B.Tech. (CSBC) I Semester
Discrete Mathematics: Unit III
Tutorial Problems (Function, Group, Ring and Field)

Function

1. If (x) = $6x^3 + 4x - 5$, find f(1), f(-2)

Ans.: f(1) = 5 , f(-2) = -61

2. Check if the following functions are odd/ev even: x^2+1, xsinx, x^2cosx, x^3

Ans.: even, even ,even & odd

3. If $f(x)=3x^4-5x^2+7$ then find f(x-1).

Ans.: $3x^4-12x^3+13x^2-2x+5$

4. If f(x)=f(3x-1) such that $f(x)=x^2-4x+11$ then find x.

Ans.: 5/4 or ½

5. If $f(x) = \sqrt{X+1}$ And $g(x) = x^2 + 2$, calculate *fog* and *gof*.

Ans.: $\sqrt{x^2+3}$ & x+3

6. Is the following function even, odd, or neither? $F(x)=12x^{11}-6x^7-5x^3$

Ans. Odd

7. If f(x)=3x+a such that f(1)=7 then find a & f(4).

Ans.: a=4, f(4)=16

8. If f(x)=f(2x+1) such that $f(x)=x^2-3x+4$ then find x.

Ans.: -1 or 2/3

9. Evaluate f(3) given that $f(x) = |x - 6| + x^2 - 1$

Ans. 11

10. Find f(x + h) - f(x) given that f(x) = a x + b

Ans.: ah

Group

1. Let $G = \{1, 2, 3, 4, 5, 6\}$ prove that $(G, \times_7)$ is a finite abelian group with respect to multiplication modulo 7
2. Let $G = \{0, 1, 2, 3, 4, 5\}$ prove that $(G, +_6)$ is a finite abelian group with respect to addition modulo 6.
3. For $_7 - \{0\}$
   - (i) Prepare composition table with respect to '×7'
   - (ii) Prove that it is an abelian group with respect to 'x7'
   - (iii) Is it cyclic?
   - (iv) Find the order of 2 & 4 and subgroups generated by these elements

4. Let G be a set of all rational numbers other than 1. Let '$*$' be the binary operation on G defined by $a * b = a + b - ab \ \forall a, b \in G$. Prove that $(G, *)$ is a group.

5. Let be a set of all positive rational numbers. Let '$*$' be the binary operation on defined by $a * b = \left(\frac{ab}{3}\right) \ \forall a, b \in Q$. Prove that $(Q, *)$ is an abelian group.

6. Let $R' = R - \{1\}$. Let '$*$' be the binary operation on R' defined by $a * b = a + b + ab \ \ \forall a, b \in R'$. Then prove that $(R', *)$ is an abelian group.

7. Let G be a set of all square matrices of type $\begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}$, where $m \in Z$, prove that G is a group under the operation of multiplication. Is it an abelian group?

8. Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7.

(i) Find multiplication table of G.

(ii) Find $2^{-1}, 3^{-1}, 6^{-1}, 1^{-1}, 4^{-1}, 5^{-1}$

9. Let G = {0, 1, 2, 3, 4, 5}

(i) Prepare composition table with respect to '$+_6$'

(ii) Prove that G is an abelian group with respect to '$+_6$'

(iii) Find the inverse of 2, 3 and 5.

(iv) Is it cyclic?

(v) Find the order of 2, 3 and sub groups generated by these elements.

10. Let G = {1, 2, 3, 4, 5 }= $_6$ − {0}.

(i) Prepare the table for multiplication mod 6.

(ii) Is G is a group under multiplication mod 6?

11. Define a group and cyclic group. Let A = {0, 3, 6, 9, 12}. Find out the table for addition modulo 15 and multiplication modulo 15. Determine whether (A, $+_{15}$) and (A, $\times_{15}$) are groups? Are they cyclic groups?

12. Let (G, *) be a group and a $\in$ G. Let f : G → G defined as f(x) = a * x * a^{-1}, x $\in$ G. Show that f is isomorphism.

13. Show that the additive group of Z_6 is isomorphic to the multiplicative group of Z_7

14. Let G be the group of integers under operation of addition, and let G' be the group of all even integers under the operation of addition. Show that the function f : G → G' defined by f(a) = 2a is an isomorphism

15. Show that the group G = (, +) is isomorphic to G' = ($^+$, ×), where $^+$ is a set of positive real numbers

16. If G is a group of all real numbers under addition and G' is the group of +ve real numbers under multiplication and mapping f : G → G' is defined by f(x) = 2^x, x $\in$ G then show that f is homomorphism

17. Let G = $_8$, for each of the following subgroups H of G, determine all the left cosets of H in G. H = {0, 2, 4, 6}

Ring and Field

1. If the addition and multiplication modulo 10 is defined on the set of congruence classes R = {0,2,4,6,8} then show that algebraic structure is a ring with unity. Is it an integral domain or field or both?
2. Prove that the set of complex numbers is a commutative ring with unity the addition and multiplication of complex numbers being two ring compositions
3. Show that the set of 2×2 matrices with entries in a ring *R* is a non-commutative ring. [Hint: since matrix multiplication is known not to be commutative.]
4. Show that ring of real numbers $(\mathbb{R},+,\times)$ and ring of complex numbers $(\mathbb{C},+,\times)$ are fields.
5. Is $(\mathbb{Z}_5,+_5,\times_5)$ a field? [Hint: $\mathbb{Z}_5$ is a commutative ring with unity and each nonzero element has inverse w.r.t. $\times_5$.]
6. Is $(\mathbb{Z}_6,+_6,\times_6)$ a field? [Hint: $\mathbb{Z}_6$ is a commutative ring with unity but each non-zero element does not have inverse w.r.t. $\times_6$.]
7. Do the following sets form integral domain with respect to ordinary addition and multiplication? If so state if they are fields.
   i. The set of numbers of the form $b\sqrt{2}$ with b is rational number.
   ii. The set of even integers.
   iii. The set of positive integers.
8. Determine if the set $M_2(\mathbb{R})$ of invertible 2×2 matrices with real entries under usual addition and multiplication is a field
9. Determine if the $\mathbb{Z}^+$ under usual subtraction and multiplication is a field