DC CIRCUITS

Topics to be covered:

Series and Parallel connection of resistor, Kirchoff's laws, Mesh and nodal Analysis, Superposition Theorem, Thevenin's Theorem, Norton's theorem

1.1 Electric Current

Flow of free electrons is called electric current. When electric pressure is applied to a copper strip, free electrons being negatively charged will start moving towards positive terminal round the circuit. This directed flow of electron is called electric current.

The ***convention current*** flows from positive terminal of source to negative terminal of source. (Opposite to the flow of electrons).

The strength of electric current I is the rate of flow of electrons i.e., charge flowing per second.

$$Current\ (I) = \frac{Q\ (coulomb)}{t\ (sec)}$$

Unit of current is ***ampere*** i.e. coulomb/second

One Ampere of current is set to flow through a wire if at any section one coulomb of charge flows in one second.

1.2 Electric Potential & Potential Difference

Electric Potential: The charged body has the capacity to do work by moving other charges either by attraction or by repulsion. This ability of the charged body to do work is called electric potential.

$$Electric\ potential\ (V) = \frac{Work\ done}{Charge} = \frac{W\ (Joule)}{Q\ (Coulomb)}$$

Unit of electric potential is ***volt,*** joule/coulomb

Potential Difference: The difference in the potentials of two charged bodies is called potential difference.

1.3 Resistance

The opposition offered by a substance to the flow of electric current is called resistance.

The resistance of a conductor has following characteristics:

a) It is directly proportional to the length of the conductor.
b) It is inversely proportional to the area of cross section of the conductor.
c) It depends on the nature of the material of the conductor.
d) It depends on the temperature of the conductor.

Hence if R is resistance of a conductor of length l, cross-sectional area A and ρ is the resistivity of the material.

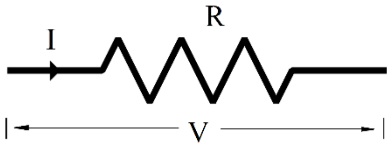
$$Resistance\ (R) = \rho \frac{l}{A}\ ohm()$$

1.4 Ohm's Law

The ratio of the potential difference (V), between the ends of a conductor to the current (I), flowing between them is constant, provided the physical condition (eg. temperature) do not change.

$$\frac{V}{I} = constant = R$$

Where R is the resistance of the conductor between the two points considered.



1.5 Electric Power

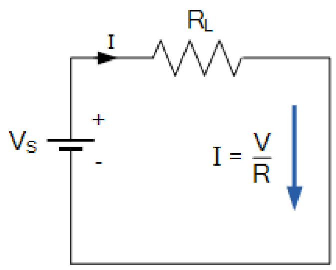
The rate at which work is done in an electric circuit is called electric power,

$$Electric\ power\ (P) = \frac{Workdone\ in\ elecric\ circuit}{Time}$$

$$P = VI = I^2R = \frac{V^2}{R}$$

1.6 DC Circuit

The closed path followed by a direct current (dc) is called a dc circuit.

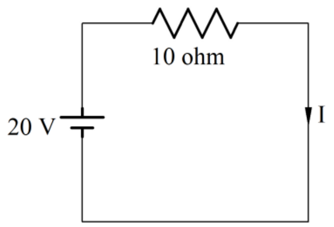


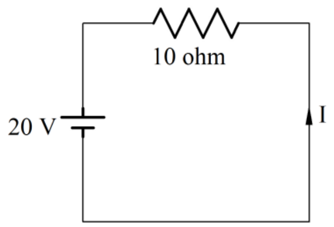
DC circuits can be classified as,

a) Series Circuit
b) Parallel Circuit
c) Series – Parallel Circuit

1.6.1 Fundamental problems based on Ohm's Law

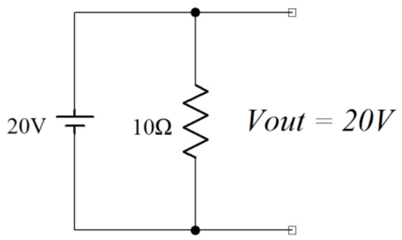
1)

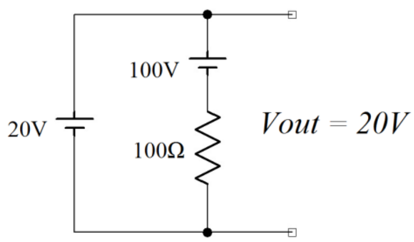


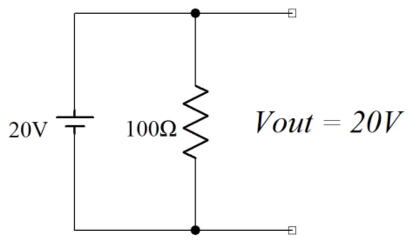


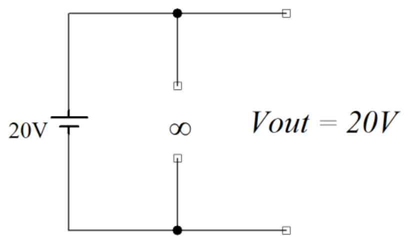
$$I = \frac{20}{10} = 2A(\downarrow) \qquad I = \frac{20}{10} = 2A\ (\downarrow) = -2A\ (\uparrow)$$

2)



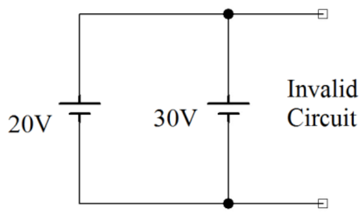


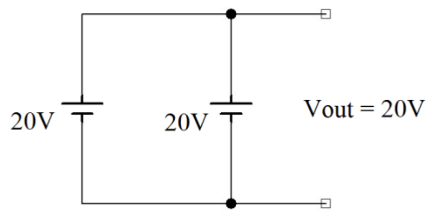


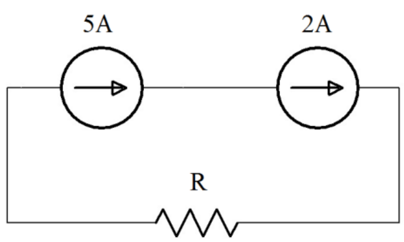


Resistance connected across, i.e. in parallel, to a voltage source is redundant resistor and can be replaced by an open circuit.

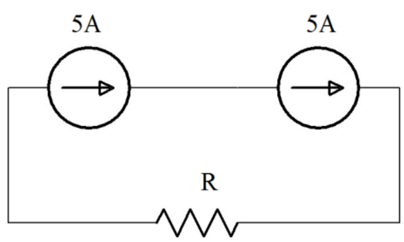
3)







Invalid Circuit



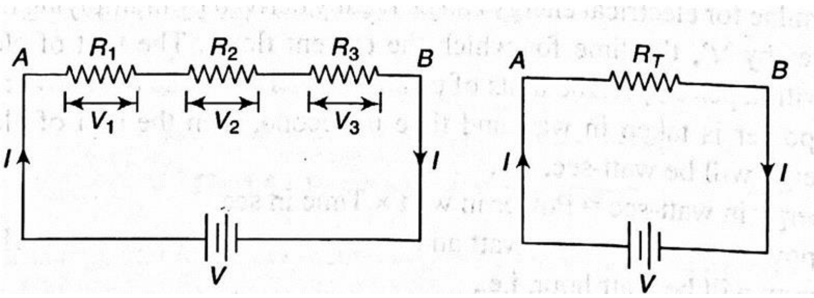
I = 5A

i) Two or more voltage sources with different magnitudes cannot be connected in parallel.
ii) Two or more voltage sources can be connected in parallel if they are of same magnitude, used in high current circuits like parallel batteries of inverter.
iii) Two or more current sources with different magnitudes cannot be connected in series.
iv) Two or more current sources can be connected in series if they are of same magnitude

1.6.2 Series Circuit

The circuit in which resistances are connected end to end so that there is only one path for the current to flow is called as series circuit.

Consider three resistances of R_1, R_2 and R_3 ohm connected in series across a battery of V volt as shown in Fig. Obviously, there is only one path for the current I, i.e., current is same throughout the circuit.



By Ohm's law,

Voltage drop across the resistance R_1, $V_1 = IR_1$
Voltage drop across the resistance R_2, $V_2 = IR_2$
Voltage drop across the resistance R_3, $V_3 = IR_3$

Now, for a series circuit, sum of voltage drops is equal to the applied voltage. So,

$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$
$$\frac{V}{I} = R_1 + R_2 + R_3$$

But V/I is the total resistance R_T between the points A and B. R_T is called the total or equivalent resistance of the three resistances. So,

$$R_T = R_1 + R_2 + R_3$$

Hence, when a number of resistances are connected in series, the total resistance is equal to the sum of the individual resistances.

The following points may be noted about a series circuit:

(i) The current flowing through each resistance is same.
(ii) The applied voltage equals the sum of different voltage drops.
(iii) The total power consumed in the circuit is equal to the sum of the powers consumed by the individual resistances.
(iv) Every resistor of the circuit has its own voltage drop.

Voltage Divider Rule

It may be observed that the source voltage in the circuit shown in Fig divides among the resistors R_1, R_2 and R_3, The voltage drop across the resistances can be obtained as

$$V_1 = IR_1 = \frac{V}{R_T} R_1 \qquad \left(\because I = \frac{V}{R_T}\right)$$

Similarly, $$V_2 = IR_2 = \frac{V}{R_T} R_2$$

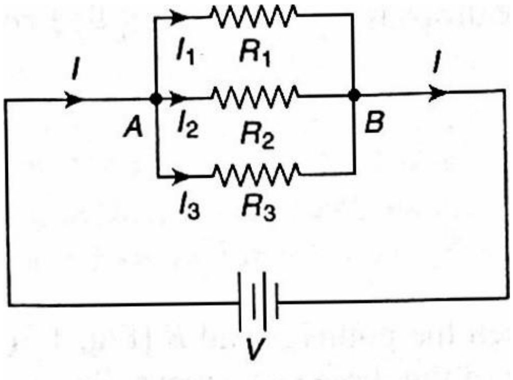
$$V_3 = IR_3 = \frac{V}{R_T} R_3$$

That is, the voltage drop across each resistor in a series circuit is directly proportional to the ratio of its resistance to the total series resistance of the circuit. By using the voltage divider rule, the proportion in which the voltage drops are distributed around a circuit can be determined.

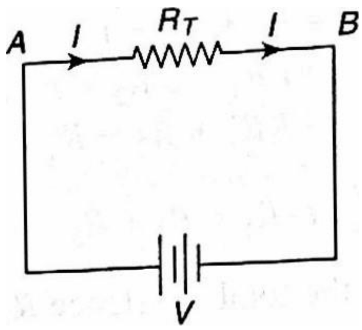
1.6.3 Parallel Circuit

The circuit in which one end of each resistance is joined to a common point and the other end of each resistance is joined to another common point, so that there are as many paths for current flow as the number of resistances, is called a parallel circuit.

Consider three resistances of R_1, R_2 and R_3 ohm connected in parallel across a battery of V volt as shown in Fig. The total current I divide into three parts: I_1 flowing through R_1, I_2 flowing through R_2 and I_3 flowing through R_3. Obviously, the voltage across each resistance is the same (i.e., V volt in this case) and there are as many current paths as the number of resistances.



(a)



By Ohm's law,

Current through the resistance R_1, $I_1 = \frac{V}{R_1}$

Current through the resistance R_2, $I_2 = \frac{V}{R_2}$

Current through the resistance R_3, $I_3 = \frac{V}{R_3}$

Now, for a parallel circuit, sum of the branch currents is equal to the total current.

So,

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \end{aligned}$$

or $$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But V/I is the total resistance R_T of the parallel resistances [see Fig. 1.8(b)] so that $I/V = 1/R_T$.

Hence, $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Hence, when a number of resistances are connected in parallel, the reciprocal of the total resistance is equal to the sum of reciprocals of the individual resistances.

The following points may be noted about a parallel circuit:

(i) The voltage drop across each resistance is same.
(ii) The total current equals the sum of the branch currents.
(iii) The total power consumed in the circuit is equal to the sum of the powers consumed by the individual resistances.
(iv) Every resistor has its own current.

Current Divider Rule

Consider a parallel circuit of Fig. 1.9. Two resistances R_1 and R_2 connected in parallel across a battery of V volt. The total current I divide into two parts I_1 and I_2.

The total resistance or equivalent resistance can be obtained as

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_2 + R_1}{R_1 R_2}$$

So, $$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

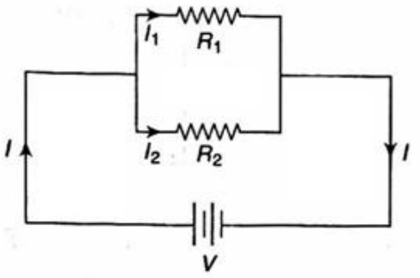


Fig. 1.9 Illustration of current divider rule

Hence, the total value of two resistances connected in parallel is equal to the product of the individual resistances divided by their sum.

The branch currents can be obtained as

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

So, $$V = IR_T = I\frac{R_1 R_2}{R_1 + R_2}$$

Current through R_1,

$$I_1 = \frac{V}{R_1}$$

$$= \frac{I\dfrac{R_1 R_2}{R_1 + R_2}}{R_1} \qquad \left(\because V = I\frac{R_1 R_2}{R_1 + R_2}\right)$$

or $$I_1 = I\frac{R_2}{R_1 + R_2}$$

Current through R_2,

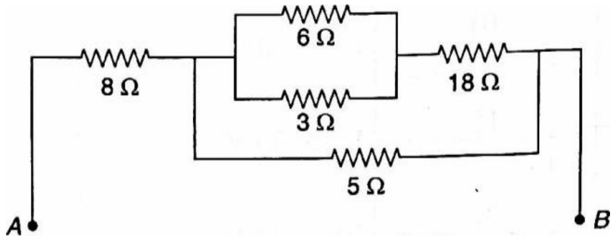
$$I_2 = \frac{V}{R_2}$$

or $$I_2 = I\frac{R_1}{R_1 + R_2} \qquad \left(\because V = I\frac{R_1 R_2}{R_1 + R_2}\right)$$

Hence, in a parallel circuit of two resistances, the current through one resistance is the line current (i.e., the total current) times the opposite resistance divided by the sum of the two resistances.

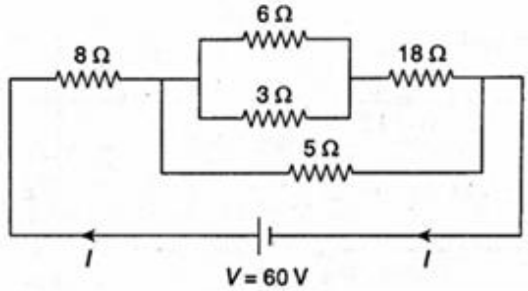
Problems:

1. Calculate the effective resistance of the circuit of Fig. and the current through 8Ω resistance, when potential difference of 60 V is applied between the points A and B.

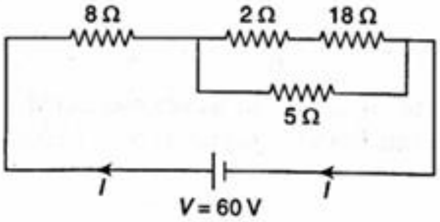


Solution

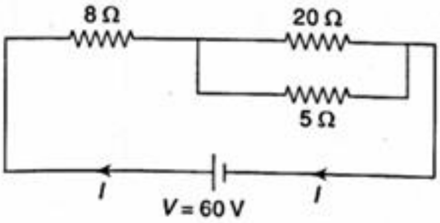
Potential difference of 60 V is applied between the points A and B. Let the current delivered by the source is I A.



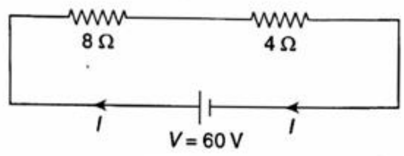
In Fig. 1.16, the resistors 6 Ω and 3 Ω, are in parallel. $\therefore 6 \parallel 3 = 2\ \Omega$.



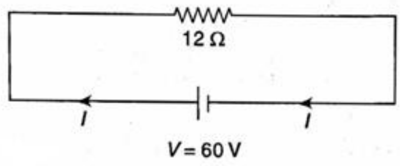
In Fig. 1.17, resistors 2 Ω and 18 Ω are in series. $\therefore 2 + 18 = 20\ \Omega$.



In Fig. 1.18, resistors 20 Ω and 5 Ω are in parallel. $\therefore 20 \parallel 5 = 4\ \Omega$



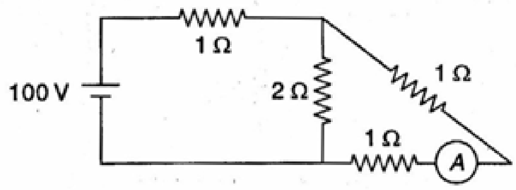
In Fig. 1.19, resistors 8 Ω and 4 Ω are in series. $\therefore 8 + 4 = 12\ \Omega$



By Ohm's law, circuit current, $I = \frac{60}{12} = 5\ \text{A}$

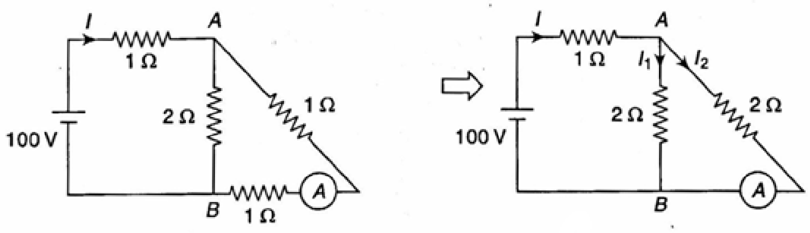
Hence, current through 8 Ω resistor, $I_{8\,\Omega} = 5\ \text{A}\ (\rightarrow)$

2. What is reading of the ammeter shown in the circuit shown?



Solution

Let the source current is I A.



The source current divides at node A. In Fig. branch current I_2 flows through ammeter. For the calculation of branch current, source current is required. Equivalent resistance across the battery can be calculated as

$$R_{eq} = 1 + (2 \| 2) = 2\ \Omega$$

By Ohm's law,

Total current,

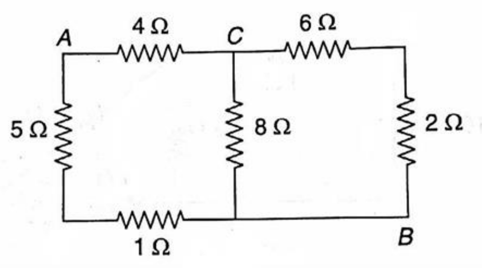
$$I = \frac{V}{R_{eq}} = \frac{100}{2} = 50A$$

By current division rule,

$$I_2 = 50 \times \frac{2}{2+2} = 25A$$

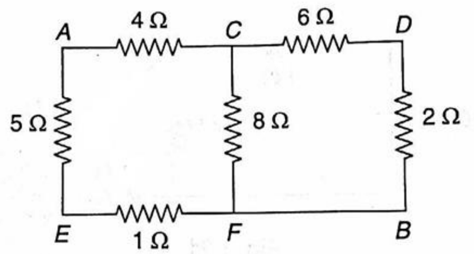
Hence, the ammeter reading is 25A.

3. Calculate the effective resistance between the points A and B in the circuit shown.

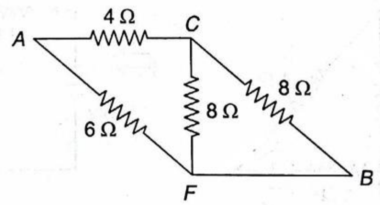


Solution

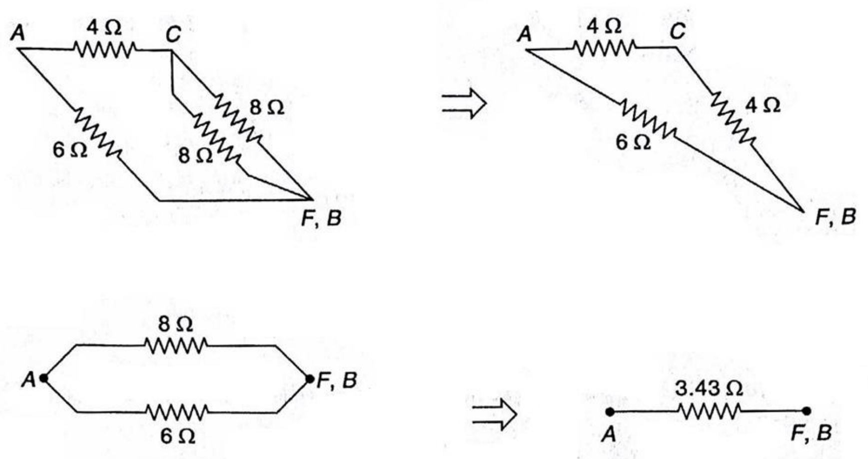
Marking the different nodes, we get the following figure:



In Fig. resistors 5 Ω and 1 Ω are in series. Also resistors 6 Ω and 2 Ω are in series.

