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C.S

M.V.N Assignment

Q1. $\mu = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 5 & 3 \\ 2 & 3 & -4 \end{bmatrix}$

$$AX = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ x_1 - 2x_3 \end{bmatrix}$$

$$A\mu = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$A\Sigma A' = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -1 & 5 & 3 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 1 \\ 1 & -23 \end{bmatrix}$$

Q2. $\mu = \begin{bmatrix} 9 \\ 11 \end{bmatrix}$ $\quad \Sigma = \begin{bmatrix} -1 & 3 \\ 3 & -2 \end{bmatrix}$ $\quad Y = X_1 - 4X_2$

Let, $a' = (1 \ -4)'$, then, $a'X = X_1 - 4X_2$

$$a'X \sim N_p(a'\mu, a'\Sigma a)$$

$$a'\mu = [1 \ \ -4] \begin{bmatrix} 9 \\ 11 \end{bmatrix} = -35$$

$$a'\Sigma a = [1 \ -4] \begin{bmatrix} -1 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \end{bmatrix} = -57$$

$\therefore$ Distribution is $\sim N(-35, -57)$

Q3. $\mu = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$ $\quad \Sigma = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -5 \\ 3 & -5 & 6 \end{bmatrix}$

For, $2X_1 + 3X_2 - X_3 + X_4$

comparing it with $c_1X_1 + c_2X_2 + c_3X_3 + c_4X_4$

$c_1 = 2$; $c_2 = 3$; $c_3 = -1$; $c_4 = 1$

Mean, vector $= (c_1 + c_2 + c_3 + c_4)\,\mu$ $= \begin{bmatrix} -15 \\ -5 \\ 5 \end{bmatrix}$

Covariance Matrix $= (c_1^2 + c_2^2 + c_3^2 + c_4^2)\,\Sigma$ $= \begin{bmatrix} 15 & -30 & 45 \\ -30 & 60 & -75 \\ 45 & -75 & 90 \end{bmatrix}$

For $X_1 - 3X_2 + 5X_3 - 2X_4$

Comparing with $b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$

$b_1 = 1$; $b_2 = -3$; $b_3 = 5$; $b_4 = -2$

$$\text{Mean Vector} = (b_1 + b_2 + b_3 + b_4)\mu = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{Covariance Matrix} = (b_1^2 + b_2^2 + b_3^2 + b_4^2)\Sigma = \begin{bmatrix} 39 & -78 & 117 \\ -78 & 156 & -195 \\ 117 & -195 & 234 \end{bmatrix}$$

Covariance between them

$$= (b_1c_1 + b_2c_2 + b_3c_3 + b_4c_4)\Sigma = \begin{bmatrix} -14 & 28 & -42 \\ 28 & -56 & 70 \\ -42 & 70 & -84 \end{bmatrix}$$

$$\text{Joint Density} = \begin{bmatrix} (\Sigma c_i^2)\Sigma & (b^T c)\Sigma \\ (b^T c)\Sigma & (\Sigma b_i^2)\Sigma \end{bmatrix}$$

$$= \begin{bmatrix} 15\Sigma & -14\Sigma \\ -14\Sigma & 39\Sigma \end{bmatrix}$$

~~$\text{Mean} = \mu_1 + \frac{S_{12}}{S_{22}}(x_2 - \mu_2)$~~

Q5. $\mu = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -2 & 0 \\ 2 & 0 & -3 \end{bmatrix}$

1. $\delta_{12} = \delta_{21} = 0$. Hence, independent

2. $\delta_{13} = \delta_{31} = 2$. Hence, X_1 & X_3 are not independent

3. $\delta_{23} = \delta_{32} = 0$. Hence, X_2 & X_3 are independent

4. $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & -2 & 0 \\ \hline 2 & 0 & -3 \end{array}\right]$ $\because$ elements are not 0
$\therefore$ (X_1, X_2) & X_3 are not independent.

5. $\left[\begin{array}{c|cc} 1 & 0 & 2 \\ \hline 0 & -2 & 0 \\ 2 & 0 & -3 \end{array}\right]$ $\because$ elements are not 0
$\therefore$ (X_2, X_3) & X are not independent.

6. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 1 \end{bmatrix}$ $AX = \begin{bmatrix} X_1 \\ 2X_1 - 3X_2 + X_3 \end{bmatrix}$ $AX \sim N(A\mu, A\Sigma A^T)$

$A\Sigma A^T = \begin{bmatrix} 1 & 4 \\ 4 & -9 \end{bmatrix}$

$\therefore$ Its clear that X_1 & $X_1 - 3X_2 + X_3$ are not independent.

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Q6 $\mu = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -2 & 0 \\ 2 & 0 & -3 \end{bmatrix}$

1. $\sigma_{12} = \sigma_{21} = 0$. Hence, X_1 & X_2 are independent.

2. $\sigma_{13} = \sigma_{31} = 2$. Hence, X_1 & X_3 are not independent.

