Module: Inventory Control

Outline

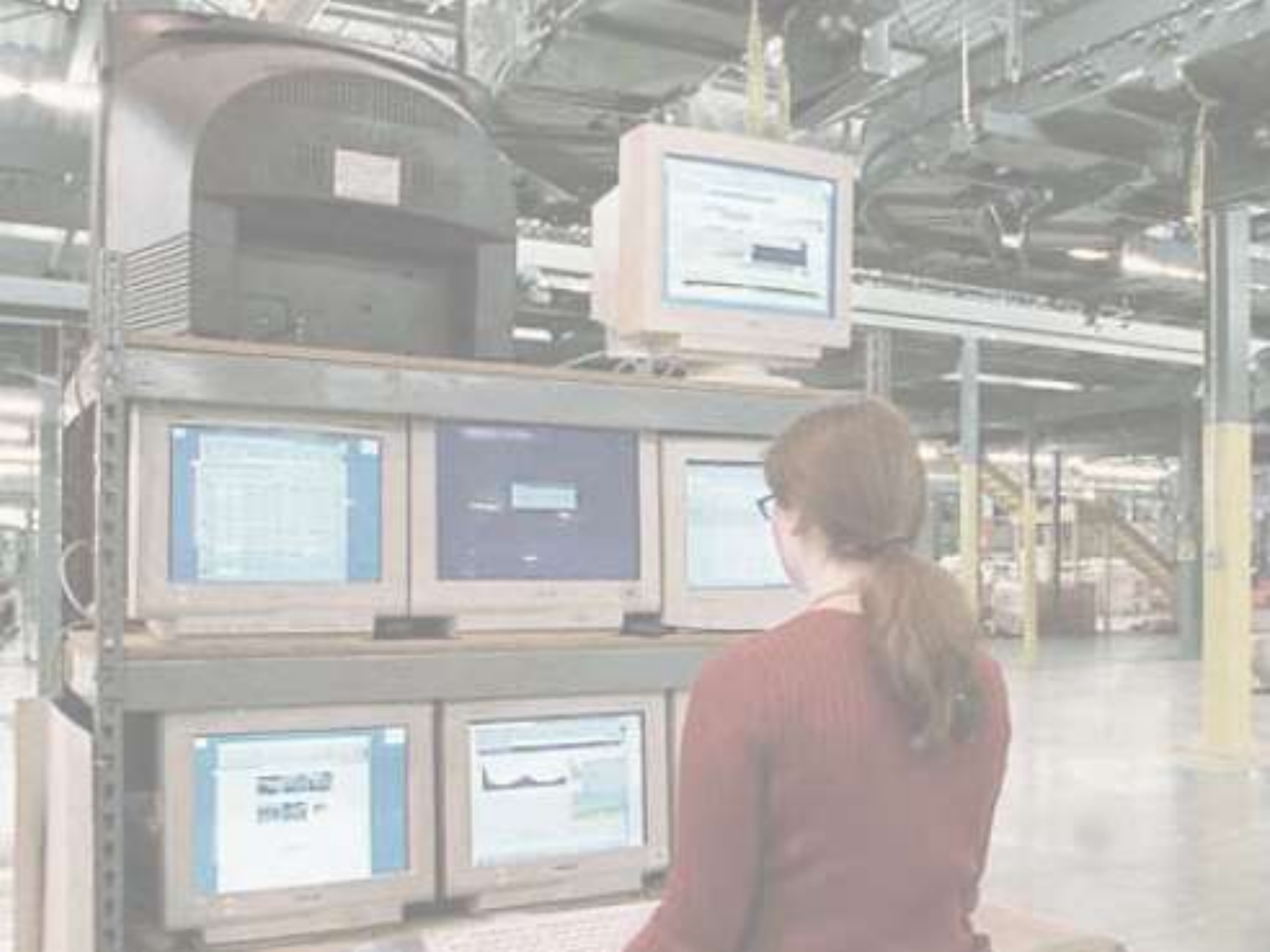
1. **Types of Inventory**
2. **Functions of Inventory**
3. **ABC Analysis**
4. **Record Accuracy**
5. **Cycle Counting**
6. **Independent vs. Dependent Demand Inventory Control Systems**
7. **Multi-Period Deterministic Inventory Models**

   I. Fixed- Order Quantity Models

   - ✓ **Economic Order Quantity (EOQ) Model.**
   - ✓ **Production Order Quantity (POQ) Model.**
   - ✓ **Quantity Discount Model.**

   II. Fixed-Time Period Models

8. **Probabilistic Models and Safety Stock**
9. **Single-Period Inventory Model**



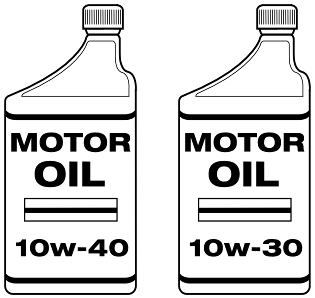
- Amazon.com started as a “virtual” retailer – no inventory, no warehouses, no overhead; just computers taking orders to be filled by others

- Growth has forced Amazon.com to become a world leader in warehousing and inventory management

Inventory Management

The objective of inventory management is to strike a balance between inventory investment and customer service

Inventory Classifications



Inventory

- Process stage
  - Raw Material
  - WIP
  - Finished Goods
- Number & Value
  - A Items
  - B Items
  - C Items
- Demand Type
  - Independent
  - Dependent
- Other
  - Maintenance
  - Repair
  - Operating

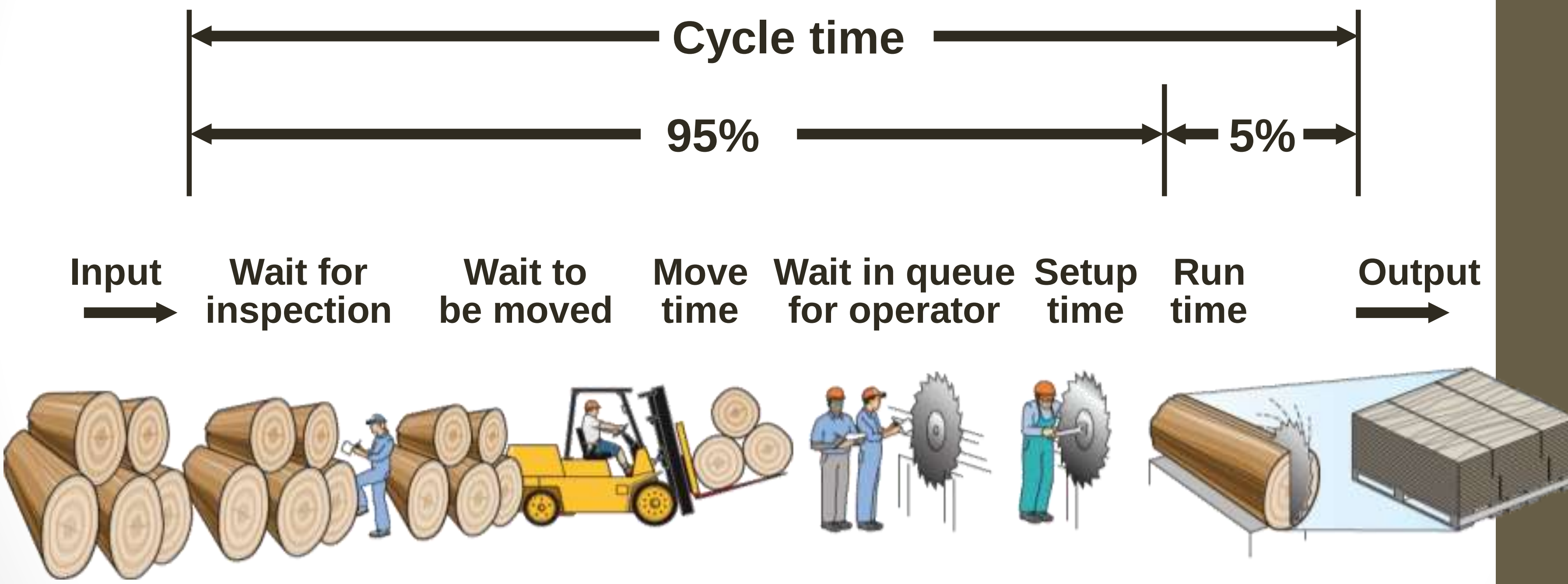
Functions of Inventory

1. To decouple various parts of the production process by covering delays
2. To protect the company against fluctuations in demand
3. To provide a selection for customers
4. To take advantage of quantity discounts
5. To hedge against inflation

Problems Caused by Inventory

- Inventory ties up working capital
- Inventory takes up space
- Inventory is prone to:
  - Damage, Pilferage and Obsolescence
- Inventory hides problems

The Material Flow Cycle



Important Issues in Inventory Management

1. **Classifying inventory items**
2. **Keeping accurate inventory records**

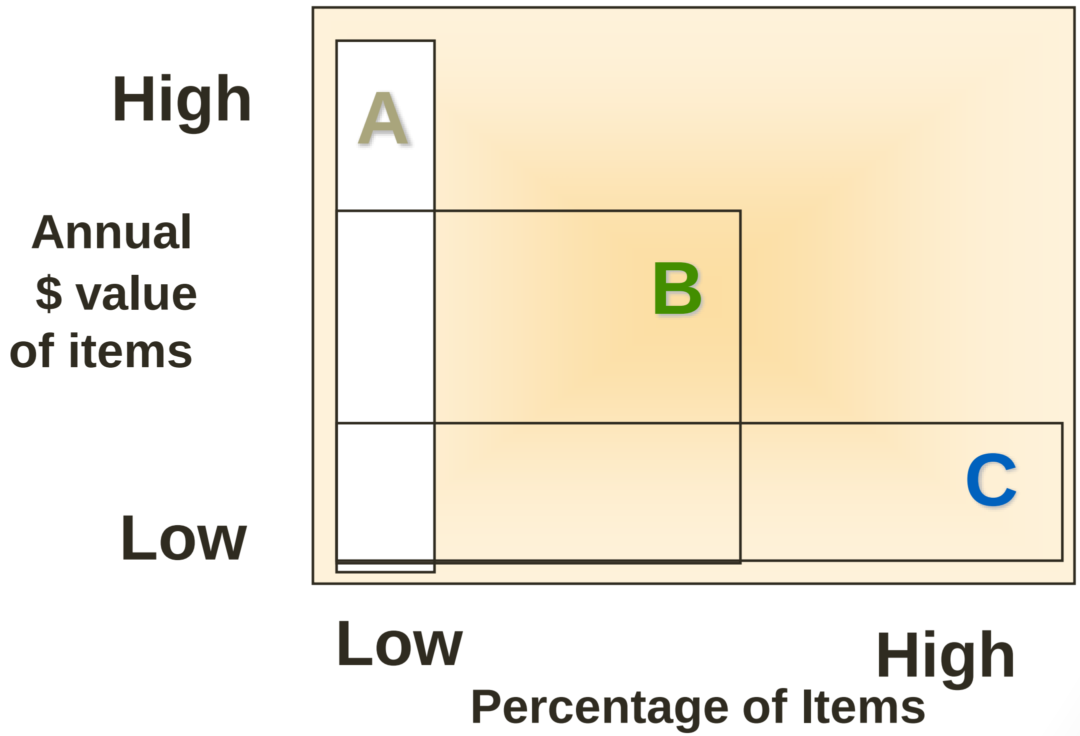
ABC Classification System

Classifying inventory according to some measure of importance and allocating control efforts accordingly.

