Tutorial 01-Set

1. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

iii. $(A \cup B)' = A' \cap B'$

iv. $(A \cap B)' = A' \cup B'$

v. $A = (A \cap B) \cup (A \cap B')$

vi. $B = (A \cap B) \cup (A' \cap B)$

vii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. Prove that using membership table.

i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

iii. $(A \cup B)' = A' \cap B'$

iv. $(A \cap B)' = A' \cup B'$

3. Let A={a,b,c}, B={d,e}, C={a,d}. Find (i) A×B(ii) B×A

Ans. (i) A×B ={(a, d),(a, e), (b, d), (b, e), (c, d), (c, e)}. (ii) B×A = {(d, a),(d, b), (d, c), (e, a) (e, b),(e, c)}.

4. In a class of 200 students who appeared certain examinations, 35 students failed in MHT-CET, 40 in AIEEE and 40 in IIT entrance, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT entrance,15 in MHT-CET and IIT entrance and 5 failed in all three examinations. Find how many students

i. did not fail in any examination.

ii. failed in AIEEE or IIT entrance.

Ans: 132 & 63.

5. From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read,

i. at least one of the newspapers.

ii. neither Marathi nor English newspaper.

iii. only one of the newspapers.

Ans: 1750, 250 & 1100.

6. Out of forty students, 14 are taking English Composition and 29 are taking

Chemistry.If five students are in both classes, a) how many students are in neither class? b) How many are in either class?

Ans: a.Two students are taking neither class. b. There are 38 students in at least one of the classes.

7. In a class of 40 students, 15 like to play cricket and football and 20 like to play cricket. How many like to play football only but not cricket?

Ans: 20

8. If A {1, 3, 5}, then write all the possible subsets of A.

Ans: all possible subsets of A are { }, {1}, {3}, {5}, {1, 3}, {3, 5}, {1, 3, 5}

9. Given three sets P, Q and R such that:

i. P = {x: x is a natural number between 10 and 16},

ii. Q = {y: y is a even number between 8 and 20} and

iii. R = {7, 9, 11, 14, 18, 20}

(i) Find the difference of two sets P and Q

(ii) Find Q - R

(iii) Find R - P

(iv) Find Q – P

Ans. P-Q = {11, 13, 15} Q – R = {10, 12, 16} R – P = {7, 9, 18, 20} Q – P = {10, 16, 18}

10. Let X={1,2,3,4,5,6} Determine whether or not each of the following is a partition of X?

i. [{1,2,3}{1,4,5,6}]

ii. [{1,2},{3,5,6}]

iii. [{1,3,5},,{2,4},{6]]

iv. [{1,3,5}{4,4,6,7}]

11. Suppose that 109 of the 150 computer science students at one of the Mumbai college take at least one of the following computer language:- VB, VC++ and Java. Suppose 45 study VB, 61 study V C ++, 53 study Java, 18 Study VB and VC ++, 53 study VC + + and Java, and 23 study VB and Java.

(i) How many students study all three languages?

(ii) How many students study only VC++?

(iii) How many students do not study any of the language?

Tutorial 2: Relations

1. Define

(i)Relation (ii) Domain of Relation

(iii) Range of Relation (iv) Inverse Relation

and give suitable example.

2. Find domain, range, matrix of relation R if

i. A={1,2,3}, B={r,s} and R={{1,r),(2,s),(3,r)}

ii. A={1,2,3,4,5} R is defined as aRb if a<b

3. Let 'R' be the relation from A = {1, 2, 3, 4} to B = {x, y, z} defined by R = {(1, y), (1, z), (3, y), (4, x), (4, z)}

a. Determine the domain and range of R.

b. Find the inverse relation R^{-1} of R.

c. Matrix of R

4. Let R and S be the relations from A = {1, 2, 3} to B = {a, b} defined by R = {(1,a),(3,a),(2,b),(3,b)} S = {(1,b), (2,b)}

a) find the composition relation SoR,

b) Find (a) R ; (b) R^{-1}; (c) R ∩ S; (e) S ; (f) S^{-1}

c) Find the Matrices $M_R, M_S, M_{\bar{R}}, M_{\bar{S}}, M_{R^{-1}}, M_{S^{-1}}, M_{RoS}, M_{R\cap S}, M_{R\cup S}$

5. Let R and S be the find the composition relation SoR,

Find R ; (b) R^{-1}; (c) R ∩ S; (e) S ; (f) S^{-1} and the Matrices $M_R, M_S, M_{\bar{R}}, M_{\bar{S}}, M_{R^{-1}}, M_{S^{-1}}, M_{RoS}, M_{R\cap S}, M_{R\cup S}$ for

i. Relations from A = {1, 2, 3} to B = {a, b} defined by R = {(1,a),(3,a),(2,b),(3,b)} S = {(1,b), (2,b)}

ii. A={1,2,3}, B={a,b,c} and C={x,y,z} Relations R and S from A to B and B to C respectively defined by: R={(1,b),(2,a),(2,c)} and S={(a,y),(b,x),(c,y),(c,z)}

6.)If A = {1, 2, 3, 4}. Determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, anti-symmetric, or transitive.

i. R= {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}

ii. R= {(1, 1), (1, 3), (1, 4), (3, 3), (2, 4), (4, 4)}

7. A = {1, 2, 3, 4} Determine whether the relation R whose matrix M_R is reflexive, irreflexive, symmetric, asymmetric, anti-symmetric, or transitive.

i. $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

ii. $M_R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

8. Define a relation on the set {a, b, c, d} that is

a) Reflexive and symmetric, but not transitive.

b) Reflexive and transitive, but not symmetric.

c) Transitive, reflexive and symmetric,

d) A symmetric and transitive.

9. Determine whether the relation R on the set A is an equivalence relation.

a) A = {a, b, c, d},R = {(a,a,), (b,a), (b,b), (c,c), (d,d), (d,c)}

b) A = {1, 2, 3, 4, 5}, R = {(1,1),(1,2),(1,3),(2,1),(2,2),(3,1),(2,3),(3,3),(4,4),(3,2),(5,5)}

c) A = {1, 2, 3, 4}, R = {(1,1),(1,2),(2,1),(2,2),(3,1),(3,3),(1,3),(4,1),(4,4)}

10. Define equivalence relation on a set. Let R be a relation on the set of integers defined by aRb iff a-b is a multiple of 5. Prove that R is equivalence relation.

11. Define equivalence relation on a set. Let R be a relation on the set of integers defined by aRb iff a-b is a multiple of 5. Prove that R is equivalence relation.

12. Let R be the following equivalence relation on the set A = {1,2,3,4, 5, 6 }: R = {(1,1), (1,5), (2,2), (2,3), (2, 6), (3,2),(3,3),(3,6),(4,4),(5,1),(5,5), (6,2), (6,3),(6,6)}Find the partitions of A induced by R, i.e., find the equivalence classes of R.