AVL TREES
Balanced Binary Search Trees

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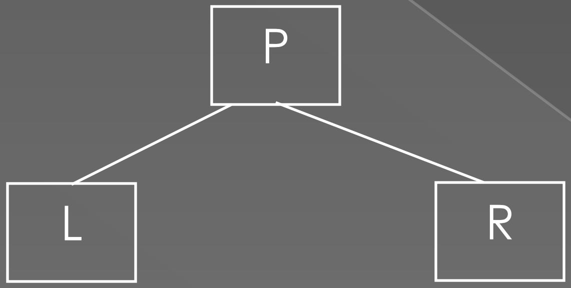
AVL TREES

- First-invented self-balancing binary search tree
- Named after its two inventors,
  1. G.M. Adelson-Velsky and
  2. E.M. Landis,
  - published it in their 1962 paper "An algorithm for the organization of information."

AVL Properties

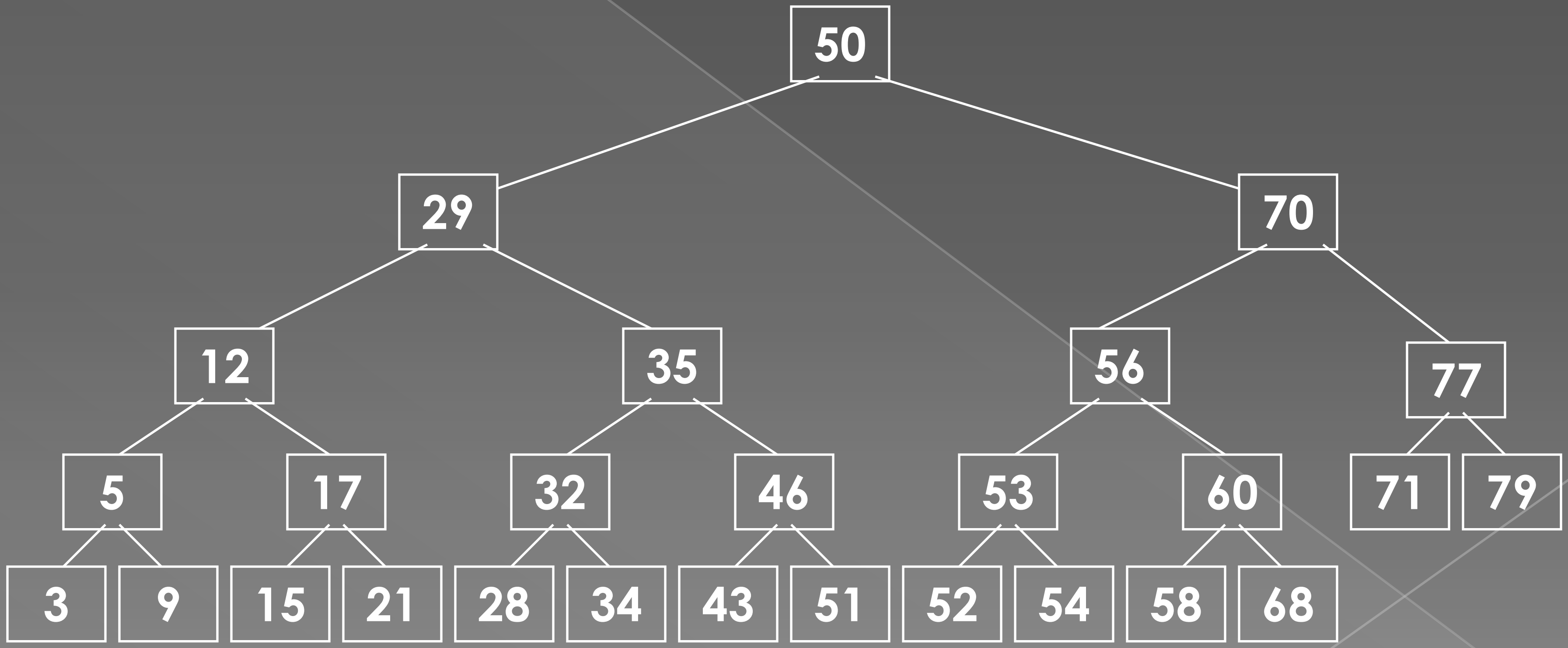
- First, its a binary search tree...



- L <= P and P <= R

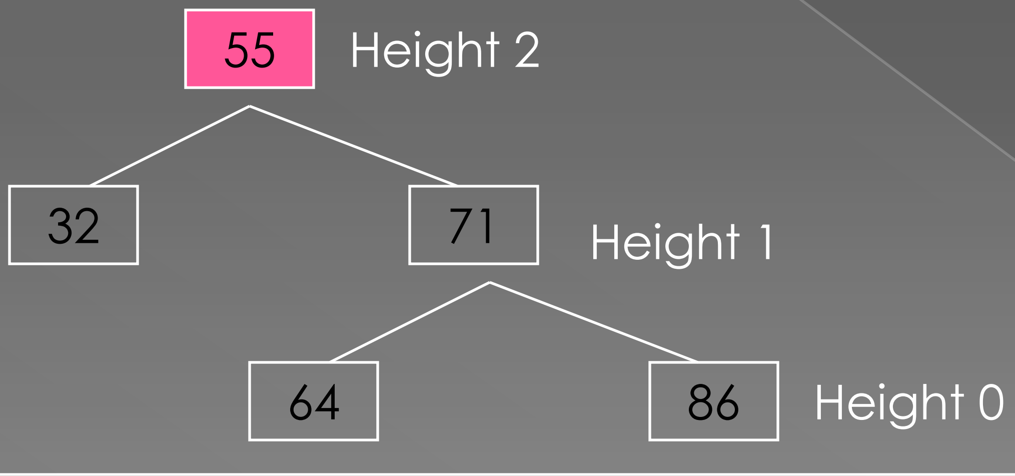
Question?

- Is this a binary tree search tree?



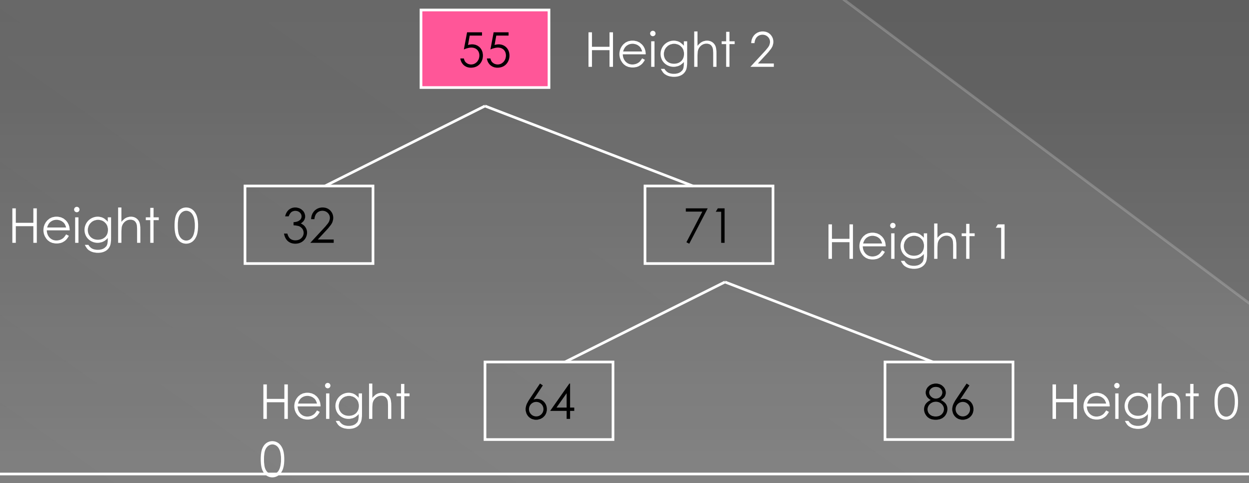
AVL Properties

- An AVL tree is a balanced binary tree
- To understand balance we need to understand the notion of **Tree Height**



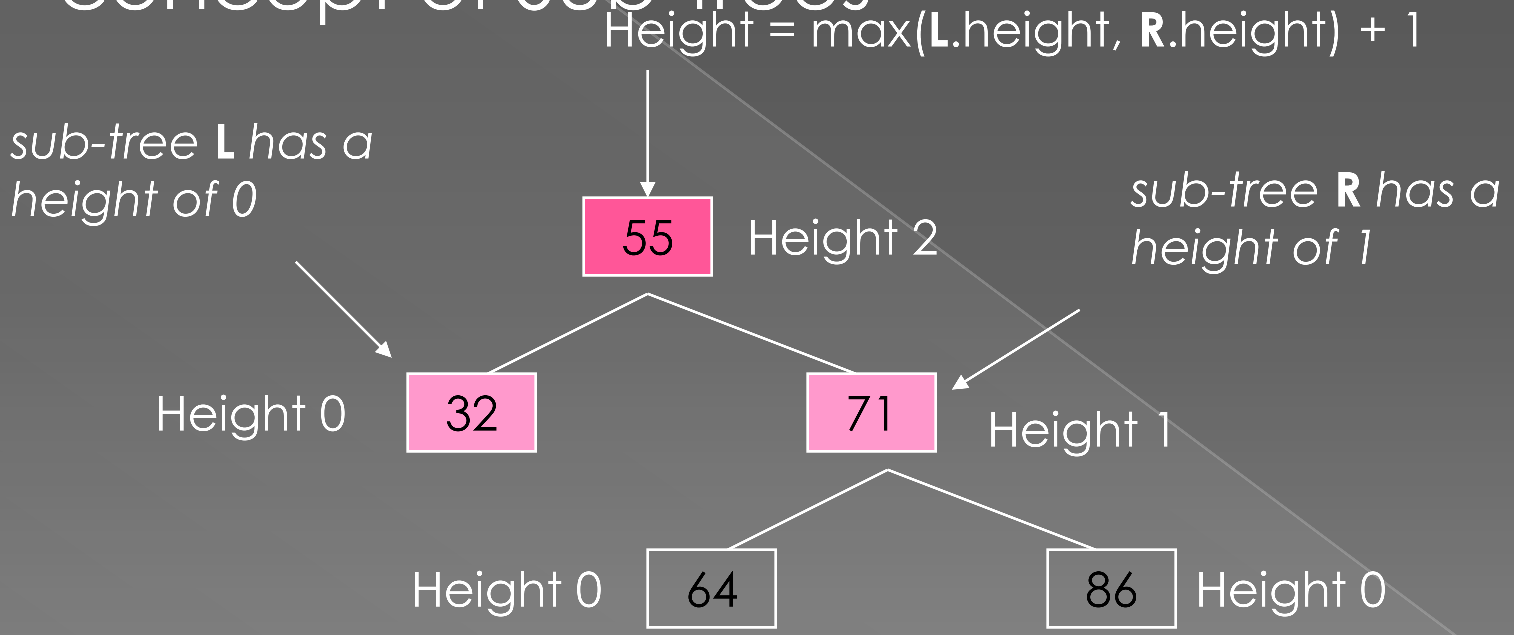
AVL Properties

- By default, nodes with no children have a height of Height of 0.



