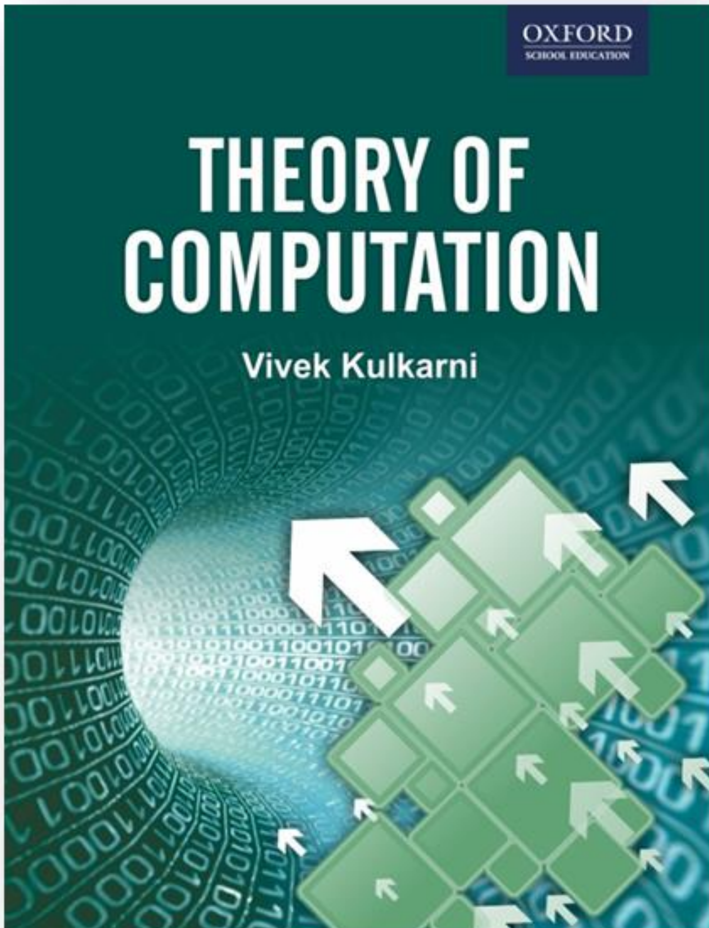

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Chapter-1: Preliminaries

Solutions for Review Questions

Q.1 Define the following and give suitable examples.

i) Cardinality of a set

ii) Closure of a set

iii) Alphabet

iv) Symbol

v) Language

vi) Word

Solution:

i) Cardinality of a set – Refer to the section 1.3.2.

ii) Closure of a set – Refer to the section 1.3.1.9.

iii) Alphabet – Refer to the section 1.2.2.

iv) Symbol – Refer to the section 1.2.1.

v) Language – Refer to the section 1.6.

vi) Word – Refer to the section 1.2.3.

Q.2 If $R = \{(a, b), (b, c), (c, a)\}$ is a relation over $\{a, b, c\}$, find R^+ and R^*.

Solution:

Let, S = {a, b, c} and R = {(a, b), (b, c), (c, a)}.

The transitive closure of a relation R, which is denoted by R^+, is defined as follows:

- If $(a, b) \in R$, then (a, b) is in R^+
- If $(a, b) \in R^+$ and $(b, c) \in R^+$, then (a, c) is in R^+

Hence, for given set S and relation R,

R^+ = {(a, b), (b, c), (c, a), (a, c), (b, a), (c, b)}

Reflexive and transitive closure of a relation R, which is denoted by R^*, is defined as follows:

$R^* = R^+ \cup \{(a, a) \mid \forall a \in S\}$, where R is a relation defined over set S.

Hence, for given set S and relation R,

R^* = {(a, b), (b, c), (c, a), (a, c), (b, a), (c, b), (a, a), (b, b), (c, c)}

Q.3 Differentiate between natural and formal languages.

Solution:

Refer to the section 1.6.1.

Q.4 Find the transitive closure and the symmetric closure of the relation:

$R = \{(1, 2), (2, 3), (3, 4), (5, 4)\}$

Solution:

Transitive closure for the given relation R (R^+) is

$R^+ = \{(1, 2), (2, 3), (3, 4), (5, 4), (1, 3), (2, 4)\}$

Symmetric closure of a relation R is defined as follows:

If $(a, b) \in R$ then (a, b) and (b, a) are in the symmetric closure of R

Thus, symmetric closure of $R = \underline{R} \cup \{(b, a) \mid (a, b) \in R\}$

Hence, symmetric closure of R is given as

Symmetric closure of $R = \{(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (5, 4), (4, 5)\}$

Q.5 Show that 'the set of real numbers R is not countable'.

Solution:

Refer to the section 1.3.3.

Q.6 Let L be a language. It is clear from its definition that $L^+ \subseteq L^*$. Under what circumstances are they equal?

Solution:

Closure of a set is defined as:

$S^* = S^0 \cup S^1 \cup S^2 \ldots,$

where, $S^0 = \{\in\}$,

and, $S^i = S^{i-1} \cdot S$; for $i > 0$.

Closure of a set is thus a repetitive concatenation of the set to itself.

Positive closure of a set S does not include $S^0 = \{\in\}$ and thus can be defined as,

$S^+ = S^1 \cup S^2 \ldots$

Language being the set, L^* here mean the set closure and L^+ means positive closure. L^+ does not include an empty string. Hence, typically $L^+ \subseteq L^*$. But, if L already contains an empty string $\in$ then these two could be equal.

Q.7 Show that if S uncountable and T is countable, then '$S - T$' is uncountable.

Solution:

Refer to the example 1.1 from the book.

Q.8 Show that any equivalence relation R on a set S partitions S into disjoint equivalent classes.

Solution:

Refer to the section 1.4.1.

Q.9 Find transitive closure and reflexive and transitive closure for the following relation set:

$R = \{(a, b), (a, c), (c, d), (a, a), (b, a)\}$

Solution:

Let, S = {a, b, c, d}.

Transitive closure of given relation set R can be given as,

$R^+ = \{(a, b), (a, c), (c, d), (a, a), (b, a), (a, d), (a, a), (b, b)\}$

Reflexive and Transitive closure of given relation set R can be given as,

$R^* = \{(a, b), (a, c), (c, d), (a, a), (b, a), (a, d), (a, a), (b, b), (c, c), (d, d)\}$

Q.10 For the sets $A = \{a, b, c, d\}$ and $B = \{c, d, e\}$, find the following:

(i) $A - B$ (ii) $A \times B$ (iii) 2^B (power set)

Solution:

i) The difference of two sets is defined as:

$\underline{A} - \underline{B} = \{\underline{x} \mid \underline{x} \in \underline{A} \text{ and } \underline{x} \notin \underline{B}\}$, or, $\underline{A} - \underline{B} = \underline{A} - (\underline{A} \cap \underline{B})$

Hence, A – B = {a, b}

ii) The Cartesian product of two sets is defined as:

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B, \forall a \,\&\, \forall b\}$

It defines the association of every element of set *A* with each element of set *B*.

Hence, $A \times B = \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, c), (c, d), (c, e), (d, c), (d, d), (d, e)\}$

i) The power set of a set *B*, *i.e.*, 2^B is the set of all subsets of *B*, including itself, and the empty set, ϕ.

$2^B = \{\phi, \{c\}, \{d\}, \{e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{c, d, e\}\}$