Unit 2

AC Circuits

Topics:

1. Generation of AC Signal
2. Alternating quantities:
    a. Instantaneous, RMS and Average values
    b. Form Factor and Crest factor
3. Analysis of AC Circuits:
    a. Resistive circuit
    b. Inductive circuit
    c. Capacitive circuit
    d. Series R-L circuit
    e. Series R-C circuit
    f. Series R-L-C circuit
4. Resonant circuit
    a. Series resonant circuit
    b. Parallel resonant circuit
5. Numericals on all of the above topics.

Unit 2
A.C.FUNDAMENTALS

2.1 : Generation of Alternating Voltage and Current:-

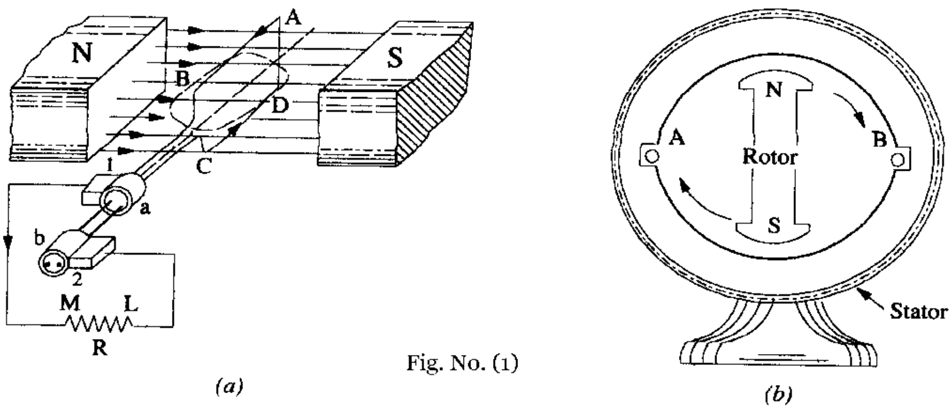


Fig. No. (1)

Alternating voltage may be generated by rotating a coil in a magnetic field as shown in fig. 1(a) or 1(b).

Consider a rectangular coil having N turns and rotating in a uniform magnetic field with an angular velocity of ω rad/sec in anticlockwise direction as shown in fig. 2. In time *t* second, this coil rotates through an angle θ = wt. Maximum flux Φ_m links the coil when its plane co-insides with the x-axis. In deflected position the plane of coil is $\Phi = \Phi_m \cos(wt)$. Hence, flux total linkages of the coil at any time are $N\Phi = N\,\Phi_m \cos(wt)$.

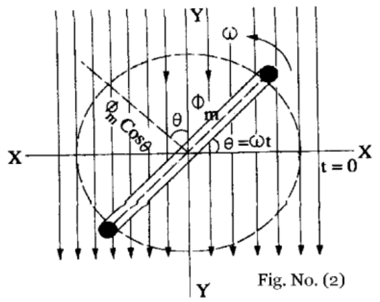


Fig. No. (2)

Equations of the Alternating voltage and current:-

According to faraday's Laws of electromagnetic induction the emf induced in the coil is given by the rate of change of flux linkage of the coil. Hence the value of the induced emf is

$$e = -\frac{d}{dt}(N\phi) \text{ Volts} \qquad = -\frac{d}{dt}(\phi_m \cos wt) \text{ Volts}$$

So $$e = -N\phi_m w(-\sin wt) \text{ Volts}$$

$$e = wN\phi_m \sin\theta \text{ Volts} \quad ----------(1)$$

When the coil has turned through 90° i.e. when θ = 90° then sin θ =1, hence e has maximum value say E_m. Then from equation (1) we get

$$E_m = \omega\, N\, B_m\, A \text{ Volts} --- (2).$$

Where B_m = maximum flux density in wb/m^2.

A = area of the coil in m^2, f = frequency in rev./sec.

Substitute this value of E_m in equations (1), we get

$$e = E_m \sin \theta = E_m \sin wt \text{ --- (3)}$$

Similarly the equations of induced current is $I = I_m \sin wt$----------------------- (4)

Provided the load is resistive, w = 2πf.

$$e = E_m \sin 2\pi ft = E_m \sin(\frac{2\pi}{T})t$$

$$\& \ I = I_m \sin 2\pi ft = I_m \sin(\frac{2\pi}{T})t$$

where T = time period of a.c. voltage or current.

The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates.

2.2 Parameters of a AC signal:

a. Cycle: -

The time taken by an alternating quantity to complete one set of positive and negative values is called its time period T.

For example a 50 Hz A.C. has a time period of ($1/50$) second.

b. Frequency (*f*): -

The number cycles / second is called the frequency of the A.C. quantity.

$f = PN$ 120 where N = revolutions in rpm and P = number of poles.

For example an alternator having 20 poles and running at 300 rpm will generate A.C. voltage and current whose frequency is $f = \frac{20\times 300}{120}$ = 50 Hz & $f = 1/T$.

c. Instantaneous amplitude v(*t*): The value of the alternating quantity at a particular instant of time is known as instantaneous value,

d. Peak Apmlitude (*Vm)*: - The maximum value positive or negative of an alternating quantity is known as its amplitude.

e. Phase (ϴ): - By phase of an A.C. current is meant the fraction of time period of that A.C current which has elapsed since the current last passed through the zero position of reference.

For example: the phase of current at point A (*T*/4 sec) , where T is time period in terms of angle it is $\Theta = \omega t = \frac{2\pi}{T}\frac{T}{4} = \pi/2$ radians (fig. No. 3).

Similarly, the phase of the rotating coil at the instant shown in fig. (4) (a) is θ = wt which is called its phase angle.

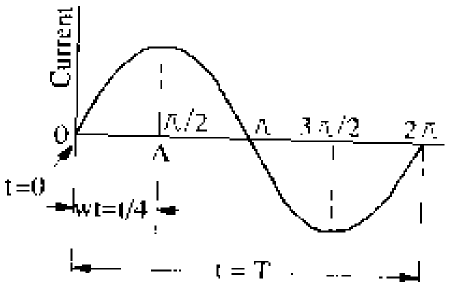


Fig 3

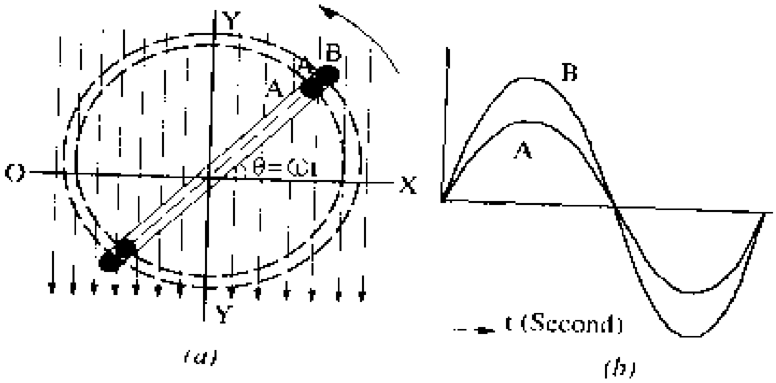


Fig 4

2.3 CONCEPT OF LEADING AND LAGGING AC QUATITIES

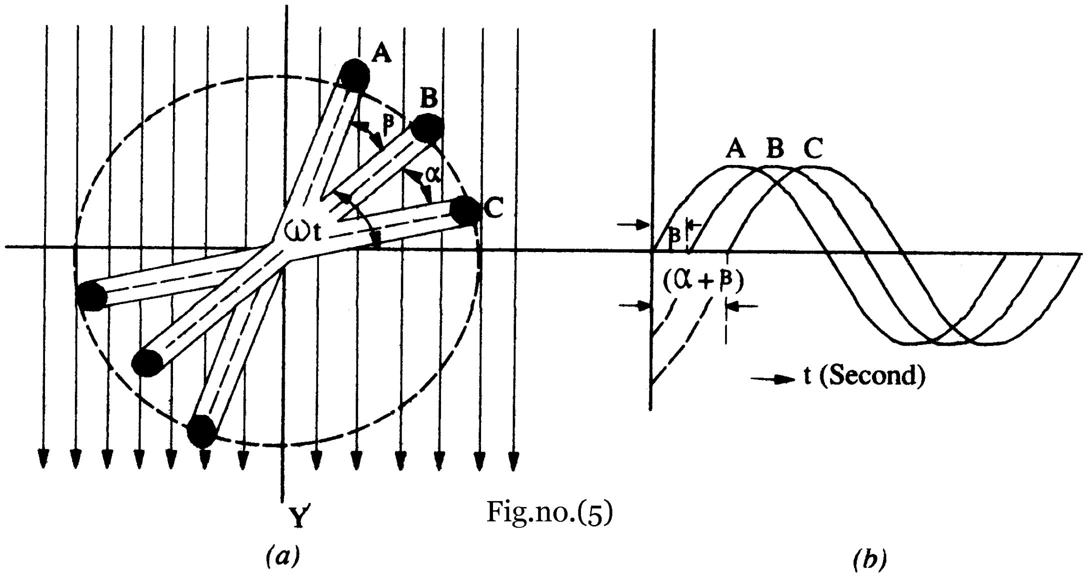


Fig.no.(5)

(a) (b)

Consider three similar single turn coils displaced from each other by angles α and β rotating in a uniform magnetic field with the same angular velocity fig. 5(a).

In this case the values of induced emf's in the three coils are the same but there is one important difference, The emf's in three phase coils do not reach their maximum or zero value simultaneously but one after another.The three sinusoidal wave are shown in fig. 5(b). It is seen that curve B and C are displaced from curve A by angles β and (α + β) respectively.

Hence it means that phase difference between A and B is β and between B and C is α, but between A and C is (α + β). The statement however, does not give indication as to which emf reaches its maximum value first.This deficiency is supplied by using the terms 'lag' and 'lead'.

A leading A.C. quantity is one which reaches its maximum or zero value earlier than the other quantity. For example in fig.5(b) B lags behind A by β & C lags behind A by (α + β) because they reach their maximum value later . The three equations for the instantaneous induced emf's are

$e_A = E_m \sin wt$ ------- reference signal
$e_B = E_m \sin (wt - \beta)$—lagging signal
$e_C = E_m \sin [wt - (\alpha + \beta)]$ –lagging signal

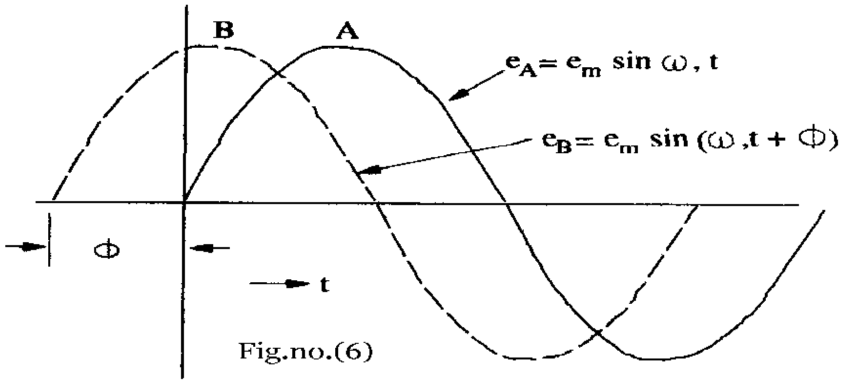


Fig.no.(6)

In fig. (6), quantity B leads A by angle Φ hence their equations are

$e_A = E_m \sin wt$ ------- reference

$e_B = E_m \sin(wt + \Phi)$---leading signal

2.4 VALUES OF ALTERNATING VOLTAGE AND CURRENT

The value of alternating voltage and current keeps on changing from instant to instant. Hence the magnitude of the alternating quantity is expressed in following ways:

a. Peak value
b. Average value
c. Rms or effective value

a. PEAK VALUE:

Peak value is the maximum value of the alternating quantity represented by V_m.

b. AVERAGE VALUE :

The average value I_{av} of an A.C. current is expressed by that steady current which transfers across any circuit. It is the arithmetic average of all the values of alternating quantities over one cycle.

MID ORDINATE METHOD:-

From fig. (7) $I_{av} = \dfrac{i_1 + i_2 + i_3 + \ldots\ldots + i_n}{n}$

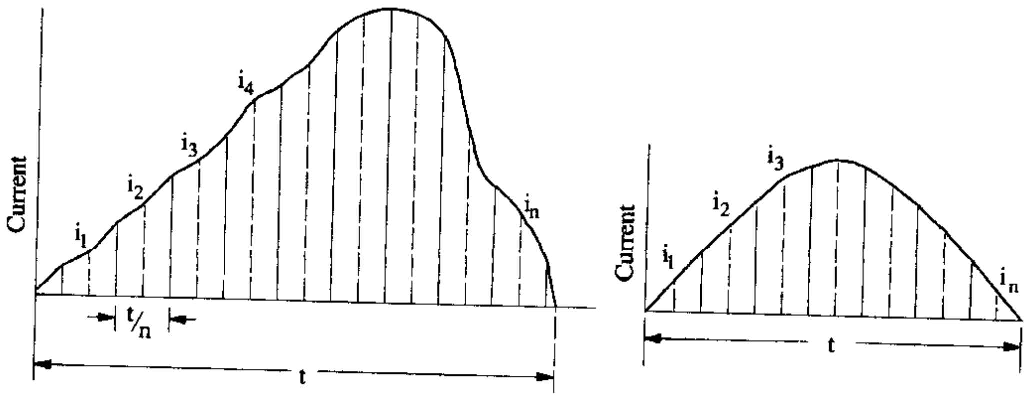


Fig. no.(7)

ANALYTICAL METHOD:-

A standard equation of a sinusoidal A.C. current is

$I = I_m \sin\theta$

Average value = $\frac{Area\ of\ half\ cycle}{base\ length\ of\ half\ cycle}$

$$I_{av} = \int_0^T \frac{i d\theta}{(T-0)} = \int_0^\pi \frac{i d\theta}{(\pi-0)}$$

$$I_{avg} = \frac{1}{\pi}\int i(\theta)d\theta$$

$$= \frac{Im}{\pi}\int_0^\pi \sin\theta d\theta = \frac{Im}{\pi}[-\cos\theta]_0^\pi = \frac{2\,Im}{\pi}$$

$$I_{avg} = \frac{2I_m}{\pi}$$

Therefore $I_{av} = 0.637I_m$

Similarly $V_{av} = 0.637V_m$

c. **ROOT MEAN SQUARE (RMS) Value:-**

RMS value is also known as the effective value of the ac quantity. It is the criterion to measure the effectiveness of an alternating current (or voltage). The RMS value of an A.C. current is given by that steady (D.C.) current which when following through a given circuit for a given time does the same work (produces the same heat) as produced by the A.C. current when flowing through the same circuit for the same time.

MID ORINATED METHOD: -

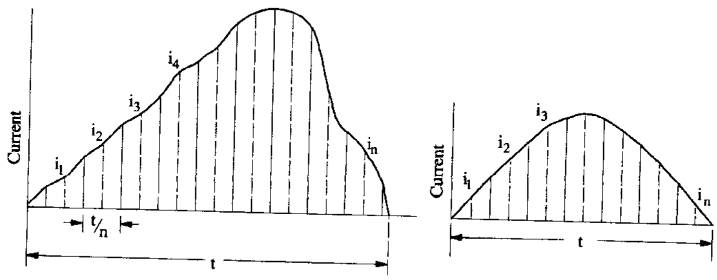


Fig. no.(7)

In fig no.(7) are shown the positive half cycles for both symmetrical sinusoidal and non- sinusoidal alternating current.

Divide time base 't' into n equal interval be respectively $I_1, I_2, I_3 \ldots\ldots\ldots I_n$, then.

$$I^2 = \frac{I_1^2 + I_2^2 + I_3^2 \ldots\ldots\ldots I_n^2}{n}$$

$$I = \frac{I_1^2+I_2^2+I_3^2 \ldots\ldots\ldots I_n^2}{n}$$

Similarly, the RMS value of alternating voltage is given by

$$V = \frac{V_1^2+V_2^2+V_3^2\ldots\ldots\ldots V_n^2}{n}$$

Analytical Method:-

The standard form of a sinusoidal alternating current is $I = I_m \sin wt = I_m \sin\theta$

The mean of the square of the instantaneous value of current over one complete cycle is

$$I^2 = \int_0^T \frac{i^2 d\theta}{(T-0)}$$ let T= 2π, the square root is

$$I = \sqrt{\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}}$$ $i = I_m \sin\theta$

Hence the rms value of alternating current is

$$I = \sqrt{\frac{I_m{}^2 \int_0^{2\pi} \sin^2\theta d\theta}{2\pi}}$$

Now $\cos 2\theta = 1 - 2\sin^2\theta$ because $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$I_{rms} = \sqrt{\frac{I_m{}^2 \int_0^{2\pi} (1-\cos 2\theta) d\theta}{4\pi}} = \sqrt{\frac{I_m}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$I_{rms} = \sqrt{\frac{I_m{}^2}{4\pi} \times 2\pi} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Hence we find for a symmetrical sine wave

$I_{rms} = 0.707\, I_m$

It should be noted that the average heating effect produced during one cycle is

$$P = I_{rms}{}^2 R = \left(\frac{Im}{\sqrt{2}}\right)^2 R = \frac{Im^2 R}{2}$$

Form Factor:-

It is defined as the ratio $Form\ Factor = \frac{RMS\ value}{Average\ value} = \frac{I_{rms}}{I_{avg}}$

For the sine wave $Form\ Factor = \frac{0.707\ I_m}{0.637\ I_m} = 1.11$

<u>Crest or Peak or Amplitude factor</u>:-

It is defined as the ratio $Peak\ Factor = \frac{Maximum\ value}{RMS\ value} = \frac{I_m}{I_{rms}}$

For the sine wave $K_a = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.41414$

Note: For sine wave: Form factor = 1.11 , Peak factor = 1.414

<u>Example: RMS VALUE OF HALF WAVE RECTIFIED AC CURRENT</u>

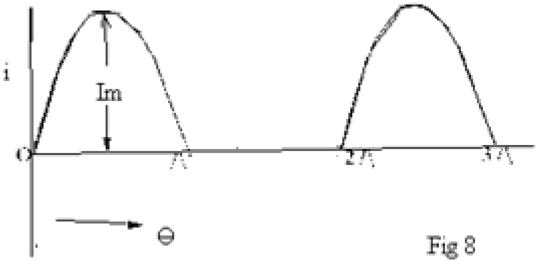


Fig 8

Half wave (HW) rectified alternating current is one whose one cycle has been suppressed i.e. one which flows for half the time during one cycle as shown in fig. (8)

The half wave rectified current equation is given as

$$i(t) = I_m sin\theta \ , 0 < \theta < \pi$$
$$= 0 \qquad , \pi < \theta < 2\pi$$

So $\quad I_{rms} = \sqrt{\frac{\int_0^{\pi} i^2 d\theta}{2\pi}} = \sqrt{\frac{\text{Im}^2 \int_0^{\pi} \sin^2\theta d\theta}{2\pi}}$

So $\quad I_{rms} = \sqrt{\frac{\text{Im}^2 \int_0^{\pi} (1-\cos 2\theta) d\theta}{4\pi}} = \sqrt{\frac{I^2{}_m}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$

So $\quad I_{rms} = \sqrt{\frac{\text{Im}^2}{4\pi} \times \pi} = \sqrt{\frac{\text{Im}^2}{4}} = \frac{I_m}{2} = 0.5\ I_m$

<u>AVERAGE VALUE OF HALF WAVE RECTIFIED AC CURRENT:</u>

$$I_{av} = \frac{1}{T}\int_0^T i d\theta = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{\text{Im}}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{\text{Im}}{2\pi} \times 2$$

$I_{av} = \dfrac{I_m}{\pi}$

FORM FACTOR OF HALF WAVE RECTIFIED AC CURRENT:

FORM FACTOR $= \dfrac{I_{rms}}{I_{av}} = \dfrac{I_m/2}{I_m/\pi} = \dfrac{\pi}{2} = 1.57$

PEAK FACTOR $= \frac{I_m}{I_{rms}} = \frac{I_m}{\frac{I_m}{2}} = 2$

2.5 PHASOR / VECTOR REPRESENTATION OF AC QUANTITY:

An sinusoidal ac signal is represented by $e(t) = E_m \sin \omega t$

Another method to represent the sine wave is **vector representation** of the sine wave.

