UNIT - III

Q1.

a. $\{(2,1,0);(3,1,-1);(0,-1,1)\}$ in R^3

$a(2,1,0)+b(3,1,-1)+c(0,-1,1) = 0$

$2a+3b=0$

$a+b-c=0$

$-b+c=0$

$$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$\therefore a = b = c = 0$

$\therefore$ *Linearly Independent*

b. $\{(1,1,-1,1);(1,-1,2,-1);(3,1,0,1)\}$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

$a(1,1,-1,1)+b(1,-1,2,-1)+c(3,1,0,1)=0$

$a+b+3c=0$

$a-b+c=0$

$-a+2b=0$

$a-b+c=0$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$a+b+3c=0 \Rightarrow a=-4b$

$-2b-2c=0 \Rightarrow b=c$

$-a+2b=0$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

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$R_3 \to 2R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \\ 0 & 0 & -8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$\therefore a = b = c = 0$

$\therefore$ Linearly Independent.

Q2. $S = \{(1, 2, -1), (1, -1, 2), (2, -1, 1)\}$

$a + b + 2c = 0$; $2a - b - c = 0$; $-a + 2b + c = 0$

$\therefore a = b = c = 0$ $\therefore$ Linearly Independent.

Let $x, y, z \in R^3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & -1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 3 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x \\ y - 2x \\ z + x \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & -6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x \\ y - 2x \\ z - 3y + 7x \end{bmatrix}$$

$$a+b+2c = x$$
$$b+3c = y-2x$$
$$-6c = z-3y+7x$$
$$\therefore \boxed{c = \frac{-7x+3y-z}{6}}$$

$$b + \frac{-7x+3y-z}{2} = y-2x$$
$$b = \frac{2y-4x+7x-3y+z}{2}$$
$$\therefore \boxed{b = \frac{3x-y+z}{2}}$$

$$a + \frac{3x-y+z}{2} + \frac{-7x+3y-z}{6} = x$$
$$a + \frac{9x-3y+3z-7x+3y+z}{6} = x$$
$$a + \frac{2x+4z}{6} = x$$
$$\therefore \boxed{a = \frac{4x-4z}{6}}$$

Q3. $\{-4+x+3x^2 ; 6+5x-2x^2 ; 8+4x+x^2\}$

$-4K_1 + K_2 x + K_3 3x^2 = 0$

$6K_1 + K_2 5x + K_3 2x^2 = 0$

$8K_1 + K_2 4x + K_3 x^2 = 0$

$$\begin{bmatrix} -4 & 1 & 3 \\ 6 & 5 & 2 \\ 8 & 4 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = 0$$

$\therefore K_1 = K_2 = K_3 = 0$ $\quad\therefore$ Linearly Independent

$a(-4+x+3x^2) + b(6+5x+2x^2) + c(8+4x+x^2) = b_0 + b_1 x + b_2 x^2$

$$\begin{bmatrix} -4 & 6 & 8 \\ 1 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ x^2 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 5 & 4 \\ -4 & 6 & 8 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ x^2 \end{bmatrix}$$

$R_2 \to R_2 + 4R_1 ; R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 26 & 24 \\ 0 & -13 & 11 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ x-4 \\ x^2-3 \end{bmatrix}$$

$R_3 \to 2R_3 + R_2$

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 26 & 24 \\ 0 & 0 & 46 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ x-4 \\ 2x^2+x-10 \end{bmatrix}$$

$a+6b-14c = 1$

$26b+24c = x-4$

$46c = 2x^2+x-10$

$$\therefore \boxed{c = \frac{2x^2+x-10}{46}}$$

$$26b + \overset{12}{\cancel{24}}\left(\frac{2x^2+x-10}{\underset{23}{\cancel{46}}}\right) = x-4$$

$$26b = \frac{23x-92-24x^2-12x+120}{23}$$

$$26b = \frac{-24x^2+11x+28}{23}$$

$$\therefore \boxed{b = \frac{-24x^2+11x+28}{598}}$$

$$a + \frac{-120x^2+55x+140}{598} + \frac{8x^2+4x-40}{46} = 1$$

$$a + \frac{-120x^2+55x+140+104x^2+52x-520}{598} = 1$$

$$a + \frac{-16x^2+107x-380}{598} = 1$$

$$\boxed{a = \frac{16x^2-107x+978}{598}}$$

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Q4. $W = \{(x, y) \in R^2 / ax + by = 0\}$

