$  by replacing  $x_m$  with  $x_m e^{-bt/2m}$ , the amplitude of the damped oscillations. By doing so, we find that

$$E(t) \approx \frac{1}{2} k x_m^2 e^{-bt/2m},$$

which tells us that, like the amplitude, the mechanical energy decreases exponentially with time.



The displacement function  $x(t)$  for the damped oscillator of figure .The amplitude, which is  $x_m e^{-bt/2m}$ , decreases exponentially with time.

## Damped physical systems can be of three types

Solution:  
damped  
oscillations

$$x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \phi)$$

$$\omega' \equiv \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

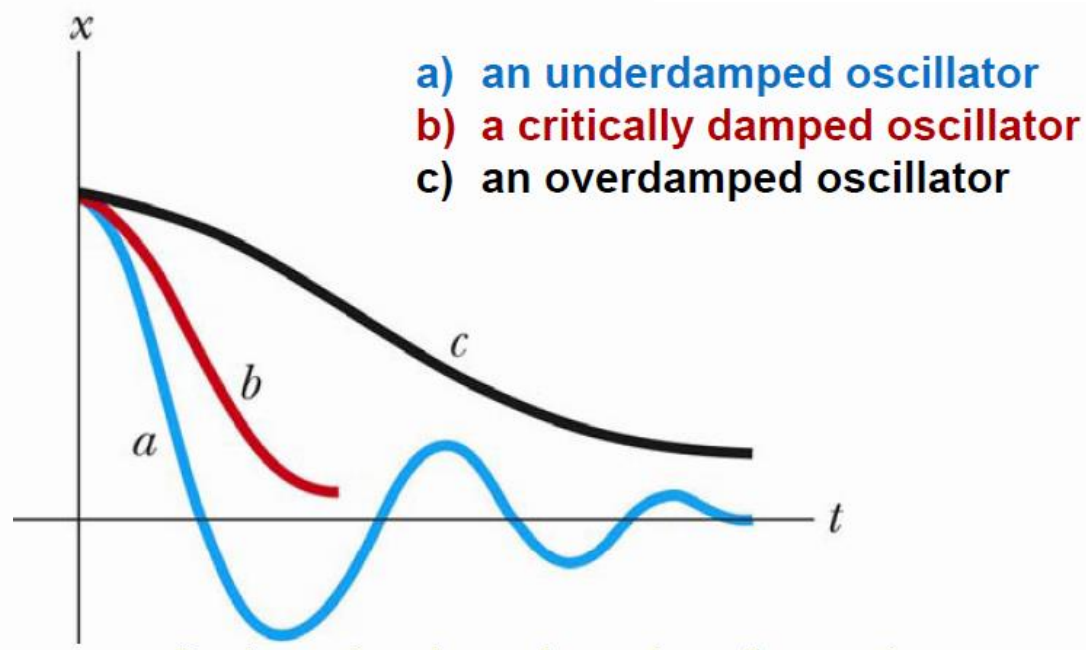
**Underdamped: small**  $b < 2\sqrt{km}$

$$\frac{b^2}{4m^2} < \frac{k}{m}, \text{ for which } \omega \text{ is positive.}$$

**Critically damped:**  $b = 2\sqrt{km}$

$$\frac{b^2}{4m^2} \approx \frac{k}{m} \equiv \omega_0^2 \text{ for which } \omega' \approx 0$$

**Overdamped:**  $\frac{b^2}{4m^2} > \frac{k}{m} \equiv \omega_0^2$  for which  $\omega'$  is imaginary



## Damped physical systems can be of three types

Solution:  
damped  
oscillations

$$x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \phi)$$

exponentially  
decaying envelope

altered  
frequency

$\omega'$  can be real  
or imaginary

$$\omega' \equiv \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

**Underdamped:**  $\frac{b^2}{4m^2} \ll \frac{k}{m} \equiv \omega_0^2$  for which  $\omega' \approx \omega_0$

- The restoring force is large compared to the damping force.
- The system oscillates with decaying amplitude

**Critically damped:**  $\frac{b^2}{4m^2} \approx \frac{k}{m} \equiv \omega_0^2$  for which  $\omega' \approx 0$

- The restoring force and damping force are comparable in effect.
- The system can not oscillate; the amplitude dies away exponentially

**Overdamped:**  $\frac{b^2}{4m^2} > \frac{k}{m} \equiv \omega_0^2$  for which  $\omega'$  is imaginary

- The damping force is much stronger than the restoring force.
- The amplitude dies away as a modified exponential



**Time Period:** of the damped harmonic oscillator is:  $T' = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$

This shows that due to damping the time period slightly increased.