In vector representation, the alternating voltage/current is represented by a vector rotating counter clockwise with the same frequency as that of a.c. quantity.

For example:

In fig. 9(a), OP is such a vector which represents $e(t) = E_m \sin \omega t$.

The maximum value of the ac voltage is OP & its angle with X-axis gives its phase.

The projection of OP on y-axis at any instant gives the instantaneous value of ac voltage thereby reproducing the ac voltage.

OM = OP sinωt

e = OP sinωt = E_m sinωt

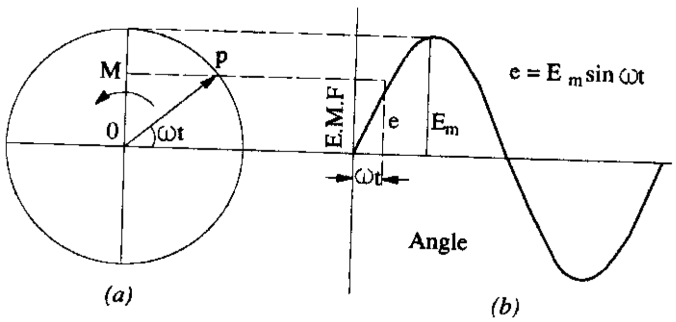


Fig 9

The line OP represents an ac voltage or current if it satisfies the following conditions.

(1) Its length should be equal to the peak or maximum of the sine wave a.c. current to a suitable scale.

(2) It should be in be in horizontal position at the same instant as the ac quantity is zero and

it is increasing.

(3) Its angular velocity should be such that it completes one revolution in the same time as taken by the ac quantity to complete one cycle.

2.6 PHASOR / VECTOR DIAGRAMS OF SINE WAVES OF SAME FREQUENCIES:

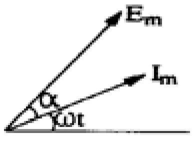
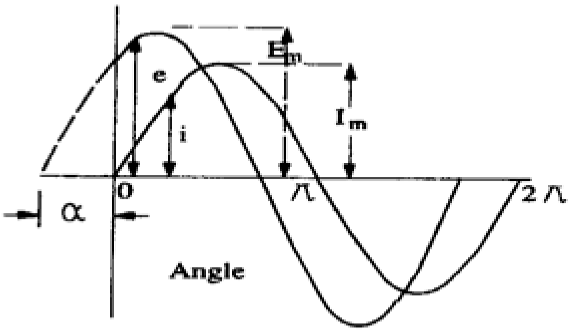


Fig. No. 10



Two or more sine waves of same frequency can be shown on the same vector diagram, because the various vector r/epresenting different waves all rotate counter-clockwise at the **same frequency** & maintain a fixed position relative to each other.

Example: e and i are two waveforms with same frequency are shown $i(t) = I_m \sin\omega t$ and $e = E_m \sin(\omega t+\alpha)$ and hence can be shown on the same phasor diagram as shown in fig. 10 a. Also note that phasor e leads the phasor i by a phase angle α.

2.7 MATHEMATICAL REPRESENTATION OF PHASORS:

The phasor can be represented mathematically in two ways:

(a) Rectangular form
(b) Polar form

(a) Rectangular form:

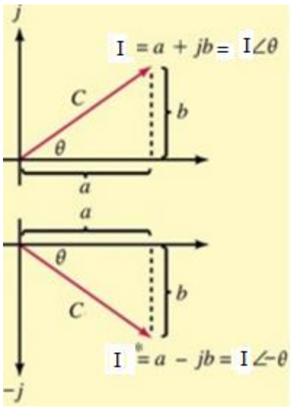


Fig. 11

The vector I is resolved into horizontal and vertical components and expressed in complex form as $\bar{I} = a \pm jb$

Rectangular form of representation is used for addition and subtraction of multiple ac quantities.

(b) Polar form:

$$\bar{I} = |I|\angle \pm \theta$$
$$|I| = \sqrt{a^2 + b^2}$$

$$\theta = tan^{-1}\frac{b}{a}$$

Numericals

I. Maximum value, Frequency, Time period, Instantaneous value, Value at a given time,

1. An alternating voltage is represented by $v(t) = 141.4\sin 377t$.
   Find (i) maximum value (ii) frequency (iii) time period (iv) instantaneous value of voltage at $t = 3$ msec.
   Ans: (i) 141.4V (ii) 60Hz (iii) 16.67ms (iv) 127.94V

The equation of alternating voltage is given as:

$v = 141.4 \sin 377t$

Comparing the given equation with standard form

$v = V_m \sin \omega t$, we have

i) Maximum value, $V_m = 141.4$ Volts

ii) Frequency, $f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60$ Hz

iii) Time period, $T = \frac{1}{f} = \frac{1}{60} = 16.67\times10^{-3}$ Second

iv) Given $v = 141.4 \sin 377t$

When $t = 3$ msec $= 3\times10^{-3}$ sec, we get

$v = 141.4 \sin(377\times3\times10^{-3})$

$v = 141.4 \sin\left[(377\times3\times10^{-3})\times\frac{180}{\pi}\right]^\circ$

So, $v = 127.94$ volts

2. An alternating current takes 3.375 msec to reach 15A for the first time after becoming instantaneously zero. The frequency of current is 40 Hz. Find the maximum value of alternating current.
   Ans: 20 A

Let the equation of alternating current is given as

$i = I_m \sin(wt)$

When $t = 3.375\times10^{-3}$ Sec ; $i = 15$ A — Given

Also, frequency, $f = 40$ Hz i.e. $w = 2\pi f = 251.327$ rad/sec.

Now $i = I_m \sin(wt)$

$15 = I_m \sin(251.327 \times 3.375\times10^{-3})$

or $15 = I_m \sin\left[(251.327\times3.375\times10^{-3})\times\frac{180}{\pi}\right]^\circ$

or $15 = I_m \times 0.7501$

Hence $I_m = 20$ A.

3. An alternating voltage of time period 0.02sec has maximum value of 12V. Write the equation for its instantaneous value. Calculate the instantaneous value of the voltage after 0.002sec, where reference is taken from the instant of zero voltage and is becoming positive. Also calculate the time required for the voltage to reach 4V for the first time.

Given:

$T = 0.02$ Sec.

$V_m = 12\ V$

Now, $f = \frac{1}{T} = \frac{1}{0.02} = 50\ Hz$

so, $\omega = 2\pi f = 314.16$ rad/sec.

so, equation of instantaneous value of alternating voltage

$v = V_m \sin \omega t$

i.e. $v = 12 \sin(314.16\, t)$

Now, when $t = 0.002$ sec ; instantaneous value, v is required.

so, $v = 12 \sin(314.16\, t)$

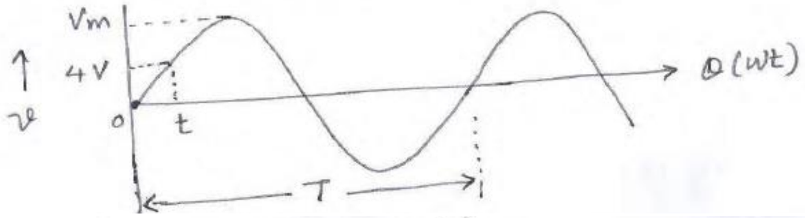
$v = 12 \sin[314.16 \times 0.002]$

$v = 12 \sin\left[(314.16 \times 0.002) \times \frac{180}{\pi}\right]^{\circ}$

$v = 12 \times 0.5878$

$v = 7.053$ Volts

Figure shows the waveform of given alternating voltage. Let the voltage become 4V for the first time after t second. Then



as $v = 12 \sin(314.16\, t)$

$4 = 12 \sin(314.16 t)$

$4 = 12 \sin\left[(314.16 t) \times \frac{180}{\pi}\right]^{\circ}$

$\sin\left[(314.16 t) \times \frac{180}{\pi}\right]^{\circ} = 0.333$

$\left[(314.16 t) \times \frac{180}{\pi}\right] = \sin^{-1} 0.333$

$\therefore t = 1.0817 \times 10^{-3}$ Sec.

4. An alternating current of frequency 60Hz has a maximum of 120A. Write down the equation for its instantaneous value. Find (i) the instantaneous value after 1/360 from the instant current is zero and is becoming positive (ii) the time taken to reach 96 A for the first time.
   Ans: (i) 103.3A (ii) 0.00245 sec

Solⁿ Given data .. $f = 60\ Hz$, $I_m = 120A$

When $t = \frac{1}{360}$ sec, $i = ?$

when $i = 96A$, $t = ?$

The instantaneous current eqⁿ is, $i = I_m \sin \omega t$

$$i = 120 \sin 2\pi f t = 120 \sin(2 \times 60 \pi t)$$

$$= 120 \sin(120\pi t)$$

① Now when $t = \frac{1}{360}$ sec,

$$i = 120 \sin\left(120 \times 180 \times \frac{1}{360}\right)$$

$$= 120 \sin 60^\circ$$

$$= \underline{103.9\ A}$$

② when $i = 96A$,

$$96 = 120 \sin(180 \times 120 t)$$

$$\frac{96}{120} = \sin(180 \times 120 t) = 0.8$$

$$\sin^{-1} 0.8 = 180 \times 120 t$$

$$180 \times 120 t = 53^\circ$$

$$t = \frac{53}{180 \times 120} = \underline{0.00245\ sec}.$$

II. RMS, Average , Peak factor, Form factor

1. Find the following parameters of the given voltage $v(t) = 200 \sin 314\, t$.
   (i) Frequency (ii) Form factor (iii) crest factor
   Ans: (i) 50 Hz (ii) Form factor = 1.11 (iii) Crest factor = 1.41

Soln. $v = V_m \sin 2\pi f t$

$2\pi f = 314$

$f = \frac{314}{2\pi} = 50\,Hz$.

For a sinusoidal waveform.

$V_{avg} = \frac{2V_m}{\pi} = \frac{2 \times 200}{3.14} = 127.38\ V$

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42$.

Form factor $= \frac{V_{rms}}{V_{avg}}$.

$= \frac{141.42}{127.38}$

$= 1.11$

Crest factor = peak factor

$= \frac{V_m}{V_{rms}}$.

$= \frac{200}{141.42}$

$= 1.41$.

2. A non sinusoidal volatage is having form factor as 1.2 and peak factor as 1.5. If the average value of the voltage is 10V. Calculate (i) rms value (ii) maximum value
   Ans: Vrms = 12V Vm = 18V

Soln. $k_f = 1.2$

$k_p = 1.5$

$V_{avg} = 10\ V$

$k_f = \frac{V_{rms}}{V_{avg}}$.

$\therefore V_{rms} = 1.2 \times 10$

$= 12\ V$.

$$K_p = \frac{V_m}{V_{rms}}.$$

$$\therefore V_m = 1.5 \times 12$$

$$= 18V.$$

3. An alternating current varying sinusoidally with a frequency of 50 c/s has an rms value of 20A. Write down the equation for the instantaneous value and find this value at (i) 0.0025 sec (ii) 0.0125 sec after passing through zero and increasing positively (iii) at what time measured from zero will the value of the instantaneous current be 14.14A ?

$f = 50$ c/s. (H_z) ; $I = 20A$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad \therefore I_m = I_{rms} \times \sqrt{2} = 20\sqrt{2}$$

$$= 28.28 A.$$

Equation of current

$$i = I_m \sin 2\pi f t$$

$$= 28.28 \sin(2 \times 180 \times 50 \times t)$$

$$= 28.28 \sin(100 \times 180 \times t)$$

a) At $t = 0.0025$ sec,

$$i = 28.28 \sin(18000 \times 0.0025)$$

$$= 28.28 \sin(45^\circ)$$

$$= 20A.$$

b) At $t = 0.0125$ sec.

$$i = 28.28 \sin(18000 \times 0.0125)$$

$$= 28.28 \sin(225^\circ)$$

$$= -20A.$$

c) $i = 28.28 \sin(100\pi t)$

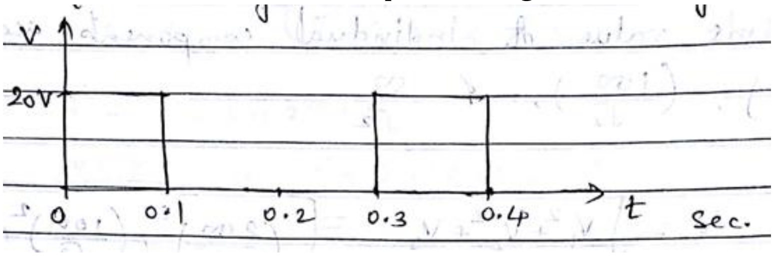
$$14.14 = 28.28 \sin 100\pi t$$

$$\sin(100\pi t) = 0.5$$

$$100\pi t = 30^\circ$$

$$\therefore t = 1.66 \times 10^{-3} \text{ sec}.$$

4. Compute the average and effective values of the square voltage wave shown in figure.



Ans: Vavg = 6.67V , Vrms = 11.5V

Sol^n - As seen for $0 < t < 0.1$, for the time interval 0 to 0.1 sec, $v = 20V$.

Similarly for $0.1 < t < 0.3$, $v = 0$

Also time period of the voltage wave is 0.3 sec.

$$V_{avg} = \frac{1}{T}\int_0^{T} v\,dt = \frac{1}{T}\left[\int_0^{0.1} v\,dt + \int_{0.1}^{0.2} v\,dt + \int_{0.2}^{0.3} v\,dt\right]$$

$$= \frac{1}{0.3}\int_0^{0.1} 20\,dt$$

$$= \frac{1}{0.3}\, 20 \times [t]_0^{0.1}$$

$$V_{avg} = \frac{20}{0.3} \times 0.1 = 6.67V.$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\,dt$$

$$= \frac{1}{0.3}\int_0^{0.1} (20)^2\,dt$$

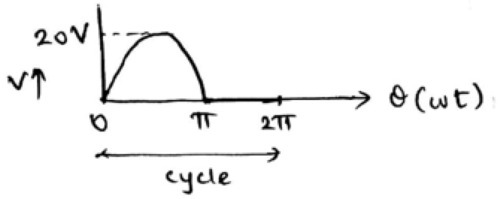
$$= \frac{400}{0.3} \times 0.1 = 133.3V.$$

$$V_{rms} = \sqrt{133.3} = 11.5V.$$

5. Find the average value of the waveform shown in figure.

Ans: Vavg = 6.366V

Solution - consider a full cycle of the given waveform.



The given voltage waveform can be divided into two intervals.

$$v = 20 \sin\theta \quad \text{for} \quad 0 < \theta < \pi$$

and $$v = 0 \quad \text{for} \quad \pi < \theta < 2\pi$$

$$V_{avg.} = \frac{\text{Area of full cycle}}{\text{Base length of full cycle}}$$

$$V_{avg.} = \frac{\int_0^{2\pi} v \, d\theta}{2\pi}$$

$$= \frac{1}{2\pi} \left\{ \int_0^{\pi} v \, d\theta + \int_{\pi}^{2\pi} v \, d\theta \right\}$$

$$= \frac{1}{2\pi} \int_0^{\pi} 20 \sin\theta \, d\theta$$

$$V_{avg.} = \frac{20}{2\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{20}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

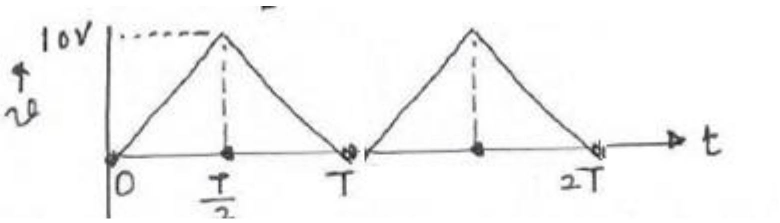
$$= \frac{20}{2\pi}\left[-\cos\pi - (-\cos 0)\right]$$

$$= \frac{20}{2\pi}\left[-(-1) - (-1)\right]$$

$$= \frac{20}{2\pi}\left[2\right]$$

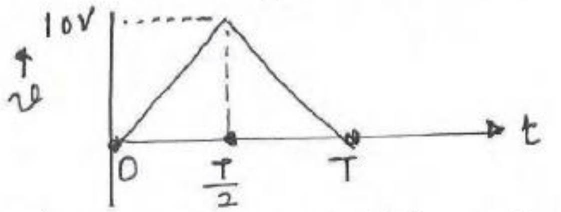
$$= 6.366 \text{ V}.$$

6. Find (i) rms value (ii) form factor for a symmetric triangular waveform shown in Fig



Ans: $V_{rms} = 5.77V$

One cycle of given waveform :



In interval $0 < t < T/2$, voltage increases with constant slope of $\frac{10}{T/2}$ V/sec. Therefore, the equation of voltage is $v = \frac{20}{T}t$.

So, $I_{average} = \dfrac{\int_0^{T/2} v\,dt + \int_{T/2}^{T} v\,dt}{T}$

$= \dfrac{2\int_0^{T/2} v\,dt}{T} \quad \left(\because \int_0^{T/2} v\,dt = \int_{T/2}^{T} v\,dt\right)$

$= \dfrac{2}{T}\int_0^{T/2} \dfrac{20}{T}t\,dt \quad \left(\because v = \dfrac{20}{T}t\right)$

$= \dfrac{2}{T}\times\dfrac{20}{T}\int_0^{T/2} t\,dt$

$= \dfrac{2}{T}\times\dfrac{20}{T}\left[\dfrac{t^2}{2}\right]_0^{T/2}$

$= \dfrac{2}{T}\times\dfrac{20}{T}\left[\left(\dfrac{T}{2}\right)^2\times\dfrac{1}{2}\right]$

$= 5$ Volts.

