Origin/Overview of Operation Research

Module 1

Operations research

Origin and Development:

The formal activities of Operations Research (OR) were initiated in England during World War II when a team of British scientists set out to make decisions regarding the best utilization of war material. Following the end of the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector. Today, OR is a dominant and indispensable *decision making tool*.

Origin and Development

- In 1948, an OR club was formed in England which later changed its name to the Operations research society of UK.
- In 1952, the operations research society of America (ORSA) was founded.
- In 1953, The institute of management science (TIMS) was founded as international society to identify, extend and unify scientific knowledge pertaining to management.

Origin and Development

- In India OR came into existence in1949, an OR unit was established at Regional Research Laboratory, Hyderabad for planning & organizing research. At the same time Prof. RS verma also setup an OR team at defense science laboratory to solve problems of store, purchase & planning.
- In 1953, Prof. P.C.Mahalanobis established an OR team in the Indian Statistical Institute, Kolkata to solve problems related to national planning & survey. In 1957 Operations Research Society of India (ORSI) was founded.

OPERATIONS RESEARCH (Definitions)

- OR is a science of Optimization; Optimization means maximizing profits and minimizing cost.
- OR is the systematic method oriented study of the basic structure, characteristic, functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decision making.

OPERATIONS RESEARCH (Definitions)

- Operations Research is the research of operations. An operation may be called a set of acts required for the achievement of a desired outcome.
- OR is a scientific approach to problem solving for executive management. (H.M. Wagner)

OPERATIONS RESEARCH (Definitions)

- OR is the art of giving bad answers to the problems, to which, otherwise, worse answers are given. (Thomas L. Saaty)
- OR is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. (Morse & Kimball)

OPERATIONS RESEARCH (Definitions)

- OR may be described as a scientific approach to decision making that involves the operations of organizational system. (FS Hiller & GJ Lieberman)
- OR is concerned with scientifically deciding how to best design and operate man-machine systems usually requiring the allocation of resources.

Characteristics of OR

- Primary focus on decision making.
- Scientific approach
- It is interdisciplinary model
- Model building and mathematical manipulation provide the methodology .
- OR is for operations economy
- Use of computers.

Methodology of OR / Main Phases Operations Research Study

1. **Formulate the Problem**

What are the objectives, controlled variables, uncontrolled variable constraints?

2. **Model Building**

This includes determination of assumptions, decision variables or parameters, constraints or restrictions, objective functions.

3. **Acquiring Input Data**

4. **Solution of the Model**

Apply suitable OR technique --- feasible & infeasible solution, optimal & non optimal solution, Unique & multiple solution.

Methodology of OR…..