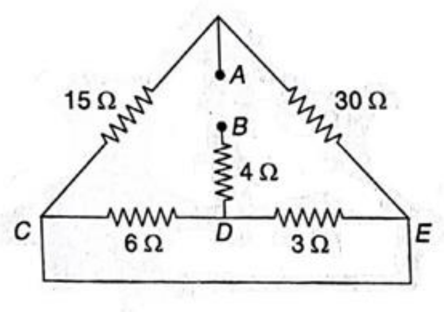


It may be noted that series resistances can be clubbed (added) together and after adding the series resistances, common nodes vanish, i.e., in Fig. , node *E* and node *D* vanish. In Fig. , node *F* and node *B* are same and by joining them, we get the following figures:

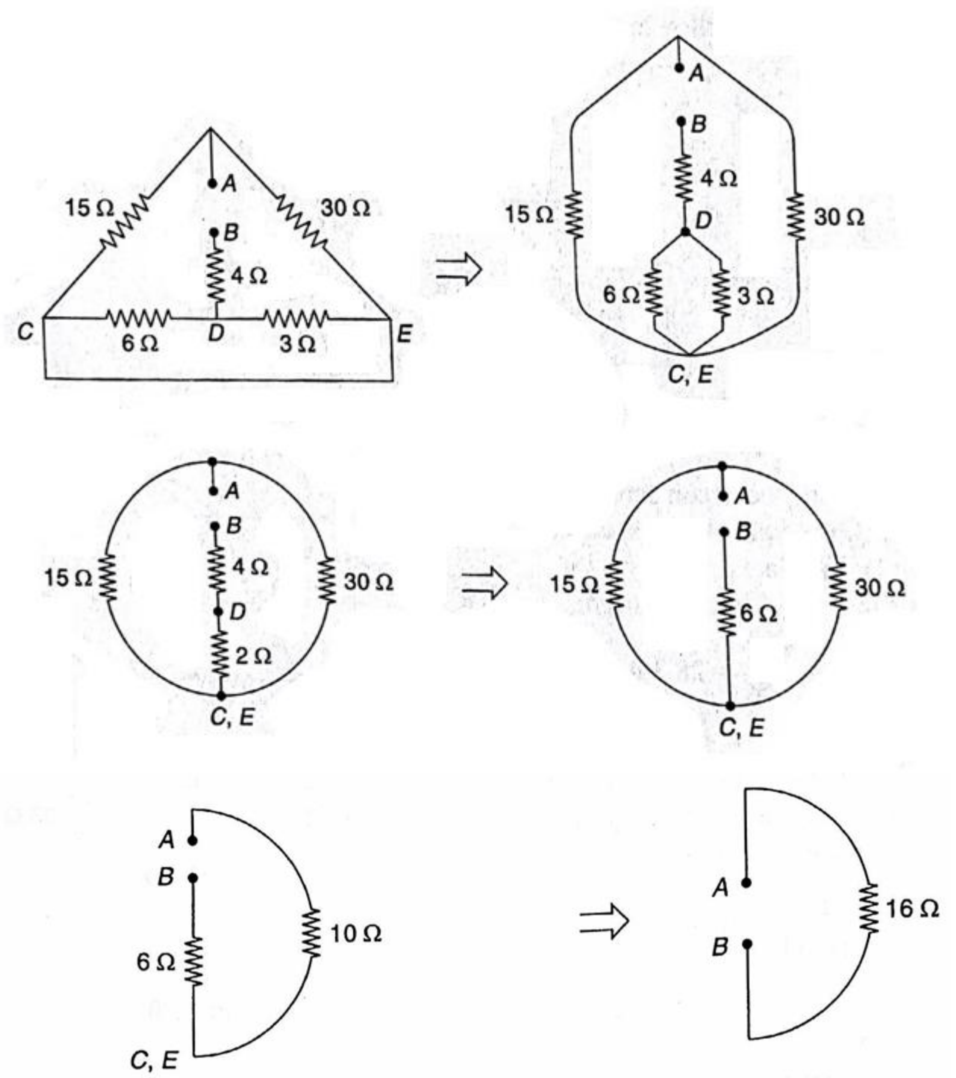


Thus, the equivalent resistance between the terminals A and B, $R_{AB} = 3.43\Omega$

4. Calculate the effective resistance RAB of network of figure shown.



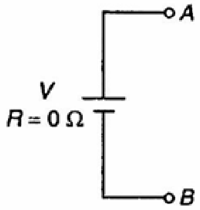
Solution

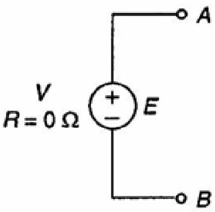


Thus, the equivalent resistance between the terminals A and B, $R_{AB} = 16\Omega$

1.7.1 Ideal Voltage Source

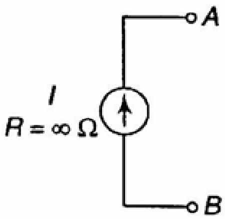
It is that voltage source whose output voltage remains absolutely constant whatever the change in load current. It has zero internal resistance. In practice, ideal voltage source is not available, and every voltage source has some internal resistance. Smaller the resistance of a voltage source, more it will approach to the ideal voltage source. The ideal voltage source can be represented by either of the symbols shown in Fig.





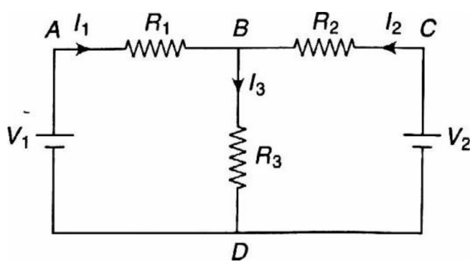
1.7.2 Ideal Current Source

It is that current source whose output current remains absolutely constant whatever the change in load resistance. Its internal resistance is infinity. At any load resistance, it supplies the constant current. In practice, ideal current source has very high resistance. Higher the resistance of a current source, more it will approach to the ideal source. The ideal current source can be represented by the symbol shown in Fig.



1.10 Kirchhoff's Laws

Consider Fig. below while discussing Kirchhoff's laws and techniques, one often comes across the terms such as active element, passive element, node, junction, etc. These are discussed below.



Active element

An active element is one that supplies electrical energy to the circuit. Thus, in Fig., V_1 and V_2 are the active elements because they supply energy to the circuit.

Passive element

A passive element is one that receives electrical energy, and then either converts it into heat (resistance) or stores in electric field (capacitance) or magnetic field (inductance). In Fig., there are three passive elements, namely R_1, R_2, and R_3. These passive elements (i.e., resistance in this case) receive energy from the active elements (i.e., V_1 and V_2) and convert it into heat.

Node

A node of network is an equi-potential surface at which two or more circuit elements are joined. Thus, in Fig., circuit elements R_1 and V_1 are joined at A and hence, A is the node. Similarly, B, C, and D are nodes.

Junction

A junction is that point in a network where three or more circuit elements are joined. In Fig., there are only two junction points, viz. B and D. That B is a junction is clear from the fact that three circuit elements R_1, R_2, and R_3 are joined at it. Similarly, point D is a junction because it joins three circuit elements R_3, V_1, and V_2. All the junctions are the nodes but all the nodes are not junctions.

Branch

A branch is the part of a network lying between two junction points. Thus, referring to Fig., there are total of three branches, viz. BAD, BCD, and BD. The branch BAD consists of R_1 and V_1, the branch BCD consists of R_2 and V_2, and branch BD merely consists of R_3.

Loop

A loop is any closed path of a network. Thus, in Fig., ABDA, BCDB, and ABCDA are the loops.

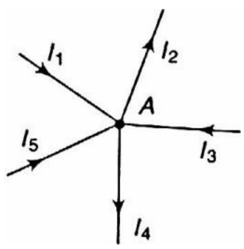
Mesh

A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig., both loops ABDA and BCDB are meshes because they cannot be further divided into other loops. However, the loop ABCDA cannot be called a mesh because it encloses two loops ABDA and BCDB. All meshes are loops but all loops are not meshes.

1.10.1 Kirchhoff's Current Law (KCL)

This law relates to the currents at the junction points of a circuit and is stated below:

"The algebraic sum of currents meeting at a junction or node in an electric circuit is zero."



Consider five conductors, carrying currents I_1, 1_2, 1_3, 1_4, and 1_5 meeting at node A as shown in Fig. An algebraic sum is one in which the sign of the quantity is taken into account. If we take the signs of the currents flowing towards node A as positive, then currents flowing away from node A will be assigned negative sign. Thus, applying Kirchhoff's current law to node A in Fig.,

$$\mathrm{I}_1 + (-\mathrm{I}_2) + \mathrm{I}_3 + (-\mathrm{I}_4) + \mathrm{I}_5 = 0$$
$$\text{Hence, } \mathrm{I}_1 - \mathrm{I}_2 + \mathrm{I}_3 - \mathrm{I}_4 + \mathrm{I}_5 = 0$$
$$Thus, I_1 + I_3 + I_5 = I_2 + I_5$$

So, Incoming currents = Outgoing currents

Thus, the above law can also be stated as:

The sum of currents flowing towards any junction in an electric circuit is equal to the sum of currents flowing away from that junction.

1.10.2 Kirchhoff's Voltage Law (KVL)

This law relates to the electromotive forces and the voltage drops in a circuit and is stated below:

"In any closed circuit or mesh, the algebraic sum of the electromotive forces and the voltage drops is equal to zero."

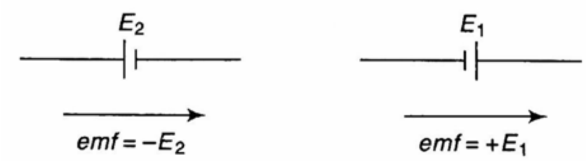
If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in potential at that point. This means that the sum of electromotive forces of all the sources met on the way, and the voltage drops in the resistances must be zero.

1.10.3 Sign Conventions

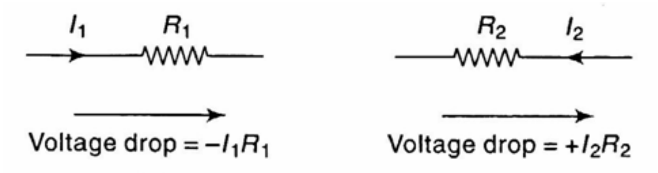
While applying Kirchhof's voltage law to a closed circuit, algebraic sums are considered. Therefore, it is very important to assign proper signs to emf's and voltage drops in the closed circuit. The following sign convention may be followed.

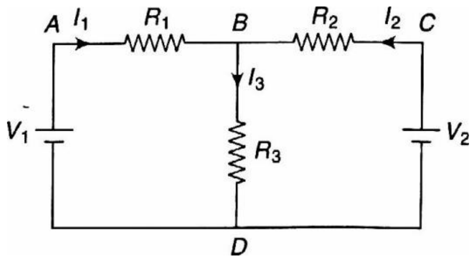
A rise in potential can be assumed positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

a) If we go from positive terminal of the battery or source to negative terminal, there is a fall in potential and so, the emf should be assigned negative sign. If we go from negative terminal of the battery or source to positive terminal, there is a rise in potential and so, the emf should be given positive sign.



b) When current flows through a resistor, there is a voltage drop across it. If we go through the resistance in the same direction as the current, there is a fall in the potential and so, the sign of this voltage drop is negative. If we go opposite to the direction of current flow, there is a rise in potential and hence, this voltage drop should be given positive sign.





Thus, applying the KVL to loop ABDA of the circuit, we get

$$V_1 - I_1R_1 - I_3R_3 = 0$$

Applying the KVL to loop ABCDA of a circuit, we get

$$V_1 - I_1R_1 + I_2R_2 - V_2 = 0$$

By using Kirchhoff's laws, we can calculate the unknown electrical quantities.

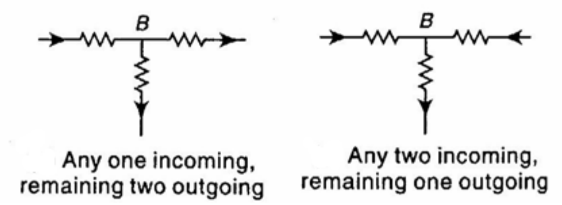
1.10.4 Solving Circuit Problems by using Kirchhoff's Laws

Kirchhoff's Voltage Law (KVL)

a. Assume unknown currents in the given circuit and show their directions by arrows.
b. By using KVL, write the equations for as many loops as the number of unknown currents.
c. In a solution if value of any unknown current comes out to be negative, it means that actual direction of the current is opposite to that of assumed direction.

Kirchhoff's Current Law (KCL)

a. Go to the junction and mark the current directions arbitrarily. For example, at junction B of a circuit shown in Fig., three branches are meeting. The directions of three branch currents can be assigned as shown below:

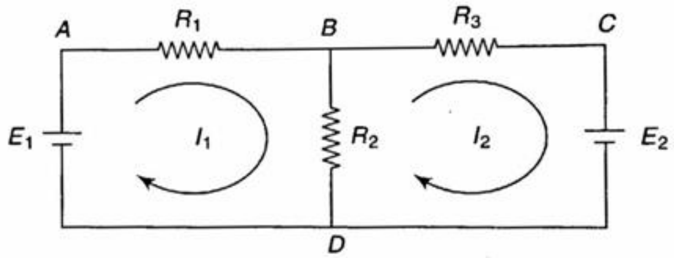


b. Name (mark) these currents using KCL so as to identify the number of unknowns.

1.10.5 Maxwell's Mesh Current Method

In this method, Kirchhoff's voltage law is applied to each mesh in terms of mesh currents instead of branch currents. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow in clockwise direction around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is applied to write equation in terms of unknown mesh currents. Once the mesh currents are known, the branch currents can be easily determined.

Maxwell's mesh current method consists of the following steps:



1. Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in clockwise direction. For example, in Fig, meshes ABDA and BCDB have been assigned mesh currents I_1 and 1_2 respectively.
2. If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of two. Thus, in Fig, there are two mesh currents I_1 and 1_2 flowing in R_2. If we go from B to D, current is $(I_1 – I_2)$ and if we go in the other direction (i.e. from D to B), current is $(I_2 – I_1)$.
3. Kirchhoff"s voltage law is applied to write equation for each mesh in terms of unknown mesh currents.
4. If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise, i.e., opposite to the assumed clockwise direction.

Consider a circuit as shown in Fig.

Applying Kirchhoff's voltage law to mesh ABDA,

$$-I_1R_1 - (I_1 - I_2)R_2 + E_1 = 0$$
$$I_1(R_1 + R_2) - I_2R_2 = E_1 \qquad (1.21)$$

Applying Kirchhoff's voltage law to mesh BCDB,

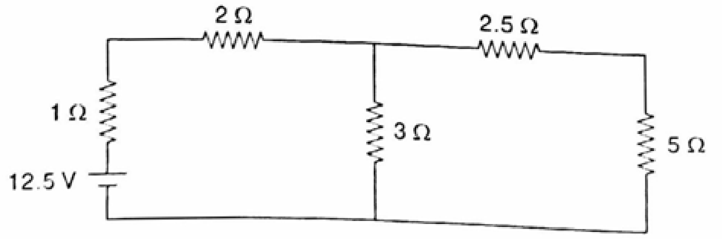
$$-I_2R_3 - (I_2 - I_1)R_2 - E_2 = 0$$
$$- I_1R_2 + (R_2 + R_3)I_2 = -E_2 \qquad (1.22)$$

Solving equation’s (1.21) and (1.22) simultaneously, mesh currents I_1 and 1_2 can be calculated. Once the mesh currents are known, the branch current can be readily obtained.

Note: Branch currents are the real currents because they actually flow in the branches and can be measured. However, mesh currents are fictitious quantities and cannot be measured directly. Hence, mesh current is concept rather than a reality.

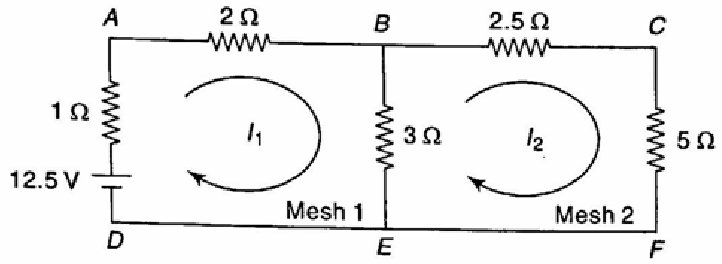
Problems
TYPE I: Simple Circuits (Only Voltage sources)

1. With the help of mesh current method, find the magnitude and direction of the current flowing through the 1Ω resistor in the network.



Solution

Marking the different nodes and assigning the separate mesh current for each mesh, we have



Applying KVL to mesh 1,

$$-2I_1 - 3(I_1 - I_2) + 12.5 - I_1 = 0$$

or $\quad -6I_1 + 3I_2 = -12.5 \qquad \text{(i)}$

Applying KVL to mesh 2,

$$-2.5I_2 - 5I_2 - 3(I_2 - I_1) = 0$$

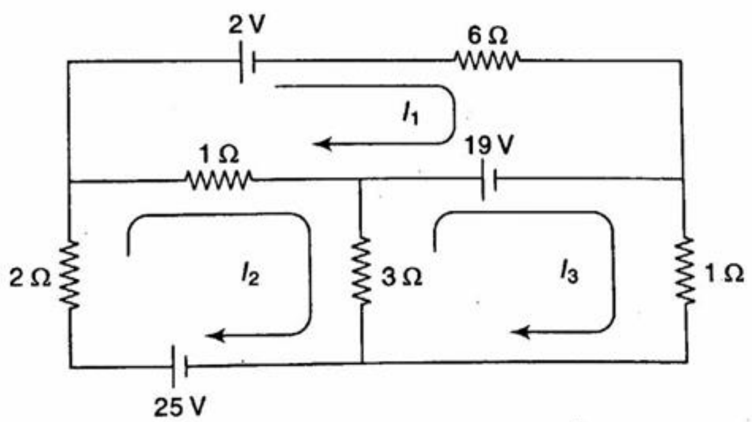
or $\quad 3I_1 - 10.5I_2 = 0 \qquad \text{(ii)}$

From Eqs (i) and (ii), we get

$$I_1 = 2.43 \text{ A}$$

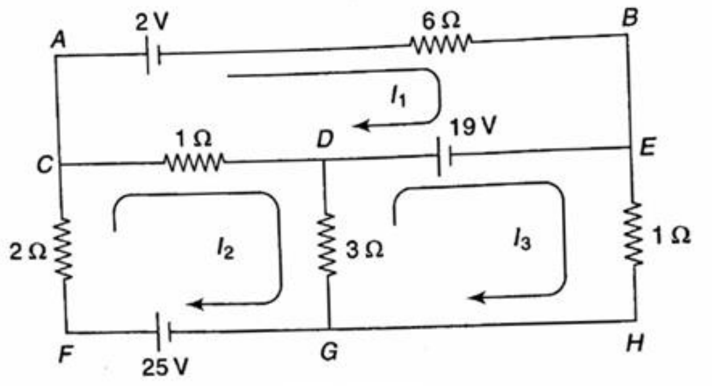
Hence, $I_{1\Omega} = 2.43 \text{ A}(\uparrow)$

2. By mesh analysis, find mesh currents I_1, I_2, and I_3 in the network.



Solution

Marking the different nodes, we get the circuit as shown



Applying KVL to mesh ABEDCA,

$$-2 - 6I_1 + 19 - 1(I_1 - I_2) = 0$$
$$7I_1 - I_2 = 17 \qquad (i)$$

Applying KVL to mesh CDGFC,

$$-(I_2 - I_1) - 3(I_2 - I_3) + 25 - 2I_2 = 0$$
$$I_1 - 6I_2 + 3I_3 = -25 \qquad (ii)$$

Applying KVL to mesh DEHGD,

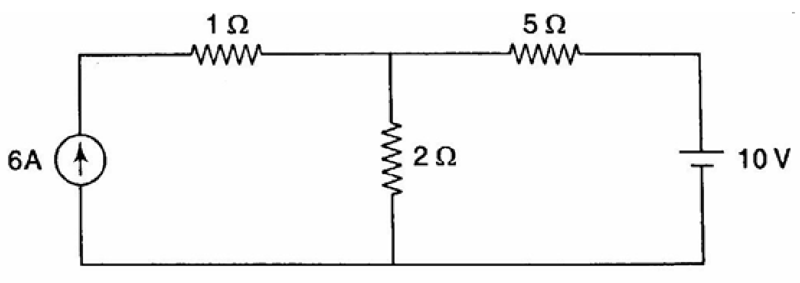
$$-19 - I_3 - 3(I_3 - I_2) = 0$$
$$3I_2 - 4I_3 = 19 \qquad (ii)$$

The values of I_1, I_2 and I_3 can be found by solving the above three simultaneous equations,

I_1 = 2.95A
I_2 = 3.65A
I_3 = -2.01A

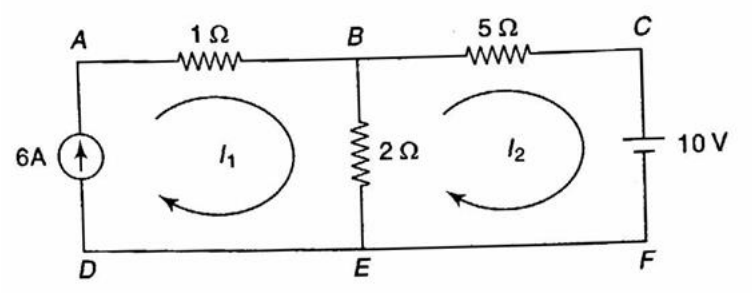
TYPE II: - Circuits with both voltage and current sources (Super mesh)

3. By mesh analysis find current through 2Ω resistor in the circuit.



Solution

Marking the different nodes and assigning the separate mesh current for each mesh, we get the following circuit:



In the circuit, current source is present in a branch not common to any other mesh. If there exists a current source in any mesh of a circuit, then Kirchhoff's voltage law cannot be applied to such mesh as voltage drop across the current source is unknown. In circuit, current source is present in a mesh ABEDA, so KVL cannot be applied to this mesh.