$Dim^n = 2$

Bases $= \{(a, 0); (0, b)\}$
$x = 1; y = 0 \quad x = 0; y = 1$

Q5. $W = \{(x, y, z) \in R^3 / ax + by + cz = 0\}$

$Dim^n = 3$

Bases $= \{(a, 0, 0); (0, b, 0); (0, 0, c)\}$
$x = 1; y = z = 0 \quad x = 0; y = 1; z = 0 \quad x = y = 0; z = 1$

Q6 $W = \left\{ \begin{bmatrix} x & y \\ 0 & x \end{bmatrix} \middle| x, y \in R \right\}$

$Dim = 2$

Bases $= \left\{ \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix}; \begin{bmatrix} 0 & y \\ 0 & 0 \end{bmatrix} \right\}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Q7 $S = \{(x,y,z) \in R^3 \mid ax = by = cz\}$

$Dim^n = 1$

Bases = $\{(0,0,0)\}$

$x = 1; y = z = 0$

Q10 $T: R^3 \to R^3$ $T(x,y,z) = \{x-y+z;\ y-z;\ 2x-5y+5z\}$

$x - y + z = 0$

$y - z = 0 \quad \Rightarrow y = z$

$2x - 5y + 5z = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 2 & -5 & 5 \end{bmatrix}$$

$x - y + z = 0$

$y - z = 0$

$\Rightarrow \boxed{y = z}$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & -3 & 3 \end{bmatrix}$$

$x - z + z = 0$

$\Rightarrow x = 0$

Rank = 2 Nullity = 1

$R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Kernel = $\{0, z, z\}$

Range = $\{r, s, 2r-3s \mid r, s \in R\}$

Q12. $T(x,y,z) = (x-y+z,\ y-z,\ 2x-5y+5z)$

$x-y+z=0$
$y-z=0$

Q12 $T(x,y,z) = (x+z,\ x+y+2z,\ 2x+y+3z)$

$x+z=0$
$x+y+2z=0$
$2x+y+3z=0$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 - R_1;\ R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x+z=0$
$y+z=0$

$\therefore y=-z,\ x=-z$

Range $=2$ Nullity $=1$

Kernel $=\{-z,-z,z\}$

Range $=\{(r,s,r+s) \mid r,s \in R\}$

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Q13 $T: R^2 \to R^3$ $\quad T(x,y) = (x, x+y, y)$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$x = 0 \; ; \; y = 0$

Kernel $= \{0,0\}$; Rank $= 2$; Nullity $= 0$ Range $= 2$

Q14. $T: R^2 \to R^2$ $\quad T(x,y) = (2x-3y, 3x-4y)$

$T(1,0) = (2,3)$
$T(0,1) = (-3,-4)$

$A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ $\quad |A| = 1$

$\because |A| \neq 0 \;\therefore$ Invertible

Let,

$2x - 3y = u$ $\qquad$ And $3x - 4y = v$

$\boxed{x = \frac{u+3y}{2}}$ $\qquad \frac{3u+9y}{2} - 4y = v$

$4y = \frac{3u+9y-2v}{2}$

$\therefore \boxed{x = \frac{12v-16u}{4}}$

$4y = \frac{3u-2v}{2} + \frac{9y}{2}$

$-\frac{y}{2} = \frac{3u-2v}{2}$

$\therefore \boxed{y = \frac{4v-6u}{2}}$

$T^{-1}(u,v) = \left(\frac{12v-16u}{4}, \frac{4v-6u}{2} \right)$

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Q15 $T: R^2 \to R^2$ $T(1,2) = (2,3)$ & $T(0,1) = (1,4)$

Let, $(x,y) = a(1,2) + b(0,1)$
$(x,y) = (a, 2a+b)$

$\therefore a = x$ $2a+b = y$
$\therefore b = y - 2x$

$T(x,y) = x\,T(1,2) + (y-2x)\,T(0,1)$
$= (x)(2,3) + (y-2x)(1,4)$
$= (2x + y - 2x,\ 3x + 4y - 8x)$
$= (y,\ 4y - 5x)$

