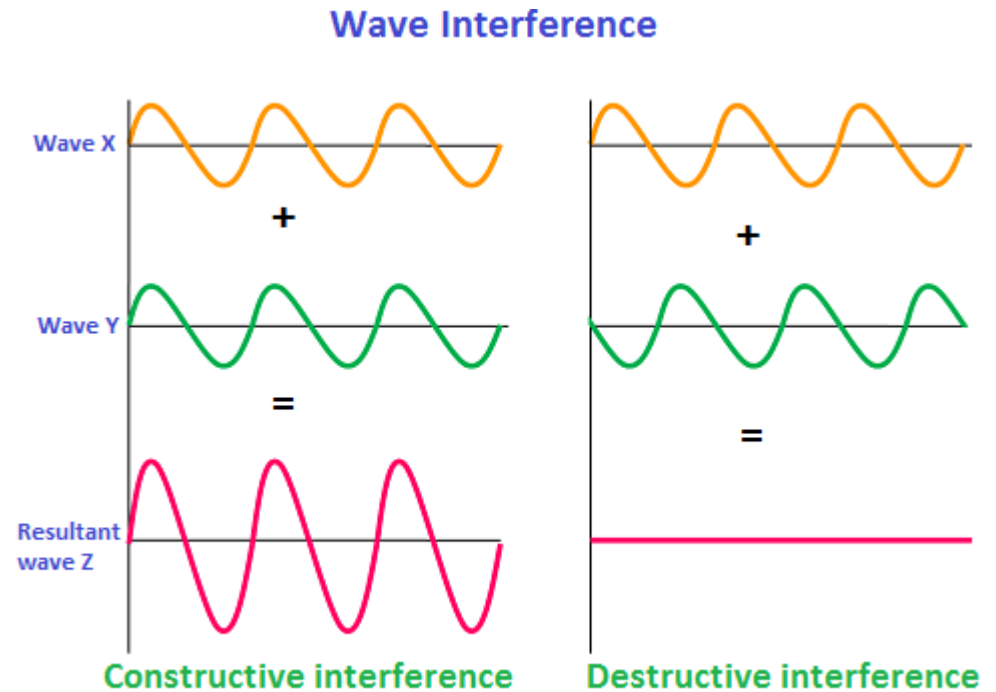
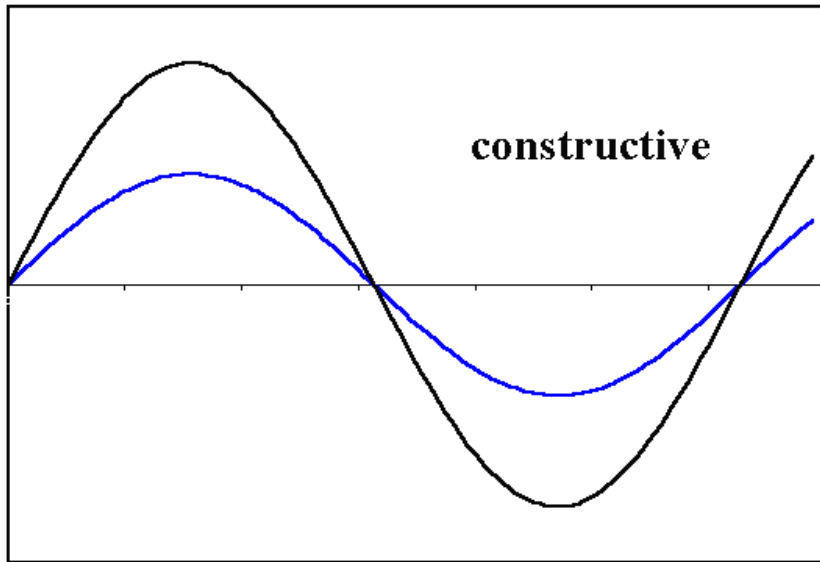


Unit 2

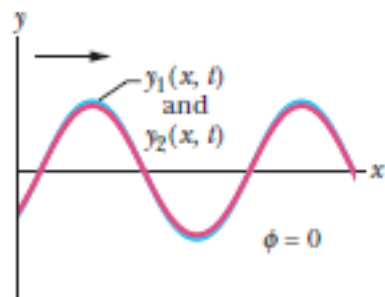
Interference and diffraction

INTERFERENCE OF LIGHT

- “When two light waves superimpose, then the resultant **amplitude/intensity** in the region of superposition is different than the amplitude of individual waves. **This modification in the distribution of intensity in the region of superposition is called interference.**”

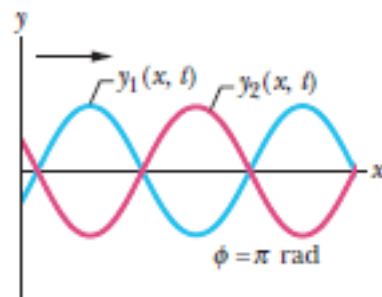


Being exactly in phase, the waves produce a large resultant wave.



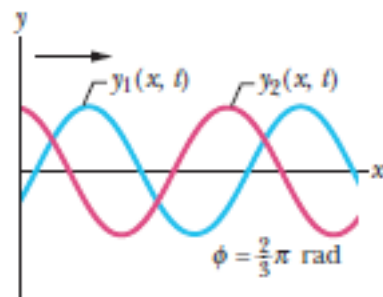
(a)

Being exactly out of phase, they produce a flat string.

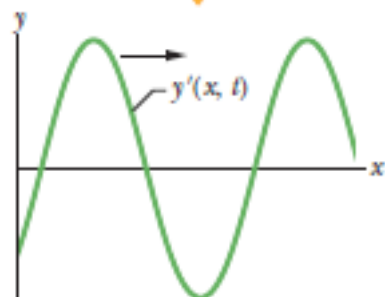


(b)

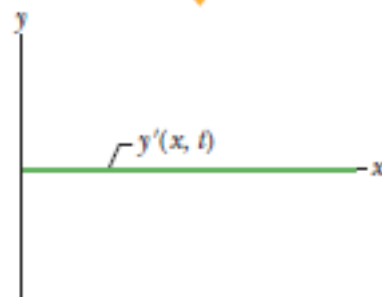
This is an intermediate situation, with an intermediate result.



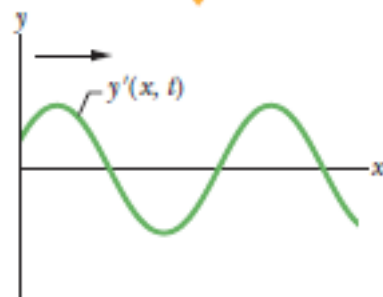
(c)



(d)



(e)



(f)

Fig. 16-13 Two identical sinusoidal waves, $y_1(x, t)$ and $y_2(x, t)$, travel along a string in the positive direction of an x axis. They interfere to give a resultant wave $y'(x, t)$. The resultant wave is what is actually seen on the string. The phase difference ϕ between the two interfering waves is (a) 0 rad or 0° , (b) π rad or 180° , and (c) $\frac{2}{3}\pi$ rad or 120° . The corresponding resultant waves are shown in (d), (e), and (f).

Conditions for sustained interference of light waves

- Two sources should continuously emit waves of **same wavelength or frequency**.
- **The amplitudes of the two interfering waves should be equal** or approximately equal.
- **The sources of light must be coherent sources.**
- **Two sources should be very narrow** as a broad source is equivalent to large number of narrow sources lying side by side which causes loss of interference pattern resulting general illumination.
- **Two sources emitting set of interfering beams must be placed very close** to each other so that wavelength interact at very small angles.

COHERENT SOURCES

- **Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and always have a constant phase difference between them.**
- Therefore, two sources must emit radiations of the same wavelength/color.
- In practice, it is impossible to have two independent sources which are coherent.
- For experimental purposes two virtual sources formed from a single source can act as coherent sources.
- The two sources must be narrow and close to each other because the wavelengths of light waves are extremely small (of the order of 10^{-7} m)

GENERATION OF COHERENT SOURCES

- **There are two ways to generate coherent sources**

a) Division of Wavefront

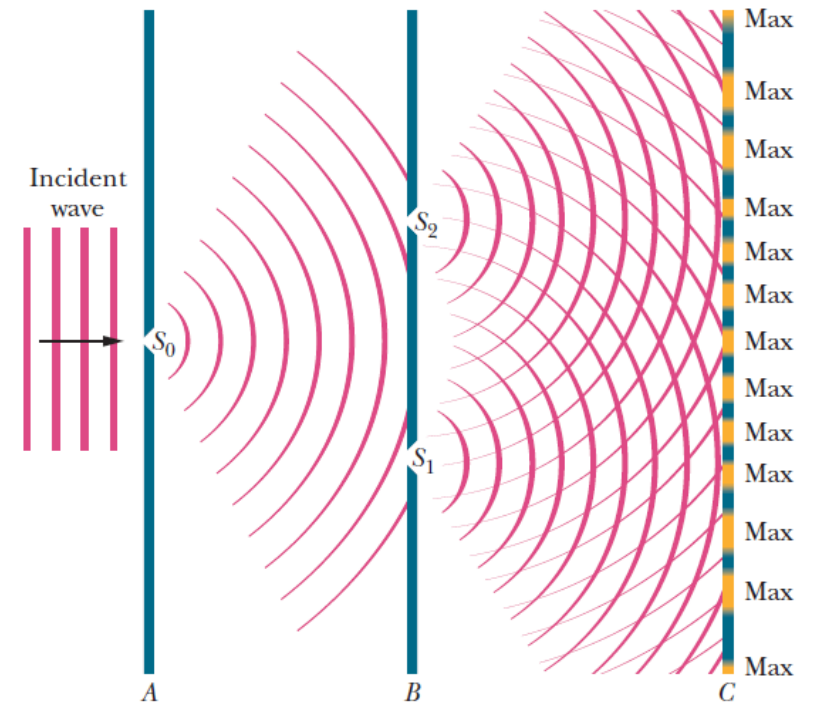
b) Division of Wave amplitude

GENERATION OF COHERENT SOURCES

- Division of Wavefront

- The wavefront originating from a source of light is divided into two parts which serves the purpose of coherent sources.
- These two parts of the same wavefront travel unequal distances and reunite at some angle to produce interference bands.
- E.g. Youngs's double slit experiment and Fresnel biprism.
- Path difference $\Delta = xd/D$
where x is the distance between two consecutive bright/dark fringes, d is the distance between two slits and D is the distance between source and screen.
- The spacing between any consecutive maxima or minima is expressed by fringe width (β).

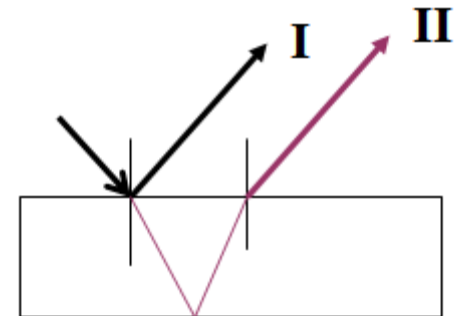
$$\beta = \lambda D/d$$



- Division of Wave Amplitude

- The amplitude of the beam is divided into two parts by partial reflection or refraction methods.
- The waves corresponding to the divided parts travel different paths and hence produce interference.
- E.g. Interference due to thin films, wedge shaped film interference, Newton's rings.
- The path difference, $\Delta = 2\mu t \cos r$, where t is the thickness of thin film, r is the angle of reflection and μ is the refractive index of the material of film.

Thin film Interference



CONDITION FOR MAXIMA AND MINIMA:

- Maximum intensity of light is observed at a point where the phase difference between the two waves reaching the point is a whole number multiple of 2π or the path difference between the two waves is a whole number multiple of wavelength (λ).
- Minimum intensity of light is observed at a point where the phase difference between the two waves reaching the point is an odd number multiple of π or the path difference between the two waves is an odd number multiple of half wavelength ($\lambda/2$).