**Relaxation Time:** It is the time taken for the total energy to decay to  $1/e$  of its initial value  $E_0$ .

>  $E = E_0 e^{\frac{-bt}{2m}}$

>  $E = E_0 e^{-1}$

>  $t_r$  is the relaxation time, then at  $t_r = t = \frac{2m}{b}$

# Energy in Damped Oscillations

## Energy in Damped Oscillations

In damped oscillations the damping force is nonconservative; the mechanical energy of the system is not constant but decreases continuously, approaching zero after a long time. To derive an expression for the rate of change of energy, we first write an expression for the total mechanical energy  $E$  at any instant:

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$

To find the rate of change of this quantity, we take its time derivative:

$$\frac{dE}{dt} = mv_x \frac{dv_x}{dt} + kx \frac{dx}{dt}$$

But  $dv_x/dt = a_x$  and  $dx/dt = v_x$ , so

$$\frac{dE}{dt} = v_x(ma_x + kx)$$

$$ma_x + kx = -b dx/dt = -bv_x, \text{ so}$$

$$\frac{dE}{dt} = v_x(-bv_x) = -bv_x^2 \quad (\text{damped oscillations})$$

The right side is **negative** whenever the oscillating body is in motion, whether the  $x$ -velocity  $v_x$  is positive or negative. This shows that as the body moves, the energy decreases, though not at a uniform rate. The term  $-bv_x^2 = (-bv_x)v_x$  (force times velocity) is the rate at which the damping force does (negative) work on the system (that is, the damping *power*). This equals the rate of change of the total mechanical energy of the system.

Similar behavior occurs in electric circuits containing inductance, capacitance, and resistance. There is a natural frequency of oscillation, and the resistance plays the role of the damping constant  $b$ .

## Undamped Oscillator

- It occurs in absence of any resistance to the variation of physical quantity.
- Amplitude remain constant
- Total energy remain constant.
- Energy is not dissipated
- Oscillation continuous to oscillate of its own and forever.

## Damped Oscillator

- It occurs in pressence of resistance to the physical quantity.
- Amplitude decreases continuously.
- Total energy decreases continuously.
- Energy is dissipated
- Oscillation comes to an end.

# **Forced Oscillations and Resonance**

# Forced Oscillations

A damped oscillator left to itself will eventually stop moving altogether. But we can maintain a constant-amplitude oscillation by applying a force that varies with time in a periodic or cyclic way, with a definite period and frequency. As an example, consider your cousin on a playground swing. You can keep him swinging with constant amplitude by giving him a little push once each cycle. We call this additional force a driving force.

## Forced oscillation

- oscillating system is driven by a periodic force  
(e.g. playing a swing)





# **Forced Oscillations and Resonance**

# Forced Oscillations (Mechanical)

A damped oscillator left to itself will eventually stop moving altogether. But we can maintain a constant-amplitude oscillation by applying a force that varies with time in a periodic or cyclic way, with a definite period and frequency. As an example, consider your cousin on a playground swing. You can keep him swinging with constant amplitude by giving him a little push once each cycle. We call this additional force a driving force.

## Forced oscillation

- oscillating system is driven by a periodic force  
(e.g. playing a swing)



A common example of a forced oscillator is a damped oscillator driven by an external force that varies periodically, such as  $F(t) = F_0 \sin \omega t$ , where  $\omega$  is the angular frequency of the driving force and  $F_0$  is a constant. In general, the frequency  $\omega$  of the driving force is different from the natural frequency  $\omega_0$  of the oscillator. Newton's second law in this situation gives

$$\sum F_x = ma_x \quad \rightarrow \quad F_0 \sin \omega t - b \frac{dx}{dt} - kx = m \frac{d^2 x}{dt^2}$$

