# TIME SERIES ANALYSIS

## MEASUREMENT OF SEASONAL VARIATIONS:

The effects of trend cycles and irregular fluctuations must be eliminated from the original time series data to obtain an estimate of seasonal variation. Once they are eliminated, we get measures of seasonal variations in the behavior of any variable. These measures are called seasonal indexes. By using such indexes, we can deseasonalise the time series. Such deseasonalised data are known as seasonally adjusted data.

### 1) Method of Simple Averages

This is the simplest method of obtaining a seasonal index. The following steps are required to calculate index:

I) Arrange the unadjusted data by years and months (or quarters if quarterly data are given)

II) Find the totals of January, February, etc.

III) Divide each total by the number of years for which data are given. For eg. If we are given monthly data for five years then total each month for five years and divide each total by 5 to obtain an average.

IV) Obtain an average of monthly averages.

V) Compute the seasonal index as follows:

$$\text{Seasonal Index for January} = \frac{Monthly\_average\_for\_January}{Average\_of\_monthly\_averages} x100$$

Ex.1 Compute the average seasonal movement for the following series:

| Year | Quarterly Production | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| 1994 | 3.5 | 3.9 | 3.4 | 3.6 |
| 1995 | 3.5 | 4.1 | 3.7 | 4.0 |
| 1996 | 3.5 | 3.9 | 3.7 | 4.2 |
| 1997 | 4.0 | 4.6 | 3.8 | 4.5 |
| 1998 | 4.1 | 4.4 | 4.2 | 4.5 |

Ex.2 Assuming that the trend is absent, determine if there is any seasonality in the data given below:

| Year | I | II | III | IV |
|---|---|---|---|---|
| 1994 | 3.7 | 4.1 | 3.3 | 3.5 |
| 1995 | 3.7 | 3.9 | 4.6 | 3.6 |
| 1996 | 4.0 | 4.1 | 3.3 | 3.1 |
| 1997 | 3.3 | 4.4 | 4.0 | 4.0 |

## 2) RATIO TO TREND METHOD

This method isolates the seasonal factor after eliminating the trend from time series. Trend is eliminated by computing the ratios, and random elements disappear when the ratios are averaged.

The steps involved in computations of index are:

I) Determine the trend values by the method of least squares.

II) Divide the original data month by month by the corresponding trend values and express them as percentage

III) Average the different values for a month.

IV) Adjust all these averages to a total of 1200.

V) If the data is distributed in quarter instead of months, adjust the respective averages to a total of 400.

It may be noted here that step (II) eliminates the trend and the values so obtained include cyclical and irregular variations. Step (III) frees the values from cyclical and irregular variations.

Ex. 1) Obtain a seasonal index for the following data by using the ratio to trend method. In applying this method, use the least square method to obtain monthly trend values.

| | Jan | Feb | Mar | April | May | June | July | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1991 | 178.2 | 156.7 | 164.2 | 153.2 | 157.5 | 172.6 | 185.9 | 185.8 | 165 | 163.6 | 169 | 183.1 |
| 1992 | 196.3 | 162.8 | 168.6 | 156.9 | 168.2 | 180.2 | 197.9 | 195.9 | 176 | 166.4 | 166.3 | 183.9 |
| 1993 | 197.3 | 173.7 | 173.2 | 159.7 | 175.2 | 187.4 | 202.6 | 205.6 | 185.6 | 175.6 | 176.3 | 191.7 |
| 1994 | 209.5 | 186.3 | 183 | 169.5 | 178.2 | 186.7 | 202.4 | 204.9 | 180.6 | 198.8 | 177.4 | 188.9 |
| 1995 | 200 | 188.7 | 187.5 | 168.6 | 175.7 | 189.4 | 216.1 | 215.4 | 191.5 | 178.5 | 178.6 | 195.6 |
| 1996 | 205.2 | 179.6 | 185.4 | 172.4 | 177.7 | 202.7 | 220.2 | 210.2 | 186.9 | 181.4 | 175.6 | 195.6 |

Ex.2) Find seasonal variations by the ratio-to-trend method from the data given below:

| Year | I | II | III | IV |
|---|---|---|---|---|
| 1990 | 30 | 40 | 36 | 34 |
| 1991 | 34 | 52 | 50 | 44 |
| 1992 | 40 | 58 | 54 | 48 |
| 1993 | 54 | 76 | 68 | 62 |
| 1994 | 80 | 92 | 86 | 82 |

**EXPONENTIAL SMOOTHING**

Exponential smoothing is method by which we can smoothen the data. It is a method of smoothing a time series by which an observation is replaced by the weighted sum of all past observations, the weights declining exponentially as the observations recede in time.

While the method of moving average consider each item of the data as equally important and attach equal weight to every item, exponential smoothing is based on the assumption that recent observations are more important predictors of the value of the next observation than the observations far in the past.

Let the subscript t refer to the time period $x_t$ to the value to the time series at time t. let a be any number between zero and one. The smoothened value of the time series at time t equals

$$x_t = ax_t + a(1-a)x_{t-1} + a(1-a)^2 x_{t-2} + a(1-a)^3 x_{t-3} + \cdots \qquad \text{---------} \ (\mathrm{I})$$

The equation will have as many terms as there are observations in the time series. If *a* were set at 0.8, then the formula would read

$$x_t = 0.8x_t + 0.16x_{t-1} + 0.032x_{t-2} + 0.0064x_{t-3} + \cdots$$

Note that each succeeding *x* value has less weight in determining the smoothed value of the time series at time *t.*

$$x_{t-1} = ax_{t-1} + a(1-a)x_{t-2} + a(1-a)^2 x_{t-3} + a(1-a)^3 x_{t-4} + \cdots$$

Multiplying the equation by *(1-a)* yields.

$$x_{t-1} - ax_{t-1} = a(1-a)x_{t-1} + a(1-a)^2 x_{t-2} + a(1-a)^3 x_{t-3} + \cdots \ \text{---------}(\mathrm{II})$$

Subtracting equation (II) from equation (I), we get

$$\boldsymbol{x_t = (1-a)x_{t-1} + ax_t}$$

Thus, the smoothened value at time *t* is the weighted average of the smoothened value for the previous period and the actual value at time *t.* The *a* in the formula is called the smoothing constant. It must lie between 0 and 1. If the smoothing constant is set close to 1, the smoothened value at time t will depend almost entirely on the value of the time series at time t. If the smoothing constant is set close to 0, the smoothened value at time t will depend mostly on the previous values of the time series.

Ex. Use Exponential Smoothing method for weekly sales of a Super Bazar for 25 weeks with the smoothing constant $a$=0.5.

| Week | Unit Sales (in '000) | Exponentially Smoothed with a=0.5 |
|---|---|---|
| 1 | 25 | |
| 2 | 13 | |
| 3 | 22 | |
| 4 | 31 | |
| 5 | 19 | |
| 6 | 13 | |
| 7 | 16 | |
| 8 | 10 | |
| 9 | 28 | |
| 10 | 22 | |
| 11 | 4 | |
| 12 | 16 | |
| 13 | 22 | |
| 14 | 19 | |
| 15 | 7 | |
| 16 | 10 | |
| 17 | 25 | |
| 18 | 13 | |
| 19 | 4 | |
| 20 | 16 | |
| 21 | 7 | |
| 22 | 22 | |
| 23 | 16 | |
| 24 | 19 | |
| 25 | 31 | |