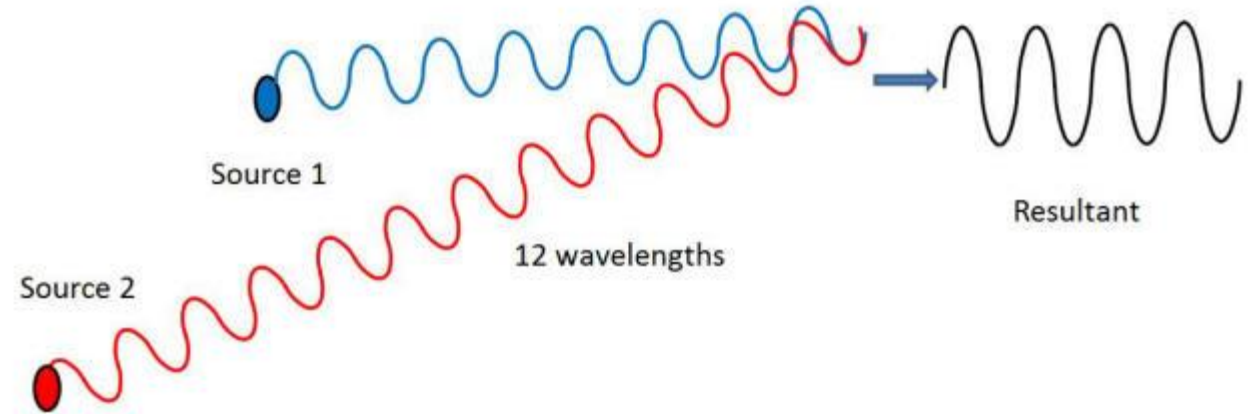
Relation between Phase Difference and Path Difference

- If the path difference between the two waves is λ , the corresponding phase difference is 2π
- Hence, for path difference x , the phase difference $\delta = 2\pi x/\lambda$

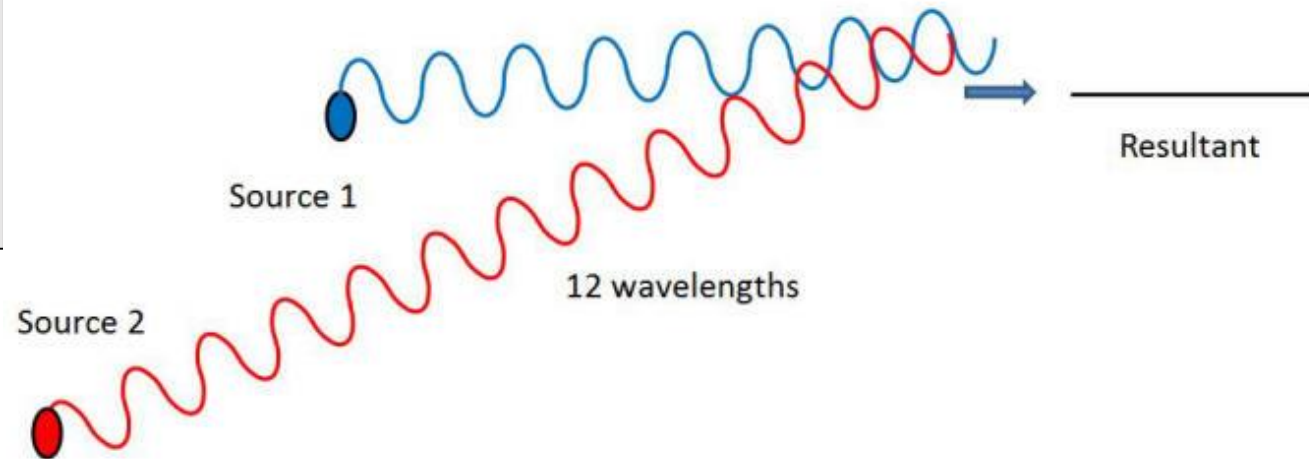
Or

$$\text{Phase difference, } \delta = (\text{path difference}) * (2\pi/\lambda)$$

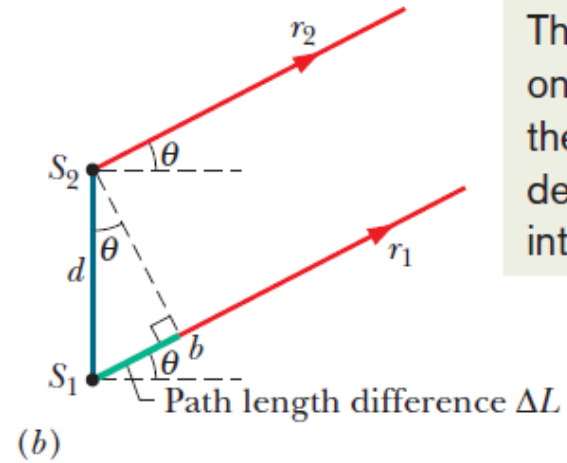
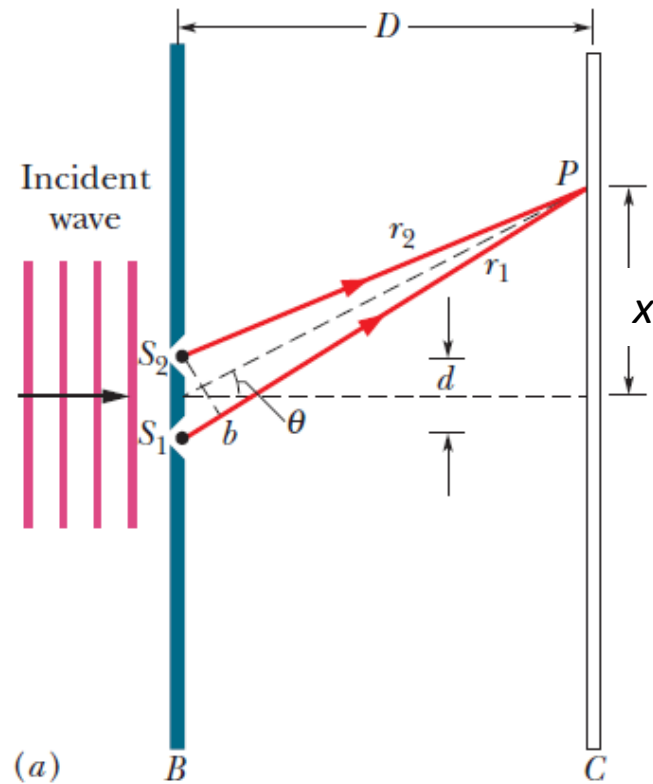
Constructive Interference



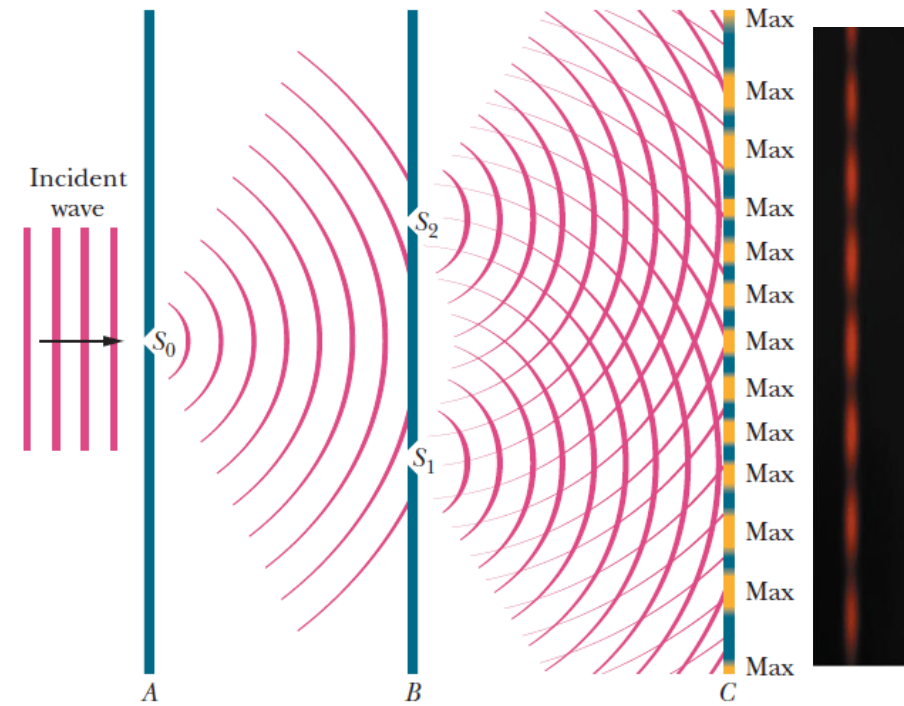
Destructive Interference



Young's Interference Experiment (Not in Syllabus)



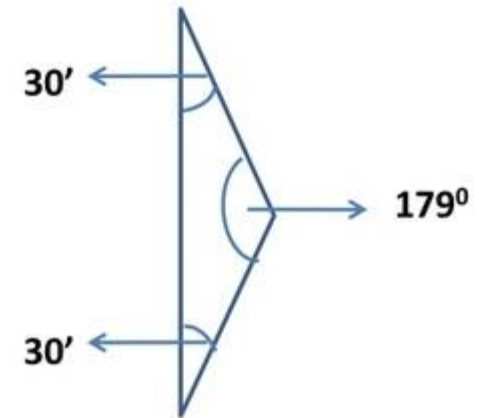
The ΔL shifts one wave from the other, which determines the interference.



FRESNEL BIPRISM INTERFERENCE

Fresnel Biprism

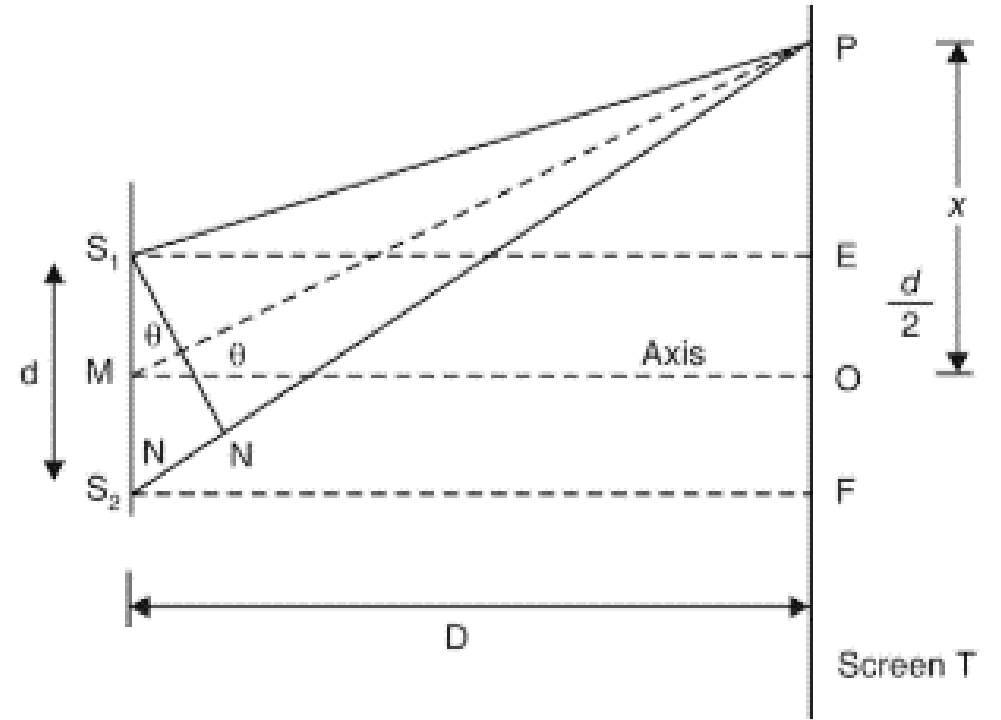
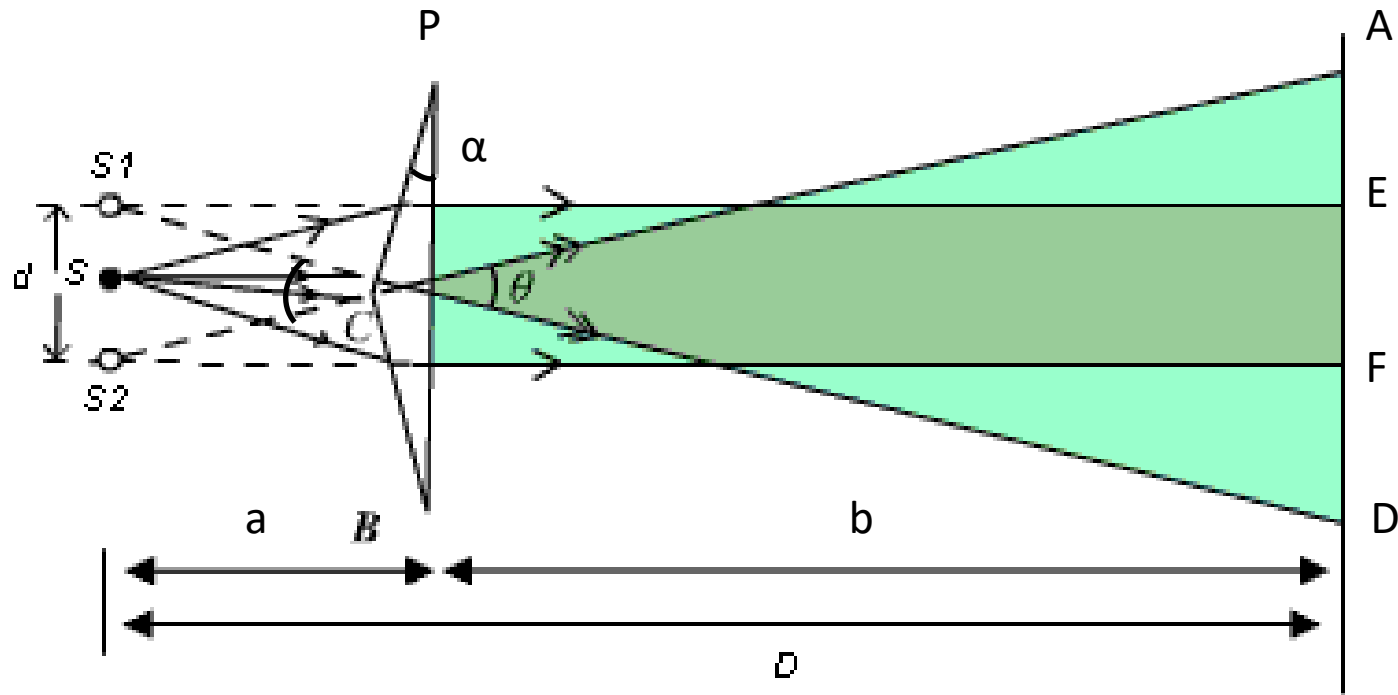
- Fresnel biprism consist of two used a biprism to obtain coherent source to produce Interference.
- The biprism consists of two prisms of very small refracting angles joined base to base.
- The biprism is made from a thin glass plate by grinding and polishing, so that it is a single prism with one of the angles about 179° (obtuse angle) and the two about $30'$ each (acute angle).



