$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}_{3\times2}$$

$$A^T A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 9 & -9 \\ -9 & 9 \end{bmatrix}.$$

Eigen Values. $\longrightarrow$ $(9-\lambda)(9-\lambda) - 81 = 0.$

$\lambda = 0, 18$

eigen vector $\rightarrow$ $\lambda = 0$, $9x - 9y = 0$

$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$\lambda = 18$, $-9x - 9y = 0$

$-9x = 9y$; $X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

vectors x_1 & x_2 are orthogonal.

normalising $\rightarrow$.

$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, $v_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

Singular Values. $\sigma_1 = 0.$, $\sigma_2 = \sqrt{18} = 3\sqrt{2}$

$V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}_{2\times2}$, $\Sigma = \begin{bmatrix} 0 & 0 \\ 0 & 3\sqrt{2} \\ 0 & 0 \end{bmatrix}_{3\times2}.$

$$u_2 = \frac{A v_2}{\sigma_2} = \frac{1}{3\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = \frac{1}{3\sqrt{2}} \begin{bmatrix} 2/\sqrt{2} \\ -4/\sqrt{2} \\ 4/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}$$

$A = U \Sigma V^T.$

$$\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 & 0 \\ 0 & -2/3 & 0 \\ 0 & 2/3 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 3\sqrt{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$