SVKM's Narsee Monjee Institute of Management Studies
Mukesh Patel School of Technology Management & Engineering

Tutorial No 1

Random Variables:

Discrete Random Variable

Q.01 If X represents total number of heads obtained, when a fair coin is tossed 4 times, find the probability distribution of X.

x_i	0	1	2	3	4
p_i	1/16	4/16	6/16	4/16	1/16

Q.02 PDF of random variable X is:

X	1	2	3	4	5	6	7
P(X)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

Find $k, P(X<5), P(X>5) P(0 \le X \le 5)$

1/8,49/64,3/32,29/32

Q.03 A RV X has the following probability distribution:

X	-2	-1	0	1	2
P(X=x)	1/5	1/5	2/5	2/15	1/15

Find the probability distribution of:

i. $V = X^2 + 1$

ii. $W = X^2 + 2X + 3$

V	1	2	5
Pi	2/5	1/3	4/15

W	2	3	6	11
Pi	3/15	3/5	2/15	1/15

Expectations, Mean and Variance(Discrete)

Q 01. An urn contains 7 white and 3 red balls. Two balls are drawn together, at random from this urn. Compute the expected number of white balls drawn

21/15

Q 02. Given the following distribution:

x	-3	-2	-1	0	1	2
P(X =x)	0.01	0.1	0.2	0.3	0.2	0.15

Find Mean and Variance

0.05,1.8475

Q.03. Find the value of k and expectation

x	0	10	15
P(X=x)	(k-6)/5	2/k	14/5k

8, 31/4

Expectations, Mean and Variance(Continuous)

Q.01 For the continuous random variable X, the probability density function given below

$$f(x) = \begin{cases} k(2-x); 0 \le x < 2 \\ kx(x-2); 2 \le x < 3. \\ 0 \ ; \ \text{otherwise} \end{cases}$$

Find k, mean and distribution function

Q.02 A daily consumption of electric power (in million kWh) is a random variable X with probability density function given below

$$f(x) = \begin{cases} kxe^{-\frac{x}{3}}; x > 0 \\ 0 \text{ ; otherwise} \end{cases}.$$

Find (i) k (ii) expectation of X (iii) Probability that on a given day, the electric consumption is more than expected value.

1/9, 6, 0.406

Q.03 The distribution function of a continuous random variable x is given by

$F(x) = 1-(1+x)e^{-x}$, $x \ge 0$. Find the probability density function, mean and standard deviation

$f(x) = xe^{-x}, 2, \sqrt{2}$

Q.04 If pdf:

$$f(x) = k.\frac{1}{1+x^2}, -\infty < x < \infty$$

Then find k and cdf

$1/\pi, \tan^{-1} x + 1/2$

Q.05 A continuous random variable has probability density function

$$f(x) = 6(x - x^2),\ 0 \le x \le 1.$$

Find mean and variance and also find $P(|x - \mu| < \sigma)$.

1/2, 1/20, 0.6264

Moments and MGF

Q.01 Find the moment generating function of a random variable X if the rth moment about the origin is given by $\mu_r' = r!$

$$\frac{1}{1-t}$$

Q.02 Find the moment generating function of the random variable X whose probability mass function is given by

X:	-2	3	1
P(X):	1/3	1/2	1/6

, Also find the first two moments about the origin.

$$\frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} + \frac{1}{6}e^{t},\quad \mu_1' = 1,\ \mu_2' = 6$$

Q.03 If a random variable has the moment generating function $M_X(t) = \dfrac{3}{3-t}$, obtain the mean and the standard deviation.