In such case, to get the required equations, the current source is expressed in terms of mesh current, i.e., current source of 6 A in the direction of mesh current I_1. We can write the equation as

$$I_1 = 6\,A \qquad (i)$$

Then apply the KVL to the remaining meshes existing without involving the branches consisting of current source. In circuit, mesh BCFEB does not contain any current source. So, applying the KVL to mesh BCFEB, we can write the equation as

$$-5I_2 - 10 - 2(I_2 - I_1) = 0$$
$$2I_1 - 7I_2 = 10 \qquad (ii)$$

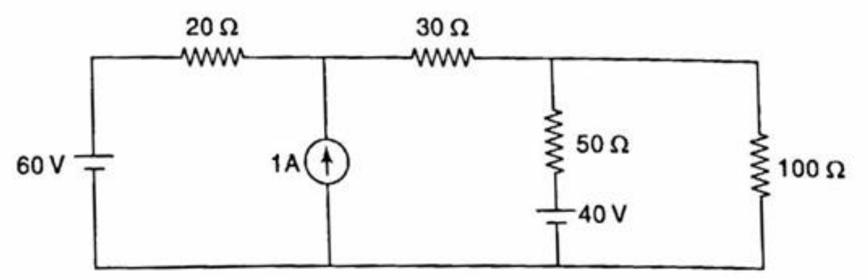
From Eqs (i) and (ii), we get

$I_2 = 0.2857$ A

Hence,

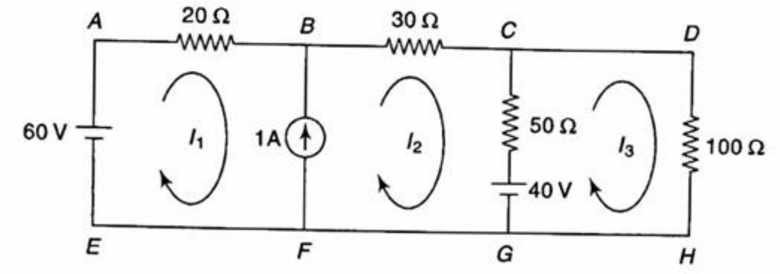
$$\boldsymbol{I_{2\Omega} = (I_1 - I_2)\ (\downarrow) = 5.71\ A\ (\downarrow)}$$

4. By mesh analysis, find the current through 100 Ω resistor in the circuit.



Solution

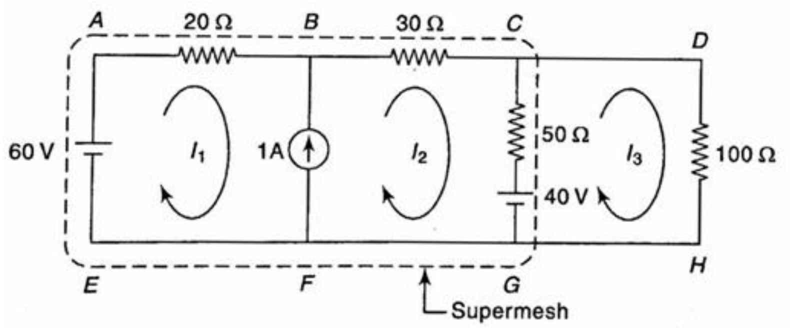
Marking the different nodes and assigning the separate mesh current for each mesh, we get the following circuit:



In this example, current source of 1A is present in a branch common to two meshes, i.e., mesh ABFEA and mesh BCGFB. When current source is present in a branch common to any two meshes, we need some manipulation and concept of supermesh can be used to solve such problems.

Supermesh is made out of two meshes. By opening the common branch in which current source is present (i.e., branch BF), we get the supermesh, i.e., supermesh ABCGFEA.

By marking the supermesh, we obtain,



In such a case, to get the required equations for the supermesh we can write the two equations, namely current equation and voltage equation. By expressing the current in the common branch in terms of mesh current, we get the current equation,

$$I_2 - I_1 = 1 \qquad \text{(i)}$$

By applying the KVL to supermesh ABCGFEA, we get the voltage equation as,

$$-20I_1 - 30I_2 - 50(I_2 - I_3) - 40 + 60 = 0$$

$$-20I_1 - 80I_2 + 50I_3 = -20 \qquad \text{(ii)}$$

Now, by applying the KVL to the remaining mesh CDHGC (which doesn't contain any current source), we have

$$-50(I_3 - I_2) - 100I_3 + 40 = 0$$

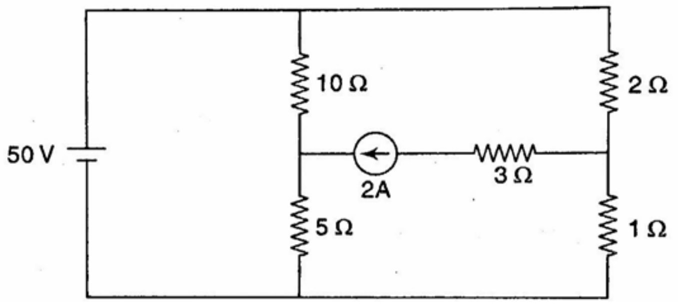
$$-50I_2 - 150I_3 = -40 \qquad \text{(iii)}$$

From equations (i), (ii) and (iii), we get

$I_3 = 0.48\ A$

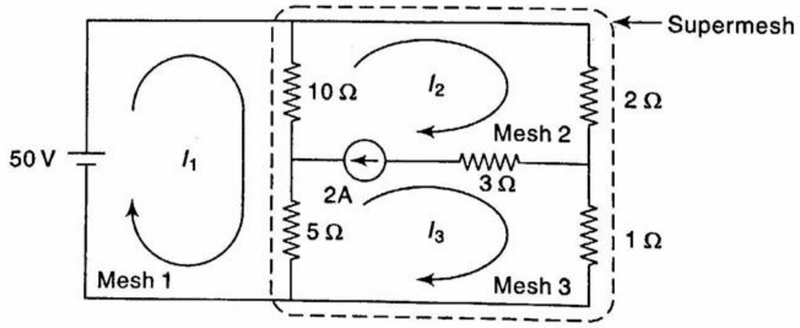
Hence, $\boldsymbol{I_{100\Omega} = 0.48A\ (\downarrow)}$

5. By mesh analysis, find the current through 5Ω resistor in the circuit.



Solution

Assigning the separate mesh current for each mesh, we obtain



Mesh 2 and mesh 3 forms a supermesh.

By expressing the current in the common branch in terms of mesh currents, we get the current equation as,

$$(I_2 - I_3) = 2 \qquad (i)$$

By applying the KVL to the supermesh, we get the voltage equation as

$$-2I_2 - I_3 - 5(I_3 - I_1) - 10(I_2 - I_1) = 0$$
$$15I_1 - 12I_2 - 6I_3 = 0 \qquad (ii)$$

Now, by applying the KVL to mesh 1 (which does not contain any current source), we have,

$$-10(I_1 - I_2) - 5(I_1 - I_3) + 50 = 0$$
$$-15I_1 + 10I_2 + 5I_3 = -50 \qquad (iii)$$

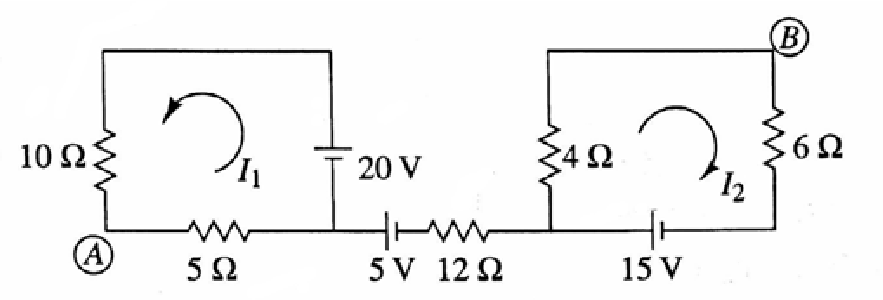
The values of I_1 and I_3 may be found by solving the above three simultaneous equations,

$I_1 = 20\ A, \qquad I_3 = 5.33\ A$

Hence,

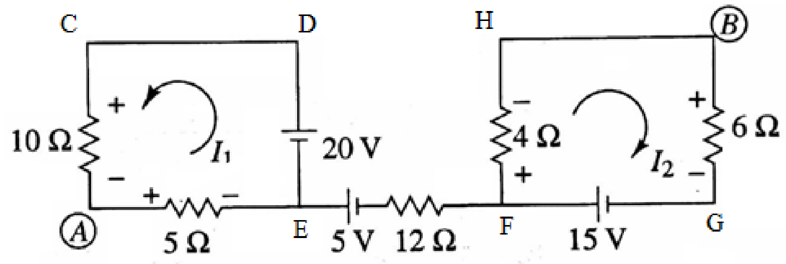
$\boldsymbol{I_{5\Omega} = (I_1 - I_3) = 14.67\ A\ (\downarrow)}$

6. In the network shown find the voltage between nodes A and B.



Solution

Marking the different nodes, we get the following circuit:



Applying KVL to Mesh ACDEA,

$- 10I_1 – 5I_1 + 20 = 0$

$$I_1 = \frac{20}{15} = 1.33\ A$$

Applying KVL to Mesh BGFHB,

$- 6I_2 + 15 – 4I_2 = 0$

$$I_2 = \frac{15}{10} = 1.5\ A$$

As seen voltage between A and B = $V_{AB} = V_A - V_B$

Writing the KVL equations for the path A to B,

$V_A – 5I_1 – 5 – 15 + 6I_2 – V_B = 0$

$V_A – 5(1.33) – 5 – 15 + 6(1.5) – V_B = 0$

$V_A – V_B = 17.65$

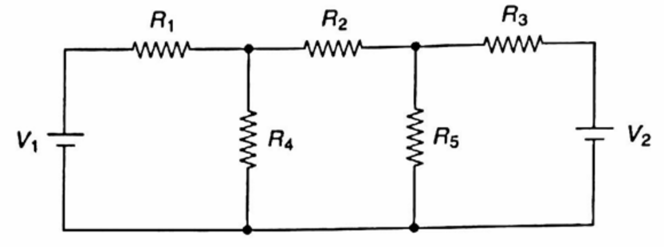
Hence,

$\mathbf{V_{AB} = 17.65\ V}$

Note: *As there is no returning path (closed path) for branch ef, no current flows through branch ef i.e.* $I_{ef} = 0$

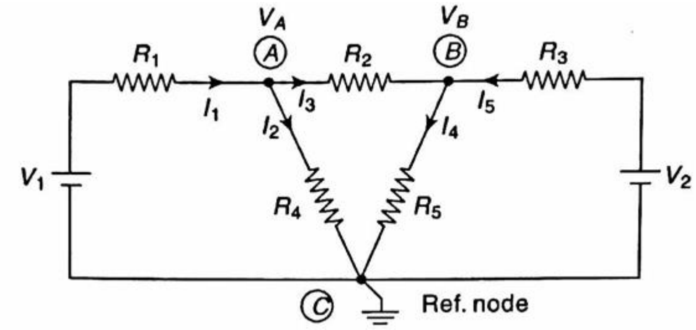
1.10.6 Nodal Analysis (Node Voltage Method)

This method is based on Kirchhoff's current law (KCL). Normally, this analysis is carried out to determine voltages of different nodes with respective to reference node. However, after determination of node voltage, currents in all branches can be determined. This method is useful where number of loops is large and hence, mesh analysis becomes lengthy. Nodal analysis also has advantage that a minimum number of equations need to be written to determine the unknown quantities.



Following steps are to be taken while solving a problem by nodal analysis. Consider the circuit shown.

Step I: Mark all nodes. Every junction of the network where three or more branches meet is regarded as a node. In a circuit, there are four nodes (marked by bold points). But lower two nodes are same, and by joining them, we get only three nodes as shown.



Step II: Select one of the nodes as reference node. Normally, for convenience, choose that node as reference where maximum elements are connected or maximum branches are meeting. Obviously, node C is selected as reference node. Reference node is also called zero potential node or datum/ground node.

Step III: Assign the unknown potentials of all nodes with respect to the reference node. For example, at nodes A and B, let the potentials are V_A and V_B with respect to the reference node.

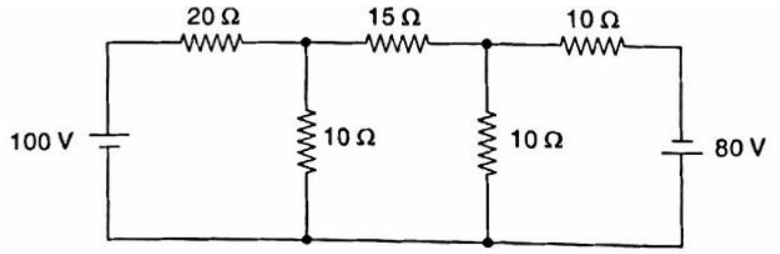
Step IV: At each node (excluding reference node), assume the unknown currents and mark their directions (choose the current directions arbitrarily).

Step V: Apply the KCL at each node and write the equations in terms of node voltages. By solving the equations, determine the node voltages. From node voltages, current in any branch can be determined.

Problems

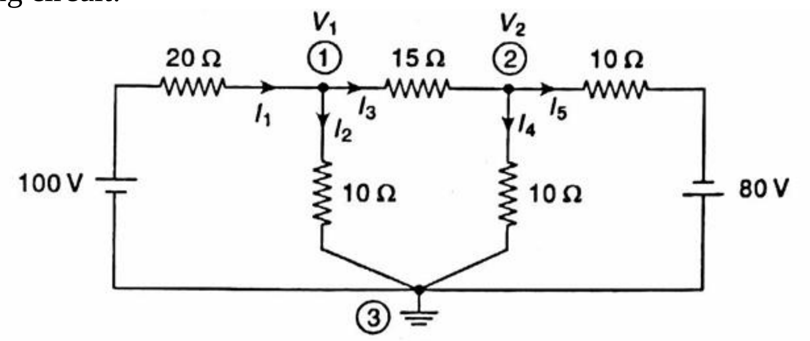
Type I: Simple circuit (Only Voltage sources)

1. By node voltage method, find the current through 15Ω resistor in the circuit.



Solution

Marking the different nodes and assigning the unknown currents, we obtain the following circuit:



Applying KCL to node 1,

$$I_1 = I_2 + I_3$$

$$\frac{100 - V_1}{20} = \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{15}$$

$$13V_1 - 4V_2 = 300 \qquad (i)$$

Applying KCL at node 2,

$$I_3 = I_4 + I_5$$

$$\frac{V_1 - V_2}{15} = \frac{V_2 - 0}{10} + \frac{V_2 - (-80)}{10}$$

$$V_1 - 4V_2 = 120 \qquad (ii)$$

From equation's (i) and (ii),

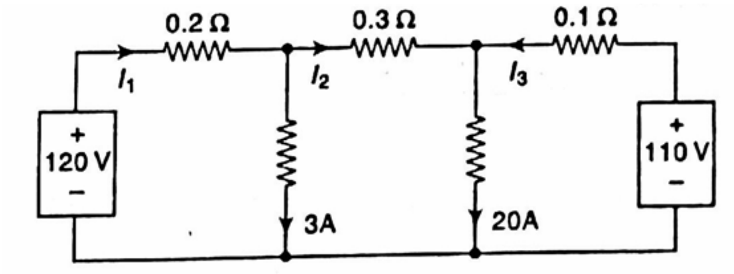
$$V_1 = 15\,V, \quad V_2 = -26.25\,V$$

Hence,

$$\boldsymbol{I_{15\Omega} = I_3 = \frac{V_1 - V_2}{15} = 2.75\,A} \ (\rightarrow)$$

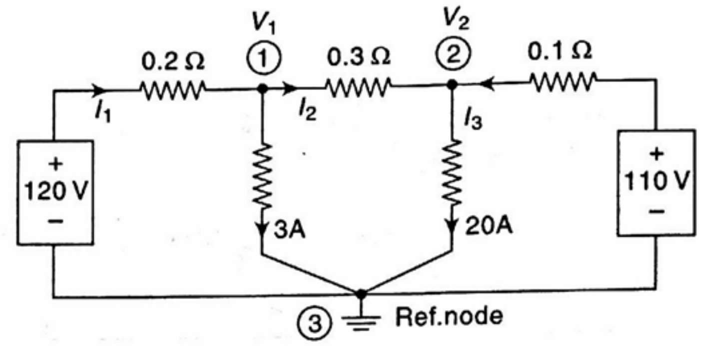
Type II: Branch currents given

2. By node voltage method, find the currents I_1, I_2 and I_3 in the circuit given.



Solution

Marking the different nodes, we get



Applying KCL at node 1,

$I_1 = 3 + I_2$

$$\frac{120 - V_1}{0.2} = 3 + \frac{V_1 - V_2}{0.3}$$

$$5V_1 - 2V_2 = 358.2 \qquad (i)$$

Applying KCL at node 2,

$I_2 + I_3 = 20$

$$\frac{V_1 - V_2}{0.3} + \frac{110 - V_2}{0.1} = 20$$

$$V_1 - 4V_2 = -324 \qquad (ii)$$

From equation’s (i) and (ii),

$V_1 = 115.6\,V, \quad V_2 = 109.9\,V$

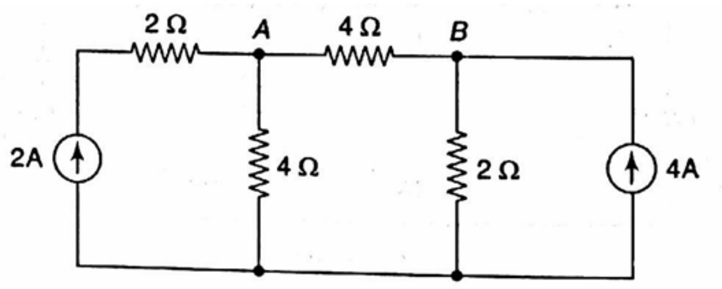
Hence,

$$\boldsymbol{I_1 = \frac{120 - V_1}{0.2} = 22\,A,\ I_2 = \frac{V_1 - V_2}{0.3} = 19\,A}$$

$$\boldsymbol{I_3 = \frac{110 - V_2}{0.1} = 1\,A}$$

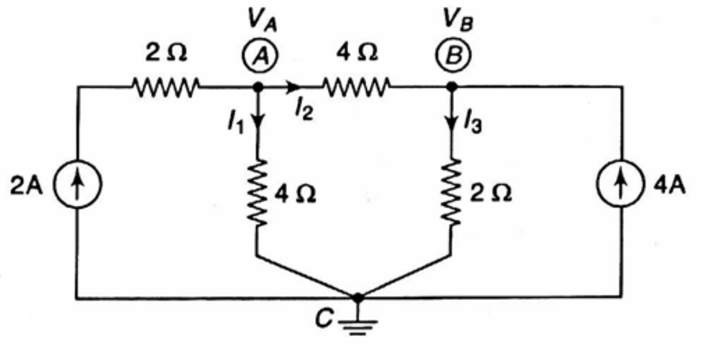
Type III: Current source in the branch

3. By nodal analysis, determine the voltages at node A and node B in the circuit.



Solution

Assigning the unknown currents, we get the circuit as shown



Applying KCL at node 1,

$$2 = I_1 + I_2$$

$$2 = \frac{V_A - 0}{4} + \frac{V_A - V_B}{4}$$

$$2V_A - V_B = 8 \qquad (i)$$

Applying KCL at node 2,

$$I_2 + 4 = I_3$$

$$\frac{V_A - V_B}{4} + 4 = \frac{V_B - 0}{2}$$

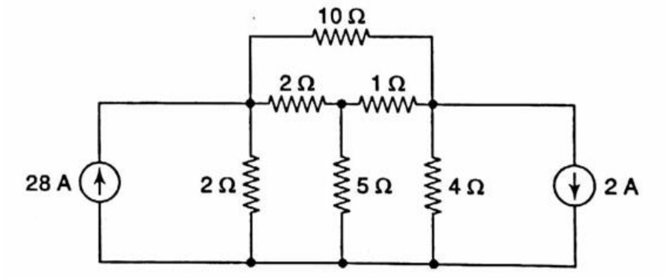
$$V_A - 3V_B = -16 \qquad (ii)$$

From equation's (i) and (ii),

$$\boldsymbol{V_A = 8\,V}$$

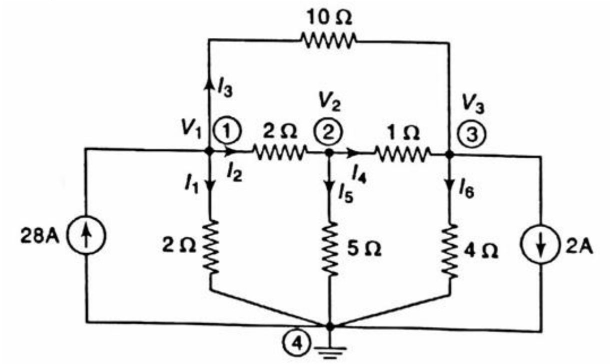
$$\boldsymbol{V_B = 8\,V}$$

4. Find the currents in the various resistors of the circuit shown.



Solution

By joining the same nodes and assigning the unknown currents, we obtain the following circuit:



Applying KCL at node 1,

$$28 = I_1 + I_2 + I_3$$

$$28 = \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{10}$$

$$11V_1 - 5V_2 - V_3 = 280 \qquad (i)$$

Applying KCL at node 2,

$$I_2 = I_4 + I_5$$

$$\frac{V_1 - V_2}{2} = \frac{V_2 - V_3}{1} + \frac{V_2 - 0}{5}$$

$$5V_1 - 17V_2 + 10V_3 = 0 \qquad (ii)$$

Applying KCL at node 3,

$$I_3 + I_4 = I_6 + 2$$

$$\frac{V_1 - V_3}{10} + \frac{V_2 - V_3}{1} = \frac{V_3 - 0}{4} + 2$$

$$V_1 + 10V_2 - 13.5V_3 = 20 \qquad (iii)$$

From equation’s (i), (ii) and (iii),

$$V_1 = 36\,V, \quad V_2 = 20\,V, \quad V_3 = 16\,V$$

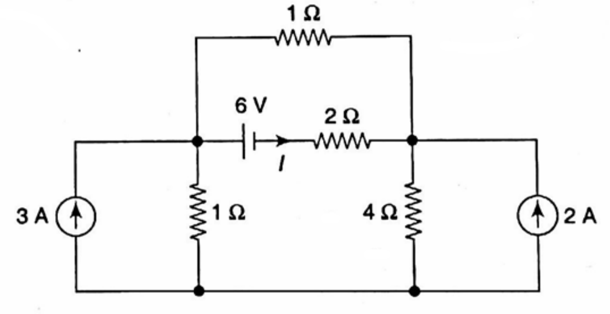
Hence,

$$\boldsymbol{I_1 = \frac{V_1 - 0}{2} = 18\,A, \qquad I_2 = \frac{V_1 - V_2}{2} = 8\,A, \qquad I_3 = \frac{V_1 - V_3}{10} = 2\,A}$$

$$\boldsymbol{I_4 = \frac{V_2 - V_3}{1} = 4\,A, \qquad I_5 = \frac{V_2 - 0}{5} = 4\,A, \qquad I_6 = \frac{V_3 - 0}{4} = 4\,A}$$

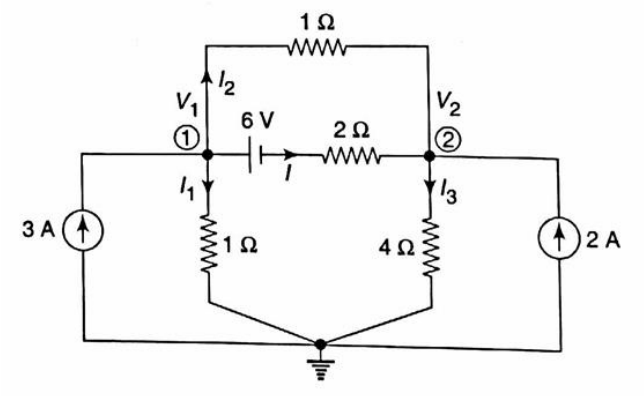
Type IV: Voltage source with series resistance between two nodes

5. Find the current I by using node-voltage analysis for the circuit given below.



Solution

Marking the different nodes and assigning the unknown currents, we obtain the following circuit:



Applying KCL at node 1,

$3 = I_1 + I + I_2$

$$3 = \frac{V_1 - 0}{1} + \frac{(V_1 - 6) - V_2}{2} + \frac{V_1 - V_2}{1}$$

$5V_1 - 3V_2 = 12 \qquad (i)$

Applying KCL at node 2,

$I_2 + I + 2 = I_3$

$$\frac{V_1 - V_2}{1} + \frac{(V_1 - 6) - V_2}{2} + 2 = \frac{V_2 - 0}{4}$$

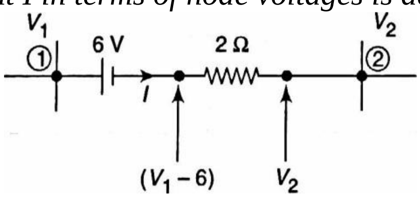
$6V_1 - 7V_2 = 4 \qquad (ii)$

From equation’s (i) and (ii),

$V_1 = 4.2353\,V, \qquad V_2 = 3.0588\,V$

$$\boldsymbol{I = \frac{(V_1 - 6) - V_2}{2} = \frac{(4.2353 - 6) - 3.0588}{2} = -2.412\,A}$$

Hint: *The expression of current I in terms of node voltages is determined as follows:*



By Ohm's law,

$$I = \frac{Potential\ difference\ across\ 2\Omega\ resistor}{Resistance}$$

The current I flows through the 2 Ω resistor from node 1 to node 2 (i.e., from right end to left end of 2 Ω resistor)

So, potential difference across 2 Ω resistor

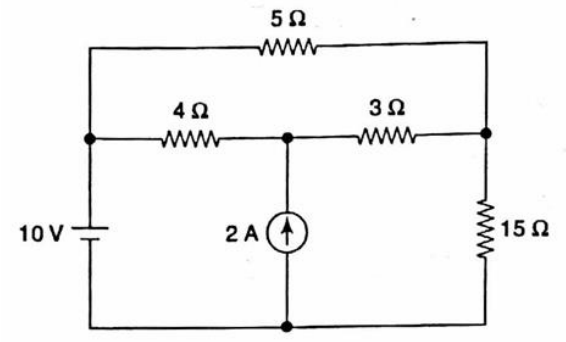
= Potential at right end of 2 Ω resistor - Potential at left end of 2 Ω resistor

= $(V_1 – 6) – V_2$

So, $\boldsymbol{I = \frac{(V_1 - 6) - V_2}{2}}$

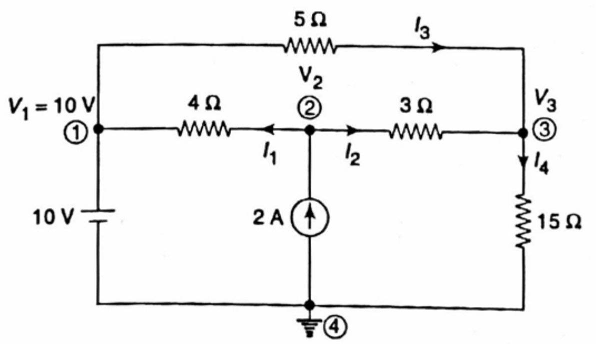
Type V: Voltage source in a branch

6. Find the current through 4Ω and 3 Ω resistances using nodal analysis in the circuit.



Solution

Marking the different nodes and assuming different currents, we get



As voltage source of 10 V is directly connected between the reference node (node 4) and non-reference node (node l) with positive polarity towards node 1, the voltage at node 1 is 10 V with respect to reference node.

Thus $V_1 = 10$

Hint: *If voltage source is connected between the reference node and non reference node, then set a voltage at non-reference node equal to the voltage source. In this case, the solution becomes simple as application of KCL at non reference node is not required. Now assigning the unknown currents at node 2 and node 3, we get above circuit. Note that as voltage at node 1 is known, application of KCL at this node is not required.*

Applying KCL at node 2,

$$2 = I_1 + I_2$$

$$2 = \frac{V_2 - 10}{4} = \frac{V_2 - V_3}{3}$$

$$7V_2 - 4V_3 = 54 \qquad (i)$$

Applying KCL at node 3,

$$I_3 + I_2 = I_4$$

$$\frac{10 - V_3}{5} + \frac{V_2 - V_3}{3} = \frac{V_3 - 0}{15}$$

$$5V_2 - 9V_3 = -30 \qquad (ii)$$

From equation's (i) and (ii),

$$V_2 = 14.09\,V, \quad V_3 = 11.16\,V$$

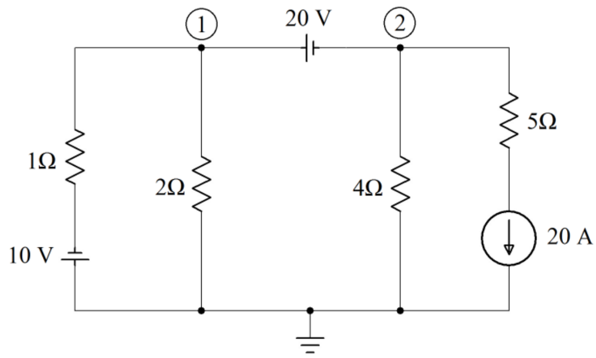
Hence,

$$\boldsymbol{I_{3\Omega} = \frac{V_2 - V_3}{3} = \frac{14.09 - 11.16}{3} = 0.98\,A}$$

$$\boldsymbol{I_{4\Omega} = \frac{V_2 - 10}{4} = \frac{14.09 - 10}{4} = 1.03\,A}$$

Type VI: Supernode (Voltage source between two nodes)

7. By nodal analysis, determine the voltages at node 1 and node 2 in the circuit.

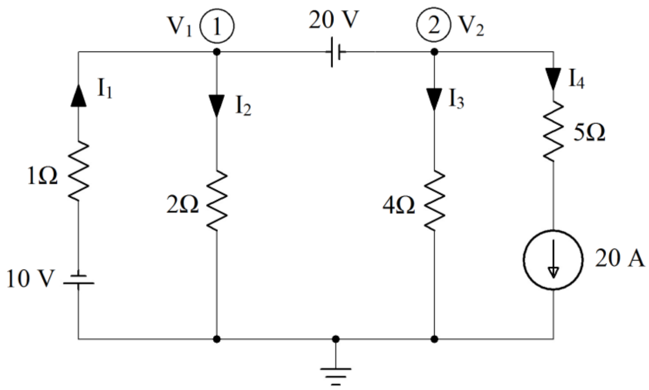


Solution

Hint:

1. *Consider supernode as a single node.*
2. *Express one node of supernode in terms of another node.*
3. *Apply KCL to supernode.*

Marking the different nodes and assuming different currents, we get



Writing equation for supernode,

$V_1 = V_2 + 20$

$V_1 - V_2 = 20$ (i)

Consider 1 and 2 as a single node,

$I_1 = I_2 + I_3 + I_4$

$$\frac{-10 - V_1}{1} = \frac{V_1}{2} + \frac{V_2}{4} + 20$$

$-30 = 1.5V_1 + 0.25V_2$ (ii)

From equation's (i) and (ii),

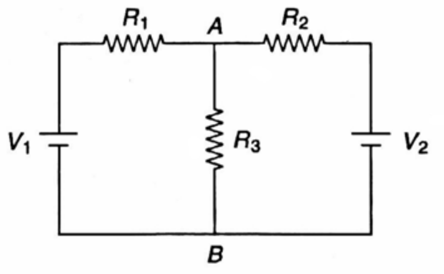
$$\boldsymbol{V_1 = -14.28\,V, \quad V_2 = -34.29\,V}$$

1.11 Superposition Theorem

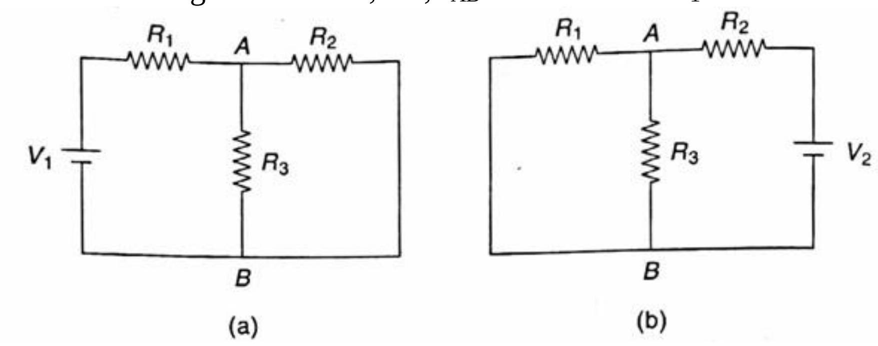
This theorem is applicable for linear and bilateral networks. According to this theorem, if there are number of sources acting simultaneously in any linear bilateral networks, then each source acts independently of the others, i.e., as if other source did not exit. Hence, this theorem may be stated as in a linear network containing more than one source, the resulted current in any branch is the algebraic sum of the currents that would be produced by each source, acting alone, all other sources of emf being replaced by their respective internal resistances.

Illustration

Consider a network shown in Fig, having two voltage sources V_1 and V_2. Let us calculate the current in branch AB of the network by using superposition theorem.



Case (i) According to superposition theorem, each source acts independently. Consider source V_1 acting independently. At this time, other sources must be replaced by internal resistances. But as internal resistance of V_2 is not given, i.e., it is assumed to be zero, V_2 must be replaced by short circuit. Thus, the circuit becomes as shown in Fig (a). Using any of the techniques, obtain the current through branch AB, i.e., I_{AB} due to source V_1 alone.



Case (ii) now, consider source V_2 alone, with V_1 replaced by a short circuit, to obtain the current through branch AB. The corresponding circuit is shown in Fig (b). Obtain I_{AB} due to V_2 alone by using any of the techniques such as mesh analysis, nodal analysis, and source transformation.

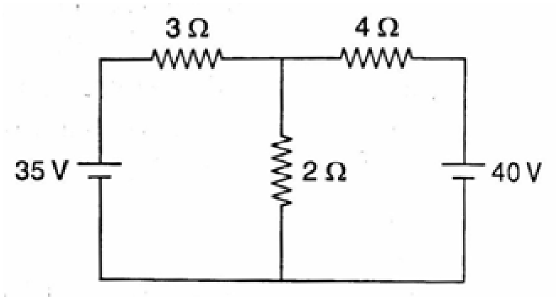
Case (iii) According to superposition theorem, the resultant current through branch AB is the sum of the currents through branch AB produced by each source acting independently.

Hence,

$$I_{AB} = I_{AB}\ \textit{Current due to}\ V_1 + I_{AB}\ \textit{Current due to}\ V_2$$

Problems

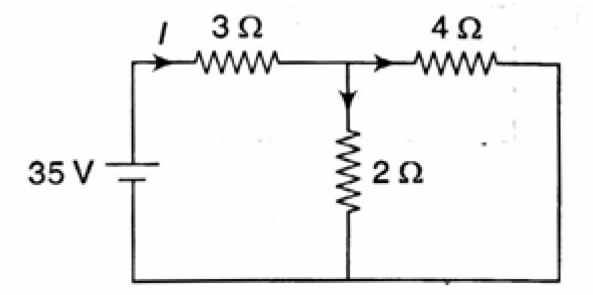
1. In the circuit shown, find the current flowing through different resistors by using super position theorem.



Solution

Step I

Considering 35 V source acting alone, replacing 40 V source by short circuit, we get the circuit as shown.



In circuit, as single source is acting, the actual directions of currents are marked. These branch currents can be calculated as follows:

The equivalent resistance across the source,

$$R_{eq} = 3 + (2 \parallel 4) = 3 + \frac{2 \times 4}{2+4} = 4.33\ \Omega$$

By ohm's law, total circuit current,

$$I = \frac{V}{R_{eq}} = \frac{35}{4.33} = 8.08\ A$$

Hence, current in 3Ω resistor, $I_{3\Omega} = 8.08\ A(\rightarrow)$

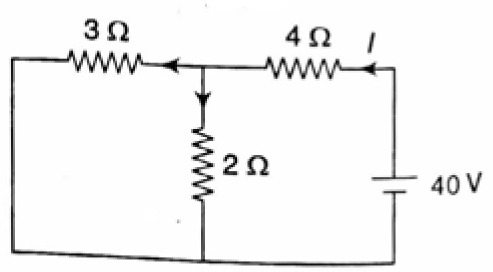
By current division rule, current in 4Ω resistor,

$$I_{4\Omega} = 8.08 \times \frac{2}{2+4} = 2.69\ A\ (\rightarrow)$$

Current in 2Ω resistor, $I_{2\Omega} = 8.08 - 2.69 = 5.39\ A\ (\downarrow)$

Step II

Considering 40 V source acting alone, replacing 35 V source by short circuit, we get the circuit as shown.



In circuit, as single source is acting, the actual directions of current are marked. These branch currents can be calculated as follows,

The equivalent resistance across the source,

$$R_{eq} = 4 + (2 \parallel 3) = 3 + \frac{2 \times 3}{2+3} = 5.2\ \Omega$$

By ohm's law, total circuit current,

$$I = \frac{V}{R_{eq}} = \frac{40}{5.2} = 7.69\ A$$

Hence, current in 4Ω resistor, $I_{4\Omega} = 7.69\ A(\leftarrow)$

By current division rule, current in 3Ω resistor,

$$I_{3\Omega} = 7.69 \times \frac{2}{2+3} = 3.08\,A\,(\leftarrow)$$

Current in 2Ω resistor, $I_{2\Omega} = 7.69 - 3.08 = 4.61\,A\,(\downarrow)$

Step III

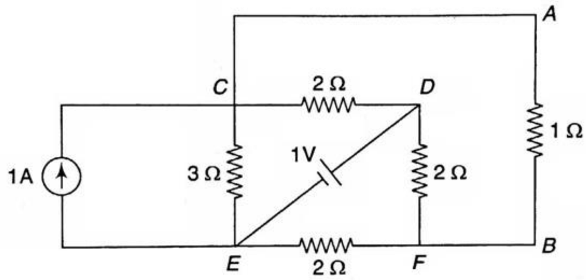
By principle of superposition, the resultant branch currents can be calculated as follows:

Current in 3Ω resistor, $\boldsymbol{I_{3\Omega} = 8.08\,A(\rightarrow) + 3.08\,A\,(\leftarrow) = 5\,A\,(\rightarrow)}$

Current in 2Ω resistor, $\boldsymbol{I_{2\Omega} = 5.39\,A(\downarrow) + 4.61\,A\,(\downarrow) = 10\,A\,(\downarrow)}$

Current in 4Ω resistor, $\boldsymbol{I_{4\Omega} = 2.69\,A(\rightarrow) + 7.69\,A\,(\leftarrow) = 5\,A\,(\leftarrow)}$

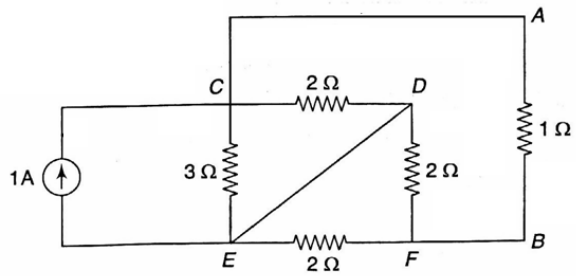
2. Determine the current in 1Ω resistor between A and B from the network shown in Fig. by superposition theorem.



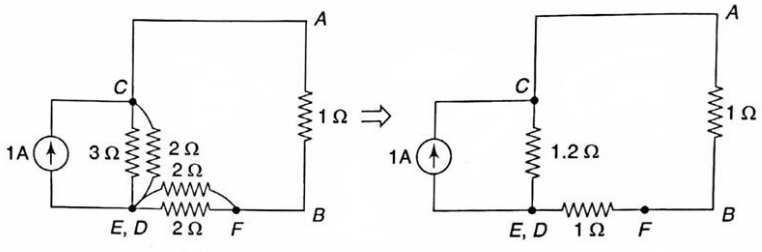
Solution

Step I

Considering 1 A source acting alone, replacing 1 V source by short circuit, we get Fig. below.



Nodes D and E are same, and by joining them, we get the circuit, as shown in Fig. (a). Replacing the parallel combinations, we get the circuit as shown in Fig. (b), where branch CE is parallel to branch CABE. The total current in the circuit is 1 A, which divides at node C.