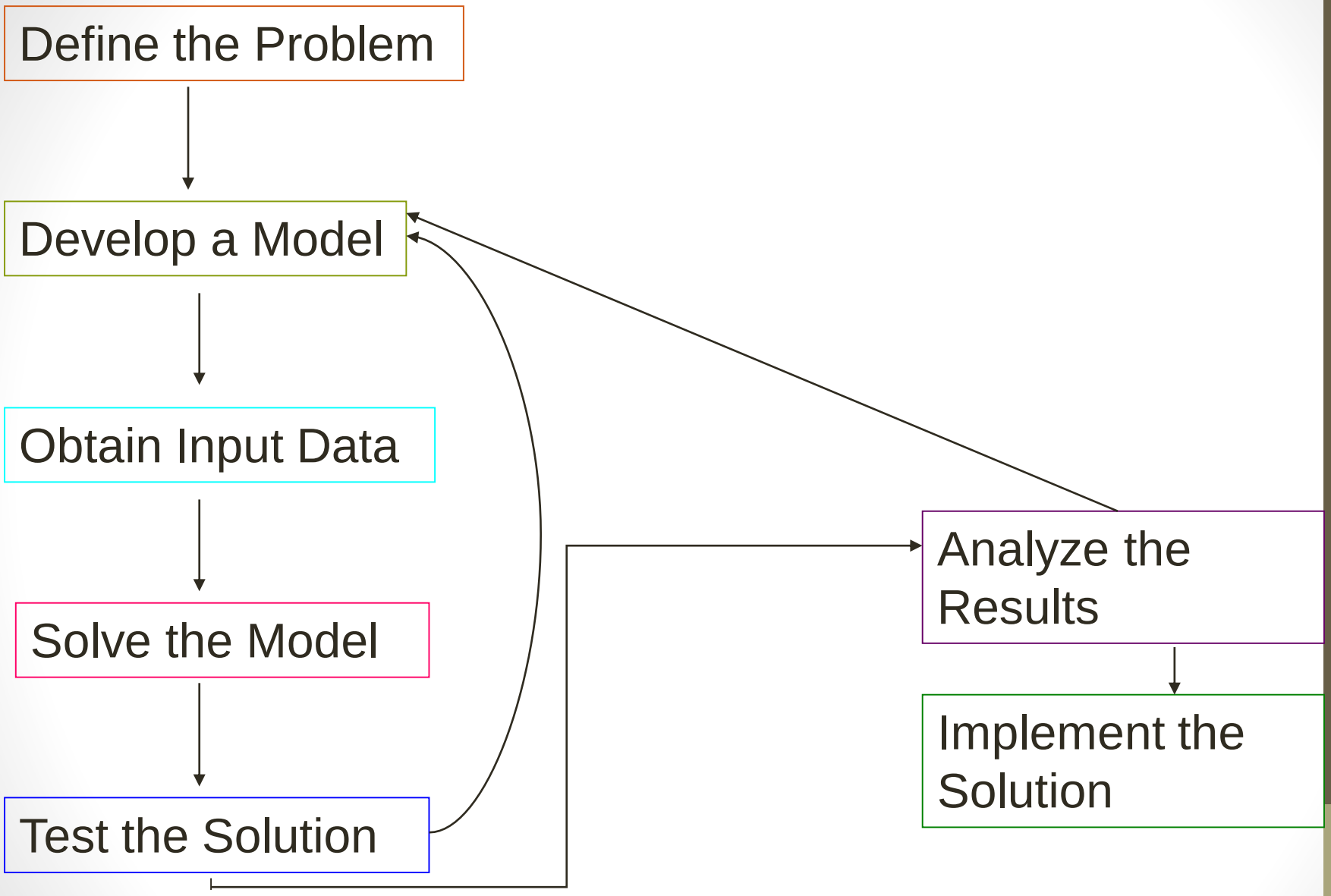
5. Model Validation (Testing the Model)

By putting values of decision variables in the model and then check the solution wheather it is valid or not.

6. Implementation and Control

Interpret solution values , put the result gained from the solution to work through organizational policies, Monitor changes and exercise control.

Operations Research Methodology



Scope of Operations Research

1. In Agriculture

- Optimum allocation of land to various crops in accordance with the climate conditions.
- Optimum distribution of water from various resources like canal for irrigation purposes.

2. In Finance

- to maximize the per capita income with minimum resources.
- to find out the profit plan for the company
- to determine the best replacement policies

3. In Industry

- for deciding optimum allocation of various limited resources such as men, machines, material, money, time etc.
- Complexity, Scattered Responsibility
- Knowledge Explosion

Scope of Operations Research

4. In Marketing

- Where to distribute the products for sale so that the total cost of transportation is minimum
- the minimum per unit sale price ?
- the size of the stock to meet the future demand ?
- how to select best adv. media w.r.t. time, cost etc. ?
- how, when & what to purchase at the minimum possible cost ?

5. In Personnel Management

- to appoint the most suitable person on minimum salary.
- to determine the best age of retirement for the employees.
- to find out the no. of persons to be appointed on full time basis when the workload is seasonal.

Scope of Operations Research

6. In production Management

- to find out the no. & size of items to be produced.
- in scheduling & sequencing the production run by proper allocation of machines.
- in calculating the optimum product mix.
- to select, locate & design the sites for the production plants

7. In L.I.C.

- What should be the minimum premium rates for various modes of policies ?
- How best the profit could be distributed in the cases of with profit policies ?

Models & modeling in OR

A MODEL is a representation of the reality. Most of our thinking of operations research in business take place in the context of models. The objective of model is not to identify all aspects of the situation but to identify significant factors and their intre-relationship. A major advantage of modelling is that it permits the decision maker to examine the behaviour of a system without interfering with as going operations.

- Model is an approximation or abstraction of real life situation.
- A model is constructed to analyze & understand the given system for the purpose of improving its performance.