Driving force =  $F(t) = A \sin \omega_d t$

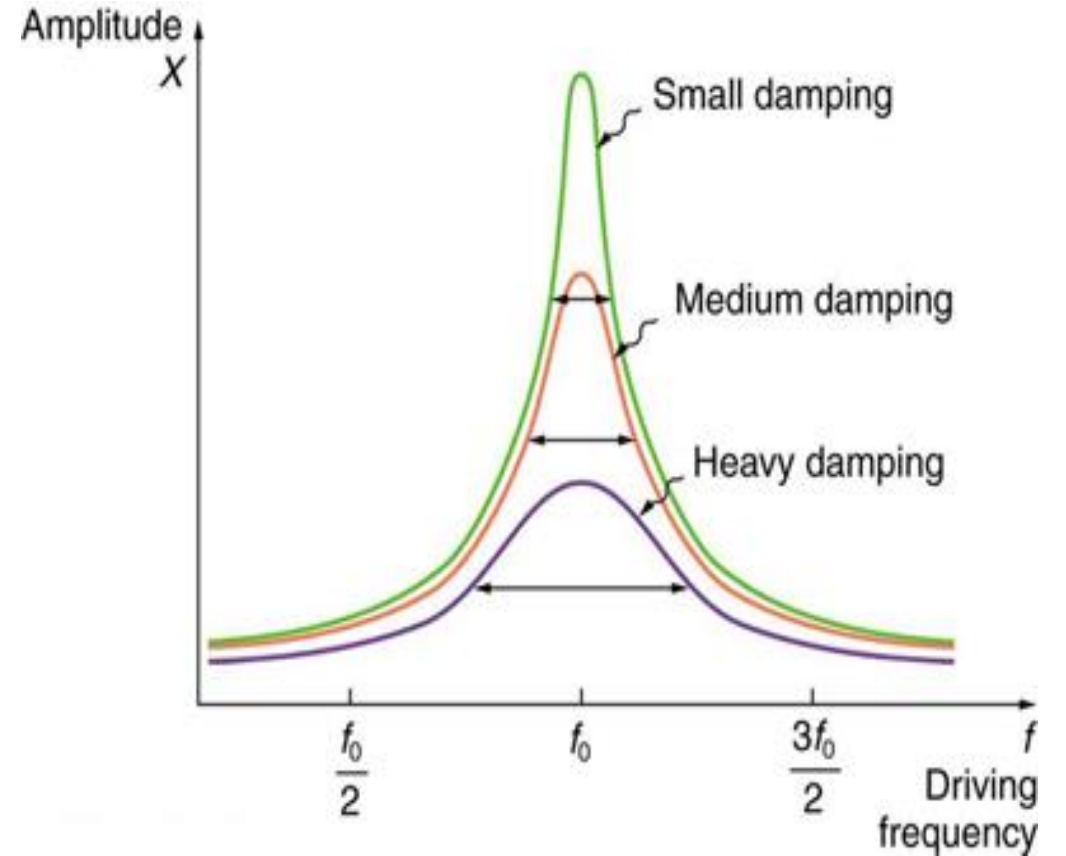
Total Eqn.

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = A \sin \omega_d t$$

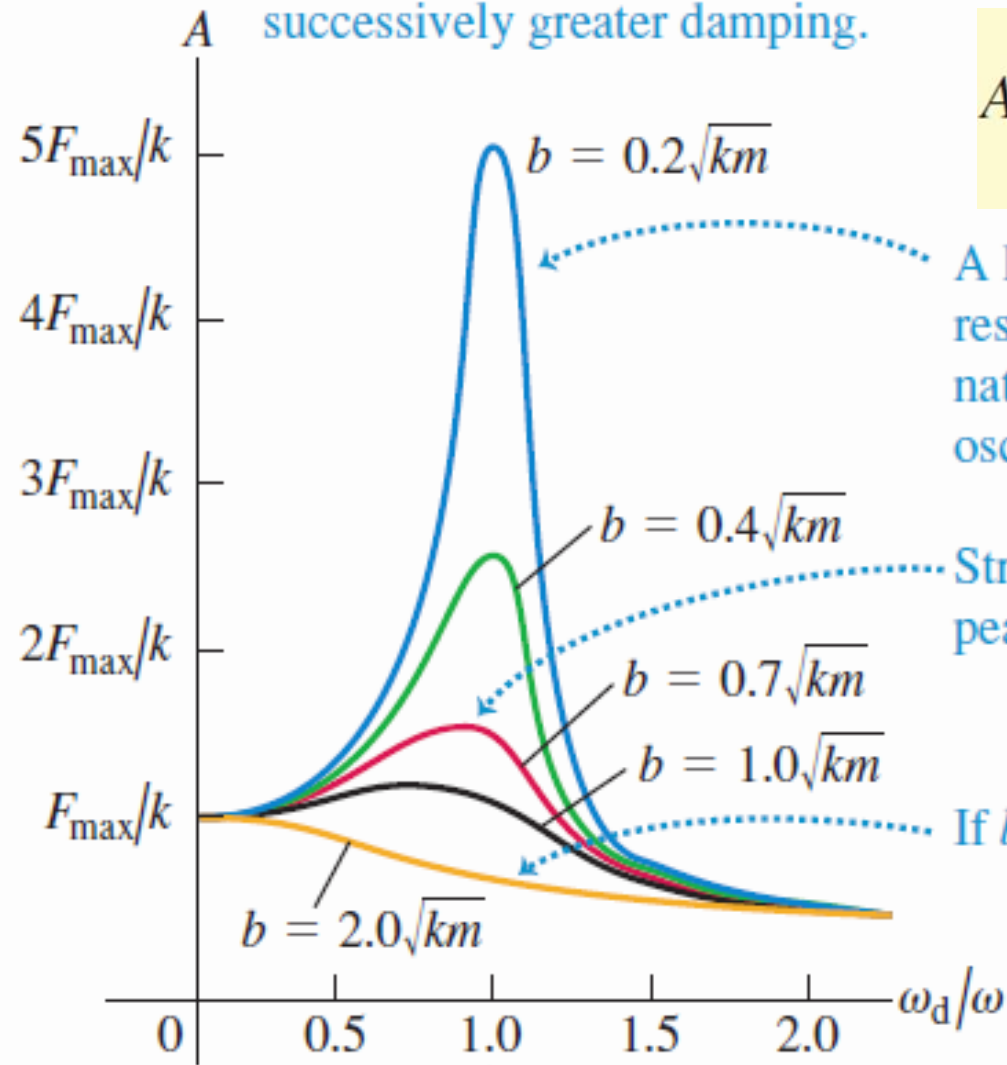
That would involve more differential equations than we're ready for, but here is the result:

