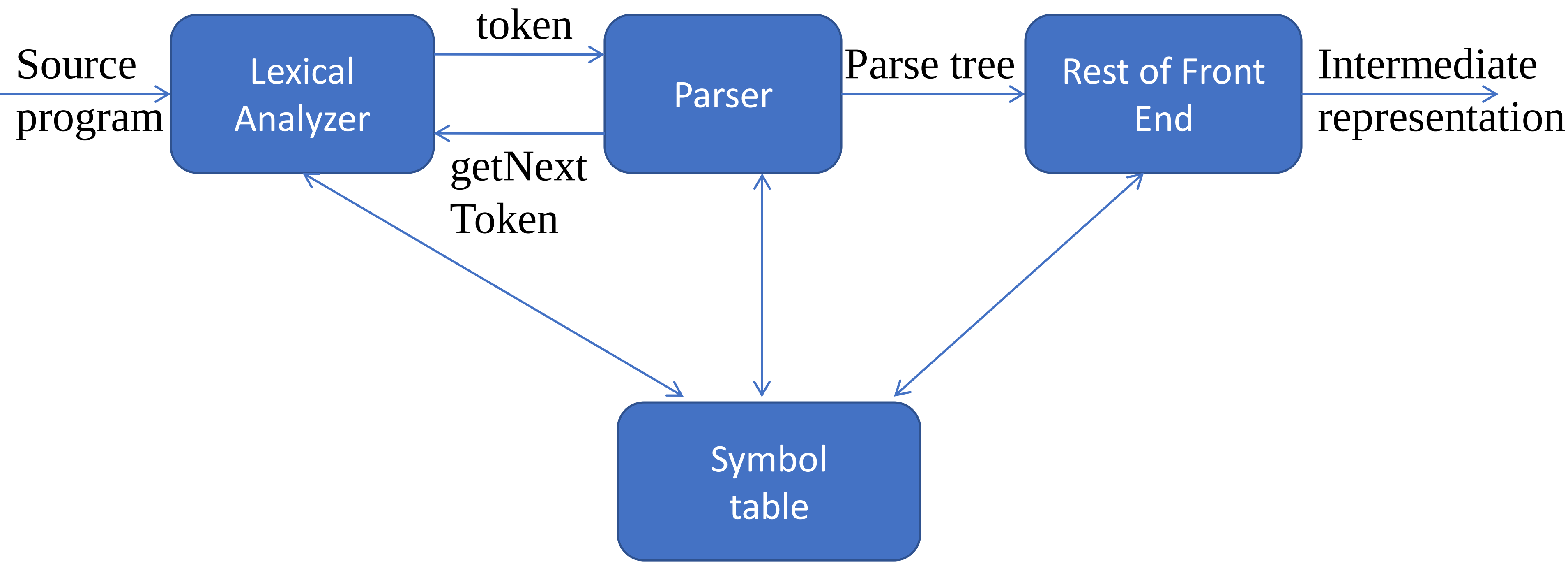
Compiler course

Syntax Analysis

Outline

- Role of parser
- Context free grammars
- Top down parsing
- Bottom up parsing
- Parser generators

The role of parser



SYNTAX ANALYSIS INTRODUCTION

- LEXICAL PHASE IS IMPLEMENTED ON FINITE AUTOMATA & FINITE AUTOMATA CAN REALLY ONLY EXPRESS THINGS WHERE YOU CAN COUNT MODULUS ON K.
- REGULAR LANGUAGES – THE WEAKEST FORMAL LANGUAGES WIDELY USED
- – MANY APPLICATIONS
- – CAN'T HANDLE ITERATION & NESTED LOOPS(NESTED IF ELSE).
- TO SUMMARIZE, THE LEXER TAKES A STRING OF CHARACTER AS INPUT AND PRODUCES A STRING
- OF TOKENS AS OUTPUT.
- THAT STRING OF TOKENS IS THE INPUT TO THE PARSER WHICH TAKES A STRING OF TOKENS AND PRODUCES A PARSE TREE OF THE PROGRAM.
- SOMETIMES THE PARSE TREE IS ONLY IMPLICIT. SO THE, A COMPILER MAY NEVER ACTUALLY BUILD THE FULL PARSE

Error handling

- Common programming errors
  - Lexical errors
  - Syntactic errors
  - Semantic errors
  - Lexical errors
- Error handler goals
  - Report the presence of errors clearly and accurately
  - Recover from each error quickly enough to detect subsequent errors
  - Add minimal overhead to the processing of correct progrms

Error-recover strategies

- Panic mode recovery
  - Discard input symbol one at a time until one of designated set of synchronization tokens is found
- Phrase level recovery
  - Replacing a prefix of remaining input by some string that allows the parser to continue
- Error productions
  - Augment the grammar with productions that generate the erroneous constructs
- Global correction
  - Choosing minimal sequence of changes to obtain a globally least-cost correction

Context free grammars

- Terminals
- Nonterminals
- Start symbol
- productions

expression -> expression + term
expression -> expression – term
expression -> term
term -> term * factor
term -> term / factor
term -> factor
factor -> (expression)
factor -> **id**

$$G=(\Sigma\ ,T,P,S)$$

Σ – IS A FINITE SET OF TERMINALS
T– IS A FINITE SET OF NON-TERMINALS P – IS A FINITE SUBSET OF PRODUCTION RULES
S – IS THE START SYMBOL

A context-free grammar has four components:

•A set of **non-terminals** (V). Non-terminals are syntactic variables that denote sets of strings. The non-terminals define sets of strings that help define the language generated by the grammar.

•A set of tokens, known as **terminal symbols** (Σ). Terminals are the basic symbols from which strings are formed.

•A set of **productions** (P). The productions of a grammar specify the manner in which the terminals and non-terminals can be combined to form strings. Each production consists of a **non-terminal** called the left side of the production, an arrow, and a sequence of tokens and/or **on- terminals**, called the right side of the production.

•One of the non-terminals is designated as the start symbol (S); from where the production begins.

The strings are derived from the start symbol by repeatedly replacing a non-terminal (initially the start symbol) by the right side of a production, for that non-terminal.

CONTEXT FREE GRAMMAR EXAMPLES

- ARITHMETIC EXPRESSIONS

E ::= T | E + T | E - T
T ::= F | T * F |T / F F
::= id | (E)

- STATEMENTS

If Statement ::= if *E* then *Statement* else *Statement*

Steps:

1.Begin with a string with only the start symbol S

2.Replace any non-terminal X in the string by the right-hand side of some production

X -> Y1…Yn

3. Repeat (2) until there are no non-terminals

Uses of grammars

E -> E + T | T

T -> T * F | F

F -> (E) | **id**

E -> TE’

E’ -> +TE’ | Ɛ

T -> FT’

T’ -> *FT’ | Ɛ

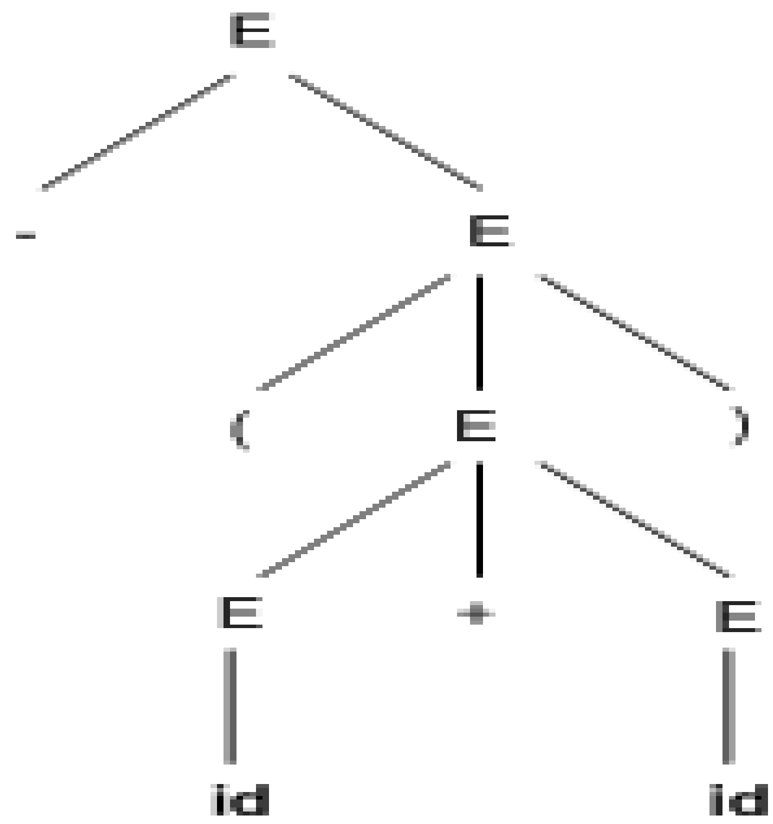
F -> (E) | **id**

Derivations

- Productions are treated as rewriting rules to generate a string
- Rightmost and leftmost derivations
  - E -> E + E | E * E | -E | (E) | **id**
  - Derivations for **–(id+id)**
    - E => -E => -(E) => -(E+E) => -(**id**+E)=>-(**id**+**id**)
- A derivation is basically a sequence of production rules, in order to get the input string. During parsing, we take two decisions for some sentential form of input:
- Deciding the non-terminal which is to be replaced.
- Deciding the production rule, by which, the non-terminal will be replaced.
- To decide which non-terminal to be replaced with production rule, we can have two options.