Classification based on Structure

- **Physical Model**
  - i) Iconic Models
  - ii) Analogue Model
- **Symbolic Models**
  - i) Verbal Models
  - ii) Mathematical Models

Models & modeling in OR

(A) PHYSICAL MODELS

These models include all forms of diagrams, drawings graphs and charts. Most of which are designed to deal with specific types of problems. By presenting significant factors and inter-relationships in pictorial term, physical models are able to a problem in a manner that facilitates analysis. There are two types of physical models which are explained as follows:

- ICONIC MODELS

An icon is an image or likeness of an object or system it represents. An iconic model the least abstract physical replica of a system, is usually based on a smaller scale than original. These models can stimulate the actual performance of a product thereby availing the treamenduous expense of designing full scale experimental models.

Iconic models have got some advantages.

- They are specified and concrete.
- They are easy to construct.
- They can be studied more easily than the system itself.

They have the disadvantages

- They are difficult to manipulate for experimental purposes.
- They cannot be used to study the changes in operation of a system.
- It is not easy to make any modification or improvement in these
- models.
- Also adjustment with changing situations cannot be done in these models.

Analogue models

In analogue models one set of properties is used to represent another set of properties. After the problem is solved, the solution is re-interpreted in terms of the original system.

For example, contour lines on a map are analogues of elevation as they represent the rise and fall of heights. Analogue models are easier to manipulate than iconic models. But they are less specific and less concrete.

SYMBOLIC MODELS

These models use symbols like letters, numbers etc.to represent the properties of the system. These models are also used to represent relationships which can be represented in a physical form. Symbolic models can be classified into two categories:

Verbal Models: These models describe a situation in written or spoken language.

Example: Written sentences, books, newspapers, journals etc.

Mathematical Models: These models represent the characteristics of a situation or reality by using a set of mathematical symbols and relationships. These models are widely used in OR due to their capacity to depict the complex relationship among the variables of a problem. Example : '+', '–', '×', '÷'.

OPERATION RESEARCH MODELS: CLASSIFICATION

I. BASED ON STRUCTURE

(1) Physical Models: These models give a physical appearance of real object in reduced or scaled up form .These are further divided into two categories:

a) **Iconic Models**: Physical or Pictorial representaion of the various aspects of the system. Ex. Blue Prints, Globe, Templates etc.

b) **Analogue Models:**These models represent a system by a set of proerties different from the original system. Ex. .Ex: A network of water pipes to show flow of current in electrical network. Level Indicator in a automobile petrol tank

(2) Symbolic Models : These models use symbols either in the form of letters or mathematical operators to represent the properties of the system. These are further classified into two types:

a) **Verbal Models:** These models used to describe a situation in written or spoken language in form of letters , words or symbols. Ex: Differential Equations representing a Dynamic system.

b) **Mathematical Models:** The decision variables of the system under consideration are represented by mathematical equations or inequations. Ex. Linear programming model to decide Product -Mix problem in manufacturing.