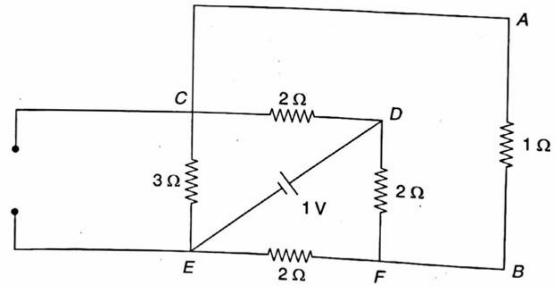
(a) (b)

Hence, by applying current division rule, current in 1Ω resistor between A and B is given as

$$I_{AB} = I_{1\Omega} = 1 \times \frac{1.2}{1.2 + (1+1)} = 0.375\,A\,(\downarrow)$$

Step II

Considering 1 V source acting alone, replacing 1 A source by open circuit, we obtain the following circuit:



Assuming the separate mesh current for each mesh, we get the circuit as shown in Fig.

Applying KVL to Mesh 1,

$$-3I_1 - 2(I_1 - I_3) - 1 = 0$$

$$5I_1 - 2I_3 = -1 \qquad (i)$$

Applying KVL to Mesh 2,

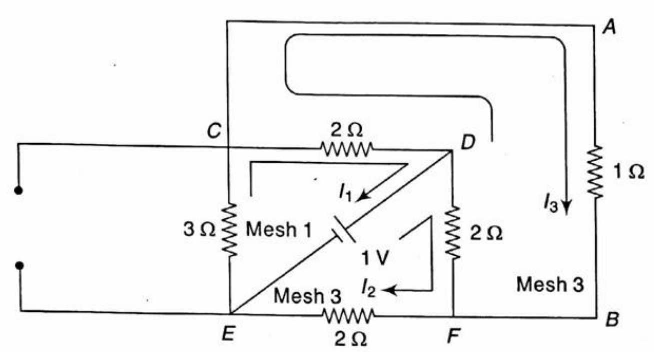
$$1 - 2(I_2 - I_3) - 2I_3 = 0$$

$$4I_2 - 2I_3 = 1 \qquad (ii)$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - I_3 - 2(I_2 - I_3) = 0$$

$$2I_1 + 2I_2 - 5I_3 = 0 \qquad (iii)$$



From equation (i), (ii) and (iii),

$$I_{AB} = I_3 = I_{1\Omega} = 0.031\ A\ (\downarrow)$$

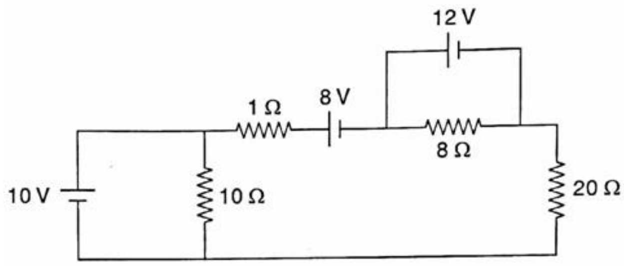
Step III

According to superposition theorem, resultant current through 1Ω resistor, i.e., through branch AB, is algebraic sum of currents due to individual sources acting alone.

Hence,

$$\boldsymbol{I_{AB} = 0.375\ A\ (\downarrow) + 0.031\ A\ (\downarrow) = 0.406\ A\ (\downarrow)}$$

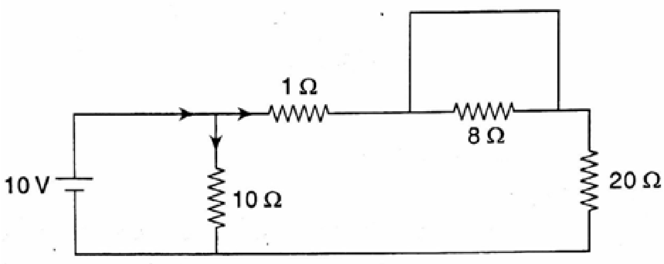
3. Determine the current in 20Ω resistor in the network shown in circuit below by superposition theorem.



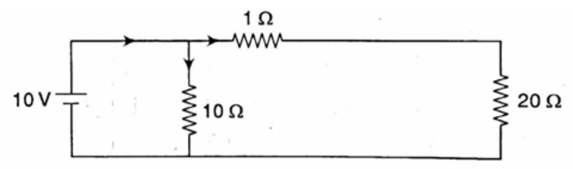
Solution

Step I:

Considering 10 V source acting alone, replacing 8 V source and 12 V source by short circuit, we obtain circuit as shown.



In the circuit above, the 8 Ω resistor gets short circuited, i.e., $I_{8\Omega} = 0A$, for circuit simplification, the 8 Ω resistor can be removed. By removing 8Ω resistor, we get the circuit as shown:

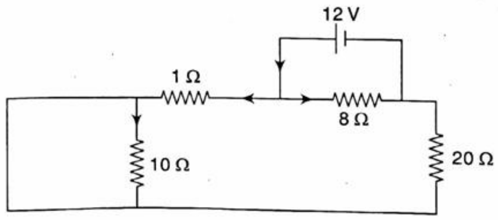


By Ohm's law

$$I_{20\Omega} = \frac{10}{1+20} = 0.476\ A(\downarrow)$$

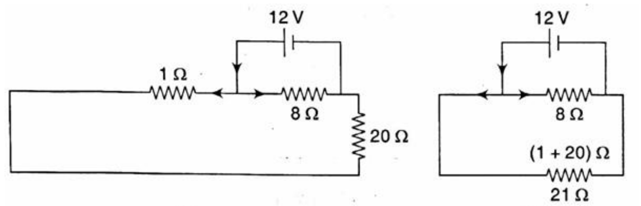
Step II:

Considering 12 V source acting alone, replacing 8 V source and 10 V source by short circuit, we get



In the circuit, the 10 Ω resistor gets short circuited, i.e., $I_{10\Omega} = 0A$, for circuit simplification, the resistor 10 Ω can be removed. By removing 10 Ω resistors, we get the circuit as shown in Fig. (a)

Now, replacing series combination (1Ω + 20Ω), we get the circuit as shown in Fig. (b)



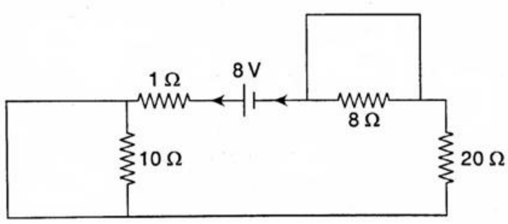
(a) (b)

By Ohm's law

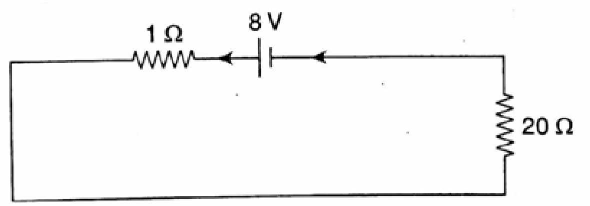
$$I_{20\Omega} = \frac{12}{1+20} = 0.571\ A(\uparrow)$$

Step III:

Considering 8V source acting alone, replacing 10V source and 12V source by short circuit, we get the circuit beside.



In the circuit of Fig. 1.233, the resistors 10 Ω and 8 Ω get short circuited. For circuit simplification, the resistors 10Ω and 8Ω can be removed. By removing 10 Ω and 8 Ω Q resistors, we get the circuit as shown in Fig.



By Ohm's law,

$$I_{20\Omega} = \frac{8}{1+20} = 0.381\ A(\uparrow)$$

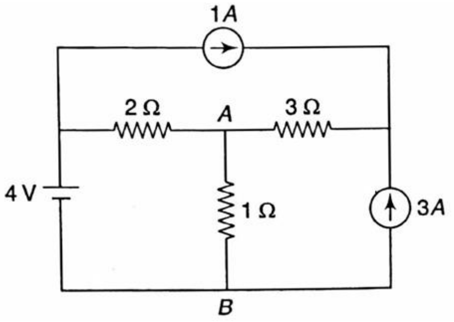
Step IV:

According to superposition theorem, resultant current through 20 Ω resistor is algebraic sum of currents due to individual sources acting alone.

Hence,

$$\boldsymbol{I_{20\Omega} = 0.476\ A(\downarrow) + 0.571\ A(\uparrow) + 0.381\ A(\uparrow) = 0.476\ A(\uparrow)}$$

4. Determine the current in 1Ω resistor in the network shown in Fig. below by superposition theorem.

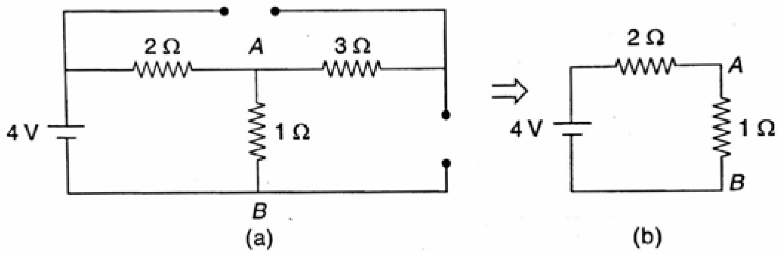


Solution

Step I:

Considering 4V source acting alone, replacing 1A source and 3A source by open circuit, we get circuit (a) and (b):

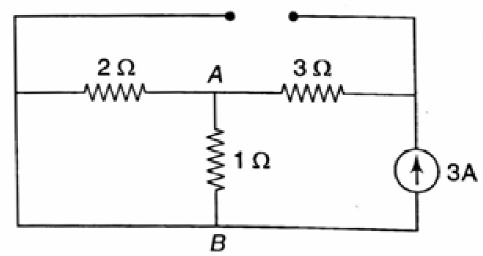
By ohm's law,



$$I_{AB} = I_{1\Omega} = \frac{4}{2+1} = 1.33\ A(\downarrow)$$

Step II:

Considering 3A source acting alone, replacing 4V source by short circuit and 1A source by open circuit, we obtain the circuit shown beside,

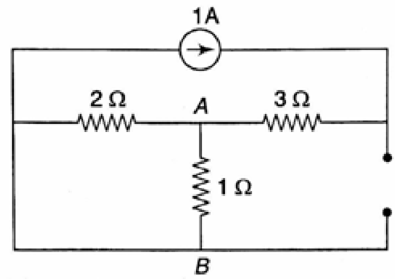


In Fig. above, current flowing through 3Ω resistor $I_{3\Omega}$ = 3A (⟵). This current divides at node A. Resistors 2Ω and 1Ω are in parallel. By current division rule,

$$I_{AB} = I_{1\Omega} = 3 \times \frac{2}{2+1} = 2\ A(\downarrow)$$

Step II:

Considering 1A source acting alone, replacing 4V source by short circuit and 3A source by open circuit, we obtain the circuit shown beside,



In Fig. above, current flowing through 3Ω resistor $I_{3\Omega}$ = 1A (⟵). This current divides at node A. Resistors 2Ω and 1Ω are in parallel. By current division rule,

$$I_{AB} = I_{1\Omega} = 1 \times \frac{2}{2+1} = 0.67\ A(\downarrow)$$

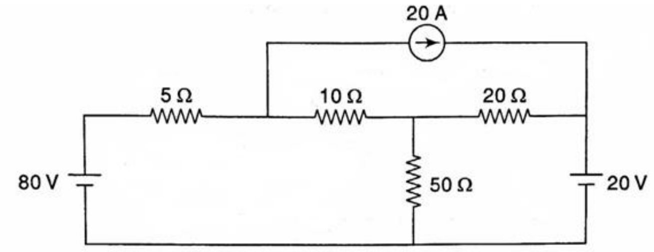
Step IV:

According to superposition theorem, resultant current through 1Ω resistor is algebraic sum of currents due to individual sources acting alone.

Hence,

$$\boldsymbol{I_{1\Omega} = 1.33\ A(\downarrow) + 2\ A(\downarrow) + 0.67\ A(\downarrow) = 4\ A(\downarrow)}$$

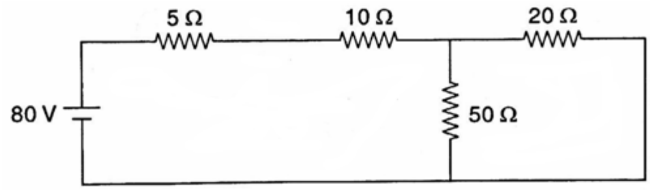
5. Determine the current in 10Ω resistor in the network shown in Fig. below by superposition theorem.



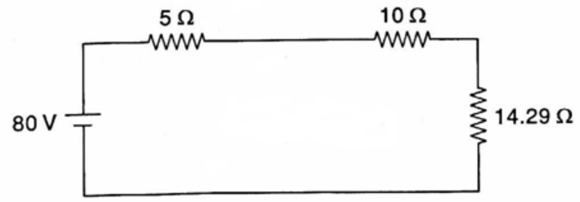
Solution

Step I

Considering 80V source acting alone, replacing 20A source by open circuit and 20V source by short circuit, we get circuit as shown.



In fig above resistors 20Ω and 50 Ω are in parallel. By circuit reduction technique, we obtain the simplified circuit as shown

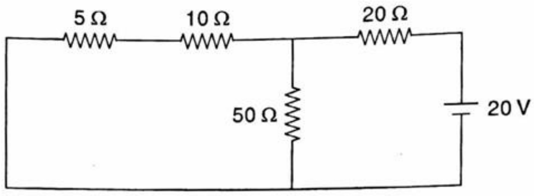


By ohm's law,

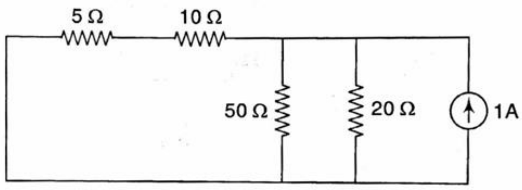
$$I_{10\Omega} = \frac{80}{5 + 10 + 14.29} = 2.73\ A(\rightarrow)$$

Step II

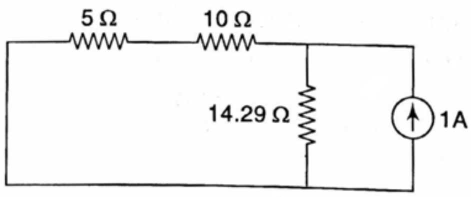
Considering 20V source acting alone, replacing 20A source by open circuit and 80V source by short circuit, we get circuit as shown.



By source transformation, i.e., converting series combination of voltage source of 20 V and resistor of 20Ω into equivalent parallel combination of current source and resistor, we get the circuit as shown in Fig.



By circuit reduction technique, we obtain the simplified circuit as shown

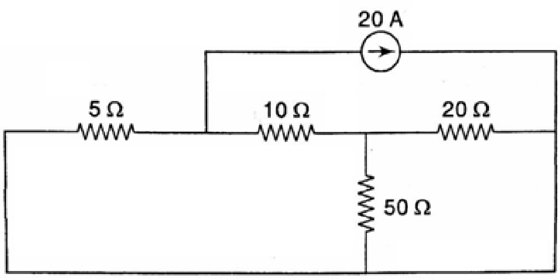


By current division rule,

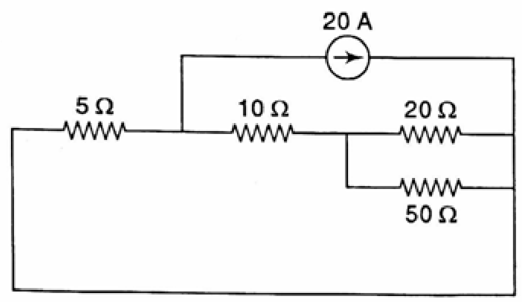
$$I_{10\Omega} = 1 \times \frac{14.29}{5 + 10 + 14.29} = 0.49\ A(\leftarrow)$$

Step III

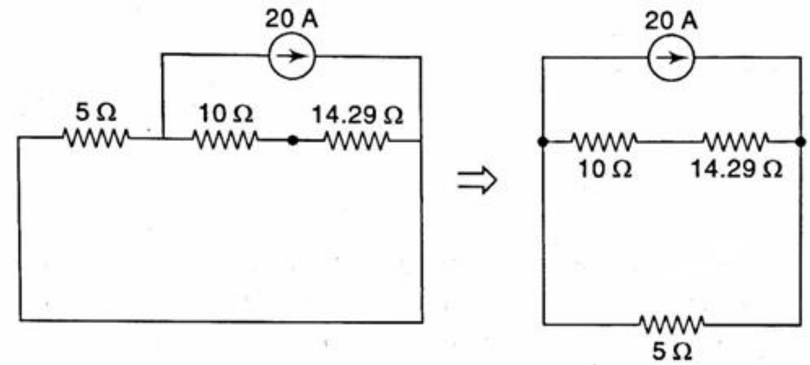
Consider 20A source acting alone, replacing 80V source and 20V source by short circuit we get circuit as shown



The circuit can be re drawn as follows



By circuit reduction technique, we obtain the simplified circuit as shown



By current division rule,

$$I_{10\Omega} = 20 \times \frac{5}{5 + 10 + 14.29} = 3.41\ A(\leftarrow)$$

Step IV:

According to superposition theorem, resultant current through 1Ω resistor is algebraic sum of currents due to individual sources acting alone.

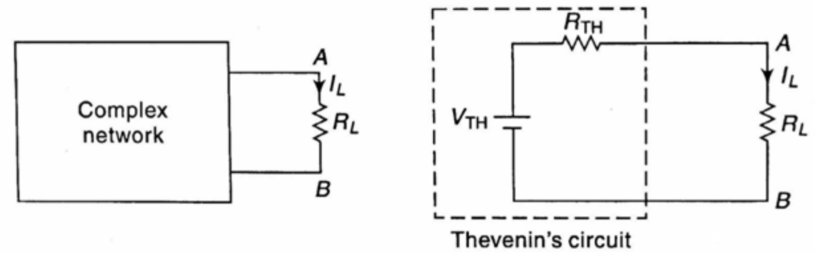
Hence,

$$\boldsymbol{I_{10\Omega} = 2.73\ A(\rightarrow) + 0.488\ A(\leftarrow) + 3.41\ A(\leftarrow) = 1.17\ A(\leftarrow)}$$

1.12 Thevenin's Theorem

Thevenin's theorem is a powerful tool in the hands of engineers to simplify a complex problem and obtain the circuit solution quickly. It reduces the complex circuit to a simple circuit. This theorem is particularly useful to find the current in a particular branch of a network as the resistance of that branch is varied while all other resistances and sources remain constant.

This theorem was first stated by French engineer M.L. Thevenin in 1883. According to this theorem, any two terminal networks, however complex, can be replaced by a single source of emf V_{TH} (called Thevenin voltage) in series with a single resistance R_{TH} (called Thevenin resistance). Figure (a) shows a complex network enclosed in a box with two terminals A and B brought out. The network in the box may consist of any number of resistors and emf sources connected in any manner. But according to Thevenin, the entire circuit behind terminals A and B can be replaced by a single source of emf V_{TH} in series with a single resistance R_{TH} as shown in Fig (b). The voltage V_{TH} is the voltage that appears across terminals A and B with load removed. The resistance R_{TH} is the resistance obtained with load removed and looking back into the terminals A and B when all the sources in the circuit are replaced by their internal resistances. Once Thevenin's equivalent circuit is obtained, current through any load R_L connected across AB can be readily obtained.



(a) (b)

Hence, Thevenin's theorem as applied to dc circuits may be stated as under:

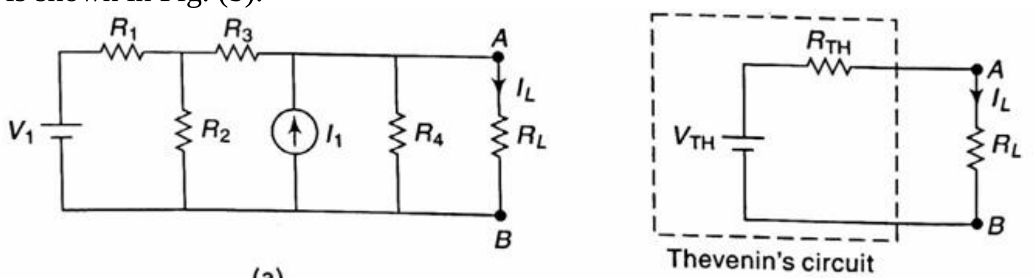
Any network having terminals A and B can be replaced by a single source of emf V_{TH} (called Thevenin voltage) in series with a single resistance R_{TH} (called Thevenin resistance).

(i) The emf V_{TH} is the voltage obtained across terminals A and B with load, if any, removed, i.e., it is open-circuited voltage between A and B.

(ii) The resistance R_{TH} is the resistance of the network measured between A and B with load removed and replacing all the voltage/current sources by their internal resistances.