$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}} \quad (\text{amplitude of a driven oscillator})$$

Graph of amplitude versus frequency for a damped oscillator when a periodic driving force is present. When the frequency of the driving force equals the natural frequency  $\omega$ , resonance occurs. Note that the shape of the resonance curve depends on the size of the damping coefficient  $b$



Each curve shows the amplitude  $A$  for an oscillator subjected to a driving force at various angular frequencies  $\omega_d$ . Successive curves from blue to gold represent successively greater damping.



$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

A lightly damped oscillator exhibits a sharp resonance peak when  $\omega_d$  is close to  $\omega$  (the natural angular frequency of an undamped oscillator).

Stronger damping reduces and broadens the peak and shifts it to lower frequencies.

If  $b \geq \sqrt{2km}$ , the peak disappears completely.

Driving frequency  $\omega_d$  equals natural angular frequency  $\omega$  of an undamped oscillator.

## Resonance

$$\omega_d = \omega$$

There is an amplitude peak at driving frequencies  $\omega_d$  close to the natural frequency  $\omega$  of the system is called **resonance**.

Phenomenon of resonance, in which an oscillating system exhibits its maximum response to a periodic driving force when the frequency of the driving force matches the oscillator's natural frequency.

We now apply this understanding to the interaction between the shaking of the ground during an earthquake and structures attached to the ground. The structure is the oscillator. It has a set of natural frequencies, determined by its stiffness, its mass, and the details of its construction. The periodic driving force is supplied by the shaking of the ground

A disastrous result can occur if a natural frequency of the building matches a frequency contained in the ground shaking. In this case, the resonance vibrations of the building can build to a very large amplitude, large enough to damage or destroy the building. This result can be avoided in two ways. The first involves designing the structure so that natural frequencies of the building lie outside the range of earthquake frequencies. (A typical range of earthquake frequencies is 0–15 Hz.)





**Quality Factor** : gives the rate of decay of energy of damped oscillator.

It is also defined as the number equal to  $2\pi$  times the ratio of the instantaneous energy of the oscillator to the energy lost during one time period after that instant.

$$Q = 2\pi \frac{\text{Energy of oscillator at any Instant}}{\text{Loss of energy during one time period}}$$

$$Q = 2\pi \frac{E}{P_d T}$$

$$\text{here } P_d = \frac{dE}{dt} \gg \frac{E}{t}$$

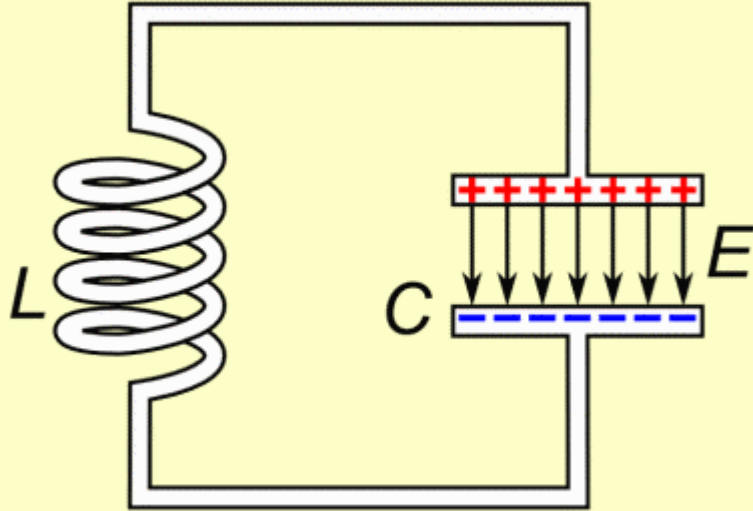
$$Q = 2\pi \frac{E}{\left(\frac{E}{t}\right)T} = 2\pi \frac{t}{T} = \omega t$$

Relaxation Time 

Quality Factor 

Damping 

## Electrical oscillator



A circuit consist of Inductor (L) and capacitor (C) serves as an electric oscillator. Suppose, a charge  $q_0$  is placed on the plates of the capacitor. As the circuit is complete (Inductance K is connected across the capacitor), the capacitor begins to discharge through the inductance coil L.

