

TRIGONOMETRY FUNCTIONS...

$$\textcircled{1} \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\textcircled{2} \cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\textcircled{3} \sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\textcircled{4} \sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\textcircled{5} \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\textcircled{6} \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\textcircled{7} \cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

$$\textcircled{8} \cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\textcircled{9} \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\textcircled{10} \sin 2x = 2\sin x \cos x = \frac{2\tan x}{1 + \tan^2 x}$$

$$\textcircled{11} \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\textcircled{13} \cos 3x = 4\cos^3 x - 3\cos x$$

$$\textcircled{14} \tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

$$\textcircled{12} \sin 3x = 3\sin x - 4\sin^3 x$$

$$(15) \cos x + \cos y = 2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2}$$

$$(16) \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$(17) \sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$(18) \sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$(19) 2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

$$(20) -2 \sin x \sin y = \cos(x+y) - \cos(x-y)$$

$$(21) 2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$(22) 2 \cos x \sin y = \sin(x+y) - \sin(x-y)$$

$$(23) \sin x = \sin y \Rightarrow x = n\pi \pm (-1)^n y$$

$$(24) \cos x = \cos y ; x = 2n\pi \pm y$$

$$(25) \tan x = \tan y ; x = n\pi + y$$

$$(26) \sin^2 x + \cos^2 x = 1$$

$$(27) 1 + \tan^2 x = \sec^2 x$$

$$(28) 1 + \cot^2 x = \operatorname{cosec}^2 x$$

INVERSE TRIGONOMETRIC FUNCTION

$$(1) \sin^{-1}(x) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) \quad (4) \cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$$

$$(2) \operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right) \quad (5) \cos^{-1}(x) = \sec^{-1}\left(\frac{1}{x}\right)$$

$$(3) \tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right) \quad (6) \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

$$(7) \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad (8) \tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$$

$$(9) \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \quad (10) \sin^{-1}(-x) = -\sin^{-1}(x)$$

$$(11) \tan^{-1}(-x) = -\tan^{-1}(x) \quad (12) \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$$

$$(13) \cos^{-1}(-x) = \pi - \cos^{-1}x \quad (14) \sec^{-1}(-x) = \pi - \sec^{-1}(x)$$

$$(15) \cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

$$(16) \sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$$

$$(17) \sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$$

$$(18) \cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2})$$

$$(19) \cos^{-1}x - \cos^{-1}y = \cos^{-1}(xy + \sqrt{1-x^2} \sqrt{1-y^2})$$

$$(20) \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad [\text{When } xy < 1]$$

$$(21) \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$(22) \tan^{-1}x - \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad [\text{When } xy > 1]$$

$$(23) \quad 2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$(24) \quad 2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$(25) \quad 2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

$$(26) \quad 2 \sin^{-1} x = \sin^{-1} 2x \sqrt{1-x^2}$$

$$(27) \quad 2 \cos^{-1} x = \cos^{-1} (2x^2 - 1)$$

$$(28) \quad 2 \cos^{-1} x = \sin^{-1} 2x \sqrt{1-x^2}$$

$$\textcircled{1} \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\textcircled{4} \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\textcircled{5} \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\textcircled{3} \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\textcircled{6} \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$\textcircled{7} \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$\textcircled{8} \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

FIRST TYPE

$$f(x) = \begin{cases} 3x & , \text{ for } x \neq a \\ 2x & , \text{ for } x = a \end{cases}$$

$$\lim_{x \rightarrow a} f(x) = f(a)$$

SECOND TYPE

$$f(x) = \begin{cases} 3x & , \text{ for } x < a \\ 2x & , \text{ for } x = a \\ x & , \text{ for } x > a \end{cases}$$

$$LHL = f(a) = RHL$$

$$(1) \frac{d}{dx} (x^n) = nx^{n-1}$$

$$(13) \frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

$$(2) \frac{d}{dx} (x) = 1$$

$$(14) \frac{d}{dx} (\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$(3) \frac{d}{dx} (\text{const}) = 0$$

$$(15) \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$(4) \frac{d}{dx} (\sin x) = \cos x$$

$$(16) \frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$(5) \frac{d}{dx} (\cos x) = -\sin x$$

$$(17) \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$(6) \frac{d}{dx} (\tan x) = \sec^2 x$$

$$(18) \frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$(7) \frac{d}{dx} (\csc^{-1} x) = -\csc^2 x$$

$$(19) \frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$(8) \frac{d}{dx} (\sec x) = \sec x \tan x$$

$$(20) \frac{d}{dx} (\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$(9) \frac{d}{dx} (\csc x) = -\csc x \cot x$$

