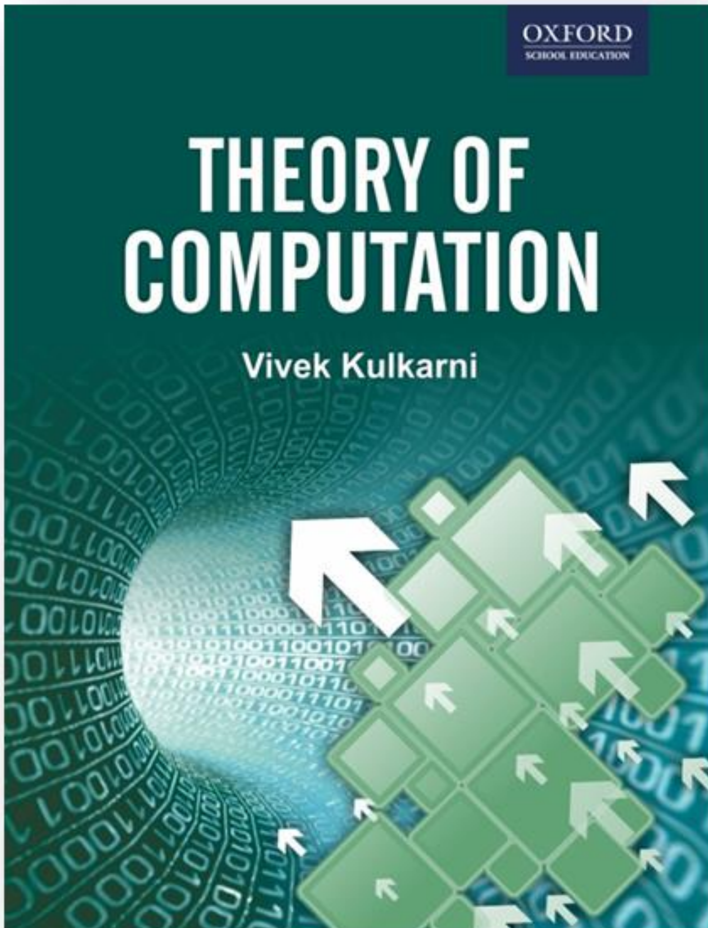


Author: Vivek Kulkarni
(vivek_kulkarni@yahoo.com)

Chapter-3: Regular Expressions

Solutions for Review Questions

Q.1 Define the following and give suitable examples:

i) Regular set

ii) Regular expression

Solution:

i) Regular set: Refer to the section 3.5.

ii) Regular expression: Refer to the section 3.2.

Q.2 Prove that the language $L = \{a^n\ b^{n+1} \mid n > 0\}$ is non-regular, using pumping lemma.

Solution:

We must not confuse the *n* in the language definition with the constant *n* of pumping lemma. Hence, we rewrite the language definition as:

$L = \{a^m b^{m+1} \mid m > 0\}$.

Step 1: Let us assume that the language *L* is a regular language. Let *n* be the constant of pumping lemma.

Step 2: Let us choose a sufficiently large string *z* such that $z = a^l b^{l+1}$, for some large $l > 0$; the length of *z* is given by: $|z| = 2l+1 \geq n$. Since we assumed that *L* is a regular language and from the language definition it is an infinite language, we can now apply pumping lemma. This means that we should be able to write *z* as: $z = uvw$.

Step 3: As per pumping lemma, every string 'uv^iw', for all $i \geq 0$ is in *L*. Further, $|v| \geq 1$, which means that *v* cannot be empty, and must contain one or more symbols.

Let us consider the case when *v* contains a single symbol from $\{a, b\}$. Hence, $z = uvw = a^l b^{l+1}$, which means that the number of *b*'s is one greater than number of a's in *z*. Therefore, as per pumping lemma, we would expect 'uv^2w' also to be a member of *L*. However, this cannot be the case, as *v* contains only a single symbol, and pumping *v* would yield different number of *a*'s and *b*'s than what is expected by the language definition. Thus, 'uv^2w' is not a member of *L*, contradicting our assumption that *L* is regular.