The potential difference across the capacitor plates is  $\xi_C = \frac{q}{C}$

The induced emf in the inductor, according the Faraday's laws is given by:

$$\xi_L = - L \frac{d^2 q}{dt^2}$$

Applying Kirchhoff's law, we get

$$\xi_L = \xi_C$$

$$L \frac{d^2 q}{dt^2} + \frac{q}{C} = 0$$

$$\frac{d^2 q}{dt^2} + \frac{q}{LC} = 0 \quad (1)$$

This is differential equation for the electrical oscillator.

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

the frequency of oscillation is

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

The solution of the differential equation (1)

$$q = q_0 \cos(\omega t + \phi)$$

where  $q_0$  is the amplitude, the maximum value of the charge on the plates of the capacitor.

## **Energy of the Electrical Oscillator**

In the electrical oscillator the energy exists as the electrical energy of the capacitor and Magnetic energy of the Inductor.

### **Energy of the capacitor:**

Total energy stored on the capacitor, when charge on plate is  $q$ ,

$$U_c = \frac{1}{2} \frac{q^2}{C}$$

### **Energy of the Inductor:**

$$U_L = \frac{1}{2} LI^2$$

**Total Energy = Electrical energy + Magnetic energy**

$$U_{em} = U_L + U_C$$

$$U_{em} = \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} LI^2$$

$$U_{em} = \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} L \frac{d^2q}{dt^2}$$

Substituting  $q = q_0 \cos(\omega t)$  we find:

$$U_{\text{em}} = \frac{1}{2} \frac{(q_0 \cos(\omega t))^2}{C} + \frac{1}{2} L (-q_0 \omega_o \sin \omega_o t)^2$$

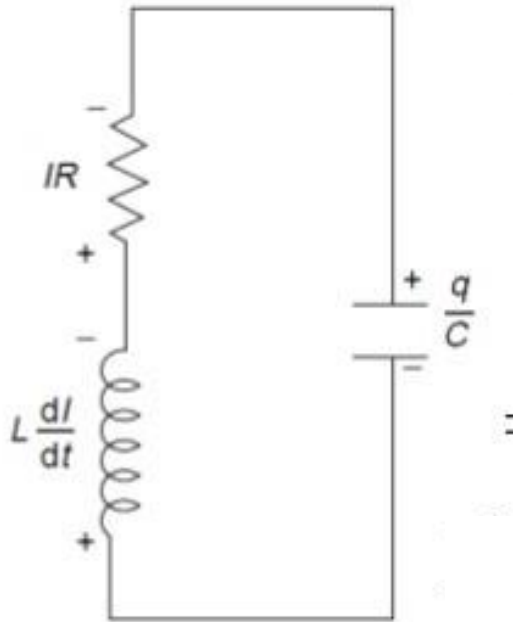
$$\text{But } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow U_{\text{em}} = \frac{1}{2} L \omega_0^2 q_o^2 (\sin^2 \omega t + \cos^2 \omega t)$$

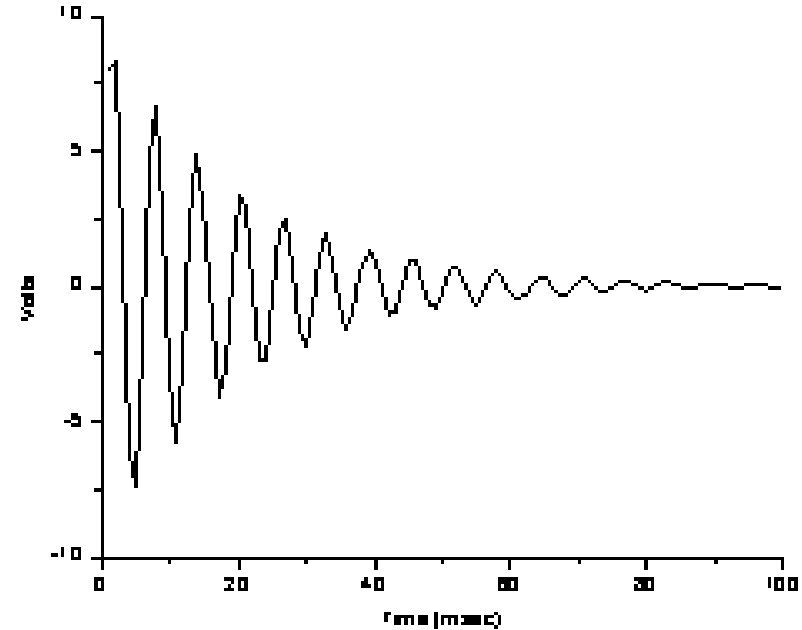
$$\Rightarrow U_{\text{em}} = \frac{1}{2} L \omega_0^2 q_o^2$$

This shows that the total energy of the oscillator is conserved.