The rms value of given waveform :

$$V_{rms} = \sqrt{\frac{\text{Area of Full Cycle of Squared wave}}{\text{Baselength of full cycle}}}$$

$$V_{rms} = \sqrt{\frac{\int_0^T v^2\,dt}{T}}$$

$$V_{rms} = \sqrt{\frac{1}{T}\left\{\int_0^{T/2} v^2\,dt + \int_{T/2}^{T} v^2\,dt\right\}}$$

$$V_{rms} = \sqrt{\frac{1}{T}\left\{2\int_0^{T/2} v^2\,dt\right\}} \quad \left(\because \int_0^{T/2} v^2\,dt = \int_{T/2}^{T} v^2\,dt\right)$$

$$V_{rms}^2 = \frac{2}{T}\int_0^{T/2}\left(\frac{20}{T}t\right)^2 dt \quad \left(\because v = \frac{20}{T}t\right)$$

$$V_{rms}^2 = \frac{2}{T}\times\left(\frac{20}{T}\right)^2\int_0^{T/2} t^2\,dt$$

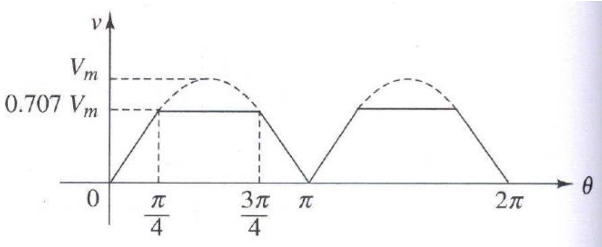
$$V_{rms}^2 = \frac{2}{T}\times\left(\frac{20}{T}\right)^2\left[\frac{t^3}{3}\right]_0^{T/2}$$

$$V_{rms}^2 = \frac{2}{T}\times\left(\frac{20}{T}\right)^2\left[\left(\frac{T}{2}\right)^3\times\frac{1}{3}\right]$$

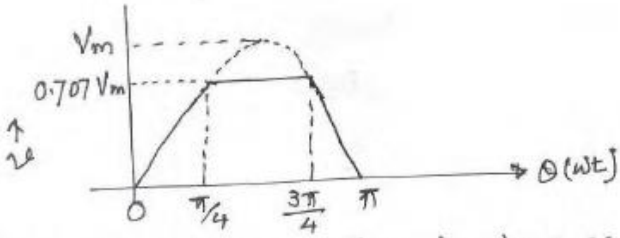
$$V_{rms}^2 = 33.33 \text{ Volt}$$

$V_{rms} = 5.77V$

7. A full rectified wave is clipped at 70.7% of its maximum value as shown in figure. Find its average and rms value



Consider one cycle of given waveform:



Now, the voltage waveform is given as follows:

Interval	Equation
$0 < \theta < \pi/4$	$v = V_m \sin\theta$
$\frac{\pi}{4} < \theta < \frac{3\pi}{4}$	$v = 0.707 V_m$
$\frac{3\pi}{4} < \theta < \pi$	$v = V_m \sin\theta$

So, average value

$$V_{average} = \frac{\int_0^{\pi} v\, d\theta}{\pi}$$

$$= \frac{\int_0^{\pi/4} v\, d\theta + \int_{\pi/4}^{3\pi/4} v\, d\theta + \int_{3\pi/4}^{\pi} v\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\left\{ \int_0^{\pi/4} V_m \sin\theta\, d\theta + \int_{\pi/4}^{3\pi/4} 0.707 V_m\, d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\, d\theta \right\}$$

$$= \frac{1}{\pi}\left\{ V_m[-\cos\theta]_0^{\pi/4} + 0.707 V_m [\theta]_{\pi/4}^{3\pi/4} + V_m[-\cos\theta]_{3\pi/4}^{\pi} \right\}$$

$$= \frac{V_m}{\pi}\left\{ [-0.7071-(-1)] + \left[0.707\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)\right] + [-(-1)-(+0.7071)] \right\}$$

$$= \frac{V_m}{\pi}\{0.2929 + 1.11 + 0.2929\}$$

$$= 0.54 \text{ Volts.}$$

Now, rms value is given as

$$V_{rms} = \sqrt{\frac{\int_0^{\pi} v^2 d\theta}{\pi}}$$

$$V_{rms} = \sqrt{\frac{\int_0^{\pi/4} v^2 d\theta + \int_{\pi/4}^{3\pi/4} v^2 d\theta + \int_{3\pi/4}^{\pi} v^2 d\theta}{\pi}}$$

$$V_{rms}^2 = \frac{1}{\pi}\left\{ \int_0^{\pi/4} (V_m \sin\theta)^2 d\theta + \int_{\pi/4}^{3\pi/4} (0.707\, V_m)^2 d\theta + \int_{3\pi/4}^{\pi} (V_m \sin\theta)^2 d\theta \right\}$$

$$= \frac{1}{\pi}\left\{ V_m^2 \int_0^{\pi/4} \sin^2\theta\, d\theta + (0.707 V_m)^2 \int_{\pi/4}^{3\pi/4} d\theta + V_m^2 \int_{3\pi/4}^{\pi} \sin^2\theta\, d\theta \right\}$$

$$= \frac{1}{\pi}\left\{ V_m^2 \int_0^{\pi/4} \frac{1-\cos 2\theta}{2}\, d\theta + (0.707 V_m)^2 \int_{\pi/4}^{3\pi/4} d\theta + V_m^2 \int_{3\pi/4}^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta \right\}$$

$$= \frac{V_m^2}{\pi}\left\{ \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4} + (0.707 V_m)^2 \left[\theta\right]_{\pi/4}^{3\pi/4} + \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_{3\pi/4}^{\pi} \right\}$$

$$= \frac{V_m^2}{\pi}\left\{ \frac{1}{2}\left[\frac{\pi}{4} - \frac{\sin 2\left(\frac{\pi}{4}\right)}{2} - \left(0 - \frac{\sin 2\times 0}{2}\right)\right] + (0.707 V_m)^2 \left[\frac{3\pi}{4} - \frac{\pi}{4}\right] + \frac{1}{2}\left[\pi - \frac{\sin 2\pi}{2} - \left(\frac{3\pi}{4} - \frac{\sin \frac{6\pi}{4}}{2}\right)\right] \right\}$$

$$= \frac{V_m^2}{\pi}\left\{ \frac{1}{2}\left[(0.7854 - 0.5) - (0 - 0)\right] + (0.707)^2 [1.5708] + \frac{1}{2}\left[3.14 - 0 - \left(2.356 - \frac{(-1)}{2}\right)\right] \right\}$$

$$= \frac{V_m^2}{\pi}\left\{ 0.1427 + 0.7852 + 0.1427 \right\}$$

$$V_{rms}^2 = 0.3409\, V_m^2$$

$$V_{rms} = 0.584\, V_m$$

8. Find the average and rms value of current given by $i(t) = 10 + 10\sin\theta$

Average Value:

$$I_{average} = \frac{\int_0^{2\pi} i(t)\,d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\left\{\int_0^{2\pi}(10+10\sin\theta)\,d\theta\right\}$$

$$= \frac{1}{2\pi}\left\{\int_0^{2\pi}10\,d\theta + \int_0^{2\pi}10\sin\theta\,d\theta\right\}$$

$$= \frac{1}{2\pi}\left\{10[\theta]_0^{2\pi} + 10[-\cos\theta]_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{10[2\pi-0] + 10[-\cos 2\pi - (-\cos 0)]\right\}$$

$$= \frac{1}{2\pi}\left\{10[2\pi-0] + 10[-1-(-1)]\right\}$$

$$= 10 \text{ A}$$

RMS Value:

$$I_{rms} = \sqrt{\frac{\int_0^{2\pi} i^2(t)\,d\theta}{2\pi}}$$

$$I^2_{rms} = \frac{1}{2\pi}\int_0^{2\pi} i^2(t)\,d\theta$$

$$I^2_{rms} = \frac{1}{2\pi}\int_0^{2\pi}(10+10\sin\theta)^2\,d\theta$$

$$I^2_{rms} = \frac{1}{2\pi}\int_0^{2\pi}(100+200\sin\theta+100\sin^2\theta)\,d\theta$$

$$I^2_{rms} = \frac{1}{2\pi}\left\{\int_0^{2\pi}100\,d\theta + \int_0^{2\pi}200\sin\theta\,d\theta + \int_0^{2\pi}100\sin^2\theta\,d\theta\right\}$$

$$I^2_{rms} = \frac{1}{2\pi}\left\{\int_0^{2\pi}100\,d\theta + 200\int_0^{2\pi}\sin\theta\,d\theta + 100\int_0^{2\pi}\frac{1-\cos 2\theta}{2}\,d\theta\right\}$$

$$I^2_{rms} = \frac{1}{2\pi}\left\{100[\theta]_0^{2\pi} + 200[-\cos\theta]_0^{2\pi} + \frac{100}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}\right\}$$

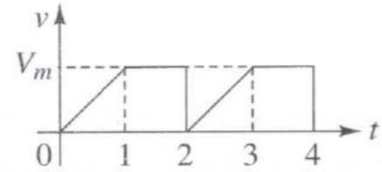
$$I^2_{rms} = \frac{1}{2\pi}\left\{100(2\pi-0) + 200[-\cos 2\pi-(-\cos 0)] + \frac{100}{2}\left[\left(2\pi-\frac{\sin 4\pi}{2}\right)-(0)\right]\right\}$$

$$I^2_{rms} = \frac{1}{2\pi}\left\{628.32 + 200[-1-(-1)] + \frac{100}{2}[6.283-0-(0)]\right\}$$

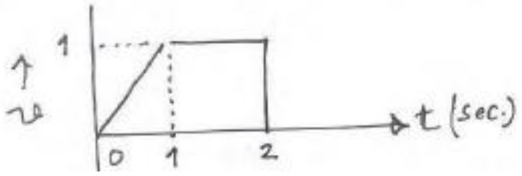
$$I^2_{rms} = 150$$

$$I_{rms} = 12.247 \text{ A}$$

9. Find the rms and average value of the waveform shown below



Consider one cycle of given waveform.



In interval 0<t<1, voltage increases with constant slop of 1 V/sec
Therefore, the equation of voltage is v = 1t.
In interval 1<t<2, voltage remains 1 volt. Therefore, the equation of voltage is v = 1

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 1\,dt\right]$$

$$= \frac{1}{2}\left\{\left[\frac{t^2}{2}\right]_0^1 + [t]_1^2\right\}$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0 + 2 - 1\right] = \frac{3}{4} = 0.75\text{ V}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 t^2 dt + \int_1^2 (1)^2 dt\right]} = \sqrt{\frac{1}{2}\left\{\left[\frac{t^3}{3}\right]_0^1 + [t]_1^2\right\}}$$

$$= \sqrt{\frac{1}{2}\left[\frac{1}{3} - 0 + 2 - 1\right]} = \sqrt{\frac{4}{6}} = 0.816\text{ V}$$

Addition of Vectors

1.

Two sinusoidal currents are given as

$$i_1 = 10\sqrt{2}\ \sin \omega t,\ i_2 = 20\sqrt{2}\ \sin(\omega t + 60°).$$

Find the expression for the sum of these currents.

Solution

Data

$$i_1 = 10\sqrt{2}\ \sin \omega t$$

$$i_2 = 20\sqrt{2}\ \sin(\omega t + 60°)$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{10\sqrt{2}}{\sqrt{2}} \angle 0° = 10\angle 0°$$

$$\bar{I}_2 = \frac{20\sqrt{2}}{\sqrt{2}} \angle 60° = 20\angle 60°$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

$$= 10\angle 0° + 20\angle 60° = 26.46\ \angle 40.89°$$

$$i = 26.46\sqrt{2}\sin(\omega t + 40.89°) = 37.42\sin(\omega t + 40.89°)$$

2.

The following three sinusoidal currents flow into the junction $i_1 = 3\sqrt{2}\ \sin\ \omega t$, $i_2 = 5\sqrt{2}\ \sin(\omega t + 30°)$ and $i_3 = 6\sqrt{2}\ \sin(\omega t - 120°)$. Find the expression for the resultant current which leaves the junction.

Solution

Data

$$i_1 = 3\sqrt{2}\sin \omega t$$

$$i_2 = 5\sqrt{2}\sin(\omega t + 30°)$$

$$i_3 = 6\sqrt{2}\sin(\omega t - 120°)$$

Writing currents i_1, i_2 and i_3 in the phasor form,

$$\bar{I}_1 = \frac{3\sqrt{2}}{\sqrt{2}} \angle 0° = 3\angle 0°$$

$$\bar{I}_2 = \frac{5\sqrt{2}}{\sqrt{2}} \angle 30° = 5\angle 30°$$

$$\bar{I}_3 = \frac{6\sqrt{2}}{\sqrt{2}} \angle -120° = 6\angle -120°$$

The resultant current which leaves the junction is given by

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 3\angle 0° + 5\angle 30° + 6\angle -120°$$

$$= 5.1\ \angle -31.9°$$

$$i = 5.1\sqrt{2}\sin(\omega t - 31.9°) = 7.21\sin(\omega t - 31.9°)$$

3.

Obtain the sum of the three voltages.

$v_1 = 147.3 \cos(\omega t + 98.1°)$
$v_2 = 294.6 \cos(\omega t - 45°)$
$v_3 = 88.4 \sin(\omega t + 135°)$

Solution

Data

$$v_1 = 147.3 \cos(\omega t + 98.1°) = 147.3 \sin(\omega t + 188.1°)$$
$$v_2 = 294.6 \cos(\omega t - 45°) = 294.6 \sin(\omega t + 45°)$$
$$v_3 = 88.4 \sin(\omega t + 135°)$$

Writing the voltages v_1, v_2 and v_3 in the phasor form,

$$\bar{V}_1 = \frac{147.3}{\sqrt{2}} \angle 188.1° = 104.16\angle 188.1°$$

$$\bar{V}_2 = \frac{294.6}{\sqrt{2}} \angle 45° = 208.31\angle 45°$$

$$\bar{V}_3 = \frac{88.4}{\sqrt{2}} \angle 135° = 62.51\angle 135°$$

Resultant voltage $\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3$

$$= 104.16\angle 188.1° + 208.31\angle 45° + 62.51\angle 135° = 176.82\angle 90°$$

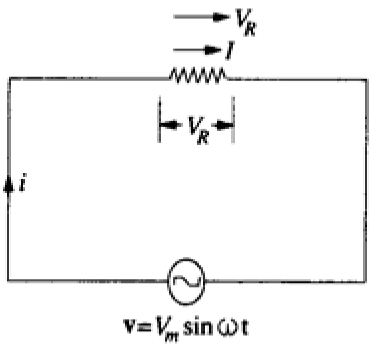
$$v = 176.82\sqrt{2} \sin(\omega t + 90°) = 250.06 \sin(\omega t + 90°)$$

ANALYSIS OF AC CIRCUITS:

The resistance , inductance and capacitance are the basic elements of any electrical network. In order to analyse any electrical circuit, it is necessary to understand the following three cases:

a. AC through pure resistor circuit
b. AC through pure inductive circuit
c. AC through pure capacitive circuit

2.8 A.C. ANALYSIS OF A RESISTIVE CIRCUIT:



The circuit diagram for resistive circuit with ac source v(t) = V_m sinωt is shown in fig. 12(a) where R= resistance.

The alternating voltage causes an alternating current i(t) to flow through the circuit given as:

$$i(t) = \frac{v(t)}{R}$$

$$i(t) = \frac{V_m sin\omega t}{R}$$

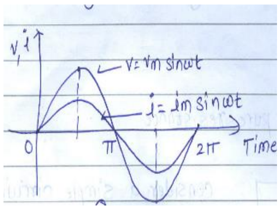
$$i(t) = \frac{V_m}{R} \sin \omega t$$

$i(t) = I_m sin\omega t$, where $I_m = \frac{V_m}{R}$

i(t) is the instantaneous value

I_m is the maximum value of the ac current.

The waveform for i(t) and v(t) is as shown below:



Instantaneous power of resistive circuit:

Power consumed by the circuit at any instant is given as the product of the voltage and current at that instant.

Instantaneous power is given as:

$$p(t) = v(t).\, i(t)$$

Where, $i(t) = \; I_m \; \sin wt$, $v(t) = \; V_m \; \sin wt$

$$P(t) = V_m \; I_m sin^2 \omega t$$

$$P(t) = \frac{V_m I_m}{2}(1 - cos2\omega t)$$

$$P(t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t$$

The instantaneous power consists of two components

a. Power consist of dc component $\frac{V_m I_m}{2}$ and
b. a fluctuating part $\frac{V_m I_m \, cos2\omega t}{2}$ of frequency

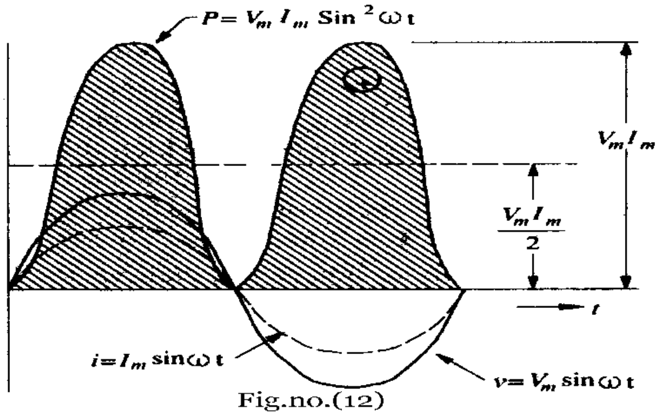


Fig. 13 Plot of instantaneous power

<u>Average power consumed by a resistive circuit:</u>

The instantaneous power consumed by the resistive circuit is

$$P(t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t$$

The average power over one complete cycle is given as

$$P_{avg = \frac{1}{2\pi}} \int_0^{2\pi} P(t) \, dt$$

Substituting $P(t)$ and solving the integration,

$$P_{avg} = \frac{V_m I_m}{2}$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} = \; V_{rms} \; I_{rms}$$

V_{rms} = rms value of the applied voltage $v(t)$

I_{rms} = rms value of the circuit current $i(t)$

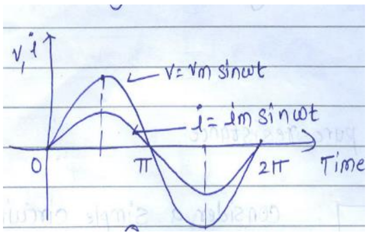
Phasor diagram:

The applied voltage and resultant circuit current for a purely resistive circuit are given by equation:

$$v(t) = V_m \sin wt$$

$$i(t) = I_m \sin wt$$

The waveform of i(t) and v(t) is given below:



The phasor diagram is shown below:

It is clear from the equation that the **applied voltage and resultant current are in phase with each other**.

1. Power factor of purely resistive circuit = 1
2. There is no phase difference between the voltage and current hence phase difference between $\bar{V}$ and $\bar{I}$ is φ = 0.
3. Power factor of the circuit is : PF=cos Φ
4. Power consumed is given as :

$$P_{consumed} = V_{rms} I_{rms} cos\varphi = V_{rms} I_{rms} . 1 = V_{rms} I_{rms}$$

2.9 A.C . ANALYSIS OF A PURELY INDUCTIVE CIRCUIT

Consider a circuit with an ac source applied to a pure inductor of value L Henrys.

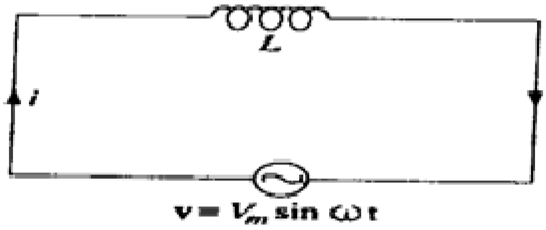


Fig. 14 : Purely inductive circuit

The applied ac voltage is given as $v(t) = V_m sin\omega t$

As a result of the applied voltage , an alternating current $i(t)$ flows through the circuit.

This alternating current sets up a alternating magnetic field around the inductance.

The changing flux links the coil and an emf is induced in it.

This emf is called as self induced emf (e=L di/dt).

The self induced emf is also called as back emf.

The back emf at every step opposes the rise or fall of current through the coil.

As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced emf only. So at any instant the self induced emf is equal and opposite to the applied voltage

$$v\,(t) = L\frac{di}{dt}$$

$$\text{Therefore, } i = \frac{V_m}{L}\int \sin wt dt$$

Since, applied voltage $v\,(t) = V_m\sin\omega t$

$$\text{So } i = \frac{V_m}{wL}(-\cos wt) = -\frac{V_m}{wL}\cos wt \quad [\text{ constant} = 0]$$

So $i = \frac{V_m}{\omega\, L}\sin(\omega t - \frac{\pi}{2}) = \frac{V_m}{X_L}\sin(\omega t - \frac{\pi}{2})$

Series RL circuit:

Applied voltage: $v\,(t) = V_m\sin\omega t$

Resultant current in a series RL circuit is $i = \frac{V_m}{X_L}\sin(\omega t - \frac{\pi}{2})$

The value of $i(t)$ is maximum when $\sin(\omega t - \frac{\pi}{2}) = 1$

Maximum current: $I_m = \frac{V_m}{X_L}$

Substituting $I_m = \frac{V_m}{X_L}$, we get

$$i(t) = I_m\sin(\omega t - \frac{\pi}{2})$$

Therefore for a purely inductive circuit the applied voltage and the resultant current are given as :

$$v(t) = V_m sin\omega t$$

$$i(t) = I_m \sin(\omega t - \frac{\pi}{2})$$

Where, $I_m = \frac{V_m}{X_L}$ is the maximum / peak current

Phasor diagram:

The applied voltage and resultant current for a purely inductive circuit is given as:

Applied voltage: $v(t) = V_m sin\omega t$

Resultant current : $i(t) = I_m \sin(\omega t - \frac{\pi}{2})$

From the above equations, it is seen that the **for a purely inductive circuit current lags behind the voltage by π/2.**

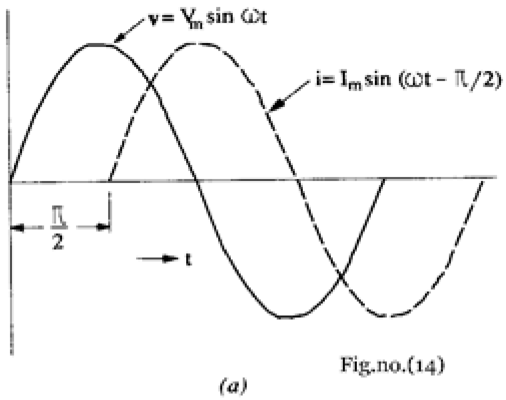


Fig. 15: Waveforms of *v*(*t*) and *i*(*t*)

Phasor diagram (RL circuit):

Applied voltage: $\bar{V} = V_{rms}\angle 0^o$

Resultant current : $\bar{I} = I_{rms}\angle -90^o$, where $I_{rms} = \frac{V_{rms}}{X_L}$, $X_L = \omega L$ and $V_{rms} = \frac{V_m}{\sqrt{2}}$ (for sinusoidal ac source).

Clearly, **The current lags behind the applied voltage by quarter cycle (90^o).**

<u>Average power consumed by an inductive circuit:</u>

Instantaneous power

$$P(t) = v(t).i(t)$$

Where, $v(t) = V_m sin\omega t$ and $i(t) = I_m \sin(\omega t - \frac{\pi}{2})$

$$P(t) = V_m I_m \sin \omega t sin(\omega t - \frac{\pi}{2})$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$P(t) = -\frac{V_m I_m}{2} sin2\omega t \quad \text{-----Instantaneous power}$$

Average power consumed by pure inductive circuit over one cycle is given as:

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} P(\theta) d\theta$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} sin2\theta \, d\theta$$

$$P_{avg} = 0$$

Average power consumed by a purely inductive circuit is 0.

That is demand of power from the supply for a complete cycle is zero.

During the positive half cycle the applied voltage, power is positive and flows from source to inductor and build up the magnetic field around inductor. That is energy supplied by source is stored in form of magnetic field of inductor.

During the negative half cycle power is negative and power flows from inductor to source, that is as current falls the magnetic field collapses and returns the stored energy back to source.

Hence the resultant power over one complete cycle of applied voltage = 0.

That is pure inductor consumes no power.

Reactive power: When power is positive, energy is put into the circuit to build the magnetic field around the inductor. When power is negative, magnetic energy is returned to the supply.

Since power supplied is equal to the power returned, net power consumed is zero. The power circulates in the circuit and is called REACTIVE POWER.

<u>Power factor of purely inductive circuit = 0</u>

Power factor: PF=cos Φ

where φ is the phase difference between the applied voltage and the resultant current.

$\Phi = 90^o$

PF=cos 90^o = 0

Power consumed is given as : $P_{consumed} = V_{rms} I_{rms} cos\varphi = V_{rms} I_{rms}. 0 = 0$

Power consumed by an inductive circuit =0.

2.10 AC ANALYSIS OF PURE CAPACITOR

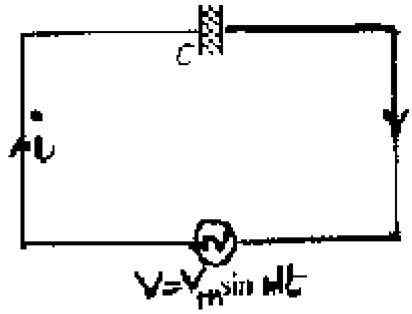


Fig. 16: Pure Capacitor circuit

Consider the circuit with an alternating voltage source connected to capacitor *C* Farad shown in Fig. 16 .

The applied voltage is given as $v(t) = V_m \sin \omega t$.

As a result of the applied voltage, an alternating current $i(t)$ will flow through the circuit.

When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction.

Charge on capacitor, $q = C.\, v(t)$

v = p.d. developed between plates at any instant,

q = charge on plate at that instant,

The current $i(t)$ flowing through the capacitive circuit is given by the rate of change of charge per unit time i.e.

$$i(t) = \frac{dq}{dt} = \frac{d\, C.v(t)}{dt} = \frac{d\,(C.V_m sin\omega t)}{dt}$$

$$i(t) = \frac{Cdv}{dt}$$

$$i(t) = \frac{C\ d\,(V_m sin\omega t)}{dt}$$

$$i(t) = \omega C V_m \cos \omega t$$

$$i(t) = \omega C V_m \sin(\omega t + \frac{\pi}{2})$$

$$i(t) = \frac{V_m}{X_c} \sin\left(\omega t + \frac{\pi}{2}\right), \text{Where } X_c = \frac{1}{\omega C}$$

$$i(t) = I_m \sin(\omega t + \frac{\pi}{2}), \text{where } I_m = \frac{V_m}{X_c}$$

X_c is known as capacitive reactance and is in ohms if C is in Farad and ω is in radians/sec.

Hence the **current in a pure capacitor leads its voltage by a quarter cycle** (π/2) as shown in fig. (17).

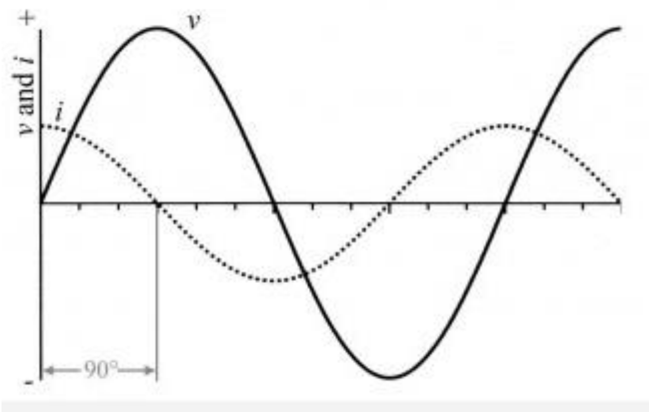


Fig. 18: Waveform of voltage and current for pure capacitor circuit

Power:-

Instantaneous power $P(t) = v(t).i(t)$

Where, $v(t) = V_m sin\omega t$ and $i(t) = I_m \sin(\omega t + \frac{\pi}{2})$

$$P(t) = V_m I_m \sin \omega t sin(\omega t + \frac{\pi}{2})$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$P(t) = \frac{V_m I_m}{2} sin2\omega t \text{----------instantaneous power}$$

Average power consumed by pure inductive circuit over one cycle is given as:

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} P(\theta)d\theta$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} sin2\theta \, d\theta$$

$$P_{avg} = 0$$

Therefore, **the average power consumed by a capacitor = 0**

When voltage rises across the plates of a capacitor, energy is supplied by the source and is stored in the capacitor in the form of electrostatic field energy. As the voltage falls , the electrostatic field collapses and returns the stored energy to the source.

Since the power supplied during positive half of ac voltage is equal to the power returned during the negative half cycle of the ac voltage, the net power consumed by the pure capacitive circuit is zero.

Phasor diagram:

Applied voltage: $v(t) = V_m \sin \omega t$

Resultant current: $i(t) = I_m \sin(\omega t + \frac{\pi}{2})$

For purely capacitive circuit current leads the applied voltage by 90 degrees.

Power factor of purely capacitive circuit = 0

Applied voltage: $v(t) = V_m \sin \omega t$

Resultant current: $i(t) = I_m \sin(\omega t + \frac{\pi}{2})$

The phase difference between the applied voltage and resultant current is φ=90^o.

Power factor for pure capacitive circuit : PF =cos Φ

PF = cos (90^o) = 0

Power consumed is given as : $P_{consumed} = V_{rms} I_{rms} cos\varphi = V_{rms} I_{rms}. 0 = 0$

Power consumed by a capacitive circuit =0.

Numerical on AC analysis of pure R, pure L and pure C circuit

1. A 50 Hz alternating voltage of 150 V is applied independently to (i) resistance of 10 Ω (ii) inductance of 0.2 H and (iii) capacitance of 50µF. Find the expression for the instantaneous current in each case.

RMS value of applied voltage, V = 150 volts & frequency (f) is 50 Hz. Hence expression is

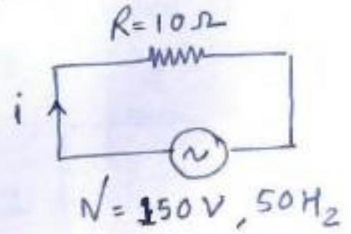
$v = V_m \sin(\omega t)$

$v = V_m \sin(2\pi f t) \qquad (\because \omega = 2\pi f)$

$v = (\sqrt{2} \times 150) \sin(2\pi \times 50\, t)$

$v = 212.13 \sin(100\pi t)$

Case (i) when a 50 Hz alternating voltage of 150V is applied to resistance of 10Ω



The rms value of circuit current,

$I = \frac{V}{R} = \frac{150}{10} = 15\ A,$

the maximum value of circuit current,

$I_m = \sqrt{2} \times I = \sqrt{2} \times 15 = 21.21$

As in case of pure resistive circuit, current is in phase with voltage, the expression of circuit current is

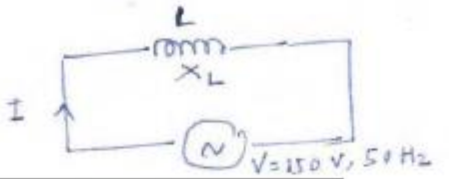
$i = I_m \sin(100\pi t)$

$i = 21.21 \sin(100\pi t)$

Case (ii) when a 50 Hz altternating voltage of 150 V is applied to inductance of 0.2 H,

$L = 0.2\ H$

$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$



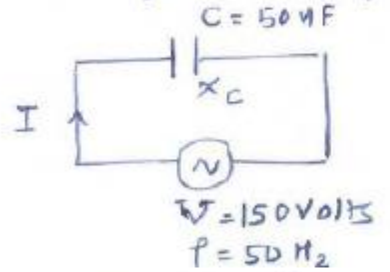
By Ohm, Law $I = \frac{V}{X_L} = \frac{150}{62.83} = 2.387$ A

Now, Maximum value of current, $I_m = \sqrt{2} \times 2.387 = 3.38$ A

In case of pure inductance, circuit current lags $90°(\pi/2)$ to applied voltage, the expression of circuit current is

$$i = I_m \sin(100\pi t - \frac{\pi}{2})$$
$$i = 3.38 \sin(100\pi t - \frac{\pi}{2})$$

Case (III) When a 50Hz, alternating voltage of 150V is applied to capacitance of 50μF



Now, $C = 50\mu F = 50 \times 10^{-6}$ F

Capacitive reactance, $X_C = \frac{1}{2\pi f C}$

$$= \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$= 63.66\ \Omega$$

Rms value of circuit current, $I = \frac{V}{X_C} = \frac{150}{63.66} = 2.356$ A

Maximum value of circuit current, $I_m = \sqrt{2} \times 2.356 = 3.33$ A

⇒ In case of pure capacitive circuit, circuit current leads $90°(\pi/2)$ to the applied voltage, the expression of current is

$$i = I_m \sin(100\pi t + \pi/2)$$
$$i = 3.33 \sin(100\pi t + \frac{\pi}{2})$$

2.11 SERIES RL AC CIRCUITS

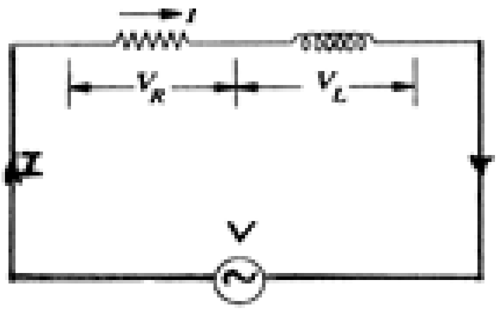


Fig. 19: Circuit diagram of series RL circuit

Consider a series circuit of a pure resistor R and a pure inductive coil L are shown in fig. 19,

Let V = rms value of the applied voltage,

I = rms value of the resultant current,

Z = Impedance of the circuit

Phasor diagram:

The applied ac voltage V results in flow of ac current I through the circuit which creates a voltage drop across resistor R and inductor L

V_R = I.R ---- where the voltage drop across R is in phase with current I

$V_L = I.X_L$ -----where voltage drop across coil L leads the current I by 90^0

The phasor diagram of series RL circuit is shown in fig 20 with current I as the reference. The voltage V_R is in phase with I and V_L leads the current I by 90^o.

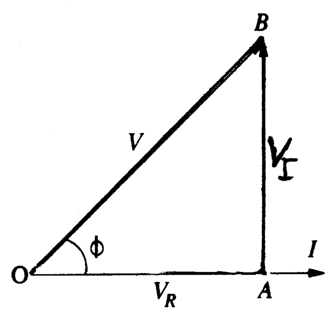


Fig. 20: Phasor diagram for series RL circuit

Voltage Triangle:

From the circuit diagram it is seen that the applied voltage $\bar{V}$ is vector sum of $\overline{V_R}$ and $\overline{V_L}$

Applied voltage :

$$\bar{V} = \overline{V_R} + \overline{V_L} = |V|\angle\varphi$$

$$\bar{V} = \bar{I}\,R + \bar{I}\,X_L = \bar{I}(R + jX_L) = \bar{I}\,\bar{Z}$$

$$|V| = \sqrt{V_R^2 + V_L^2}$$

$$\varphi = tan^{-1}\left(\frac{V_L}{V_R}\right)$$

Resultant current:

$$\bar{I} = \frac{\bar{V}}{\bar{\bar{Z}}} = \frac{V\angle 0^o}{|Z|\angle\varphi} = \frac{V}{|Z|}\angle -\varphi^o$$

$\bar{I} = |I| \angle -\varphi$, where $|I| = \frac{V}{|Z|}$ and

$$, \qquad \varphi = tan^{-1}\left(\frac{V_L}{V_R}\right) = tan^{-1}\left(\frac{X_L}{R}\right)$$

Hence if the applied voltage is $v(t) = V_m \sin\omega t$
In other words, **current I lags behind the applied voltage V by an angle Φ**.
Then current equation is $i(t) = I_m \sin(\omega t - \varphi), where\ I_m = \frac{V_m}{|Z|}$

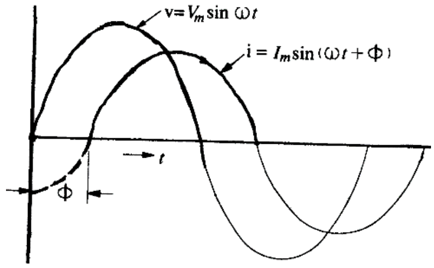


Fig. 21: Waveform of applied voltage and resultant current

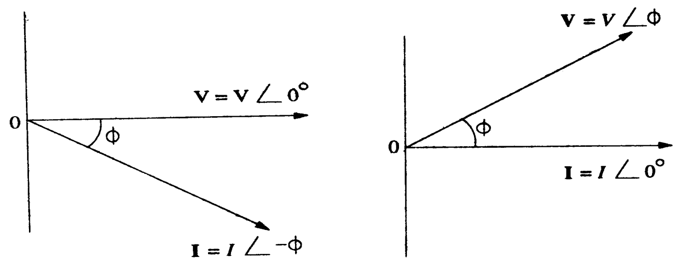


Fig. 22: Phasor diagram of applied voltage and resultant current

<u>**Impedance Triangle**</u>:

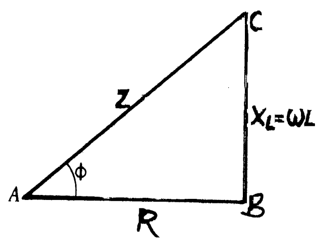


Fig. 23: Impedance triangle of series RL circuit

<u>**Circuit Impedance**</u>:

$$\bar{Z} = R + jX_L = (|Z|\angle\varphi\ )\ \Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}\ \Omega, \qquad \varphi = tan^{-1}\left(\frac{X_L}{R}\right)$$

Power: -

Instantaneous power consumed by series RL circuit is given as

$$P(t) = v(t).i(t)$$

Where, $v(t) = V_m \sin \omega t$ and $i(t) = I_m \sin(\omega t - \varphi)$

$$P(t) = V_m \sin \omega t . I_m \sin(\omega t - \varphi)$$

$$P(t) = \frac{V_m I_m (cos\varphi - \cos(2\omega t - \varphi))}{2}$$

$$P(t) = \frac{V_m I_m cos\varphi}{2} - \frac{V_m I_m \cos(2\omega t - \varphi)}{2}$$

This power consists of two parts :

(1) Real power : A constant part $\frac{V_m I_m cos\varphi}{2}$

(2) A pulsating component: $\frac{V_m I_m \cos(2\omega t - \varphi)}{2}$ which has a frequency twice that of the voltage and current. It does not contribute to actual power, since its average value over a complete cycle is zero.