Experimental arrangement

- The biprism is mounted suitably on an optical bench
- A monochromatic light source such as sodium vapor lamp illuminates a vertical slit S.
- The biprism is placed in such a way that its refracting edge is parallel to the length of the slit S.
- A cylindrical wavefront impinges on both prisms.
- The top portion of wavefront is refracted downwards and appears to have emanated from the virtual image S_1 .
- The lower segments, falling on the lower part of the biprism is refracted upwards and appears to have emanated from the virtual source S_2 .
- The virtual sources S_1 and S_2 are coherent.

FRESNEL BIPRISM



Optical Path Difference between the Waves at P

Let the point P be at a distance x from O (Fig.6.8). Then

$$PE = x - d / 2 \quad \text{and} \quad PF = x + d / 2,$$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right] - \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right]$$

$$\therefore (S_2P)^2 - (S_1P)^2 = 2xd$$

$$\text{or} \quad S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

We can approximate that $S_2P \cong S_1P \cong D$.

$$\therefore \quad \text{Path difference} = S_2P - S_1P = \frac{xd}{D} \quad (6.6)$$

We now find out the conditions for observing bright and dark fringes on the screen.

- **Condition for bright fringes**

The condition for finding a bright fringe at P is that

$$S_2P - S_1P = m\lambda$$

This implies that

$$\frac{xd}{D} = m\lambda$$

where m is called order of the fringe. The bright fringe at the center O corresponds to the $m = 0$; and called zero order fringe. The first order is $m = 1$, second order $m = 2 \dots$ so on

- **Condition for dark fringes**

The condition for finding a bright fringe at P is that

$$S_2P - S_1P = (2m+1) \lambda/2$$

This implies that

$$\frac{xd}{D} = (2m+1) \lambda/2$$

The first order dark fringe is $m = 1$, second order dark fringe $m = 2 \dots$ so on

Separation between Neighbouring Bright Fringes

The m^{th} order bright fringe occurs when

$$x_m = \frac{m\lambda D}{d}$$

and the $(m + 1)^{\text{th}}$ order bright fringe occurs when

$$x_{m+1} = \frac{(m + 1)\lambda D}{d}$$

The bright fringe separation, β is given by

$$\beta = x_{m+1} - x_m = \frac{\lambda D}{d}$$

The same result will be obtained for dark fringes. Thus, neighbouring bright and dark fringes are separated by the same amount everywhere on the screen. The separation β is called the *fringe width*.

The width of the dark or bright fringe is given by equ.(6.9).

$$\beta = \frac{\lambda D}{d}$$

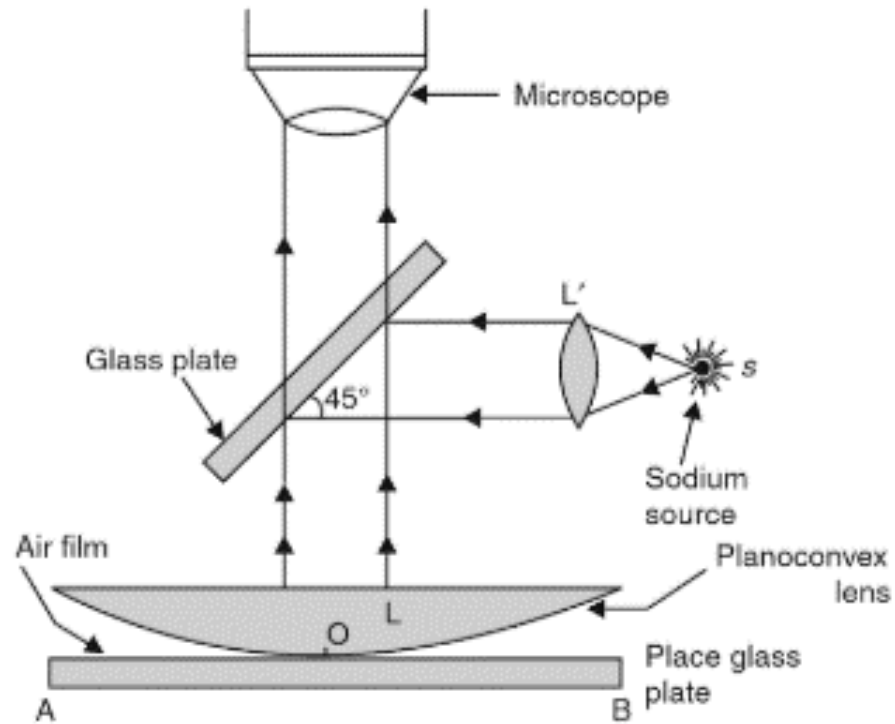
where $D(= a + b)$ is the distance of the sources from the eye-piece.

Newton's Ring

Newton's Ring

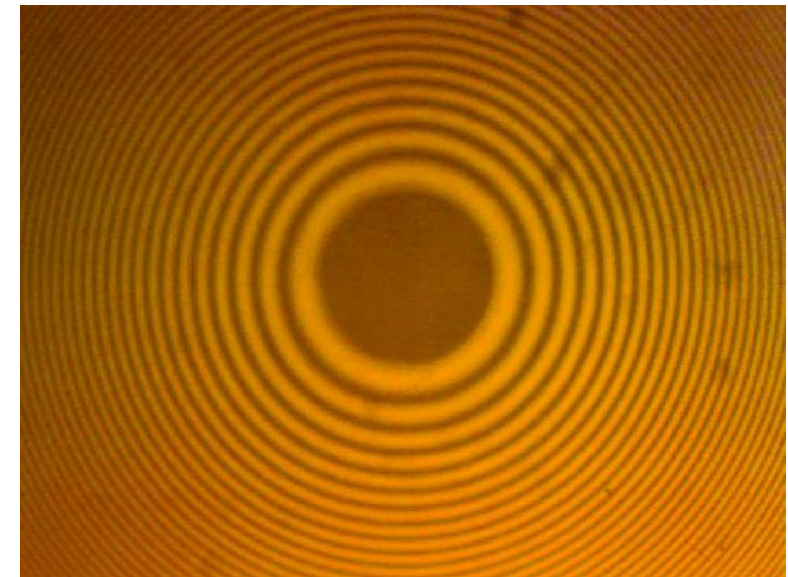
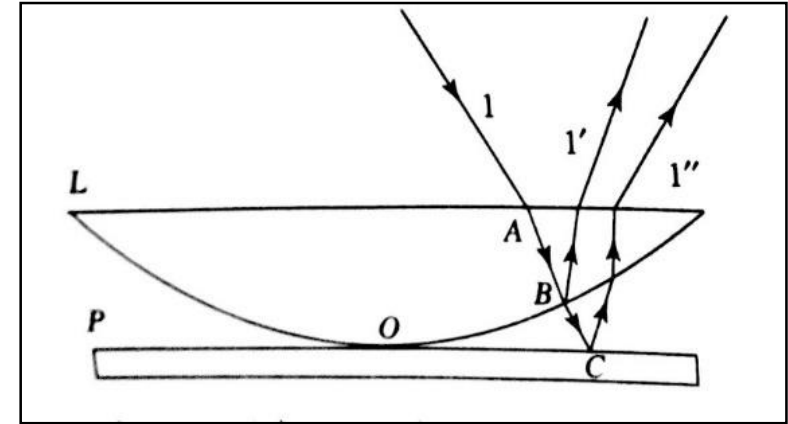
- When two glass plates are kept inclined to each other, a wedge-shaped air film is formed and bright and dark fringes are observed when the plates are illuminated by monochromatic light.
- If a glass plate and a plano convex lens is used, again a wedge-shaped air film is formed but with circular geometry.
- The illumination by monochromatic light will produce bright and dark rings.
- This phenomenon was explained by Newton and hence known as Newton's rings.
- A plano convex lens with its convex surface is placed on a plane glass plate; an air film with gradually increasing thickness is produced between the two surfaces.
- At the point of contact of the lens and plane surface, the thickness of the film is zero and becomes maximum at the edge of the lens and plane surfaces.
- A monochromatic source, incident normally is used to obtain bright and dark concentric rings around a point of contact between lens and the glass plate.

Experimental arrangement for observing newton's Ring



EXPLANATION OF THE FORMATION OF THE RINGS

- Newton's rings are formed due to interference between the two rays $1'$ and $1''$, as a result of reflection from the top and bottom surfaces of the air film formed between the lens and the plate.
- Monochromatic ray 1 of light falls normally on the lens-plate system at the point A.
- At the point B on the glass-air boundary, the light gets partially reflected out as ray $1'$ without any phase change.
- The remaining part is refracted along BC and reflected at the point C with a phase change of π radians and emerges out as ray $1''$.
- The two reflected rays are derived from the same ray 1 and hence produce interference.
- For a very small wedge angle θ , and for normal incidence, $r = 0$, the path difference between the two reflected waves $1'$ and $1''$ is $[2\mu t + \lambda/2]$
- At the point of contact $t = 0$, so the path difference is $(\lambda/2)$.
- This is the condition for minimum intensity and hence the central spot is dark.
- The condition for n^{th} maxima is
- $2\mu t + \lambda/2 = n\lambda, n = 1, 2, 3, \dots$



- To calculate the diameter of dark and bright rings, consider GCH to be the plano-convex lens placed on a glass plate AB.
- Let R be the radius of curvature of the lens.
- Point C is the point of contact between plate AB and lens GCH
- Regions GCA and HCB is the wedge-shape circular air film.
- Newton's rings are formed due to this air film.
- Let r be the radius of Newton's rings corresponding to constant film thickness t, the locus of which forms a locus of points of a circle with center on point C.
- Now from the property of circle,

$$IE \times IF = IC \times ID$$

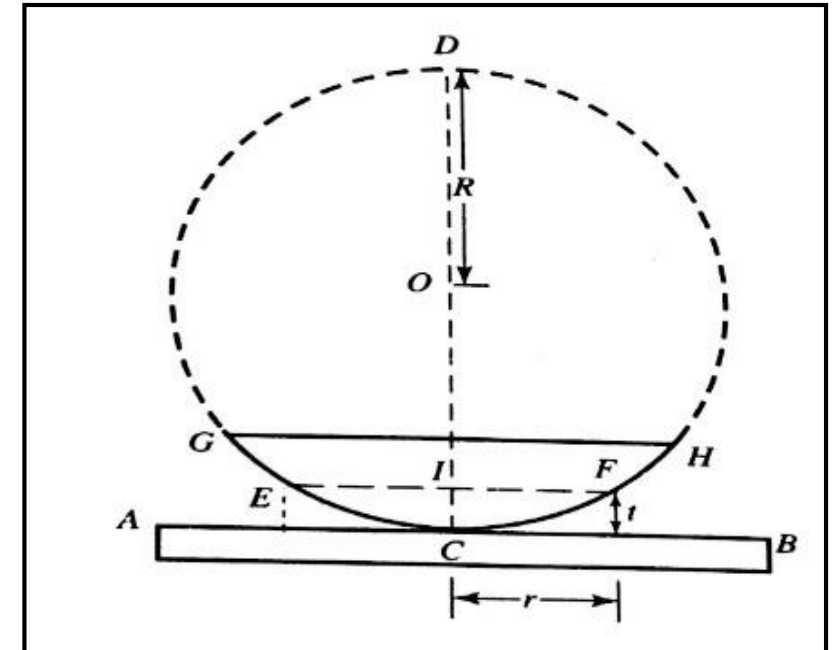
- But $IE = IF = r$, radius of the rings,

$$IC = t \text{ and } ID = 2R - t$$

- Therefore,

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$



- $r^2 = 2Rt$ $[t \ll R \text{ and hence } t^2 \text{ can be neglected compared to } 2Rt]$
- Now if D_n is the diameter of the nth ring, $D_n^2 = 4r^2$
- Therefore,

$$D_n^2/4 = 2Rt$$

Or

$$2t = D_n^2/4R$$

For Bright Rings

$$2\mu t = (2n - 1)\lambda/2$$

$$D_n^2/4R = (2n - 1)\lambda/2\mu$$

$$D_n^2 = (2n - 1) \cdot 2\lambda R/\mu$$

$$D_n \propto \sqrt{2n-1}$$

$$D_n \propto \sqrt{\lambda}$$

$$D_n \propto \sqrt{R}$$

For Dark Rings

$$2\mu t = n\lambda$$

$$D_n^2/4R = n\lambda/\mu$$

$$D_n^2 = 4n\lambda R/\mu$$

$$D_n \propto \sqrt{n}$$

$$D_n \propto \sqrt{\lambda}$$

$$D_n \propto \sqrt{R}$$

- This shows that the diameter of the rings is proportional to square root of λ and R .
- Also the diameter of bright and dark rings is proportional to $\sqrt{2n-1}$ and \sqrt{n} respectively.
- Therefore the diameter of the bright rings reduces faster than dark rings.
- So as the order of rings increases, thinner and sharper rings are obtained.