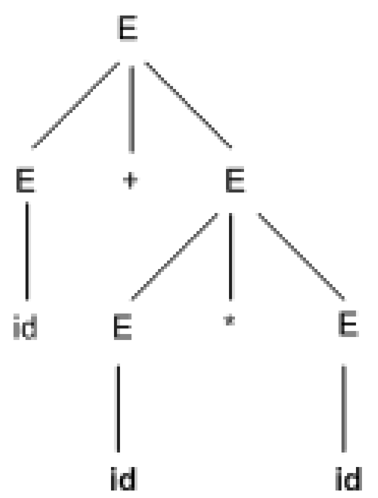
Parse trees
A parse tree is a graphical depiction of a derivation. It is convenient to see how strings are derived from the start symbol. The start symbol of the derivation becomes the root of the parse tree.

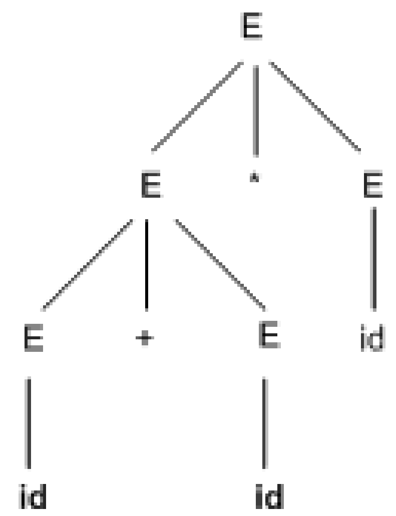
- -(**id**+**id**)
- E => -E => -(E) => -(E+E) => -(**id**+E)=>-(**id**+**id**)



Ambiguity

- For some strings there exist more than one parse tree
- Or more than one leftmost derivation
- Or more than one rightmost derivation
- Example: id+id*id





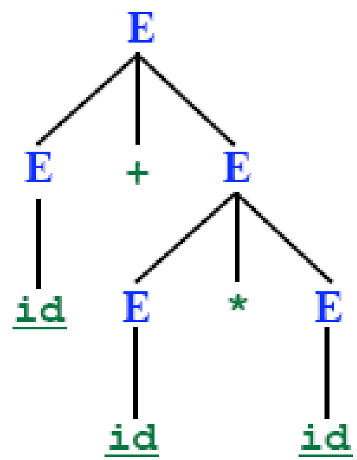
AMBIGUOUS GRAMMAR

1. E → E + E
2. → E * E
3. → (E)
4. → - E
5. → ID

Input: `id+id*id`

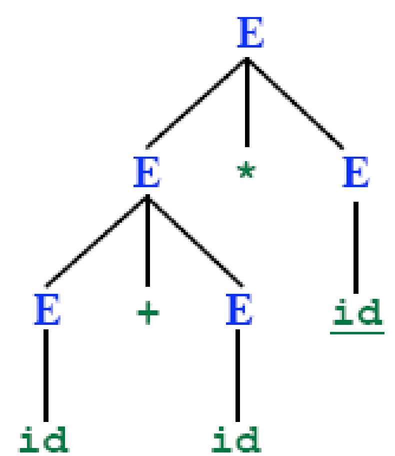
Leftmost Derivation #1

E
⇒ E+E
⇒ id+E
⇒ id+E*E
⇒ id+id*E
⇒ id+id*id



Leftmost Derivation #2

E
⇒ E*E
⇒ E+E*E
⇒ id+E*E
⇒ id+id*E
⇒ id+id*id



AMBIGUOUS GRAMMAR

- More than one Parse Tree for some sentence.
- The grammar for a programming language may be ambiguous
- Need to modify it for parsing.

- Also: Grammar may be left recursive.
- Need to modify it for parsing.

ELIMINATION OF AMBIGUITY

- Ambiguous

- A Grammar is ambiguous if there are multiple parse trees for the same sentence.

- Disambiguation

- Express Preference for one parse tree over others
  - Add disambiguating rule into the grammar

RESOLVING PROBLEMS: AMBIGUOUS **GRAMMARS**

Consider the following grammar segment:

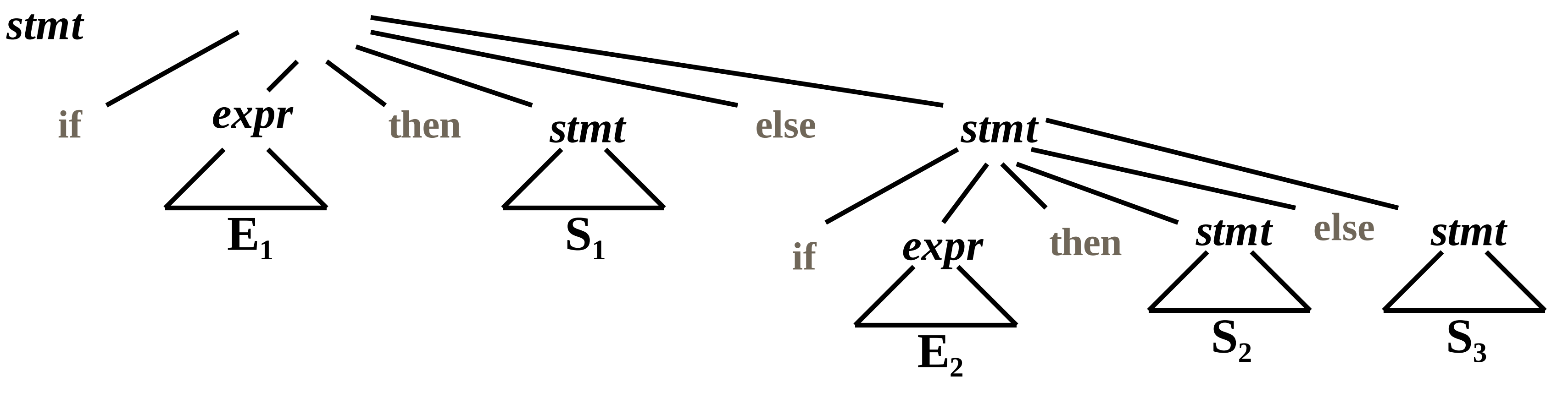
$$stmt \rightarrow \textbf{if } expr \textbf{ then } stmt$$

$$|\quad \textbf{if } expr \textbf{ then } stmt \textbf{ else } stmt$$

$$|\quad \textbf{other} \quad \textbf{(any other statement)}$$

If E1 then S1 else if E2 then S2 else S3

simple parse tree:



EXAMPLE : WHAT HAPPENS WITH THIS STRING?

If E_1 then if E_2 then S_1 else S_2

How is this parsed ?

if E_1 then
if E_2 then
S_1
else
S_2

vs.

if E_1 then
if E_2 then
S_1
else
S_2

$G = (V, T, P, S)$

$E \rightarrow E + E \mid E * E \mid id$

$V = \{E\}$

$T = \{+, *, id\}$.

$\underline{id + id * id}$

LMD

$E \Rightarrow E + E$

$\Rightarrow id + E$

$\Rightarrow id + E * E$

$\Rightarrow id + id * E$

$\Rightarrow id + id * id$

RMD:

$E \Rightarrow E + E$

$\Rightarrow E + E * E$

$\Rightarrow E + E *$

$\Rightarrow E + id *$

$\Rightarrow id + id *$

mD

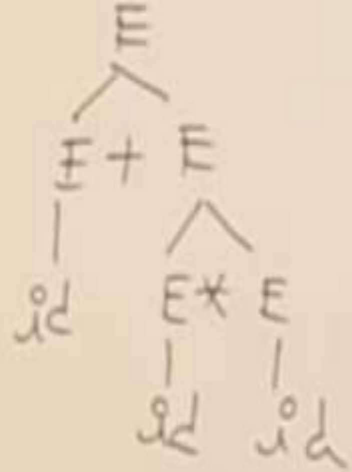
$E \Rightarrow E * E$

$\Rightarrow \underline{E + E} * E$

$\Rightarrow id + E * E$

$\Rightarrow id + id * E$

$\Rightarrow id + id * id$



RmD: $E \Rightarrow E * E$

$\Rightarrow E * id$

$\Rightarrow E + E * id$

$\Rightarrow E + id * id$

$\Rightarrow id + id * id$

D

$E \Rightarrow E * E$

$\Rightarrow \underline{E + E} * E$

$\Rightarrow id + E * E$

$\Rightarrow id + id * E$

$\Rightarrow id + id * id$

RmD: $E \Rightarrow E * E$

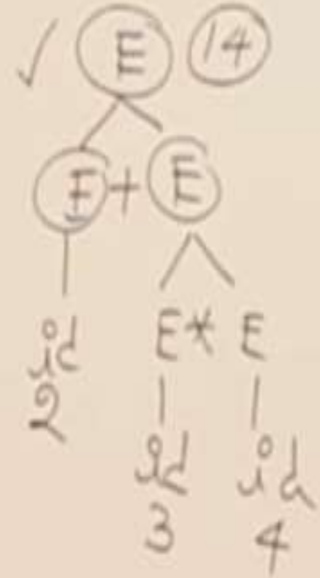
$\Rightarrow E * id$

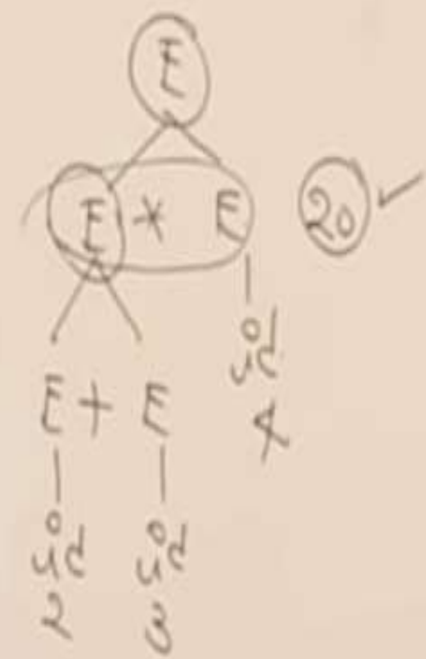
$\Rightarrow E + E * id$

$\Rightarrow E + id * id$

$\Rightarrow id + id * id$

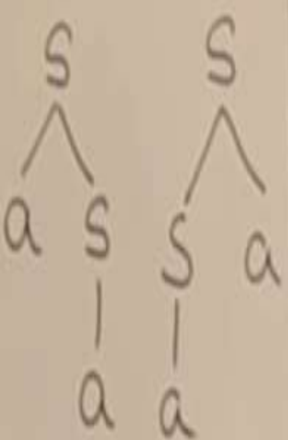
2+3*4





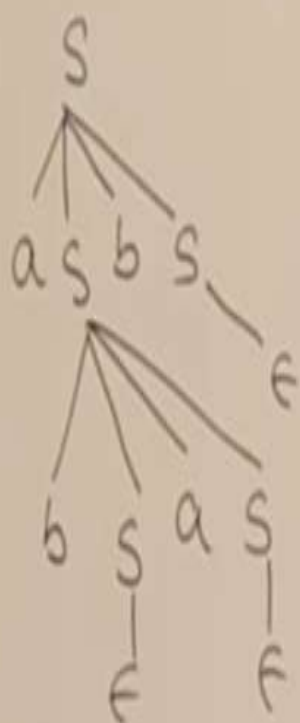
$\to aS/Sa/a$

$w = aa$



$S \to aSbS/bSaS/\epsilon$

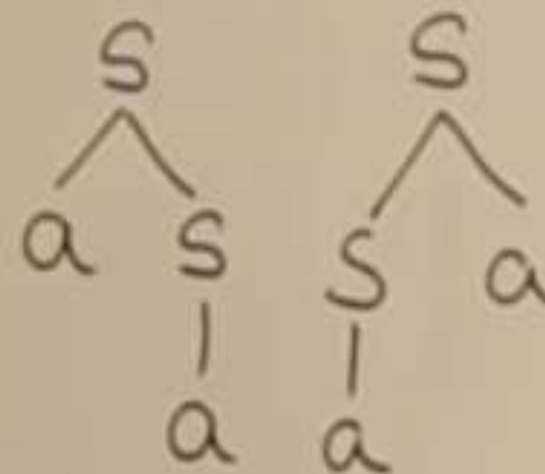
$w = abab$



$R \to R+R/RR/R^*/a/b/c$

✓ $S \to aS/Sa/a$

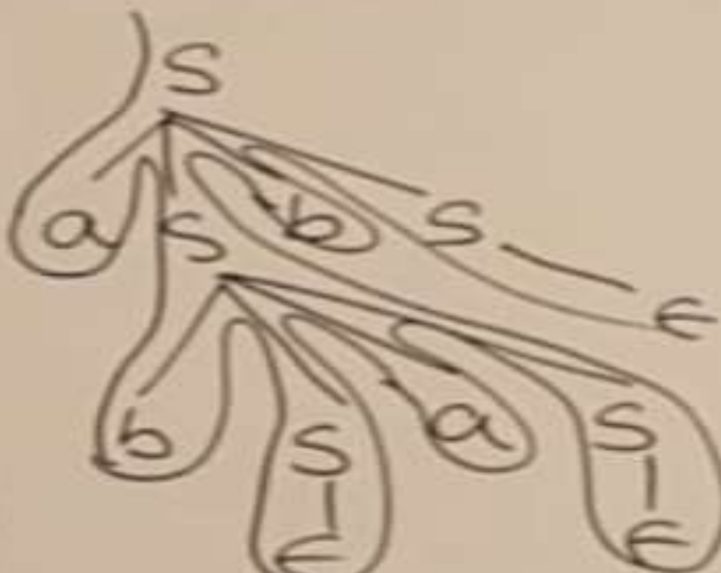
$w = aa$



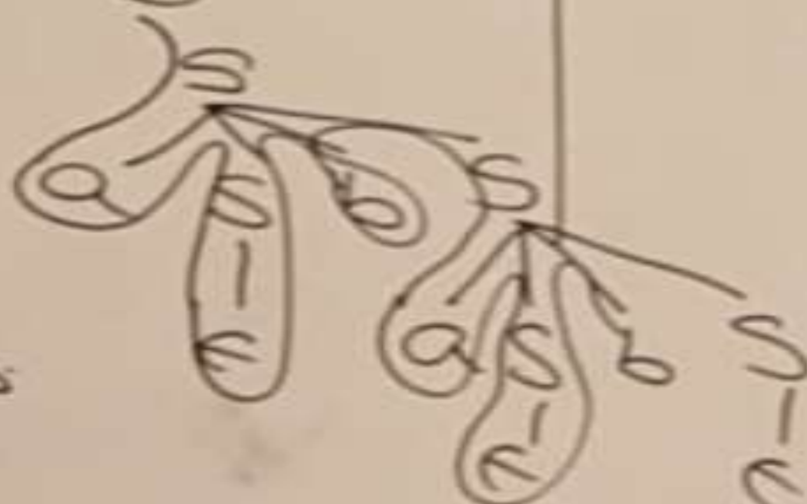
$S \rightarrow aSbS \,/\, bSaS \,/\, \epsilon$