A - very important

B - mod. important

C - least important



ABC Worked Example

- Item Usage and Value

Table 9.2 Item usage and value

Item type	Purchase price (£)	Annual sales (items per year)
a	8	1,250
b	18	450
c	30	75
d	25	10
e	3	280
f	4	80
g	18	45
h	7	250
i	12	150
j	26	30

ABC Worked Example

Table 9.3 Annual usage values

Item type	Purchase price (£)	Annual sales (items per year)	Annual spend (price × number)
a	8	1,250	10,000
b	18	450	8,100
c	30	75	2,250
d	25	10	250
e	3	280	840
f	4	80	320
g	18	45	810
h	7	250	1,750
i	12	150	1,800
j	26	30	780

ABC Worked Example

Table 9.4 Ascending percentage usage values

Item type	Purchase price (£)	Annual sales (items per year)	Annual spend (price × number)	Percentage spend (%)	Cumulative spend (%)
a	8	1,250	10,000	37.2	37.2
b	18	450	8,100	30.1	67.3
c	30	75	2,250	8.4	75.7
i	12	150	1,800	6.7	82.3
h	7	250	1,750	6.5	88.8
e	3	280	840	3.1	92.0
g	18	45	810	3.0	95.0
j	26	30	780	2.9	97.9
f	4	80	320	1.2	99.1
d	25	10	250	0.9	100.0
			26,900	100.0	

ABC Worked Example

- ABC Chart Showing Classifications

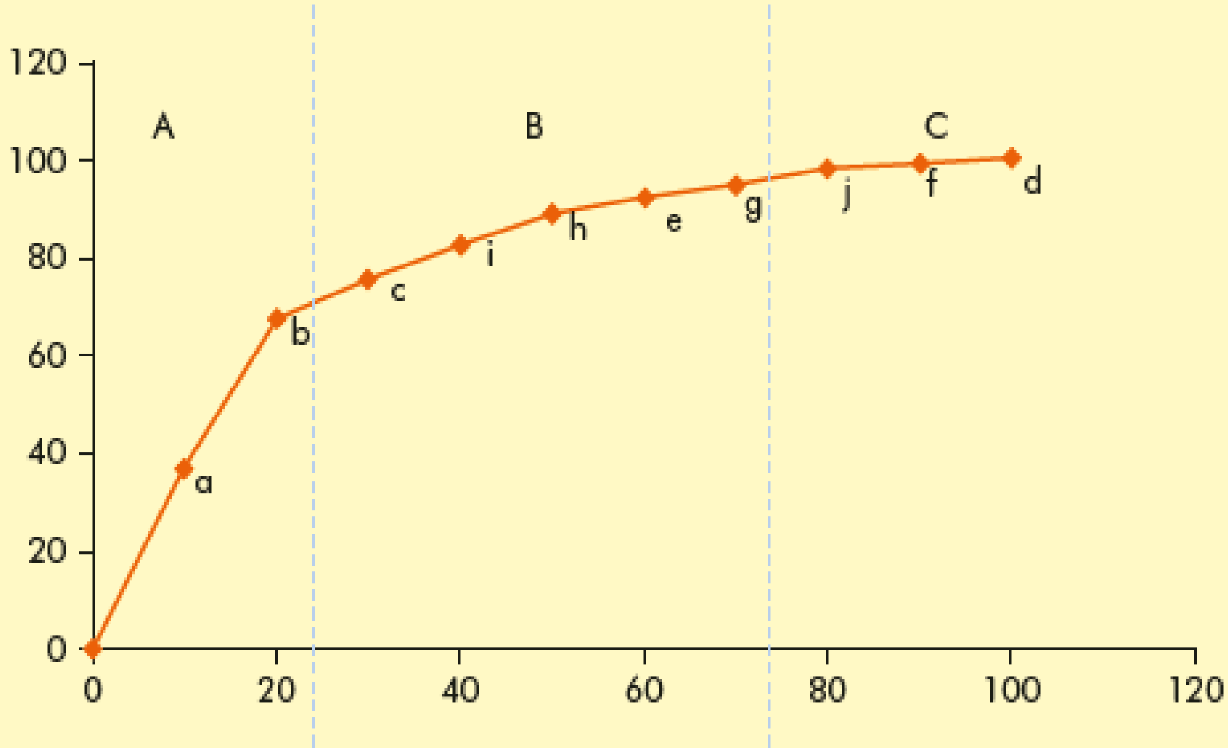


Figure 9.5 ABC chart showing classifications

ABC Classification System

- **Policies employed for A items may include**
  - **More emphasis on supplier development**
  - **Tighter physical inventory control**
  - **More care in forecasting**

Inventory Record Accuracy &Cycle Counting

- Items are counted and records are updated on a periodic basis
- Often used with ABC analysis to determine the cycle

  (frequency of counting)

- Eliminates shutdowns and interruptions
- Maintains accurate inventory records

Cycle Counting

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days)

Item Class	Quantity	Cycle Counting Policy	Number of Items Counted per Day
A	500	Each month	500/20 = 25/day
B	1,750	Each quarter	1,750/60 = 29/day
C	2,750	Every 6 months	2,750/120 = 23/day
Total	5000		77/day

Record Accuracy and Inventory Counting Systems

- *Periodic Inventory Counting System*

Physical count of items is made at periodic intervals (weekly, monthly or yearly)

- *Perpetual (continual) Inventory Counting System*

Computer System that keeps track of removals from inventory continuously, thus monitoring current levels of each item (Bar code Technology)

Independent and Dependent Demand Inventory Management Systems

- **Independent demand** - the demand for the item is independent of the demand for any other item in inventory

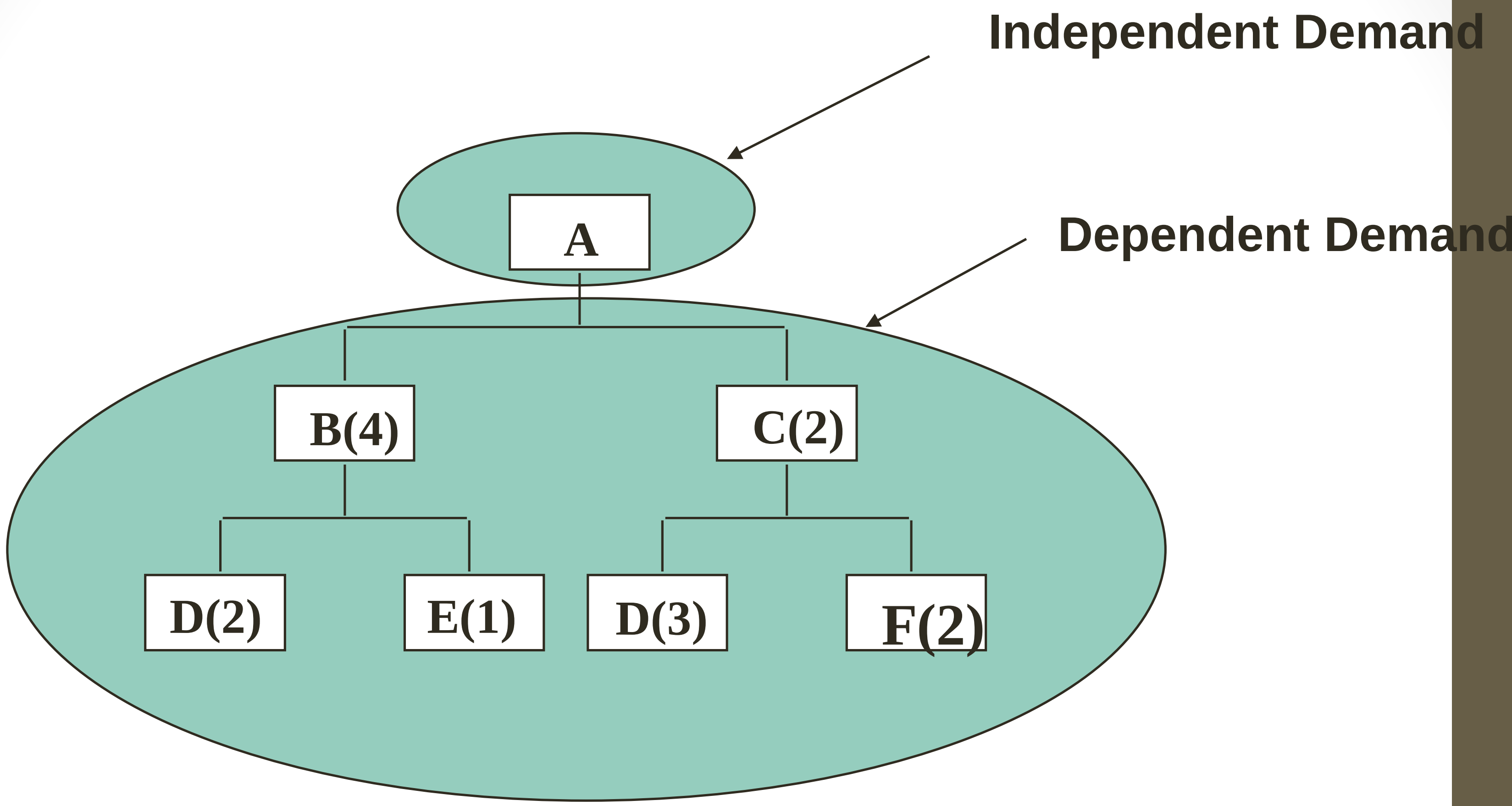
- **Dependent demand** - the demand for the item is dependent upon the demand for some other item in the inventory

Examples
for Independent Versus Dependent Demand

- Independent demand – finished goods, items that are ready to be sold such as computers, cars.
  - Forecasts are used to develop production and purchase schedules for finished goods.

- Dependent demand – components of finished products (computers, cars) such as chips, tires and engine
  - Dependent demand inventory control techniques utilize material requirements planning (MRP) logic to develop production and purchase schedules (Ch14)

Inventory



Independent demand is uncertain.
That is why it is forecasted.
Dependent demand is certain and it is calculated.

Regardless of the nature of demand (independent, dependent) two fundamental issues underlie all inventory planning:

How Much to Order?
When to order?

Independent Demand Inventory Models to Answer These Questions

1) Single-Period Inventory Model:

One time ordering decision such as selling t-shirts at a football game, newspapers, fresh bakery products. Objective is to balance the cost of running out of stock with the cost of overstocking. The unsold items, however, may have some salvage values.

2) Multi-Period Inventory Models

- Fixed-Order Quantity Models:

  Each time a fixed amount of order is placed.

  - Economic Order Quantity (EOQ) Model
  - Production Order Quantity (POQ) Model
  - Quantity Discount Models

- Fixed-Time Period Models

  Orders are placed at specific time intervals.

Key Inventory Terms

- *Lead time*: time interval between ordering and receiving the order
- *Holding (carrying) costs*: cost to carry an item in inventory for a length of time, usually a year (heat, light, rent, security, deterioration, spoilage, breakage, depreciation, opportunity cost,…, etc.,)
- *Ordering costs*: costs of ordering and receiving inventory (shipping cost, preparing invoices, cost of inspecting goods upon arrival for quality and quantity, moving the goods to temporary storage)
- *Set-up Cost:* cost to prepare a machine or process for manufacturing an order
- *Shortage costs*: costs when demand exceeds supply, the opportunity cost of not making a sale

Basic EOQ Model

Important assumptions

1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are ordering and holding
6. Stockouts can be completely avoided