AVL Properties

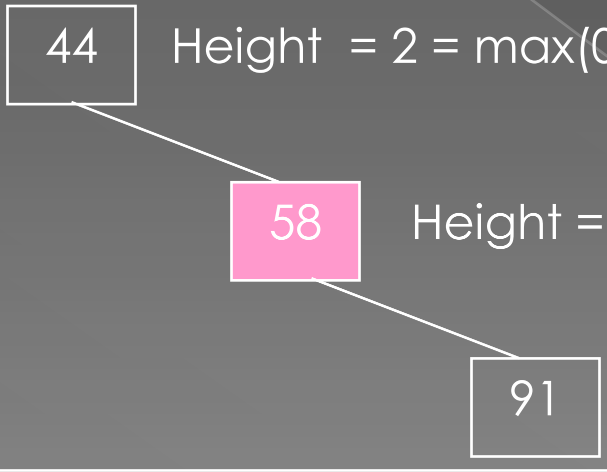
- But, we must also understand the concept of Sub-trees


Height = max(**L**.height, **R**.height) + 1
*sub-tree **L** has a height of 0*
55
Height 2
*sub-tree **R** has a height of 1*
Height 0
32
71
Height 1
Height 0
64
86
Height 0

AVL Properties

- Also empty sub-trees have a Height of -1

Height = max(**L**.height, **R**.height) + 1



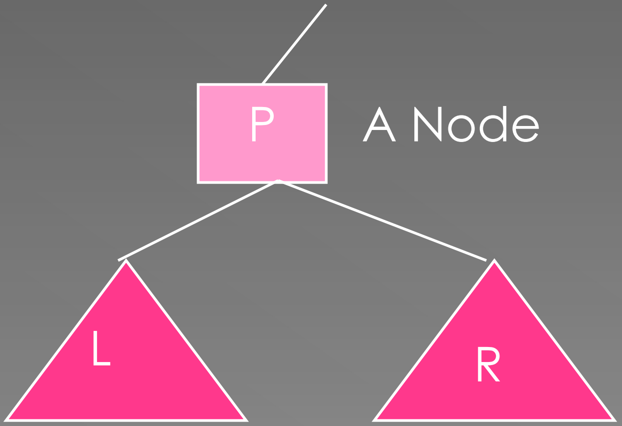
44 Height = 2 = max(0, 1) + 1

58 Height = 1 = max(-1, 0) + 1

91 Height = 0 = max(-1,-1) + 1

AVL Properties

- Anyway, the AVL Balance Property is as follows...
- For ALL nodes, the Height of the Left and Right Sub-trees can only differ by 1.



$$|L.height - R.height| \leq 1$$

Correcting Imbalance

1. After every insertion
2. Check to see if an imbalance was created.
   - All you have to do backtrack up the tree
3. If you find an imbalance, correct it.
4. As long as the original tree is an AVL tree, there are only 4 types of imbalances that can occur.

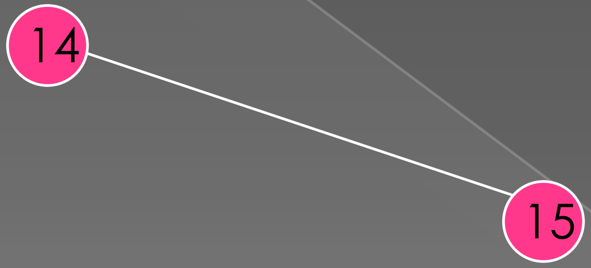
Imbalances

- Left-Left (Single Rotation)
- Right-Right(Single Rotation)
- Left-Right (Double Rotation)
- Right-Left(Double Rotation)

AVL Tree Rotations

Single rotation: insert 14, 15, 16, 13, 12, 11, 10

- First insert 14 and 15:

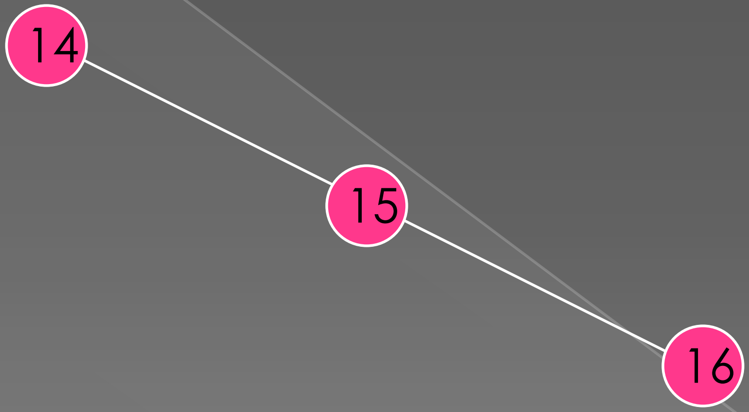


- Now insert 16.

AVL Tree Rotations

Single rotations:

- Inserting 16 causes AVL violation:

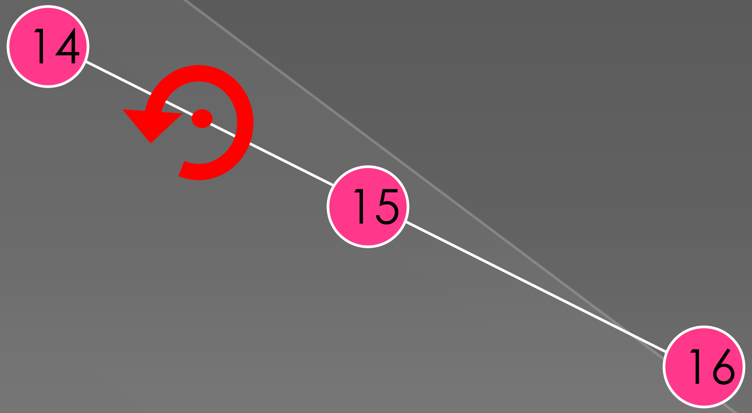


- Need to rotate.

AVL Tree Rotations

Single rotations:

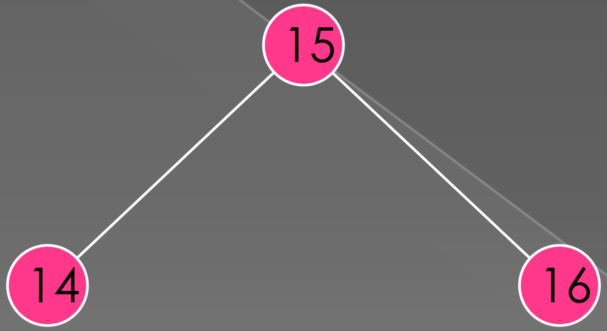
- Rotation type:



AVL Tree Rotations

Single rotations:

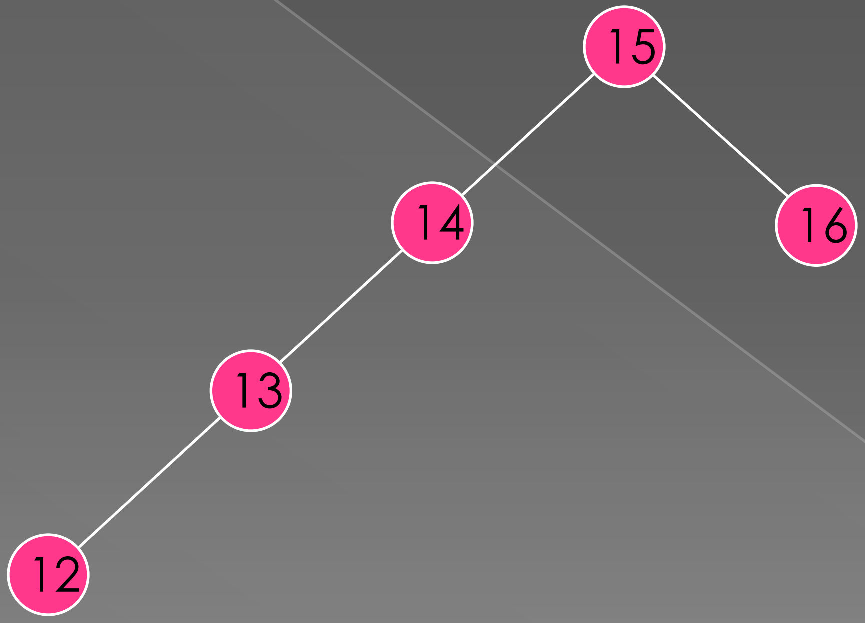
- Rotation restores AVL balance:



AVL Tree Rotations

Single rotations:

- Now insert 13 and 12:

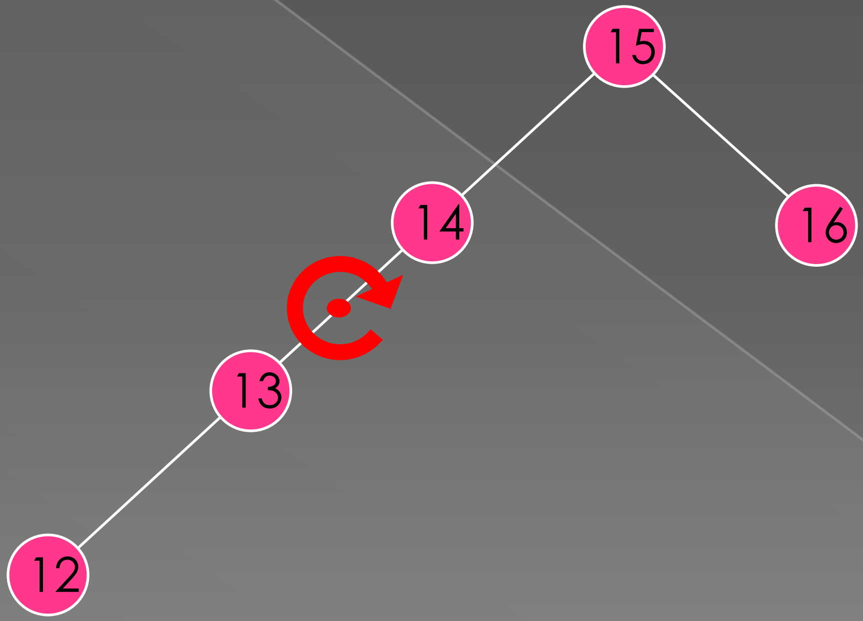


- AVL violation - need to rotate.

AVL Tree Rotations

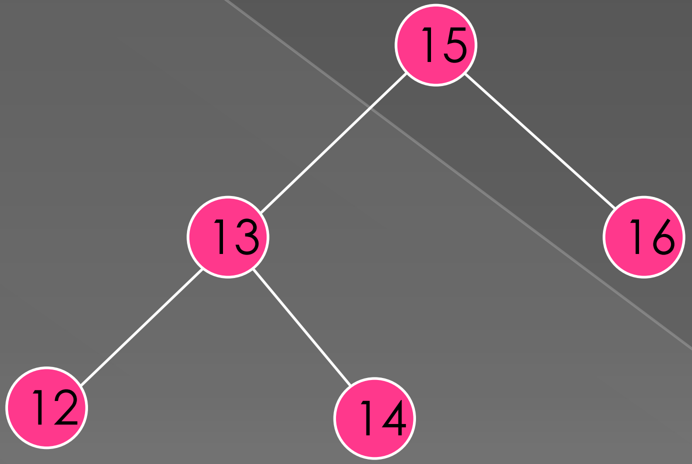
Single rotations:

- Rotation type:



AVL Tree Rotations

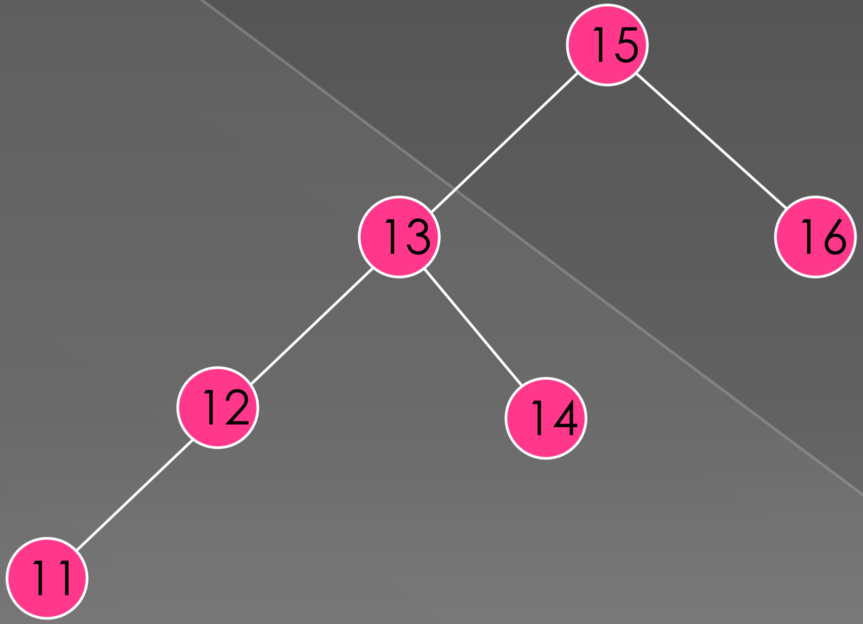
Single rotations:



- Now insert 11.