Let us now consider the case when *v* contains both the symbols, i.e., *a* as well as *b*. The sample *v* could be written as '*ab*', or '*aabb*', and so on. When we try to pump *v* multiple times, such as, for example, $v^2 = abab$, or $v^2 = aabbaabb$, and so on, we find that even *a*'s can follow *b* in the string, which is against the language definition '$a^m b^{m+1}$', according to which, a's are followed by b's, and not vice versa. Thus, 'uv^2w' is not a member of L, contradicting our assumption that L is regular.

Hence, language $L = \{a^m b^{m+1} \mid m > 0\}$ is non-regular.

Q.3 Explain in brief the applications of finite automata.

Solution:

Refer to the section 3.8.

Q.4 Construct the NFA with $\in$-transitions, which accepts the language defined by:

$(ab + ba)^* \, aa \, (ab + ba)^*$

Also convert this to a minimized DFA.

Solution:

Refer to the example 3.27 form the book.

Q.5 Construct regular expressions defined over the alphabet $\Sigma = \{a, b\}$, which denote the following languages:

i) All strings without a double a.

ii) All strings in which any occurrence of the symbol b, is in groups of odd numbers.

iii) All strings in which the total number of a's is divisible by 2.

Solution:

i) Strings without double a means strings without two consecutive a's. Hence, the required RE is,

$$(a + \in) \cdot (b + ba)^*$$

ii) Here, b's exist in groups of odd numbers, i.e., 1, 3, 5 and so on. Hence, the RE is,

$$a^* \, b \, (bb)^* \, a^*$$

iii) Here, we require even number of a's. The required RE is,

$$(b^* \cdot a \cdot b^* \cdot a \cdot b^*)^* + b^*$$

Q.6 Check the following regular expressions for equivalence and justify:

(i) $R_1 = (a + bb)^* (b + aa)^*$

$R_2 = (a + b)^*$

(ii) $R_1 = (a + b)^* abab^*$

$R_2 = b^* a (a + b)^* ab^*$

Solution:

i) Let us write languages denoted by R_1 and R_2 as below.

L(R_1) = {$\in$, a, b, aa, ab, bb, abb, baa, bba, ... }

L(R_2) = {$\in$, a, b, aa, ab, ba, bb, ... }

Given regular expressions R_1 and R_2 are not equal as the strings produced by them are not same. For example, string '*ba*' cannot be generated using regular expression R_1 which can be produced by R_2.

ii) Let us write languages denoted by R_1 and R_2 as below.

L(R_1) = {aba, aaba, baba, abab, ababb, ababa, ... }

L(R_2) = { aa, baa, baaa, baba, baab, ... }

Given regular expressions R_1 and R_2 are not equal as the strings produced by them are not same. For example, string 'aa' cannot be produced by Regular Expression R_1 which can be produced by R_2.

Q.7 Describe in English the sets denoted by the following regular expressions:

(i) $(a + \in) (b + ba)^*$

(ii) $(0^*1^*)^*$

Solution:

i) Let us write language denoted by the given RE.

L(R) = {$\in$, a, b, ab, ba, bb, aba, abb, bbb, baba, abab, ...}

Given language consists of strings where two consecutive *a*'s cannot occur.

ii) Let us write language denoted by the given RE.