## Damped Oscillations (Electrical )



LCR circuit



**LCR (damped oscillator) circuit diagram and voltage decay of an LCR circuit**

A circuit consist of Inductor (L), capacitor (C) and the resistance serves as damped electrical oscillator. Suppose, a charge  $q_0$  is placed on the plates of the capacitor.

The potential difference across the capacitor plates is  $= \frac{q}{C}$

The induced emf in the inductor, according the Faraday's laws is given by:

$$\xi_L = -L \frac{d^2 q}{dt^2}$$

The potential across the resistance  $\xi_R = RI = R \frac{dq}{dt}$

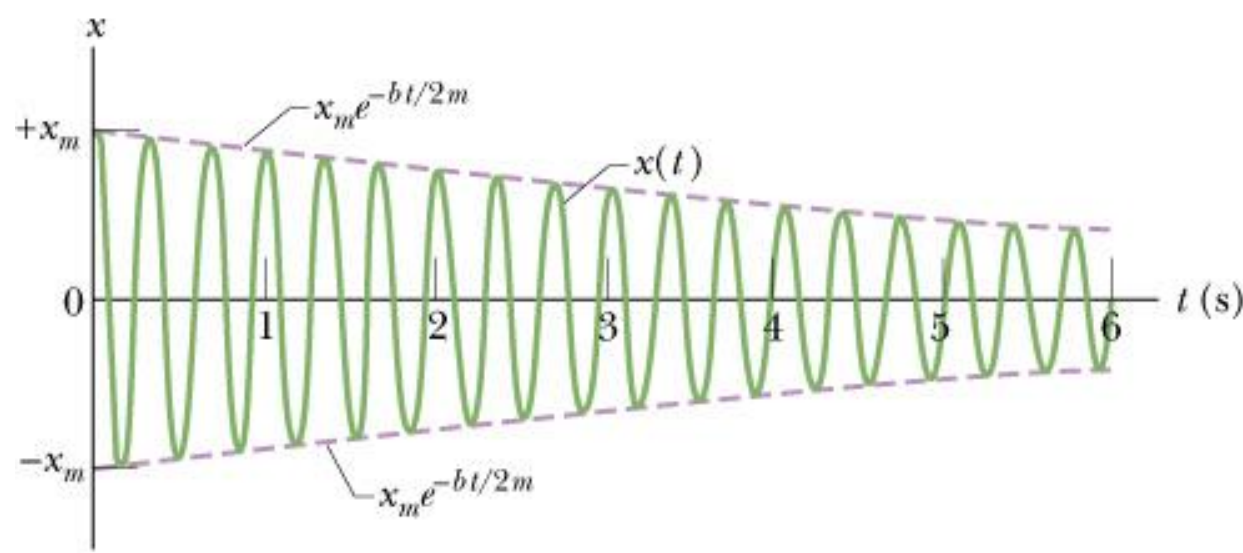
Applying Kirchhoff's law, we get

$$\xi_L + \xi_C + \xi_R = 0$$

$$L \frac{d^2 q}{dt^2} + \frac{q}{C} + R \frac{dq}{dt} = 0$$

$$\frac{d^2 q}{dt^2} + \frac{q}{LC} + \frac{R}{L} \frac{dq}{dt} = 0 \tag{1}$$

This is differential equation for the DAMPED electrical oscillator.