Average power consumed : $=P_{avg=}\ \frac{V_m I_m cos\varphi}{2} = V_{rms} I_{rms} cos\varphi$

The **average power consumed** by the current is given by the product of V and that component of the current I which is in phase with V. So

$P_{avg} = V_{rms}.I_{rms}.\cos\Phi$.

The term **cosΦ is called the power factor** (p.f.) of the circuit.

True Power W= $V_{rms}.I_{rms} \cos\Phi$ (Watts)

It should be noted that **power consumed is due to ohmic resistance** only because **pure reactance does not consume any active power**.

<u>Power Factor :</u>

Power factor = PF = cos φ
From phasor diagram : PF = cosΦ= R/Z,
Therefore, average power consumed by the series RL circuit is
$P_{avg} = V_{rms}.I_{rms}.\cos\Phi = V_{rms}.I_{rms}.(R/Z)$
= (V/Z). (I.R)
$P = I^2R$ where, I = V/Z

Power triangle (Active, Reactive and Apparent Power):-

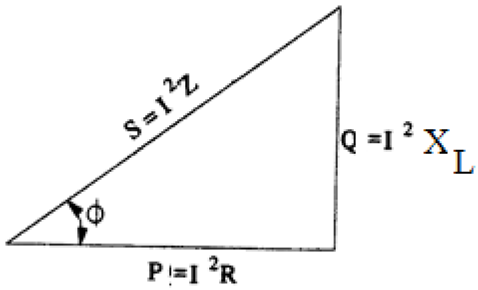


Fig.24: Power triangle for series RL circuit

The series R-L circuit draw a current of I when an alternating voltage of rms value V is applied to it.
For series RL circuit the resultant current lags behind the applied voltage by Φ.
The applied power VI is consumed by the resistor and inductor.
The powers drawn by the circuit are as under:

1). **Apparent power (S):-** It is given by the product of rms values of **applied voltage** and circuit and circuit current.

$$S = V_{rms}.I_{rms} = (I_{rms}.Z).\ I_{rms} = I_{rms}{}^{2}.Z \text{ volt-amperes (VA).}$$

2). **Active power (P or W):** - It is the power, which is **actually dissipated in the circuit resistance.**

$$P = I_{rms}{}^{2}R = V_{rms}\ I_{rms} \cos\Phi \text{ watts.}$$

3). **Reactive Power (Q):** - It is the **power developed in the inductive reactance** of the circuit.

$$Q = I_{rms}.\ I_{rms}.X_L = I_{rms}{}^{2}Z.\sin\Phi = I_{rms}.(\ I_{rms}.Z).\sin\Phi$$
$$= V_{rms}.\ I_{rms} \sin\Phi \text{ volt-ampere reactive (VAR)}$$

These three powers are shown in the power triangle of fig. 24.,
from where it can be seen that $S^2 = P^2 + Q^2$ or $S = (P^2 + R^2)^{1/2}$

Numericals of RL circuit

1.

An alternating voltage of 80 + j60 V is applied to a circuit and the current flowing is 4 – j2 A. Find the (a) impedance, (b) power consumed, (c) phase angle, and (d) power factor.

Solution:

Data $\bar{V} = 80 + j60$ V

$\bar{I} = 4 - j2$ A

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{80 + j60}{4 - j2} = \frac{100\angle 36.87^\circ}{4.47\angle -26.56^\circ} = 22.37\ \angle 63.43^\circ\ \Omega$$

Impedance $Z = 22.37\ \Omega$

Phase angle $\phi = 63.43^\circ$

Power factor $\text{pf} = \cos\phi = \cos(63.43^\circ) = 0.447$ (lagging)

Power consumed $P = VI\cos\phi$

$= 100 \times 4.47 \times 0.447 = 199.81$ W

2.

The voltage and current in a circuit are given by $\bar{V} = 150\ \angle 30^\circ$ V and $\bar{I} = 2\ \angle -15^\circ$ A. If the circuit works on a 50-Hz supply, determine the power factor, power loss, impedance, resistance, and reactance considering the circuit as a simple series circuit.

Solution:

Data $\bar{V} = 150\ \angle 30^\circ$ V

$\bar{I} = 2\angle -15^\circ$ A

$f = 50$ Hz

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{150\angle 30^\circ}{2\angle -15^\circ}$$

$= 75\ \angle 45^\circ\ \Omega = 53.03 + j53.03\ \Omega$

Impedance $Z = 75\ \Omega$

Resistance $R = 53.03\ \Omega$

Reactance $X = 53.03\ \Omega$

Power factor $\text{pf} = \cos\phi = \cos(45^\circ) = 0.707$ (lagging)

Power loss $P = VI\cos\phi$

$= 150 \times 2 \times 0.707 = 212.1$ W

3.

When a sinusoidal voltage 120 V (rms) is applied to a series R-L circuit, it is found that there occurs a power dissipation of 1200 W and a current flow given by $i(t) = 28.3 \sin(314t - \phi)$. Find the circuit resistance and inductance.

Solution:

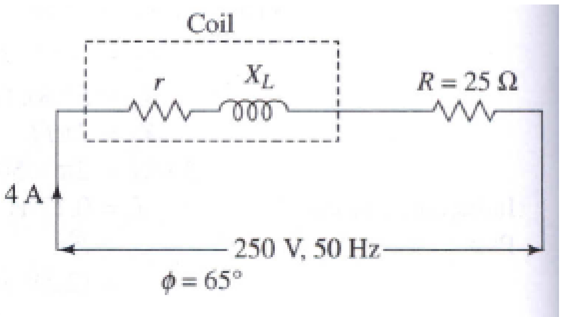
Data

$$V = 120 \text{ V}$$
$$i(t) = 28.3 \sin(314t - \phi)$$
$$P = 1200 \text{ W}$$
$$I = \frac{28.3}{\sqrt{2}} = 20.01 \text{ A}$$
$$P = VI \cos\phi$$
$$1200 = 120 \times 20.01 \times \cos\phi$$
$$\cos\phi = 0.499$$
$$\phi = 60.02^\circ$$
$$Z = \frac{V}{I} = \frac{120}{20.01} = 6\ \Omega$$
$$\overline{Z} = Z\angle\phi = 6\angle 60.02^\circ = 3 + j5.2\ \Omega$$
$$\text{Resistance } R = 3\ \Omega$$
$$\text{Reactance } X_L = 5.2\ \Omega$$
$$X_L = \omega L$$
$$5.195 = 314 \times L$$
$$\text{Inductance } L = 0.0165 \text{ H}$$

4.

A resistance of 25 Ω is connected in series with a choke coil. The series combination when connected across a 250-V, 50-Hz supply, draws a current of 4-A which lags behind the voltage by 65°. Calculate (i) total power, (ii) power consumed by resistance, (iii) power consumed by choke coil, and (iv) resistance and inductance of the coil.

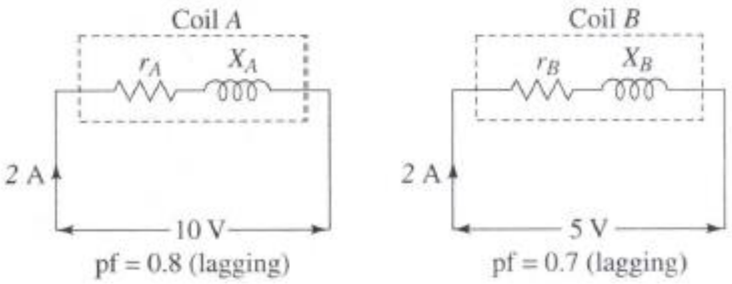
Solution:



Total impedance	$Z = \dfrac{250}{4} = 62.5\ \Omega$
	$\bar{Z} = Z\angle\phi = 62.5\ \angle$ [illegible]
But	$\bar{Z} = (R + r) + jX_L$
	$X_L = 56.64\ \Omega$
	$R + r = 26.41$
Resistance of coil	$r = 26.41 - 25 = 1.41\ \Omega$
Total power	$P = I^2 (R + r) = (4)^2 \times 26.41 = 422.56\ \text{W}$
Power consumed by resistance	$P_R = I^2 R = (4)^2 \times 25 = 400\ \text{W}$
Power consumed by choke coil	$P_{coil} = I^2 r = (4)^2 \times 1.41 = 22.56\ \text{W}$
	$X_L = 2\pi f L$
	$56.64 = 2\pi \times 50 \times L$
Inductance of coil	$L = 0.18\ \text{H}$

5.

A coil A takes 2 A at a power factor of 0.8 lagging with an applied p.d. of 10 V. A second coil B takes 2 A with a power factor of 0.7 lagging with an applied voltage of 5 V. What voltage will be required to produce a total current of 2 A with coils A and B in series? Find the power factor in this case.



Solution:

For coil A,

$$\phi_A = \cos^{-1}(0.8) = 36.87^\circ$$

$$Z_A = \frac{10}{2} = 5\ \Omega$$

$$\bar{Z}_A = 5\ \angle 36.87^\circ$$

$$= 4 + j3\ \Omega$$

$$r_A = 4\ \Omega$$

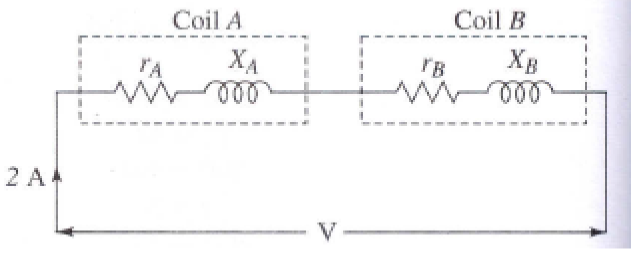
$$X_A = 3\ \Omega$$

For coil B,

$$\phi_B = \cos^{-1}(0.7) = 45.57°$$

$$Z_B = \frac{5}{2} = 2.5\ \Omega$$

$$\bar{Z}_B = 2.5\ \angle 45.57°$$



$$\bar{Z}_B = 1.75 + j1.78\ \Omega$$

$$r_B = 1.75\ \Omega$$

$$X_B = 1.78\ \Omega$$

When coils A and B are connected in series,

$$\bar{Z} = r_A + jX_A + r_B + jX_B = 4 + j3 + 1.75 + j1.78$$

$$= 5.75 + j4.78 = 7.48\ \angle 39.74°\ \Omega$$

$$Z = 7.48\ \Omega$$

$$\phi = 39.74°$$

$$V = Z \,.\, I = 7.48 \times 2 = 14.96\ \text{V}$$

$$\text{pf} = \cos\phi = \cos(39.74°) = 0.77\ (\text{lagging})$$

2.12 A.C. analysis of RC circuit

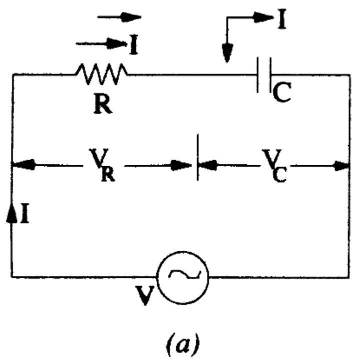


Fig. 25. Series connected RC circuit

The circuit diagram of a series connected resistor *R* (Ω) and capacitor *C* (Farad) is shown in Fig. 25.

Let V = rms value of the applied voltage,
I = rms value of the resultant current,
Z = Impedance of the circuit

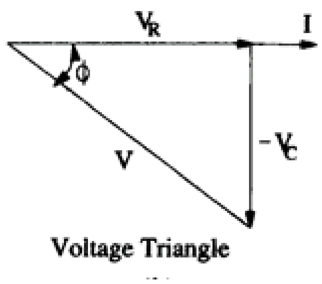
Phasor diagram:

The applied voltage V results in flow of current I through the circuit which creates a voltage drop across resistor *R* and capacitor *C*

$V_R = I.R$ ---- where the voltage drop across R is in phase with current I

$V_c = I.X_c$ -----where voltage drop across capacitor C lags the current I by 90^0

The phasor diagram of series RC circuit is shown in fig 26 with current I as the reference. The voltage V_R is in phase with I and Vc lags the current I by 90^o.



Voltage Triangle:

From the circuit diagram it is seen that the applied voltage $\bar{V}$ is vector sum of $\overline{V_R}$ and $\overline{V_c}$

Applied voltage :

$$\bar{V} = \overline{V_R} + \overline{V_c} = |V|\angle\varphi$$

$$\bar{V} = \bar{I}\,R + \bar{I}\,X_c = \bar{I}(R - jX_c) = \bar{I}\,\bar{Z}$$

$$|V| = \sqrt{V_R^2 + V_c^2}$$

$$\varphi = tan^{-1}\left(\frac{V_c}{V_R}\right)$$

Resultant current:

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V\angle 0^o}{|Z|\angle -\varphi} = \frac{V}{|Z|}\angle + \varphi$$

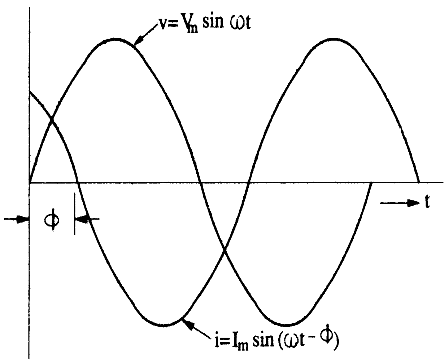
$$\bar{I} = |I| \angle + \varphi, \text{ where } |I| = \frac{V}{|Z|} \text{ and}$$

$$\varphi = tan^{-1}\left(\frac{V_c}{V_R}\right) = tan^{-1}\left(\frac{X_c}{R}\right)$$

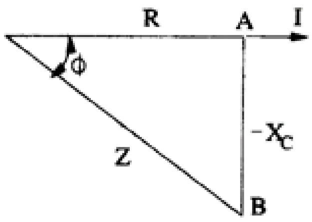
Hence if the applied voltage is $v(t) = V_m \sin \omega t$

In other words, **current I leads the applied voltage V by an angle Φ.**

Then current equation is $i(t) = I_m \sin(\omega t + \varphi), where\ I_m = \frac{V_m}{|Z|}$



Impedance Triangle:



Impedance Triangle

Circuit Impedance:

$$\bar{Z} = R - jX_c = (|Z|\angle - \varphi)\ \Omega$$

$$|Z| = \sqrt{R^2 + X_c^2}\ \Omega, \qquad \varphi = tan^{-1}\left(\frac{X_c}{R}\right)$$

Power: -

Instantaneous power consumed by series RL circuit is given as

$$P(t) = v(t).i(t)$$

Where, $v(t) = V_m \sin \omega t$ and $i(t) = I_m \sin(\omega t + \varphi)$

$$P(t) = V_m \sin \omega t \,.\, I_m \sin(\omega t + \varphi)$$

$$P(t) = \frac{V_m I_m (cos\varphi - \cos(2\omega t + \varphi))}{2}$$

$$P(t) = \frac{V_m I_m cos\varphi}{2} - \frac{V_m I_m \cos(2\omega t + \varphi)}{2}$$

This power consists of two parts :

(1) Real power : A constant part $\frac{V_m I_m cos\varphi}{2}$

(2) A pulsating component: $\frac{V_m I_m \cos(2\omega t+\varphi)}{2}$ which has a frequency twice that of the voltage and current. It does not contribute to actual power, since its average value over a complete cycle is zero.

Average power consumed : $=P_{avg}= \frac{V_m I_m cos\varphi}{2} = V_{rms} I_{rms} cos\varphi$

The **average power consumed** by the current is given by the product of V and that component of the current I which is in phase with V. So

$$\mathbf{P_{avg} = V_{rms}.I_{rms}.cos\Phi.}$$

The term **cosΦ is called the power factor** (p.f.) of the circuit.

True Power W= $V_{rms}.I_{rms}$ cosΦ (Watts)

It should be noted that **power consumed is due to ohmic resistance** only because **pure reactance does not consume any active power**.

Power Factor :

Power factor = PF = cos φ
From phasor diagram : PF = cosΦ= V_R / V = R/Z.
Therefore, average power consumed by the series RC circuit is
$P_{avg} = V_{rms}.I_{rms}.cos\Phi = V_{rms}.I_{rms}$.(R/Z)
= (V/Z). (I.R)
P = I^2R where, I = V/Z

Active, Reactive and Apparent Power:-

Power triangle:

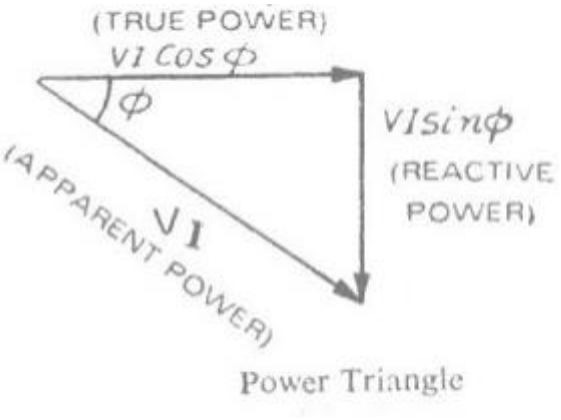


Fig.26: Power triangle for series RC circuit

The series R-C circuit draw a current of I when an alternating voltage of rms value V is applied to it. For series RC circuit the resultant current lags behind the applied voltage by Φ.

The powers drawn by the circuit are as under:

1). **Apparent power (S):-** It is given by the product of rms values of **applied voltage** and circuit and circuit current.

$$S = V_{rms}.I_{rms} = (I_{rms}.Z).\ I_{rms} = I_{rms}{}^{2}.Z \text{ volt-amperes (VA).}$$

2). **Active power (P or W):** - It is the power, which is actually **dissipated in the circuit resistance.**

$$P = I_{rms}{}^{2}R = V_{rms}\ I_{rms} \cos\Phi \text{ watts.}$$

3). **Reactive Power (Q):** - It is the **power developed in the capacitive reactance** of the circuit.

$$Q = I_{rms}.\ I_{rms}.X_L = I_{rms}{}^{2}Z.\sin\Phi = I_{rms}.(\ I_{rms}.Z).\sin\Phi$$

$$= V_{rms}.\ I_{rms} \sin\Phi \text{ volt-ampere reactive (VAR)}$$

These three powers are shown in the power triangle of fig. 24.,
from where it can be seen that $S^2 = P^2 + Q^2$ or $S = (P^2 + R^2)^{1/2}$

Numericals of RC circuit

Q.1)

The voltage applied to a circuit is e = 100 sin(wt+30) and current flowing in the circuit is

i = 15sin(wt+60). Determine the impedence, resistance, reactance, power and power factor.

Data : $e = 100 \sin(\omega t + 30^\circ)$

$i = 15 \sin(\omega t + 60^\circ)$

$$\overline{E} = \frac{100}{\sqrt{2}} \angle 30^\circ \text{ V}$$

$$\overline{I} = \frac{15}{\sqrt{2}} \angle 60^\circ \text{ A}$$

$$\overline{Z} = \frac{\overline{E}}{\overline{I}} = \frac{\frac{100}{\sqrt{2}}\angle 30^\circ}{\frac{15}{\sqrt{2}}\angle 60^\circ} = 6.67 \angle -30^\circ = 5.77 - j3.33\ \Omega$$

$Z = 6.67\ \Omega$

$R = 5.77\ \Omega$

$X_C = 3.33\ \Omega$

$= \cos\phi = \cos(30^\circ) = 0.866$ (leading)

$$P = VI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5 \text{ W}$$

Q.2)

A series circuit consumes 2000W at 0.5 leading power factor, when connected to 230 V, 50 Hz ac supply. Calculate i) kVA ii) kVAR iii) Current

Solution:

Data :

$$P = 2000\ W$$
$$pf = 0.5\ \text{(leading)}$$
$$V = 230\ V$$
$$P = VI \cos\phi$$
$$2000 = 230 \times I \times 0.5$$
$$\therefore \quad I = 17.39\ A$$
$$S = VI = \frac{P}{\cos\phi} = \frac{2000}{0.5} = 4000\ VA = 4\ kVA$$
$$\phi = \cos^{-1}(0.5) = 60^\circ$$
$$Q = VI \sin\phi = 230 \times 4 \times \sin(60^\circ) = 3.464\ kVAR$$

Q.3)

A resistor R in series with a capacitance C is connected to a 240 V, 50 Hz ac supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and maximum stored energy in C.

Data : $V = 240\ V$ $V_R = 100\ V$

$P = 300\ W$ $f = 50\ Hz$

$$P = \frac{V_R^2}{R}$$
$$300 = \frac{(100)^2}{R}$$
$$\therefore \quad R = 33.33\ \Omega$$
$$P = I^2R$$
$$300 = I^2 \times 33.33$$
$$\therefore \quad I = 3\ A$$
$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$
$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(80)^2 - (33.33)^2} = 72.72\ \Omega$$
$$X_C = \frac{1}{2\pi fC}$$
$$72.72 = \frac{1}{2\pi \times 50 \times C}$$
$$\therefore \quad C = 43.77\ \mu F$$

Q.4)

A capacitor of 35 μF is connected in series with a variable resistor. The circuit is connected across 50 Hz mains. Find the value of resistor for a condition when the voltage across the capacitor is half the supply voltage.

Data : $C = 35\ \mu F$ $\quad f = 50\ Hz$

$V_C = \frac{1}{2} V$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 35 \times 10^{-6}} = 90.946\ \Omega$$

$$V_C = \frac{1}{2} V$$

$$X_C \cdot I = \frac{1}{2} Z \cdot I$$

$$\therefore \quad X_C = \frac{1}{2} Z$$

$$Z = 2X_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$(2X_C)^2 = R^2 + X_C^2$$

$$3X_C^2 = R^2$$

$$R^2 = 3 \times (90.946)^2 = 24813.35$$

$$\therefore \quad R = 157.5\ \Omega$$

2.13 SERIES RLC CIRCUIT

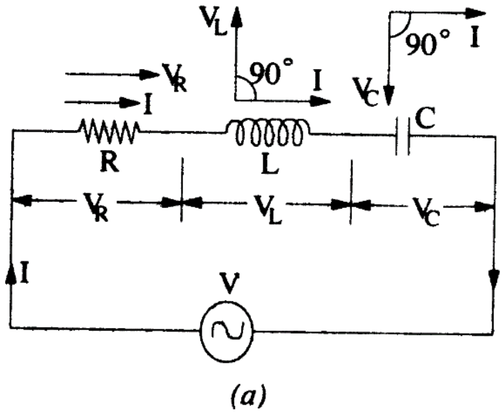


Fig. 27 Series RLC circuit

Consider the circuit consisting of resistor R (Ω), inductor L (H) and capacitor C (F) conneced in series to a ac source.

Let V = rms value of the applied voltage,
I = rms value of the resultant current,
Z = Impedance of the circuit

Let V_R = IR = voltage drop across R- **in phase with I**

$V_L = IX_L$ = voltage drop across L – **leading I by 90^0**

$V_C = IX_C$ = voltage drop across C – **lagging I by 90^0**

Where, $X_L = 2\pi f L$ and $X_c = \frac{1}{2\pi f C}$.

From the circuit diagram it is seen that : $\bar{V} = \overline{V_R} + \bar{V_L} + \bar{V_c}$

$$\bar{V} = \bar{I}\,(R + j\,X_L - jX_c) = \bar{I}\,\bar{Z}$$

Where, $\bar{Z} = R + j\,X_L - jX_c$

$$\bar{Z} = |Z|\,\angle\varphi$$

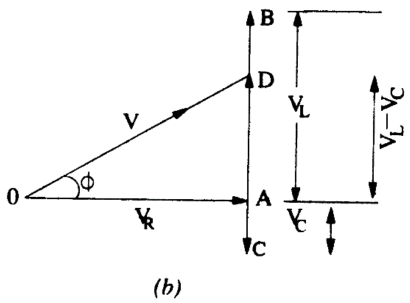
As frequency varies the value of X_L and X_c varies resulting in the following cases:

Case (i) $X_L > X_c$

$$\bar{V} = \bar{I}\,R + j\bar{I}\,(X_L - X_C) = \bar{I}\,R + j\bar{I}\,X \; where \; X = X_L - X_C$$

$$|Z| = \sqrt{R^2 + (X_L - X_c)^2} \text{ and } \varphi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Voltage triangle:



$$\bar{V} = \overline{V_R} + \bar{V_L} + \bar{V_c}$$
$$\bar{V} = \bar{I}\,R + jI\,(X_L - X_c)$$
$$\bar{V} = \bar{V}_R + j\,(\bar{V}_L - \bar{V}_c)$$
$$\bar{V} = |V|\,\angle\varphi$$

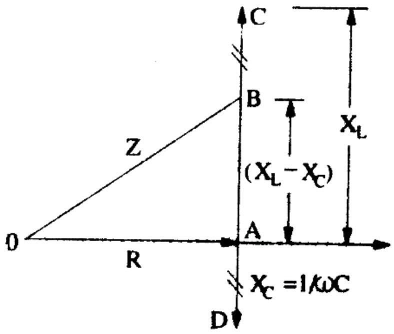
$|V| = \sqrt{V_R^2 + (V_L - V_c)^2}$ and $\angle\varphi = tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

If , v(t) = Vm Sin wt ; i(t)= Im Sin (wt - ϕ)
i.e I lags V by angle ϕ
For series RLC circuit with $X_L > Xc$, the circuit is inductive circuit and the resultant current lags the applied voltage by an angle $\varphi = tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = tan^{-1}\left(\frac{X_L - X_c}{R}\right)$

Impedance triangle:

$$\bar{Z} = |Z|\,\angle\varphi$$

$|Z| = \sqrt{R^2 + (X_L - X_c)^2}$ and $\varphi = tan^{-1}\left(\frac{X_L - X_c}{R}\right)$



Phase angle Φ is given by
tanΦ = $(X_L - X_C)$ / R = X / R
Power factor cosΦ = R / Z

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

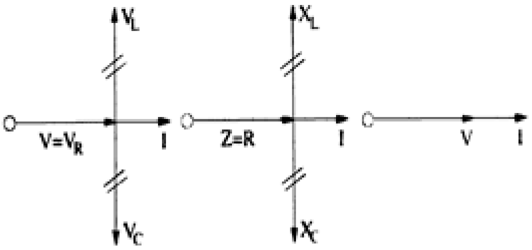
Power consumed = $V_{rms}I_{rms}cos\emptyset$

Case(ii): $X_L = Xc$
When $X_L = Xc$
Also $V_L = Vc$ (or) $IX_L = IXc$

So V_L and Vc cancel each other and the resultant is zero. So $V = V_R$ in such a case, the circuit is purely resistive in nature.

$$\bar{V} = \bar{I}\,R$$

Circuit is purely resistive and hence applied voltage and resultant current are in phase.



Power factor cosΦ = R / Z=R/R=1

Power consumed = $V_{rms}I_{rms}$

<u>Case(iii): X_L < Xc</u>

When X_L < Xc

Also V_L < Vc (or) IX_L < IXc

Hence the resultant of V_L and Vc will directed towards Vc i.e current is said to be capacitive in nature Form voltage triangle:

$$V = \sqrt{(V_R^2 + (V_c - V_L)^2)} = \sqrt{((IR)^2 + (IX_c - IX_L)^2)}$$

$$V = I\sqrt{(R^2 + (X_c - X_L)^2)}$$

$$V = IZ$$

$$Z = \sqrt{(R^2 + (Xc - X_L)^2)}$$

If , V = Vm Sin wt ; i = Im Sin (wt + ϕ)

i.e I lags V by angle ϕ

Power factor cosΦ = R / Z

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

Power consumed = $V_{rms}I_{rms}cos\emptyset$

Impedance:

In general, for RLC series circuit impedance is given by,

Z = R + j X

X = X_L – Xc = Total reactance of the circuit

If $X_L > X_c$; X is positive & circuit is Inductive

If $X_L < X_c$; X is negative & circuit is Capacitive

If $X_L = X_c$; X =0 & circuit is purely Resistive

tan ϕ = [(X_L - Xc)/R]

cos ϕ = [R/Z]

Z = √ (R^2 + (X_L - Xc) ^{2})

Power and power triangle:

The average power consumed by circuit is,

Pavg = (Average power consumed by R) + (Average power consumed by L) + (Average power consumed by C)

Average power consumed by inductor and capacitor = 0.

$$P_{avg} = \text{Power taken by } R = I^2R = I(IR) = VI$$

$$V = V \cos \phi$$
$$P = VI \cos \phi$$

Thus, for any condition, $X_L > X_C$ or $X_L < X_C$ General power can be expressed as

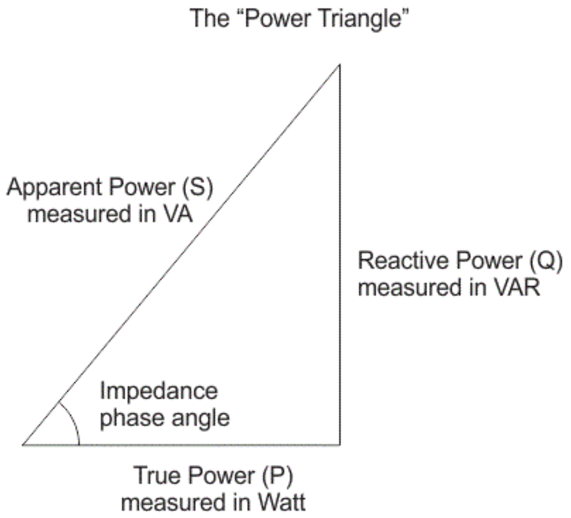
P = Voltage x Current component in phase with voltage

Power triangle:

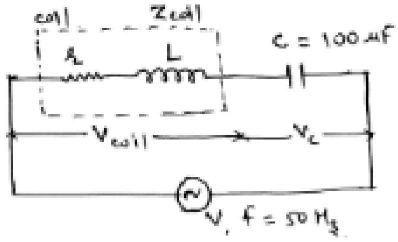
S = Apparent power = $I^2Z = VI$

P = Real or True power = $VI \cos \phi$ = Active power

Q = Reactive power = $VI \sin \phi$



Q.1.) A coil of 0.6 pf is in series with a 100 µF capacitor and is connected to a 50 Hz supply. The potential difference across the coil is equal to the potential difference across the capacitor. Find inductance and resistance of the coil.



Soln - Given, $p.f_{coil} = 0.6$, $C = 100\ \mu F$, $f = 50\ Hz$

$V_{coil} = V_c$; $X_c = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}}$

$I Z_{coil} = I X_c$ $= 31.83\ \Omega$

$\therefore Z_{coil} = X_c = 31.83\ \Omega$

$$P.f. = \frac{R}{Z_{coil}} = 0.6$$

$$R = 0.6 \times Z_{coil} = 0.6 \times 31.83$$

$$= 19.098\ \Omega$$

$$Z_{coil} = \sqrt{R^2 + X_L^2}$$

$$31.83 = \sqrt{(19.098)^2 + X_L^2}$$

$$X_L = 25.46\ \Omega = 2\pi f L$$

$$25.46 = 2 \times 3.14 \times 50 \times L$$

$$\therefore L = 0.081\ H$$

Q.2)

A coil of resistance 3 Ω and inductance of 0.22 H is connected in series with imperfect capacitor. When such a series circuit is connected across 200 V, 50 Hz supply, it has been observed that their combined impedance is (3.8 + j6.4) Ω. Calculate resistance and capacitance of imperfect capacitor.

Data : $\overline{Z} = 3.8 + j6.4\ \Omega$

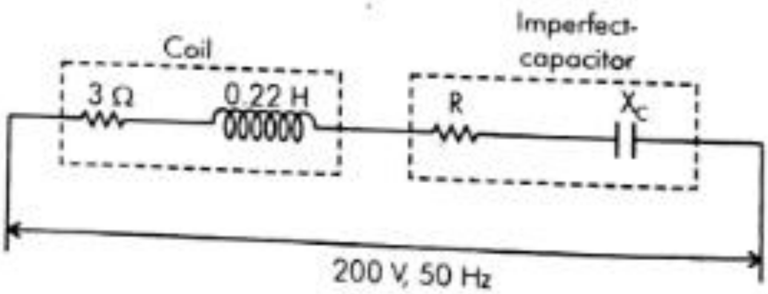


Fig. 3.59

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.22 = 69.12\ \Omega$$

Total impedance $\overline{Z} = 3 + j69.12 + R - jX_C$

$$= (3 + R) + j\,(69.12 - X_C)$$

$$3 + R = 3.8$$

$$\therefore \quad R = 0.8\ \Omega$$

$$69.12 - X_C = 6.4$$

$$\therefore \quad X_C = 62.72\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$62.72 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 50.75\ \mu F$$

Q.3) A R-L-C series circuit has a current which lags the applied voltage by 45^0. The voltage across the inductance has maximum value equal to twice the maximum value of voltage across the capacitor. Voltage across the inductance is 300 sin(1000t) and R = 20Ω. Find the value of inductance and capacitance.

Data : $v_L = 300 \sin(1000t)$

$$R = 20\ \Omega$$

$$\phi = 45^\circ$$

$$V_{Lmax} = 2\ V_{Cmax}$$

$$\sqrt{2}V_L = 2\sqrt{2}\ V_C$$

$$I \times X_L = 2\ I \times X_C$$

$$X_L = 2\ X_C$$

$$\cos\phi = \frac{R}{Z}$$

$$\cos(45^\circ) = \frac{20}{Z}$$

$$\therefore\ Z = 28.28\ \Omega$$

For series R-L-C circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$(28.28)^2 = (20)^2 + (2X_C - X_C)^2$$

$$799.76 = 400 + X_C^2$$

$$X_C = 20\ \Omega$$

$$X_L = 2\ X_C = 40\ \Omega$$

$$X_L = \omega L$$

$$40 = 1000 \times L$$

$$\therefore\ L = \frac{40}{1000} = 0.04\ H$$

$$X_C = \frac{1}{\omega C}$$

$$20 = \frac{1}{1000 \times C}$$

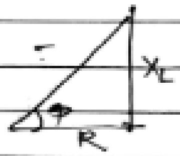
$$\therefore\ C = 50\ \mu F.$$

Q.4)

Two impedances Z_1 & Z_2 having same numerical value are connected in series. If Z_1 is having power ~~generation~~ factor of 0.866 lagging & Z_2 is having pf of 0.8 leading, calculate pf of the series combination.

Soln. $pf_1 = 0.866$ (lagging). $pf_2 = 0.8$ (leading)

$Z_1 = Z_2 = Z$



$\cos\phi_1 = 0.866$

$\phi_1 = \cos^{-1} 0.866$

$= 30°$

$\cos\phi =$

$\phi_2 = \cos^{-1} 0.8 = 36.87°$.

$\bar{Z}_1 = Z\angle\phi_1 = Z\angle 30° = 0.866Z + j0.5Z$

$\bar{Z}_2 = Z\angle\phi_2 = Z\angle 36.87° = 0.8Z - j0.6Z$

For series combination,

$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 0.866Z + j0.5Z + 0.8Z - j0.6Z$

$= 1.666Z - j0.1Z$

$\bar{Z} = Z(1.666 - j0.1)$

$= 1.666Z \angle -3.43°$.

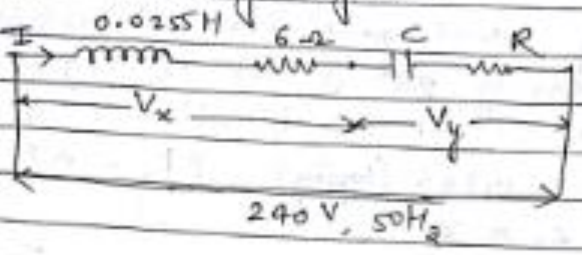
$pf = \cos\phi$

$= \cos(3.43°)$

$= 0.9982$ (leading).

Q.5)

Find the values of R & C so that $V_x = 3V_y$, V_x & V_y are in quadrature. Ref. following figure.