L(R) = {$\in$, 0, 1, 00, 11, 01, 10, 000, 111, ...}

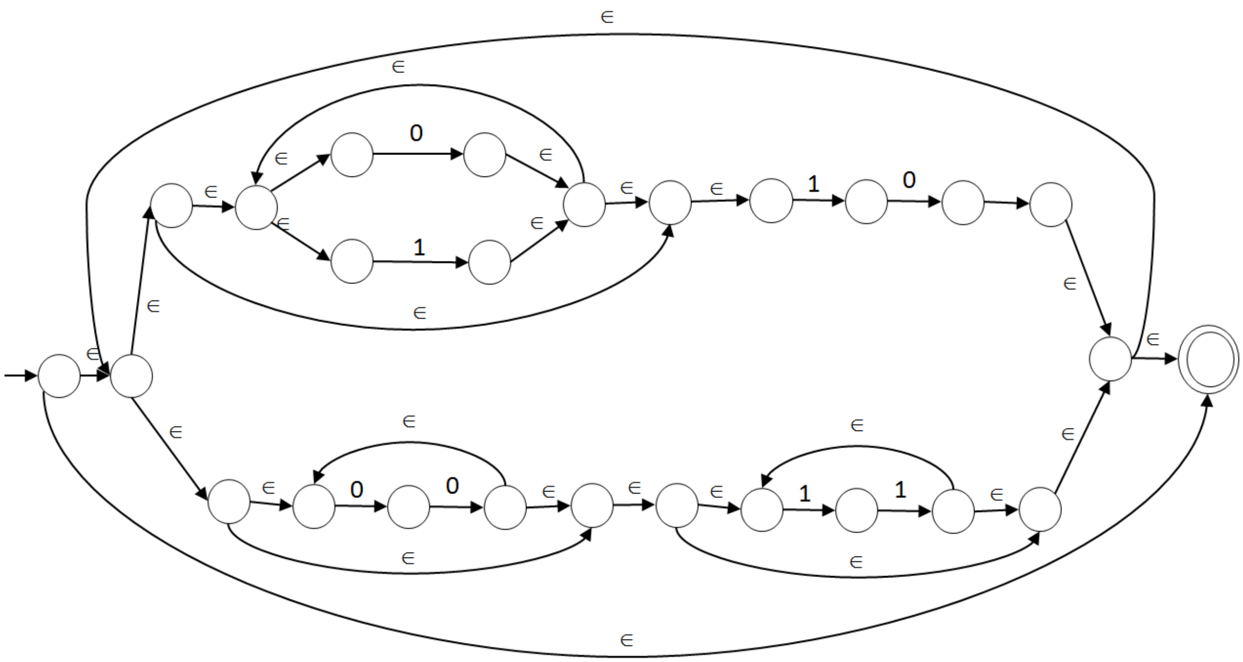
Given language consists of strings where any combination of 0's and 1's can be observed.

Q.8 Construct an NFA with $\in$-moves, which accepts the language defined by:

$[(0 + 1)^* 10 + (00)^* (11)^*]^*$

Solution:

The NFA with ∈-moves is,



Q.9 Let R_1 and R_2 be two regular expressions. With the help of transition diagrams, illustrate the three operations (+, ·, *) on R_1 and R_2.

Solution:

Refer to the section 3.4.2.1.

Q.10 Show that the regular expressions, $(a^* \, bbb)^* \, a^*$ and $a^* \, (bbba^*)^*$, are equivalent.

Solution:

Let, $R_1 = (a^* \, bbb)^* \, a^*$ and $R_2 = a^* \, (bbba^*)^*$.

Let us write language denoted by R_1 as,

L(R_1) = {∈, a, aa, aaa, bbb, aaaa, abbb, bbba abbba, ...}

Let us write language denoted by R_2 as,

L(R_2) = {∈, a, aa, aaa, bbb, aaaa, abbb, bbba, abbba, ...}

As we can see that languages denoted by regular expressions are same, i.e., L(R_1) = L(R_2). Therefore, regular expressions R_1 and R_2 are equivalent.

Q.11 Give a regular expression for representing all strings over $\{a, b\}$ that do not include the sub-strings '*bba*' and '*abb*'.

