Principle Component Analysis (PCA)

Construct a sample Covariance matrix S for the data given below

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 2 & 4 & 5 & 1 \end{pmatrix}$$

Find the eigen vector of S that points the most significant direction of the data

PCA

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 2 & 4 & 5 & 1 \end{pmatrix}$$

Avg of row 1 = 2

Avg of row 2 = 3.

Centred matrix $A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 1 & 2 & -2 \end{pmatrix}$

Covariance matrix $S = \frac{AA^T}{n-1} = \frac{AA^T}{3}$ (n = no. of columns)

$$AA^T = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 1 & 2 & -2 \end{pmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 10 \end{bmatrix}$$

$$S = \frac{AA^T}{3} = \begin{pmatrix} 2/3 & 2/3 \\ 2/3 & 10/3 \end{pmatrix}.$$

Eigen values $\begin{pmatrix} 2/3 - \lambda & 2/3 \\ 2/3 & \frac{10}{3} - \lambda \end{pmatrix} = \left(\frac{2}{3} - \lambda\right)\left(\frac{10}{3} - \lambda\right) - \frac{4}{9}.$

$$= \lambda^2 - 4\lambda + 1.77 = 0.$$

$\lambda = 3.49, 0.506.$

To find eigen vector corresponding to $\lambda = 3.49$.

$$-2.82x + 0.666y = 0.$$

$$x = \frac{0.666}{2.82} y.$$

For $y = 1$, $x = 0.236$.

normalising. (0.236, 1) (dividing by 1.0274)

we get the direction as. (0.2297, 0.9733)

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PCA

$$\begin{pmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \end{pmatrix}$$

Avg of row 1 = 11

Avg of row 2 = 6.25

Centred matrix $A = \begin{pmatrix} 2 & -2 & -4 & 4 \\ 1.75 & 0.75 & 2.25 & -0.25 \end{pmatrix}$

Covariance matrix $S = \frac{AA^T}{n-1} = \frac{AA^T}{3}$.

$$AA^T = \begin{pmatrix} 2 & -2 & -4 & 4 \\ 1.75 & 0.75 & 2.25 & -0.25 \end{pmatrix} \begin{bmatrix} 2 & 1.75 \\ -2 & 0.75 \\ -4 & 2.25 \\ 4 & -0.25 \end{bmatrix}$$

$$S = \frac{AA^T}{3} = \frac{1}{3}\begin{pmatrix} 40 & -8 \\ -8 & 8.75 \end{pmatrix} = \begin{pmatrix} 13.33 & -2.66 \\ -2.66 & 2.916 \end{pmatrix}$$

Eigen values of S

$$\begin{pmatrix} 13.33-\lambda & -2.66 \\ -2.66 & 2.916-\lambda \end{pmatrix} = (13.33-\lambda)(2.916-\lambda) - 7.0756.$$

$$= 38.870 - 16.246\lambda + \lambda^2 - 7.0756$$

$$= \lambda^2 - 16.246\lambda + 31.7944$$

$$\lambda = 13.97, \ 2.275$$

Eigen vector corresponding to larger eigen value $\lambda = 13.97$

$$-0.64x - 2.66y = 0.$$

$$x = -\frac{2.66}{0.64}y.$$

For $y = 1$, $x = -4.156$

normalising $(-4.156, 1)$ (dividing by 4.2746)

we get the direction as $(-0.97, 0.23)$