AVL Tree Rotations

Single rotations:

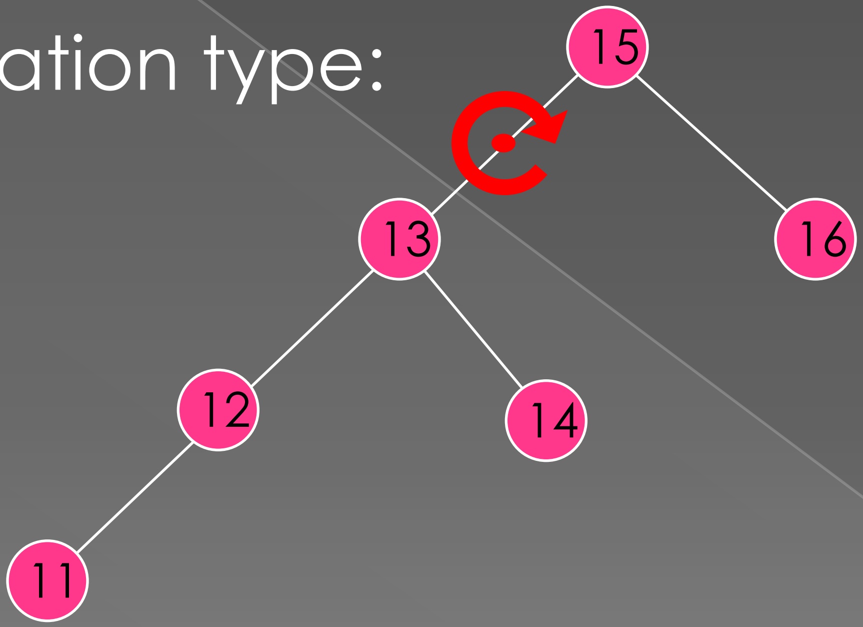


- AVL violation – need to rotate

AVL Tree Rotations

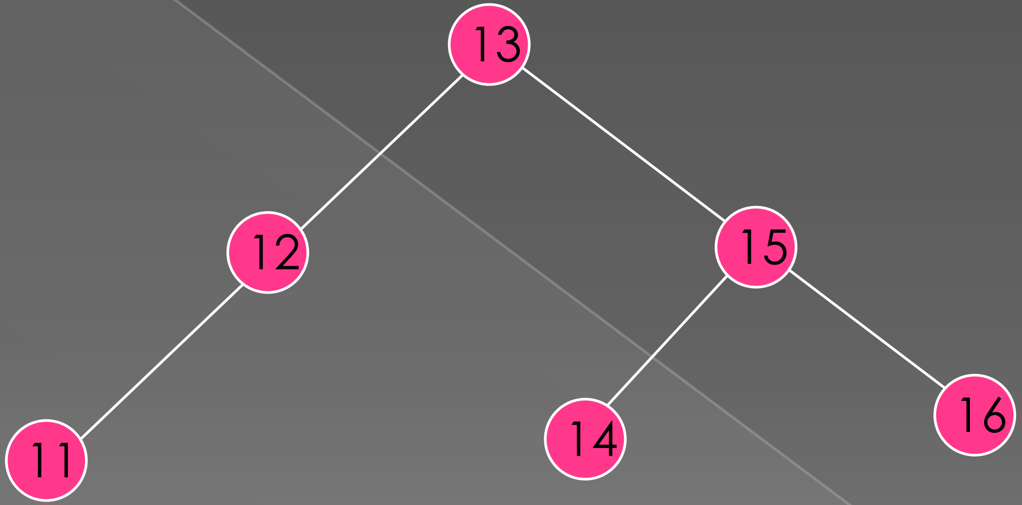
Single rotations:

- Rotation type:



AVL Tree Rotations

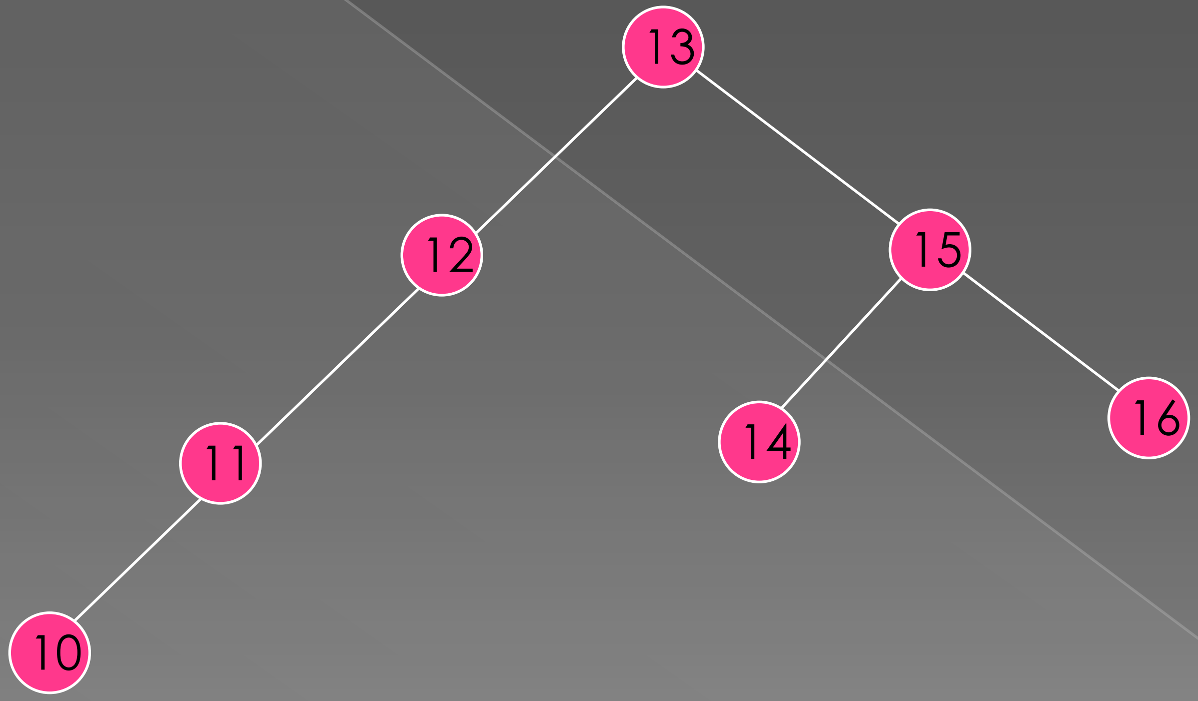
Single rotations:



- Now insert 10.

AVL Tree Rotations

Single rotations:

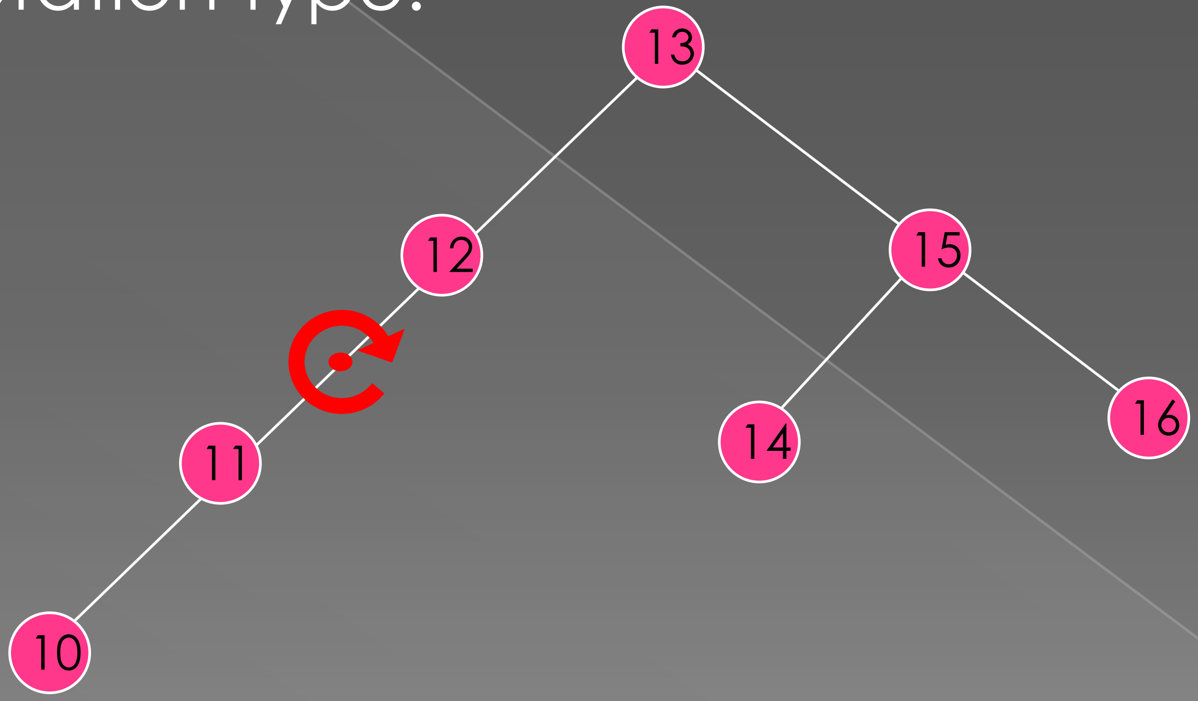


- AVL violation – need to rotate

AVL Tree Rotations

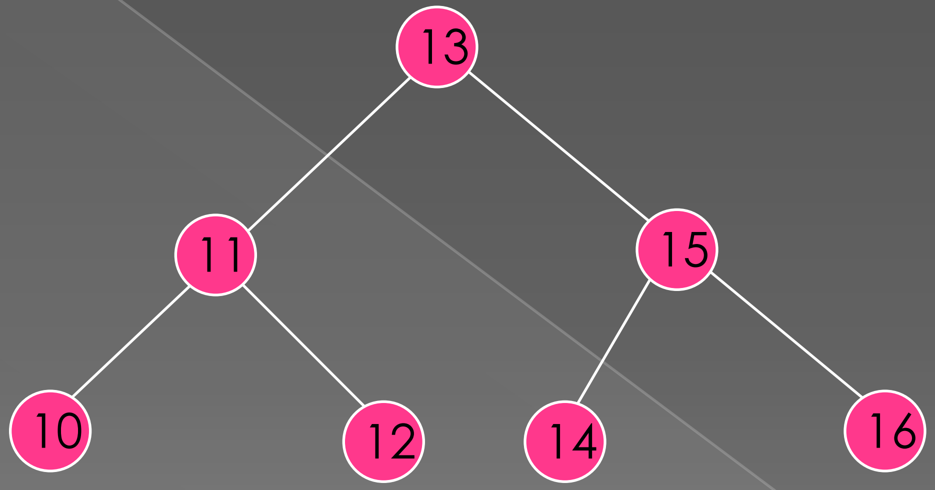
Single rotations:

- Rotation type:



AVL Tree Rotations

Single rotations:

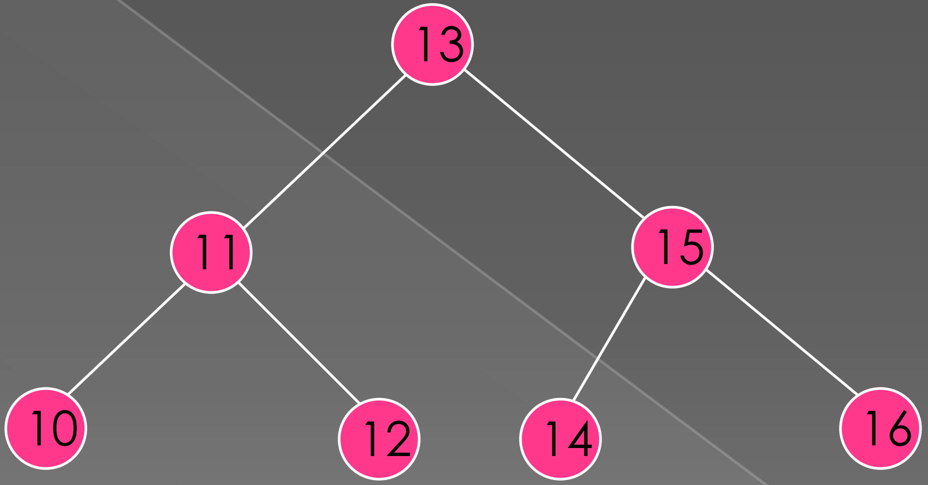


- AVL balance restored.