Solution:

This essentially requires no consecutive b's. The RE can be written as,

$(a + \in)(b + ba)^*$

Q.12 Consider the two regular expressions:

$R_1 = a^* + b^*$

$R_2 = ab^* + ba^* + b^* a + (a^* b)^*$

(i) Find a string corresponding to R_1 but not to R_2.

(ii) Find a string corresponding to R_2 but not to R_1.

(iii) Find a string corresponding to both R_1 and R_2.

Solution:

(i) aaaaaa

(ii) abbbbbbb

(iii) a

Q.13 Construct an NFA for the regular expression, $(a / b)^* ab$. Convert the NFA to its equivalent DFA and validate the answer with suitable examples.

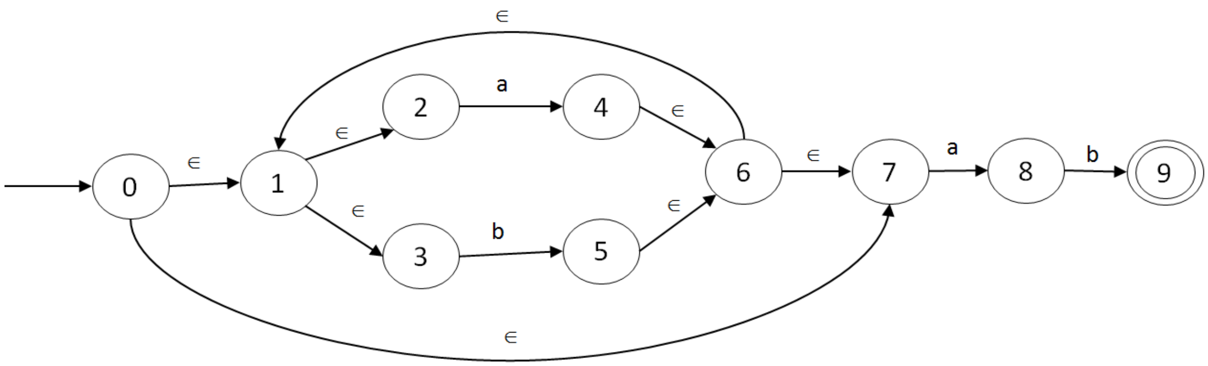
Solution:

It is expected to construct a DFA that recognizes the regular set:

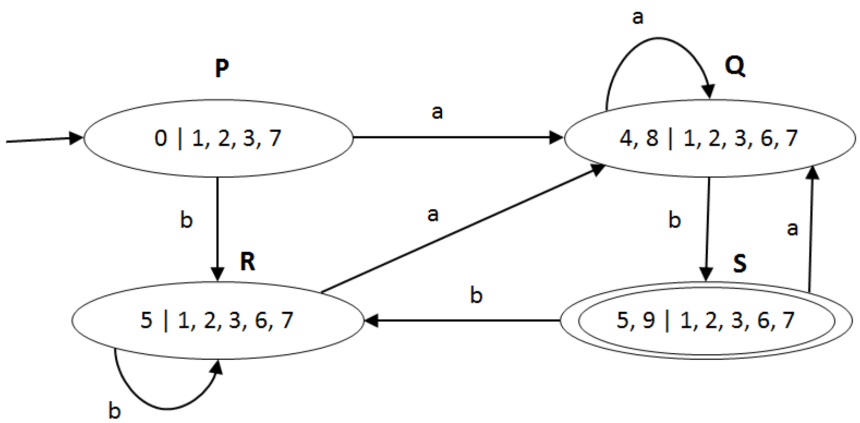
$R = (a/b)^* \cdot a \cdot b$

Let us first build the NFA with $\in$-moves and the convert the same to DFA.

The TG for NFA with $\in$-moves is as follows,



Let us convert this NFA with ∈-moves to its equivalent DFA using a direct method.