Illustration

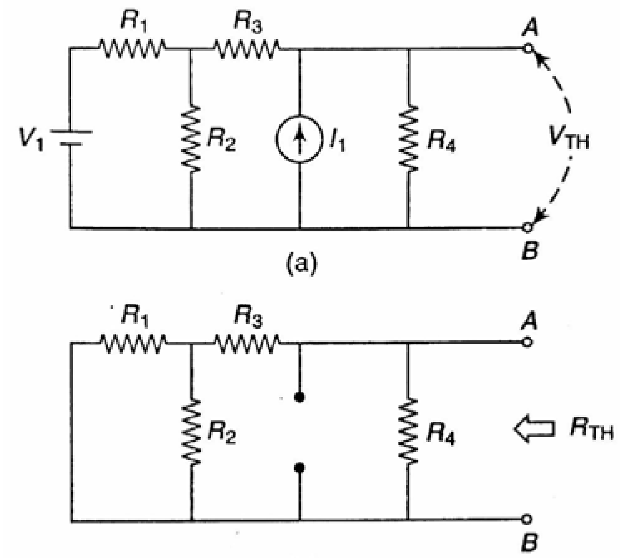
The concept of Thevenin's equivalent circuit across the load terminals can be explained by considering the circuit shown in Fig. (a). The Thevenin's equivalent circuit is shown in Fig. (b).



(a) (b)

Note: The internal resistance of an ideal voltage source is zero and that of an ideal current source is infinite. Hence, while finding out R_{TH}, voltage sources are replaced by short circuits and current sources by open circuits.

The voltage V_{TH} is obtained across the terminals A and B with R_L removed. Hence, V_{TH} is also called open circuit Thevenin's voltage. The circuit to be used to calculate V_{TH} is shown in Fig. (a). While R_{TH} is the equivalent resistance obtained as viewed through the terminals A and B with R_L removed and replacing voltage source short circuit and current source by open circuit as shown in Fig. (b).



(b)

While obtaining V_{TH}, any of the network simplification techniques can be used. When the circuit is replaced by Thevenin's equivalent across the load resistance, the load current can be obtained as

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Steps to apply for Thevenin's Theorem

Step 1: Remove the branch resistance through which current is to be calculated.

Step 2: Calculate the voltage across these open circuited terminals, by using any one of the network simplification techniques. This is V_{TH}.

Step 3: Calculate R_{TH} as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all sources by their internal resistances.

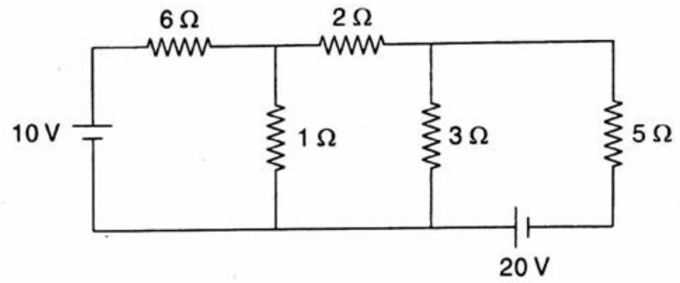
Step 4: Draw the Thevenin's equivalent circuit showing source V_{TH} with the resistance R_{TH} in series with it.

Step 5: Reconnect the branch resistance. Let it be R_L. The required current through the branch is given by

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

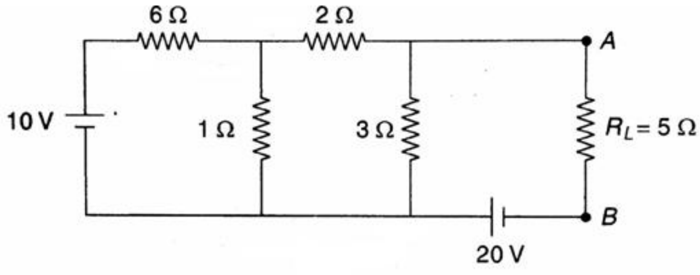
Problems

1. Determine the current through 5Ω resistor in the network shown in Fig. by Thevenin's theorem.



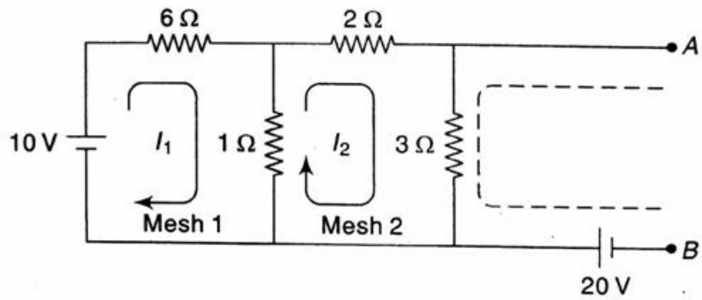
Solution

Current through 5Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.



Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:



In Fig., voltage appears across the load terminals A and B, which is called Thevenin's voltage (V_{TH}). The voltage V_{TH}, i.e., V_{AB}, can be calculated as follows:

Select any path from A to B and marked the selected path by dotted line as shown in Fig. By using any circuit simplification techniques, calculate the current through all the resistances present in the selected path. Then travel through the selected path from B to A and add all voltage drops and emf's algebraically.

In Fig., 3Ω resistor is present in the selected path, and for calculation of V_{TH}, current through the 3Ω resistor is required. By using mesh analysis, $I_{3\Omega}$ can be calculated.

Applying KVL to mesh 1

$- 6I_1 - (I_1 - I_2) + 10 = 0$

$7I_1 - I_2 = 10$ (i)

Applying KVL to mesh 2

$- 2I_2 - 3I_2 - (I_2 - I_1) = 0$

$I_1 - 6I_2 = 0$ (ii)

Solving (i) and (ii),

$I_2 = 0.244\ A$

Hence, $I_{3\Omega} = 0.244\ A\ (\downarrow)$

Thus, $V_{TH} = V_{AB}$

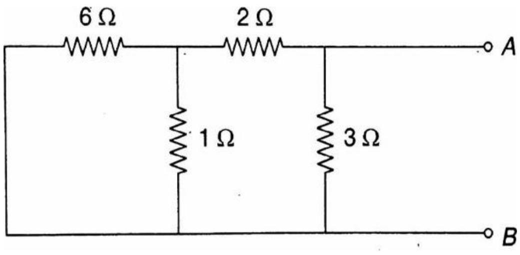
$= 20 + 3I_2$

$= 20 + 3(0.244)$

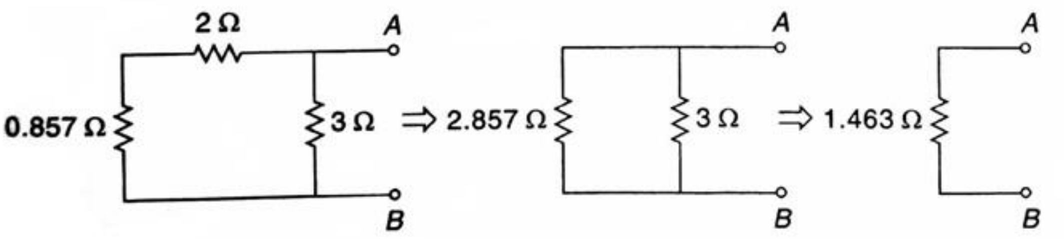
$= \mathbf{20.732\ V}$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuit, we get the following network:



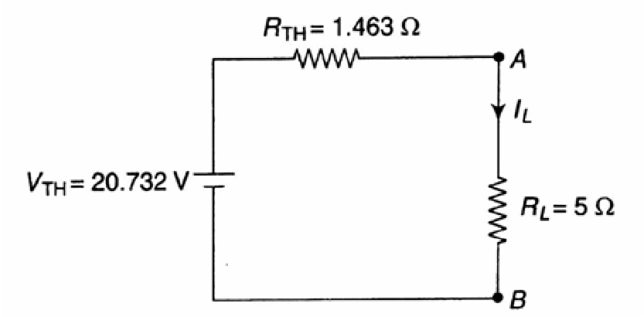
In the circuit equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. By series parallel circuit reduction techniques, we obtain the circuit as shown below.



Thus, $\mathbf{R_{TH} = R_{AB} = 1.463\ \Omega}$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as follows:



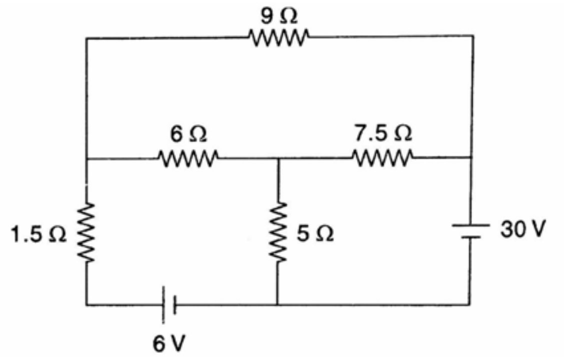
By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence,

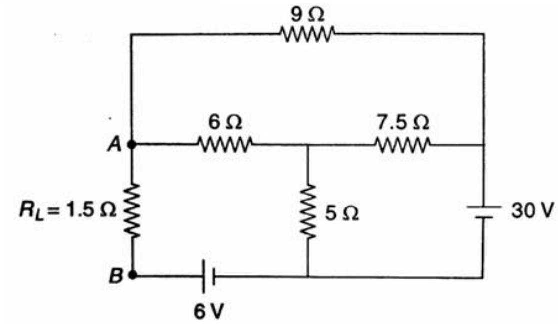
$$\boldsymbol{I_L = I_{5\Omega} = \frac{20.732}{5 + 1.463} = 3.21\ A\ (\downarrow)}$$

2. Determine the current through 1.5Ω resistor in the network shown in Fig. by Thevenin's theorem.



Solution

Current through 1.5 Ω resistor is required. This resistance can be called load resistance R_L. It's terminals A and B are called load terminals.



Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the circuit as shown in Fig. In Fig., voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. As this path contains the resistors 5 Ω and 6 Ω currents through these resistances are required. By using mesh analysis, these required currents can be calculated.

Applying KVL to mesh 1,

$$-9I_1 - 7.5(I_1 - I_2) - 6I_1 = 0$$

$$-22.5I_1 + 7.5I_2 = 0 \qquad \text{(i)}$$

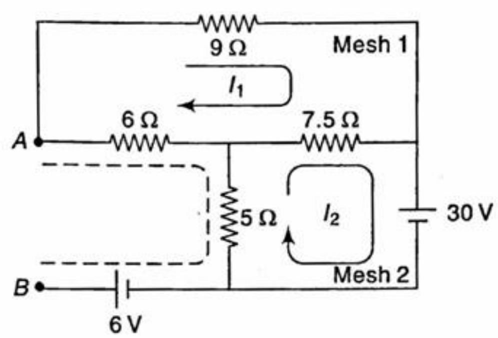
Applying KVL to mesh 2,

$$-7.5(I_2 - I_1) - 30 - 5I_2 = 0$$

$$7.5I_1 - 12.5I_2 = 30 \qquad \text{(ii)}$$

Solving (i) and (ii),

$$I_1 = -1\ \text{A}, \qquad I_2 = -3\ \text{a}$$



Hence,

$$V_{TH} = V_{AB}$$

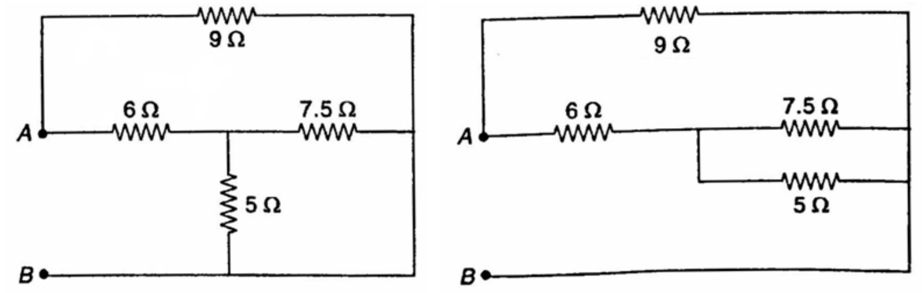
$$= -6 - 5I_2 - 6I_1$$

$$= -6 - 5(-3) - 6(-1)$$

$$= \mathbf{15\ V}$$

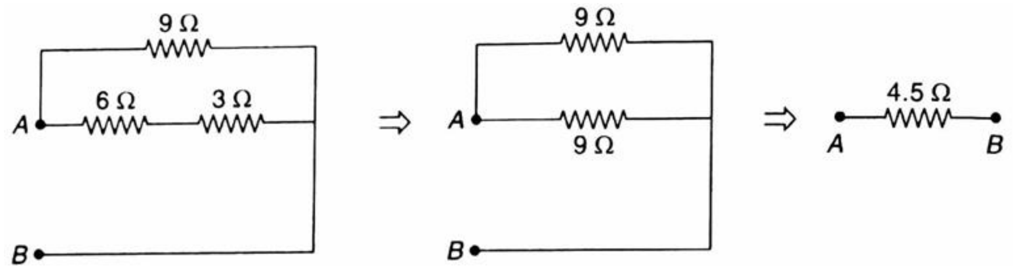
Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuits, we get the circuit as shown in Fig (a). In Fig (a)., equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. The circuit can be redrawn as shown in Fig (b). Below,



(a) (b)

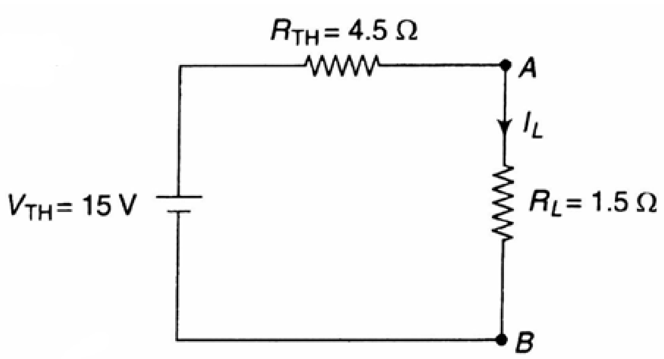
By series parallel circuit reduction techniques, we obtain the circuit as shown below.



Thus, $\mathbf{R_{TH} = R_{AB} = 4.5\ \Omega}$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as follows:



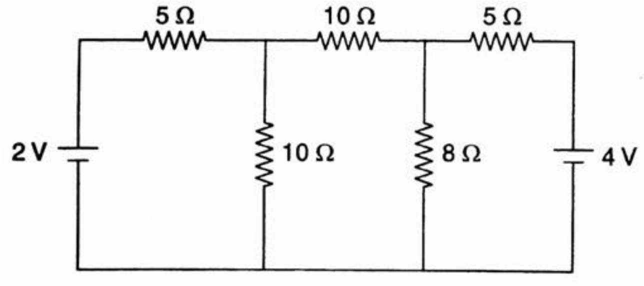
By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

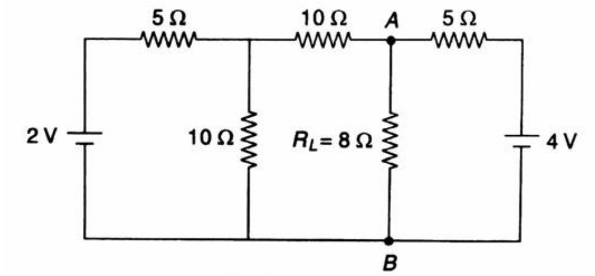
Hence,

$$\boldsymbol{I_L = I_{1.5\Omega} = \frac{15}{1.5 + 4.5} = 2.5\ A}\ (\downarrow)$$

3. Determine the current through 8 Ω resistor in the network shown in Fig. by Thevenin's theorem.



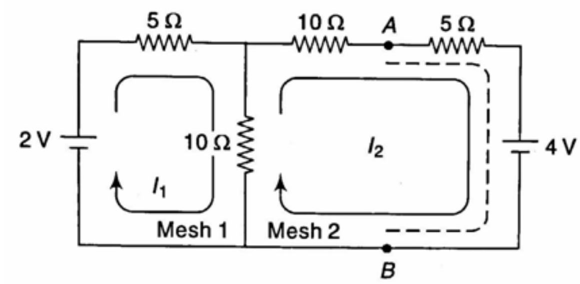
Solution



Current through 8 Ω resistor is required. This resistance can be called load resistance R_L. It's terminals A and B are called load terminals.

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:



In fig. beside, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. As this path contains the resistor 5Ω, current through this resistance is required. By using mesh analysis, this required current can be calculated as follows.

Applying KVL to mesh 1,

$$-5I_1 - 10(I_1 - I_2) + 2 = 0$$

or $$-15I_1 + 10I_2 = -2 \qquad \text{(i)}$$

Applying KVL to mesh 2,

$$-10I_2 - 5I_2 - 4 - 10(I_2 - I_1) = 0$$

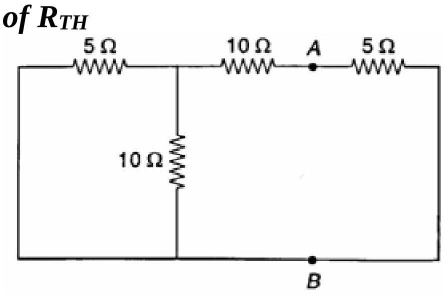
or $$10I_1 - 25I_2 = 4 \qquad \text{(ii)}$$

Solving (i) and (ii),

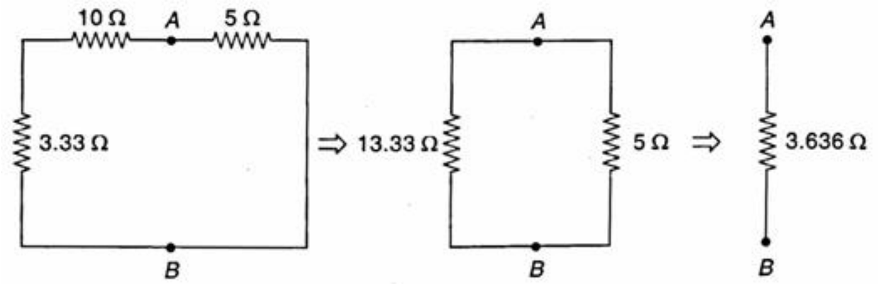
$$I_2 = -0.145 \text{ A}$$

Hence, $$V_{TH} = V_{AB}$$
$$= +4 + 5I_2$$
$$= +4 + 5(-0.145)$$
$$= 3.275 \text{ V}$$

Step II: Calculation of R_{TH}



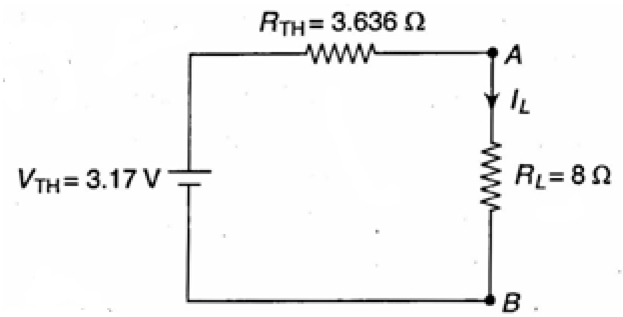
In Fig. above, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. By series-parallel circuit reduction techniques, we modify the circuit as shown below



Thus, $\mathbf{R_{TH} = R_{AB} = 3.636\ \Omega}$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as follows:



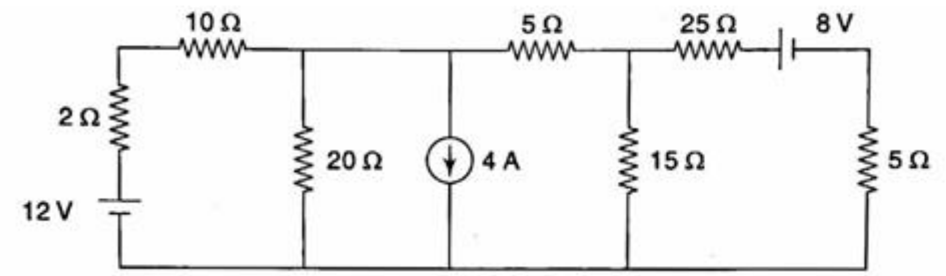
By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence,

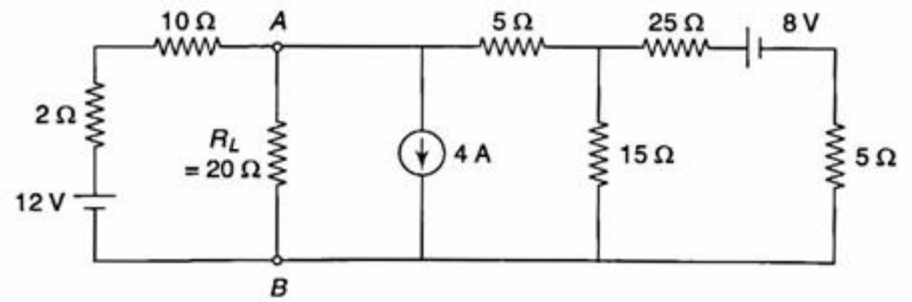
$$\boldsymbol{I_L = I_{8\Omega} = \frac{3.275}{8 + 3.636} = 0.281\ A}\ (\downarrow)$$

4. Using Thevenin's theorem, obtain the power drawn by 20Ω resistor in the network shown below.



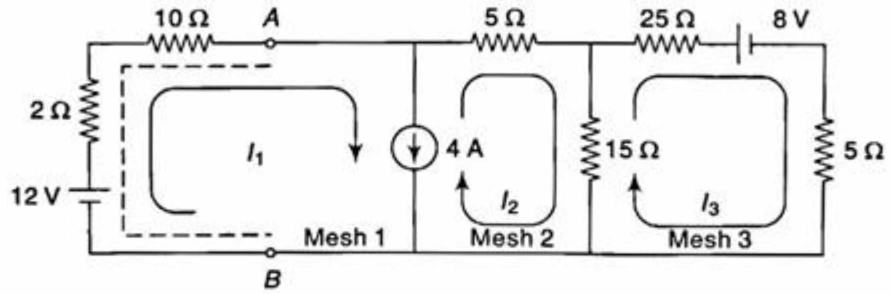
Solution

Power drawn by 20 Ω resistor can be calculated as $P_{20\Omega} = I^2_{20\Omega} \times 20$. Thus, current through 20 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.



Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:



In Fig. above, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. As this path contains the resistors 2Ω and 10Ω, currents through these resistances are required. By using mesh analysis, these required currents can be calculated; Mesh 1 and mesh 2 form a supermesh.

By expressing the current in the common branch, we get the current equation as

$$(I_l - I_2) = 4 \qquad (i)$$

By applying the KVL to the supermesh, we get the voltage equation as

$$-10 I_1 - 5I_2 - 15(I_2 - I_3) + 12 - 2I_1 = 0$$

$$-12I_1 - 20I_2 + 15I_3 = -12 \qquad (ii)$$

Now, by applying the KVL to mesh 3, we have

$$-25I_3 - 8 - 5I_3 - 15(I_3 - I_2) = 0$$

$$15I_2 - 45I_3 = 8 \qquad (iii)$$

Solving, (i), (ii) and (iii), we get

$$I_1 = 2.57 \text{ A}$$

Hence,

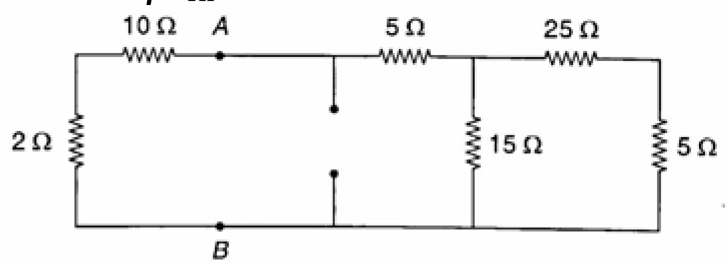
$$\mathbf{V_{TH} = V_{AB}}$$

$$= 12 - 2I_1 - 10I_1$$

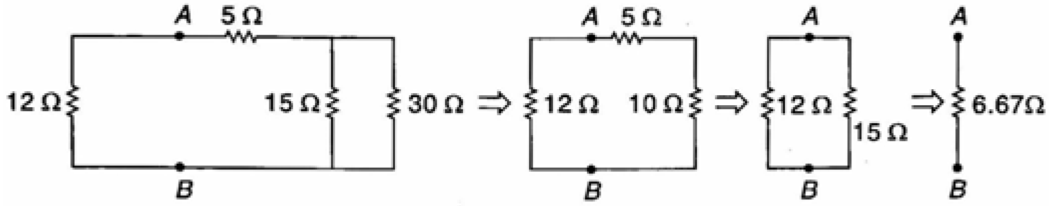
$$= 12 - 2(2.57) - 10(2.57)$$

$$\mathbf{= -18.84 \text{ V}}$$

Step II: Calculation of R_{TH}



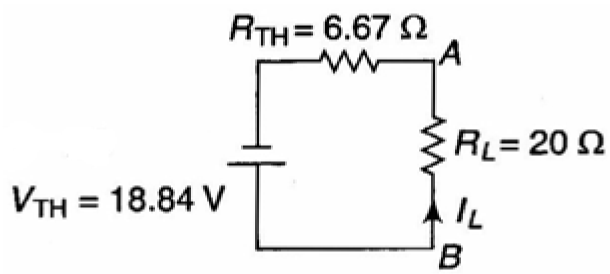
In Fig. above, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. In Fig., 10Ω and 2Ω resistors are in series. Also 25Ω and 5Ω resistors are in series.



Thus, $\mathbf{R_{TH} = R_{AB} = 6.67\Omega}$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as follows:



By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

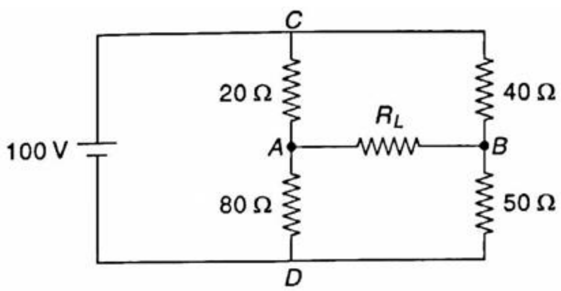
Hence,

$$\boldsymbol{I_L = I_{20\Omega} = \frac{18.84}{20 + 6.67} = 0.706\,A\ (\uparrow)}$$

Power drawn by 20Ω resistor,

$$\boldsymbol{P_{20\Omega} = I^2_{20\Omega} \text{ X } 20 = (0.706)^2 \text{ X } 20 = 9.97\ W}$$

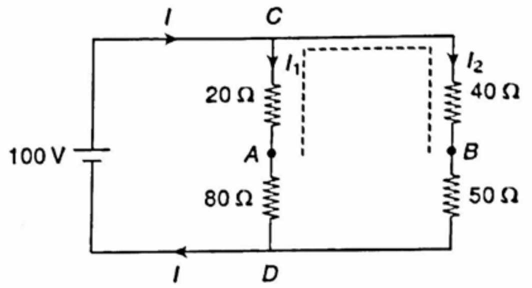
5. Find the current in R_L in the network shown in Fig., when R_L takes up values 5Ω, 10Ω, and 20Ω.



Solution

Step I: Calculation of V_{TH}

Current through R_L is required. Its terminals A and B are called load terminals. Removing the load resistance from the network, we get the following network:



In Fig. beside, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH} i.e. V_{AB}, the selected path from A to B is marked by dotted line in Fig. As this path contains the resistors 20Ω and 40Ω, currents through these resistances are required. In Fig, the actual directions of currents are marked. The 100 V source produces the total current I A, which divides at node C. Let current through branch CAD is I_1 and current through branch CBD is I_2.

By Ohm's Law,

$$I_1 = \frac{100}{20+80} = 1\ A$$

$$I_2 = \frac{100}{40+50} = 1.111\ A$$

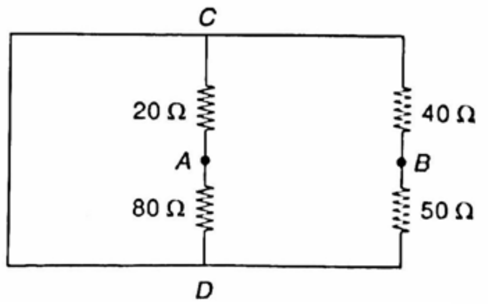
Hence, $\mathbf{V_{TH} = V_{AB}}$

$= 40I_2 - 20I_1$

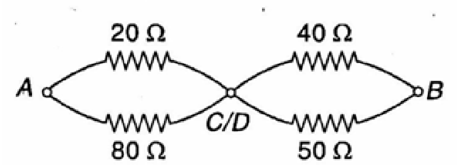
$= 40(1.111) - 20(1)$

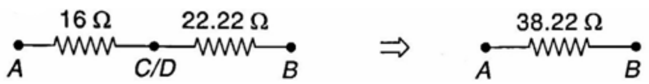
$= \mathbf{24.44\ V}$

Step II: Calculation of R_{TH}



In Fig., equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. Nodes C and D are same and by shorting them, the circuit can be redrawn as follow. By series-parallel circuit reduction techniques, we get the following circuit:

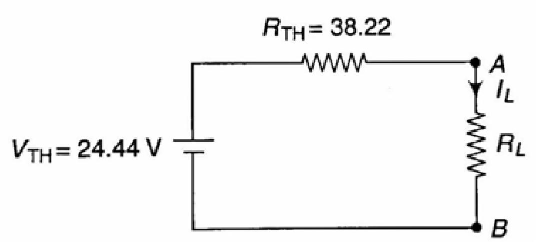




Thus, $\mathbf{R_{TH} = R_{AB} = 38.22\ \Omega}$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as follows:



When R_L = 5 Ω

$$\boldsymbol{I_L = I_{5\Omega} = \frac{24.44}{5+38.22} = 0.565\ A(\rightarrow)}$$

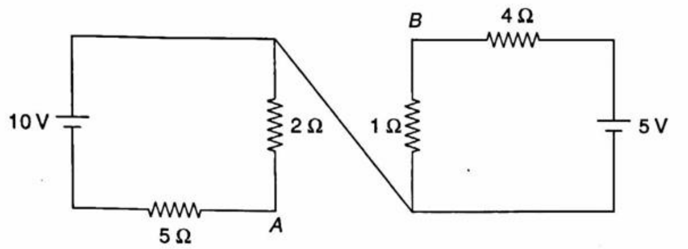
When R_L = 10 Ω

$$\boldsymbol{I_L = I_{10\Omega} = \frac{24.44}{10+38.22} = 0.507\ A(\rightarrow)}$$

When R_L = 20 Ω

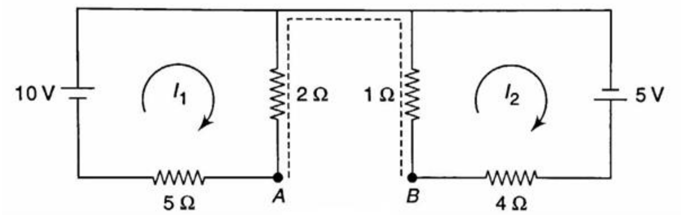
$$\boldsymbol{I_L = I_{20\Omega} = \frac{24.44}{20+38.22} = 0.4198\ A(\rightarrow)}$$

6. For the circuit shown in Fig., find the Thevenin's equivalent circuit across A-B.



Solution

For simplicity, the circuit shown in Fig. can be redrawn as shown in Fig



Step I: Calculation for V_{TH}

In Fig. beside, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH} i.e. V_{AB}, the selected path from A to B is marked by dotted line in Fig. As this path contains the resistors 1Ω and 2Ω, current through these resistors are required. Let the loop currents are marked as I_1 and I_2.

From Fig.,

$$I_1 = \frac{10}{2+5} = 1.43\ A$$

$$I_2 = \frac{5}{4+1} = 1\ A$$

Hence, $\mathbf{V_{TH} = V_{AB}}$

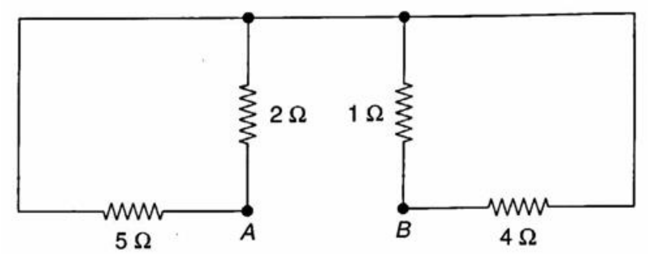
$= -1I_2 - 2I_1$

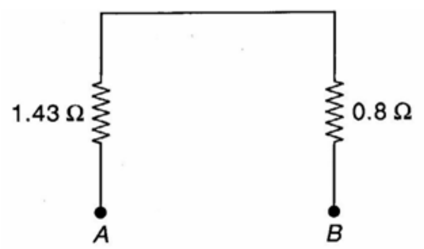
$= -1 \times 1 - 2 \times 1.43$

$= \mathbf{-3.86\ V}$

Step II: Calculation of R_{TH}

In Fig., replacing the sources with their internal resistances, we get the circuit as shown in Fig. In Fig., equivalent resistance across the load terminal A to B is called Thevenin's resistance R_{TH}.

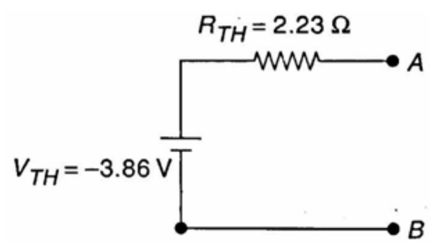




Thus, $\mathbf{R_{TH} = R_{AB} = 1.43 + 0.8 = 2.23\ \Omega}$

Step III: Thevenin's equivalent circuit

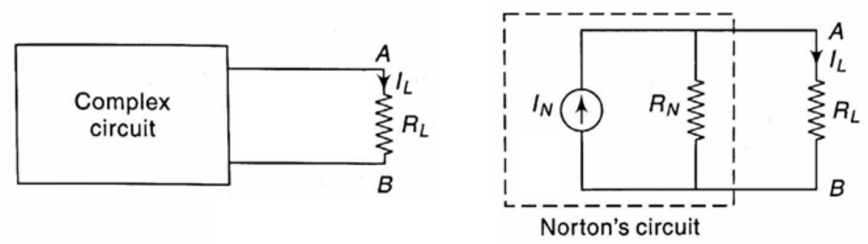
Thevenin's equivalent circuit can be drawn as shown:



1.13 Norton's Theorem

Norton's theorem is converse of Thevenin's theorem. Norton's equivalent circuit uses a current source instead of voltage source and a resistance R_N (which is same as R_{TH}) in parallel with the source instead of being in series with it.

E.L. Norton, an engineer employed by the Bell Laboratory, USA, first developed this theorem. According to this theorem, any two-terminal network can be replaced by a single current source of magnitude I_N (called Norton current) in parallel with a single resistance R_N (called Norton resistance). Figure shows a complex network enclosed in a box with two terminals A and B brought out. The network in the box may contain any number of resistors and sources connected in any fashion. But according to Norton, the entire circuit behind terminals AB can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. The current I_N is equal to the current that would flow when terminals A and B are short circuited, i.e., I_N, is equal to the current flowing through short-circuited terminals AB. The resistance R_N is the same as Thevenin's resistance R_{TH}, i.e., R_N is the resistance measured at AB with load removed and replacing all sources by their internal resistances.



Hence, Norton's theorem as applied to dc circuits may be stated as under:

Any complex network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N.

i) The output I_N of the current source is equal to the current that would flow through AB when A and B are short circuited.

ii) The resistance R_N is the resistance of the network measured between A and B with load removed and replacing the source with their internal resistances.

Steps to apply Norton's theorem

Step 1: Short the branch resistance through which current is to be calculated.

Step 2: Obtain the current through this short-circuited branch, using any of the network-simplification techniques. This current is Norton's current I_N.

Step 3: Calculate R_N as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all sources by their internal resistances.

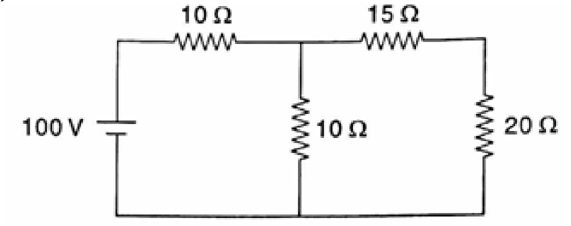
Step 4: Draw the Norton's equivalent circuit showing current source I_N, with the resistance R_N in parallel with it.

Step 5: Reconnect the branch resistance. Let it be R_L. The required current through the branch is given by

$$I_L = I_N \times \frac{R_N}{R_L + R_N}$$

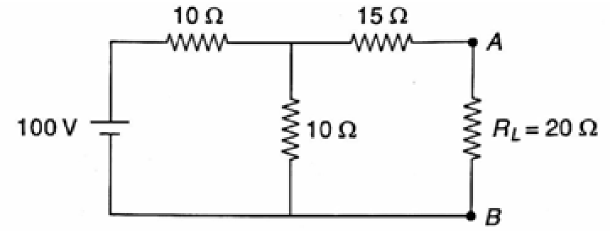
Problems

1. By Norton's theorem, find the current in 20 Ω resistor in the network shown in Fig.



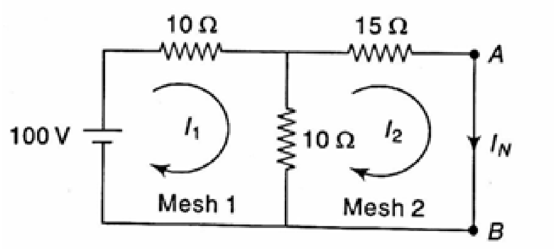
Solution

Current through 20 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.



Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the modified network as shown in Fig.



In Fig., current flowing through the short circuit placed across the load terminals A and B is called Norton current I_N. This current can be calculated by mesh analysis as shown below.

Applying the KVL to mesh l,

$$-10I_1 - 10(I_1 - I_2) + 100 = 0$$

$$-20I_1 + 10I_2 = -100 \qquad (i)$$

Applying the KVL to mesh 2,

$$-15I_2 - 10(I_2 - I_1) = 0$$

$$10I_1 - 25I_2 = 0 \qquad (ii)$$

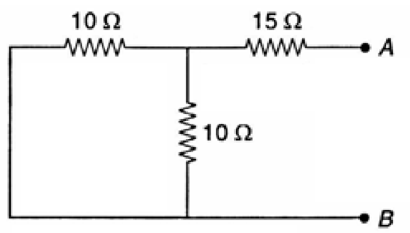
Solving equation (i) and (ii),

$$I_2 = 2.5\ A$$

Hence, ***I_N = 2.5 A, from A to B***

Step II: Calculation of R_N

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the network as shown in Fig.

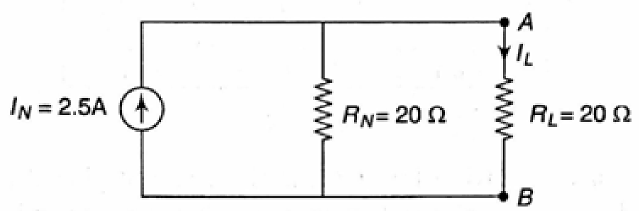


In fig equivalent resistance across the load terminals A and B is called Norton's resistance R_N.

Thus, $\boldsymbol{R_N = R_{AB} = (10 \parallel 10) + 15 = 20\ \Omega}$

Step III: Calculation of load current

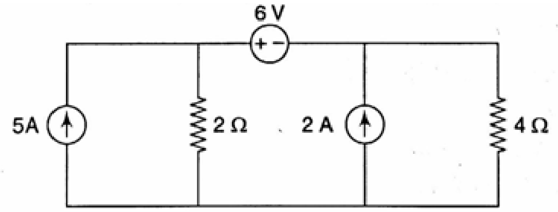
Norton's equivalent circuit can be drawn as shown in fig



By current division rule,

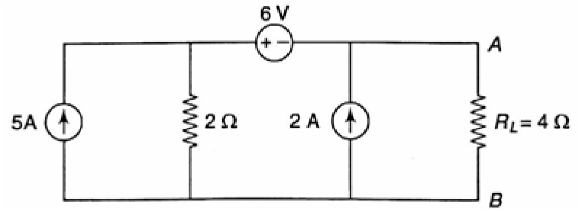
$$\boldsymbol{I_L = I_{20\Omega} = 2.5 \times \frac{20}{20 + 20} = 1.25\ A}\ (\downarrow)$$

2. By Norton's theorem, find the current in 4 Ω resistor in the network shown in Fig.



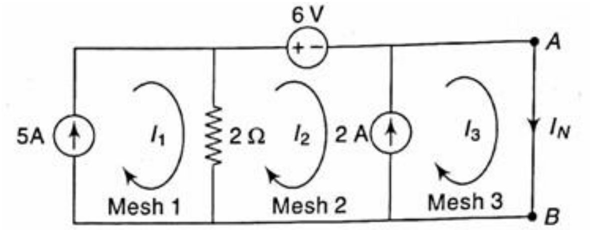
Solution

Current through 4 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.



Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the modified circuit as shown in Fig.



In Fig., current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N. This current can be calculated by mesh analysis as follows.

In mesh l, current source of 5A is in the direction of mesh current I_1.

So, $I_1 = 5$ A (i)

Mesh 2 and mesh 3 forms a supermesh.