AVL Tree Rotations

Double rotations: insert 1, 2, 3, 4, 5, 7, 6, 9, 8

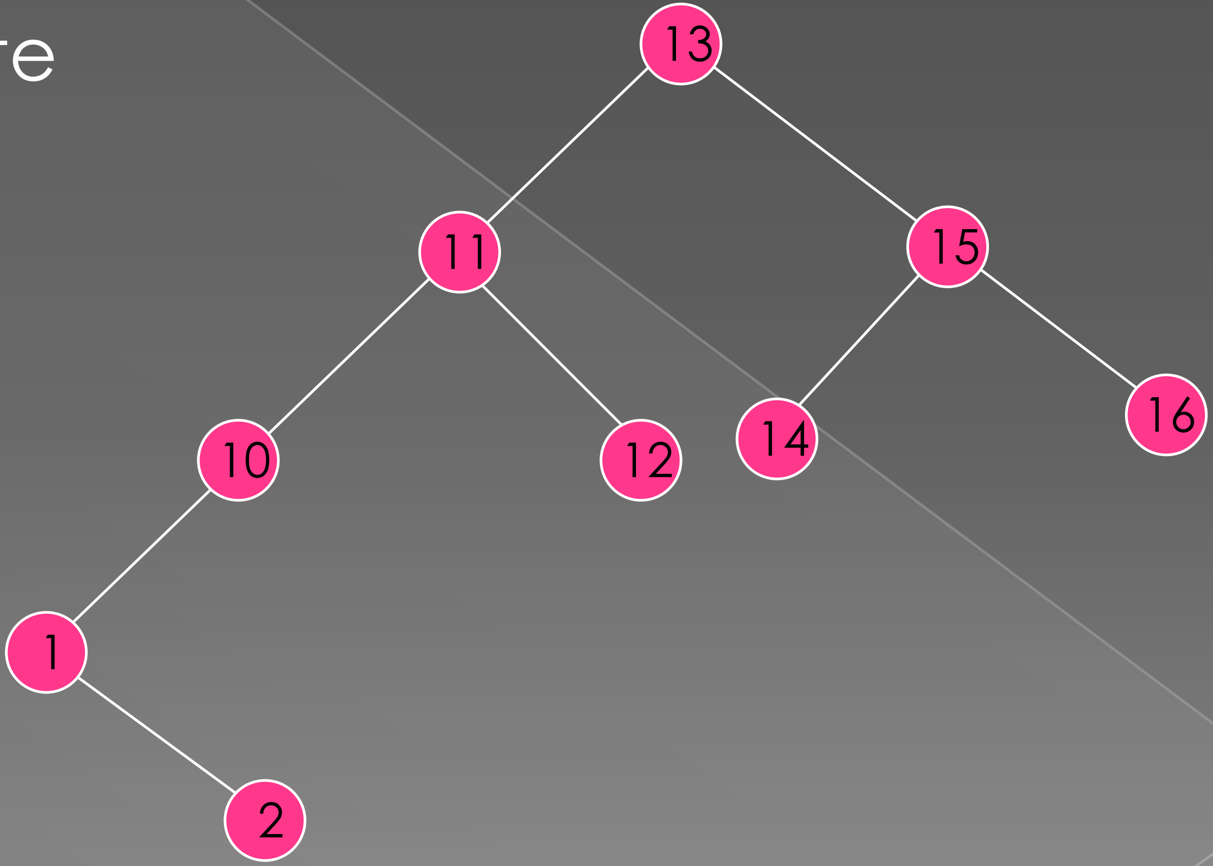
- First insert 1 and 2:



AVL Tree Rotations

Double rotations:

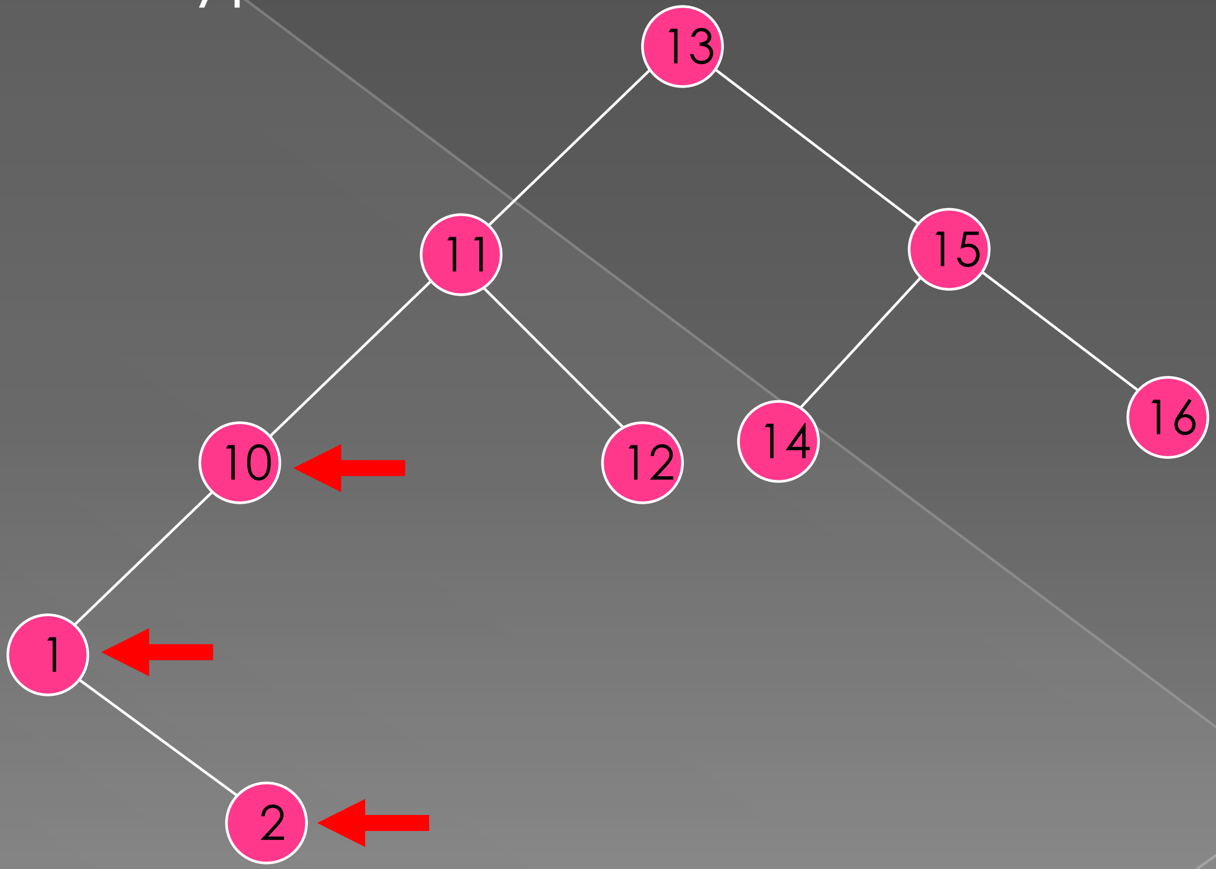
- AVL violation - rotate



AVL Tree Rotations

Double rotations:

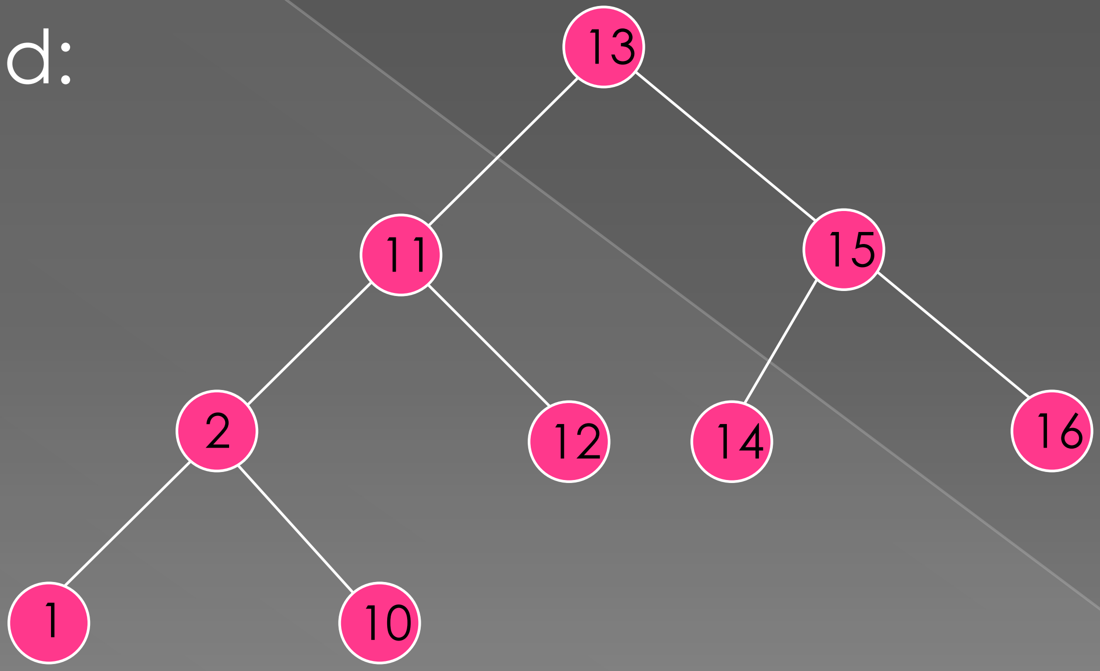
- Rotation type:



AVL Tree Rotations

Double rotations:

- AVL balance restored:

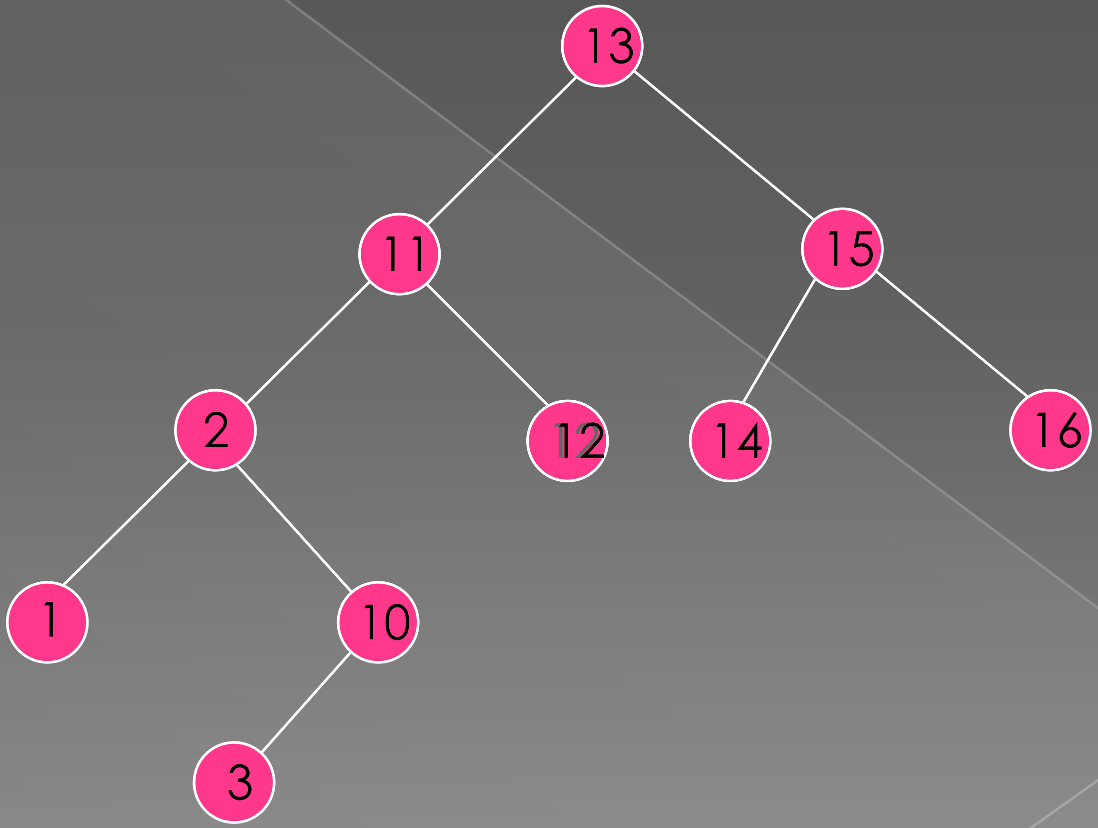


- Now insert 3.

AVL Tree Rotations

Double rotations:

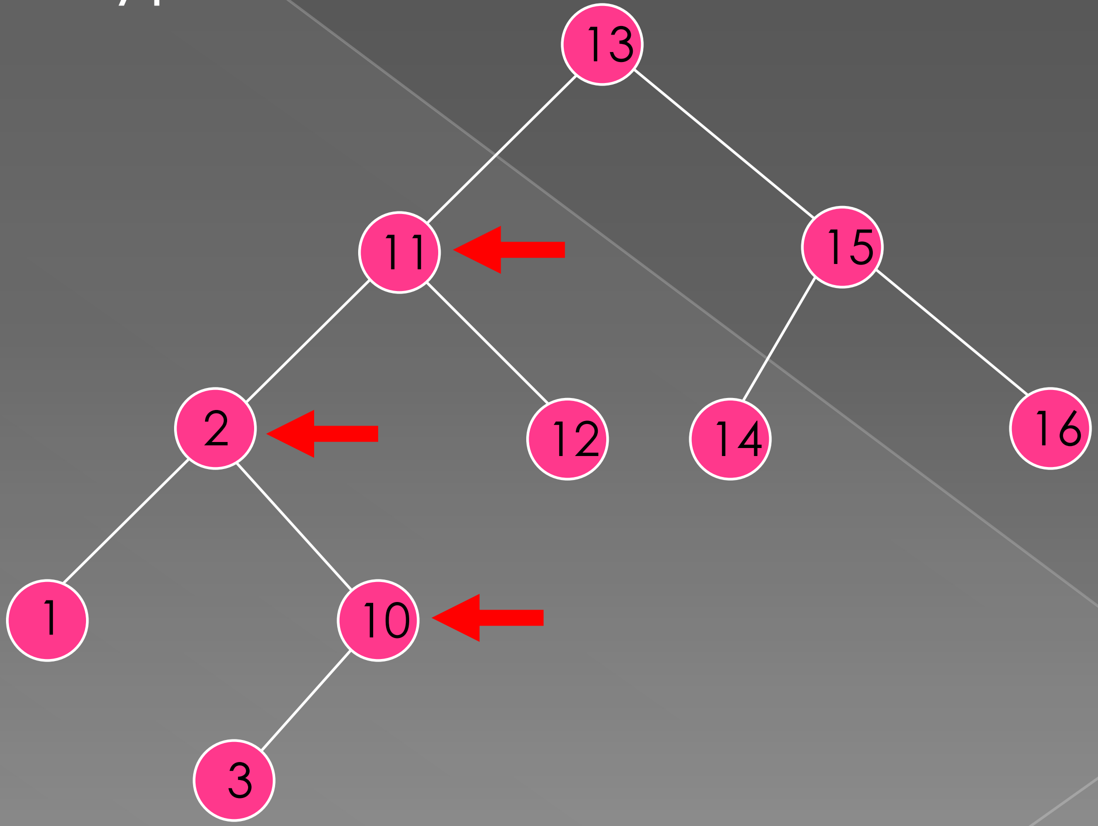
- AVL violation – rotate:



AVL Tree Rotations

Double rotations:

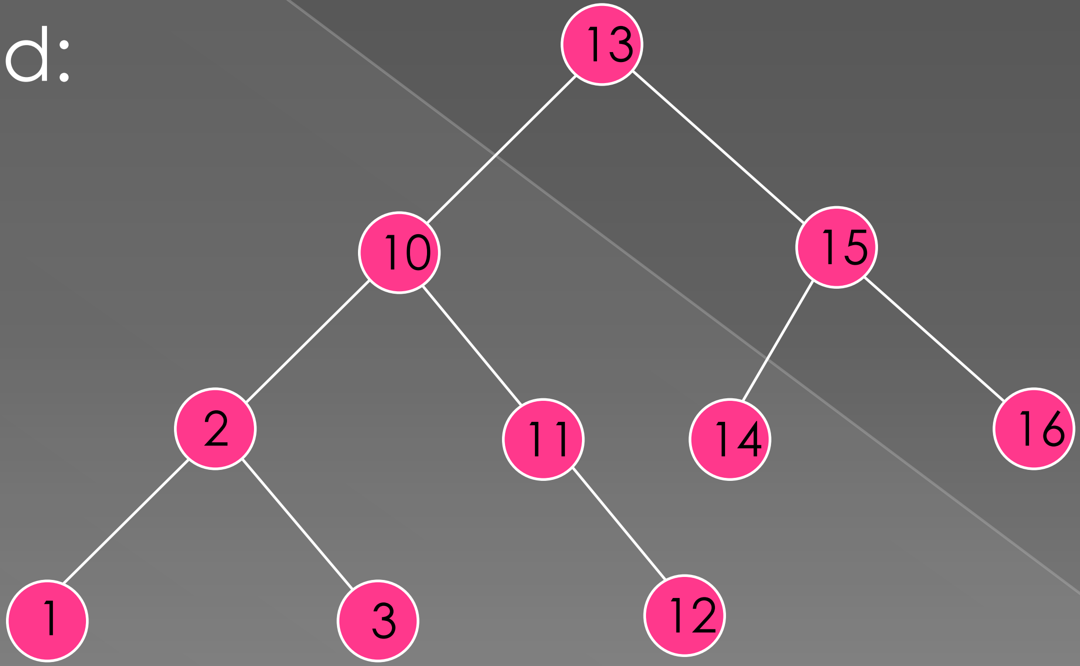
- Rotation type:



AVL Tree Rotations

Double rotations:

- AVL balance restored:

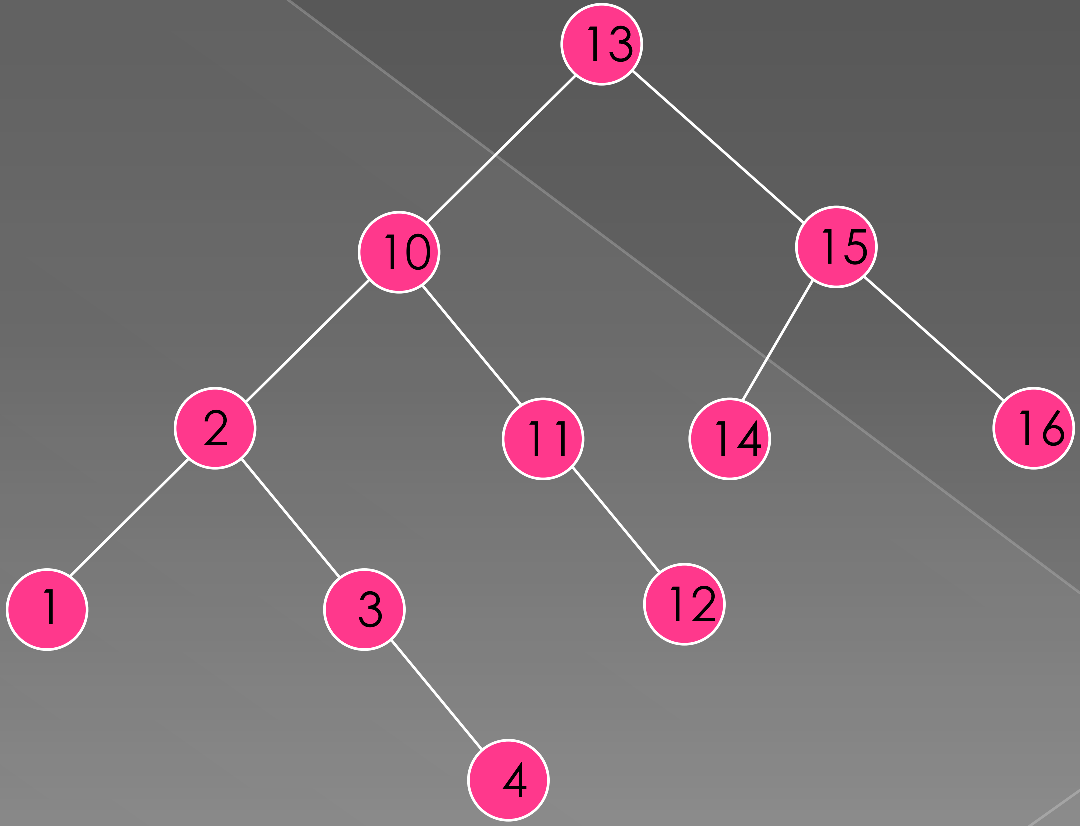


- Now insert 4.