OPERATION RESEARCH MODELS: CLASSIFICATION

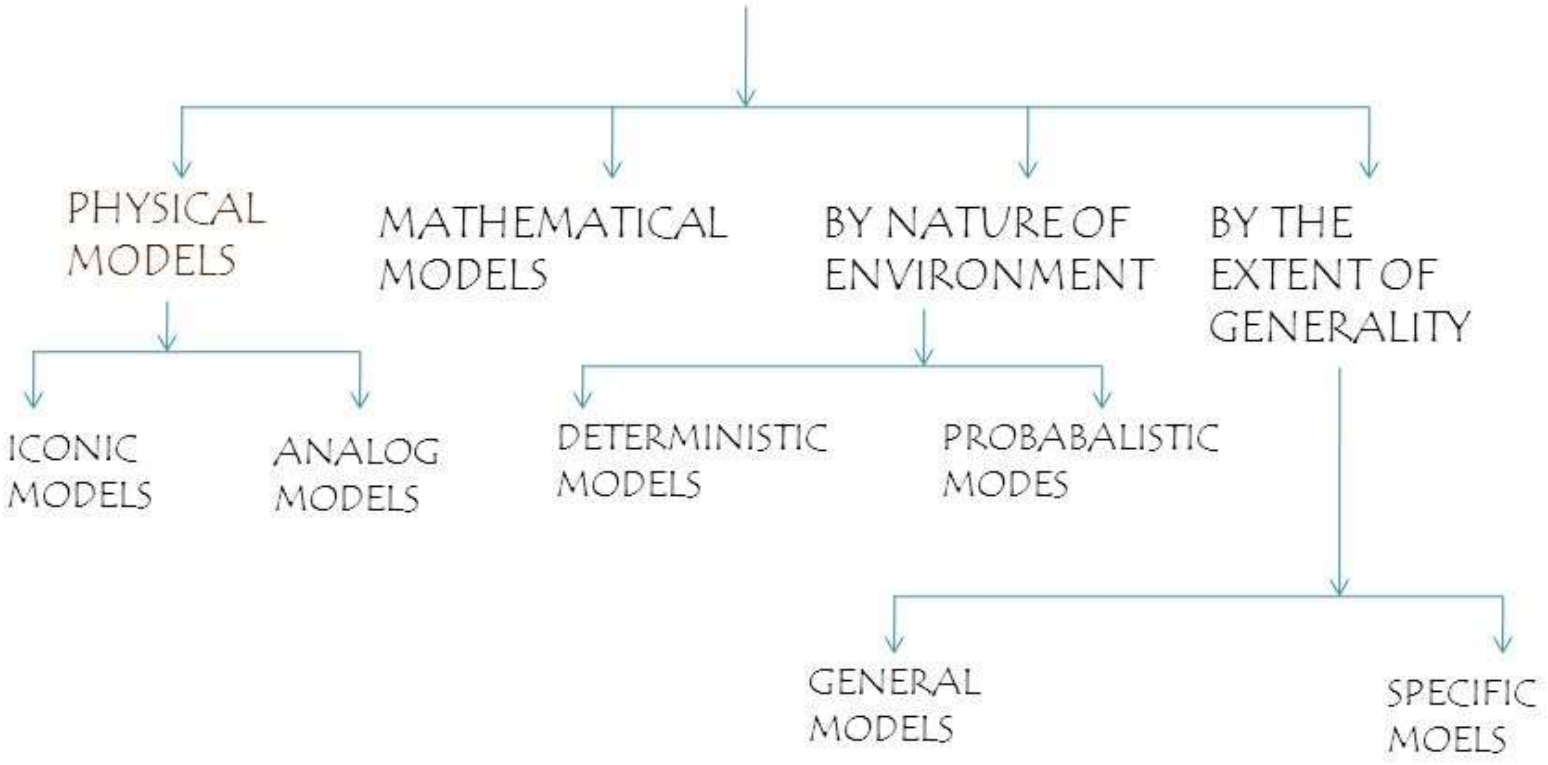
- **BASED ON METHOD OF SOLUTION**

  (1) Analytical Model: These have a specific mathematical structure and can be solved by analytical and mathematical techniques. Ex. Any optimisation model such as inventory models, waiting lines etc.

  (2) Iterative or Heuristic Model: In these models solution is obtained from the conclusion of previous step.Ex. Simplex Method for LPP.

  (3) Simulation Models: A computer assisted mathematical representation of real life problem under certain assumptions. Ex. Monte-Carlo Simulation , Use of Random Numbers, Forecasting Models.

TYPES OF OR MODELS



Models in OR(different classification schemes)

- Degree of abstraction
  - Mathematical models
  - Language models
  - Concrete models
- Function
  - Descriptive models
  - Predictive models
  - Normative models
- Time Horizon
  - Static models
  - Dynamic models
- Structure
  - Iconic or physical models
  - Analogue or schematic models
  - Symbolic or mathematical models
- Nature of environment
  - Deterministic models
  - Probabilistic models
- Extent of generality
  - General model
  - Specific models

Models & modeling in OR

Classification based on Purpose or Function

- **Descriptive Models**
- **Predictive Models**
- **Normative (Optimization) Models**

Models & modeling in OR

Classification Based on Time Reference

- Static Models
- Dynamic Models

Classification based on degree of Certainty

- Deterministic Models
- Probabilistic Models

Models & modeling in OR

Classification based on Method of Solution

- **Heuristic Models**
- **Analytical Models**
- **Simulation Models**

Applications of OR Techniques

- **Kellogg's**

  - The largest cereal producer in the world.
  - LP-based operational planning (production, inventory, distribution) system **saved $4.5 million in 1995.**

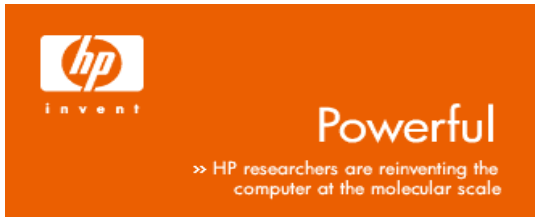
Applications of OR Techniques

- **Procter and Gamble**

  - A large worldwide consumer goods company.
  - Utilised integer programming and network optimization worked in concert with Geographical Information System (GIS) to re-engineering product sourcing and distribution system for North America.
  - Saved over $200 million in cost per year.

Applications of OR Techniques

- **Hewlett-Packard**



  - Robust supply chain design based on advanced inventory optimization techniques.
  - Realized savings of over $130 million in 2004

DECISION THEORY

Introduction

- Decision Theory represents a general approach to decision making which is suitable for a wide range of operations management decisions.
- Decision theory is a body of knowledge and related analytical techniques of different degrees of formality designed to help a decision maker choose among a set of alternatives in light of their possible consequences.

Introduction

- Decision theory is the combination of descriptive and perspective business modeling approach to classify degree of knowledge. The degree of knowledge is usually divided into four categories:

Ignorance → Uncertainty → Risk → Certainty

Decision theory elements

1. Decision Alternatives
2. States of Nature
3. Payoff

1. Decision Alternatives / Course of Action (Strategies)

There is a finite number of decision alternatives available with the decision maker at each point in time when a decision is made. The number and type of such alternatives may depend on the previous decisions made and what has happened subsequent to those decisions. All the alternatives are under control and known to the decision maker.

2. States of Nature / Subsequences / Events / Scenarios

These are the future conditions not under the control of decision maker. A state of nature can be a state of Economy (eg. Inflation), a weather condition, a political state etc.

Most relevant states of nature may be identified through some technique such as **scenario analysis** i.e. there may be certain possible states of nature which may not have

2. States of Nature.....

a serious impact on the decision and other could be quite serious. In scenario analysis, various knowledgeable section of people are interviewed – stakeholders, long-time managers, etc., to determine the most relevant states of nature to the decision.

2. States of Nature.....

The states of nature are mutually exclusive and collectively exhaustive with respect to any decision problem. The states of nature may be described numerically such as, demand of 100 units of an items or non-numerically such as, students/employees strike etc.

3. Payoff

A numerical value or outcome resulting from each possible combination of alternatives and states of nature. The payoff values are always conditional values because of unknown states of nature.

Note:- Payoff is measured within a specified period, eg. After one year. Payoff can be measured in terms of money market share.

Payoff matrix:- Tabular arrangement of conditio outcomes (payoff) value.

General form of Payoff Matrix

		Courses of Action (Alternatives)				
States of Nature	Probability	S_1	S_2	S_3	…..	S_n
N_1	P_1	P_{11}	P_{21}	P_{31}	……	P_{m1}
N_2	P_2	P_{21}	P_{22}	P_{32}	……	P_{m2}
N_3	P_3	P_{31}	P_{32}	P_{33}	……	P_{m3}
.	.	.	.	.		.
.	.	.	.	.		.
.	.	.	.	.		.
N_m	P_m	P_{m1}	P_{m2}	P_{m3}	……	P_{mn}

Construction of Payoff matrix

ABC firm manufactures four types of products. The fixed and variable costs (in Rs.) are given below:

Product	Fixed cost	Variable cost
a1	20,000	20
a2	30,000	10
a3	35,000	09
a4	40,000	05

Construction of Payoff matrix

The likely demand of products is given below:

bad demand (d1)=2,000

avg. demand (d2)= 6,000

good demand (d3)=8,000

v. good demand (d4)=9,000

If the sale price of each type of products is 30 rs., then prepare the payoff matrix.