$X_L = 2\pi f L$

$= 2 \times 3.14 \times 50 \times 0.0255$

$= 8\ \Omega$

$\bar{Z}_x = \underbrace{6}_{R = 6 \text{ given}} + \overbrace{j8}^{X_L} = \underline{10 \angle 53.16^\circ\ \Omega}$

$V_x = 3V_y$

$I Z_x = 3 I Z_y$

$Z_x = 3 Z_y \quad \therefore Z_x = 10 \ \& \ Z_y = \frac{10}{3}$

V_x & V_y are in quadrature i.e phase angle between V_x & V_y is 90°.

Hence angle between Z_x & Z_y will be 90°. The impedance Z_y is capacitive in nature.

$\bar{Z}_y = Z_y \angle -\phi$

$\bar{Z}_y = \frac{10}{3} \angle 53.16 - 90^\circ$

$= 3.33 \angle -36.84^\circ$

$\bar{Z}_y = 2.66 - j2\ \Omega$

$R = 2.66$ & $X_c = 2\ \Omega$

$X_c = \frac{1}{2\pi f c}$

$\therefore c = \frac{1}{2\pi f X_c} = \frac{1}{2 \times \pi \times 50 \times 2} = \underline{1.59\ mF}$

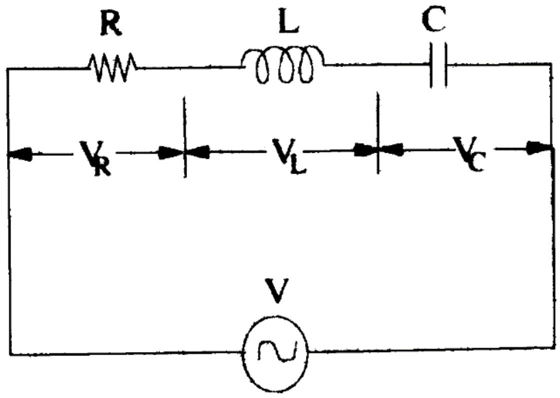
2.14 SERIES RLC RESONANT CIRCUIT

Resonance: Ability of the circuit to select a particular frequency.

Application: Radio receiver to select a particular frequency transmitted by the station and to eliminate the frequency received from other stations.

Theory:

Consider the series RLC circuit shown in figure:



(a)

Net reactance of the circuit : X= ($X_L - X_C$) ohm

Impedance of the circuit : Z = $(R^2 + (X_L - X_C)^2)^{½} = (R^2 + X^2)^{½}$

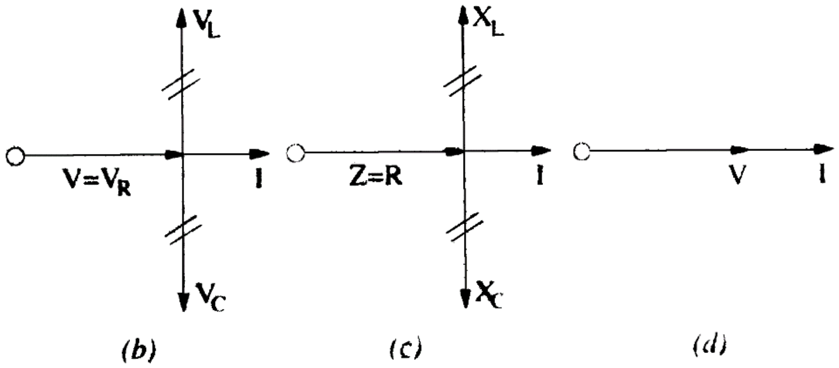
Where, $X_L = 2\pi f L$ Ω and $X_c = \frac{1}{2\pi f C}$ Ω

The inductive reactance X_L is directly proportional to frequency and the capacitive reactance X_C is inversely proportional to frequency. Therefore by changing the supply frequency X_L and X_C are frequency dependent.

If for some frequency of the applied voltage $X_L = X_C$ (in magnitude), then X= 0 & Z = R. At this frequency the impedance is minimum and the current $I = V/Z$ is maximum. This frequency is known as resonant frequency.

Effects at resonant frequency:

1. Net reactance $X_L - X_C = 0$
2. Net impedance is minimum Z=R ohm
3. The current in the circuit is maximum $I = V/Z = V/R$
4. Since current is maximum , power absorbed by the circuit will also be maximum
5. $V_L = I.X_L$ and $V_C = I.X_C$ and the two voltage drops are equal in magnitude but opposite in phase. Hence, they cancel out each other. The two reactance's taken together act as short circuit since no voltage develops across them. The applied voltage V drops entirely across R so that V = V_R as shown in figure below.
6. Power factor at resonance =1.



<u>Resonant frequency</u>:

The frequency at which net reactance is zero given from the relation

$X_L - X_C = 0$ or $X_L = X_C$ or $\omega L = 1/\omega C$

$$\omega^2 = \frac{1}{LC}$$

$$(2\pi f_o)^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

<u>Variation of impedance z with frequency:</u>

a. **At resonant frequency** : Supply frequency $f = f_o$, the circuit is at resonance and resistive, $Z = R$.
b. **At low frequencies** : Supply frequency $f > f_o$, $X_L > X_C$, the circuit is inductive and $Z = R + j\,X$.
c. **At high frequencies**: Supply frequency $f < f_o$, $X_L < X_C$, the circuit is 61apacitive and $Z = R - j\,X$

Resonance curve (series RLC circuit):

The curve between current versus frequency is known as resonance curve.

It has low value on both sides of resonant frequency (for $f > f_o$ and $f < f_o$, , I = V/Z , where $Z = \sqrt{R^2 + X^2}$ and Z is large). At resonant frequency $f = f_o$ current is maximum ($I_0 = V / R$) as shown by the peaked curve. Hence, maximum power is dissipated under resonant conditions.

The shape of the resonance curve depends on the value of resistor R.

For circuits with **low values of R** the resonance curve is sharply peaked and such a circuit is said to be sharply resonant or **highly selective**.

On the other hand, circuits with **high value of R** have flat resonance curve and are said to **have poor selectivity**.

Bandwidth of a series RLC resonant circuit:-

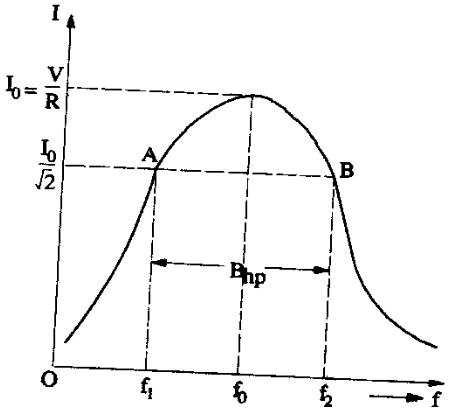


Fig. No.(31)

Bandwidth of a circuit is given by the range of frequencies which lie between two points on either side of the resonant frequencies f_o where current falls to $1/\sqrt{2}$ of its maximum value at resonance.

Narrower the bandwidth, higher the selectivity of the circuit. A

As shown in figure the bandwidth is given by $\Delta f = (f_2 - f_1)$ Hz or $\Delta\omega = (\omega_2 - \omega_1)$ rad/sec.

This range of frequencies (bandwidth), current is equal to or greater than $I_0 / \sqrt{2}$ where $I_0 = V/R$ --- maximum current at resonance.

For series resonant circuit bandwidth is given as $BW = \frac{R}{2\pi L}\ (Hz) = \frac{R}{L}\ rad/sec$

Bandwidth = R / L (rad/sec)

f_1 and f_2 are the frequencies at which the current is exactly = $I_0 / \sqrt{2}$.

These frequencies f_1 and f_2 are called as the upper and the lower cutoff frequencies respectively.

$$f_2 = f_r + \frac{BW}{2}$$

$$f_1 = f_r - \frac{BW}{2}$$

Half Power Frequencies :

Power at resonance frequency = I_o^2R

Note : at resonance current through series RLC circuit is maximum (Io)

Half power frequencies are the frequencies at which the power is ½ of the power at resonance.

$$P_{f_1, f_2} = \frac{Power\ at\ resonance}{2} = \frac{I_o^2R}{2}$$

Value of ω_1 and ω_2 :

$I_m = \frac{V}{R}$ at resonance.

I = V/Z = $\dfrac{V}{\left[R^2 + (wL - \frac{1}{wc})^2\right]^{1/2}}$ at any frequency

At half power points A & B: I = $\dfrac{Io}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} * \dfrac{V}{R}$

So $\dfrac{1}{\sqrt{2}} * \dfrac{V}{R}$ = $\dfrac{V}{\left[R^2 + (wL - \frac{1}{wc})^2\right]^{1/2}}$

Or R = $\pm(wL - 1/wC)$ = $\pm$ X

It shows that of half power points, net reactance is equal to the resistance.

Since resistance equals reactance, p.f. of the circuit at these points is = $\dfrac{1}{\sqrt{2}}$ = 0.707, though leading at point A and lagging at point B.

Hence R^2 = $(wL - 1/wC)^2$

So, ω = $\pm\dfrac{R}{2L} \pm \sqrt{\dfrac{R^2}{4L^2} + \dfrac{1}{LC}}$ = $\pm\alpha \pm \sqrt{\alpha^2 + w_0{}^2}$

Where $\alpha = \dfrac{R}{2L}$ & ω_o = $\dfrac{1}{\sqrt{LC}}$

Since $R^2/4L^2$ is much less than $\dfrac{1}{\sqrt{LC}}$, neglecting $R^2/4L^2$

So ω= $\dfrac{R}{2L} \pm \dfrac{1}{\sqrt{LC}}$ = $\dfrac{R}{2L} \pm \omega_o$,

Since only positive values of ω_0 are considered

It is obvious that f_o is the centre frequency between f_1 & f_2 also $w_1 = w_o - \Delta w/2$ &

$\omega_2 = \omega_o + \Delta w/2$. As stated above, bandwidth is measure of circuits selectivity, Narrower the band width, higher the selectivity, & vice versa.

Half power frequencies: $\omega_{1,2} = \dfrac{R}{2L} \pm \dfrac{1}{\sqrt{LC}}$

$$\omega_{1,2} = \frac{R}{2L} \pm \omega_o$$

ω_o is the resonant frequency.

Q-Factor of series circuit:

In the case of series R-L-C circuit it is defined as equal to the voltage magnification in the circuit at resonance.

At resonance $I_0 = \frac{V}{R} = I_{max}$,

Since $X_L = X_c$

$V_L = V_c = I_0 X_L = I_0 X_C$

Supply voltage V = IR

So Voltage magnification = $\frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R} = \frac{X_C}{R} = \frac{1}{\omega_0 CR}$

So Q –factor $= \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \tan\phi$

Where Q is power factor of the coil

Since, $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Or $2\pi f_0 = \frac{1}{LC}$ so Q = $\frac{1}{R}\sqrt{\frac{L}{C}}$

In fact, Q of series circuit may be written as :

$$Q = \frac{\omega_0}{\text{Band width}}$$

$$= \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Voltage Magnification Factor (Q):

Voltage across the capacitor at resonance

$$V_o = I_o X_C = \frac{I_0}{\omega_0 C} = \frac{V}{R}\sqrt{\frac{L}{C}} = V \cdot Q$$

Similarly : Voltage across the inductor at resonance is:

$$V_L = I_o \omega_o L = \frac{1}{R}\sqrt{\frac{L}{C}} = V \cdot Q$$

Therefor at resonance : $V_L = V_c$ = V. Q , where Q is the voltage magnification factor / quality factor.

Numericals on Series RLC Resonance Circuit

Q.1)

An RLC ckt with the resistance of 10Ω inductance of 0.2 H & capacitor of 40mf. supplied with a 100V supply at variable frequency. Find the following

i) freq. at which resonance takes place

ii) current iii) power iv) power factor

v) voltage across RLC at resonant freq.

vi) Q- factor vii) Half power points.

Sol^n.

① $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 40\times10^{-3}}} = 56.3\ Hz$

② $I_0 = \frac{V}{R} = \frac{100}{10} = 10\ A$

③ $P = I^2 R = 1000\ W$

④ power factor = 1

⑤ voltage across RLC

$V_R = IR = 10 \times 10 = 100\ V$

$V_L = I X_L = 10 \times 2 \times 3.14 \times 0.2 = 707.5$

$V_c = I X_c$ and $X_L = X_c = 70.75$

$\therefore V_c = 707.5$

⑥ Q. factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = 7.07$

⑦ $f_1 = f_0 - \frac{R}{4\pi L} = 52.32\ Hz$

$f_2 = f_0 + \frac{R}{4\pi L} = 60.3\ Hz$

Q.2)

A series resonant circuit has a impedance of 500 Ω at resonant frequency. Cut off frequencies are 10 KHz and 100 Hz. Determine (i) resonant frequency (ii) value of R-L-C (iii) quality factor at resonant frequency.

Data : $R = 500\ \Omega$

$f_1 = 100\ \text{Hz}$

$f_2 = 10\ \text{KHz}$

$$BW = f_2 - f_1 = 10 \times 10^3 - 100 = 9900\ \text{Hz}$$

$$f_2 = f_0 - \frac{BW}{2}$$

$$f_2 = f_0 + \frac{BW}{2}$$

Adding equations (1) and (2),

$$f_1 + f_2 = 2f_0$$

$$\therefore \quad f_0 = \frac{f_1 + f_2}{2} = \frac{10000 + 100}{2} = 5050\ \text{Hz}$$

$$BW = \frac{R}{2\pi L}$$

$$9900 = \frac{500}{2\pi L}$$

$$\therefore \quad L = 8.038\ \text{mH}$$

$$X_{L_0} = 2\pi f_0 L = 2\pi \times 5050 \times 8.038 \times 10^{-3} = 255.05\ \Omega$$

At resonance $X_{L_0} = X_{C_0} = 255.05\ \Omega$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$255.05 = \frac{1}{2\pi \times 5050 \times C}$$

$$\therefore \quad C = 0.12\ \mu F$$

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{500}\sqrt{\frac{8.038 \times 10^{-3}}{0.12 \times 10^{-6}}} = 0.5176$$

Q.3)

A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to 230 V, 50 Hz supply, the maximum current obtained by varying the inductance is 2 A. The voltage across the capacitor is 500 V. Calculate the resistance, inductance and capacitance of the circuit.

Data : $V = 230\ \text{V}$

$f_0 = 50\ \text{Hz}$

$I_0 = 2\ \text{A}$

$V_{C_0} = 500\ \text{V}$

Resistance $R = \frac{V}{I_0} = \frac{230}{2} = 115\ \Omega$

$$X_{C_0} = \frac{V_{C_0}}{I_0} = \frac{500}{2} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times 50 \times C}$$

$\therefore$ Capacitance $C = 12.73\ \mu F$

At resonance $X_{C_0} = X_{L_0}$

$$X_{L_0} = 250\ \Omega$$

$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \times 50 \times L$$

$\therefore$ Inductance $L = 0.795$ H

Q.4)

A series R-L-C circuit has the following parameter values: $R = 10\ \Omega$, $L = 0.014$ H, $C = 100\ \mu F$.

Compute the following:

(a) Resonance frequency in rad/sec
(b) Quality factor of the circuit
(c) Bandwidth
(d) Lower and upper frequency points of the bandwidth
(e) Maximum value of the voltage appearing across the capacitor if the voltage $v = 1 \sin 1000t$ is applied to the R-L-C circuit.

Solution

(a) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.014 \times 100 \times 10^{-6}}} = 845.15$ rad/sec

(b) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{0.014}{100 \times 10^{-6}}} = 1.183$

(c) $\text{BW} = \dfrac{R}{L} = \dfrac{10}{0.014} = 714.29$ rad/sec

(d) Lower and upper frequency points of the bandwidth

$$\omega_1 = \omega_r - \frac{\text{BW}}{2} = 845.15 - \frac{714.29}{2} = 488 \text{ rad/sec}$$

$$\omega_2 = \omega_r + \frac{\text{BW}}{2} = 845.15 + \frac{714.29}{2} = 1202.3 \text{ rad/sec}$$

(e) Applied voltage, $v_1 = 1 \sin 1000t$

So, $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} = 0.707$

At resonance, voltage that appears across the capacitor is maximum and given by

$$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{0.707}{10}\sqrt{\frac{0.014}{100 \times 10^{-6}}} = 0.837 \text{ V}$$

Q.5)

A series circuit consists of a 0.1 μF capacitor, an inductive coil having resistance of 22 Ω, inductance 0.1 H, and a non-inductive resistance of 68 Ω. Find the resonant frequency of the circuit. If the circuit is connected to a supply of 20 V ac at the resonant frequency, calculate: (a) the current in the circuit, (b) the voltage drop across each component, (c) the power consumed by the circuit, and (d) Q-factor of the circuit.

Solution

The conditions of the example are shown in Fig. 2.118.

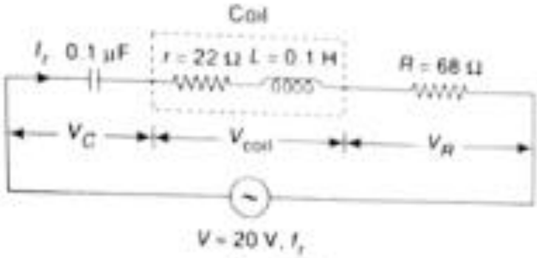


Fig. 2.118

As supply frequency is resonant frequency, the circuit is under resonance condition. So, current flowing through the circuit,

$$I_r = \frac{V}{R+r} = \frac{20}{68+22} = 0.222 \text{ A}$$

Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\times 0.1\times 10^{-6}}} = 10000 \text{ rad/sec}$$

As $\omega_r = 2\pi f_r$,

$$f_r = \frac{\omega_r}{2\pi} = \frac{10000}{2\pi} = 1591.5 \text{ Hz}$$

Now, capacitive reactance,

$$X_C = \frac{1}{\omega_r C} = \frac{1}{10000\times 0.1\times 10^{-6}} = 1000\ \Omega$$

Voltage across the capacitor,

$$V_C = I_r X_C = 0.222 \times 1000 = 222 \text{ V}$$

Voltage across the non-inductive resistance,

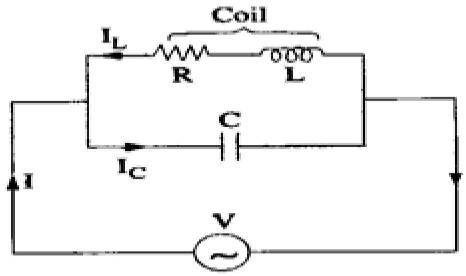
$$V_R = I_r R = 0.222 \times 68 = 15.096 \text{ V}$$

Voltage across the inductive coil:

Inductive reactance,

$$X_L = \omega_r L = 10000 \times 0.1 = 1000\ \Omega$$

2.15 : PARALLEL A.C. RESONANT CIRCUIT



The most common parallel resonant circuit is a coil a in parallel with a capacitor as shown in figure.