AVL Tree Rotations

Double rotations:

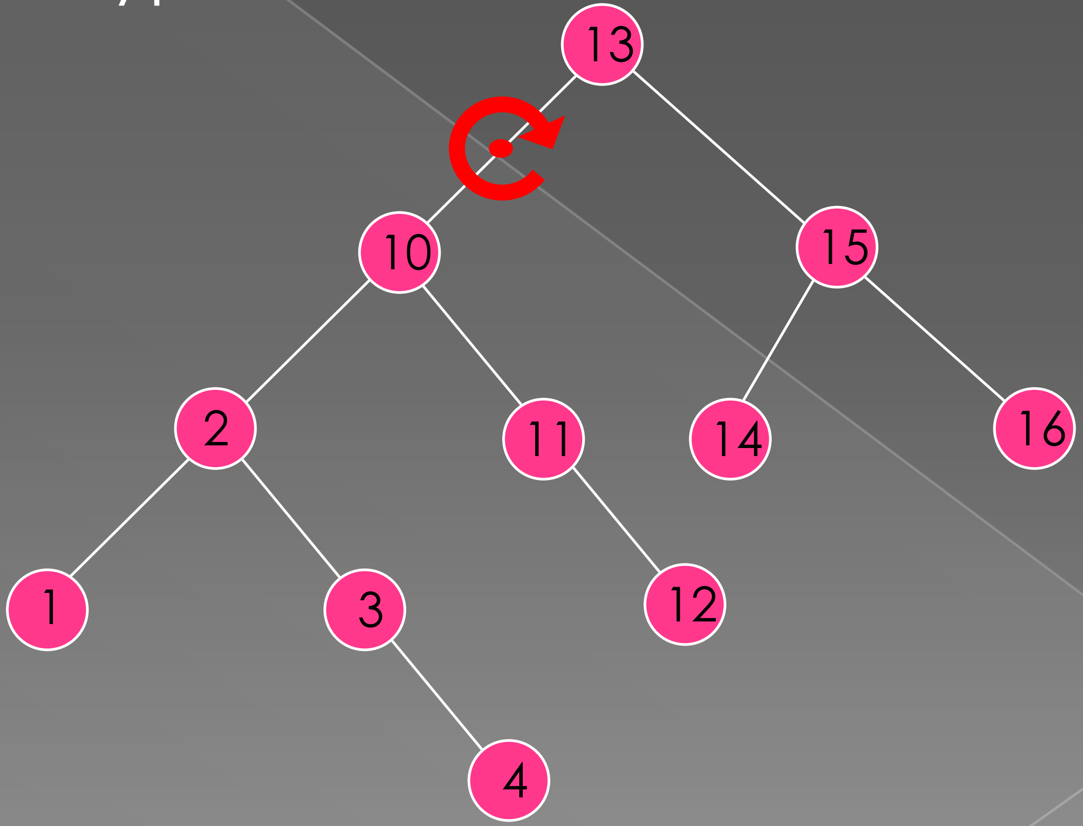
- AVL violation - rotate



AVL Tree Rotations

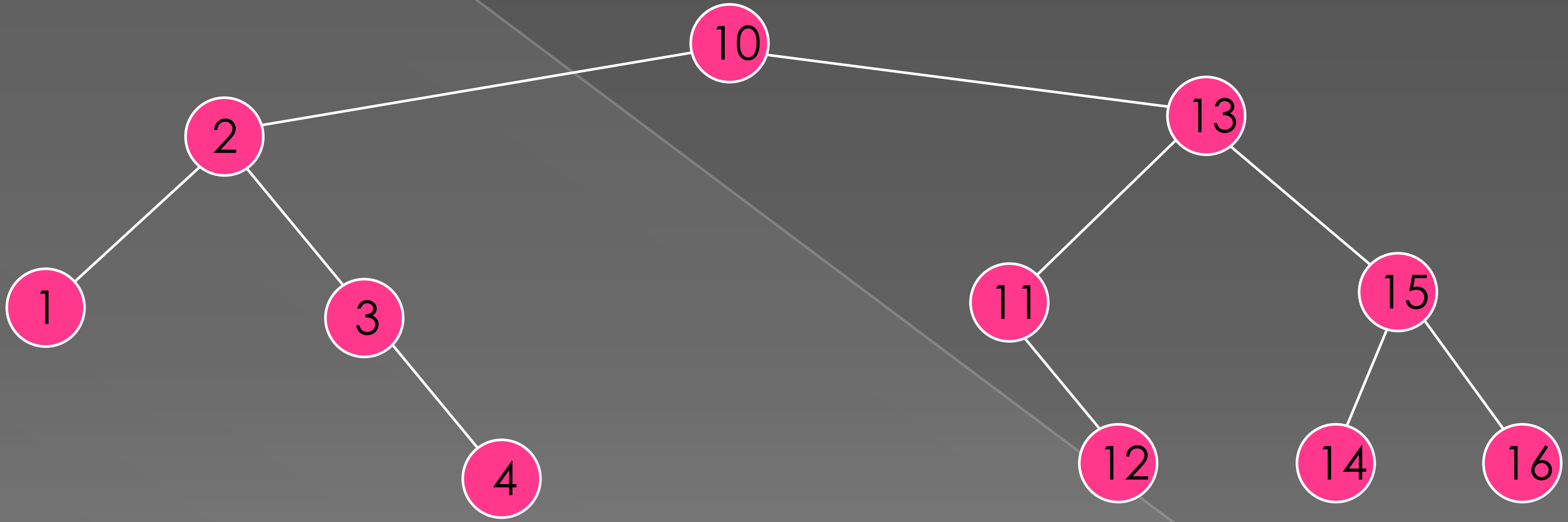
Double rotations:

- Rotation type:



AVL Tree Rotations

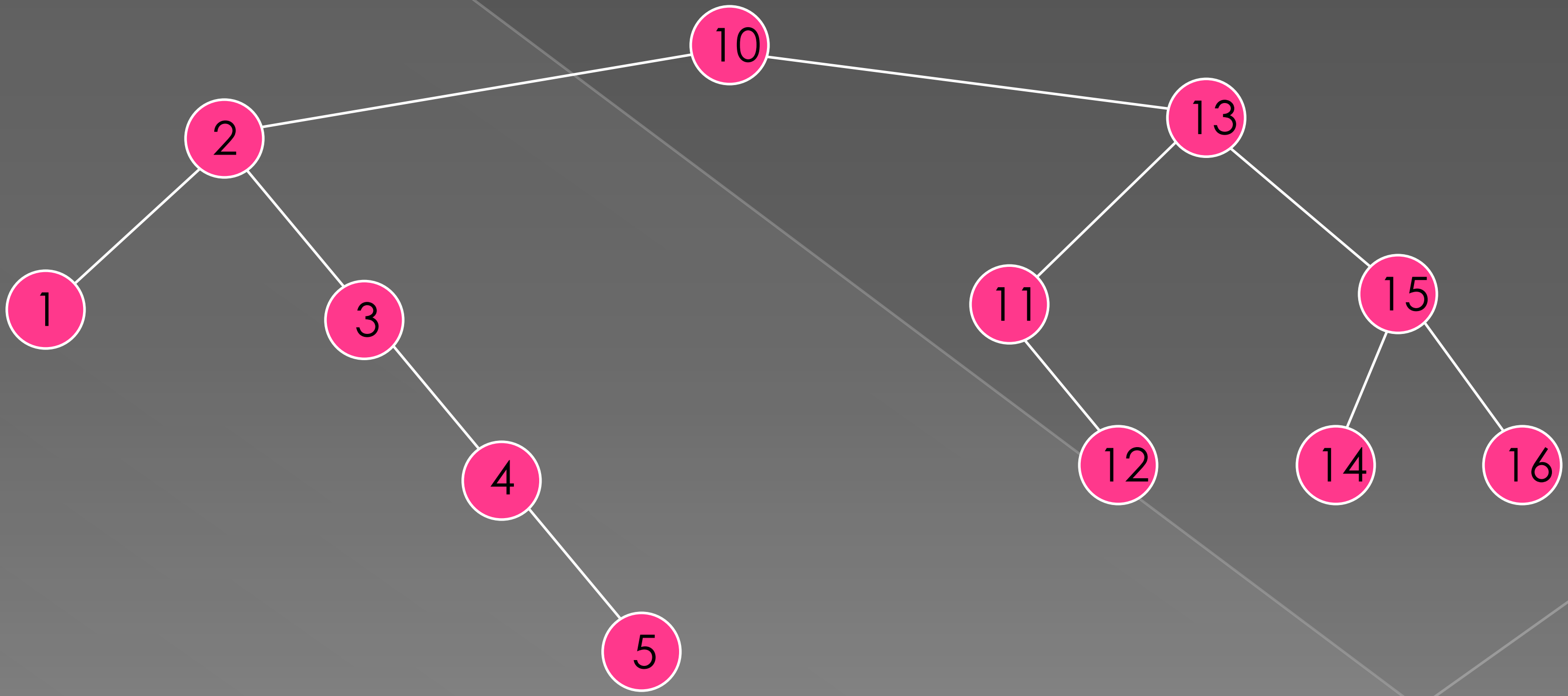
Double rotations:



- Now insert 5.

AVL Tree Rotations

Double rotations:

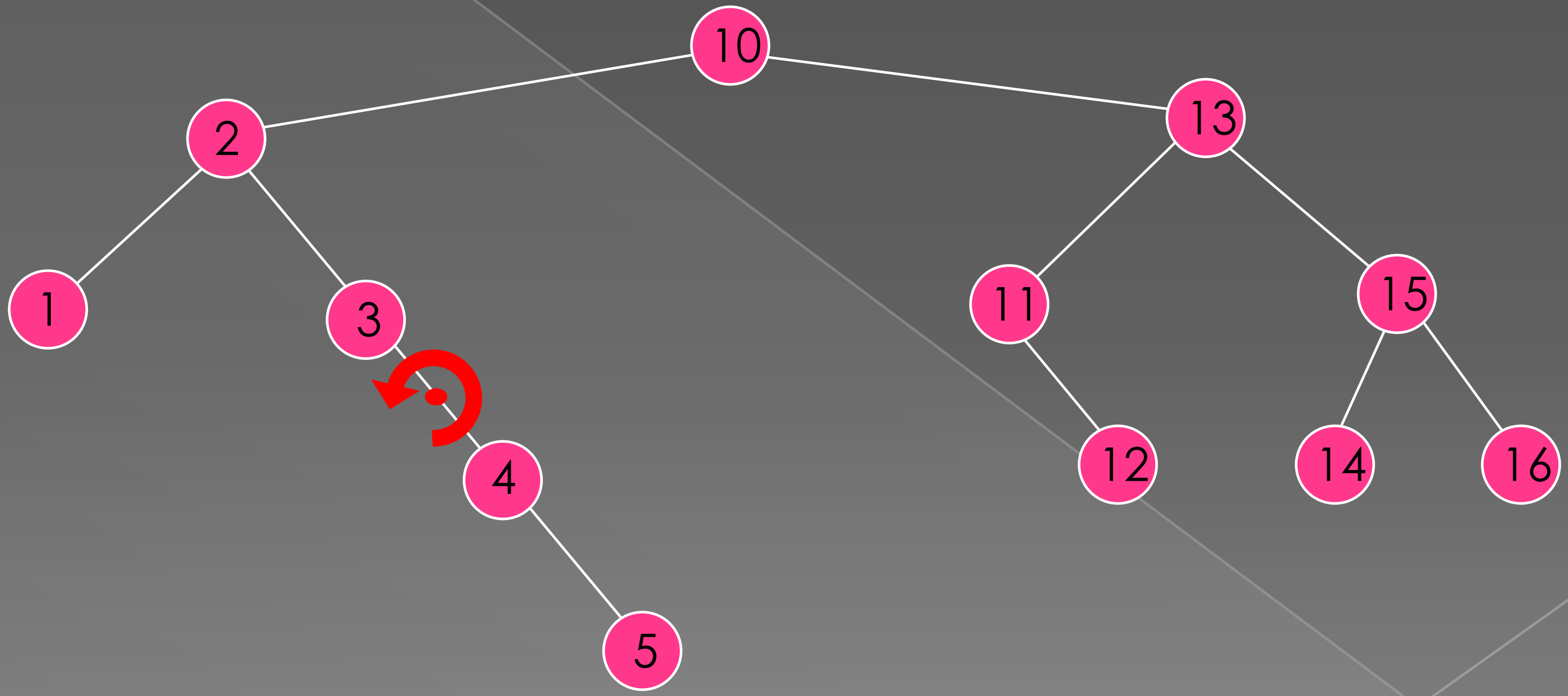


- AVL violation – rotate.