Construction of Payoff matrix

We know that

Payoff = Sales revenue – cost

The calculations for payoff for each pair of alternative demand (course of action) & the types of product (state of nature) are :

d1a1= 2000*30 – (20,000+2000*20)=0

d1a2= 2000*30 – (30,000+2000*10)=10,000

d1a3= 2000*30 – (35,000+2000*9)=7,000

d1a4= 2000*30 – (40,000+2000*5)=10,000

Construction of Payoff matrix

d2a1= 6000*30 – (20,000+6000*20)=40,000

d2a2= 6000*30 – (30,000+6000*10)=90,000

d2a3= 6000*30 – (35,000+6000*9)=91,000

d2a4= 6000*30 – (40,000+6000*5)=1,10,000

d3a1= 8000*30 – (20,000+8000*20)=60,000

d3a2= 8000*30 – (30,000+8000*10)=1,30,000

d3a3= 8000*30 – (35,000+8000*9)=1,60,000

d3a4= 8000*30 – (40,000+8000*5)=1,60,000

Construction of Payoff matrix

d4a1= 9000*30 – (20,000+9000*20)=70,000

d4a2= 9000*30 – (30,000+9000*10)=50,000

d4a3= 9000*30 – (35,000+9000*9)=1,54,000

d4a4= 9000*30 – (40,000+9000*5)=1,85,000

Construction of Payoff matrix

Payoff values are:

Product Type	Alternative demand			
	d1	d2	d3	d4
a1	0	40,000	60,000	70,000
a2	10,000	90,000	1,30,000	50,000
a3	7,000	91,000	1,60,000	1,54,000
a4	10,000	1,10,000	1,60,000	1,85,000

Steps of Decision Making Process

1. Identify and define the problem.
2. Listing of all possible future states (states of nature).
3. Identify all the courses of actions (alternatives decisions).
4. Expressing the payoffs resulting from each course of action and state of nature.
5. Apply an appropriate mathematical decision theory model to select best course of action (alternative decision).

Decision-making Environments

There are four types of decision making environments:

1. Certainty
2. Uncertainty
3. Risk
4. Conflict

Decision making under certainty

In this case the decision maker has complete information of consequences of every decision choice with certainty. The decision maker select an alternative that yield the largest return or payoff for the future.

In this case the decision-maker is unable to specify the probabilities with which the various states of nature or future events will occur. In it possible states of nature are known.

In the absence of knowledge about the probability of any state of nature occurring, the decision-maker must arrive at a decision only on the actual conditional payoff values, together with a policy.

Decision making under uncertainty

Criteria for decision-making under uncertainty

1. Optimism (Maximax or Minimin) criterion
2. Pessimism (Maximin or Minimax) criterion
3. Equal Probabilities (Laplace) criterion
4. Coefficient of optimism (Hurwicz) criterion
5. Regret (salvage) criterion

Decision making under uncertainty

1. **Optimism (Maximax or Minimin) Criteria**

   In it the decision-maker ensures that he should not miss the opportunity to achive the largest possible profit (maximax) or lowest possible cost (minmin). So, he/she selects the alternative course of action the represents the maximum of the maxima payoffs.

Decision making under uncertainty

Steps for optimism criterion

1. Locate the maximum (or minimum) payoff values corresponding to each alternative.
2. Select an alternative with best anticipated payoff value (maximum for profit & minimum for cost).

It is also called **optimistic decision criteria**.

Decision making under uncertainty

2. **Pessimism (Maximin or Minimax) criterion**

In this criteria the decision-maker ensures that he would earn no less (or pay no more) than some specified amount. So, he/she selects the best alternative that represents the maximum of the minima (minima of the maxima in case of loss) payoff in case of profits.

Decision making under uncertainty

Steps for optimism criterion

1. Locate the minimum (or maximum in case of profit) payoff value in case of loss/cost corresponding to each alternative.
2. Select an alternative with best anticipated payoff value.

Decision tree analysis

Decision trees are used to select the best course of action in situations where you face uncertainty. Many business decisions fall into this category. For example, a manufacturer must decide how much inventory to build before knowing precisely what demand will be. A litigant must choose between accepting an out-of-court settlement or risking a trial. A speculator must decide to buy an asset before knowing if it can be sold for a profit.

Decision tree analysis

In all of these cases, the decision-maker faces an unknown that seems to make it impossible to choose the right option with any certainty. Although the decision-maker does not know what the outcome of the unknown will be, he or she generally has some knowledge about what the possible outcomes are and how likely each is to occur.

This information can be used to select the option that is most likely to yield favorable results. Decision trees make this type of analysis easy to apply.

Format of a Decision tree