Mean=1/3, S.D=1/3

Q.04 A random variable X has the probability distribution $P(X = x) = \frac{1}{8}\,{}^3C_X,\ X = 0,1,2,3$, Find the moment generating function of X and then find mean and variance.

$$\frac{1}{8}(1+e^t),\quad 3/2, 3/4$$

Q.05 Find the moment generating function of a random variable having density function $f(x) = \begin{cases} e^{-x} & x \ge 0 \\ 0 & otherwise \end{cases}$ and determine the first four moments about the origin.

$$\frac{1}{1-t},\ |t| < 1,\ \mu = 1, \mu_2' = 2, \mu_3' = 6, \mu_4' = 24$$

Q.06 A random variable X has the following probability distribution

X=x :	0	1	2
P(X=x):	1/3	1/3	1/3

, Find the moment generating function, first four raw moments and the first four central moment.

$$(a)\frac{1}{3}(1+e^t+e^{2t}),\ \text{(b) } 1, 5/3, 9/3, 17/3,\ \ (c) 0, 2/3, 0, 2/3$$

(PMF, Cumulative distribution function)

1. A discrete random variable X has the probability mass function given below

X	-2	-1	0	1	2	3
$P(X=x)$	0.2	k	0.1	$2k$	0.1	$2k$

Find k, mean and distribution function. (3/25, 6/25)

2. For the continuous random variable X, the probability density function given below $f(x) = \begin{cases} k(2-x); 0 \le x < 2 \\ kx(x-2); 2 \le x < 3 \\ 0 \; ; \; \text{otherwise} \end{cases}$. Find k, mean and distribution function.

3. A daily consumption of electric power (in million kWh) is a random variable X with probability density function given below $f(x) = \begin{cases} kxe^{-\frac{x}{3}}; x > 0 \\ 0 \; ; \text{otherwise} \end{cases}$. Find (i) k (ii) expectation of X (iii) Probability that on a given day, the electric consumption is more than expected value. (1/9, 6, 0.406)

4. The distribution function of a continuous random variable x is given by $F(x) = 1 - (1+x)e^{-x}$, $x \ge 0$. Find the probability density function, mean and standard deviation. $\left(f(x) = xe^{-x}, 2, \sqrt{2}\right)$

5. Find the value of k from the following data

X	0	10	15
P(X=x)	$(k-6)/5$	$2/k$	$14/5k$

(k=8)

6. The time a person has to wait for a bus at a bus-stop is a random variable has the following distribution function $F(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{3}, & 0 \le x \le 1 \\ \frac{1}{3}, & 1 \le x \le 3 \\ \frac{x}{9}, & 3 \le x \le 9 \\ 1, & x \ge 9 \end{cases}$.

A. Find the probability density function.

B. Find the mean and the variance.

7. A continuous random variable has probability density function $f(x) = 6(x - x^2)$, $0 \le x \le 1$. Find mean and variance and also find $P(|x - m| < \sigma)$. (1/2, 1/20, 0.6264)

8. If a function of variable x is $f(x) = \begin{cases} xe^{-\frac{x^2}{2}}; x \ge 0 \\ 0 \; ; \text{otherwise} \end{cases}$. Prove that (i) $f(x)$ is a probability density function. (ii) Obtain distribution function $F(x)$.
$\left(\int_0^\infty xe^{-x^2/2}dx = \int_0^\infty e^{-x^2/2}dx = 1 \;\; (ii) F(x) = 1 - e^{-x^2/2}\right)$

9. A random variable X has the following probability function

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	2k	0.3	3k

Find (i) k (ii) $P(-2 < X < 1)$ (iii) obtain the distribution function of X. (0.1, 0.6, 0.3)

10. A probability mass function of a random variable X is given by $P(X = x) = kx^3;\ \ x = 1, 2, 3, 4$. Find (i) k (ii) $P(\frac{1}{2} < X < \frac{5}{2})$ (iii) cumulative probability distribution of X.

11. A random variable X has the following probability density function $f(x) = \begin{cases} \frac{1}{3e^{-\frac{x}{3}}}; & x > 0 \\ 0 \text{ ; otherwise} \end{cases}$.

Find (i) $P(X > 3)$ (ii) MGF of X (iii) $E(X)$ and $Var(X)$.

