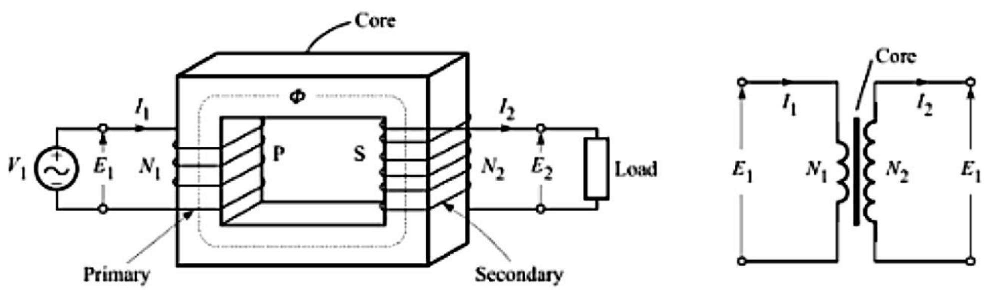
Transformer

❖ Principle of Operation

A transformer operates on the principle of ***mutual induction*** between *two coils*. Figure 13.1*a* shows the general construction of a transformer. The vertical portions of the steel-core are termed ***limbs***, and the top and bottom portions are called *yokes*. The two coils P and S, having N_1 and N_2 turns, are wound on the limbs. These two windings are electrically unconnected but are linked with one another through a magnetic flux in the core. The coil P is connected to the supply and is therefore called *primary*; coil *S* is connected to the load and is termed the *secondary*.

Basically, two principles are involved in the operation of a transformer. Firstly, an electric current produces a magnetic field (electromagnetism), and secondly, a changing magnetic field within a coil induces an emf across the ends of the coil (electromagnetic induction). A changing current in the primary circuit creates a changing magnetic field; in turn, this magnetic field induces a voltage in the secondary circuit. Thus, energy is transferred from one circuit to the other.



(*a*) Construction. (*b*) Circuit symbol.

Fig. 13.1 *A transformer.*

Figure 13.1*b* shows the circuit symbol of a transformer. The thick line denotes the iron core. By having different ratios N_1/N_2 of the two windings, power at lower or at higher voltage can be obtained. When $N_2 > N_1$, the transformer is called a ***step up*** transformer; and when $N_2 < N_1$, the transformer is called a ***step down*** transformer.

❖ EMF Equation

Consider a sinusoidally varying voltage V_1 applied to the primary of the transformer shown in Fig. 13.1*a*. Due to this voltage, a sinusoidally varying magnetic flux is set up in the core, which can be represented as

$$\Phi = \Phi_m \sin \omega t = \Phi_m \sin 2\pi f t \tag{13.1}$$

where Φ_m is the peak value of the flux and f is the frequency of sinusoidal variation of flux. As per the law of electromagnetic induction, the induced emf in a winding of N turns is given as

$$e = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(\Phi_m \sin \omega t) = -N\omega\Phi_m \cos \omega t = \omega N\Phi_m \sin(\omega t - \pi/2) \tag{13.2}$$

Thus, the peak value of the induced emf is $E_m = \omega N\Phi_m$. Therefore, the rms value of the induced emf E is given as

$$E = \frac{E_m}{\sqrt{2}} = \frac{\omega N\Phi_m}{\sqrt{2}} = \frac{2\pi f N\Phi_m}{\sqrt{2}} = 4.44 fN\,\Phi_m$$

or

$$\boxed{E = 4.44\, fN\Phi_m} \tag{13.3}$$

This equation, known as ***emf equation of transformer***, can be used to find the emf induced in any winding (primary or secondary) linking with flux Φ.

❖ Effect of Frequency

The emf of a transformer at a given flux increases with frequency (see Eq. 13.3). By operating at higher frequencies, transformers can be made physically more compact because a given core is able to transfer more power without reaching saturation, and fewer turns are needed to achieve same impedance. However, properties such as core losses and conductor skin effect* also increase with frequency. Aircraft and military equipments employ 400-Hz power supplies which reduces core and winding weight.

❖ Ideal Transformer

Conditions for Ideal Transformer

(*i*) The permeability (μ) of the magnetic circuit (the core) is infinite, i.e., the magnetic circuit has zero reluctance so that no mmf is needed to set up the flux in the core.

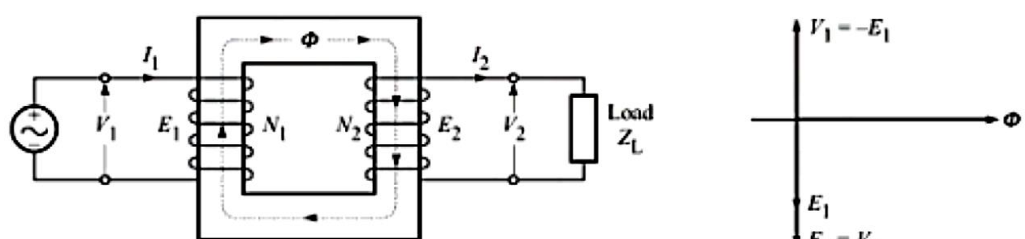
(*ii*) The core of the transformer has no losses.

(*iii*) The resistance of its windings is zero, hence no I^2R losses in the windings.

(*iv*) Entire flux in the core links both the windings, i.e., there is no *leakage flux*.

Thus, an ideal transformer has no losses and stores no energy. However, an ideal transformer has no physical existence. But, the concept of ideal transformer is very helpful in understanding the working of an actual transformer.

Consider an ideal transformer whose secondary is connected to a load Z_L and primary is supplied from an ac source V_1 (Fig. 13.2*a*). The voltage across the load is V_2. The primary and secondary windings of the ideal transformer have zero impedance. Hence, the induced emf E_1 in the primary exactly counter balances the applied voltage V_1, that is, $V_1 = -E_1$. Also, the induced emf E_2 is the same as voltage V_2, that is, $E_2 = V_2$. Here, E_1 is called ***counter emf*** or ***back emf*** induced in the primary, and E_2 called ***mutually induced emf*** in the secondary.



(*a*) The circuit.   (*b*) The phasor diagram.

Fig. 13.2 *Ideal transformer.*

Figure 13.2*b* shows the phasor diagram of the ideal transformer. We have taken flux Φ as reference phasor, as it is common to both the primary and secondary. As per Eq. 13.2, the induced emfs E_1 and E_2 lag flux Φ by 90°. The voltage V_1 is equal and opposite to emf E_1. Thus, the applied voltage V_1 leads the flux Φ by 90°. According to the first condition of ideality, the reluctance of the magnetic circuit is zero and hence the required magnetising current to produce flux Φ is also zero.

❖ Transformation Ratio

The *ratio of secondary voltage to the primary voltage is known as* ***transformation ratio*** or ***turns-ratio***. It is denoted by letter K. Let N_1 and N_2 be the number of turns in primary and secondary windings, and E_1 and E_2 be the rms values of the primary and secondary induced emfs. Using Eq. 13.3, we can write

$$E_1 = 4.44 f N_1 \Phi_m \tag{13.4}$$

and $$E_2 = 4.44\, f N_2 \Phi_m \tag{13.5}$$

Then, the *transformation ratio* or *turns-ratio* can be expressed as

$$K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \tag{13.6}$$

Thus, the side of the transformer with the larger number of turns has the larger voltage. Indeed, *the voltage per turn is constant for a given transformer*. By selecting K properly, the transformation of voltage can be done from any value to any other convenient value. There can be two* cases:

(*i*) When $K > 1$ (i.e., $N_2 > N_1$), $V_2 > V_1$; the device is known as ***step-up transformer.***

(*ii*) When $K < 1$ (i.e., $N_2 < N_1$), $V_2 < V_1$; the device is known as ***step-down transformer.***

In general, a transformer can have more than 2 windings. The windings of a three-winding transformer are called *primary*, *secondary* and *tertiary*. The primary is connected to an ac supply. Different loads may be connected across the secondary and tertiary**. The induced emf in a winding is still proportional to its number of turns,

$$E_1 : E_2 : E_3 :: N_1 : N_2 : N_3$$

❖ Volt-Amperes

Consider again the two-winding transformer of Fig. 13.2*a*. For an ideal transformer, the current I_1 in the primary is just sufficient to provide mmf I_1N_1 to overcome the demagnetising effect of the secondary mmf I_2N_2. Hence,

$$\therefore \qquad I_1 N_1 = I_2 N_2 \quad \text{or} \quad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{K} \tag{13.7}$$

Thus, we find that *the current is transformed in the reverse ratio of the voltage*. If the voltage is stepped up ($V_2 > V_1$), then the current is stepped down ($I_2 < I_1$) by the same factor. That is, the side of the transformer with the *larger* number of turns has the *smaller* current. For example, a step-up transformer would have a primary with few turns of thick wire (small voltage, large current) and the secondary would have many turns of thin wire (large voltage, small current).

Combining Eqs. 13.5 and 13.7, we have

$$E_1 I_1 = E_2 I_2$$

Hence, *in an ideal transformer the input VA and output VA are identical.*

❖ Impedance Transformation

Equations 13.6 and 13.7 reveal a very useful property of transformers, called *impedance transformation*. Figure 13.3 shows an ideal transformer. It has N_1 and N_2 turns in its primary and secondary windings. A load impedance Z_L is connected across its secondary, and an equivalent impedance Z_{eq} is defined at its primary.

The equivalent impedance Z_{eq} as faced by a source V_1 is given as

$$Z_{eq} = \frac{V_1}{I_1} = \frac{V_1 \times (V_2 I_2)}{I_1 \times (V_2 I_2)} = \left(\frac{V_1}{V_2}\right) \times \left(\frac{I_2}{I_1}\right) \times \left(\frac{V_2}{I_2}\right) = \left(\frac{1}{K}\right) \times \left(\frac{1}{K}\right) \times Z_L$$

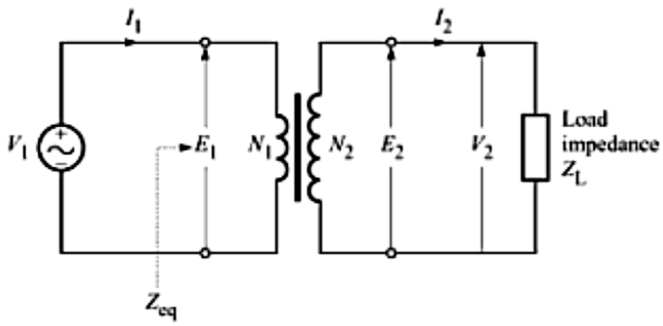


Fig. 13.3 *The transformer changes the impedance Z_L to equivalent impedance Z_{eq}.*

or $$Z_{eq} = Z_L/K^2 \qquad (13.8)$$

Therefore, the impedance is transformed in inverse proportion to the *square* of the turns-ratio. The concept of impedance transformation is used for *impedance matching*. As per maximum power transfer theorem, the load impedance has to be properly matched with the source impedance, as illustrated in Example 13.3 given below.

❖ Transformer at No-Load

In actual practice, a transformer can never satisfy any of the conditions specified above for the ideal transformer. Nevertheless, the concept of ideal transformer is helpful to understand the working of an actual transformer. We shall consider these conditions one by one, and see in what way a practical transformer deviates from the ideal transformer. In this Section, we shall consider only the first two ideality conditions. The remaining two conditions shall be considered in Section 13.7.

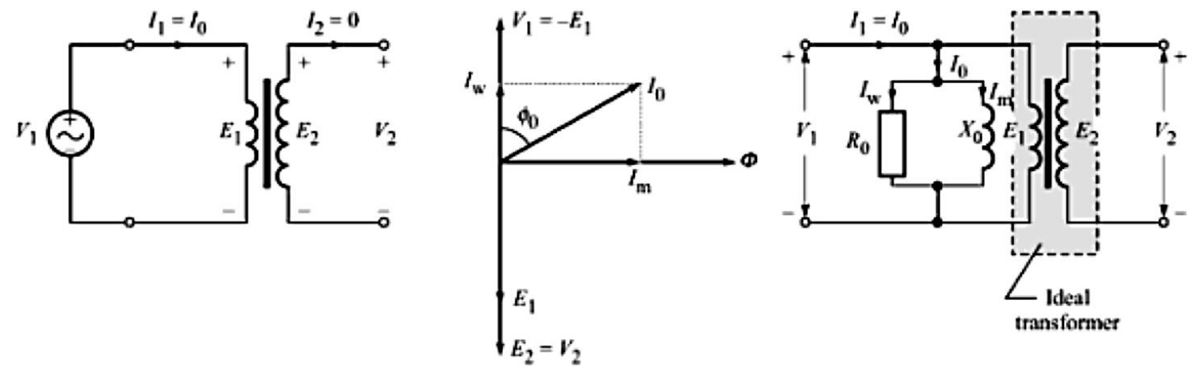
Consider a transformer with its primary connected to an alternating voltage source V_1, and no load connected across its secondary (Fig. 13.5*a*). With no closed circuit, the current in the secondary winding is zero. If the transformer were truly ideal, the primary current too would be zero, as per Eq. 13.7. But, in practice there does flow a little ***no-load current*** I_0 in the primary. This current I_0 is also called the ***exciting current*** of the transformer. Following are the two reasons why the no-load current I_0 flows in the primary.

(1) Effect of Magnetisation

Consider the *first ideality condition*. No magnetic material can have infinite permeability so as to offer zero reluctance to the magnetic circuit. Hence, in a practical transformer a finite mmf is needed to establish magnetic flux in the core. As a result, an in-phase ***magnetising current*** I_m in the primary is needed to set up flux Φ in the core. The current I_m is purely reactive and lags the voltage V_1 by 90°. This effect is modelled by an inductive reactance X_0 in parallel with the ideal transformer, as shown in the equivalent circuit of Fig. 13.5*c*.

The flux Φ induces emfs E_1 and E_2 in the primary and the secondary windings. As per Eq. 13.2, both these emfs lag flux Φ by 90°, as shown in the phasor diagram of Fig. 13.5*b*.

As the current I_2 in the secondary is zero (no load connected), the voltage drop in the secondary winding is zero. Hence, $V_2 = E_2$. The induced emf E_1 counter balances the applied voltage V_1 and establishes an electrical equilibrium. If the third and fourth ideality conditions (i.e., the effect of the resistance of the winding and the leakage of flux) are ignored, the magnitude of V_1 will be the same as that of emf E_1. Thus, $V_1 = -E_1$.



(*a*) The circuit. (*b*) The phasor diagram. (*c*) The equivalent circuit.

Fig. 13.5 *Transformer on no load.*

(2) Effect of Core Losses

Let us now consider the *second ideality condition*. There exist two reasons (*hysteresis* and *eddy current*) for the energy loss in the core of the transformer. The source must supply enough power to the primary to meet the core losses. These losses are proportional to the square of the core flux. Since the core flux is proportional to the applied voltage V_1, the iron loss can be represented by a resistance R_0 in parallel with the ideal transformer, as shown in the equivalent circuit of Fig. 13.5*c*. The ***core-loss current*** I_w flowing through R_0 is in phase with the applied voltage V_1, as shown in the phasor diagram of Fig. 13.5*b*.

Thus, we find that the no-load current I_0 has two components, I_m and I_w. The magnetising current I_m lags voltage V_1 by 90° and the loss component I_w is in phase with voltage V_1. The angle ϕ_0 is the *no-load phase angle*. Thus, from the phasor diagram of Fig. 13.5*b*, we have

$$I_0 = \sqrt{I_w^2 + I_m^2}\,; \qquad \phi_0 = \tan^{-1}(I_m/I_w); \qquad \text{and} \qquad \text{Input power} = V_1 I_w = V_1 I_0 \cos\phi_0$$

In the equivalent circuit shown in Fig. 13.5*c*, the no-load current I_0 is divided into two parallel branches. The component I_w flowing through resistance R_0 accounts for the core loss, and the component I_m flowing through reactance X_0 represents magnetising current. The R_0-X_0 parallel circuit is called ***exciting circuit*** of the transformer.

Since the core losses occur in the iron core, these are also called ***iron losses.*** These losses have two components: (*i*) hysteresis loss, and (*ii*) eddy-current loss.

(i) Hysteresis Loss When alternating current flows through the windings, the core material undergoes cyclic process of magnetisation and demagnetisation. It is found that there is a tendency of the flux density *B* to *lag behind* the field strength *H*. This tendency is called *hysteresis**. The effect of this phenomenon on the core material can be best understood from the *B*-*H* plot shown in Fig. 13.6.

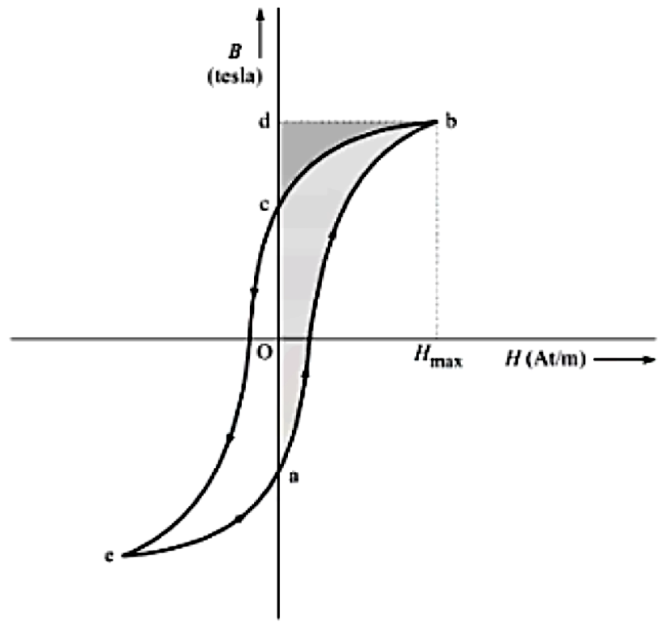


Fig. 13.6 *Hysteresis loop and energy relationship per half-cycle.*

During positive half-cycle, when H increases from zero to its positive maximum value, the energy is stored in the core. This energy is given by the area abda. However, when H decreases from its positive maximum value to zero, the energy is released which is given by the area bdcb. The difference between these two energies is the net loss and is dissipated as heat in the core. Thus, as H varies over one complete cycle, the total energy loss (per cubic metre) is represented by the area abcea of the hysteresis loop. The hysteresis loss (usually expressed in watts) is given as

$$P_h = K_h B_m^n f V \tag{13.9}$$

where K_h = hysteresis coefficient whose value depends upon the material
($K_h = 0.025$ for cast steel, $K_h = 0.001$ for silicon steel)
B_m = maximum flux density (in tesla)
n = a constant, $1.5 \leq n \leq 2.5$ depending upon the material
f = frequency (in hertz)
V = volume of the core material (in m^3)

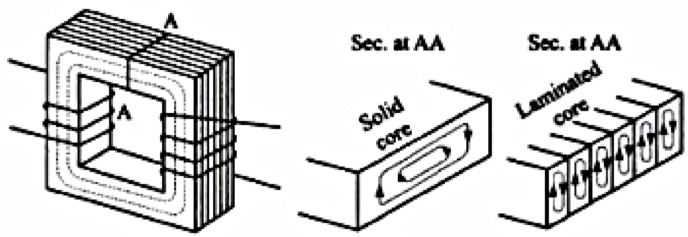
This loss can be minimized by selecting suitable ferromagnetic material for the core.

(ii) Eddy-Current Loss Ferromagnetic materials are also good conductors. A solid core made from such a material constitutes single short-circuited turns throughout its entire length. Eddy currents therefore circulate within the core in a plane normal to the alternating magnetic flux (shown with dotted lines in Fig. 13.7*a*). These currents (shown in Fig. 13.7*b*) may be quite high since the resistance of the iron is quite low. This results in unnecessary heating of the core and loss of power. The eddy-current loss (in watts) is given by

$$P_e = K_e B_m^2 f^2 t^2 V \tag{13.10}$$

where K_e = a constant dependent upon the material
t = thickness of laminations (in metre)

The eddy-current loss can be minimized by dividing the solid iron core into thin sheets or *laminations* insulated from one another (Fig. 13.7*c*). The path of the induced eddy currents in the core is broken by the insulating material between the sheets. The eddy currents and hence the eddy-current loss is thus substantially reduced.



(*a*) (*b*) (*c*)

Fig. 13.7 *Laminated core helps in reducing the eddy currents.*

Note that the eddy-current loss varies as the square of the frequency, whereas the hysteresis loss varies directly with the frequency. The total iron loss is given as

$$P_i = P_h + P_e \tag{13.11}$$

❖ Losses in transformer

(i) Core Losses or Iron Losses

Eddy current loss and hysteresis loss depend upon the magnetic properties of the material used for the construction of core. Hence these losses are also known as **core losses** or **iron losses**.

- **Hysteresis loss in transformer**: Hysteresis loss is due to reversal of magnetization in the transformer core. This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. It can be given by, Steinmetz formula:

  $W_h = \eta B_{max}^{1.6} f V$ (watts)

  where, η = Steinmetz hysteresis constant

  V = volume of the core in m^3

- **Eddy current loss in transformer**: In transformer, AC current is supplied to the primary winding which sets up alternating magnetizing flux. When this flux links with secondary winding, it produces induced emf in it. But some part of this flux also gets linked with other conducting parts like steel core or iron body or the transformer, which will result in induced emf in those parts, causing small circulating current in them. This current is called as eddy current. Due to these eddy currents, some energy will be dissipated in the form of heat.

(ii) Copper Loss in Transformer

Copper loss is due to ohmic resistance of the transformer windings. Copper loss for the primary winding is $I_1^2R_1$ and for secondary winding is $I_2^2R_2$. Where, I_1 and I_2 are current in primary and secondary winding respectively, R_1 and R_2 are the resistances of primary and secondary winding respectively. It is clear that Cu loss is proportional to square of the current, and current depends on the load. Hence copper loss in transformer varies with the load.

❖ Transformer on Load

Let us examine what happens when a load is connected to the secondary of the transformer. Note that for simplicity we are still considering a partially ideal transformer (i.e., a transformer satisfying only the *ideality conditions* (*iii*) and (*iv*) stated on page 368). Before connecting the load, there exists a flux Φ in the core due to the no-load current I_0 flowing in the primary. On connecting the load, a current I_2 flows through the secondary, as shown in Fig. 13.10. The magnitude and phase of I_2 with respect to the secondary voltage V_2 depends upon the nature of the load.

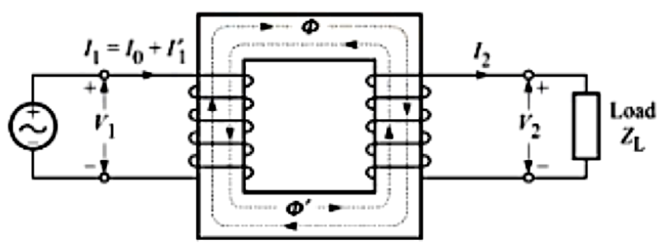


Fig. 13.10 *Transformer on load.*

The current I_2 sets up a flux Φ in the core, which opposes the main flux Φ. This momentarily weakens the main flux, and the primary back emf E_1 gets reduced. As a result, the difference $V_1 - E_1$ increases and more current is drawn from the supply. This again increases the back emf E_1, so as to balance the applied voltage V_1. In this process, the primary current increases by I'_1. This current is known as ***primary balancing current***, or ***load component of primary current***. Under such a condition, the secondary ampere-turns must be counterbalanced by the primary ampere-turns. That is, $N_1 I'_1 = N_2 I_2$. Hence, we have

$$I'_1 = \left(\frac{N_2}{N_1}\right) I_2 = K I_2 \tag{13.12}$$

The total primary current I_1 is the phasor sum of the no-load current I_0 and the primary balancing current I'_1. That is,

$$\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}'_1 \tag{13.13}$$

❖ Transformer Testing

Open and short circuit tests are performed on a transformer to determine the:

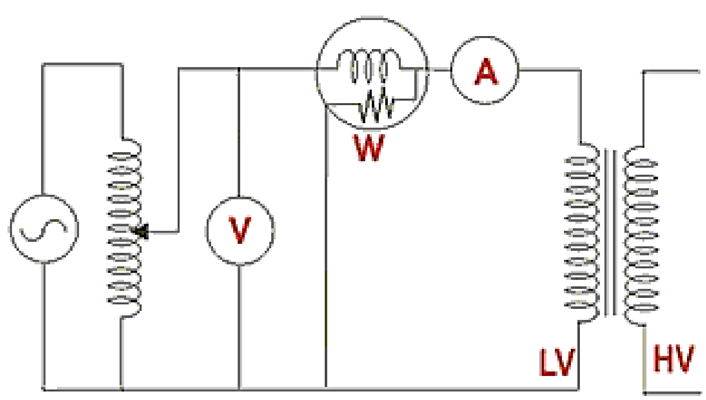
1. Equivalent circuit of transformer
2. Voltage regulation of transformer
3. Efficiency of transformer

The power required for **open circuit tests and short circuit tests on a transformer** is equal to the power loss occurring in the transformer.

➢ Open Circuit Test on Transformer

The connection diagram for **open circuit test on transformer** is shown in the figure. A voltmeter, wattmeter, and an ammeter are connected in LV side of the transformer as shown. The voltage at rated frequency is applied to that LV side with the help of a variac of variable ratio auto transformer.

The HV side of the transformer is kept open. Now with the help of variac, applied voltage gets slowly increased until the voltmeter gives reading equal to the rated voltage of the LV side. After reaching rated LV side voltage, we record all the three instruments reading (Voltmeter, Ammeter and Wattmeter readings).



Open Circuit Test on Transformer

The ammeter reading gives the no load current I_O. As no load current I_O is quite small compared to rated current of the transformer, the voltage drops due to this current that can be taken as negligible.

Since voltmeter reading V_1 can be considered equal to the secondary induced voltage of the transformer, wattmeter reading indicates the input power during the test. As the transformer is open circuited, there is no output, hence the input power here consists of core losses in transformer and copper loss in transformer during no load condition. But as said earlier, the no-load current in the transformer is quite small compared to the full load current so, we can neglect the copper loss due to the no-load current. Hence, can take the wattmeter reading as equal to the core losses in the transformer.

Let us consider wattmeter reading is P_o.

$$P_o = \frac{V_1^2}{R_m}$$

Where, R_m is shunt branch resistance of transformer.

If, Z_m is shunt branch impedance of transformer.

$$Then,\ Z_m = \frac{V_1}{I_e}$$

Therefore, if shunt branch reactance of transformer is X_m,

$$Then,\ \left(\frac{1}{X_m}\right)^2 = \left(\frac{1}{Z_m}\right)^2 - \left(\frac{1}{R_m}\right)^2$$

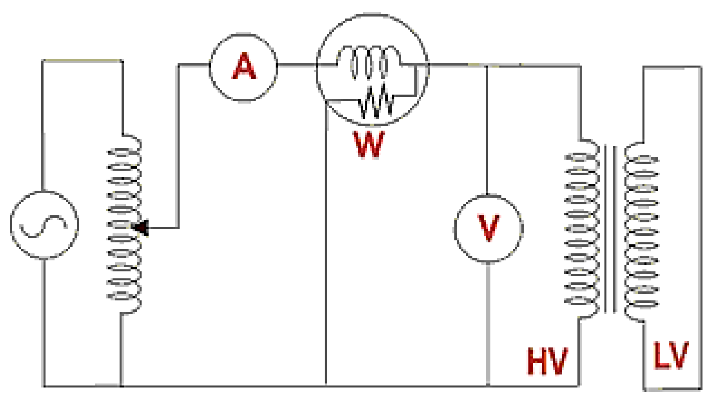
These values are referred to the LV side of the transformer due to the tests being conducted on the LV side of transformer. These values could easily be referred to HV side by multiplying these values with square of transformation ratio.

Therefore, it is seen that the **open circuit test on transformer** is used to determine core losses in transformer and parameters of the shunt branch of the equivalent circuit of the transformer.

- **Short Circuit Test on Transformer**

The connection diagram for **short circuit test on transformer** is shown in the figure. A voltmeter, wattmeter, and an ammeter are connected in HV side of the transformer as shown. The voltage at rated frequency is applied to that HV side with the help of a variac of variable ratio auto transformer. We short-circuit the LV side of the transformer. Now with the help of variac applied voltage is slowly increased until the wattmeter, and an ammeter gives reading equal to the rated current of the HV side.

After reaching rated current of HV side, we record all the three instruments reading (Voltmeter, Ammeter and Watt-meter readings). The ammeter reading gives the primary equivalent of full load current I_L. As the voltage applied for full load current in short circuit test on transformer is quite small compared to the rated primary voltage of the transformer, the core losses in transformer can be taken as negligible here.



Short Circuit Test on Transformer

Let's say, voltmeter reading is V_{sc}. The watt-meter reading indicates the input power during the test. As we have short-circuited the transformer, there is no output; hence the input power here consists of copper losses in the transformer. Since the applied voltage V_{sc} is short circuit voltage in the transformer and hence it is quite small compared to the rated voltage, so, we can neglect the core loss due to the small applied voltage. Hence the wattmeter reading can be taken as equal to copper losses in the transformer. Let us consider wattmeter reading is P_{sc}.

$$P_{sc} = R_e I_L^2$$

Where, R_e is equivalent resistance of transformer.

If, Z_e is equivalent impedance of transformer.

$$Then,\ Z_e = \frac{V_{sc}}{I_L}$$

Therefore, if equivalent reactance of transformer is X_e.

$$Then,\ X_e^2 = Z_e^2 - R_e^2$$

These values are referred to the HV side of the transformer as the test is conducted on the HV side of the transformer. These values could easily be converted to the LV side by dividing these values with the square of transformation ratio.

Hence the **short-circuit test of a transformer** is used to determine copper losses in the transformer at full load. It is also used to obtain the parameters to approximate the equivalent circuit of a transformer.