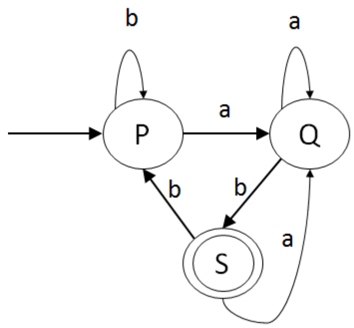
We have relabelled the states as well. Let us see if we can minimize it. The STF for the DFA looks like,

Q\∑	a	b
P	Q	R
Q	Q	S
R	Q	R
* S	Q	R

We can see that states *P* and *R* are equivalent. Hence, we can replace *R* by *P* and get rid of *R*. The reduced STF is,

Q\∑	a	b
P	Q	P
Q	Q	S
* S	Q	P

The TG for the equivalent DFA is,



Q.14 Define the term: regular language.

Solution:

Refer to the section 3.5.

Q.15 Write short note on: pumping lemma for regular sets.

Solution:

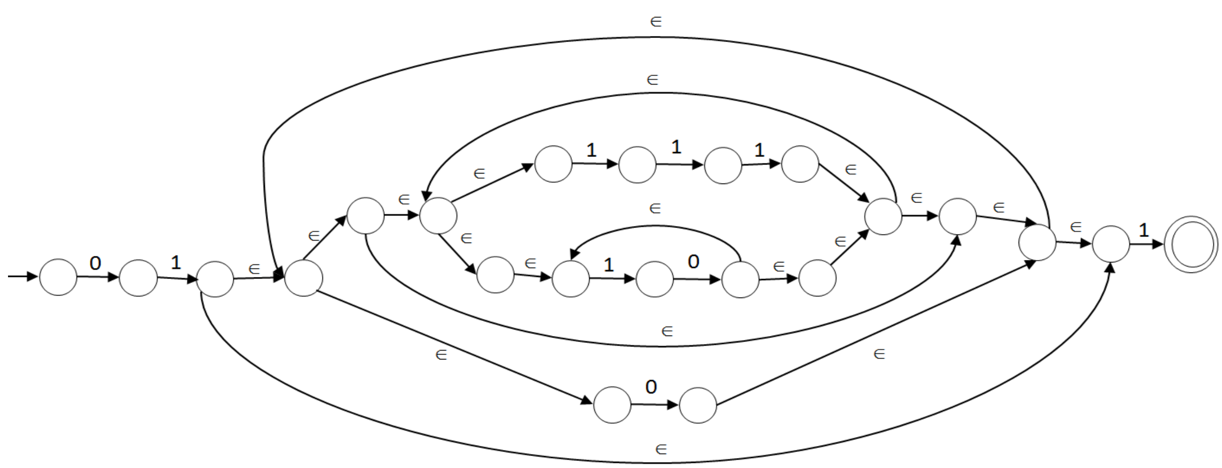
Refer to the section 3.6.

Q.16 Construct an NFA (Q, Σ, δ, q_0, F) for the following regular expression:

$01[((10)^+ + 111)^* + 0]^*\ 1$

Solution:

NFA can be drawn as below.



Q.17 Prove that the regular expressions given below are equivalent.

(i) $(a^* \, bbb)^* \, a^*$

(ii) $a^* \, (bbb \, a^*)^*$

Solution:

Let, $R_1 = (a^* \, bbb)^* \, a^*$ and $R_2 = a^* \, (bbba^*)^*$.

Let us write language denoted by R_1 as,

$L(R_1) = \{\in$, a, aa, aaa, bbb, aaaa, abbb, bbba abbba, ...$\}$

Let us write language denoted by R_2 as,

$L(R_2) = \{\in$, a, aa, aaa, bbb, aaaa, abbb, bbba, abbba, ...$\}$

As we can see that languages denoted by regular expressions are same, i.e., $L(R_1) = L(R_2)$. Therefore, regular expressions R_1 and R_2 are equivalent.

Q.18 Describe the language accepted by the following finite automaton.

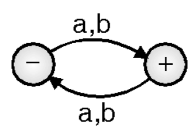


Figure 3.38: Example DFA

Solution:

Regular expression can be written as,

$(a + b) \cdot ((a + b) (a + b))^*$

Q.19 Describe as simply as possible in English the language represented by: (0/1)* 0.

Solution:

Let us write the language denoted by the regular expression.

$L(R) = \{0, 00, 10, 000, 110, 0000, 1110, 010, 0110, ...\}$

Given language consists of all the strings over {0, 1} that ends with a *0*.

Q.20 Construct an NFA that recognizes the regular expression $(a \, / \, b)^* \cdot a \cdot b$. Convert it to a DFA, and draw the state transition table.

Solution:

Refer to the answer for Q.13 above.

Q.21 Construct a regular expression corresponding to the state diagram shown below, using Arden's theorem.

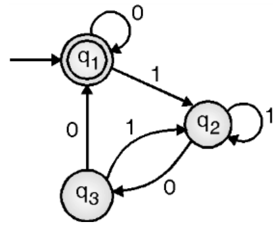


Figure 3.39: Example FA

Solution:

Refer to the example 3.41 from the book.

Q.22 Is the following language regular? Justify.

$L = \{0^p\ 1^p\ p^{p+q}\ \mid p \geq 1, q \geq 1\}$

Solution:

We need to show that the following language is non-regular using Pumping lemma,

$L = \{0^p\ 1^p\ p^{p+q}\ \mid p \geq 1, q \geq 1\}$

As we observe the length of every string from the language L = 2p + 2q = 2 (p + q) is even.

Step 1: Let us assume that the language L is a regular language. Let n be the constant of pumping lemma.

Step 2: Let us choose a sufficiently large string z, such that $z = xx$, where $x = 0^p1^q\mathrm{P}^{p+q}$, for some large p, q > 0; the length of z is given by: $|z| = 2\ (p + q) \geq n$.

Since we assumed that L is a regular language and from the language definition it is an infinite language, we can now apply pumping lemma. Hence, we should be able to write z as: $z = uvw$.

Step 3: As per pumping lemma, every string 'uv^iw', for all $i \geq 0$, is in L. Further, $|v| \geq 1$, which means that v cannot be empty, and must contain one or more symbols.

Let us consider the case when v contains a single symbol from {0, 1}. We assume $z = uvw = xx = 0^p1^q\text{P}^{p+q}0^p1^q\text{P}^{p+q}$. As per pumping lemma, we would expect 'uv^2w' also to be a member of L. However, this cannot be the case as v contains only a single symbol; hence, pumping v would cause the first x in string 'xx' to end with v, and the second x of string 'xx' to begin with v. For example, for $z = 0^p1^q\text{P}^{p+q}0^p1^q\text{P}^{p+q}$, after pumping $v = 0$ once, we get, $z_1 = 0^p1^q\text{P}^{p+q}\ 00\ 0^p1^q\text{P}^{p+q}$, which cannot be represented as a concatenation of two equal sub-strings. Thus, uv^2w is not a member of L, as it modifies the string of the form xx to $xvvx$ rather than $xvxv$. This contradicts our assumption that L is regular.

Let us now consider the case when v contains both the symbols, i.e., 0 as well as 1. The sample v could be written as 01, or 100, and so on. When we try to pump v multiple times, we obtain strings of the form, xv^2v^2x, xv^3v^3x, and so on, which is against the language definition xx—every string is represented as concatenation of two equal sub-strings. Thus, uv^iw, for all $i \geq 0$ is not a member of L. This contradicts our assumption that L is regular.

Hence, language L is non-regular.

Q.23 Construct the regular expression and finite automata for: $L = L_1 \cap L_2$ over alphabet $\{a, b\}$, where:

L_1 = all strings of even length

L_2 = all strings starting with b

Solution:

Let us list down L_1 and L_2 for given conditions.

L_1 = {∈, aa, bb, ab, ba, abab, aaab, aabb, abbb, baaa, bbbb, baba, bbabab, ...}

L_2 = {b, bb, ba, baa, bbb, baab, baaaa, babbb, ... }

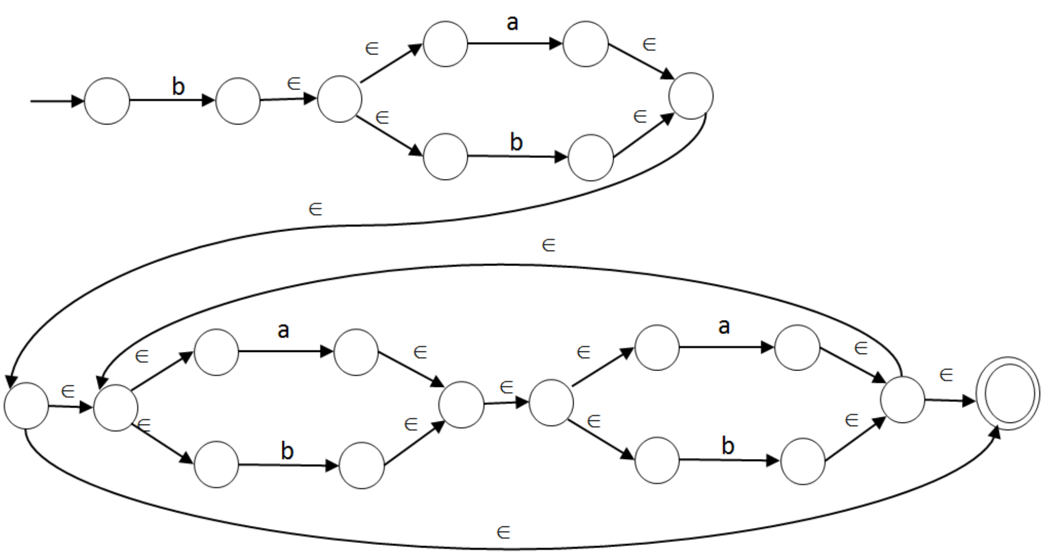
Now, as $L = L_1 \cap L_2,$

L = {bb, ba, baaa, bbbb, baab, baba, ... }

Hence, regular expression for L can be given as,

$r = b\ (a + b)\ [(a + b)\ (a + b)]^*.$

Let us construct the NFA with ∈-moves as shown in the diagram below.



Q.24 Which of the following are true? Explain.

(1) $baa \in a^* b^* a^* b^*$

(2) $b^*a^* \cap a^* b^* = a^* \cup b^*$

(3) $a^* b^* \cap b^* c^* = \phi$

(4) $abcd \in [a (cd)^* b]^*$

Solution:

i) Let L be the language denoted by the given RE, then,

$L = \{\in$, a, b, ab, aa, aba, abab, ba, baa, baab, ... }

As 'baa' string belongs to language produced by given RE.

Hence, $baa \in a^* b^* a^* b^*$ is TRUE.

ii) Let $R_1 = b^*a^*$ then $L_1 = \{\in$, b, a, ba, bb, aa, bbb, aaa, baa, bba, ...}.

Let $R_2 = a^*b^*$ then $L_2 = \{\in$, a, b, ab, bb, aa, bbb, aaa, abb, aab, ...}.

Therefore, $L_1 \cap L_2 = \{\in$, a, b, aa, bb, aaa, bbb, ... }.