$T(x,y) = 0$
$y = 0$ $(0, -5)$ $y = 1$ $(1,4)$
$x = 0$

$A = \begin{bmatrix} 0 & -5 \\ 1 & 4 \end{bmatrix}$ $|A| = 5$

$\therefore |A| \neq 0 \therefore$ Invertible

Q16. $T: R^2 \to R^2$ $T(x,y) = (2x+y,\ 3x-5y)$

$T(1,0) = (2,3)$

$T(0,1) = (1,-5)$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$$

$|A| = -13$ $\because |A| \neq 0$ $\therefore$ Invertible

Let,

$2x + y = u$ And $3x - 5y = v$

$x = \dfrac{u-y}{2}$

$\dfrac{3u-3v}{2} - 5y = v$

$5y = \dfrac{3u-5v}{2}$

$\therefore \boxed{y = \dfrac{3u-5v}{10}}$

Let,

$2x+y = u$ And $3x-5y = v$

$x = \dfrac{u-y}{2}$

$\dfrac{3u-3y}{2} - 5y = v$

$\dfrac{3u-3y-10y}{2} = v$

$\dfrac{3u-13y}{2} = v$

$\boxed{y = \dfrac{3u-2v}{13}}$

$x = \dfrac{u - \frac{3u-2v}{13}}{2}$

$= \dfrac{10u+2v}{26}$

$\boxed{x = \dfrac{5u+v}{13}}$

Q11. $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & 0 \\ 0 & c+d \end{pmatrix}$

$$\begin{bmatrix} a+b & 0 \\ 0 & c+d \end{bmatrix} = 0$$

$a = -b \,;\, c = -d$

$\therefore$ Kernel $= \begin{bmatrix} -b & b \\ -d & d \end{bmatrix}$

Range $= 2$ Nullity $= 2$ Dimension $- 4$

Q12.

a.

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \\ \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \end{array}$$

$U_1 = V_1$

$= \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$U_2 = V_2 - \dfrac{\langle V_2, U_1 \rangle}{\|U_1\|^2} \cdot U_1$

$= \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} - \dfrac{2-1+1}{\sqrt{1+1+1+1}} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$

$$\cancel{q_3 = \frac{V_3}{\|V_3\|}} = \frac{1}{\ }$$

Q17.

a.

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \\ \begin{matrix} \underbrace{\ \ }_{x_1} & \underbrace{\ \ }_{x_2} & \underbrace{\ \ }_{x_3} \end{matrix} \end{array}$$

$$V_1 = x_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$V_2 = x_2 - \left(\frac{V_1 \cdot x_2}{V_1 \cdot V_1} \right) \cdot V_1$$

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} - \frac{2-1+1}{1+1+1+1} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3/2 \\ 3/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$V_3 = x_3 - \left(\frac{V_1 \cdot x_3}{V_1 \cdot V_1}\right) V_1 - \left(\frac{V_2' \cdot x_3}{V_2' \cdot V_2'}\right) V_2$$

$$= \begin{bmatrix} 2 \\ 2 \\ 1 \\ 2 \end{bmatrix} - \frac{2-2-1+2}{1+1+1+1} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} - \frac{6+6+1+2}{9+9+1+1} \begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ 1 \\ 2 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} - \frac{15}{20} \begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

$$Q = \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 3 & 0 \\ -1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$A = QR \Rightarrow Q^T A = R$

$R = \Big[$

$$q_1 = \frac{v_1}{\|v_1\|} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad q_2 = \frac{V_2}{\|V_2\|} = \frac{1}{2\sqrt{5}}\begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\sqrt{5}/20 \\ 3\sqrt{5}/20 \\ \sqrt{5}/20 \\ \sqrt{5}/20 \end{bmatrix}$$

$$q_3 = \frac{V_3}{\|V_3\|} = \frac{1}{\sqrt{6}}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\sqrt{6}/6 \\ 0 \\ \sqrt{6}/6 \\ 2\sqrt{6}/3 \end{bmatrix}$$

$$Q = [\,q_1 \;\; q_2 \;\; q_3\,]$$

$$= \begin{bmatrix} 1/2 & 3\sqrt{5}/20 & -\sqrt{6}/6 \\ -1/2 & 3\sqrt{5}/20 & 0 \\ -1/2 & \sqrt{5}/20 & \sqrt{6}/6 \\ 1/2 & \sqrt{5}/10 & \sqrt{6}/3 \end{bmatrix}$$