Application of Newton's Ring Experiment

a) TO DETERMINE WAVELENGTH OR RADIUS OF CURVATURE OF LENS:

- Let the diameter of n^{th} and $(n+m)^{\text{th}}$ dark rings are D_n and D_{n+m}

$$D_n^2 = 4n\lambda R \quad \text{and} \quad D_{n+m}^2 = 4(n+m)\lambda R$$

- $D_{n+m}^2 - D_n^2 = 4(n+m)\lambda R - 4n\lambda R = 4m\lambda R$

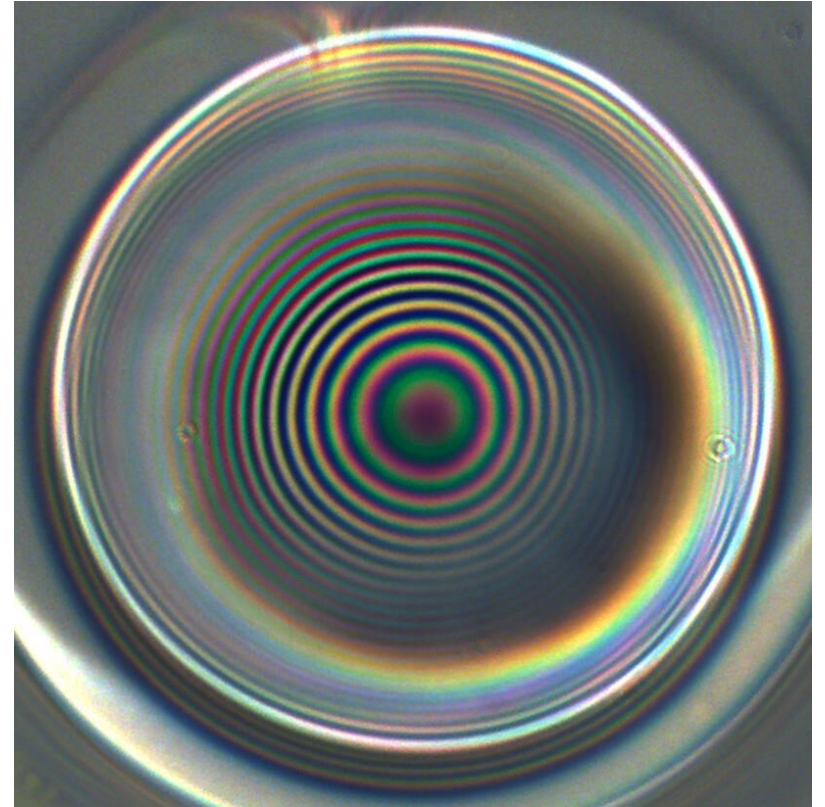
- $\lambda = [D_{n+m}^2 - D_n^2] / 4mR$ and $R = [D_{n+m}^2 - D_n^2] / 4m\lambda$

b) TO DETERMINE THE REFRACTIVE INDEX (μ) OF A LIQUID

- In air medium, let the diameter of n th and $(n+m)$ th dark rings are D_n and D_{n+m} respectively
- $D_n^2 = 4n\lambda R$ and $D_{(n+m)}^2 = 4(n+m)\lambda R$
- $D_{(n+m)}^2 - D_n^2 = 4(n+m)\lambda R - 4n\lambda R = 4m\lambda R$
- Now, let the diameter of dark rings with liquid of refractive index μ be
- $d_n^2 = 4n\lambda R/\mu$ and $d_{(n+m)}^2 = 4(n+m)\lambda R/\mu$
- $d_{(n+m)}^2 - d_n^2 = 4m\lambda R/\mu$
- $\mu = [D_{(n+m)}^2 - D_n^2] / [d_{(n+m)}^2 - d_n^2]$

NEWTON'S RINGS WITH WHITE LIGHT

- If we use white light like a mercury source, colored rings are obtained.
- In this case, the diameters of different rings are different for different colors as it depends on $\sqrt{\lambda}$.
- So the first few colored rings are seen clearly, after which overlapping of colors occurs and the rings cannot be seen distinctly.



Diffraction

Diffraction

What it is?

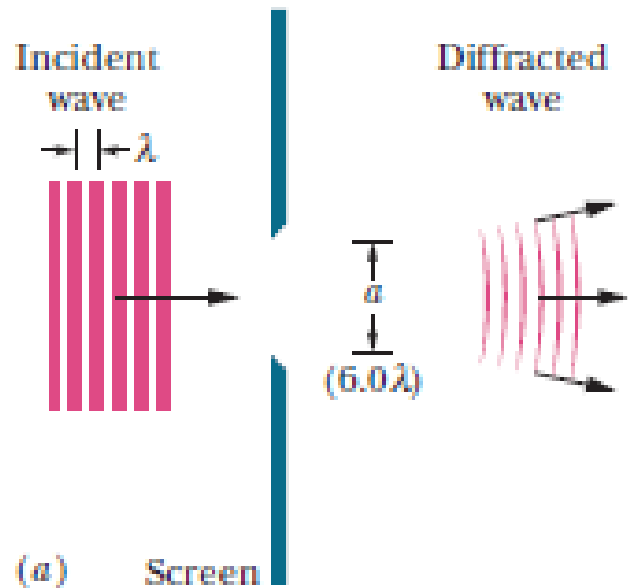
- When light falls on obstacles or small apertures whose size is comparable with wavelength of light, there is a deviation from straight line, the light bends around the corner of the obstacle/aperture and enters in geometrical shadow. This bending of light is called **DIFFRACTION**.

How to observe diffraction?

- Diffraction is produced when the wavelength of wave is similar to the size of the obstacles or aperture.
- For example, the diffraction of sound waves is commonly observed because the wavelength of sound is similar to the size of obstacles (in meters)
- Light will diffract around a single slit or obstacles.
- The resulting pattern of bright and dark fringes on a screen is called a diffraction pattern.
- This pattern arises because different points along a slit create wavelets that interfere with each other just as a double slit would.

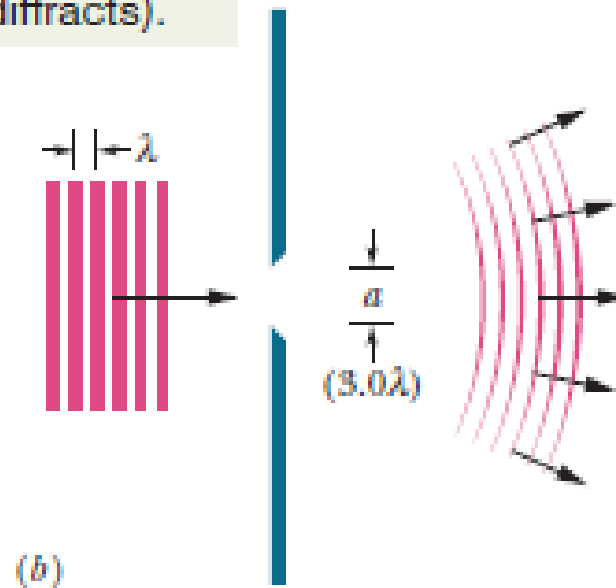
Dependence of the phenomenon on wavelength

$$a \gg \lambda$$

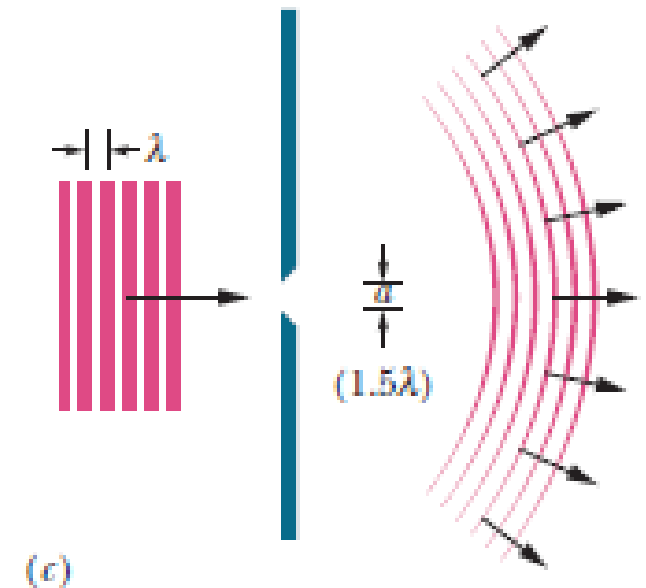


$$a = \lambda$$

A wave passing through a slit flares (diffracts).

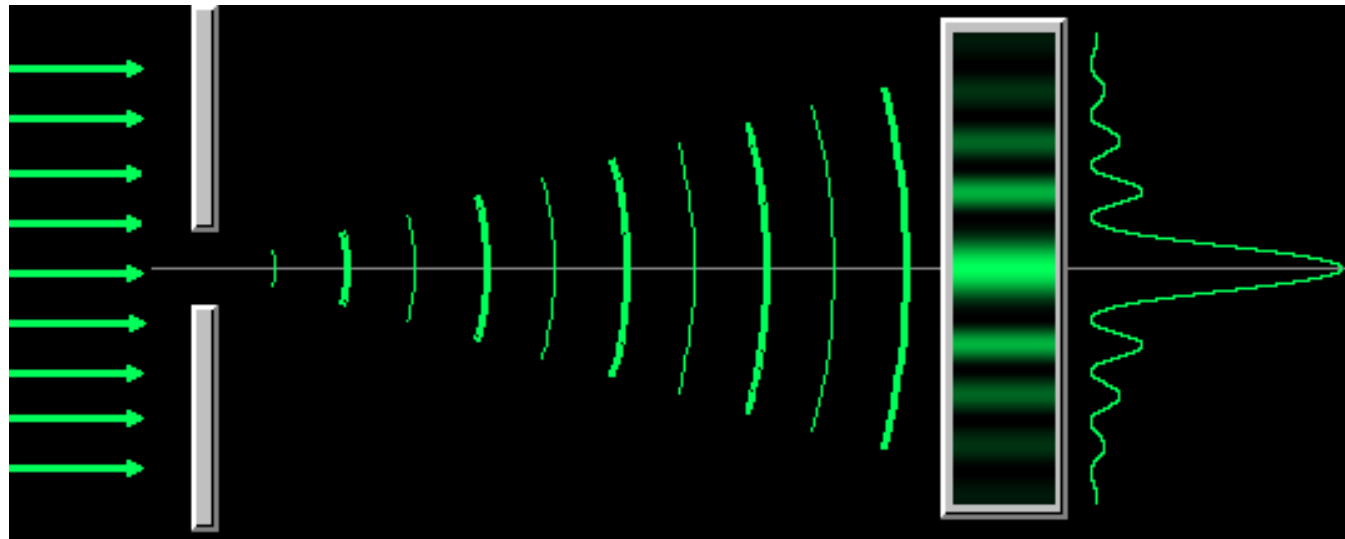


$$a \ll \lambda$$



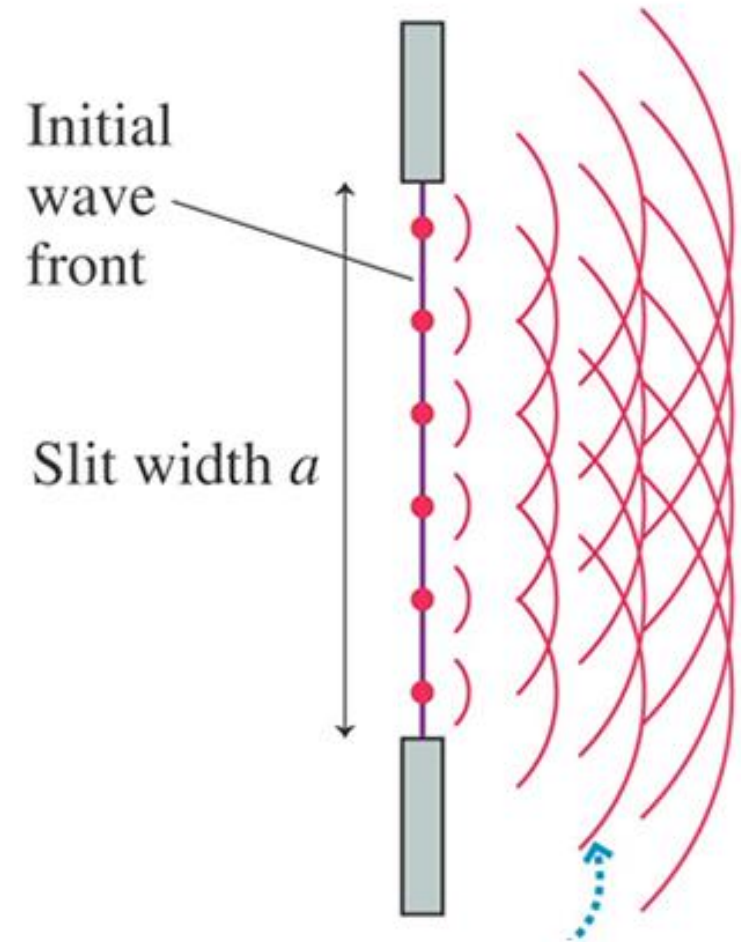
Diffraction pattern

- A light passing through a tiny opening such as slit or aperture produces alternate region of darkness and brightness beyond the region of geometric shadow. Such alternate bright and dark bands are known as the **diffraction pattern**.
- The central bright portion is called central maximum and it is bounded on either side by a series of secondary maxima separated by dark bands called minima. The intensity of the bright bands keep on decreasing on moving outwards from the central maximum.



Reason for diffraction pattern (Huygens -Fresnel principle)

- Diffraction occurs due to the interference of secondary wavelet.
- The portion of wavefront that is incident on the opaque portion of the screen is obstructed while a the portion of wavefront that is allowed to pass through the aperture.
- Every point on this portion of the wavefront acts as a center of secondary wavelets.
- Constructing the envelop of these secondary wavelets, wave spreads into the region of geometric shadow bending around the edges of the aperture.



