

Principle Component Analysis (PCA)

Construct a sample Covariance matrix S for the data given below

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 2 & 4 & 5 & 1 \end{pmatrix}$$

Find the eigen vector of S that points the most significant direction of the data

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 2 & 4 & 5 & 1 \end{pmatrix}$$

PCA

Ang of row 1 = 2

Ang of row 2 = 3

Centred matrix $A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 1 & 2 & -2 \end{pmatrix}$

Covariance matrix $S = \frac{AA^T}{n-1} = \frac{AA^T}{3}$ ($n = \text{no. of columns}$)

$$AA^T = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 1 & 2 & -2 \end{pmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 10 \end{bmatrix}$$

$$S = \frac{AA^T}{3} = \begin{pmatrix} 2/3 & 2/3 \\ 2/3 & 10/3 \end{pmatrix}$$

Eigen values $\begin{pmatrix} 2/3 - \lambda & 2/3 \\ 2/3 & 10/3 - \lambda \end{pmatrix} = (2/3 - \lambda)(10/3 - \lambda) - \frac{4}{9}$
 $= \lambda^2 - 4\lambda + 1.77 = 0.$

$$\lambda = 3.49, 0.506.$$

To find eigen vector corresponding to $\lambda = 3.49$.

$$-2.82x + 0.666y = 0.$$

$$x = \frac{0.666y}{2.82}$$

For $y=1$, $x = 0.236$.

normalising. $(0.236, 1)$ (dividing by 1.0274)

we get the direction as $(0.2297, 0.9733)$

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PCA

$$\begin{pmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \end{pmatrix}$$

$$\text{Avg of row 1} = 11$$

$$\text{Avg of row 2} = 6.25$$

$$\text{Centred matrix } A = \begin{pmatrix} 2 & -2 & -4 & 4 \\ 1.75 & 0.75 & 2.25 & -0.25 \end{pmatrix}$$

$$\text{Covariance matrix } S = \frac{AA^T}{n-1} = \frac{AA^T}{3}$$

$$AA^T = \begin{pmatrix} 2 & -2 & -4 & 4 \\ 1.75 & 0.75 & 2.25 & -0.25 \end{pmatrix} \begin{bmatrix} 2 & 1.75 \\ -2 & 0.75 \\ -4 & 2.25 \\ 4 & -0.25 \end{bmatrix}$$

$$S = \frac{AA^T}{3} = \frac{1}{3} \begin{pmatrix} 40 & -8 \\ -8 & 8.75 \end{pmatrix} = \begin{pmatrix} 13.33 & -2.66 \\ -2.66 & 2.916 \end{pmatrix}$$

Eigen values of S

$$\begin{pmatrix} 13.33 - \lambda & -2.66 \\ -2.66 & 2.916 - \lambda \end{pmatrix} = (13.33 - \lambda)(2.916 - \lambda) - 7.0756$$
$$= 38.870 - 16.246\lambda + \lambda^2 - 7.0756$$
$$= \lambda^2 - 16.246\lambda + 31.7944$$

$$\lambda = 13.97, 2.275$$

Eigen vector corresponding to larger eigenvalue $\lambda = 13.97$

$$-0.64x - 2.66y = 0$$

$$x = -\frac{2.66}{0.64}y$$

$$\text{For } y = 1, \quad x = -4.156$$

normalising. $(-4.156, 1)$ (dividing by 4.2746)
we get the direction as $(-0.97, 0.23)$