$$A = QR \Rightarrow Q^T A = Q^T Q R \Rightarrow Q^T A = R$$

$$R = \begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ 3\sqrt{5}/10 & 3\sqrt{5}/10 & \sqrt{5}/10 & \sqrt{5}/10 \\ -\sqrt{6}/6 & \cancel{\sqrt{6}/6}\;0 & \sqrt{6}/6 & \sqrt{6}/3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1/2 \\ 0 & \sqrt{5} & 3\sqrt{5}/2 \\ 0 & 0 & \sqrt{6}/2 \end{bmatrix}$$

b. $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\quad\ \ x_1 \quad x_2 \quad x_3$

$$V_1 = X_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$V_2 = X_2 - \frac{V_1 X_2}{V_1 \cdot V_1} \cdot X_1$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{1+1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$V_3 = X_3 - \frac{V_2 X_3}{V_2 \cdot V_2} \cdot V_2 - \frac{V_1 X_2}{V_1 \cdot V_1} \cdot V_1$$

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} - \frac{2}{1+1} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \frac{1}{1+1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 0 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

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$$q_1 = \frac{V_1}{\|V_1\|} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2\\0\\\sqrt{2}/2\end{bmatrix}$$

$$q_2 = \frac{V_2}{\|V_2\|} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2\\0\\-\sqrt{2}/2\end{bmatrix}$$

$$q_3 = \frac{V_3}{\|V_3\|} = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\2\\1\end{bmatrix} = \begin{bmatrix}\sqrt{6}/6\\\sqrt{6}/3\\\sqrt{6}/6\end{bmatrix}$$

$$Q = \begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2 & \sqrt{6}/6\\ 0 & 0 & \sqrt{6}/3\\ \sqrt{2}/2 & -\sqrt{2}/2 & \sqrt{6}/6\end{bmatrix}$$

$$A = QR \Rightarrow Q^T A = Q^T Q R \Rightarrow Q^T A = R$$

$$R = \begin{bmatrix}\sqrt{2}/2 & 0 & \sqrt{2}/2\\ \sqrt{2}/2 & 0 & -\sqrt{2}/2\\ \sqrt{6}/6 & -\sqrt{6}/3 & \sqrt{6}/6\end{bmatrix}\begin{bmatrix}1 & 1 & 2\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}\sqrt{2} & \sqrt{2}/2 & \sqrt{2}\\ 0 & \sqrt{2}/2 & \sqrt{2}\\ \sqrt{6}/3 & \sqrt{6}/6 & 0\end{bmatrix}$$

C. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\quad\;\; X_1 \quad X_2 \quad X_3$

$V_1 = X_1$

$\Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$$V_2 = X_2 - \frac{V_1 X_2}{V_1 \cdot V_1} \cdot V_1$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1+1+1}{1+1+1+1} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{3}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$V_3 = X_3 - \frac{V_1 X_3}{V_1 \cdot V_1} \cdot V_1 - \frac{V_2 X_3}{V_2 \cdot V_2} \cdot V_2$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1+1+1}{1+1+1+1} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1+1}{9+1+1+1} \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{3}{4}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/4 \\ -11/12 \\ 1/12 \\ 1/12 \end{bmatrix} = \begin{bmatrix} -3 \\ -11 \\ 1 \\ 1 \end{bmatrix}$$

$$q_1 = \frac{V_1}{\|V_1\|} = \frac{1}{\sqrt{1+1+1+1}}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix} = \cancel{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$q_2 = \frac{V_2}{\|V_2\|} = \frac{1}{\sqrt{9+1+1+1}}\begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{2\sqrt{3}}\begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{12}}{4} \\ \sqrt{12}/12 \\ \sqrt{12}/12 \\ \sqrt{12}/12 \end{bmatrix}$$

$$q_3 = \frac{V_3}{\|V_3\|} = \frac{1}{\sqrt{9+121+1+1}}\begin{bmatrix} -3 \\ -11 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{132}}\begin{bmatrix} -3 \\ -11 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{132}/44 \\ -\sqrt{132}/12 \\ \sqrt{132}/132 \\ \sqrt{132}/132 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & -\sqrt{12}/4 & -\sqrt{132}/44 \\ 1 & \sqrt{12}/12 & -\sqrt{132}/12 \\ 1 & \sqrt{12}/12 & \sqrt{132}/132 \\ 1 & \sqrt{12}/12 & \sqrt{132}/132 \end{bmatrix}$$