Inventory Usage Over Time

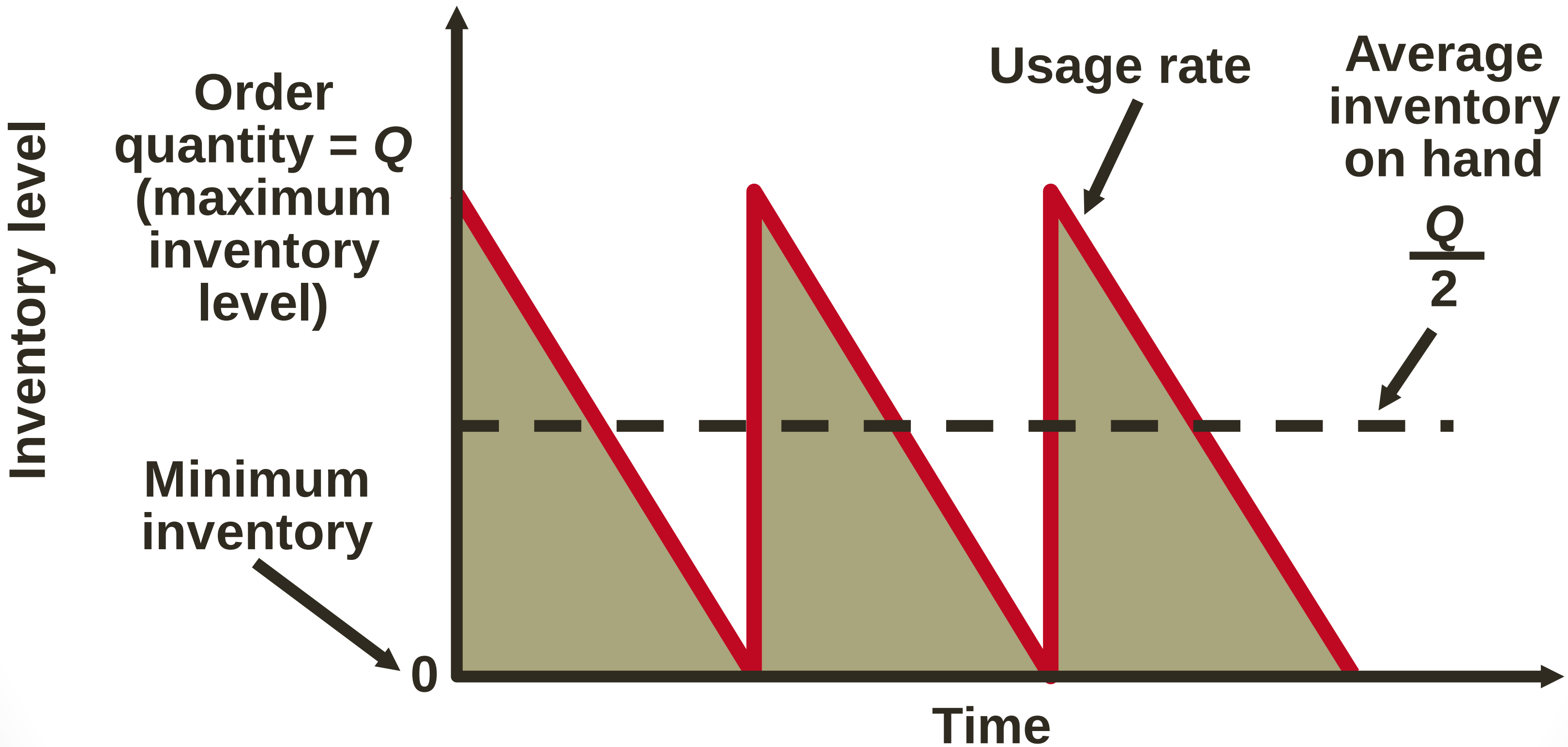
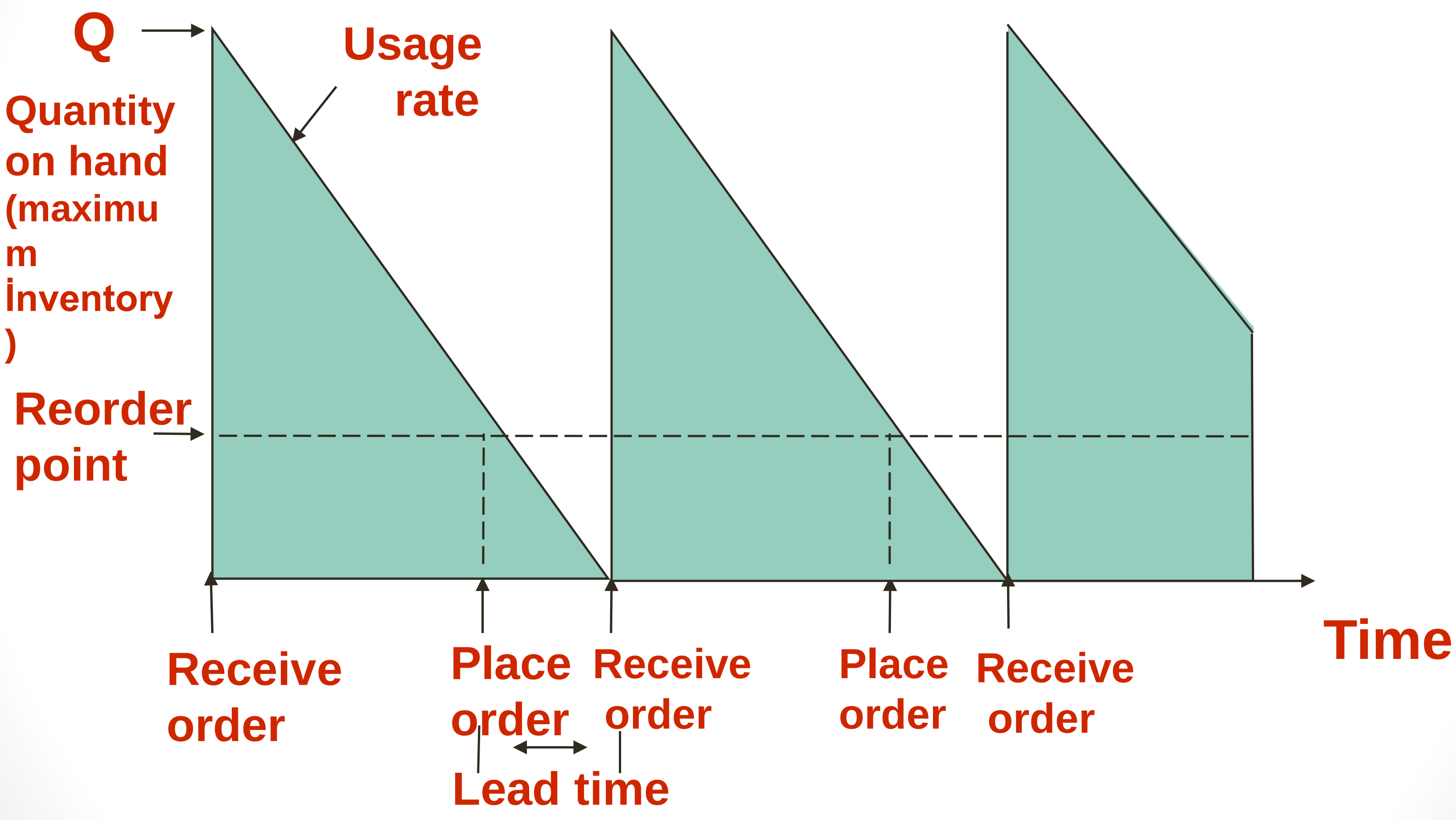


Figure 12.3

The Inventory Cycle



Minimizing Costs

Objective is to minimize total costs

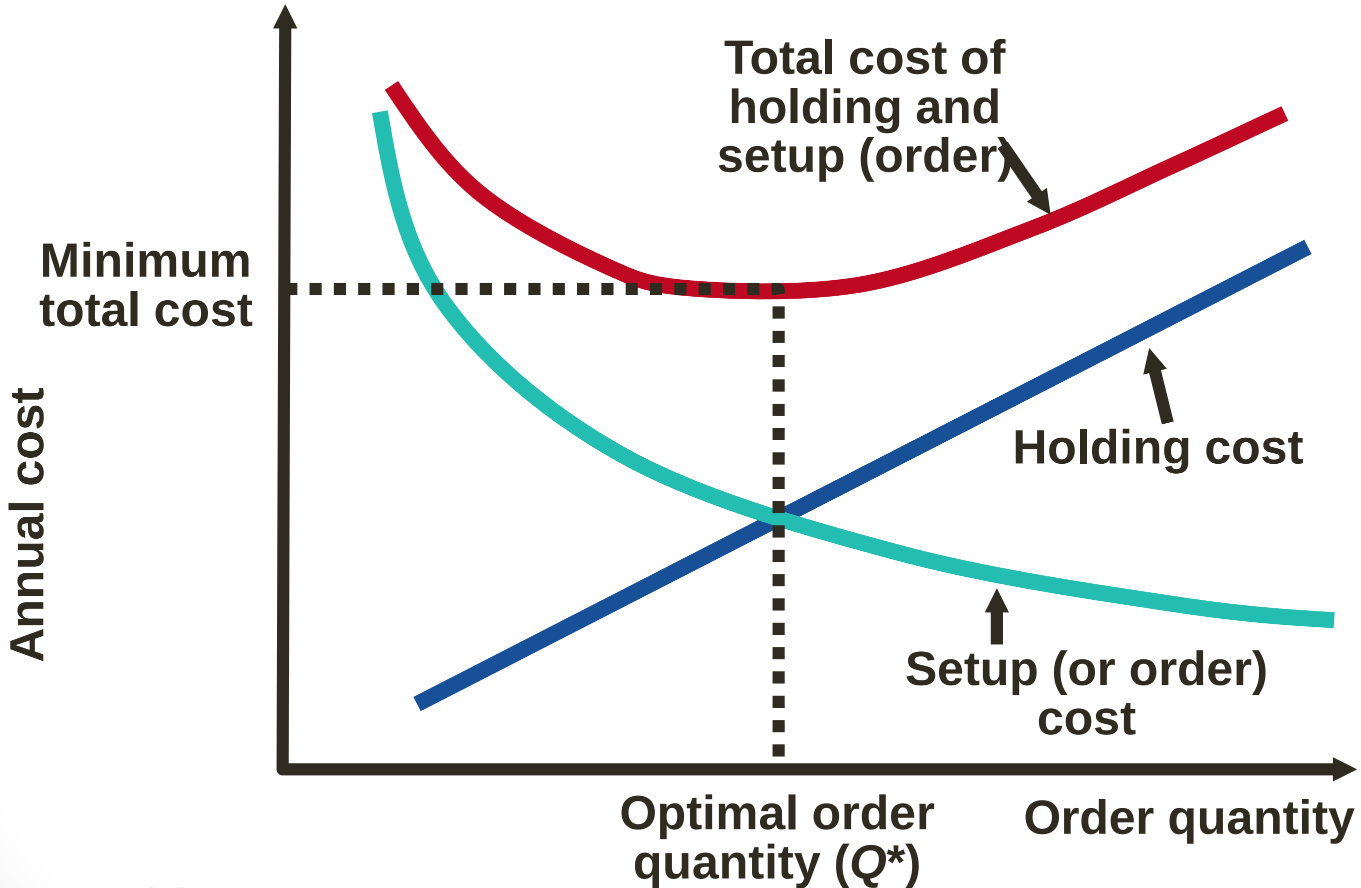


Table 12.4(c)

The EOQ Model

$$\text{Annual setup cost} = \frac{D}{Q}S$$

Q = Order Quantity
*Q** = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual setup cost = (Number of orders placed per year) x (Setup or order cost per order)

$$= \left(\frac{\text{Annual demand}}{\text{Order Quantity}}\right)\left(\begin{array}{c}\text{Setup or order}\\ \text{cost per order}\end{array}\right)$$

$$= \left(\frac{D}{Q}\right)(S)$$

The EOQ Model

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Order Quantity
Q^* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual holding cost = (Average inventory level)
x (Holding cost per unit per year)

$$= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year})$$

$$= \left(\frac{Q}{2}\right)(H)$$

The EOQ Model

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Order Quantity
Q^* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Optimal order quantity is found when annual setup cost equals annual holding cost or we take the derivative of the total cost function and set the derivative (slope) equal to zero and solve for **Q**

Solving for Q^*

$$\frac{D}{Q}S = \frac{Q}{2}H$$

$$2DS = Q^2H$$

$$Q^2 = 2DS/H$$

$$Q^* = \sqrt{2DS/H}$$

An EOQ Example

Determine optimal number of needles to order (Q)

D = 1,000 units per year

S = \$10 per order

H = \$.50 per unit per year

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

An EOQ Example

Determine expected number orders per year (N)

D = 1,000 units

*Q** = 200 units

S = $10 per order

H = $.50 per unit per year

$$\text{Expected number of orders} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1{,}000}{200} = 5 \text{ orders per year}$$

An EOQ Example

Determine expected time between orders (T)

D = 1,000 units
S = $10 per order
H = $.50 per unit per year

Q^* = 200 units
N = 5 orders per year

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{N}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ Example

Determine total annual cost:

D = 1,000 units

S = \$10 per order

H = \$.50 per unit per year

Q^* = 200 units

N = 5 orders per year

T = 50 days

Total annual cost = Setup cost + Holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$TC = (5)(\$10) + (100)(\$.50) = \$50 + \$50 = \$100$$

Robust Model

- The EOQ model is robust
- It works even if all parameters and assumptions are not met