$$q(t) = Qe^{-Rt/2L} \cos(\omega t + f)$$

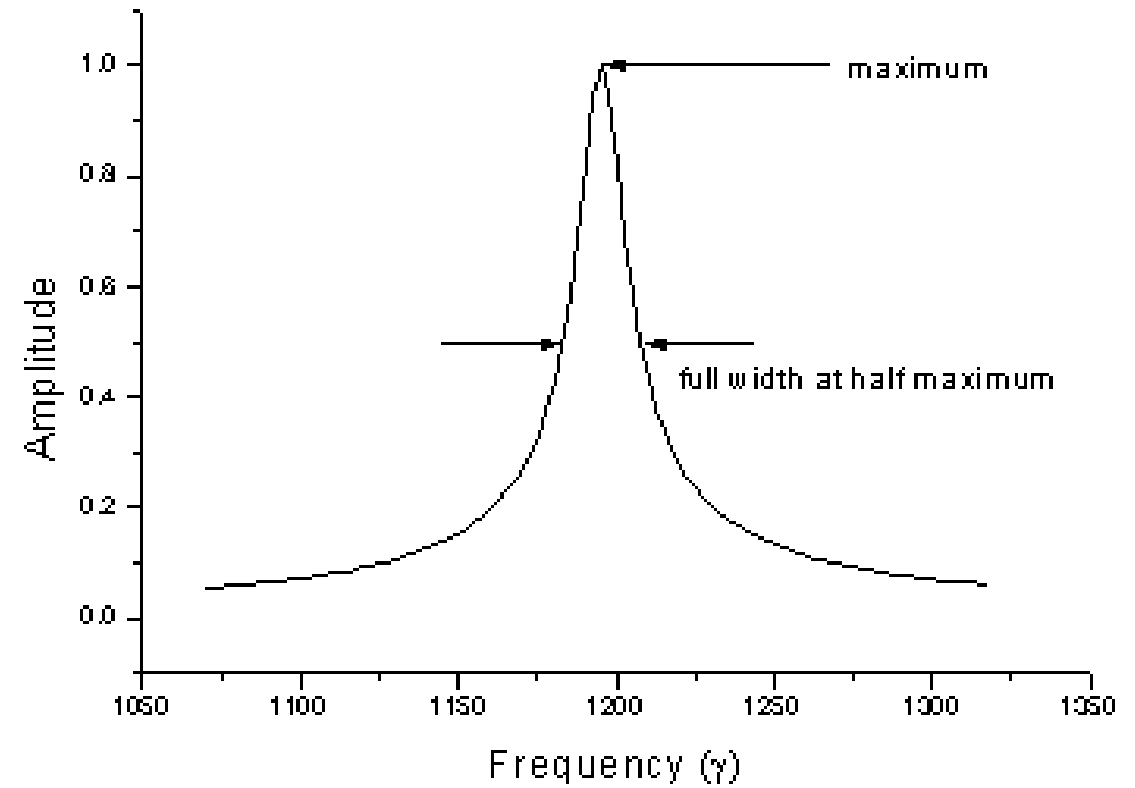
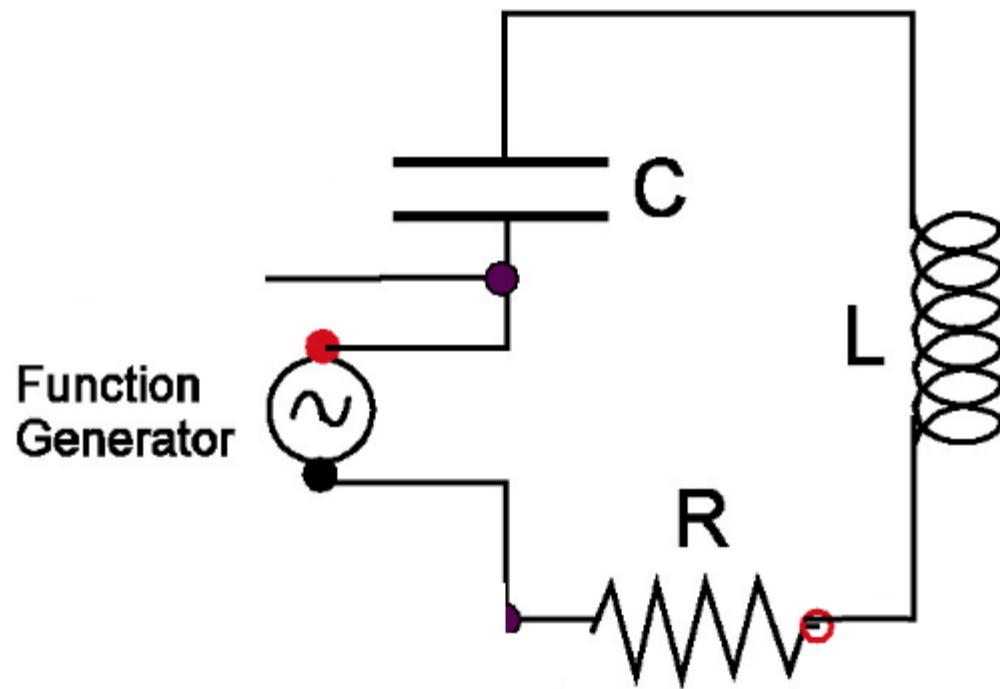
$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

The equations above describe a harmonic oscillator with an exponentially decaying amplitude  $Qe^{-Rt/2L}$ . The angular frequency of the damped oscillator

$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$  is always smaller than the angular frequency  $\omega = \sqrt{\frac{1}{LC}}$  of the undamped oscillator.



# Forced Oscillator



**The amount of charge on the capacitor in this circuit is described by the differential equation**

$$L \frac{d^2 Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = E_m \cos \omega t$$

*This equation is similar to the equation for the forced mechanical oscillator.)* The solution for the current ( $I = dQ/dt$ ) in the circuit is

$$I = I_m \cos (\omega t + \phi)$$

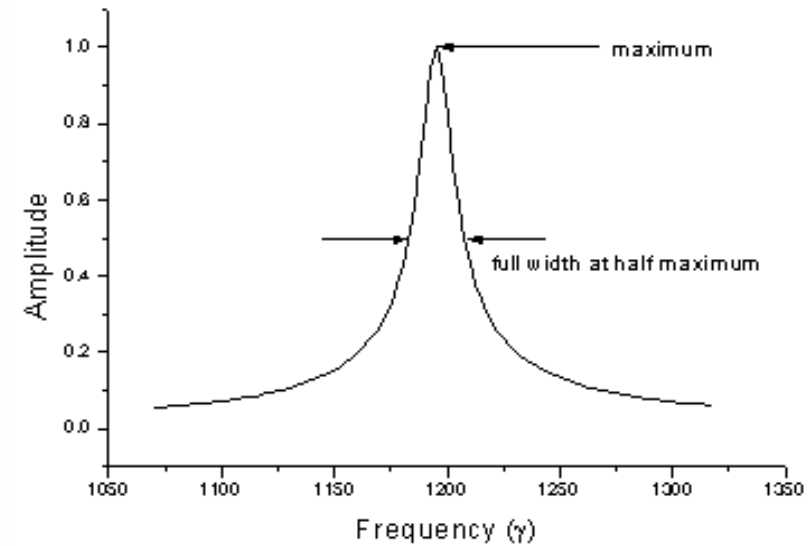
$$I_m = \frac{V_m}{D}$$

with

$$D = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$I_m$  is a maximum when  $D$  is a minimum, which occurs when the term in parenthesis above is zero and the driving frequency  $\omega$  obeys the relation

$$\omega L = \frac{1}{\omega C} \quad i.e., \quad \omega = \sqrt{\frac{1}{LC}} = \omega_R$$



These relationships show that the current is greatest when the angular frequency  $\omega$  of the driving force equals the resonant frequency of the circuit  $\omega_R$ .



**Gang capacitor in radio**

## Practice questions

### Block-spring SHM

Ques.1) A block whose mass  $m$  is 680 g is fastened to a spring whose spring constant  $k$  is 65 N/m. The block is pulled a distance  $x = 11$  cm from its equilibrium position at  $x = 0$  on a frictionless surface and released from rest at  $t = 0$ .

- (a) What are the angular frequency, the frequency and the period of the resulting motion?
- (b) What is the amplitude of the oscillation?
- (c) What is the maximum speed of the oscillating block?
- (d) What is the magnitude  $a_m$  of the maximum acceleration of the block?
- (e) What is the phase constant  $\phi$  for the motion?
- (f) What is the displacement function  $x(t)$  for the spring-block system?

## Practice questions

Ques.2) At  $t = 0$ , the displacement  $x(0)$  of the block in a linear oscillator is  $-8.50$  cm [Read as  $x$  at time zero]. The block's velocity  $v(0)$  then is  $-0.920$  m/s and its acceleration  $a(0)$  is  $+47.0$  m/s<sup>2</sup>.

What is the angular frequency  $\omega$  of this system.

What are the phase constant  $\phi$  and amplitude  $x_m$ ?

Ques.3) A block with a mass of 200 g is connected to a light horizontal spring of force constant 5.00 N/m and is free to oscillate on a horizontal, frictionless surface.

- a) If the block is displaced 5.00 cm from equilibrium and released from rest. Find the period of its motion.
- b) Determine the maximum speed and maximum acceleration of the block.
- c) Express the position, velocity and acceleration of this object as function of time, assuming that  $\phi = 0$

Ques.4) A particle oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation

$$X = (4.00 \text{ m}) \cos \left( \pi t + \frac{\pi}{4} \right)$$

Where t is in seconds.

- a) Determine the amplitude, frequency and period of the motion.
- b) Calculate the velocity and acceleration of the particle at any time t.
- c) What are the position and the velocity of the particle at time  $t = 0$ .



Ques.5) For the damped oscillator  $m = 250 \text{ g}$ ,  $k = 85 \text{ N/m}$  and  $b = 70 \text{ g/s}$

- a) What is the period of the motion?
- b) How long does it take for the amplitude of the damped oscillations to drop to half its initial value?
- c) How long does it take for the mechanical energy to drop to one-half its initial value?

A spring is hung with an object and vibrated. For the vibration frequency to double the original vibration frequency, then the mass of the object is changed to...

- A. twice the mass of the original load
- B. four times the mass of the original load
- C. half the load mass time
- D. a quarter of the original load mass

An LCR series circuit with inductance  $1\text{mH}$ , capacitance  $100\text{mF}$  and resistance  $1\text{K Ohm}$  are connected to AC voltage source. Find the frequency of source for which current through the resistor is maximum.