$A = QR$

$Q^T A = R$

$$R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\sqrt{13}/4 & \sqrt{13}/13 & \sqrt{13}/13 & \sqrt{13}/13 \\ \frac{-\sqrt{132}}{44} & \frac{\sqrt{132}}{12} & \frac{\sqrt{132}}{132} & \frac{\sqrt{132}}{132} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 4 & 3 & 2 \\ 0 & \sqrt{13}/4 & \sqrt{13}/6 \\ \frac{-4\sqrt{132}}{33} & \frac{-13\sqrt{132}}{132} & \frac{\sqrt{132}}{66} \end{bmatrix}$$

Q8. a. $V = (3, 1, -2)$ $\quad u_1 = (1, 1, 1)$ $\quad u_2 = (1, -1, 0)$

$$\text{Proj}(V) = \left(\frac{u_1 \cdot V}{u_1 \cdot u_1}\right) \cdot u_1 + \left(\frac{u_2 \cdot V}{u_2 \cdot u_2}\right) \cdot u_2$$

$$= \left(\frac{3+1-2}{1+1+1}\right)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \left(\frac{3-1}{1+1}\right)\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \left(\frac{2}{3}\right)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 \\ -1/3 \\ 2/3 \end{bmatrix}$$

b. $V=(1,2,3)$; $U_1=(2,-2,1)$; $U_2=(-1,1,4)$

$$Proj(V)=\left(\frac{U_1 V}{U_1\cdot U_1}\right)U_1+\left(\frac{U_2 V}{U_2\cdot U_2}\right)\cdot U_2$$

$$=\left(\frac{2-4+3}{4+4+1}\right)\begin{bmatrix}2\\-2\\1\end{bmatrix}+\left(\frac{-1+2+12}{1+1+16}\right)\begin{bmatrix}-1\\1\\4\end{bmatrix}$$

$$=\left(\frac{1}{9}\right)\begin{bmatrix}2\\-2\\1\end{bmatrix}+\left(\frac{13}{18}\right)\begin{bmatrix}-1\\1\\4\end{bmatrix}$$

$$=\begin{bmatrix}-1/2\\1/2\\3\end{bmatrix}$$

Q9. $x-y+2z=0$ $\quad V=\begin{bmatrix}3 & -1 & 2\end{bmatrix}$

$x=y-2z$

$$\begin{bmatrix}y-2z\\y\\z\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}y+\begin{bmatrix}-2\\0\\1\end{bmatrix}z$$

$U_1=\begin{bmatrix}1\\1\\0\end{bmatrix}$ $V=\begin{bmatrix}-2\\0\\1\end{bmatrix}$ are basis of W. But, they are not orthogonal so we ~~use~~ find Non-zero Vector in W,

Let, $W=\begin{bmatrix}x\\y\\z\end{bmatrix}$ is vector orthogonal to U_1

$\therefore U_1.W=0$ $\quad \therefore x+y=0$

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and $x - y + 2z = 0$

$\therefore x = -2$

$\therefore y = 2$

$$W = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = U_2 ,$$

$$\therefore U_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad U_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{Proj}(V) = \left(\frac{U_1 \cdot V}{U_1 \cdot U_1}\right) U_1 + \left(\frac{U_2 \cdot V}{U_2 \cdot U_2}\right) U_2$$

$$= \frac{2}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 1/3 \\ -2/3 \end{bmatrix}$$

~~Proj~~ (V).

Q18. $(x_1, x_2) = T(y_1, y_2) = (2y_1 - 3y_2, 4y_1 + y_2)$

$(y_1, y_2) = T(z_1, z_2) = (z_1 - 2z_2, 2z_1 + 3z_2)$

$(x_1, x_2) = T(y_1, y_2)$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$X = AY$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$Y = BAZ$

$X = ABZ$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 & -13 \\ 8 & -5 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$(x_1, x_2) = (-4z_1 - 13z_2, 8z_1 - 5z_2)$

Q19. $Y = AX$

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$3 = 2a + 2b$

$2 = 2c + 2d$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$2 = 4a - b$

$3 = 4c - d$

$a = 7/10 \quad b = 4/5 \quad c = 4/5 \quad d = 1/5$

$$A = \begin{bmatrix} 7/10 & 4/5 \\ 4/5 & 1/5 \end{bmatrix}$$

$Y = AX$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 7/10 & 4/5 \\ 4/5 & 1/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$