3. $\sigma_{23} = \sigma_{32} = 0$. Hence, X_2 & X_3 are independent.

4. $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & -2 & 0 \\ \hline 2 & 0 & -3 \end{array}\right]$ $\because$ Elements are not 0
$\therefore$ (X_1, X_2) & X_3 are not independent

5. $\left[\begin{array}{c|cc} 1 & 0 & 2 \\ \hline 0 & -2 & 0 \\ 2 & 0 & -3 \end{array}\right]$ $\because$ Elements are not 0
$\therefore$ X_1 & (X_2, X_3) are not independent.

6. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ $AX = \begin{bmatrix} X_1 \\ X_1 - 2X_2 + 3X_3 \end{bmatrix}$ $AX \sim N(A\mu, A\Sigma A^T)$

$A\Sigma A^T = \begin{bmatrix} 1 & 7 \\ 7 & -22 \end{bmatrix}$

$\therefore$ Its clear that X_1 & $X_1 - 2X_2 + 3X_3$ are not independent.

Q7. $\mu = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & 2 \\ -1 & 2 & -1 \end{bmatrix}$

3. $Y = 2X_1 + 3X_2 - X_3$

$\mu_* = 2\mu_{x_1} + 3\mu_{x_2} + -\mu_{x_3}$

$= 2(-1) + 3(2) - 1(3)$

$= 1$

$\delta_*^2 = [2 \quad 3 \quad -1] \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & 2 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = 1$

4. $P(Y > 56) = P\left(Z > \frac{56-1}{\sqrt{1}}\right)$

$= P(Z > 56) = \underline{\underline{1}}$

1. $P(X_1 > 5) = P\left(Z > \frac{5-(-1)}{1}\right) = P(Z > 6) = 1 - \phi(6)$

$= \underline{\underline{1}}$

2. $P(X_2 > 9) = P\left(Z > \frac{9-(2)}{\sqrt{-2}}\right) = P(Z > -3.5) = \underline{\underline{0.0002}}$

Q8. $\mu = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

1. $P(X_1 > 7) = P\left(Z > \frac{7-(-1)}{1}\right) = P(Z>8)$
$= \underline{\underline{1}}$

2. $P(X_2 > 11) = P\left(Z > \frac{11-2}{1}\right) = P(Z>9)$
$= \underline{\underline{1}}$

3. $Y = X_1 + X_2 - 2X_3)$
$\mu_{Y} = \mu_{X_1} + \mu_{X_2} - 2\mu_{X_3}$
$= -1 + 2 - 2(1)$
$= -1$

$$\delta_Y^2 = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = 6$$

4. $P(Y > 49) = P\left(Z > \frac{49-(-1)}{\sqrt{6}}\right) = P(Z > 20.41)$
$= \underline{\underline{1}}$

Q4. $\mu = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & 5 & 2 \\ 5 & -2 & -1 \\ 2 & -1 & -3 \end{bmatrix}$

For, $4X_1 + 3X_2 - 2X_3 + X_4$
Comparing it with $C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4$
$C_1 = 4;\ C_2 = 3;\ C_3 = -2;\ C_4 = 1$

$$\text{Mean Vector}, \cancel{\mu} = [C_1+C_2+C_3+C_4]\,\mu = \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$

$$\text{CoVariance Matrix}, \cancel{\Sigma} = [C_1^2+C_2^2+C_3^2+C_4^2]\,\Sigma = \begin{bmatrix} 30 & 150 & 60 \\ 150 & -60 & -30 \\ 60 & -30 & -90 \end{bmatrix}$$

For, $X_1 - 2X_2 + 3X_3 - 4X_4$

Comparing it with $b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$

$b_1 = 1$; $b_2 = -2$; $b_3 = 3$; $b_4 = -4$

$$\text{Mean Vector} = [b_1+b_2+b_3+b_4]\,\mu = \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$$

$$\text{CoVariance Matrix} = [b_1^2+b_2^2+b_3^2+b_4^2]\,\Sigma = \begin{bmatrix} 30 & 150 & 60 \\ 150 & -60 & -30 \\ 60 & -30 & -90 \end{bmatrix}$$

CoVariance Between Them,

$$[b_1C_1+b_2C_2+b_3C_3+b_4C_4]\,\Sigma = \begin{bmatrix} -12 & -60 & -24 \\ -60 & 24 & 12 \\ -24 & 12 & 36 \end{bmatrix}$$

$$\text{Joint Density} = \begin{bmatrix} (\Sigma C_i^2)\,\Sigma & (b^T c)\,\Sigma \\ (b^T c)\,\Sigma & (\Sigma b_i^2)\,\Sigma \end{bmatrix}$$

$$= \begin{bmatrix} 30\,\Sigma & -12\,\Sigma \\ -12\,\Sigma & 42\,\Sigma \end{bmatrix}$$