By expressing the current in the common branch, we get the current equation,

$(I_3 - I_2) = 2$ (ii)

By applying the KVL to supermesh, we get the voltage equation as

$-6 - 2(I_2 - I_1) = 0$

$2I_1 - 2I_2 = 6$ (iii)

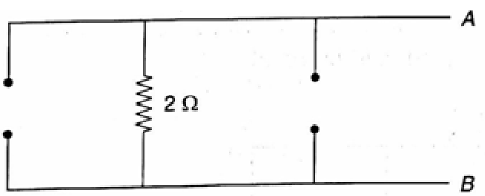
Solving equation's (i), (ii) and (iii),

$I_3 = 4$A

Hence, $\mathbf{I_N = 4\ A}$, from A to B

Step II: Calculation of R_N

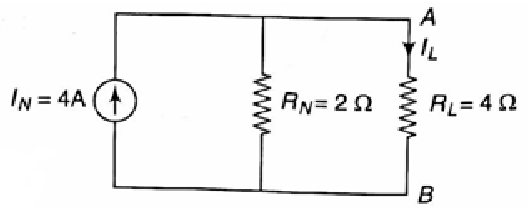
Removing the load resistance from the network and replacing the voltage source by short circuit and current sources by open circuits, we get the network as shown in Fig.



Thus, $\mathbf{R_N = R_{AB} = 2\ \Omega}$

Step III: Calculation of load current

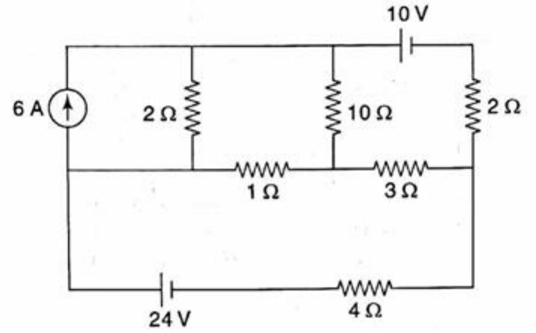
Norton's equivalent circuit can be drawn as shown in fig



By current division rule,

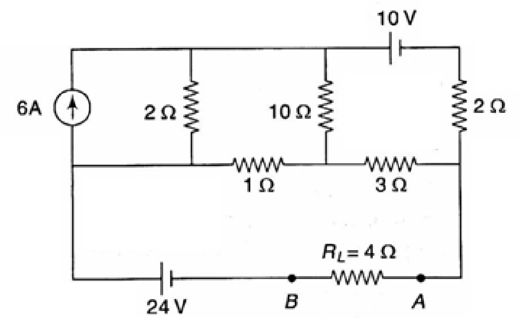
$$I_L = I_{4\Omega} = 4 \times \frac{2}{2+4} = 1.33\ A\ (\downarrow)$$

3. By Norton's theorem, find the current in 4 Ω resistor in the network shown in Fig.



Solution

Current through 4 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

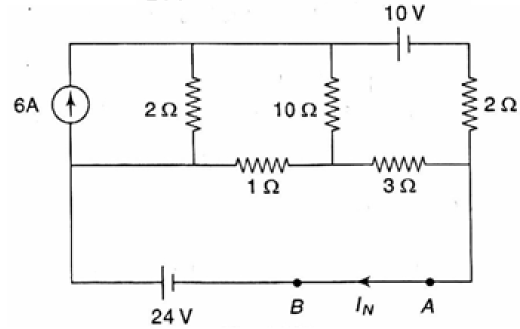


Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the modified circuit as shown in Fig.

In Fig., current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N.

By source transformation, i.e., converting parallel combination of current source of 6 A and resistor of 2Ω into equivalent series combination of voltage source and resistor, we get the modified network as shown in Fig.



Applying KVL to mesh 1,

$$-2I_1 - 10(I_1 - I_2) - (I_1 - I_3) + 12 = 0$$

or $$-13I_1 + 10I_2 + I_3 = -12 \quad \text{(i)}$$

Applying the KVL to mesh 2,

$$-10(I_2 - I_1) - 10 - 2I_2 - 3(I_2 - I_3) = 0$$

or $$10I_1 - 15I_2 + 3I_3 = 10 \quad \text{(ii)}$$

Applying the KVL to mesh 3,

$$-(I_3 - I_1) - 3(I_3 - I_2) + 24 = 0$$

or $$I_1 + 3I_2 - 4I_3 = -24 \quad \text{(iii)}$$

24 V B I_N A

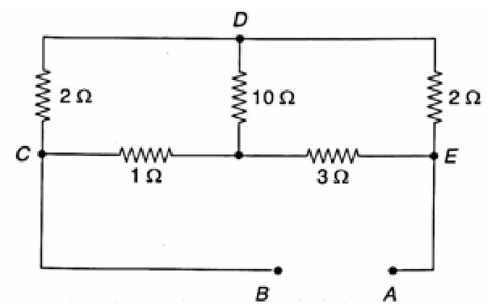
Solving equation's (i), (ii) and (iii),

I_3 = 12.39A

Hence, I_N **= 12.39 A,** from A to B

Step II: Calculation of R_N

Removing the load resistance from the network and replacing the voltage source by short circuit and current sources by open circuits, we get the network as shown in Fig.



Converting the star connection formed by 1Ω, 10Ω, and 3Ω resistors (Y CDE) into equivalent delta connection, i.e., Y CDE → Δ CDE, we get the circuit as shown in Fig.

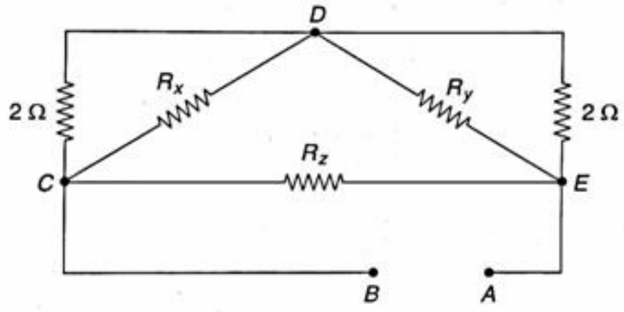
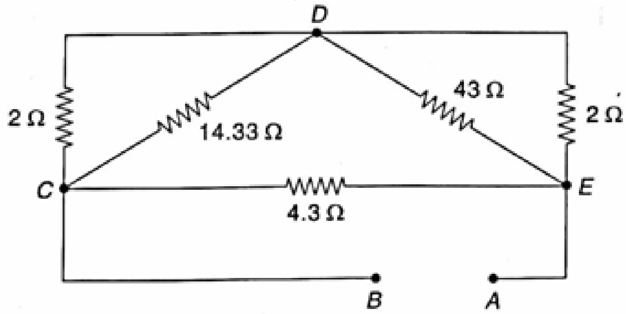


Fig. 1.344

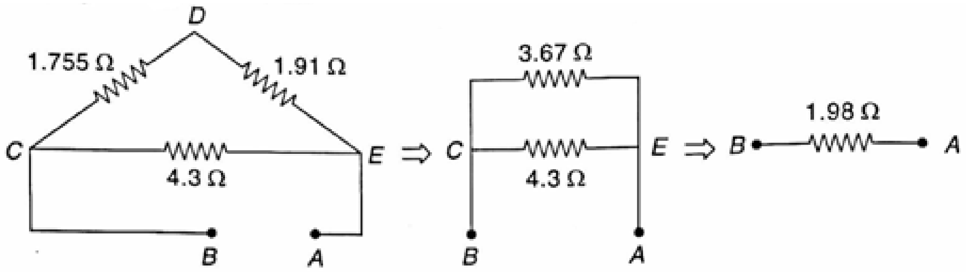
We have $R_x = 1 + 10 + \frac{1\times10}{3} = 14.33\ \Omega$, $R_y = 10 + 3 + \frac{10\times3}{1} = 43\ \Omega$,

$R_z = 1 + 3 + \frac{1\times3}{5} = 4.3\ \Omega$

The simplified network is shown in fig below,



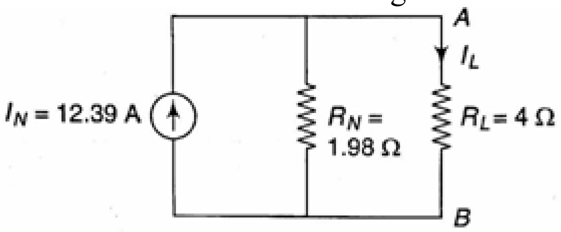
By series-parallel circuit reduction techniques, we get the following network:



Thus, $\mathbf{R_N = R_{AB} = 1.98\ \Omega}$

Step III: Calculation of load current

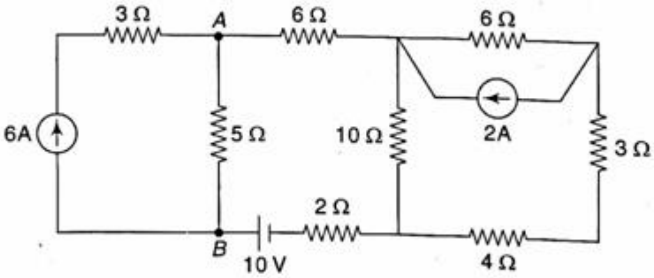
Norton's equivalent circuit can be drawn as shown in fig



By current division rule,

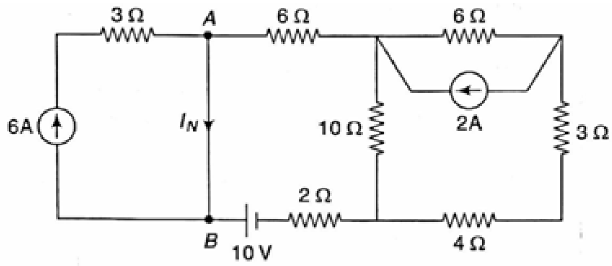
$$\boldsymbol{I_L = I_{4\Omega} = 12.39 \times \frac{1.98}{1.98 + 4} = 4.1\ A} \ (\leftarrow)$$

4. By Norton's theorem, find the current in 5 Ω resistor in the network shown in Fig.



Solution

Step I: Calculation of I_N



Current through 5Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

Removing the load resistance from the network and short circuiting the load terminals, we get the modified circuit as shown in Fig.

In Fig., current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N.

By source transformation, i.e., converting parallel combination of current source of 2A and resistor of 6Ω into equivalent series combination of voltage source and resistor, we get the modified network as shown in Fig.

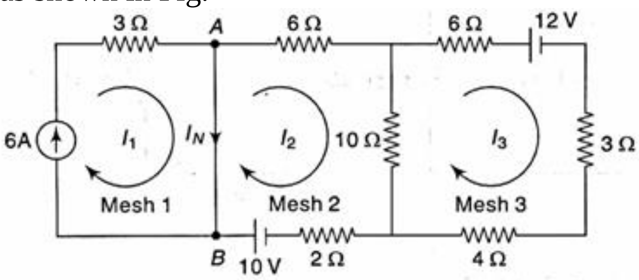


Fig. 1.350

The current I_N can be calculated by mesh analysis.

In mesh 1, current source of 6 A is in the direction of mesh current I_1.

So, $\quad I_1 = 6 \qquad$ (i)

Applying the KVL to mesh 2,

$$6I_2 - 10(I_2 - I_3) - 2I_2 + 10 = 0$$

or $\quad -18I_2 + 10I_3 = -10 \qquad$ (ii)

Applying the KVL to mesh 3,

$$-6I_3 - 12 - 3I_3 - 4I_3 - 10(I_3 - I_2) = 0$$

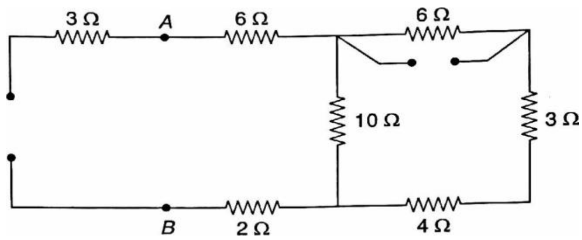
or $\quad 10I_2 - 23I_3 = 12 \qquad$ (iii)

Solving equation's (i), (ii) and (iii),

$I_1 = 6$ A, $I_2 = 0.35$ A

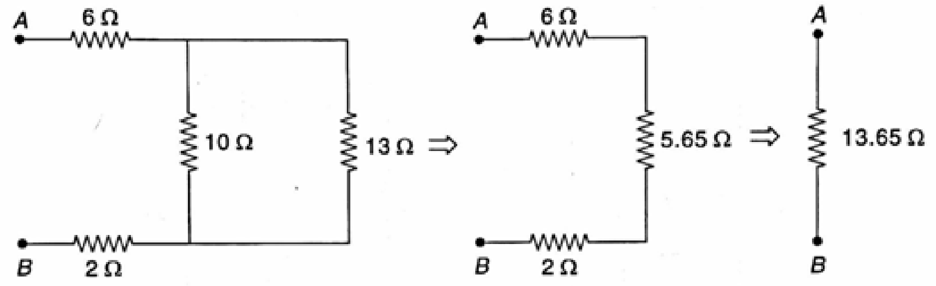
Hence, $\mathbf{I_N = (I_1 - I_2) = 5.65}$ **A,** from A to B

Step II: Calculation of R_N



Removing the load resistance from the network and replacing the voltage source by short circuit and current sources by open circuits, we get circuit as shown

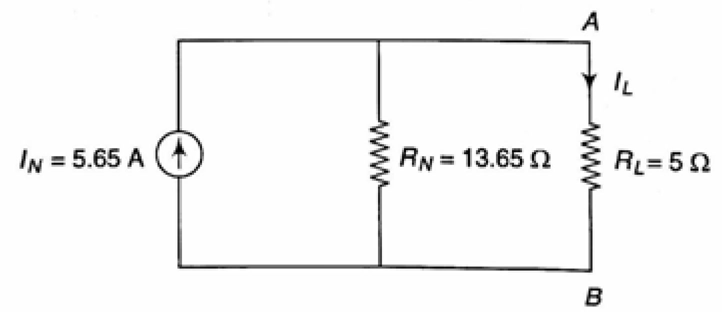
By series-parallel circuit reduction techniques, we get the network as shown in Fig.



Thus, $\mathbf{R_N = R_{AB} = 13.65\ \Omega}$

Step III: Calculation of load current

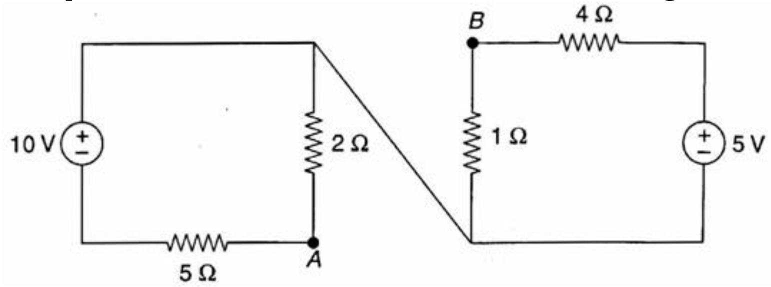
Norton's equivalent circuit can be drawn as shown in fig



By current division rule,

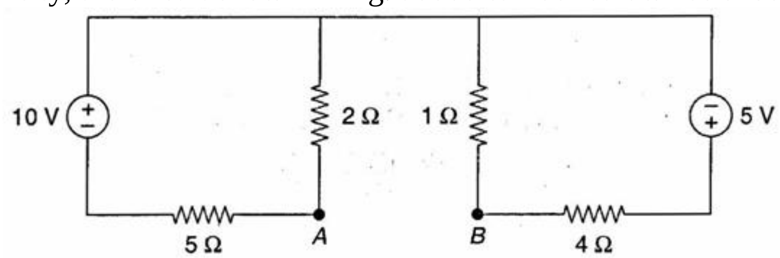
$$\boldsymbol{I_L = I_{5\Omega} = 5.65 \times \frac{13.65}{13.65 + 5} = 4.14\ A}\ (\downarrow)$$

5. Obtain Norton's equivalent circuit across A and B as shown in Fig.

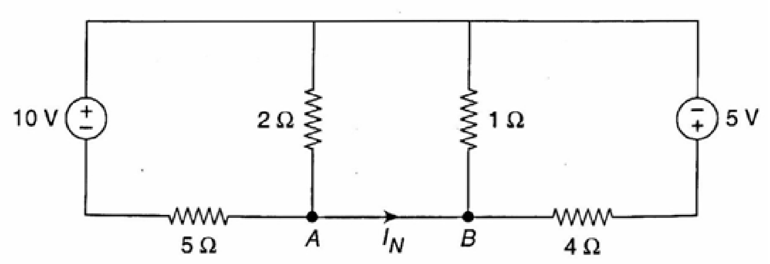


Solution

For simplicity, the circuit shown in Fig. above can be redrawn as shown in Fig. below



Step I: Calculation of I_N



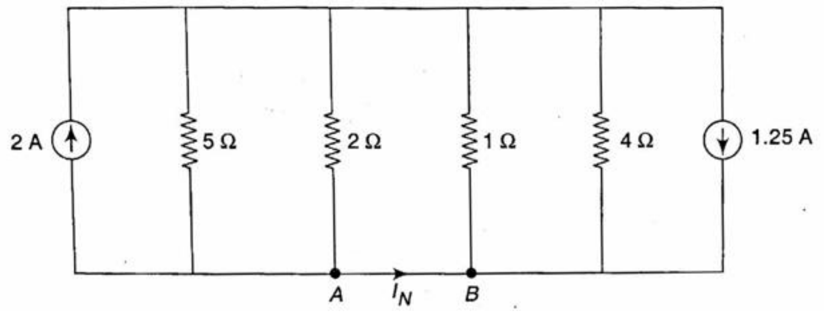
In Fig. above, current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N

In the circuit, there are two combinations as follows:

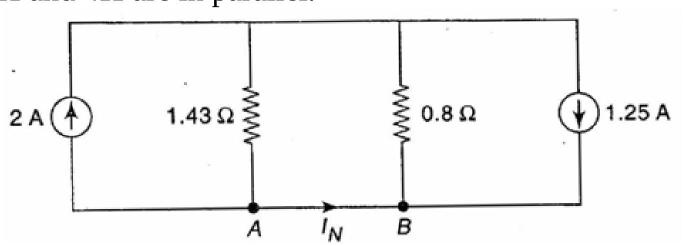
(i) Series combination of voltage source of 10 V and resistor of 5Ω

(ii) Series combination of voltage source of 5 V and resistor of 4Ω

Converting the above combinations into equivalent combinations, we get the simplified circuit as follows:



In Fig. above, all elements are in parallel. The resistors 5Ω and 2Ω are in parallel. Similarly resistors 1Ω and 4Ω are in parallel.

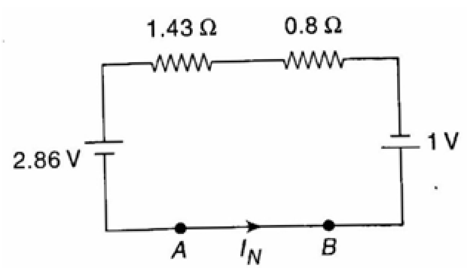


In the circuit of Fig. above, there are two combinations as follows:

(i) Parallel combination of current source of 2 A and resistor of 1.43Ω

(ii) Parallel combination of current source of I .25 A and resistor of 0.8Ω

Converting the above combinations into equivalent combinations, we get the simplified circuit as follows:



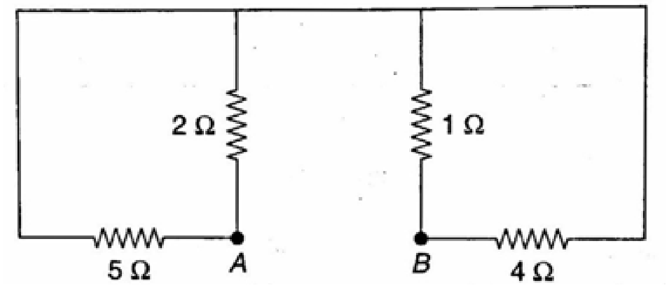
By applying KVL to above circuit

$2.86 + 1.43I_N + 0.8I_N + 1 = 0$

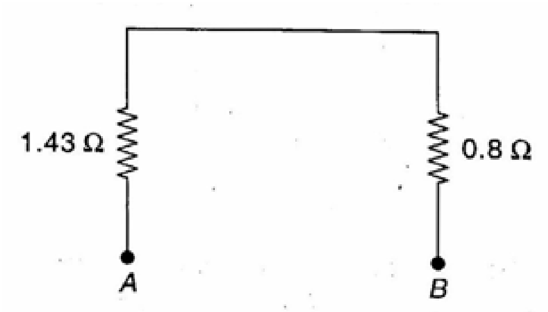
$\mathbf{I_N = -1.73\ A}$

Step II: Calculation of R_N

Replacing the voltage sources by short circuit, we get the network as shown in Fig. Below In circuit, equivalent resistance across load terminals A and B is called Norton's resistance R_N.



In Fig. above, resistors 5Ω and 2Ω are in parallel. Also resistors 1Ω and 4Ω are in parallel.



Thus, $\mathbf{R_N = R_{AB} = 1.43 + 0.8 = 2.23\ \Omega}$

Step III: Norton's equivalent circuit

Norton's equivalent circuit can be drawn as follows: