TUTORIAL:UNIT-III

1. Examine the following sets of vectors for linear dependence/independence

   a) $\{(2,1,0),(3,1,-1),(0,-1,1)\}$ in $\mathbb{R}^3$

   b) $\{(1,1,-1,1),(1,-1,2,-1),(3,1,0,1)\}$ in $\mathbb{R}^4$

2. Check that the set $S=\{(1,2,-1),(1,-1,2),(2,-1,1)\}$ is a basis of vector space $\mathbb{R}^3(\mathbb{R})$ or not?

3. Show that the set $S=\{-4+x+3x^2, 6+5x+2x^2, 8+4x+x^2\}$ is basis of vector space P_2

4. Let $W=\{(x,y)\in\mathbb{R}^2 / ax+by=0\}$ Show that W is subspace of $\mathbb{R}^2$

5. Let $W=\{(x,y,z)\in\mathbb{R}^3 : ax+by+cz=0\}$. Show that W is subspace of $\mathbb{R}^3$.

6. Determine whether the set W given below $W=\left\{\begin{bmatrix} x & y \\ 0 & x \end{bmatrix} \middle| x,y\in\mathbb{R}\right\}$ is a vector subspace of M_{22} where M_{22} represents the vector space of matrices of order 2.

7. Let $S=\{(x,y,z)\in\mathbb{R}^3 / ax=by=cz\}$ Show that S is subspace of $\mathbb{R}^3$

8. Find the orthogonal projection of v onto the subspace W spanned by the vectors u_i.

   a) $v=(3,1,-2), u_1=(1,1,1), u_2=(1,-1,0)$

   b) $v=(1,2,3), u_1=(2,-2,1), u_2=(-1,1,4)$

9. Let W be the plane in $\mathbb{R}^3$ with equation $x-y+2z=0$ and let $v=\begin{bmatrix}3\\-1\\2\end{bmatrix}$. Find the orthogonal projection of v onto W and the component of v orthogonal to W.

10. Find the range and kernel of $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x-y+z, y-z, 2x-5y+5z)$.

11. Find the kernel and range of T where T:$M_{22}\to M_{22}$ is defined by: $T\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a+b & 0 \\ 0 & c+d \end{pmatrix}$

12. Find the range and kernel of $T: R^3\to R^3$, defined by $T(x,y,z)=(x+z, x+y+2z, 2x+y+3z)$.

13. Find the range and kernel of $T:\mathbb{R}^2\to\mathbb{R}^3$ defined by $T(x,y)=(x,x+y,y)$.

14. Let $T: R^2\to R^2$ be defined by $T(x,y)=(2x-3y, 3x-4y)$. Show that T is invertible and find T^{-1}.

15. Show that there is a unique linear map $T: R^2 \to R^2$ for which $T(1,2) = (2,3)$ and $T(0,1) = (1,4)$. Find a formula for $T(x,y)$. Is T invertible?

16. Show that T: $R^2 \to R^2$, defined by T(x,y)=(2x+y,3x-5y) is invertible. Also find out T^{-1}

17. Find QR factorization of

a) $A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$,

b) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and

c) $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

18. Express each of the following transformations

$(x_1, x_2) = T(y_1, y_2) = (2y_1 - 3y_2, 4y_1 + y_2)$

$(y_1, y_2) = T(z_1, z_2) = (z_1 - 2z_2, 2z_1 + 3z_2)$

in the matrix form and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2.

19. Find a linear transformation $Y = AX$ which carries $X_1 = (2,2)'$ and $X_1 = (4,-1)'$ to $X_1 = (3,2)'$ and $X_1 = (2,3)'$ respectively.

20. Let S = {(1, 1, 0, -1), (1, 2, 1, 3), (1, 1, -9, 2), (16, -13, 1, 3)} consisting of the vectors of R^4.

a) Prove that S is an orthogonal set of R^4. Hence Show that it is basis of R^4.
b) Find the coordinates of an arbitrary vector $(3, 8, 1, 9)$ in R^4 relative to the basis S.