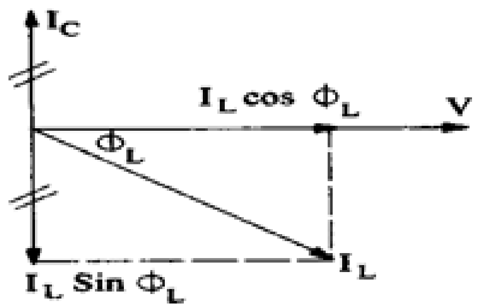
The parallel resonant circuit will resonate when:

1. Power factor =1.
2. Net reactance of the circuit =0.
3. Current is minimum
4. Impedance is maximum

Phasor diagram:

The coil current I_L has two components

a. The reactive component ($I_L sin\varphi_L$)
b. The active component ($I_L cos\varphi_L$)
   Where φ_L is the power factor of the coil ($\varphi_L = cos^{-1}\frac{R}{Z_L}$)



At resonance :

The circuit is resistive that is current and voltage is in phase

Therefore; $I_C - I_L \sin \Phi_L = 0$

$I_L \sin \Phi_L = I_C$

Now $I_L = \frac{V}{Z_L}$

$\sin \Phi_L = \frac{X_L}{Z_L}$

& $I_C = \frac{V}{X_C}$

Hence, **condition for resonance becomes** : $I_L \sin \Phi_L = I_C$

$$\frac{V}{Z_L}\frac{X_L}{Z_L} = \frac{V}{X_C}$$

Impedance of the parallel resonant circuit at resonance is **$Z_L{}^2 = X_L / X_C$**

Resonant frequency of a parallel resonant circuit:

Since, $X_L = \omega L$; $X_C = \dfrac{1}{wC}$

Therefore, $\dfrac{wL}{wC} = Z_L{}^2$

$$\frac{L}{C} = Z_L{}^2 = R^2 + X_L{}^2$$

$$\frac{L}{C} = Z_L{}^2 = R^2 + (2\pi f_0)^2 L^2$$

So $(2\pi f_0)^2 L^2 = \dfrac{L}{C} - R^2$

Or $2\pi f_0 = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$ Or $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

This is the resonant frequency and is given in Hz if R is in Ohm; L is in Henry & C in farad.

If R is negligible then $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

<u>Current at resonance</u>:

At resonance:

$I_C = I_L \sin \Phi_L$

The two out of phase component of current cancel eachother and hence the current flowing through the circuit at resonance is given as:

$I = I_L \cos \Phi_L$

Where, $I_L = \frac{V}{Z_L}$ and $\cos \Phi_L = R / Z_L$

Therefore substituting the values of I_L and $\cos \Phi_L$ in :

$I = I_L \cos \Phi_L$

$I = (V/Z_L) . (R/Z_L)$

$I = VR/Z_L{}^2$

Since at resonance : $Z_L{}^2 = \dfrac{L}{C}$

Therefore substituting $Z_L{}^2 = \dfrac{L}{C}$

we get $I = \dfrac{VR}{L/C} = \dfrac{V}{L/CR}$

Dynamic impedance of parallel resonant circuit: $Z_r = \frac{L}{CR}\ \Omega$

The denominator L/CR is known as the equivalent or **dynamic impedance** of the parallel circuit at resonance. It should be noted that impedance is resistive only.

Since current is minimum at resonance, L/CR must therefore; represent the maximum impedance of the circuit.

In fact parallel resonance is a condition of maximum impedance or minimum admittance. Current at resonance is minimum.

<u>Q-Factor of a parallel circuit:</u>

At parallel resonance, the current circulating between the two branches (I_L or I_c) is many times greater than the line current

This current amplification produced at resonance in parallel resonant circuit is called **CURRENT MAGNIFICATION FACTOR** or **Q factor**.

Q factor is defined as the ratio of the current circulating between its two branches to the line current drawn from the supply.

$$Q\ factor = \frac{I_L\ or\ I_C}{I}$$

Q-factor = I_C/I ; $where, I_C = \dfrac{V}{X_C} = wCV$ & at resonance , $I = \dfrac{V}{L/CR}$

Therefore, Q = $\dfrac{I_c}{I} = \dfrac{wCV}{\dfrac{V}{L/CR}} = \dfrac{wL}{R} = \dfrac{2\pi foL}{R} = \tan\phi$

Where Φ is the power factor angle of the coil.

Where f_r is the resonant frequency given as : $f_o = \frac{1}{2\pi\sqrt{LC}}$

Therefore, Q = $\dfrac{2\pi f_0 L}{R} = \dfrac{2\pi L}{R} \cdot \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

It should be noted that in series circuit, Q-factor gives the voltage magnification, where as in parallel circuits, it gives the current magnification.

Point to be Remembers:-

At resonance

(1) Net susceptance =0

(2)Reactive or wattles component of line current is zero.

(3) Dynamic impedance = $\dfrac{L}{CR}$ Ohm.

(4) Line current at resonance is minimum , $I_0 = \dfrac{V}{L/CR}$ in phase with applied voltage.

(5) Power factor of the circuit is unity.

Comparison of series & parallel resonance:-

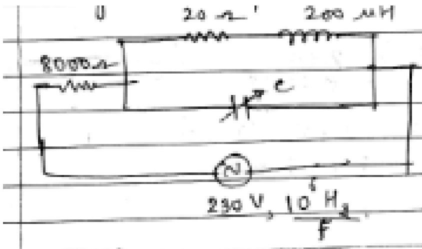
Item	Series Circuit (R-L-C)	Parallel circuit (R-L with C)
(1) Impedance at resonance	Minimum	Maximum
(2) Current at resonance	Maximum = $\frac{V}{R}$	Minimum = $\dfrac{V}{L/CR}$
(3) Effective impedance	R	L/CR
(4) Power factor at resonance	Unity	Unity
(5) Resonant frequency	$\dfrac{1}{2\pi\sqrt{LC}}$	$\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{L^2}}$
(6) It magnifies	Voltage	Current
(7) Magnification	wL/R	wL/R

Numericals on Parallel Resonance

Q.1)

A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit. Calculate (a) the value of capacitance at resosnance (b) Q factor of the circuit (c) dynamic impedance of the cirucit (d) total circuit current.

Solution



$C_0 = ?$

$Q = ?$

$Z_{dynamic} = ?$

$I_{Total} = ?$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Here R = R of resist. of coil.

$$10^6 = \frac{1}{2 \times 3.14}\sqrt{\frac{1}{200\times10^{-6}C} - \frac{(20)^2}{(200\times10^{-6})^2}}$$

$$C = 126.65 \times 10^{-12}\,F = 126.65\,pF$$

$$\boxed{Q_0 = \frac{\omega L}{R}}$$

$$Q_0 = \frac{2\pi f L}{R}$$

$$= \frac{2 \times 3.14 \times 10^6 \times 200 \times 10^{-6}}{20}$$

$$= 62.83$$

Dynamic impedance, $Z = \frac{L}{CR}$

$$Z = \frac{200\times10^{-6}}{126.65\times10^{-12} \times 20}$$

$$= 78.958\,k\Omega$$

Total equivalent impedance of the ckt at resonance

$$= 78958 + 8000$$

$$= 86958\,\Omega$$

Total ckt current $= \frac{230}{86958} = \frac{V}{R}$

$$= 2.645 \times 10^{-3}\,A$$

$$= 2.645\,mA.$$

Q.2)

A coil of 20 Ω resistance has an inductance of 0.2 H is connected in parallel wi condenser of 100 μF capacitance. Calculate the frequency at which this circuit behave as a non inductive resistance. Find also the value of dynamic resistance.

Data : R = 20 Ω L = 0.2 H

C = 100 μF

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 100 \times 10^{-6}} - \left(\frac{20}{0.2}\right)^2}$$

$$= 31.83 \text{ Hz}$$

$$\text{Dynamic resistance} = \frac{L}{CR}$$

$$= \frac{0.2}{100 \times 10^{-6} \times 20} = 100\ \Omega$$

Q.3)

A coil takes a current of 1 A at 0.3 pf when connected to a 100 V, 50 Hz supply. Determine the value of the capacitance, which when connected in parallel with the coil, will reduce the line current to a minimum. Calculate the impedance of the parallel circuit at 50 Hz.

Solution

When a coil is connected to 100 V, 50 Hz supply, it takes a current of 1 A at 0.3 pf

$(pf)_{coil} = 0.3$

or $\cos \phi_L = 0.3$

or $\phi_L = 72.542$ (ϕ_L is the phase angle of the coil)

Now, capacitor is connected across the coil

We need a value of C, so that the line current reduces to a minimum value, i.e., parallel resonance occurs at $f = 50$ Hz.

At resonance, $I_C = I_L \sin \phi_L$

So, $I_C = 1 \times \sin(72.542)$

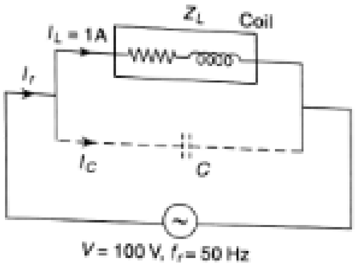
or $I_C = 0.954$ A

Now, $X_C = \frac{V}{I_C} = \frac{100}{0.954} = 104.82\ \Omega$

So, $\frac{1}{2\pi f_r C} = 104.82$

or $\frac{1}{2\pi \times 50 \times C} = 104.82$

or $C = 30.37 \times 10^{-6}$ F
$= 30.37$ μF



At resonance,

$I_r = I_L \cos \phi_L$

or $I_r = 1 \times \cos 72.542$

or $I_r = 0.3$ A

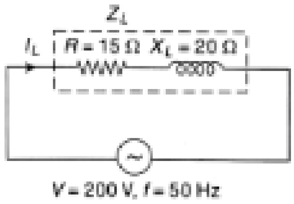
Dynamic impedance of the circuit,

$$Z_r = \frac{V}{I_r} = \frac{100}{0.3} = 333.33\ \Omega$$

Q.4)

A circuit has $X_L = 20\ \Omega$ at 50 Hz, its resistance being 15 Ω. For an applied voltage of 200 V at 50 Hz, calculate (i) the pf, (ii) the current, (iii) the value of shunting capacitance to bring the resultant current into phase with the applied voltage, and (iv) the resultant current in case (iii).

Solution



Impedance of the circuit, $Z_L = \sqrt{(15)^2 + (20)^2}$

$= 25\ \Omega$

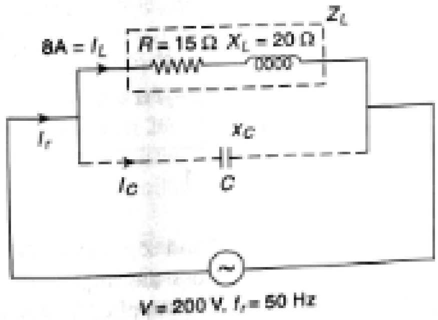
(i) $(\text{pf})_{coil} = \cos\phi_L = \dfrac{R}{Z_L} = \dfrac{15}{25}$

$= 0.6$ lagging

So, $\phi_L = \cos^{-1} 0.6 = 53.15$

(ii) Current, $I_L = \dfrac{V}{Z_L} = \dfrac{200}{25} = 8$ A

(iii) Now, capacitance is connected in parallel with the above circuit



We need a value of C, so that resulting current comes in phase with the applied voltage, i.e., pf = 1 (unity). In other words, parallel resonance occurs at $f = 50$ Hz.

At resonance,

$I_C = I_L \sin\phi_L$

So, $I_C = 8 \times \sin(53.13)$

or $I_C = 6.4$ A

Now, $X_C = \dfrac{V}{I_C} = \dfrac{200}{6.4} = 31.25\ \Omega$

So, $\dfrac{1}{2\pi f_r C} = 31.25$

or $\dfrac{1}{2\pi \times 50 \times C} = 31.25$

So, $C = \dfrac{1}{2\pi \times 50 \times 31.25}$

or $C = 101.859 \times 10^{-6}$ F

$= 101.859\ \mu$F

(iv) The circuit current at resonance,

$I_r = I_L \cos\phi_L$

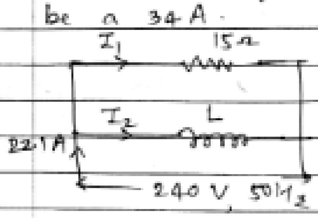
or $I_r = 8 \times 0.6$

or $I_r = 4.8$ A

PARALLEL AC CIRCUIT

Q.1)

When a 240 V, 50 Hz supply is fed to a 15 Ω resistor in parallel with an inductor, the total current is 22.1 A. What value must the freq. have for the total current be a 34 A.



V = 240 V

R = 15 Ω

I = 22.1 A

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{240\angle 0^\circ}{15\angle 0^\circ} = 16\angle 0^\circ \xrightarrow{\text{rect}} = 16$$

$$= 16 \text{ A}.$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{\bar{V}}{X_L\angle\phi} = \frac{240\angle 0^\circ}{X_L\angle 90^\circ}$$

$$= \frac{240}{X_L}\angle 0-90^\circ \xrightarrow{\text{rect.}} = -j\frac{240}{X_L} \text{ A}.$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

$$\bar{I} = 16 - j\frac{240}{X_L} = 22.1$$

$$\therefore \sqrt{(16)^2 + \left(\frac{240}{X_L}\right)^2} = 22.1$$

$$256 + \frac{57600}{X_L^2} = 488.41$$

$$X_L = 15.74\ \Omega$$

$$X_L = 2\pi f L$$

$$\therefore L = \frac{X_L}{2\pi f} = \frac{15.74}{2\times 3.14\times 50}$$

$$L = 0.05 H$$

Let the new freq. be f, then

$$\sqrt{(16)^2 + \left(\frac{240}{2\pi f \times 0.05}\right)^2} = 34$$

Squaring both sides

$$256 + \frac{57600}{0.0987 f^2} = 1156$$

$$f = 25.47 \text{ Hz}$$

Q.2)

A resistance of 20 Ω and a pure coil of inductance 31.8 mH are connected in parallel across 230 V, 50 Hz supply. Find (i) the line current, (ii) power factor, and (iii) power consumed by the circuit.

Solution

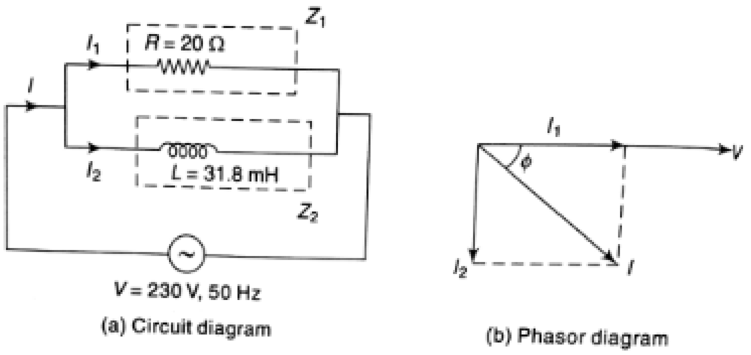


Fig. 2.123

Given: $V = 230$ V

$f = 50$ Hz

$L = 31.8 \times 10^{-3}$ H $\Rightarrow$ $X_L = 2\pi fL = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Branch current, $I_1 = \dfrac{V}{R} = \dfrac{230}{20} = 11.5$ A

The current I_1 is in phase with the applied voltage.

Branch current, $I_2 = \dfrac{V}{X_L} = \dfrac{230}{10} = 23$ A

The current I_2 lags behind the applied voltage by 90°.

So, line current, $I = \sqrt{I_1^2 + I_2^2}$

$= \sqrt{(11.5)^2 + (23)^2}$

$= 25.71$ A

From phasor diagram,

$$\text{pf} = \cos\phi = \frac{I_1}{I} = \frac{11.5}{25.71} = 0.447 \text{ lag}$$

Q.3)

An inductive coil of resistance 20 Ω and inductance 0.2 H is connected in parallel with 200 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with supply voltage. Also find the value of this current. Draw the phasor diagram.

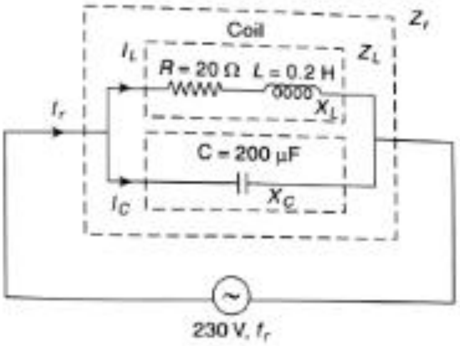


Fig. 2.165

Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ Hz}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 200 \times 10^{-6}} - \frac{20^2}{(0.2)^2}}$$

$$= \frac{1}{2\pi}\sqrt{25000 - 10000}$$

$$= \frac{1}{2\pi}\sqrt{15000}$$

$$= 19.49 \text{ Hz}$$

Dynamic impedance of the circuit,

$$Z_r = \frac{L}{CR}$$

$$= \frac{0.2}{200 \times 10^{-6} \times 20}$$

$$= 50\ \Omega$$

Circuit current at resonance,

$$I_r = \frac{V}{Z_r} = \frac{230}{50} = 4.6 \text{ A}$$

For phasor diagram, we need to calculate the values of branch currents (I_L and I_C) and phase angle of the coil (ϕ_L).

Now, $$I_L = \frac{V}{Z_L} = \frac{230}{\sqrt{(20)^2 + (2\pi \times 19.49 \times 0.2)^2}} = \frac{230}{31.62} = 7.274 \text{ A}$$

$$I_C = \frac{V}{X_C} = \frac{230}{\left(\frac{1}{2\pi \times 19.49 \times 200 \times 10^{-6}}\right)} = \frac{230}{40.83} = 5.63 \text{ A}$$

Phase angle of the coil,

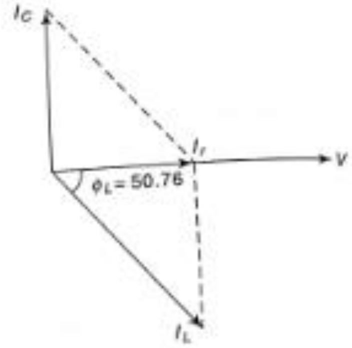
$$\phi_L = \tan^{-1}\frac{X_L}{R}$$

$$= \tan^{-1}\frac{(2\pi f_r L)}{R}$$

$$= \tan^{-1}\frac{(2\pi \times 19.49 \times 0.2)}{20}$$

$$= \tan^{-1}\frac{24.49}{20}$$

So, $\phi_L = 50.76$



Take V as reference phasor.

We know that $\bar{I}_r = \bar{I}_L + \bar{I}_C$.

Now, $I_L = 7.274$ A

$I_C = 5.63$ A

$I_r = 4.6$ A

$\phi_L = 50.76°$