AVL Tree Rotations

Single rotations:

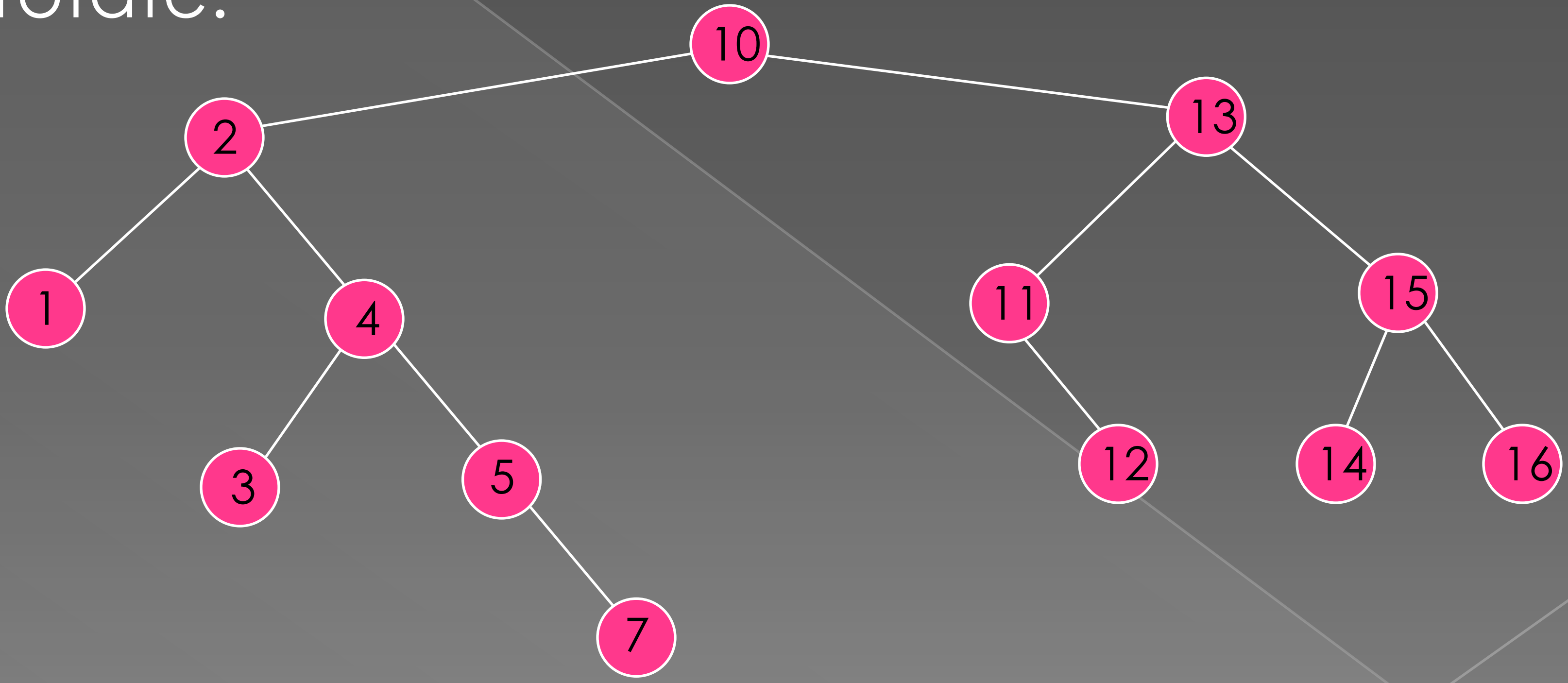
- Rotation type:



AVL Tree Rotations

Single rotations:

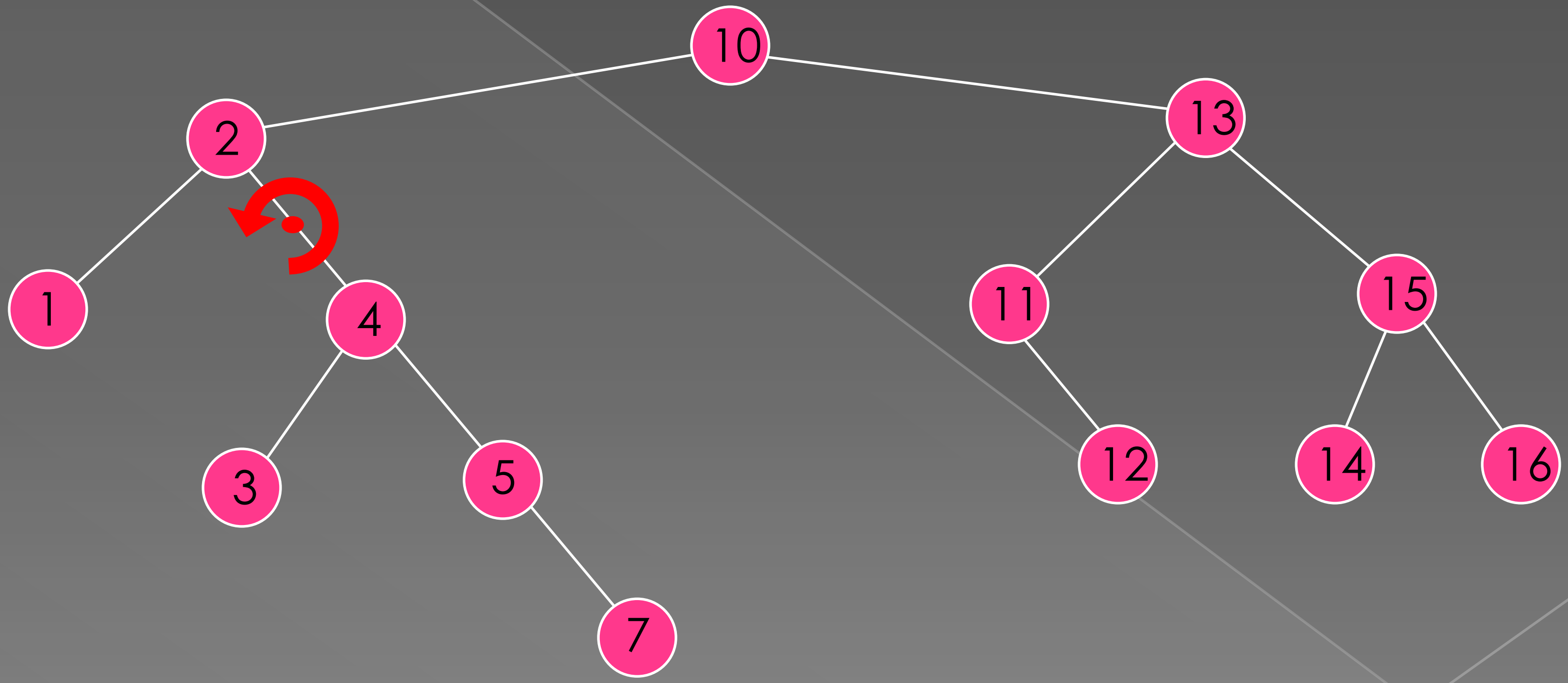
- AVL violation – rotate.



AVL Tree Rotations

Single rotations:

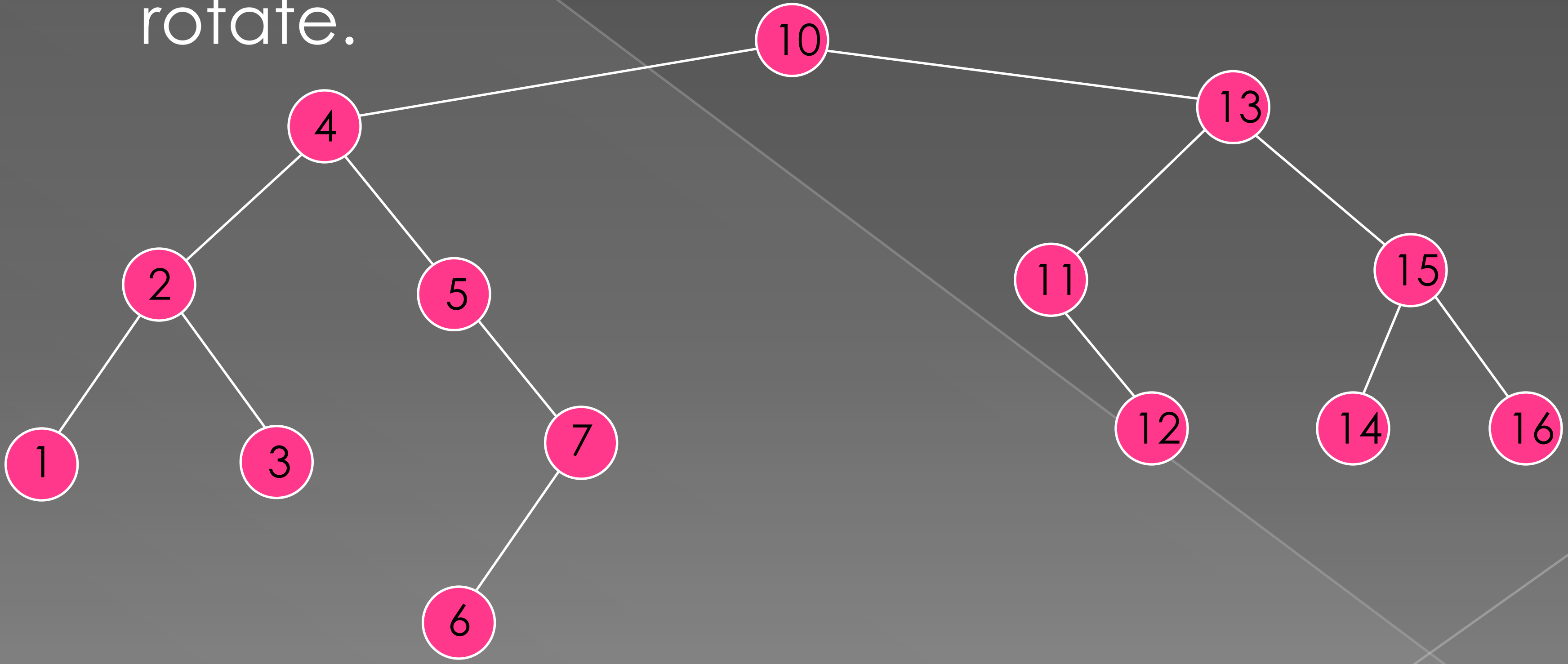
- Rotation type:



AVL Tree Rotations

Double rotations:

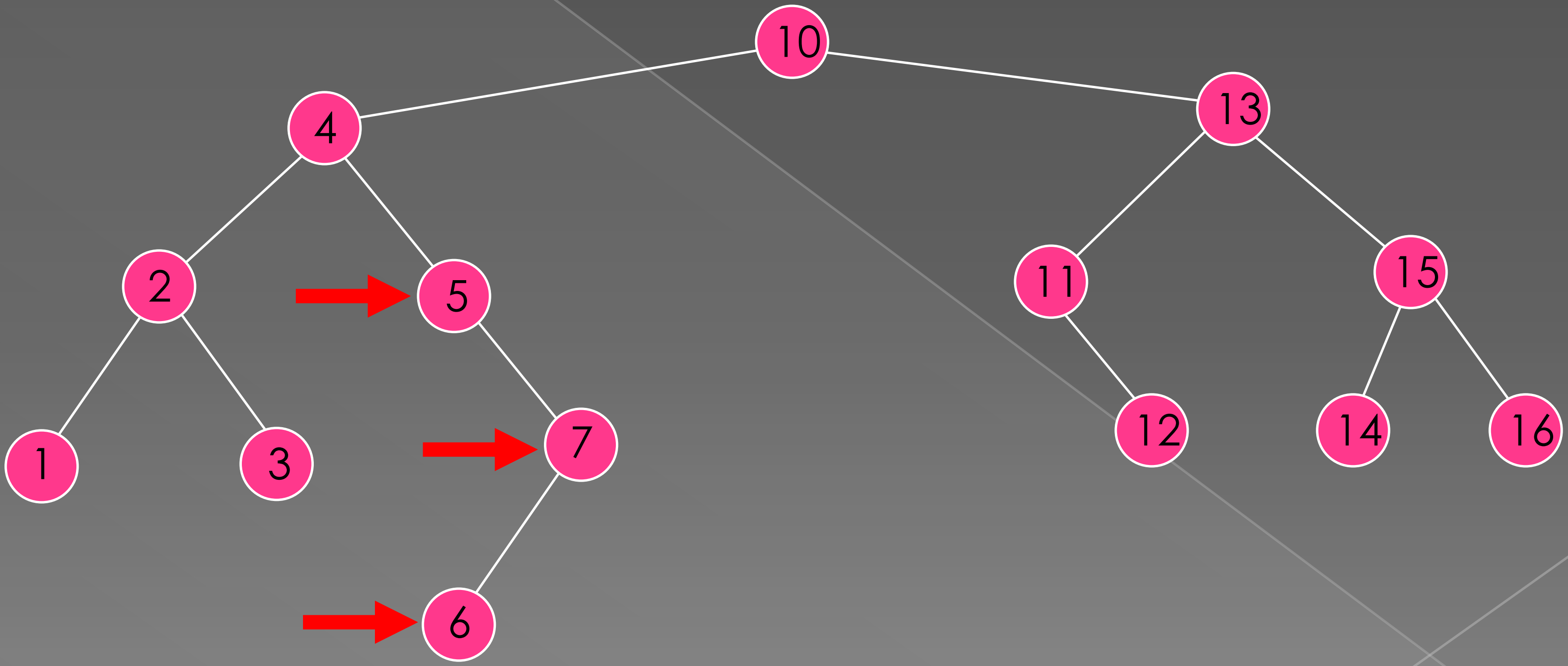
- AVL violation - rotate.



