Assignment 1
Operation Research
BTech Sem IV (20-21 Batch)

Submission on 30 January 2021

A) Solve the two problems discussed in class on Regional Planning and Scheduling problem

B) Solve the problems as given below:

1. The Metalco Company desires to blend a new alloy of 40 percent tin, 35 percent zinc, and 25 percent lead from several available alloys having the following properties: (3.4.14)

	Alloy				
Property	**1**	**2**	**3**	**4**	**5**
Percentage of tin	60	25	45	20	50
Percentage of zinc	10	15	45	50	40
Percentage of lead	30	60	10	30	10
Cost ($/lb)	77	70	88	84	94

The objective is to determine the proportions of these alloys that should be blended to produce the new alloy at a minimum cost.

(a) Formulate a linear programming model for this problem.

(b) Solve this model by the simplex method.

3.4-14.

(a) Let x_i be the amount of Alloy i used for $i = 1, 2, 3, 4, 5$.

$$\begin{array}{lll} \text{mi} \quad \text{nimize} & C = & 77x_1 + 70x_2 + 88x_3 + 84x_4 + 94x_5 \\ \text{subject} \quad \text{to} & & 60x_1 + 25x_2 + 45x_3 + 20x_4 + 50x_5 = 40 \\ & & 10x_1 + 15x_2 + 45x_3 + 50x_4 + 40x_5 = 35 \\ & & 30x_1 + 60x_2 + 10x_3 + 30x_4 + 10x_5 = 25 \\ & & x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ \text{and} & & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

(b)

	Contribution Toward Required Amount							Required
Requirement	Alloy 1	Alloy 2	Alloy 3	Alloy 4	Alloy 5	Total		Amount
% tin	60	25	45	20	50	40	=	40
% zinc	10	15	45	50	45	35	=	35
% lead	30	60	10	30	10	25	=	25
%total	1	1	1	1	1	1	=	1
Unit Cost	$77	$70	$88	$84	$94	$82.43		
Solution	0.0435	0.2826	0.6739	0	0			

2. A cargo plane has three compartments for storing cargo: front, center, and back. These compartments have capacity limits on both *weight* and *space*, as summarized below: (3.4.15)

Compartment	**Weight Capacity (Tons)**	**Space Capacity (Cubic Feet)**
Front	12	7,000
Center	18	9,000
Back	10	5,000

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.
The following four cargoes have been offered for shipment on an upcoming flight as space is available:
Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight.
(a) Formulate a linear programming model for this problem.
(b) Solve this model by the simplex method to find one of its multiple optimal solutions.

3.4-15.

(a) Let x_{ij} be the number of tons of cargo type $i = 1, 2, 3, 4$ stowed in compartment $j =$ F (front), C (center), B (back).

maximize $P = 320(x_{1F} + x_{1C} + x_{1B}) + 400(x_{2F} + x_{2C} + x_{2B}) + 360(x_{3F} + x_{3C} + x_{3B}) + 290(x_{4F} + x_{4C} + x_{4B})$

subject to

$$x_{1F} + x_{2F} + x_{3F} + x_{4F} \leq 12$$
$$x_{1C} + x_{2C} + x_{3C} + x_{4C} \leq 18$$
$$x_{1B} + x_{2B} + x_{3B} + x_{4B} \leq 10$$
$$x_{1F} + x_{1C} + x_{1B} \leq 20$$
$$x_{2F} + x_{2C} + x_{2B} \leq 16$$
$$x_{3F} + x_{3C} + x_{3B} \leq 25$$
$$x_{4F} + x_{4C} + x_{4B} \leq 13$$
$$500x_{1F} + 700x_{2F} + 600x_{3F} + 400x_{4F} \leq 7,000$$
$$500x_{1C} + 700x_{2C} + 600x_{3C} + 400x_{4C} \leq 9,000$$
$$500x_{1B} + 700x_{2B} + 600x_{3B} + 400x_{4B} \leq 5,000$$
$$\tfrac{1}{12}(x_{1F} + x_{2F} + x_{3F} + x_{4F}) - \tfrac{1}{18}(x_{1C} + x_{2C} + x_{3C} + x_{4C}) = 0$$
$$\tfrac{1}{12}(x_{1F} + x_{2F} + x_{3F} + x_{4F}) - \tfrac{1}{10}(x_{1B} + x_{2B} + x_{3B} + x_{4B}) = 0$$

and

$$x_{1F}, x_{2F}, x_{3F}, x_{4F}, x_{1C}, x_{2C}, x_{3C}, x_{4C}, x_{1B}, x_{2B}, x_{3B}, x_{4B} \geq 0$$