Distinction between Interference and diffraction

| Interference | Diffraction |
|-----------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| Interference is the result of interaction of light coming from different wavefronts originating from the source | Diffraction is the result of interaction of light coming from different parts of the same wavefronts |
| Interference fringes may or may not be of same width | Diffraction fringes are not of same width |
| Regions of minimum intensity are perfectly dark | Regions of minimum intensity are not perfectly dark |
| All bright bands are of same intensity | All bright bands are not of same intensity |

Two types of diffraction

We can define two distinct types of diffraction:

- (a) **Fresnel diffraction** is produced when light from a point source meets an obstacle, the waves are spherical and the pattern observed is a fringed image of the object.
- (b) **Fraunhofer diffraction** occurs with plane wave-fronts with the object effectively at infinity. The pattern is in a particular direction and is a fringed image of the source.

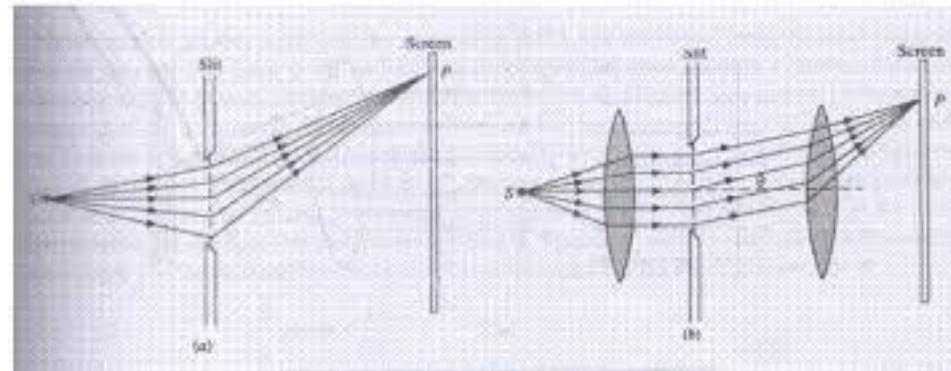
Difference between Fraunhofer and fresnel Diffraction

Fraunhofer diffraction

- Source and screen are at infinite distance from slit
- Incident wavefront on the aperture is plane
- Two biconvex lenses are required to study diffraction in laboratory
- It has many application is designing the optical instrument
- The maximas and minimas are well defined

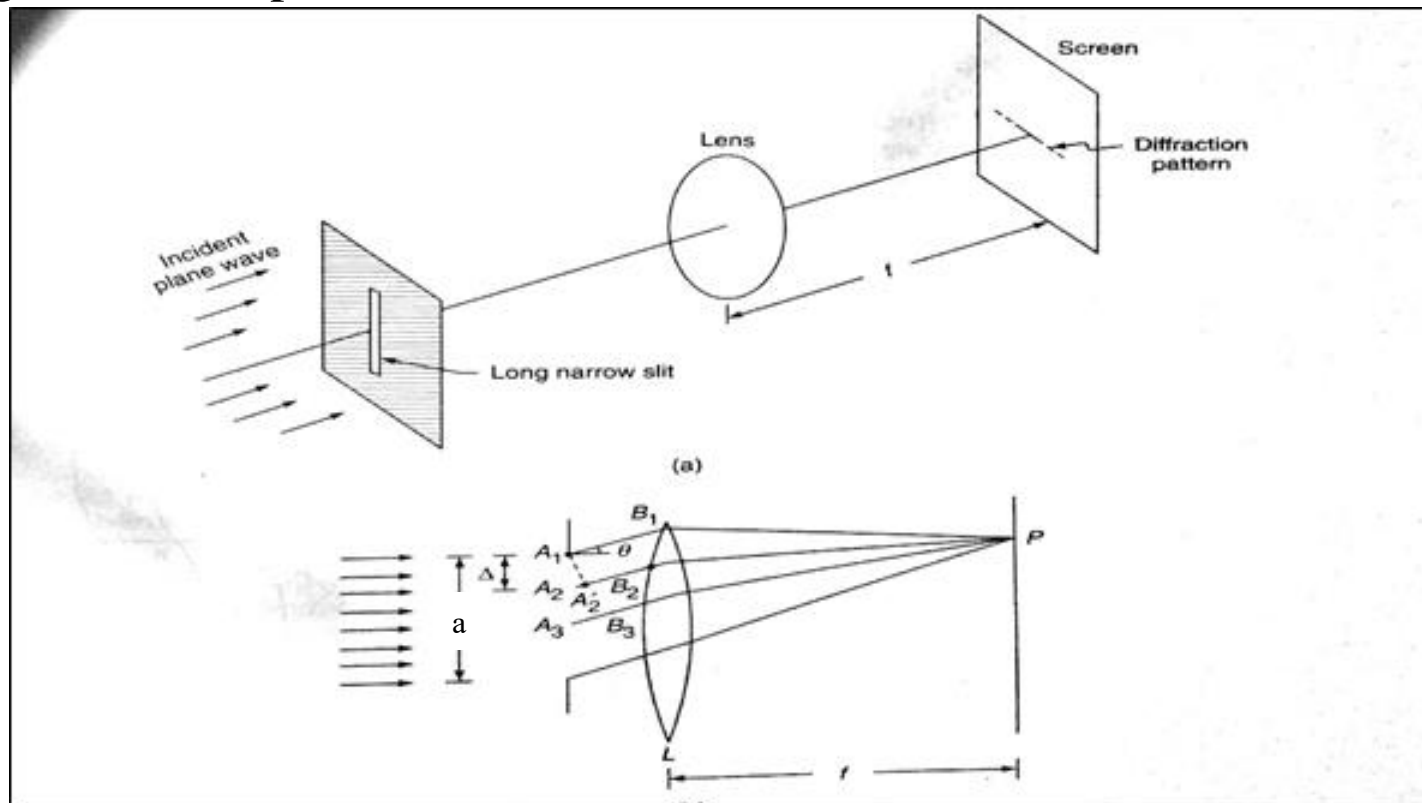
Fresnel diffraction

- Source and screen are at finite distance from slit
- Incident wavefront on the aperture is either spherical or cylindrical
- No lenses are required to study diffraction in the laboratory
- It has less application in designing the optical instrument
- The maximas and minimas are not well defined.

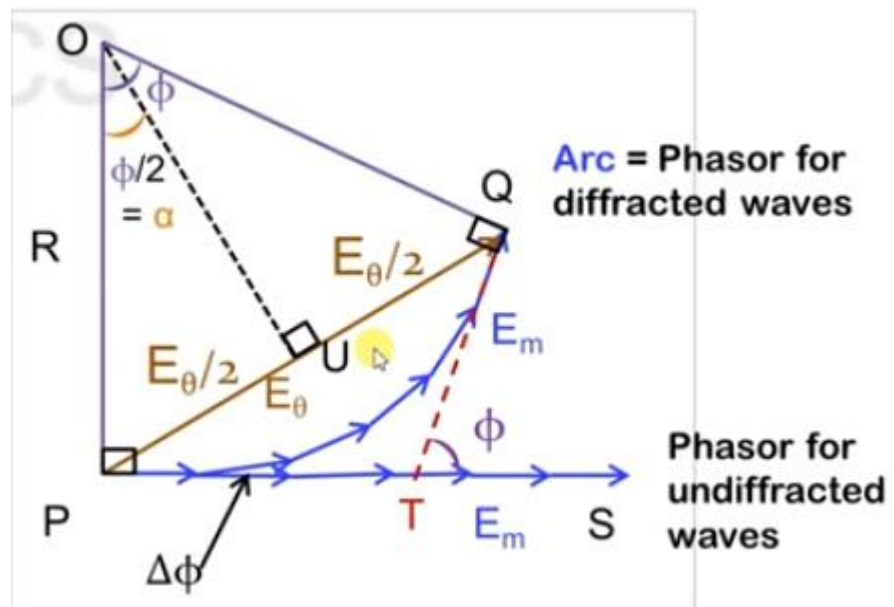
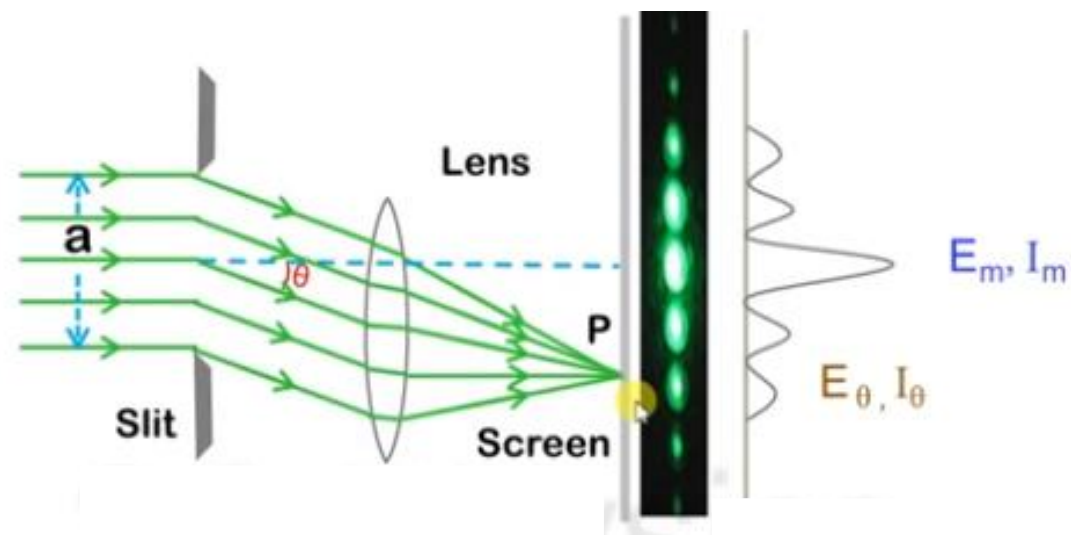
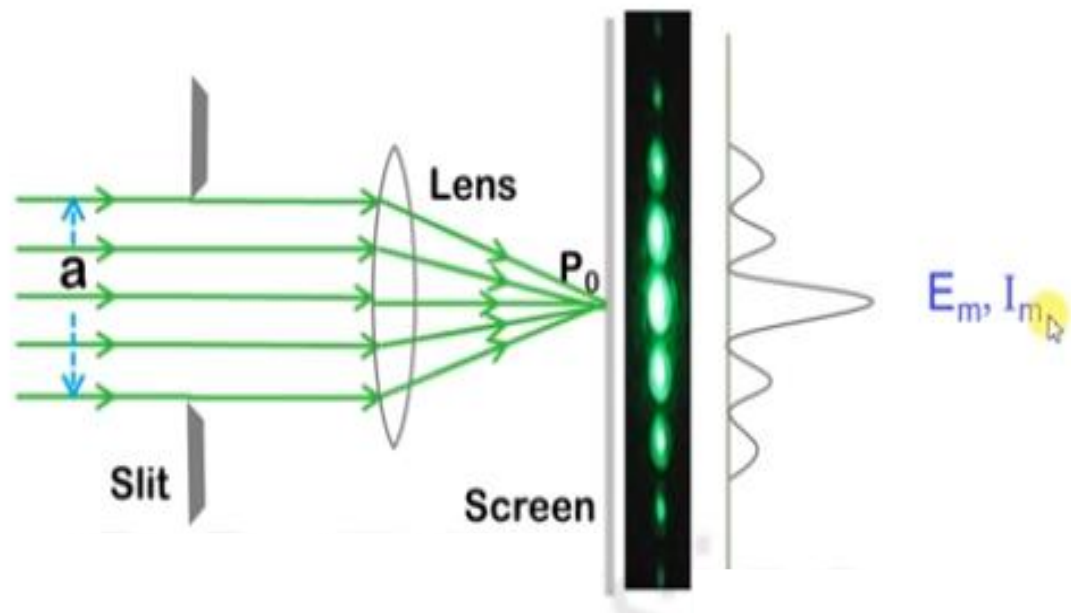


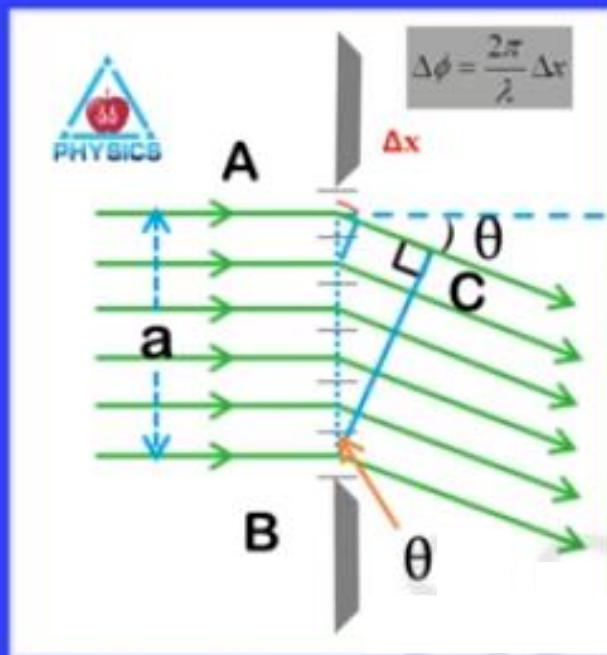
FRAUNHOFER DIFFRACTION AT SINGLE SLIT

- Let us assume that a plane wave is incident normally on the slit with width b and the intensity distribution on the focal plane of lens L is to be calculated.
- The slit is considered to have a large number of equally spaced point sources A_1, A_2, A_3, \dots and each point on the slit is a source of Huygen's secondary wavelets which interfere with the wavelets emanating from other points.