AVL Tree Rotations

Double rotations:

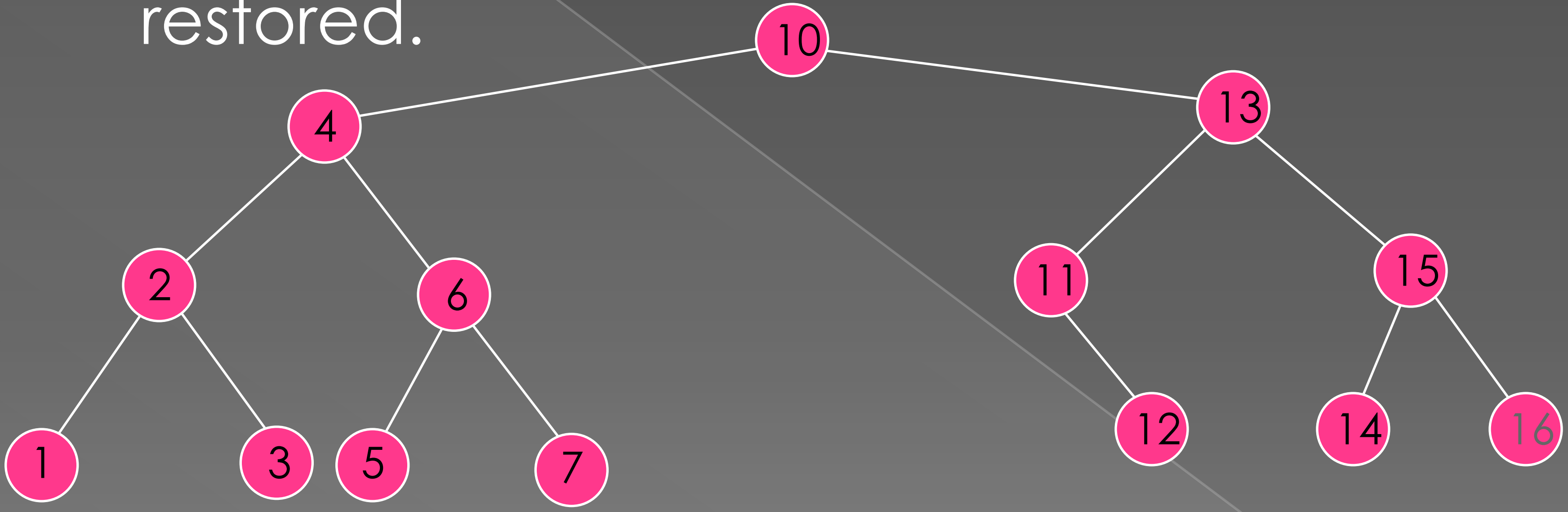
- Rotation type:



AVL Tree Rotations

Double rotations:

- AVL balance restored.

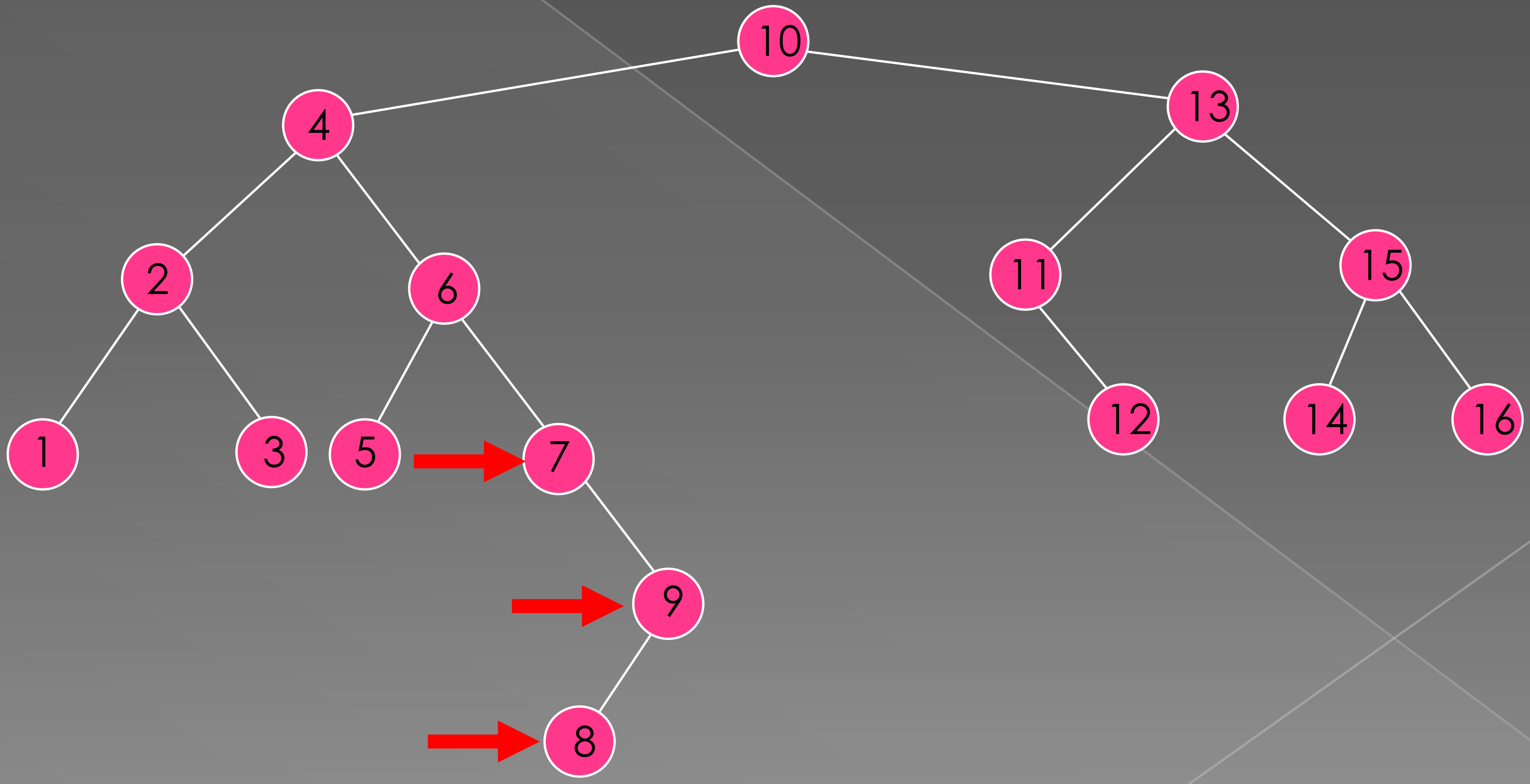


- Now insert 9 and 8.

AVL Tree Rotations

Double rotations:

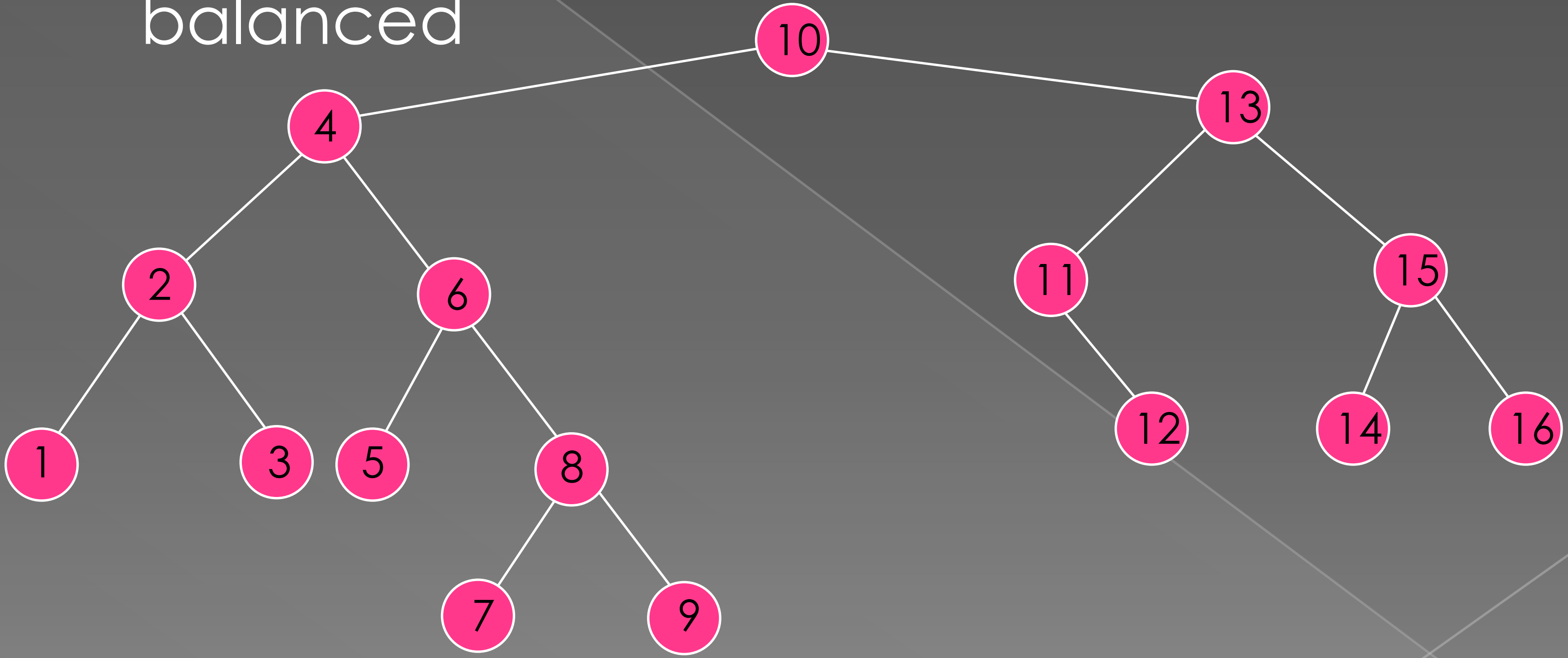
- Rotation type:



AVL Tree Rotations

Final tree:

- Tree is almost perfectly balanced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