(b)

Resource Usage Per Unit of Each Activity

Resource	1F	1C	1B	2F	2C	2B	3F	3C	3B	4F	4C	4B	Totals		Resource Available
Front Wt.	1	0	0	1	0	0	1	0	0	1	0	0	12	≤	12
Center Wt.	0	1	0	0	1	0	0	1	0	0	1	0	18	≤	18
Back Wt.	0	0	1	0	0	1	0	0	1	0	0	1	10	≤	10
Cargo 1 Wt.	1	1	1	0	0	0	0	0	0	0	0	0	15	≤	20
Cargo 2 Wt.	0	0	0	1	1	1	0	0	0	0	0	0	12	≤	16
Cargo 3 Wt.	0	0	0	0	0	0	1	1	1	0	0	0	0	≤	25
Cargo 4 Wt.	0	0	0	0	0	0	0	0	0	1	1	1	13	≤	13
Space Front	500	0	0	700	0	0	600	0	0	400	0	0	7000	≤	7000
Space Center	0	500	0	0	700	0	0	600	0	0	400	0	9000	≤	9000
Space Back	0	0	500	0	0	700	0	0	600	0	0	400	5000	≤	5000

Contribution Toward Required Amount

Requirement	1F	1C	1B	2F	2C	2B	3F	3C	3B	4F	4C	4B	Totals		Required Amount
%F=%C	0.0833	-0.0556	0	0.0833	-0.0556	0	0.0833	-0.0556	0	0.08333	-0.0556	0	0	=	0
%F=%B	0.0833	0	-0.1	0.0833	0	-0.1	0.0833	0	-0.1	0.0833	0	-0.1	0	=	0
Unit Profit	320	320	320	400	400	400	360	360	360	290	290	290	$ 13,330		
Solution	**0**	**5**	**10**	**7.33333**	**4.167**	**0.000**	**0**	**0**	**0**	**4.66667**	**8.333**	**0.000**			

Cargo	Weight (Tons)	Volume (Cubic Feet/Ton)	Profit ($/Ton)
1	20	500	320
2	16	700	400
3	25	600	360
4	13	400	290

3. You are given the following data for a linear programming problem where the objective is to maximize the profit from allocating three resources to two nonnegative activities. (3.5.2)

Resource	Resource Usage per Unit of Each Activity		Amount of Resource Available
	Activity 1	Activity 2	
1	2	1	10
2	3	3	20
3	2	4	20
Contribution per unit	\$20	\$30	

Contribution per unit = profit per unit of the activity.

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model.

(c) Display the model on an Excel spreadsheet.

(d) Check the following solutions: ($x1$, $x2$) = (2, 2), (3, 3), (2, 4), (4, 2), (3, 4), (4, 3). Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?

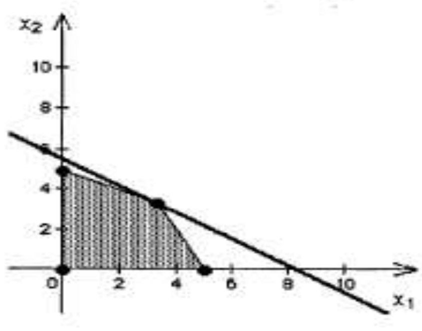
(e) Use the QM to solve the model by the simplex method.

(a) maximize $P = 20x_1 + 30x_2$

subject to

$$
\begin{aligned}
2x_1 + x_2 &\le 10 \\
3x_1 + 3x_2 &\le 20 \\
2x_1 + 4x_2 &\le 20 \\
x_1, x_2 &\ge 0
\end{aligned}
$$

(b) Optimal Solution: $(x_1^*, x_2^*) = \left(3\frac{1}{3}, 3\frac{1}{3}\right)$ and $P^* = 166.67$



(c) - (e)

Resource	Resource Usage Per Unit of Each Activity		Totals		Resource Available
	Activity 1	Activity 2			
1	2	1	10	≤	10
2	3	3	20	≤	20
3	2	4	20	≤	20
Unit Profit	20	30	\$ 166.67		
Solution	**3.333**	**3.333**			

(d)

(x_1, x_2)	Feasible?	P
$(2, 2)$	Yes	\$100
$(3, 3)$	Yes	\$150
$(2, 4)$	Yes	\$160 Best
$(4, 2)$	Yes	\$140
$(3, 4)$	No	
$(4, 3)$	No	

4. Ed Butler is the production manager for the Bilco Corporation which produces three types of spare parts for automobiles. The manufacture of each part requires processing on each of two machines, with the following processing times (in hours): (3.5.3)

Machine	Part A	Part B	Part C
1	0.02	0.03	0.05
2	0.05	0.02	0.04

Each machine is available 40 hours per month. Each part manufactured will yield a unit profit as follows:

	Part A	Part B	Part C
Profit	$300	$250	$200

Ed wants to determine the mix of spare parts to produce in order to maximize total profit.