$R \rightarrow R+R \,/\, RR \,/\, R^* \,/$

$\omega = abab$



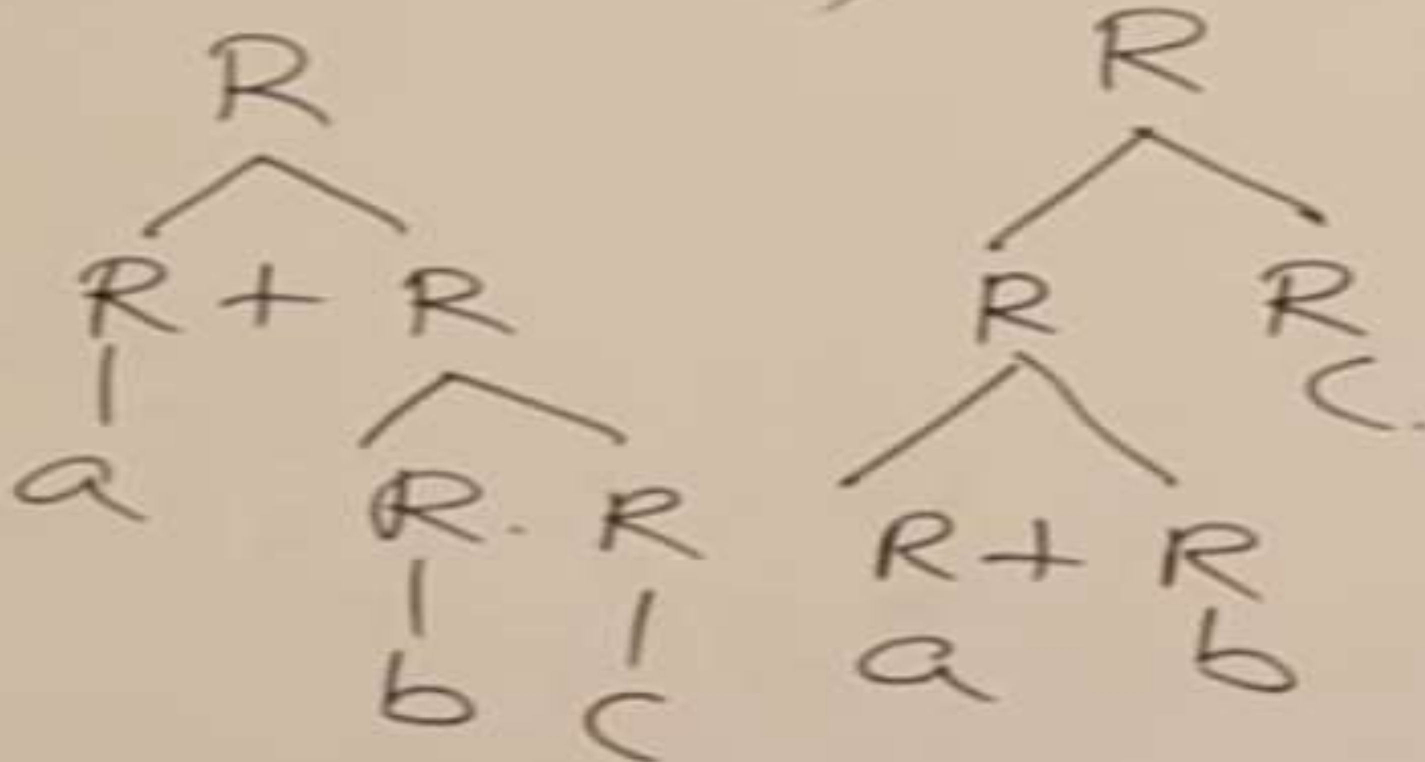
abab



abab

$R \to R+R \,/\, RR \,/\, R^* \,/\, a \,/\, b \,/\, c$

$a+(bc)$

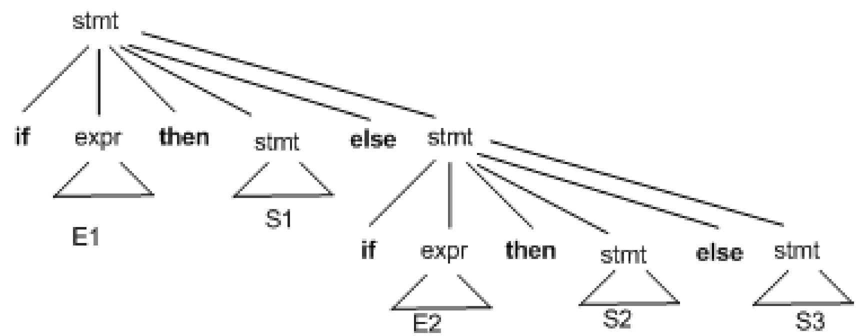


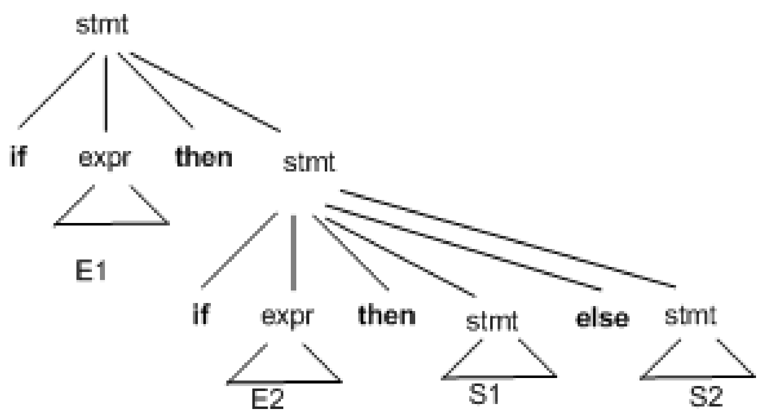
Elimination of ambiguity

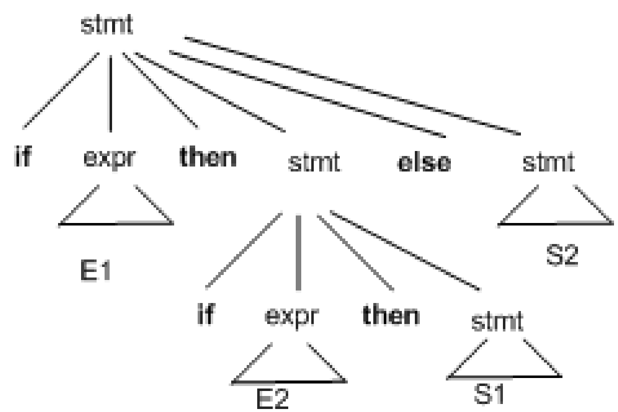
stmt → If expr **then** stmt

| If expr **then** stmt **else** stmt

| **other**







Elimination of ambiguity (cont.)

- Idea:
  - A statement appearing between a **then** and an **else** must be matched

stmt → matched_stmt
  | open_stmt

matched_stmt → **If** expr **then** matched_stmt **else** matched_stmt
  | **other**

open_stmt → **If** expr **then** stmt
  | **If** expr **then** matched_stmt **else** open_stmt

REMOVING AMBIGUITY

Take Original Grammar:

stmt → **if** ***expr*** **then** ***stmt***
| **if** ***expr*** **then** ***stmt*** **else** ***stmt***
| **other** **(any other statement)**

Rule: Match each **else** with the closest previous unmatched **then**.

Revise to remove ambiguity:

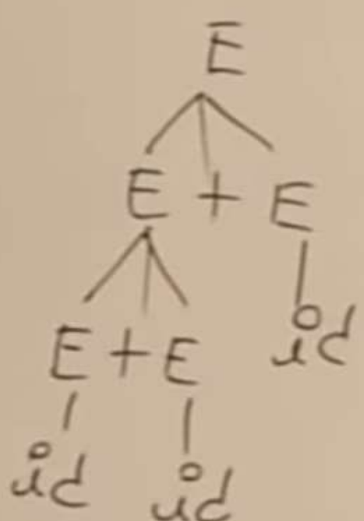
stmt → ***matched_stmt*** | ***unmatched_stmt***

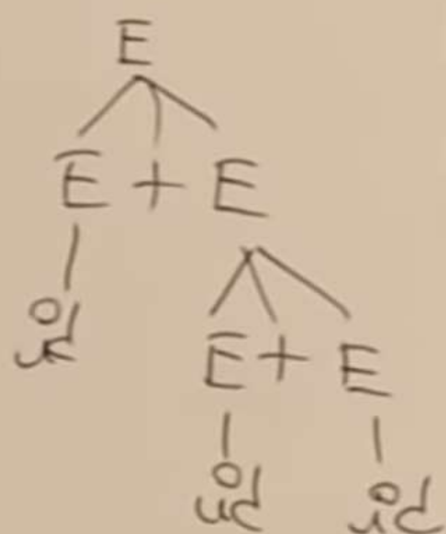
matched_stmt → **if** ***expr*** **then** ***matched_stmt*** **else** ***matched_stmt*** | **other**

unmatched_stmt → **if** ***expr*** **then** ***stmt***
| **if** ***expr*** **then** ***matched_stmt*** **else** ***unmatched_stmt***

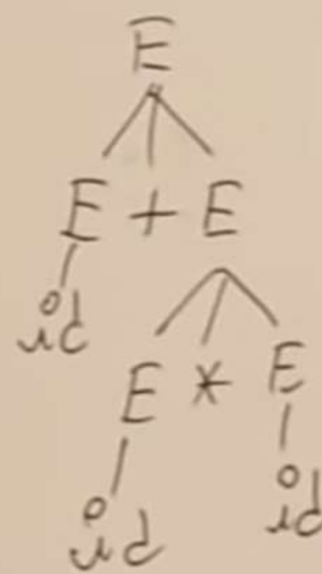
$$E \rightarrow E + E \mid E * E \mid id$$