Because the total cost curve is relatively flat in the area of the EOQ

Minimizing Costs

Objective is to minimize total costs

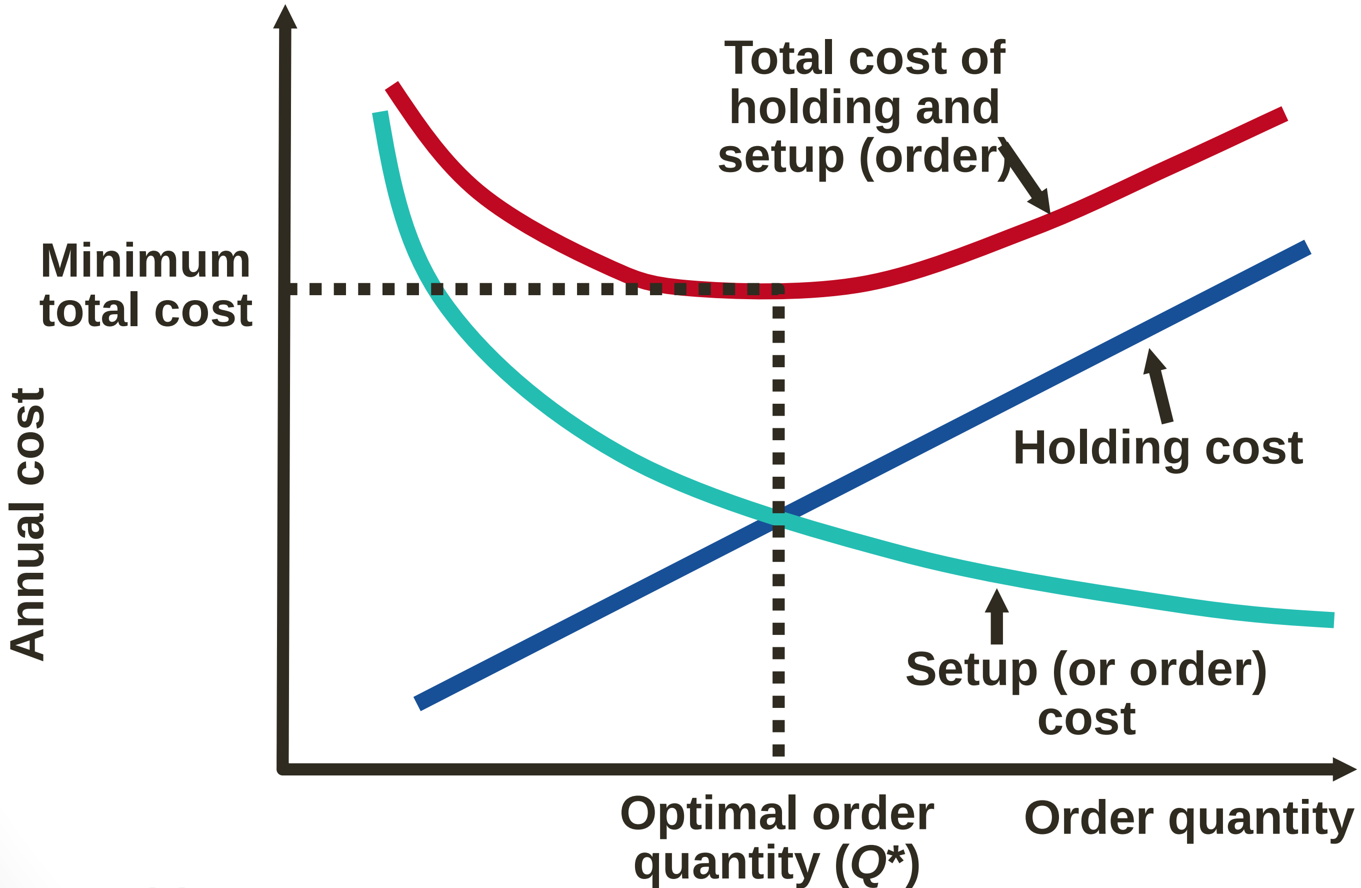


Table 12.4(c)

An EOQ Example

Suppose Management underestimates demand by 50%

D = 1,000 ~~units~~ 1,500 units
S = $10 per order
H = $.50 per unit per year

Q^* = 200 units
N = 5 orders per year
T = 50 days

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}500}{200}(\$10) + \frac{200}{2}(\$.50) = \$75 + \$50 = \$125$$

An EOQ Example

Actual EOQ for new demand is 244.9 units

D = ~~1,000 units~~ 1,500 units $\quad$ **Q^* = 244.9 units**

S = $10 per order $\quad$ **N = 5 orders per year**

H = $.50 per unit per year $\quad$ **T = 50 days**

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$TC = \$61.24 + \$61.24 = \$122.48$$

Only 2% less than the total cost of $125 when the order quantity was 200

Production Order Quantity (POQ) Model

- The third assumption of EOQ model is relaxed: Receipt of inventory is not instantaneous and complete
- Units are produced and used/or sold simultaneously
- Production is done in batches or lots
- Capacity to produce a part exceeds the part's usage or demand rate
- Hence, inventory builds up over a period of time after an order is placed

Production Order Quantity Model

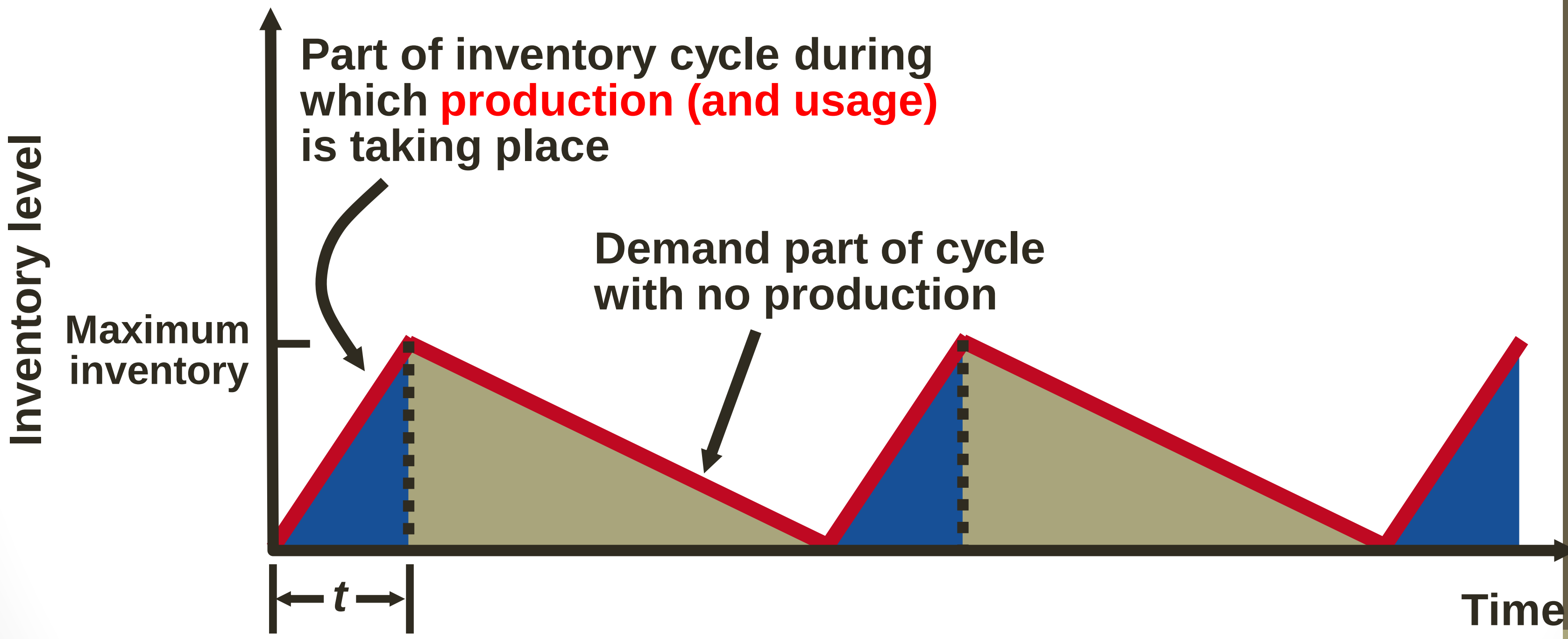


Figure 12.6

Production Order Quantity Model

Q = Order Quantity
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\left(\begin{matrix}\text{Annual inventory} \\ \text{holding cost}\end{matrix}\right) = (\text{Average inventory level}) \times \left(\begin{matrix}\text{Holding cost} \\ \text{per unit per year}\end{matrix}\right)$$

$$\left(\begin{matrix}\text{Annual inventory} \\ \text{level}\end{matrix}\right) = (\text{Maximum inventory level})/2$$

$$\left(\begin{matrix}\text{Maximum} \\ \text{inventory level}\end{matrix}\right) = \left(\begin{matrix}\text{Total produced during} \\ \text{the production run}\end{matrix}\right) - \left(\begin{matrix}\text{Total used during} \\ \text{the production run}\end{matrix}\right)$$

$$= pt - dt$$

Production Order Quantity Model

Q = Order Quantity
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]H$$

Production Order Quantity Model

Q = Order Quantity
H = Holding cost per unit per year
D = Annual demand
p = Daily production rate
d = Daily demand/usage rate

$$\text{Setup cost} = (D/Q)S$$

$$\text{Holding cost} = \frac{1}{2}HQ[1 - (d/p)]$$

$$(D/Q)S = \frac{1}{2}HQ[1 - (d/p)]$$

$$Q^2 = \frac{2DS}{H[1 - (d/p)]}$$

$$Q_p^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

Production Order Quantity Example

D = 1,000 units
S = \$10
H = \$0.50 per unit per year

p = 8 units per day
d = 4 units per day
\# of days plant is open=250

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50[1 - (4/8)]}} = \sqrt{80{,}000}$$

$= 282.8$ or 283 hubcaps

Production Order Quantity Model

Note:

$$d = 4 = \frac{D}{\text{Number of days the plant is in operation}} = \frac{1{,}000}{250}$$

When annual data are used the equation becomes

$$Q^* = \sqrt{\frac{2DS}{H\left(1 - \frac{\text{annual demand rate}}{\text{annual production rate}}\right)}}$$

Quantity Discount Models

- These models are used where the price of the item ordered varies with the order size.
- Reduced prices are often available when larger quantities are ordered.
- The buyer must weigh the potential benefits of reduced purchase price and fewer orders that will result from buying in large quantities against the increase in carrying cost caused by higher average inventories.
- Hence, three is trade-off is between reduced purchasing and ordering cost and increased holding cost

Total Costs with Purchasing Cost

TC = Annual carrying cost + Annual ordering cost + Purchasing cost

$$TC = \frac{Q}{2}H + \frac{D}{Q}S + PD$$

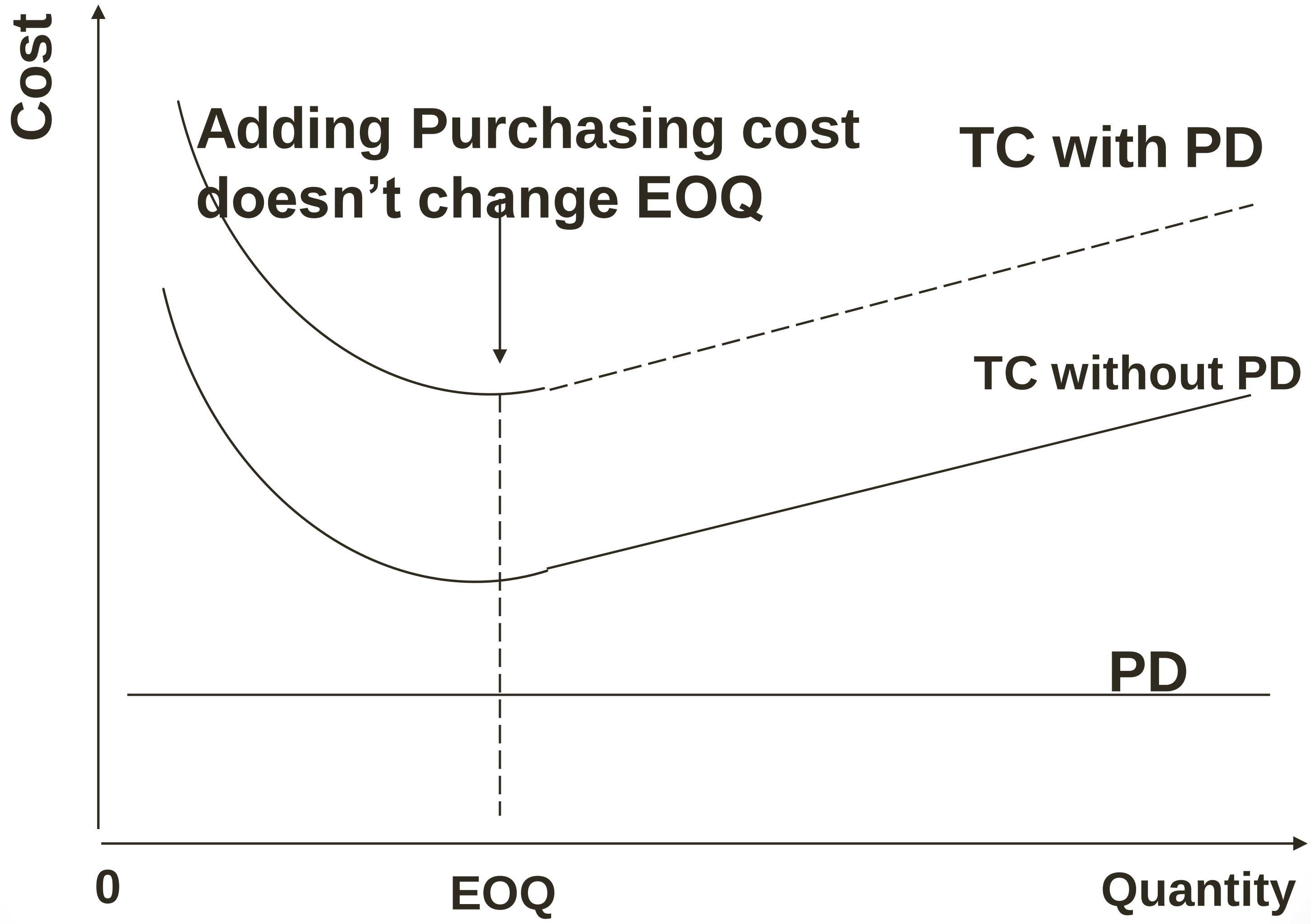
Where P is the unit price.

Remember that the basic EOQ model **does not take into consideration the purchasing cost**.

Because this model works under the assumption of **no quantity discounts**, price per unit is the same for all order size.

Note that including purchasing cost would merely increase the total cost by the amount P times the demand (D). See the following graph.

Total Costs with Purchasing Cost



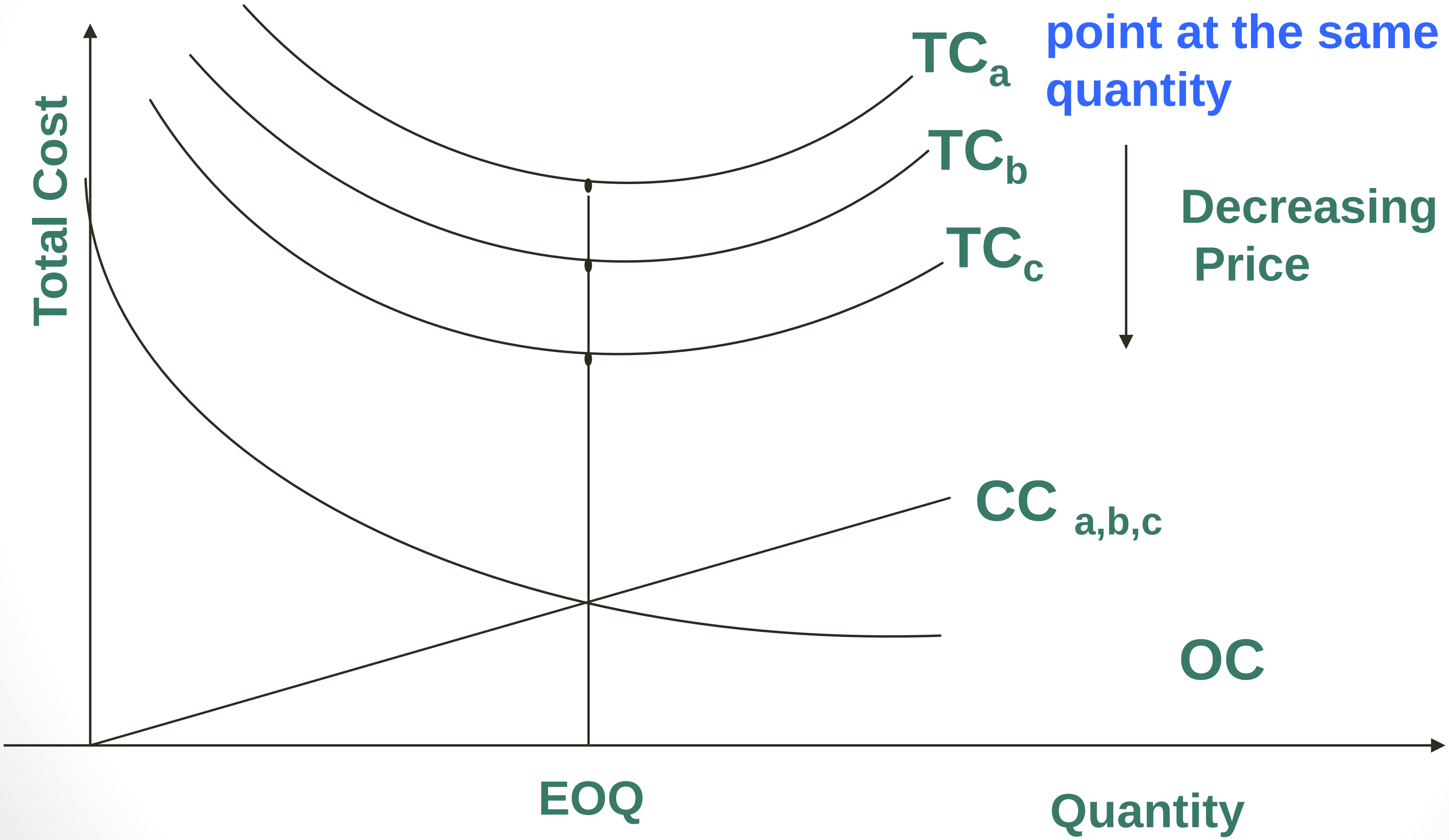
Quantity Discount Models

- There are two general cases of quantity discount models:

1. Carrying costs are ***constant*** (e.g. $2 per unit).
2. Carrying costs are stated as a ***percentage of purchase price*** (20% of unit price)

Total Cost with Constant Carrying Costs

In this case there is a single minimum point; all curves will have their minimum point at the same quantity



EOQ when carrying cost is constant

1. Compute the common minimum point by using the basic economic order quantity model.

2. Only one of the unit prices will have the minimum point in its feasible range since the ranges do not overlap. Identify that range:

   a. if the feasible minimum point is on the lowest price range, that is the optimal order quantity.

   b. if the feasible minimum point is any other range, compute the total cost for the minimum point and for the price breaks of all lower unit cost.

Compare the total costs; the quantity that yields the lowest cost is the optimal order quantity.

Quantity Discount Model with Constant Carrying Cost

QUANTITY	PRICE
1 - 49	$1,400
50 - 89	1,100
90+	900

S = $2,500
H = $190 per computer
D = 200

$$Q_{opt} = \sqrt{\frac{2SD}{H}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ PCs}$$

For $Q = 72.5$

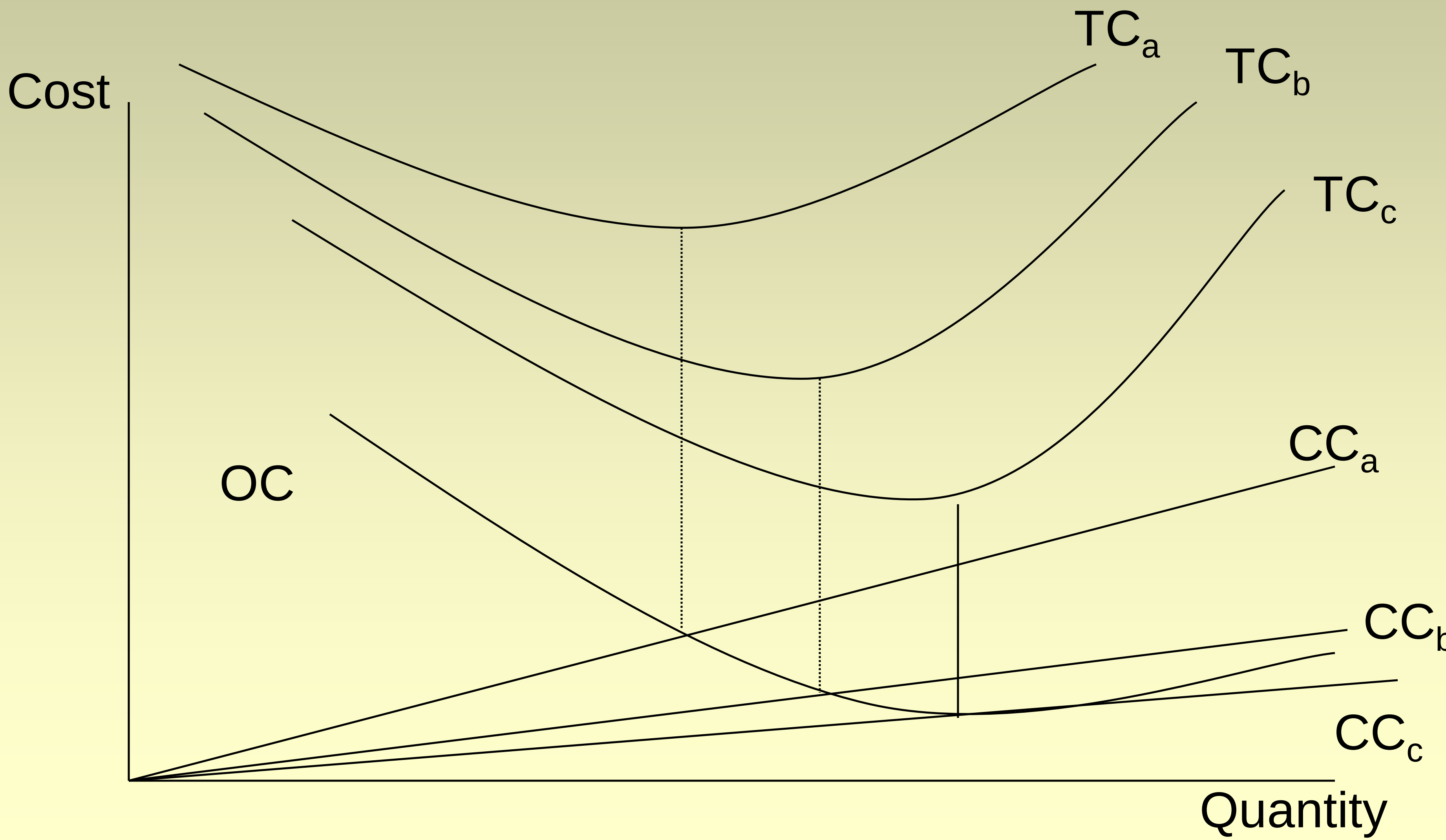
$$TC = \frac{SD}{Q_{opt}} + \frac{H\,Q_{opt}}{2} + PD = \$233{,}784$$

For $Q = 90$

$$TC = \frac{SD}{Q} + \frac{H\,Q}{2} + PD = \$194{,}105$$

Total Cost with **varying** Carrying Costs

When carrying cost is expressed as a percentage of the unit price, each curve will have different minimum point.



EOQ when carrying cost is a percentage of the unit price

1. Beginning with the lowest unit price, compute the minimum points for each price range until you find a feasible minimum point (i.e., until a minimum point falls in the quantity range of its price).

2. If the minimum point for the lowest unit price is feasible, it is the optimal order quantity. If the minimum point is not feasible in the lowest price range, compare the total cost at the price break for all lower prices with the total cost of the feasible minimum point. The quantity which yields the lowest total cost is the optimum

Quantity Discount Models

A typical quantity discount schedule, Inventory
Carrying cost is 20% of unit price

Discount Number	Discount Quantity	Discount (%)	Discount Price (*P*)
1	0 to 999	no discount	\$5.00
2	1,000 to 1,999	4	\$4.80
3	2,000 and over	5	\$4.75

Table 12.2

When carrying costs are specified as a percentage of unit price, the total cost curve is broken into different total cost curves for each discount range

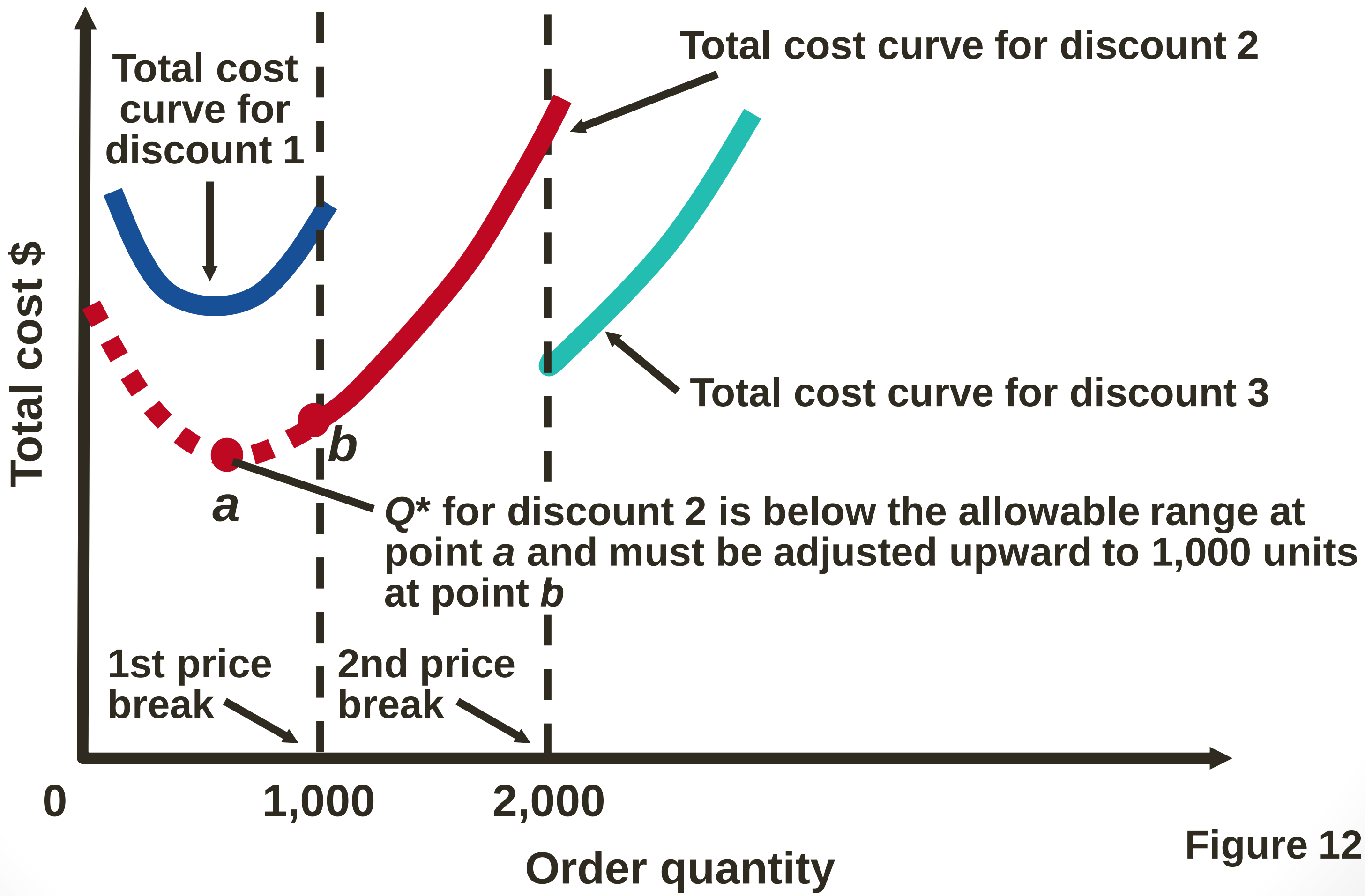


Figure 12.7

Quantity Discount Example

Calculate Q^* first for the lowest price range

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = 718 \text{ cars/order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = 714 \text{ cars/order}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$

Quantity Discount Example

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = \cancel{714 \text{ cars/order}}$$

1,000 — adjusted

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = \cancel{718 \text{ cars/order}}$$

2,000 — adjusted

Quantity Discount Example

Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	$5.00	700	$25,000	$350	$350	$25,700
2	$4.80	1,000	$24,000	$245	$480	$24,725
3	$4.75	2,000	$23.750	$122.50	$950	$24,822.50

Table 12.3

Choose the price and quantity that gives the lowest total cost

Buy 1,000 units at $4.80 per unit

When to Reorder with EOQ Ordering

- The EOQ models answer the equation of how much to order, but not the question of when to order. The reorder point occurs when the quantity on hand drops to predetermined amount.
- That amount generally includes expected demand during lead time.
- In order to know when the reorder point has been reached, a perpetual inventory is required.
- The goal of ordering is to place an order when the amount of inventory on hand is sufficient to satisfy demand during the time it takes to receive that order (i.e., lead time)

When to Order: Reorder Points (Make sure demand and lead time are expressed in the same time units)

- **If the demand and lead time are both constant, the reorder point (ROP) is simply:**

$$\textbf{ROP} = \begin{pmatrix}\text{Demand}\\ \text{per day}\end{pmatrix}\begin{pmatrix}\text{Lead time for a}\\ \text{new order in days}\end{pmatrix}$$

$$= d \times L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve

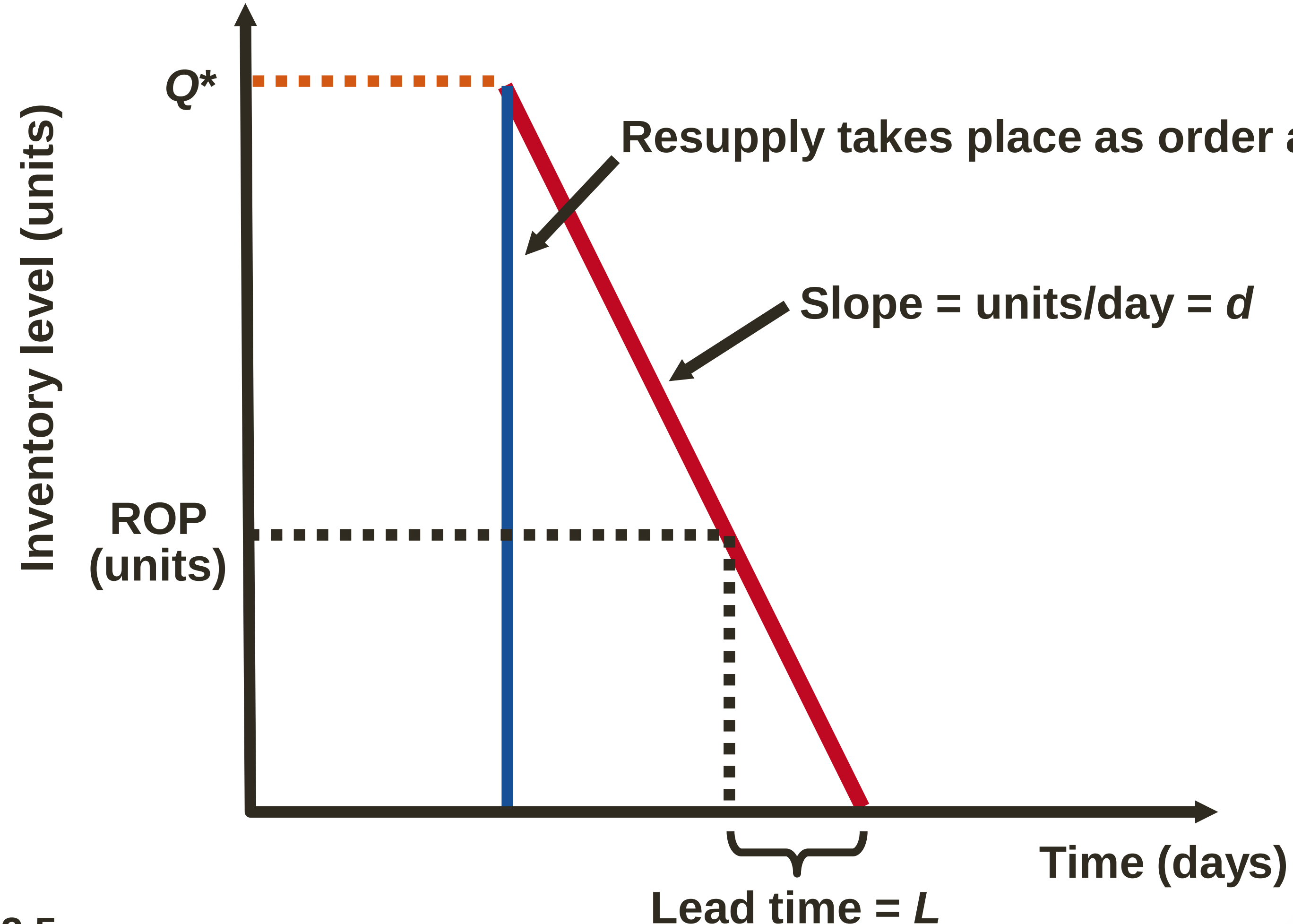


Figure 12.5

Reorder Point Example

Demand = 8,000 iPods per year
250 working day year
Lead time for orders is 3 working days

$$d = \frac{D}{\text{Number of working days in a year}}$$

$$= 8{,}000/250 = 32 \text{ units}$$

$$\text{ROP} = d \times L$$

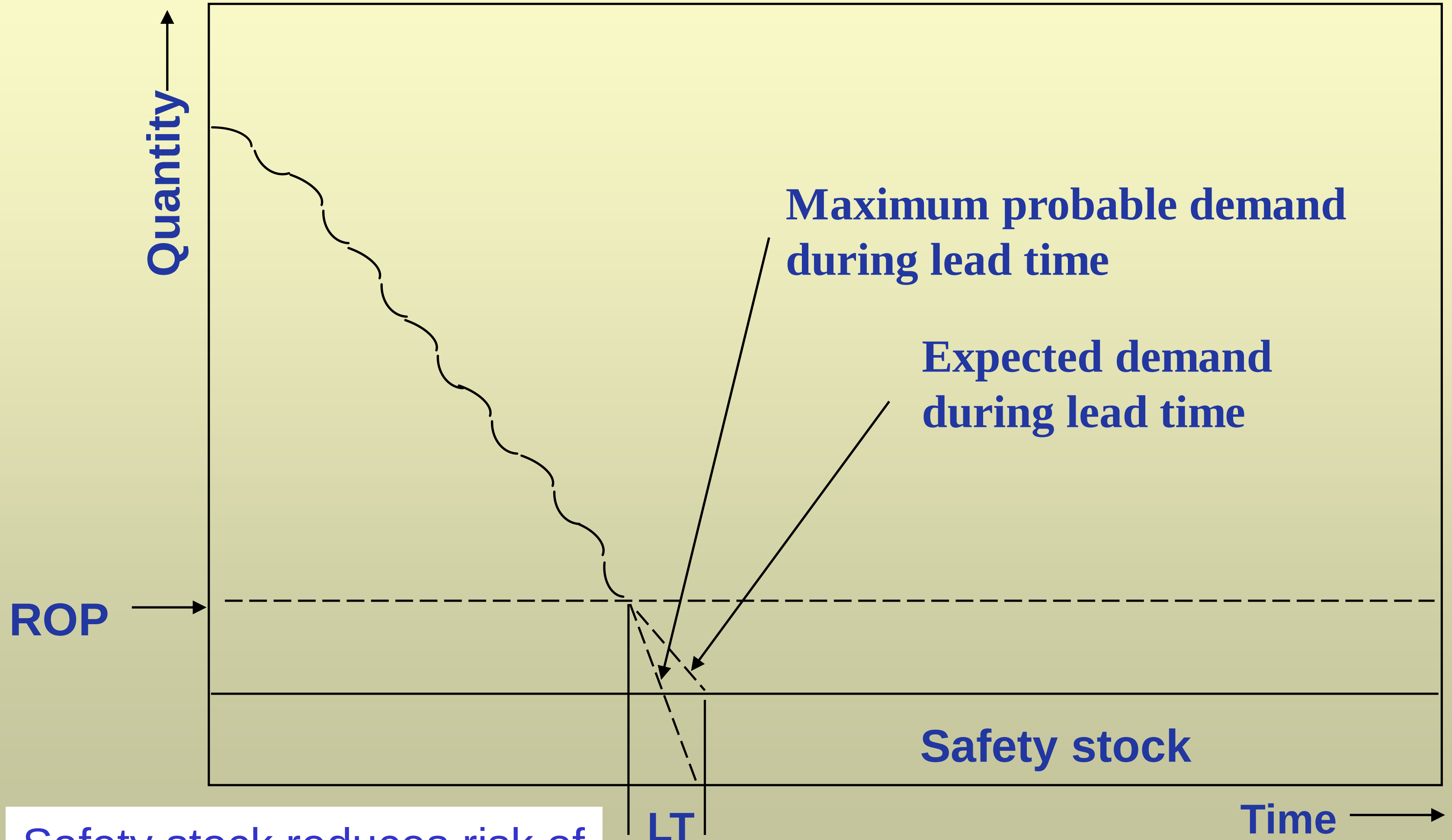
$$= 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units}$$

When to reorder

- When variability is present in demand or lead time, it creates the possibility that actual demand will exceed expected demand.
- Consequently, it becomes necessary to carry additional inventory, called "safety stock", to reduce the risk of running out of stock during lead time. The reorder point then increases by the amount of the safety stock:

ROP = expected demand during lead time + safety stock (SS)

Safety Stock



Safety stock reduces risk of stockout during lead time

Safety stock

- Because it costs money to hold safety stock, a manager must carefully weigh the cost of carrying safety stock against the reduction in stockout risk it provides.
- The **customer service level** increases as the risk of stockout decreases.
- The order cycle **“service level”** can be defined as the probability that demand will not exceed supply during lead time. A service level of 95% implies a probability of 95% that demand will not exceed supply during lead time.

Safety Stock

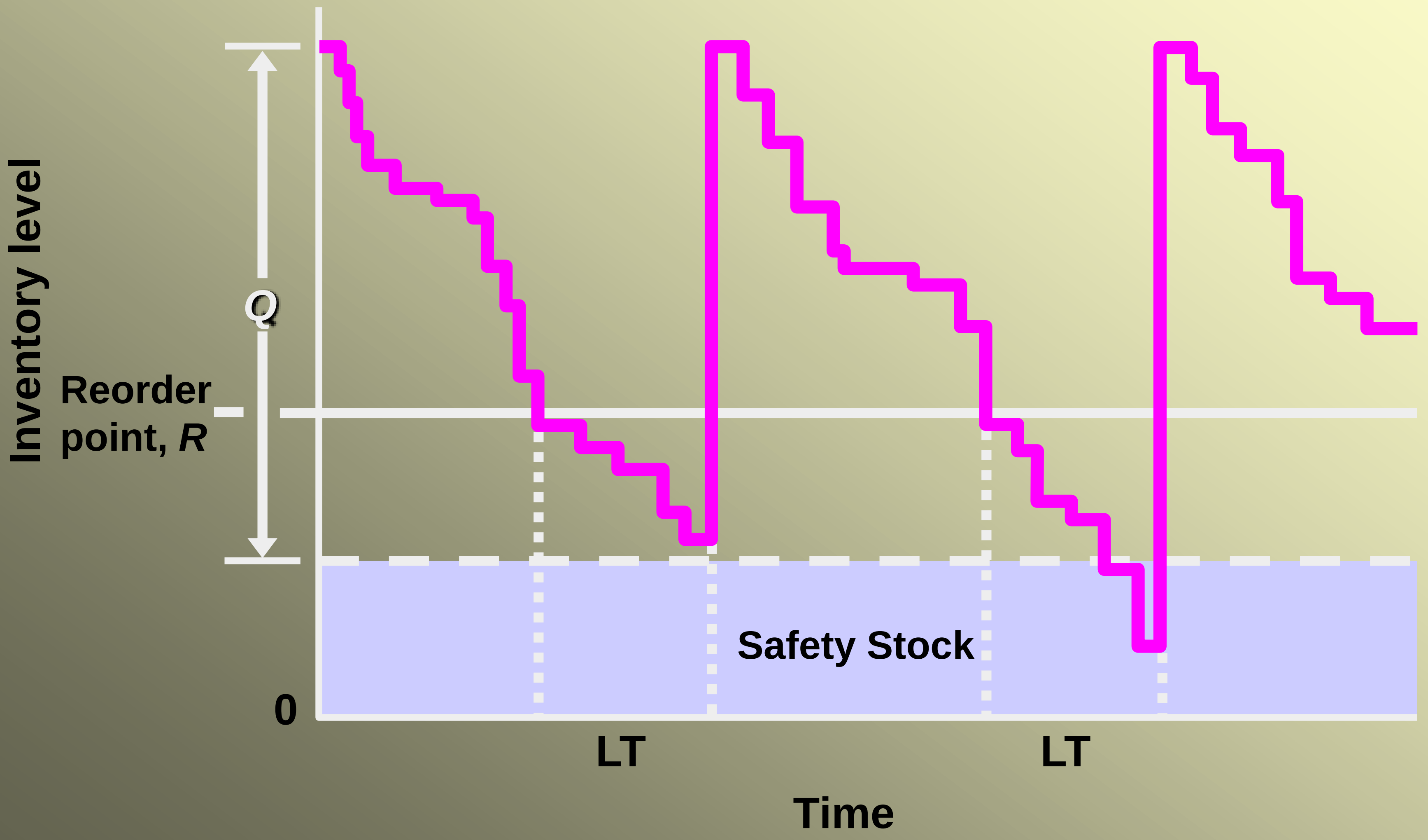
- The "risk of stockout" is the complement of "service level"

Service level = 1 - Probability of stockout

- Higher service level means more safety stock
- More safety stock means higher ROP

ROP = expected demand during lead time + safety stock (SS)

Reorder Point with a Safety Stock



Probabilistic Models to Determine ROP and Safety Stock (When Stockout Cost/Unit is known)

- Use safety stock to achieve a desired service level and avoid stockouts

$$\text{ROP} = d \times L + ss$$

Annual stockout costs = **the sum of the units short for each demand level** x **the probability of that demand level** x **the stockout cost/unit** x **the number of orders per year (Equation 12-12)**

EXAMPLE 10 (pg.531): Probabilistic demand, constant lead time, stockout cost/unit is known

David Rivera Optical has determined that its reorder point for eyeglass frames is 50 ($d \times L$) units. Its carrying cost per frame per year is $5, and stockout (or lost sale) cost is $40 per frame. The store has experienced the following probability distribution for inventory demand during the lead time (reorder period). The optimum number of orders per year is six.

	Number of Units	Probability
	30	.2
	40	.2
ROP →	50	.3
	60	.2
	70	.1
		1.0

How much safety stock should David Rivera keep on hand?

EXAMPLE 10 (pg.531): Probabilistic demand, constant lead time, stockout cost/unit is known

APPROACH ▶ The objective is to find the amount of safety stock that minimizes the sum of the additional inventory holding costs and stockout costs. The annual holding cost is simply the holding cost per unit multiplied by the units added to the ROP. For example, a safety stock of 20 frames, which implies that the new ROP, with safety stock, is $70(= 50 + 20)$, raises the annual carrying cost by $\$5(20) = \$100$.

However, computing annual stockout cost is more interesting. For any level of safety stock, stockout cost is the expected cost of stocking out. We can compute it, as in Equation (12-12), by multiplying the number of frames short (Demand − ROP) by the probability of demand at that level, by the stockout cost, by the number of times per year the stockout can occur (which in our case is the number of orders per year). Then we add stockout costs for each possible stockout level for a given ROP.

Safety Stock Example (Stochastic demand and constant lead time)

ROP = 50 units　　　　Stockout cost = $40 per frame

Opt. # of Orders per year (N) = 6

Carrying cost = $5 per frame per year　(SS ???)

NUMBER OF UNITS	PROBABILITY
30	.2
40	.2
ROP → 50	.3
60	.2
70	.1
	1.0

Safety Stock Example

ROP = 50 units
Orders per year = 6

Stockout cost = $40 per frame
Carrying cost = $5 per frame per year

SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	(20)($5) = $100	$0	$100
10	(10)($5) = $ 50	(10)(.1)($40)(6) = $240	$290
0	$ 0	(10)(.2)($40)(6) + (20)(.1)($40)(6) = $960	$960

A safety stock of 20 frames gives the **lowest total cost**

ROP = 50 + 20 = 70 frames

Probabilistic Models to Determine ROP and Safety Stock (when the cost of stockouts cannot be determined)

- ✓ **Desired service levels are used to set safety stock**

$$ROP = \text{demand during lead time} + Z\sigma_{dLT}$$

where Z = Number of standard deviations below (or above) the mean

σ_{dLT} = Standard deviation of demand during lead time

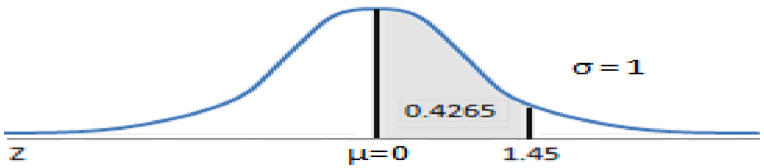
From non-standard normal to standard normal

- X is a normal random variable with mean μ, and standard deviation σ
- Set $Z=(X-\mu)/\sigma$

Z=standard unit or z-score of X

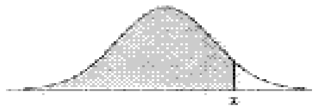
Then Z has a standard normal distribution with mean 0 and standard deviation of 1.

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

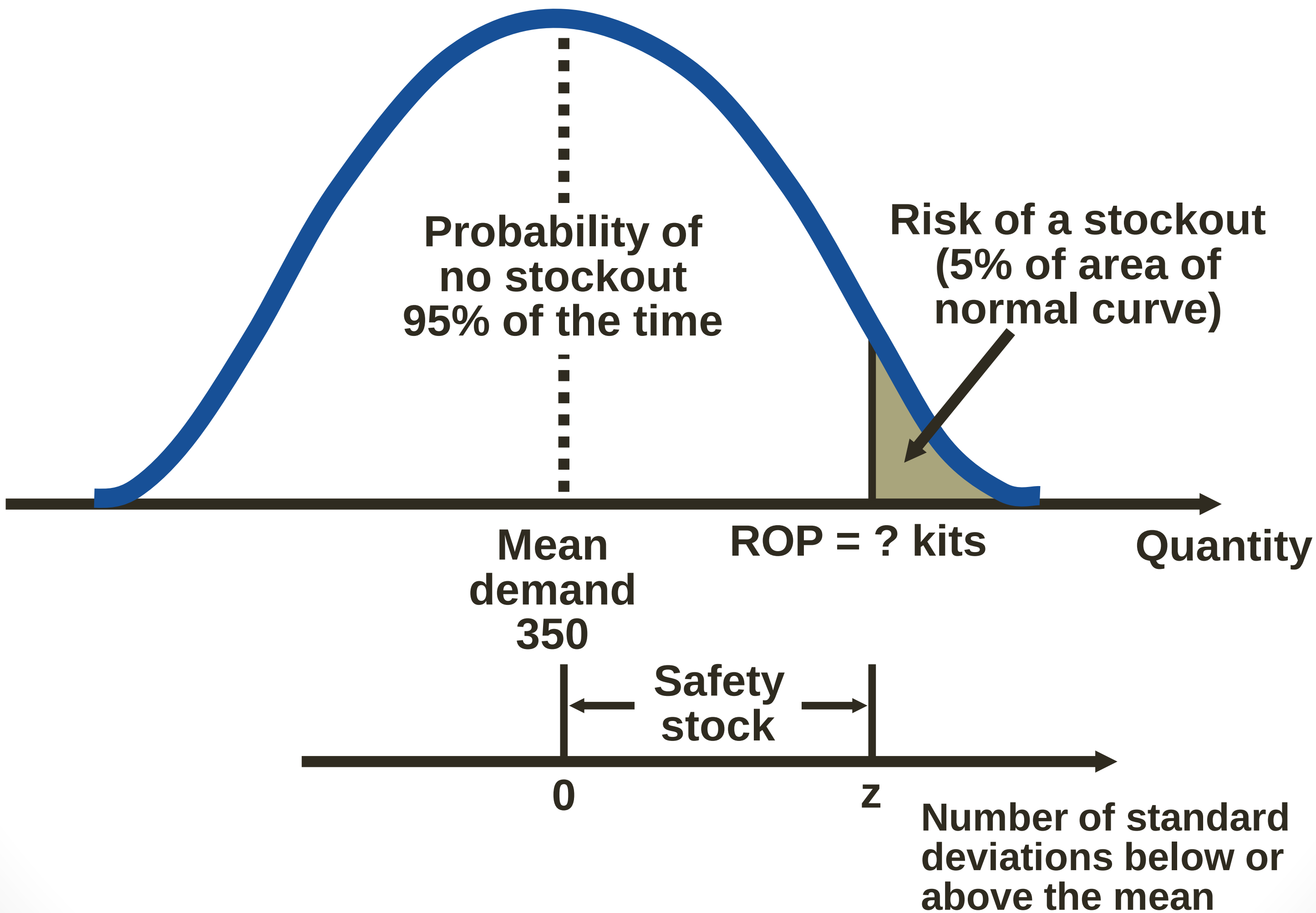
Tables of the Normal Distribution



Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Probabilistic Demand



Probabilistic Example

μ =**Average demand during lead time** = 350 resuscitation kits

σ_{dLT} =**Standard deviation of demand during lead time** = 10 kits

Z =5% stockout policy (service level = 95%)

Using Appendix I, for an area under the curve of 95%, the Z = 1.65

Safety stock = $Z\sigma_{dLT}$ = 1.65(10) = 16.5 kits

Reorder point = Expected demand during lead time + Safety stock

= 350 kits + 16.5 kits of safety stock

= 366.5 or 367 kits

Probabilistic Demand

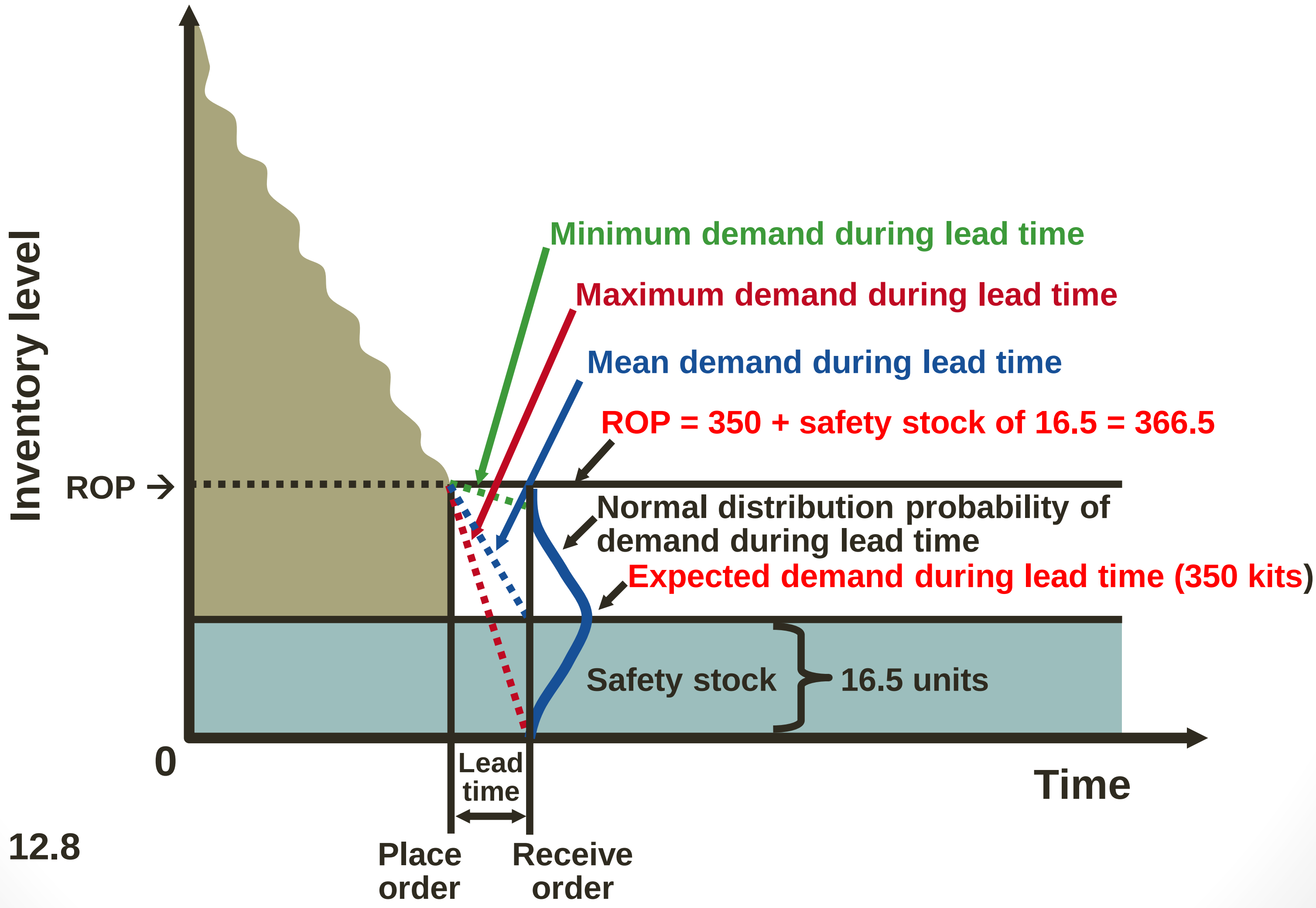


Figure 12.8

Other Probabilistic Models to determine SS and ROP

- When data on demand during lead time is not available, there are other models available
  1. When demand per day is variable and lead time (in days) is constant
  2. When lead time (in days) is variable and demand per day is constant
  3. When both demand per day and lead time (in days) are variable

Demand per day is variable and lead time (in days) is constant

ROP =(*Average* daily demand)

* Lead time in days) + $Z\sigma_{dLT}$

where $\sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$

σ_d= standard deviation of demand per day

Example

Average daily demand (normally distributed) = 15
Lead time in days (constant) = 2
Standard deviation of daily demand = 5
Service level = 90%

Z for 90% = 1.28
From Appendix I

$$\begin{aligned} ROP &= (15 \text{ units x } 2 \text{ days}) + Z\sigma_{dLT} \\ &= 30 + 1.28(5)(\sqrt{2}) \\ &= 30 + 9.02 = 39.02 \approx 39 \end{aligned}$$

Safety stock is about 9 computers

Lead time (in days) is variable and demand per day is constant

ROP = (Daily demand * *Average* lead time in days) + Z * (Daily demand) * σ_{LT}

where σ_{LT} = Standard deviation of lead time in days

Example

Daily demand (constant) = 10
Average lead time = 6 days
Standard deviation of lead time = σ_{LT} = 1
Service level = 98%, so Z (from Appendix I) = 2.055

ROP = (10 units x 6 days) + 2.055(10 units)(1)
= 60 + 20.55 = 80.55

Reorder point is about 81 cameras

Both demand per day and lead time (in days) are variable

ROP = (Average daily demand x Average lead time) + $Z\sigma_{dLT}$

where σ_d = Standard deviation of demand per day

σ_{LT} = Standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma_{LT}^2}$$

Example

Average daily demand (normally distributed) = 150
Standard deviation = σ_d = 16
Average lead time 5 days (normally distributed)
Standard deviation = σ_{LT} = 1 day
Service level = 95%, so Z = 1.65 (from Appendix I)

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.65\sigma_{dLT}$$

$$\sigma_{dLT} = \sqrt{\left(5 \text{ days} \times 16^2\right) + \left(150^2 \times 1^2\right)} = \sqrt{\left(5 \times 256\right) + \left(22{,}500 \times 1\right)}$$

$$= \sqrt{\left(1{,}280\right) + \left(22{,}500\right)} = \sqrt{23{,}780} \approx 154$$

$$\text{ROP} = (150 \times 5) + 1.65(154) \approx 750 + 254 = 1{,}004 \text{ packs}$$

Single-Period Inventory Model

Used to handle ordering of perishables (fresh fruits, flowers) and other items with limited useful lives (newspapers, spare parts for specialized equipment).

Single-Period Inventory Model

- In a single-period model, items are received in the beginning of a period and sold during the same period. The unsold items are not carried over to the next period.
- The unsold items may be a total waste, or sold at a reduced price, or returned to the producer at some price less than the original purchase price.
- The revenue generated by the unsold items is called the <u>salvage value.</u>

Single Period Model

- Only one order is placed for a product
- Units have little or no value at the end of the sales period

C_s = Cost of shortage = Cost of understocking

= Sales price/unit – Cost/unit = lost profit

C_o = Cost of overage = Cost of overstocking

= Cost/unit – Salvage value

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single Period Example 15, pg.536

Chris Ellis's newsstand, just outside the Smithsonian subway station in Washington, DC, usually sells 120 copies of the *Washington Post* each day. Chris believes the sale of the *Post* is normally distributed, with a standard deviation of 15 papers. He pays 70 cents for each paper, which sells for \$1.25. The *Post* gives him a 30-cent credit for each unsold paper. He wants to determine how many papers he should order each day and the stockout risk for that quantity.

Single Period Example 15, pg.536

Average demand = μ = 120 papers/day

Standard deviation = σ = 15 papers

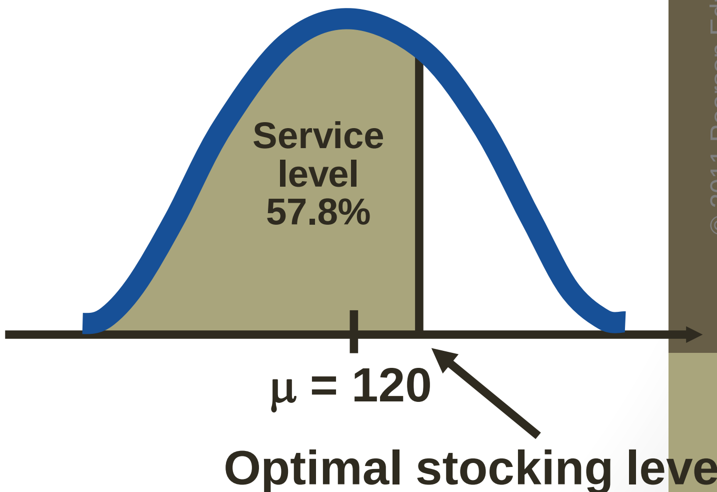
C_s = cost of shortage = \$1.25 - \$.70 = \$.55

C_o = cost of overage = \$.70 - \$.30 = \$.40

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

$$= \frac{.55}{.55 + .40}$$

$$= \frac{.55}{.95} = .578$$



Single Period Example

From Appendix I, for the area .578, $Z \cong .20$

The optimal stocking level

$= 120$ copies $+ (.20)(\sigma)$

$= 120 + (.20)(15) = 120 + 3 = 123$ papers

The stockout risk = 1 – service level

$= 1 – .578 = .422 = 42.2\%$

Fixed-Period (P) Systems

- Orders placed at the end of a fixed period
- Inventory counted only at the end of period
- Order brings inventory up to target level
  - Only relevant costs are ordering and holding
  - Lead times are known and constant
  - Items are independent of one another

Fixed-Period (P) Systems, also called Periodic Review System

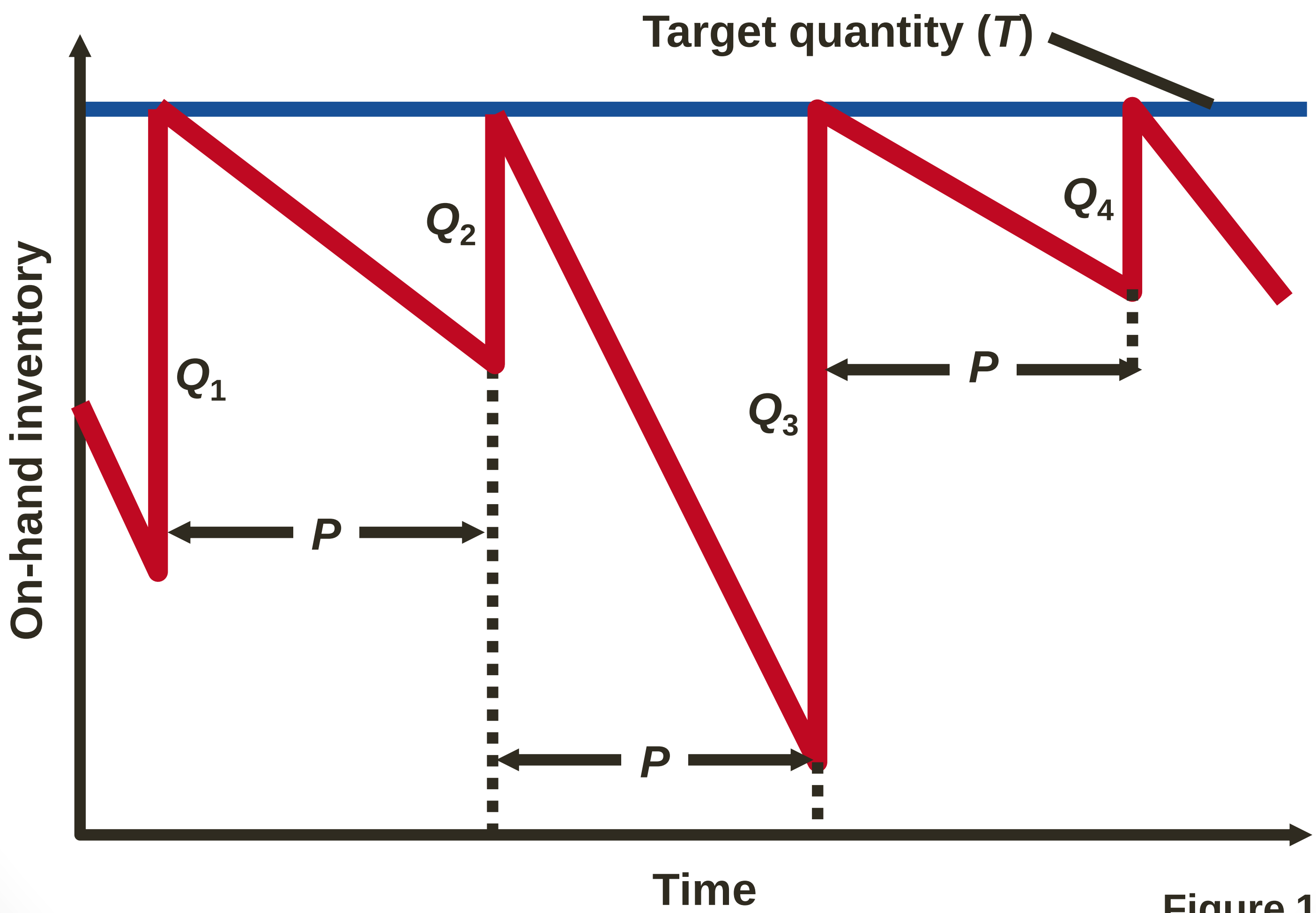


Figure 12.9

Fixed-Period Systems

- Inventory is only counted at each review period
- May be scheduled at convenient times
- Appropriate in routine situations
- May result in stockouts between periods
- May require increased safety stock