$$(21) e^{\log_e x} = x$$

$$(10) \frac{d}{dx} (e^x) = e^x$$

$$(22) \log ab = \log a + \log b$$

$$(23) \log \left(\frac{a}{b} \right) = \log a - \log b$$

$$(11) \frac{d}{dx} (a^x) = a^x \log_e a$$

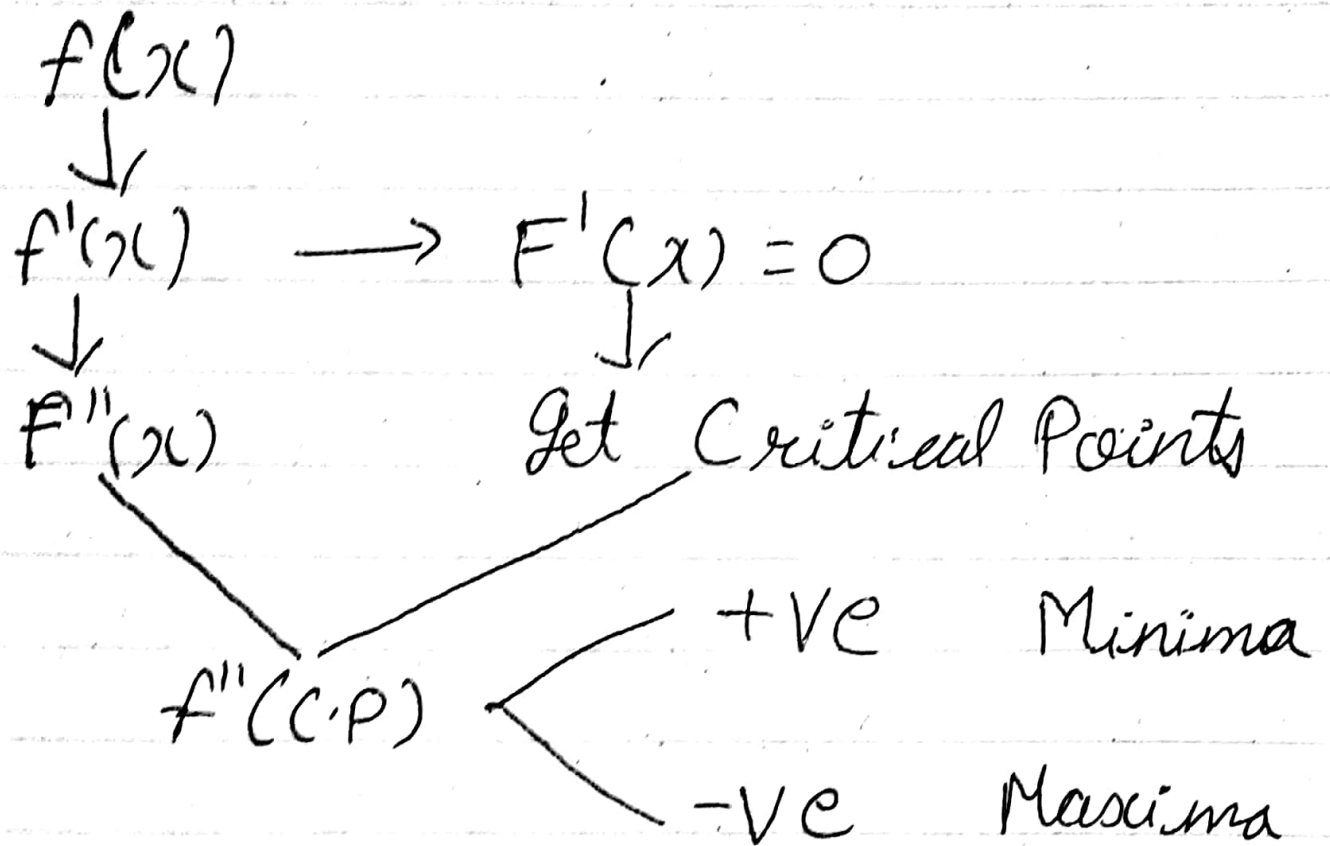
$$(24) \log b^a = a \log b$$

$$(12) \frac{d}{dx} (\log x) = \frac{1}{x}$$

$$(25) \log_b a = \frac{\log a}{\log b}$$

	0°	$\frac{\pi}{6} = 30^\circ$	$\frac{\pi}{4} = 45^\circ$	$\frac{\pi}{3} = 60^\circ$	$\frac{\pi}{2} = 90^\circ$	$\pi = 180^\circ$	$\frac{3\pi}{2} = 270^\circ$	$2\pi = 360^\circ$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D	0	N.D	0
cot	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	-1 N.D	0	N.D
cosec	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	N.D	-1	N.D
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D	-1	N.D	1