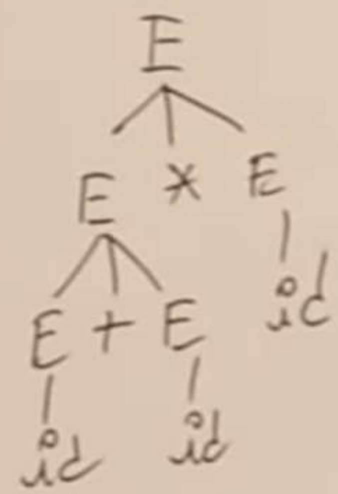
$\boxed{id + id + id}$

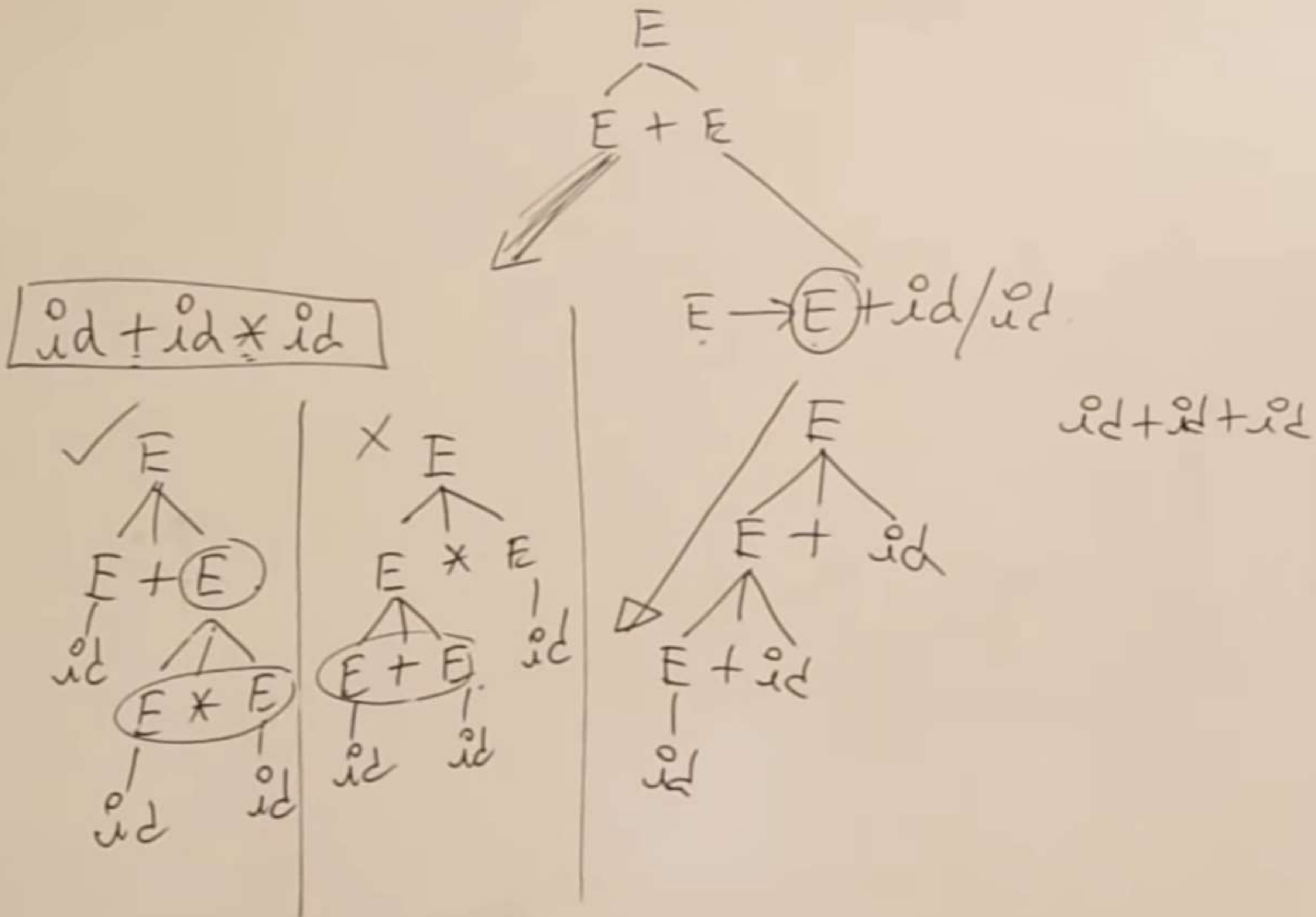


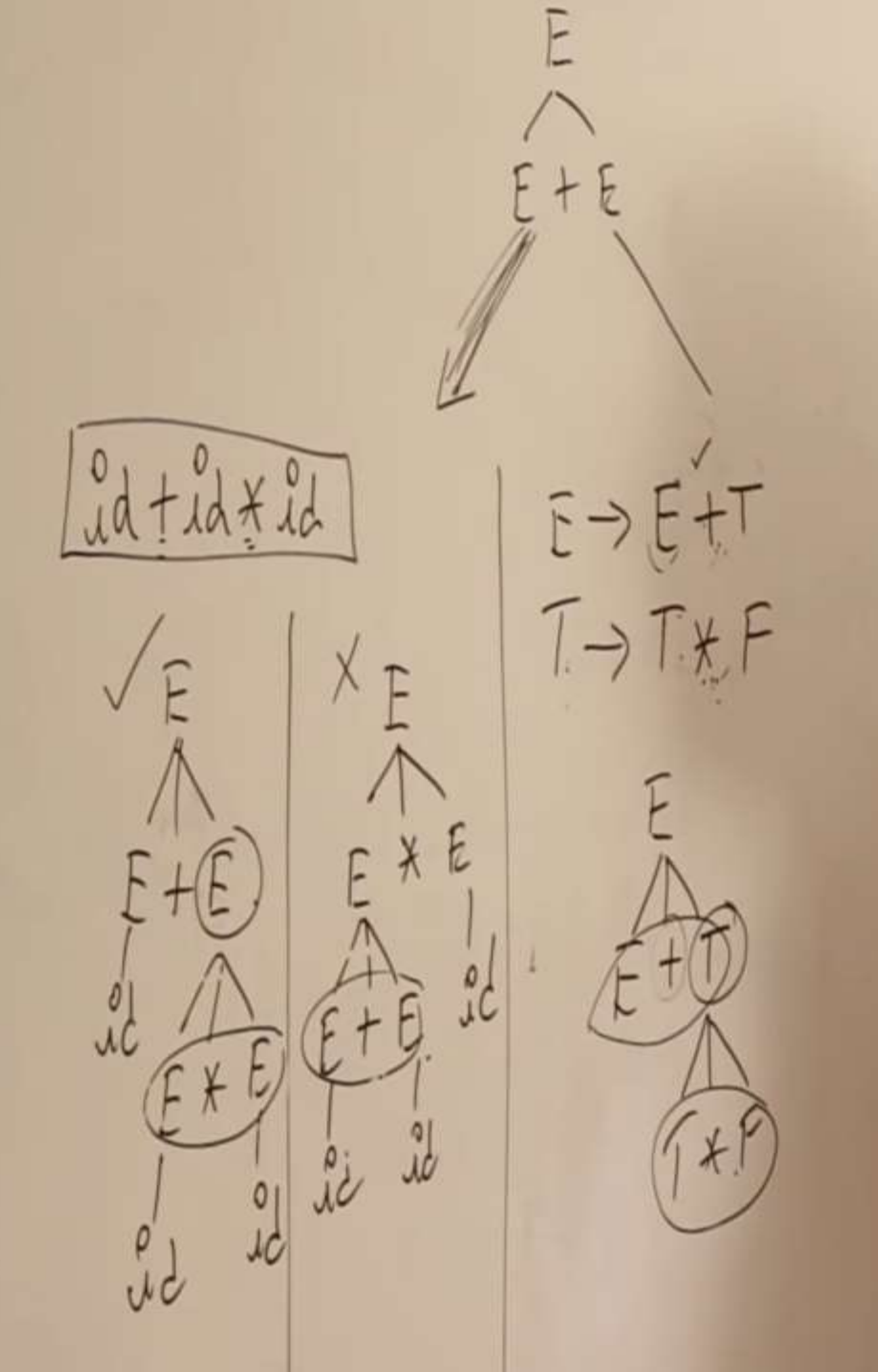


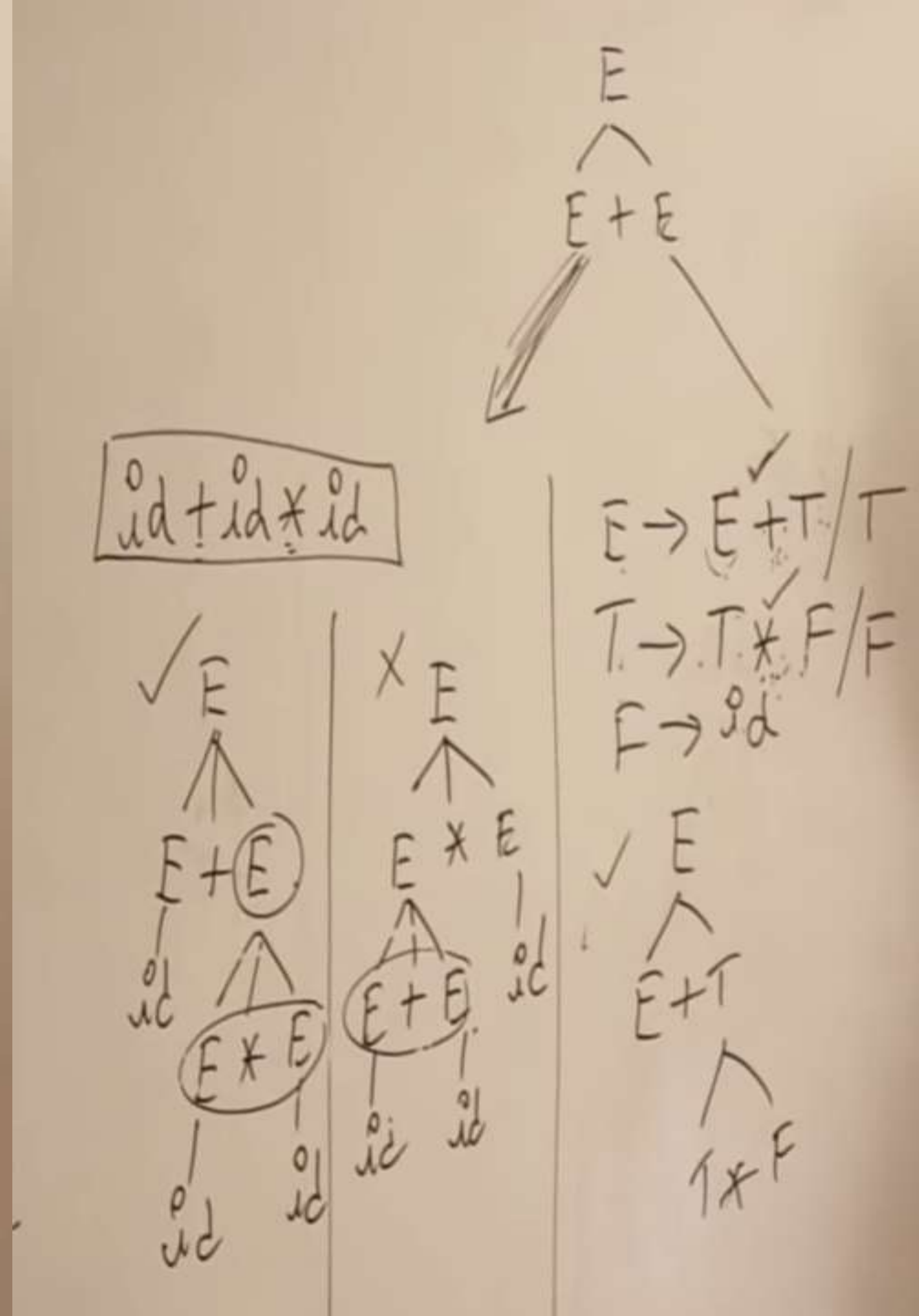
$\boxed{id + id * id}$

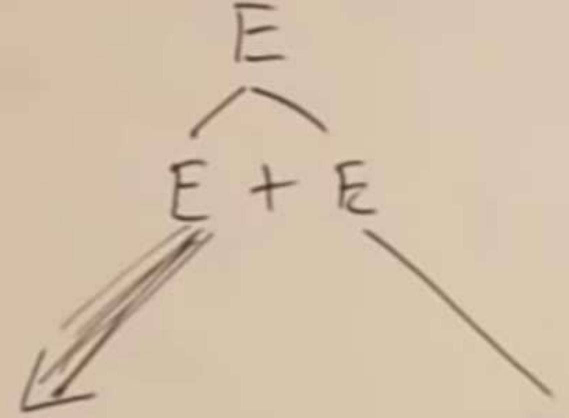




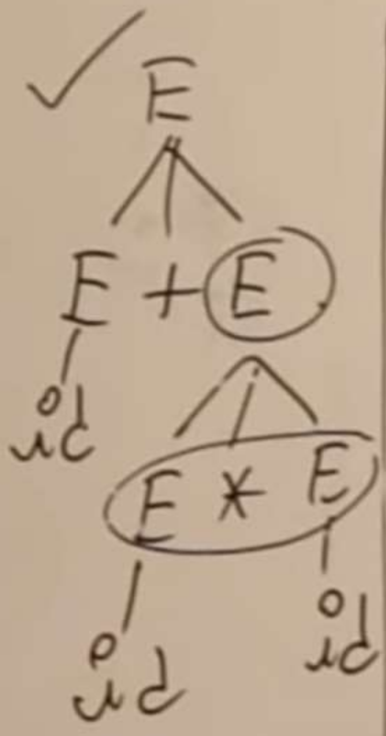

id+id*id
E → E+id/id
id+id+id

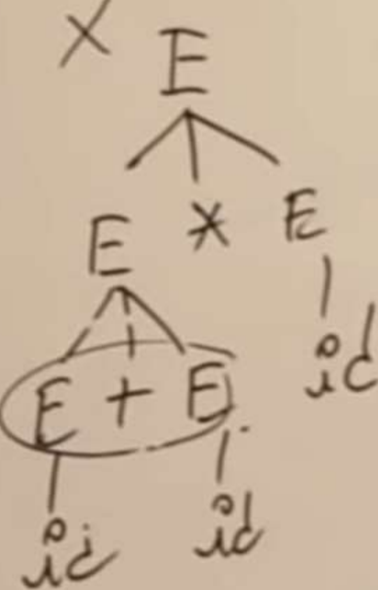

E
E + E
id + id * id
E → E + T
T → T * F
E
E + E
id
E * E
id
id
X
E
E * E
E + E
id
id
id
E
E + T
T * F