12. A continuous random variable X has the p.d.f. defined by $f(x) = A + Bx, 0 \le x \le 1$. Also the mean of the distribution is 1/3, find A and B. (A=2, B=-2)

13. Find k and then $E(X)$ if X has the p.d.f. $f(x) == \begin{cases} kx(2-x); 0 \le x \le 2, k > 0 \\ \quad 0 \ ; \ \text{otherwise} \end{cases}$. (3/4, 1, 1/5)

14. Let X be a continuous random variable with p.d.f. $f(x) = kx(1-x)\ , 0 \le x \le 1$. Find k and determine a number b such that $P(X \le b) = P(X \ge b)$.

15. The probability mass function for a random variable X is

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	$2k$	0.3	k

Find $k, E(X), V(X), P(X < 2)$. (k=0.1, 0.8, 2.16, 2/5)

16. If X and Y are discrete random variables with $E(X) = 2, E(Y) = 3, V(X) = 4, V(Y) = 1$. Find $E(X - Y),\ V(X - Y), E(X - Y)^2$.

17. If $X_1$and $X_2$are discrete random variable with $E(X_1) = 4, E(X_2) = -2, V(X_1) = 9, V(X_2) = 4$. Find $E(2X_1 + X_2 - 3)$ and $V(2X_1 + X_2 - 3)$. (3, 40)

18. Find the expectation of number of failures proceeding the first success in an infinite series of independent trials with constant probabilities p and q of success and failure respectively. (q/p)

19. A player tosses 3 fair coins. He wins Rs. 15 if 3 heads appear; Rs. 6 if 2 heads appear and Rs. 2 if 1 head appears. On the other hand, he loses Rs. 20 if 3 tails appear. Find the expected gain of the player.

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MGF, Raw moment & Central Moment

20. Find the moment generating function of random variable having the following probability density function $f(x) = \begin{cases} e^{-(x-5)}; x \geq 5 \\ 0 \ ; \ \text{otherwise} \end{cases}$. Also find the mean and variance. $\left(\frac{e^{5t}}{1-t}, 6, 1\right)$
21. If X denotes the outcome when a fair die is tossed, find M.G.F. of X and hence, find the mean and variance of X. (7/2, 35/12)

22. A continuous random variable has probability density function $f(x) = \begin{Bmatrix} 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{Bmatrix}$
Find (i) E(x), (ii) $E(X^2)$ (iii) Var (x) (iv) S.D. of X. (1/2, ½, ¼, ½)

23. Find the moment generating function of random variable having the following probability density function $f(x) = \begin{cases} x \ ; 0 \leq x \leq 1 \\ 2 - x; 1 \leq x < 2 \\ 0 \ ; \ \text{otherwise} \end{cases}$. Also find the mean and variance.
24. A random variable X has the following probability function $f(x) = \begin{cases} ke^{-kx}; x > 0, k > 0 \\ 0 \ ; \ \text{otherwise} \end{cases}$. Find the moment generating function and hence mean and variance. (1/k, $1/k^2$)

25. Let X be a random variable with the following probability density function $f(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \cdots$. Find the moment generating function about origin. Hence, find the mean and variance of X. (2, 2)

26. Find the MGF of the exponential distribution $f(x) = \frac{1}{c}e^{-x/c}, 0 \leq x \leq \infty, c > 0$, Hence find its mean and S.D. $[(1 - ct)^{-1}, c, c^2]$
27. Let X be a random variable with $E(X) = 15$ and $V(X) = 25$. Find the positive value of a and b such that $Y = aX - b$ has expectation 0 and variance 1. (a=1/5, b=3)
28. Find the moment generating function of random variable having the following probability density function $f(x) = \begin{cases} 2; 0 \leq x \leq \frac{1}{2} \\ 0 \ ; \ \text{otherwise} \end{cases}$. Also find the mean and variance. $\left(\frac{2(e^t-1)}{t}, \frac{1}{4}, \frac{1}{48}\right)$

29. The first four moments of a distribution about the value 4 are - 1.5, 17, -30 and 108. Calculate the moments about the mean.