Maxima And Minima



$$\text{Minimum Value} = f(C.P)$$

INTEGRATION

$$\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C, & n \neq -1 \\ \log |x| + C, & n = -1 \end{cases} \quad \int (a+bx)^n dx = \begin{cases} \frac{(a+bx)^{n+1}}{(n+1)b} + C, & n \neq -1 \\ \frac{1}{a} \log |a+bx| + C, & n = -1 \end{cases}$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cot x dx = \log |\sin x| + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \tan x dx = -\log |\cos x| + C = \log |\sec x| + C$$

$$\int \operatorname{cosec}^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$$

$$\int \sec x dx = \log |\sec x + \tan x| + C$$

$$\int \operatorname{cosec} x dx = \log |\operatorname{cosec} x - \cot x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\log a} + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C, |x| > 1$$

$$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log |x + \sqrt{a^2 + x^2}| + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log |x + \sqrt{a^2 + x^2}| + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\int k \cdot f(x) dx = k \int f(x) dx$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int k \{f(x) \pm g(x)\} dx = k \int f(x) dx \pm k \int g(x) dx$$

$$\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + C$$

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C$$

$$\int \frac{f'(x)}{[f(x)]^n} dx = \frac{[f(x)]^{n+1}}{-n+1} + C$$

$$\int f(x) \cdot g(x) dx = f(x) \cdot [\int g(x) dx] - \int f'(x) \cdot [\int g(x) dx] dx$$

$$\int 1 dx = x + C$$

$$\int \frac{1}{x} dx = \log x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int a^{\log_a x} = x + C$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$$

$$\int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + C$$

$$\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C$$

$$\int \sec x dx = \log \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| + C$$

$$\int \operatorname{cosec} x dx = \log \left| \tan \frac{x}{2} \right| + C$$

$$\int \frac{-1}{\sqrt{1-x^2}} = -\cot^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} = \cos^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

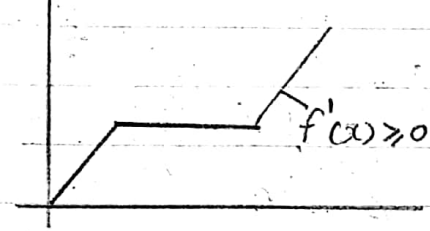
$$\int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \left(\frac{x}{a} \right) + C = \frac{1}{a} \operatorname{cosec}^{-1} \left(\frac{x}{a} \right) + C$$

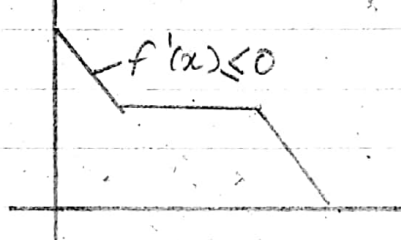
$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2} [a \sin bx - b \cos bx] + C$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2} [a \cos bx + b \sin bx] + C$$

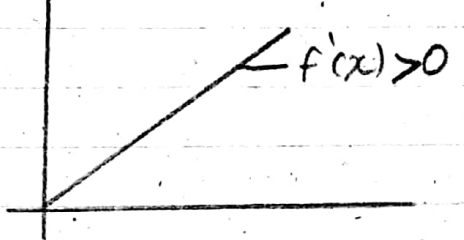
Increasing Function



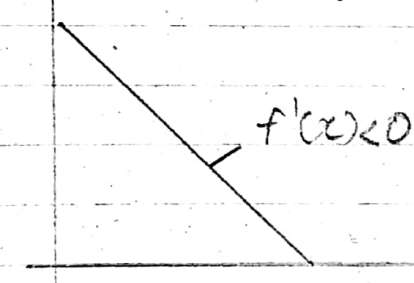
Decreasing Function



Strictly Increasing Function



Strictly Decreasing Function



Approximation

$$y = f(x) \quad \text{--- (1)}$$

$$y + \Delta y = f(x + \Delta x) \quad \text{--- (2)}$$

$$\text{(2) - (1)}$$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\Delta y = \frac{dy}{dx} \Delta x$$

$$\Delta f \approx f(a+h) - f(a) = h f'(a)$$

Rolle's Theorem

Let $f(x)$ be any function $[a, b]$

① $f(x)$ is defined and continuous $[a, b]$

② $f(x)$ is derivable (a, b)

③ $f(a) = f(b)$, then there exist a point $c \in (a, b)$ such that $f'(c) = 0$

Mean Value Theorem

$f(x)$ = any function $[a, b]$

① $f(x)$ is defined and continuous $[a, b]$

② $f(x)$ is derivable at (a, b) then there exist a point c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$