E
E + E
id + id * id
E → E + T / T
T → T * F / F
F → id
E
E + E
id
E * E
id
id
X
E
E * E
E + E
id
id
id
E
E + T
T * F



$\boxed{id + id * id}$





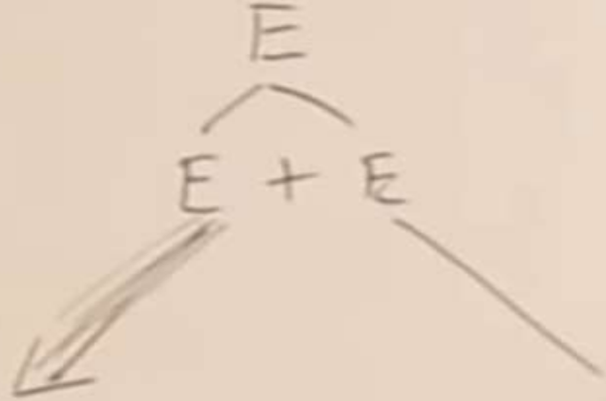
E → E + T / T

T → T * F / F

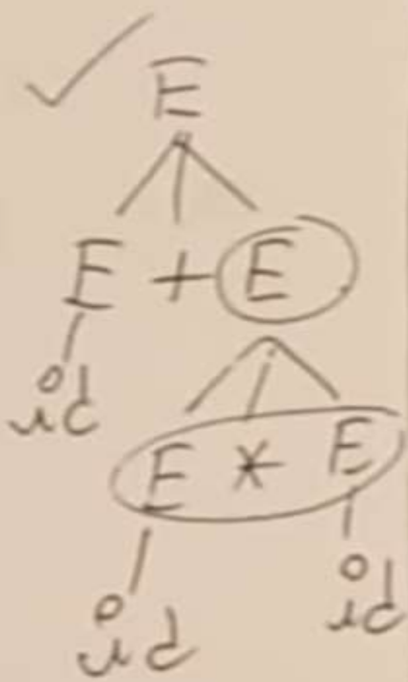
F → id

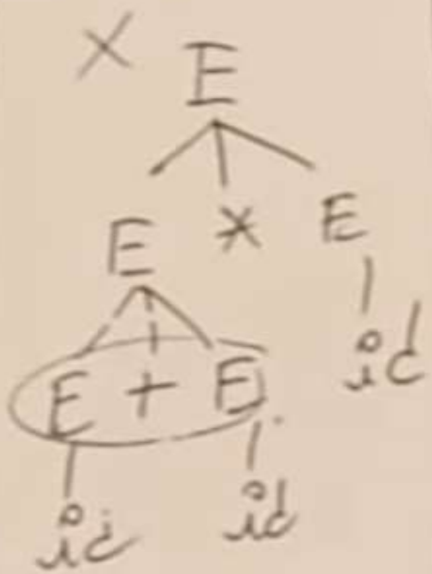
2↑3↑2.

$2^{3^{2}}$



$\boxed{id + id * id}$





$E \rightarrow E + T \,/\, T$

$T \rightarrow T * F \,/\, F$

$F \rightarrow G \uparrow F \,/\, G$

$G \rightarrow id$

$+, *, \uparrow$

$$R \to R+R$$
$$/ RR$$
$$/ R*$$
$$/ a$$
$$/ b$$
$$/ c$$

$$E \to E+T/T$$
$$T \to TF/F$$
$$F \to F*/a/b/c.$$

$bExp \rightarrow bExp$ or $bExp$

$\quad /\ bExp$ and $bExp$

$\quad /$ not $bExp$

$\quad /$ True

$\quad /$ False.

$E \rightarrow E$ or $F \,/\, F$

$F \rightarrow F$ and $G \,/\, G$

$G \rightarrow$ Not G / True / False.

$A \rightarrow A\$B / B$

$B \rightarrow B\#C / C$

$C \rightarrow C@D / D.$

$D \rightarrow d$

$\$ \gtrdot \$$

$\# \gtrdot \#$

$@ \gtrdot @$

$\$ \lessdot \# \lessdot @$

$E \rightarrow E * F$

$/ F + E$

$/ F$

$F \rightarrow F - F$

$/ id$

$* \doteq +$

$\$ \gtrdot \$$

$\# \gtrdot \#$

$@ \gtrdot @$

$\$ \lessdot \# \lessdot @$

$* \gtrdot *$

$+ \lessdot +$

Elimination of left recursion

- A grammar becomes left-recursive if it has any non-terminal ‘A’ whose derivation contains ‘A’ itself as the left-most symbol.
- Left-recursive grammar is considered to be a problematic situation for top-down parsers. Top-down parsers start parsing from the Start symbol, which in itself is non-terminal.
- So, when the parser encounters the same non-terminal in its derivation, it becomes hard for it to judge when to stop parsing the left non-terminal and it goes into an infinite loop.
- A grammar is left recursive if it has a non-terminal A such that there is a derivation A=> Aα
- Top down parsing methods cant handle left-recursive grammars
- A simple rule for direct left recursion elimination:
  - For a rule like:
    - A -> A α|β
  - We may replace it with
    - A -> β A’
    - A’ -> α A’ | ε

Left recursion elimination (cont.)

- There are cases like following
  - S -> Aa | b
  - A -> Ac | Sd | ε
- Left recursion elimination algorithm:
  - Arrange the nonterminals in some order A1,A2,…,An.
  - For (each i from 1 to n) {
    - For (each j from 1 to i-1) {
      - Replace each production of the form Ai-> Aj γ by the production Ai -> $\delta 1 \gamma$ | $\delta 2 \gamma$ | … | $\delta k \gamma$ where Aj-> $\delta 1$ | $\delta 2$ | … | δk are all current Aj productions
      - }
      - Eliminate left recursion among the Ai-productions
    - }

RESOLVING DIFFICULTIES : LEFT RECURSION

A left recursive grammar has rules that support the derivation : $A \Rightarrow^{+} A\alpha$, for some α.

Top-Down parsing can't reconcile this type of grammar, since it could consistently make choice which wouldn't allow termination.

$A \Rightarrow A\alpha \Rightarrow A\alpha\alpha \Rightarrow A\alpha\alpha\alpha$ … etc. $A \rightarrow A\alpha \mid \beta$

Take left recursive grammar:

$$A \rightarrow A\alpha \mid \beta$$

To the following:

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \in$$

WHY IS LEFT RECURSION A PROBLEM ?

Consider:

$$E \rightarrow E + T \quad | \quad T$$
$$T \rightarrow T * F \quad | \quad F$$
$$F \rightarrow (E) \quad | \quad id$$

Derive : id + id + id

$$E \Rightarrow E + T \Rightarrow$$

How can left recursion be removed ?

$$E \rightarrow E + T \;|\; T$$

What does this generate?

$$E \Rightarrow E + T \Rightarrow T + T$$

$$E \Rightarrow E + T \Rightarrow E + T + T \Rightarrow T + T + T$$

. . .

How does this build strings ?

What does each string have to start with ?

RESOLVING DIFFICULTIES : LEFT RECURSION (2)

Informal Discussion:

Take all productions for <u>A</u> and order as:

$\mathbf{A \to A\alpha_1 \mid A\alpha_2 \mid \ldots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n}$

Where no β_i begins with A.

Now apply concepts of previous slide: $\mathbf{A \to \beta_1 A' \mid \beta_2 A' \mid \ldots \mid \beta_n A'}$

$\mathbf{A' \to \alpha_1 A' \mid \alpha_2 A' \mid \ldots \mid \alpha_m A' \mid \in}$

For our example:

$\mathbf{E \to E + T \mid T}$

$\mathbf{T \to T * F}$

$\mathbf{\mid F}$

$\mathbf{F \to (E}$

$\mathbf{) \mid id}$

$\mathbf{E \to TE'}$

$\mathbf{E' \to + TE' \mid \in}$

$\mathbf{F \to (E) \mid id}$

$\mathbf{T \to FT'}$

$\mathbf{T' \to * FT' \mid \in}$

RESOLVING DIFFICULTIES : LEFT RECURSION (3)

Problem: If left recursion is two-or-more levels deep, this isn't enough

$$\left.\begin{array}{l} S \rightarrow Aa \mid b \\ A \rightarrow Ac \mid Sd \mid \in \end{array}\right\} \qquad S \Rightarrow Aa \Rightarrow Sda$$

Algorithm:

Input: **Grammar G with ordered Non-Terminals $A_1, ..., A_n$**

Output: **An equivalent grammar with no left recursion**

1. **Arrange the non-terminals in some order A_1=start NT,A_2,…A_n**
2. `for` *i* `:= 1 to` *n* `do begin`

   `for` *j* `:= 1 to` *i* `- 1  do begin`

   replace each production of the form $A_i \rightarrow A_j\gamma$

   by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$

   where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ are all current A_j productions;

   `end`

   eliminate the immediate left recursion among A_i productions

USING THE ALGORITHM

Apply the algorithm to: $A_1 \to A_2a \mid b \mid \in$

$A_2 \to A_2c \mid A_1d$

i = 1

For A_1 there is no left recursion

i = 2

for j=1 to 1 do

Take productions: $A_2 \to A_1\gamma$ **and replace with**

$A_2 \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma \mid$ **where** $A_1 \to \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ **are A_1 productions**

In our case $A_2 \to A_1d$ **becomes** $A_2 \to A_2ad \mid bd \mid d$

What's left: $A_1 \to A_2a \mid b \mid \in$

$A_2 \to A_2c \mid A_2ad \mid bd \mid d$

Are we done ?

USING THE ALGORITHM (2)

No ! We must still remove A_2 left recursion !

$$A_1 \rightarrow A_2a \mid b \mid \in$$

$$A_2 \rightarrow A_2c \mid A_2ad \mid bd \mid d$$

Recall:

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$$

$$A \rightarrow \beta_1A' \mid \beta_2A' \mid \ldots \mid \beta_nA'$$

$$A' \rightarrow \alpha_1A' \mid \alpha_2A' \mid \ldots \mid \alpha_m A' \mid \in$$

$$A_1 \rightarrow A_2a \mid b \mid \in$$

$$A_2 \rightarrow bdA_2' \mid dA_2'$$

$$A_2' \rightarrow c A_2' \mid adA_2' \mid \in$$

Apply to above case. What do you get ?

REMOVING DIFFICULTIES : $\in$-MOVES

Transformation: **In order to remove $A \rightarrow \in$ find all rules of the form $B \rightarrow uAv$ and *add* the rule $B \rightarrow uv$ to the grammar G.** **Why does this work ?**

Examples:

$E \rightarrow TE'$

$T_{\in} \rightarrow^{E'} F \rightarrow T'^{+} \; TE' \,|$

$F_{\in} \; {}^{T}\!\rightarrow' \rightarrow (E^{*})^{FT} | \; i' d |$

$A_1 \rightarrow A_2 \, a \mid b$

$A_2 \rightarrow bd \, A_2' \mid A_2'$

$A_2' \rightarrow c \, A_2' \mid bd \, A_2' \mid \in$

A is Grammar $\in$-free if:

1. It has no $\in$ -production **or**
2. There is exactly one $\in$ -production $S \rightarrow \in$ and then the start symbol S does not appear on the right side of any production.

REMOVING DIFFICULTIES : CYCLES

How would cycles be removed ?

Make sure every production is adding some terminal(s) (except a single $\in$ -production in the start NT)…

e.g.

$\mathbf{S \rightarrow SS \mid (S) \mid \in}$

Has a cycle: $\mathbf{S \Rightarrow \underbrace{SS}_{S \rightarrow \in} \Rightarrow S}$

Transform to:

$\mathbf{S \rightarrow S (S) \mid (S) \mid \in}$

Re

LR

A()
{
A()
α.
}

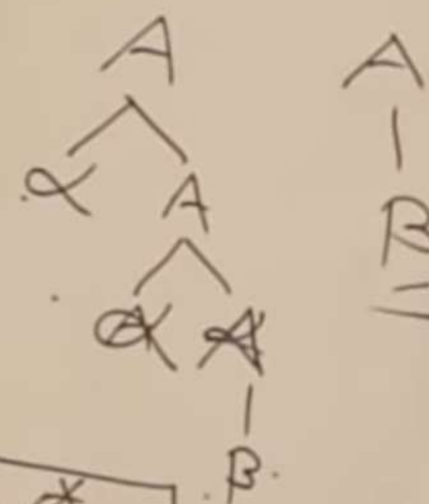
$A \to A\alpha / \beta$

$\beta\alpha^*$

RR

$A \to \alpha A / \beta$

A()
{
α
A()
}



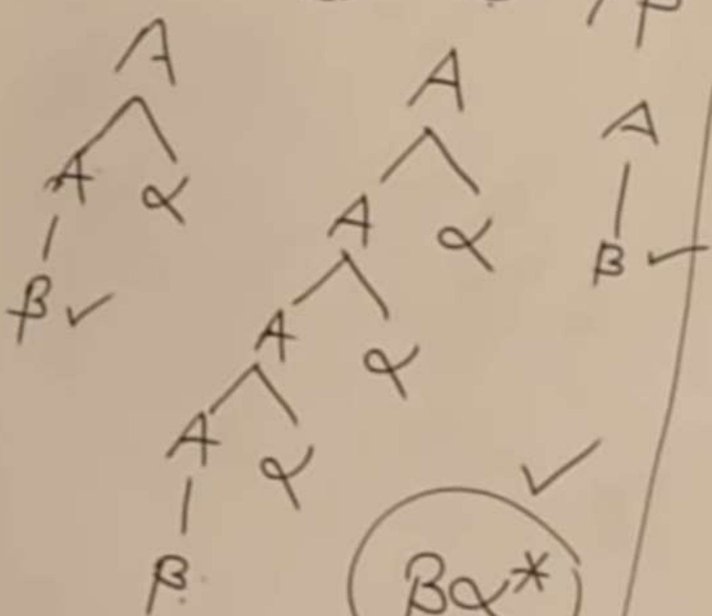
$\boxed{\alpha^* \beta}$

A()
{
A()
α.
}

Re

LR

$A \to A\alpha / \beta$

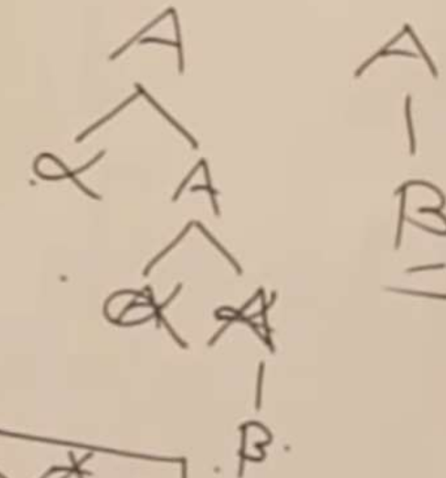


$\beta\alpha^*$

RR

$A \to \alpha A / \beta$

A()
{
α
A()
}



$\boxed{\alpha^* \beta}$

Re

TOP

x^{RR}

$\beta\alpha^*$

$A \to \beta\alpha^*$

$A \to \beta A'$

$A' \to \epsilon / \alpha A'$

β

A

$\beta\alpha^*$

$A \to \beta\alpha^*$

$$\boxed{\begin{aligned} A &\to \beta A' \\ A' &\to \epsilon / \alpha A' \end{aligned}} \Longleftrightarrow \boxed{A \to \alpha A / \beta}$$

TOP

Re

RR

$\boxed{\underbrace{E}_{A} \rightarrow \underbrace{E}_{A}\underbrace{+T}_{\alpha} / \underbrace{T}_{\beta}}$

$\beta\alpha^*$

$A \rightarrow \beta\alpha^*$

$$\boxed{\begin{aligned} &E \rightarrow TE' \\ &E' \rightarrow \epsilon / +TE' \end{aligned}}$$

$$\boxed{\begin{aligned} &A \rightarrow \beta A' \\ &A' \rightarrow \epsilon / \alpha A' \end{aligned}} \iff \boxed{A \rightarrow A\alpha / \beta}$$

TOP

x RR

$\beta\alpha^*$

$A \rightarrow \beta\alpha^*$

$$\boxed{\begin{array}{l} A \rightarrow \beta A' \\ A' \rightarrow \epsilon \,/\, \alpha A' \end{array}} \iff \boxed{A \rightarrow A\alpha \,/\, \beta}$$

$$\underbrace{S}_{A} \rightarrow \underbrace{S}_{A}\underbrace{0S1S}_{\alpha} \,/\, \underbrace{01}_{\beta}.$$

$S \rightarrow 01S'$

$S' \rightarrow \epsilon \,/\, 0S1SS'$

$$S \to (L) / x$$

$$\underbrace{L}_{A} \to L\underbrace{,S}_{\alpha} / \underbrace{S}_{\beta}$$

$$A \to A\alpha / \beta$$

$$\Downarrow$$

$$A \to \beta A'$$

$$A' \to \alpha A' / \epsilon$$

$$\boxed{\begin{aligned} L &\to SL' \\ L' &\to ,SL' / \epsilon \end{aligned}}$$

Left factoring

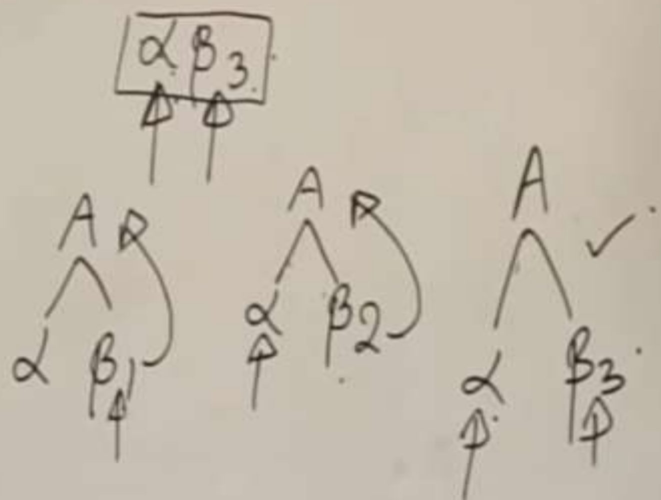
- If more than one grammar production rules has a common prefix string, then the top-down parser cannot make a choice as to which of the production it should take to parse the string in hand
- Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive or top-down parsing.
- Then it cannot determine which production to follow to parse the string as both productions are starting from the same terminal (or non-terminal). To remove this confusion, we use a technique called left factoring.
- Left factoring transforms the grammar to make it useful for top-down parsers. In this technique, we make one production for each common prefixes and the rest of the derivation is added by new productions.

- Consider following grammar:
  - Stmt -> **if** expr **then** stmt **else** stmt
  - | **if** expr **then** stmt
- On seeing input **if** it is not clear for the parser which production to use
- We can easily perform left factoring:
  - If we have A->αβ1 | αβ2 then we replace it with
    - A -> αA'
    - A' -> β1 | β2

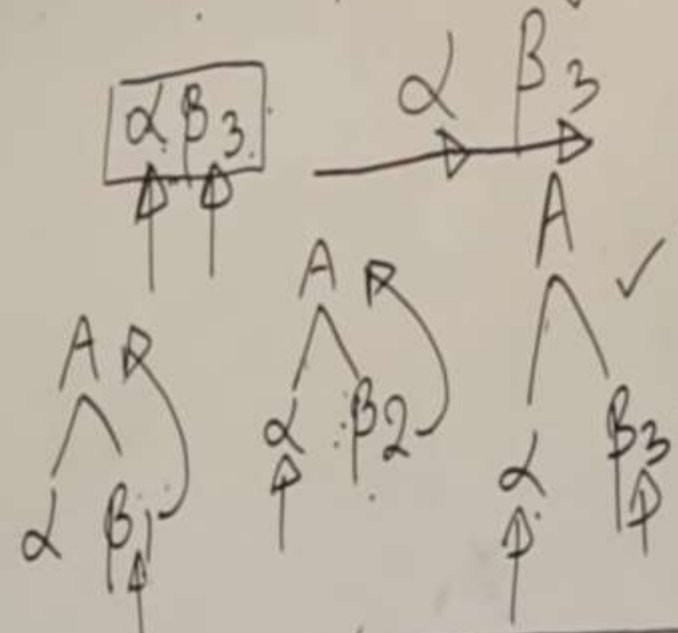
Left factoring (cont.)

- Algorithm
  - For each non-terminal A, find the longest prefix α common to two or more of its alternatives. If α <> ε, then replace all of A-productions A-> $\alpha\ \beta 1 \mid \alpha\ \beta 2 \mid \ldots \mid \alpha\ \beta n \mid \gamma$ by
    - A -> α A' | γ
    - A' -> $\beta 1 \mid \beta 2 \mid \ldots \mid \beta n$
- Example:
  - S -> I E t S | i E t S e S | a
  - E -> b

$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3$

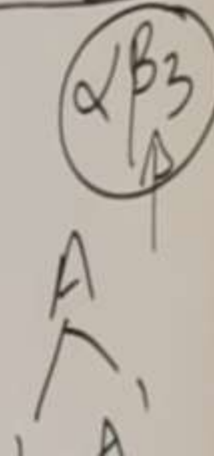


$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3$



$A \rightarrow \alpha A'$

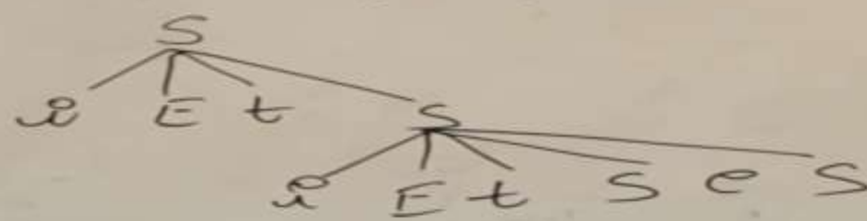
$A' \rightarrow \beta_1 \mid \beta_2 \mid \beta_3$

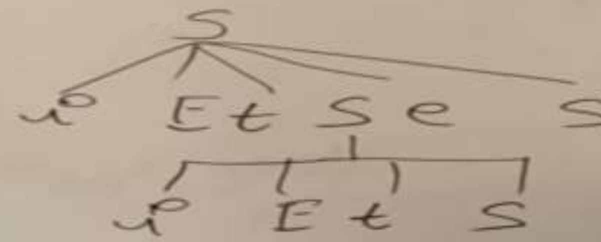


$$S \rightarrow \underline{iEtS}$$
$$\quad / \underline{iEtS}eS$$
$$\quad / a$$
$$E \rightarrow b$$

$$S \rightarrow \underline{iEtS}S' / a$$
$$S' \rightarrow \epsilon / eS$$
$$E \rightarrow b$$

iEtiEtSeS.





$S \rightarrow iEtS$

$\quad / iEtSeS$

$\quad / a$

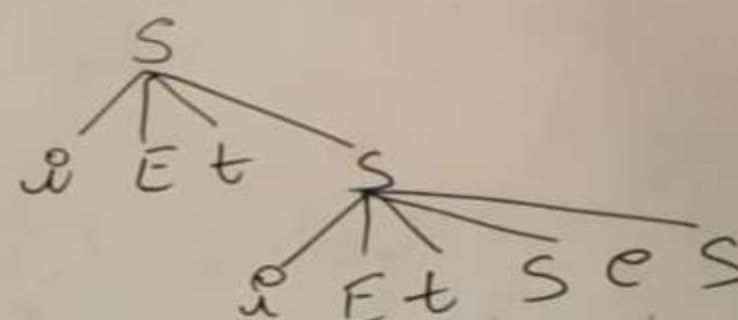
$E \rightarrow b$

$S \rightarrow iEtSS' / a$

$S' \rightarrow \epsilon / eS$

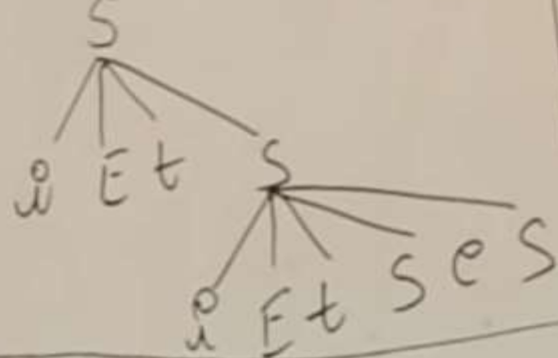
$E \rightarrow b$

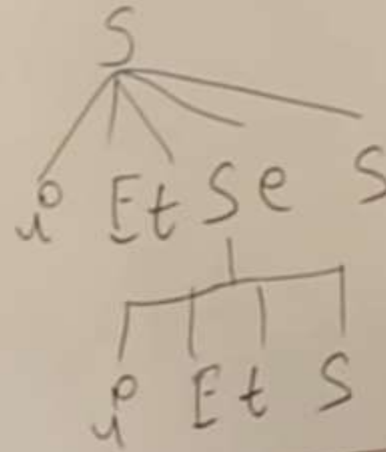
iEtiEtSeS.



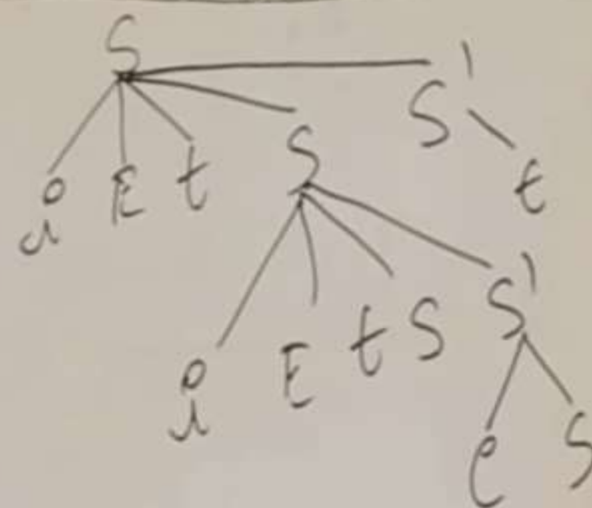
$S \rightarrow iEtS$
$\quad / iEtSeS$
$\quad / a$
$E \rightarrow b$

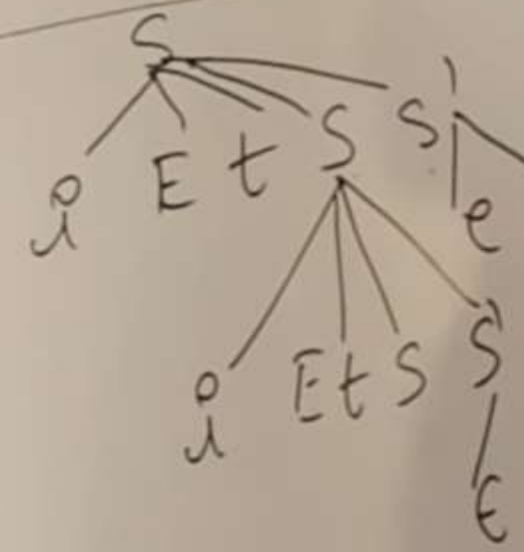
$iEtiEtSeS$





$S \rightarrow iEtSS' / a$
$S' \rightarrow \epsilon / eS$
$E \rightarrow b$





$S \to \underline{a}SSbS$
$\quad / \underline{a}SaSb$
$\quad / \underline{a}bb$
$\quad / b$

$S \to bSSaaS$
$\quad / bSSaSb$
$\quad / bSb$
$\quad / a$

EXAMPLES OF LEFT FACTORING

1. S -> iEtS|iEtSES|a E ->b

2. S-> aSSbS|aSaSb|abb|b

3. S-> bSSaaS|bSSaSb|bSb|a