THE F-TEST OR THE VARIANCE RATIO TEST

The F-test is named in honour of the great statistician R.A. Fisher. The object of the F-test is to find out whether the two independent estimates of population variance differ significantly, or whether the two samples may be regarded as drawn from the normal populations having the same variance. For carrying out the test of significance, we calculate the ratio F. F is defined as :

$$F = \frac{S_1^2}{S_2^2}, \text{ where } S_1^2 = \frac{(X_1 - \overline{X}_1)^2}{n_1 - 1}$$

$$\text{and} \qquad S_2^2 = \frac{\Sigma (X_2 - \overline{X}_2)^2}{n_2 - 1}$$

It should be noted that S_1^2 is always the larger estimate of variance, *i.e.*, $S_1^2 > S_2^2$

$$F = \frac{\text{Larger estimate of variance}}{\text{Smaller estimate of variance}}$$

$$\upsilon_1 = n_1 - 1 \text{ and } \upsilon_2 = n_2 - 1$$

υ_1 = degrees of fredom for sample having larger variance.

υ_2 = degrees of freedom for sample having smaller variance.

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Null hypothesis, $H_0 : \mu_1 = \mu_2$, *i.e.,* the average weekly food expenditures of the two populations of shoppers are equal.

Alternative Hypothesis, $H_1 : \mu_1 \neq \mu_2$. (Two-tailed)

Test Statistic. Since samples are large, under H_0, the test statistic is :

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}} \sim N(0, 1)$$

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$\sqrt{n}$ $\sqrt{n}$ ...(14·9)

14·8·3. Test of Significance for Single Mean. We have proved that if x_i, $(i = 1, 2, \ldots, n)$ is a random sample of size n from a normal population with mean μ and variance σ^2, then the sample mean is distributed normally with mean μ and variance σ^2/n, *i.e.*, $\bar{x} \sim N(\mu, \sigma^2/n)$. However, this result holds, *i.e.*, $\bar{x} \sim N(\mu, \sigma^2/n)$, even in random sampling from non-normal population provided the sample size n is large [*c.f.* Central Limit Theorem]. Thus for large samples, the *standard normal variate* corresponding to $\bar{x}$ is :

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Under the *null hypothesis* H_0, that the sample has been drawn from a population with mean μ and variance σ^2, *i.e.*, there is no significant difference between the sample mean ($\bar{x}$) and population mean (μ), the test statistic (for large samples), is :

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1) \qquad \ldots(14{\cdot}9a)$$

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$\sqrt{n}$

Tests of Significance of Small Samples :

1. To test the significance of the mean of a random sample

$$t = \frac{(\bar{X} - \mu)}{S} \sqrt{n}$$

where $\nu = n - 1$

2. To test the significance of the difference of the means of two samples

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

where $\nu = n_1 + n_2 = 2$

3. To test the difference of the means of two samples which are not independent.

$$t = \frac{\bar{d}\sqrt{n}}{S}$$

where $\nu = n - 1$

4. To test the significance of an observed correlation coefficient

$$t = \frac{r}{\sqrt{1 - r^2}} \times \sqrt{n - 2}$$

where $\nu = n - 2$

The Variance Ratio Test—F-Test :

$$F = \frac{S_1^2}{S_2^2}$$

where $S_1^2 = \dfrac{\Sigma(X_1 - \bar{X}_1)^2}{n_1 - 1}$

and $\quad S_2^2 = \dfrac{\Sigma(X_1 - \bar{X}_1)^2}{n_2 - 1}$

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LIST OF FORMULAE

Partial Correlation Coefficients	$r_{12\cdot3} = \dfrac{r_{12} - r_{13}\, r_{23}}{\sqrt{1 - r^2_{13}}\,\sqrt{1 - r^2_{23}}}$ $r_{13\cdot2} = \dfrac{r_{13} - r_{12}\, r_{23}}{\sqrt{1 - r^2_{12}}\,\sqrt{1 - r^2_{23}}}$ $r_{23\cdot1} = \dfrac{r_{23} - r_{12}\, r_{13}}{\sqrt{1 - r^2_{12}}\,\sqrt{1 - r^2_{13}}}$
Multiple Correlation Coefficient	$R_{1\cdot23} = \sqrt{\dfrac{r^2_{12} + r^2_{13} - 2\, r_{13}\, r_{12}\, r_{23}}{1 - r^2_{23}}}$
Multiple Regression of X_1 on X_2 and X_3	$X_{1\cdot23} = a_{1\cdot23} + b_{12\cdot3}\, X_2 + b_{13\cdot2}\, X_3$ $b_{12\cdot3} = \dfrac{\sigma_1}{\sigma_2} \times \dfrac{r_{12} - r_{13}\, r_{23}}{1 - r^2_{23}}$
Partial Regression Coefficient	$b_{13\cdot2} = \dfrac{\sigma_1}{\sigma_2} \times \dfrac{r_{13} - r_{12}\, r_{23}}{1 - r^2_{23}}$

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