- Hence, for n number of point sources, $a = (n-1) \Delta$.
- Now let us calculate the resultant field produced by these n sources at the point P, P being an arbitrary point receiving parallel rays making an angle θ with the normal to the slit.
- For an incident plane wave, the points A1, A2..... are in phase and, therefore the additional path traversed by the disturbance emanating from the point A2 will be A2A2' where A2 is the foot of perpendicular drawn from A1 on A2B2.
- If the diffracted rays make an angle θ with the normal to the slit then the path difference would be
- $A2A2' = \Delta \sin \theta$
- Corresponding Phase difference is,
$$\phi = \frac{2\pi}{\lambda} \Delta \sin \theta$$



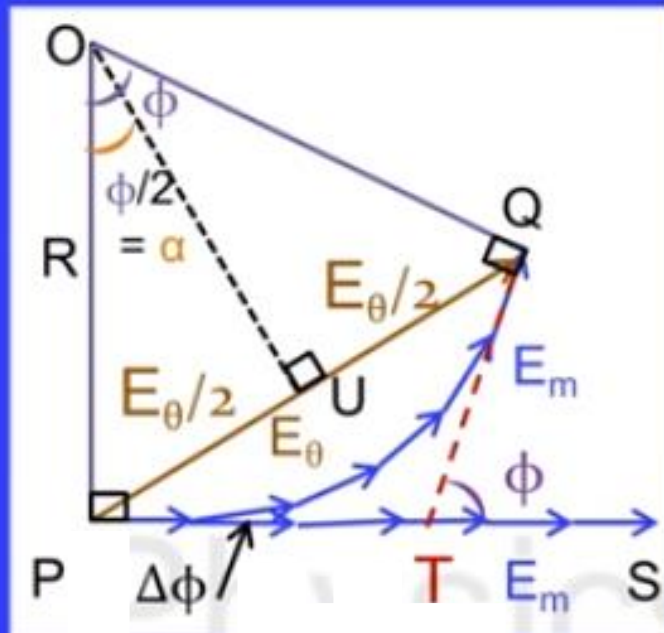


Let the path diff. betⁿ two nearby diffracted waves = Δx

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} \cdot \text{Path difference}$$

Therefore, the phase diff. $\Delta\phi$ betⁿ two nearby diffracted waves =

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$



Total path difference $\Delta = \Delta x + \Delta x + \dots = AC$

Therefore, $\Delta = AC$

From $\triangle ABC$, $\sin \theta = \frac{AC}{AB}$

$$\therefore AC = AB \sin \theta$$

$$\therefore p.d. AC = a \sin \theta$$

$$\therefore \text{phase diff. } \phi = \frac{2\pi}{\lambda} (a \sin \theta)$$

$$\text{Let } \alpha = \frac{\phi}{2} \text{ --- (1)}$$

for simplicity.

$$\therefore \alpha = \frac{\pi}{\lambda} (a \sin \theta)$$

From $\triangle PUO$,

$$\sin \alpha = \frac{E_{\theta/2}}{R}$$

$$\frac{E_{\theta}}{2} = R \sin \alpha$$

$$E_{\theta} = 2R \sin \alpha \text{ --- (1)}$$

For sector OPQ ,

$$\phi = \frac{\text{length of arc}}{\text{radius}}$$

$$\therefore \phi = \frac{E_m}{R}$$

$$\therefore R = \frac{E_m}{\phi}$$

$$\therefore R = \frac{E_m}{2\alpha}$$

Put R in eq.(1)

$$E_{\theta} = 2 \frac{E_m}{2\alpha} \sin \alpha$$

$$E_{\theta} = \frac{E_m}{\alpha} \sin \alpha$$

$$\therefore E_{\theta} = E_m \frac{\sin \alpha}{\alpha}$$

This is the resultant amplitude at an angle θ

where

$$\alpha = \frac{\pi}{\lambda} (a \sin \theta)$$

Now squaring amplitude equation,

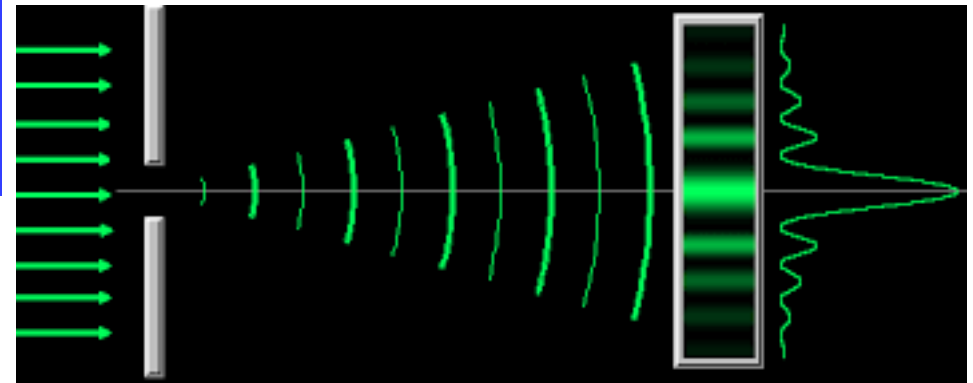
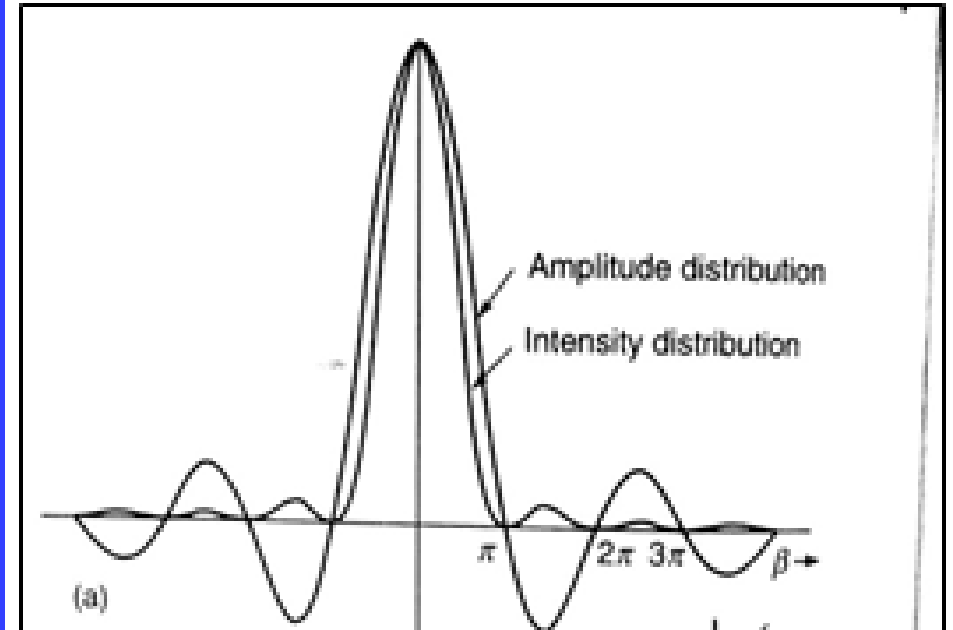
$$\therefore E_{\theta}^2 = E_m^2 \frac{\sin^2 \alpha}{\alpha^2} \text{---(4)}$$

We know
intensity of light \propto amplitude 2

$$I \propto E^2$$

$$\therefore I_{\theta} = I_m \frac{\sin^2 \alpha}{\alpha^2} \text{---(5)}$$

This is the resultant intensity at an angle θ



Case(i) Principal Maximum:

The value of R will be maximum.

$$R = A_0 \frac{\sin \alpha}{\alpha}; \quad \alpha = \frac{\pi a \sin \theta}{\lambda}$$

$$\Rightarrow I = \left(A_0 \frac{\sin \alpha}{\alpha} \right)^2$$

For maximum intensity $\frac{\sin \alpha}{\alpha} \rightarrow 1$ for $\alpha \rightarrow 0$

so $\sin \theta = 0 \gg \theta = 0$ **NO diffraction**

Case(ii) Minimum Intensity:

The value of R will be minimum. $\gg R=0 \gg$

$\sin \alpha = 0$ **BUT** $\alpha \neq 0$

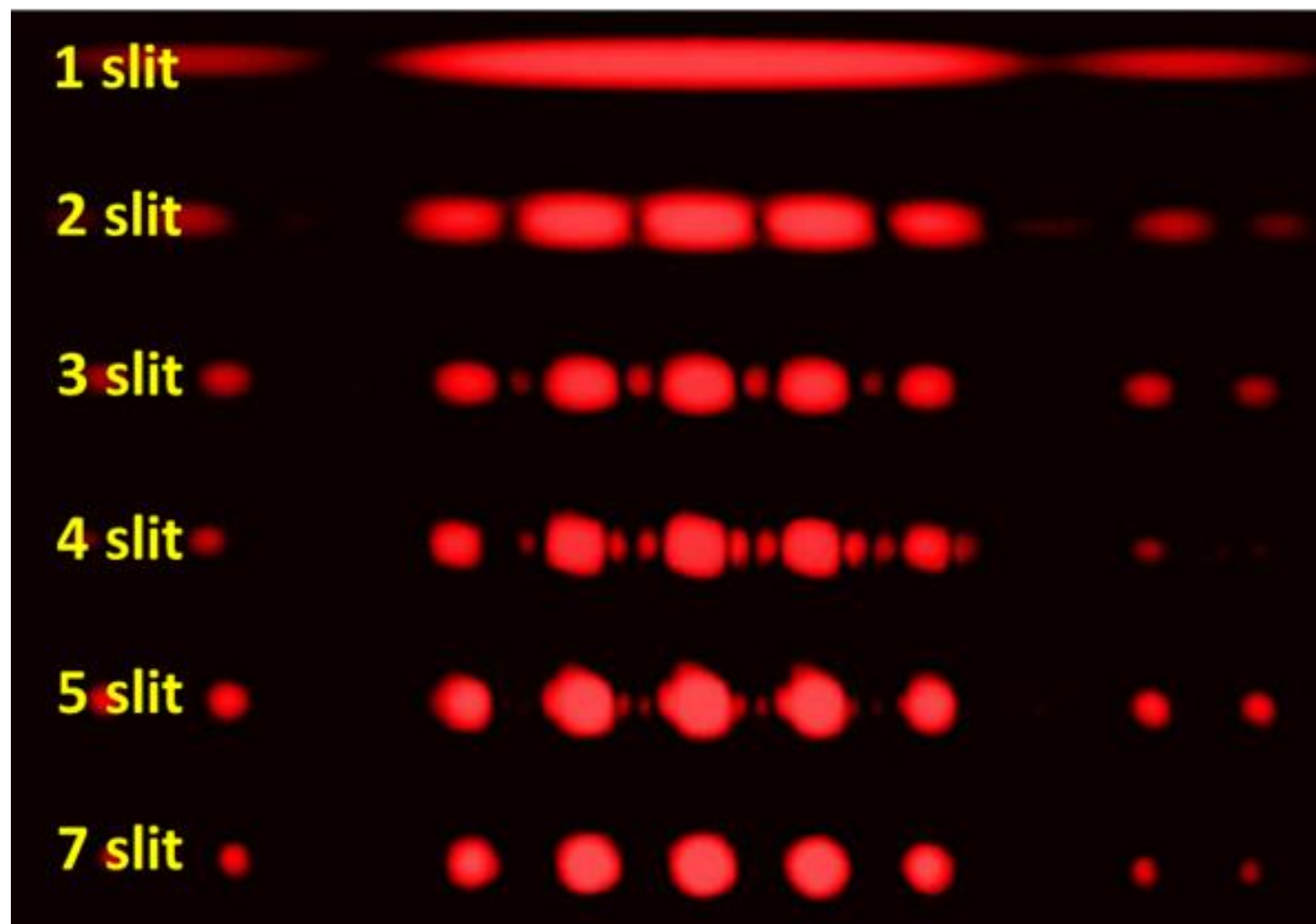
$$\Rightarrow \alpha = \pm \pi n \quad \{n=1,2,3,\dots\}$$

$$\Rightarrow \frac{\pi a \sin \theta}{\lambda} = \pm \pi n \quad \{n=1,2,3,\dots\}$$

Case(iii) Secondary Maxima: There are weak secondary maxima between equally spaced minima in addition to principal maxima at $\alpha=0$

(3) Secondary Maxima: There are weak secondary maxima between equally spaced minima in addition to principal maxima at $\alpha=0$

$$\begin{aligned} \frac{dI_\theta}{d\alpha} &= A_0^2 \frac{2 \sin \alpha (\alpha \cos \alpha - \sin \alpha)}{\alpha^2} = 0 \\ \Rightarrow \tan \alpha &= \alpha \end{aligned}$$



DIFFRACTION GRATING

- **Diffraction Grating is an arrangement consisting of a large number of parallel slits of same width separated by equal opaque spaces.**
- Gratings are fabricated by ruling equidistant parallel lines on a glass plate with the help of a fine diamond point.
- The lines act as opaque spaces and the incident light cannot pass through them.
- The space between the two lines is transparent to light and acts as a slit.

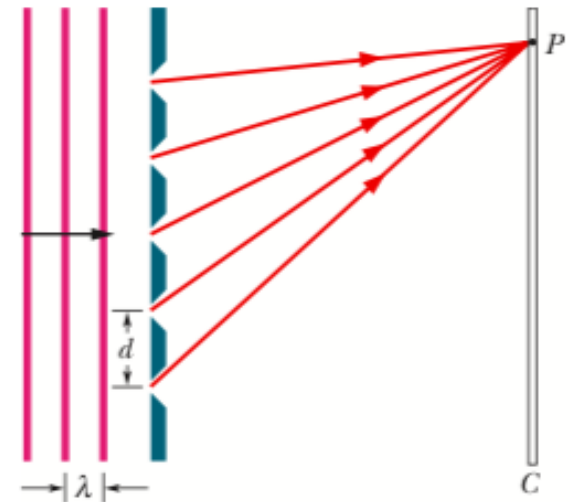


Fig. 36-18 An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen C .

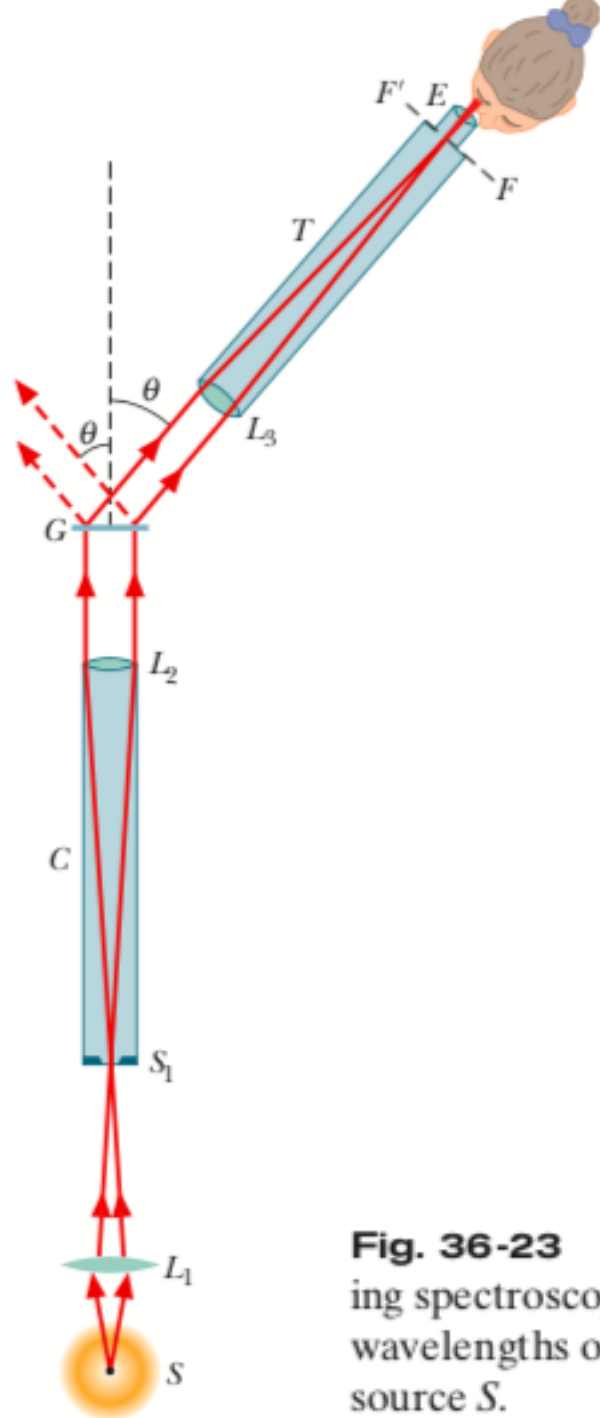


Fig. 36-23 A simple type of grating spectroscope used to analyze the wavelengths of light emitted by source S .

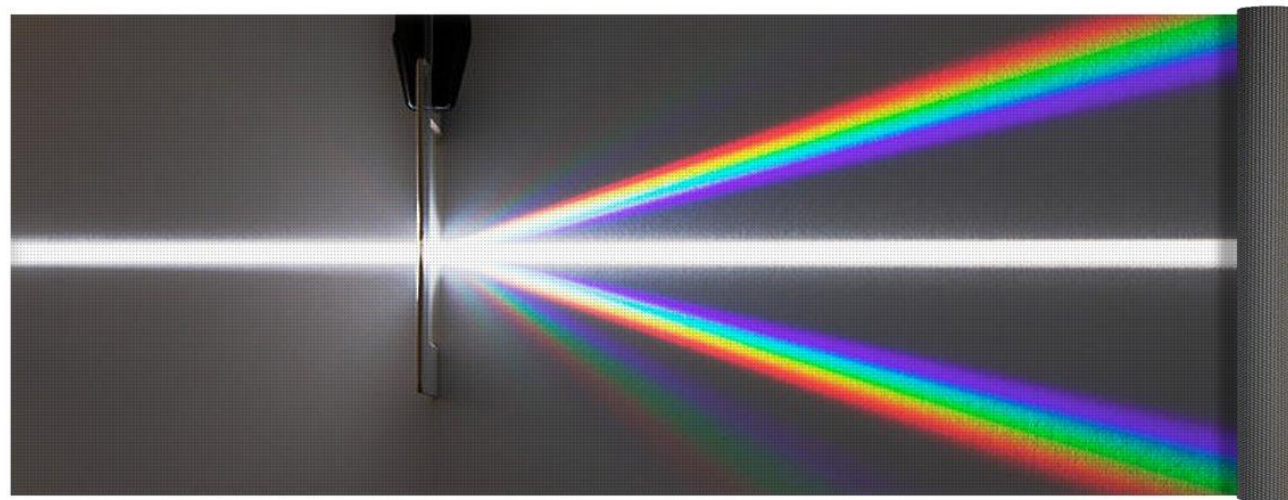


Fig. 36-25 The visible emission lines of cadmium, as seen through a grating spectroscope. (*Department of Physics, Imperial College/Science Photo Library/Photo Researchers*)

CHARACTERISTICS OF GRATING

- **Grating Spectrum**
- **Dispersive power of the Grating**
- **Resolving Power of the Grating**

Grating Spectrum

- For N slit diffraction pattern we have seen that the positions of the principal maxima are given by

$$d\sin\theta = m\lambda \quad (m = 0, 1, 2, \dots)$$

- **This relation is called as Grating Equation**
- It can be used to study the dependence of the angle of diffraction θ on the wavelength λ
- **The zeroth principal maximum occurs at $\theta = 0$ irrespective of the wavelength.**
- If we are using a polychromatic source (white light) then the central maximum will be of the same color as the source itself.
- **For m other than zero the angles of diffraction are different for different wavelengths and therefore, various spectral components appear at different positions.**
- Thus by measuring the angles of diffraction for various colors one can determine the values of the wavelengths.

Dispersive power of the Grating

Dispersion

To be useful in distinguishing wavelengths that are close to each other (as in a grating spectroscope), a grating must spread apart the diffraction lines associated with the various wavelengths. This spreading, called **dispersion**, is defined as

$$D = \frac{\Delta\theta}{\Delta\lambda} \quad (\text{dispersion defined}). \quad (36-29)$$

Here $\Delta\theta$ is the angular separation of two lines whose wavelengths differ by $\Delta\lambda$. The greater D is, the greater is the distance between two emission lines whose wavelengths differ by $\Delta\lambda$. We show below that the dispersion of a grating at angle θ is given by

$$D = \frac{m}{d \cos \theta} \quad (\text{dispersion of a grating}). \quad (36-30)$$

Thus, to achieve higher dispersion we must use a grating of smaller grating spacing d and work in a higher-order m . Note that the dispersion does not depend on the number of rulings N in the grating. The SI unit for D is the degree per meter or the radian per meter.

Resolving Power

To *resolve* lines whose wavelengths are close together (that is, to make the lines distinguishable), the line should also be as narrow as possible. Expressed otherwise, the grating should have a high **resolving power** R , defined as

$$R = \frac{\lambda_{\text{avg}}}{\Delta\lambda} \quad (\text{resolving power defined}). \quad (36-31)$$

Here λ_{avg} is the mean wavelength of two emission lines that can barely be recognized as separate, and $\Delta\lambda$ is the wavelength difference between them. The greater R is, the closer two emission lines can be and still be resolved. We shall show below that the resolving power of a grating is given by the simple expression

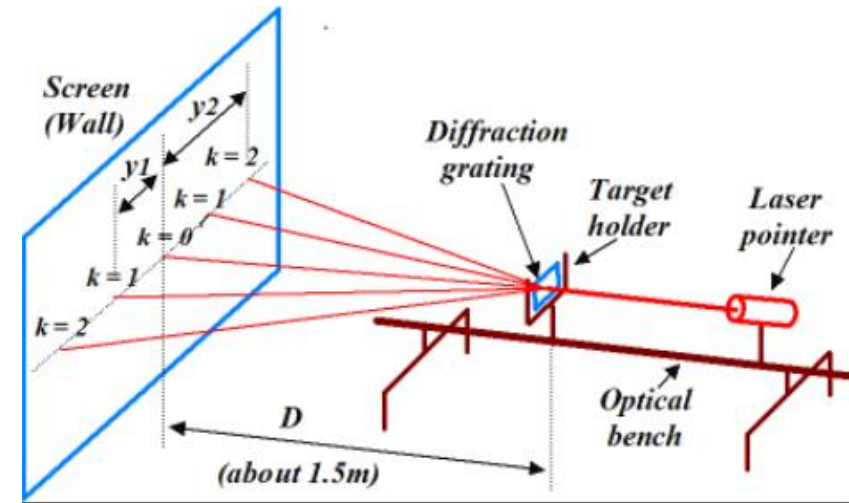
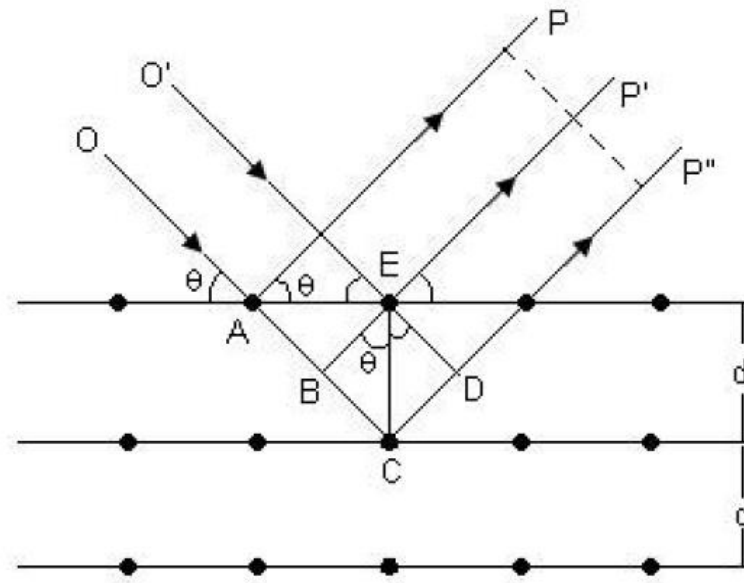
$$R = Nm \quad (\text{resolving power of a grating}). \quad (36-32)$$

To achieve high resolving power, we must use many rulings (large N).

- The diffraction grating is an immensely useful tool for the separation of the spectral lines associated with atomic transitions. It acts as a "super prism", separating the different colors of light much more than the dispersion effect in a prism.
- **Reflective Gratings** are wavelength-selective filters.
- In **optical communications**, they are used for
 - Wavelength Selection: Splitting and/or combining optical signals
 - Pulse Compression: Normally as reflectors in external cavity DBR lasers

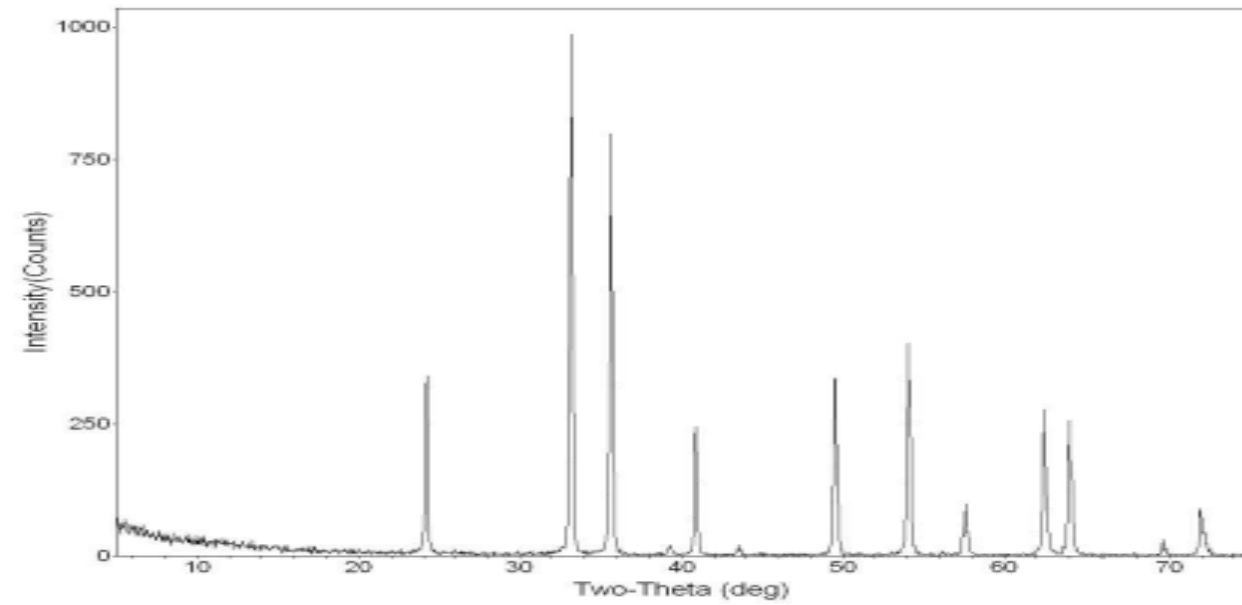
The inter-atomic spacing in crystals is of the order of 1\AA . Because of the short wavelength (comparable to the inter-planer distance), X-rays are scattered by adjacent atoms in crystals which can interfere and give rise to diffraction effects. When X-rays enter into a crystal, each atom acts as a diffraction centre and crystal as a whole acts like a three dimensional diffraction grating. The diffraction pattern so produced can tell us much about the internal arrangement of atoms in crystal.

$$2d \sin \theta = n\lambda$$



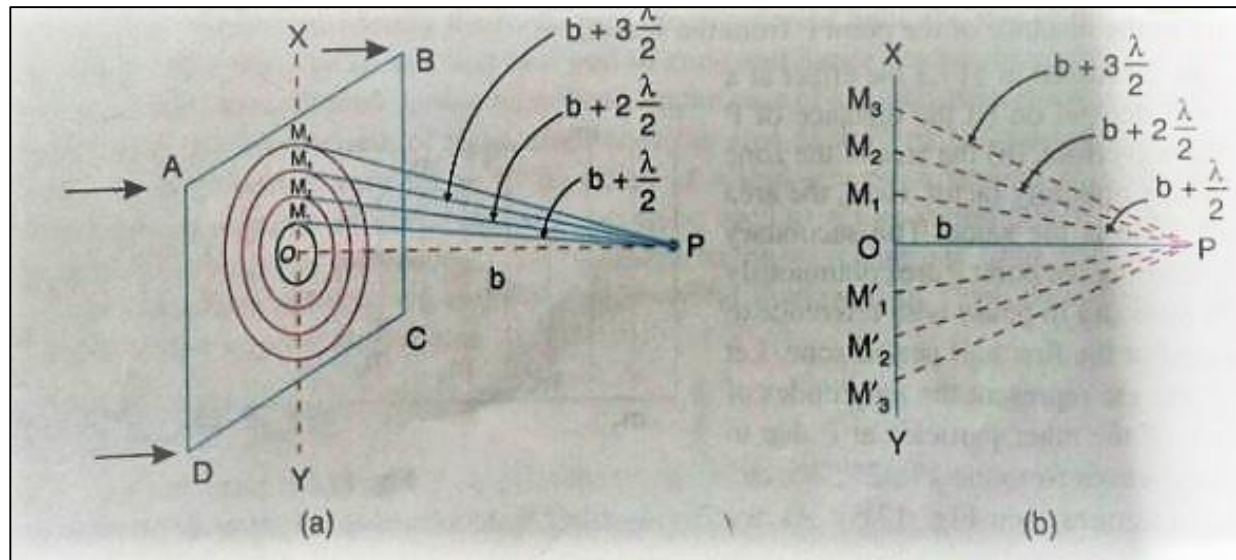


XRD



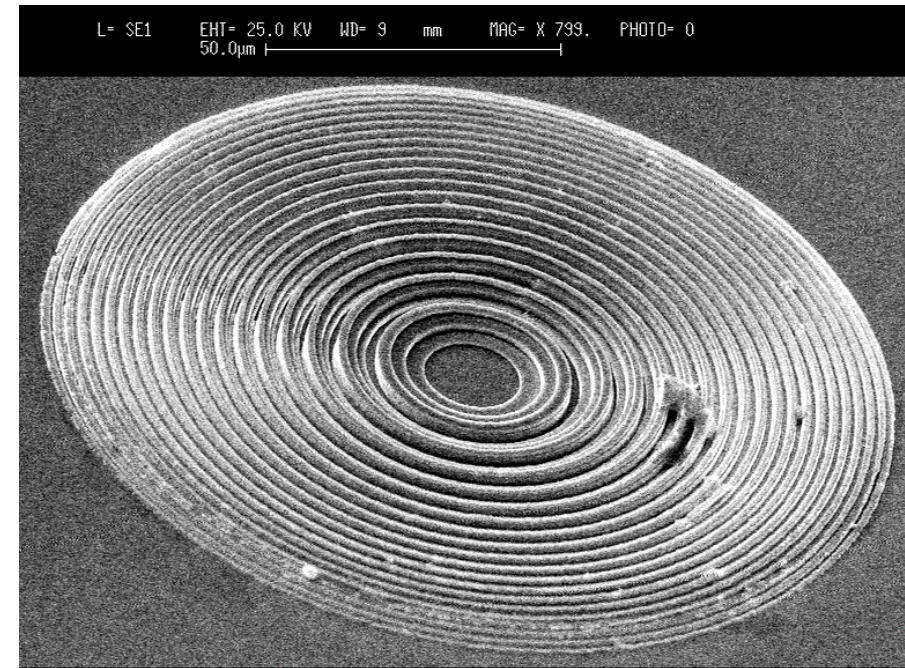
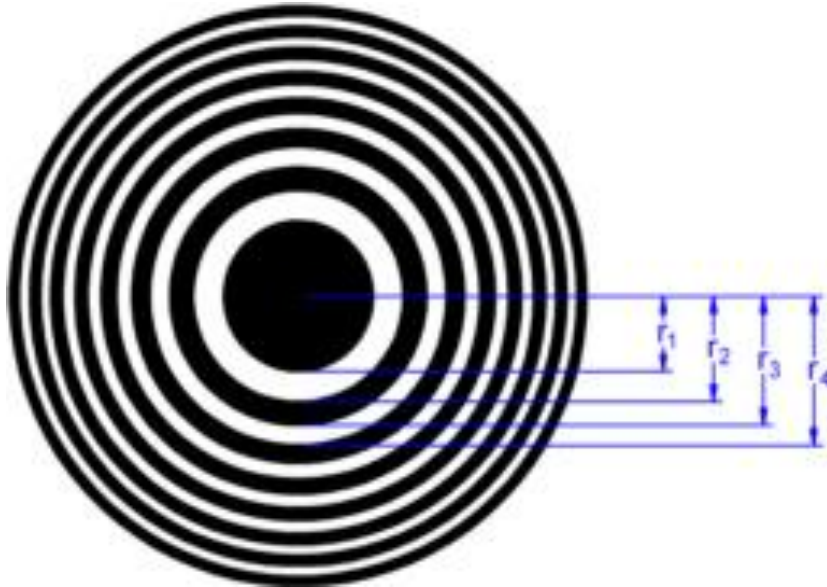
Fresnel's Half Period Zones

- According to Fresnel's the entire wavefront can be divided into a large number of parts of zones which are known as Fresnel's half period Zones. The resultant effect at any point on screen is due to the combined effect of all secondary waves from the various Zones.
- Or Fresnel's half period zones are the thin annular zones of a primary wavefront in which the secondary wavelets from any two corresponding points of the neighboring zones differ in path by $\lambda/2$.
- Let us consider a plane wavefront ABCD coming from a distant monochromatic source of light. Let it be perpendicular to the plane of paper and P be an external point. Draw a perpendicular OP from P on the wavefront. The O is called the pole of the wavefront with respect to the point P. Let $PO = b$. Now with P as center and radii equal to $\left(b + \frac{\lambda}{2}\right)$, $\left(b + \frac{2\lambda}{2}\right)$, $\left(b + \frac{3\lambda}{2}\right)$ etc. construct spheres which will cut out circular areas of radii OM_1 , OM_2 , OM_3 etc. on the wavefront. Each circular area differs from its neighbor by a path equal to $\frac{\lambda}{2}$ or phase π . These circular areas between OM_1 , OM_2 , OM_3 are called Fresnel's half period zones or elements.
- The area enclosed by the first circle of radius OM_1 is called the first half period zone, and so on.



The Fresnel Zone Plate

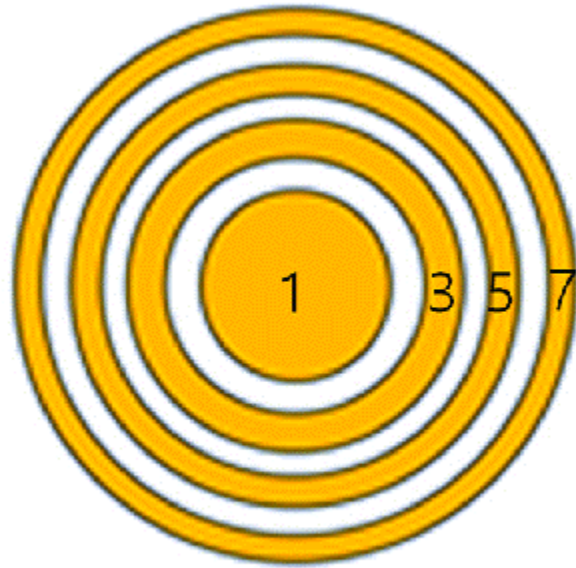
- Zone plate is a specially designed transparent plate on which circles, whose radii are proportional to the square roots of the natural numbers $1, 2, 3, \dots$ are drawn according to the theory of Fresnel's half period zones. The alternate annular zones thus formed are blocked i.e. made opaque so as to cut off light due to even numbered zones or that due to odd numbered zones. Such a plate behaves like a convex lens and produces an image of a source of light at a suitable distance. **It provide an experimental confirmation of Fresnel's theory of half-period zones.**



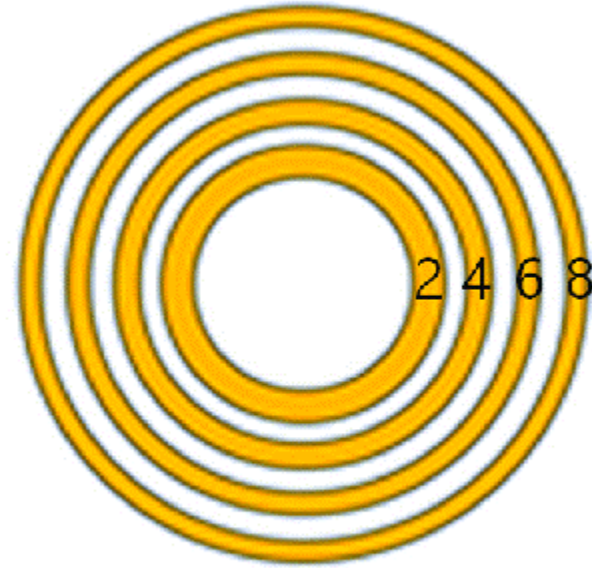
Types of plates of half period zone

Negative zone plate: A zone plate in which even zones are transparent and odd zones are opaque is known as a negative zone plate Fig(a).

Positive zone plate: A zone plate in which odd zones are transparent and even zones are opaque is known as a positive zone plate Fig(b).



(a)
Negative Z.P.



(b)
Positive Z.P.

Comparison of Convex lens and zone plate

Convex Lens

- Convex lens works on the principle of refraction of light.
- The focal length f of the convex lens is given by $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$, where μ is the refractive index of the material of the lens and R_1, R_2 the radii of curvature of the two surfaces of the lens.
- The focal length of a lens is directly proportional to the wavelength λ i.e. the focal length of a lens is more for red than that of violet. ($f_r > f_v$)
- The rays of light from different parts of the lens reach the focus at the same time.
- A lens produces only one image on an object.

Zone plate

- Zone plates work on the principle of diffraction.
- The focal length of zone plate is given by $f = \frac{r_n^2}{n\lambda}$, where r_n is the radius of the n th zone and λ the wavelength of light used.
- The focal length is inversely proportional to wavelength i.e. focal length for violet is more than that of red. ($f_r < f_v$)
- Light from consecutive zones will only reach the focus one period later.
- It produces a number of images of the object. The intensity of the images goes on decreasing as the distance between the screen and zone plate decreases.

Temporal and spatial coherence

- The important condition to observe interference is coherence.
- Coherence means that two or more electromagnetic waves of the same frequency, nearly the same amplitude and always have a constant phase difference between them.
- In general the phase between two electromagnetic waves can vary from point to point (in space) or change from instant to instant (in time). Therefore, there are two times of coherence (Temporal and Spatial coherence)
- **Temporal coherence:** This type of coherence refers to the correlation between the field at a point and the field at the same point at a later time. i.e. the relation between $E(x,y,z,t_1)$ and $E(x,y,z,t_2)$. If the phase difference changes many times and in an irregular way during the shortest period of observation the wave is said to be non-coherent. Also known as Longitudinal Coherence.
- **Spatial coherence:** the waves at different points in space are said to be space coherent, if they preserve a constant difference over any time t . This is possible even when two beams are individually time incoherent, as long as any phase change in one of the beams is accompanied by a simultaneous equal phase change in the other beam. Also known as Transverse Coherence.
- Time coherence is a characteristic of a single beam of light whereas space coherence concerns the relationship between two separate beams of light.

Coherence time and Coherence length

- **Coherence time** is defined as the longest time interval over which the phase undergoes change in a regular way.
- **Coherence length:** Coherence length (ℓ_{coh}) is defined as the spatial extent over which the wave train has predictable phase.

$$\ell_{\text{coh}} = c\Delta t$$

Relationship between coherence length and frequency bandwidth

The coherence time is the reciprocal of the bandwidth. The **coherence time** is given by:

$$(\Delta t)_c = 1 / \Delta \nu$$

where **$\Delta \nu$ is the light bandwidth** (the width of the spectrum). Sunlight is temporally very incoherent because its bandwidth is very large (the entire visible spectrum). Lasers can have coherence times as long as about a second, which is amazing; that's $>10^{14}$ cycles!