(a) Formulate a linear programming model for this problem.

(b) Display the model on the QM

(c) Make three guesses of your own choosing for the optimal solution.

Use the Software to check each one for feasibility and, if feasible, to find the value of the objective function.

(a) maximize $P = 300A + 250B + 200C$

subject to

$0.02A + 0.03B + 0.05C \leq 40$

$0.05A + 0.02B + 0.04C \leq 40$

and $A, B, C \geq 0$

(b)

Resource	Resource Usage Per Unit of Each Activity: Part A	Part B	Part C	Total		Resource Available
Machine 1	0.02	0.03	0.05		≤	40
Machine 2	0.05	0.02	0.04		≤	40
Unit Profit	$300	$250	$200			
Solution						

(c) Many answers are possible.

(A, B, C)	Feasible?	P
(500, 500, 300)	No	
(350, 1000, 0)	Yes	$355,000
(400, 1000, 0)	Yes	$370,000 Best

(d)

Resource	Resource Usage Per Unit of Each Activity: Part A	Part B	Part C	Total		Resource Available
Machine 1	0.02	0.03	0.05	40	≤	40
Machine 2	0.05	0.02	0.04	40	≤	40
Unit Profit	$300	$250	$200	381818		
Solution	363.6363636	1090.909091	0			

5. Joyce and Marvin run a day care for preschoolers. They are trying to decide what to feed the children for lunches.
They would like to keep their costs down, but also need to meet the nutritional requirements of the children. They have already decided to go with peanut butter and jelly sandwiches, and some combination of graham crackers, milk, and orange juice. The nutritional content of each food choice and its cost are given in the table below. (3.4.17)

Food Item	Calories from Fat	Total Calories	Vitamin C (mg)	Protein (g)	Cost (¢)
Bread (1 slice)	10	70	0	3	5
Peanut butter (1 tbsp)	75	100	0	4	4
Strawberry jelly (1 tbsp)	0	50	3	0	7
Graham cracker (1 cracker)	20	60	0	1	8
Milk (1 cup)	70	150	2	8	15
Juice (1 cup)	0	100	120	1	35

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice).
Joyce and Marvin would like to select the food choices for each child which minimize cost while meeting the above requirements.

(a) Formulate a linear programming model for this problem.
(b) Solve this model by the simplex method (solve using QM).

(a) Let B = slices of bread, P = tablespoons of peanut butter, S = tablespoons of strawberry jelly, G = graham crackers, M = cups of milk, and J = cups of juice.

$$
\begin{array}{ll}
\text{minimize} & C = 5B + 4P + 7S + 8G + 15M + 35J \\
\text{subject to} & 70B + 100P + 50S + 60G + 150M + 100J \geq 400 \\
& 70B + 100P + 50S + 60G + 150M + 100J \leq 600 \\
& 10B + 75P + 20G + 70M \leq 0.3(70B + 100P + 50S + 60G + 150M + 100J) \\
& 3S + 2M + 120J \geq 60 \\
& 3B + 4P + G + 8M + J \geq 12 \\
& B = 2 \\
& P \geq 2S \\
& M + J \geq 1 \\
\text{and} & B, P, S, G, M, J \geq 0
\end{array}
$$

(b)

	Resource Usage Per Unit of Each Activity								Resource
Resource	Bread	PB	Jelly	Crackers	Milk	Juice	Total		Available
Min Calories	70	100	50	60	150	100	400	≥	400
Max Calories	70	100	50	60	150	100	400	≤	600
Fat	-11	45	-15	2	25	-30	0	≤	0
Vitamin C	0	0	3	0	2	120	60	≥	60
Protein	3	4	0	1	8	1	13.95	≥	12
Bread	1	0	0	0	0	0	2	=	2
PB&J	0	1	-2	0	0	0	0	≥	0
Liquid	0	0	0	0	1	1	1	≥	1
Unit Cost	5	4	7	8	15	35	47.31		
Solution	2	0.57	0.29	1.04	0.52	0.48			