$a^* = \{\in$, a, a, aa, aaa, aaaa, ... } and $b^* = \{\in$, b, b, bb, bbb, bbbb, ... }

Hence, $a^* \cup b^* = \{\in$, a, b, aa, bb, aaa, bbb, ... }

Therefore, $b^*a^* \cap a^* b^* = a^* \cup b^*$ is TRUE.

iii) Let $R_1 = a^*b^*$ then $L_1 = \{\in, a, b, ab, bb, aa, bbb, aaa, ...\}$ and

Let $R_2 = b^* c^*$ then $L_2 = \{\in, b, c, bc, bb, cc, bbb, ccc, ...\}$ then

Therefore, $L_1 \cap L_2 = \{\in, b, bb, bbb, ...\} \neq \phi$

Hence, $a^* b^* \cap b^* c^* = \phi$ is FALSE.

iv) Let L be the language denoted by the given RE, $[a (cd)^* b]^*$ then,

$L = \{\in, ab, abab, acdb, acdcdb, acdbacdb, abacdb, ... \}$

'abcd' does not belong to language L.

Therefore, $abcd \in [a (cd)^* b]^*$ is FALSE.

Q.25 Construct the regular expressions for the following DFAs:

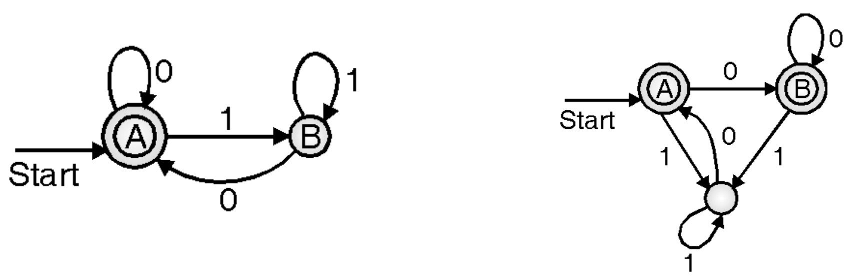


Figure 3.40: Example DFAs

Solution:

i) The state equations for the given DFA are:

$A = \in + A0 + B0$

$B = A1 + B1$

$B = A11^*$ ... using Arden's Theorem

Substituting for B in A,

$A = \in + A0 + A11^*0$

$= \in + A (0 + 11^*0)$

$= \in (0 + 11^*0)^*$ ... using Arden's Theorem

Hence, $A = (0 + 11^*0)^*$

A being the final state, regular expression for the given DFA is $(0 + 11^*0)^*$.

ii) Let the third state label be *C*.

The state equations for the given DFA are:

$A = \in + C0$

$B = A0 + B0$

$C = A1 + B1 + C1$

Let us try to simplify the equations.

$B = A0 + B0$

$= A00^*$ ... using Arden's Theorem

Substituting B in C we get,

$C = A1 + A00^*1 + C1$

$= A\,(1 + 00^*1) + C1$

$= A\,(1 + 00^*1)\,1^*$ ... using Arden's Theorem

Substituting C in A we get,

$A = \in + C0$

$= \in + A\,(1 + 00^*1)\,1^*\,0$

$= ((1 + 00^*1)\,1^*\,0)^*$

Therefore, $B = ((1 + 00^*1)\,1^*\,0)^*\,00^*$

Both *A* and *B* are the final states for the DFA. Hence, the regular expression pertaining to the DFA is,

$A + B$

$= ((1 + 00^*1)\,1^*\,0)^*\,(\in + 00^*)$

Q.26 Which of the following languages are regular sets? Justify your answer.

(i) $\{0^{2n} \mid n \geq 1\}$

(ii) $\{0^m\, 1^n\, 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$

Solution:

(i) It is given that $n \geq 1$.

For n=1, $0^{2n} = 0^2$, length = 2

For n=2, $0^{2n} = 0^4$, length = 4

For n=3, $0^{2n} = 0^6$, length = 6

Hence, length of each string is multiples of 2 which is even length.

The language $\{0^{2n} \mid n \geq 1\}$ is a regular language that can be denoted by the regular expression, $(00)^+$.

(ii) $\{0^m 1^n 0^{m+n} \mid m \geq 1$ and $n \geq 1\}$ is not a regular set. Refer to the answer for the question 3.22 above.

Q.27 Find out whether given languages are regular or not:

(1) $L = \{ww \mid w \in \{0, 1\}^*\}$

(2) $L = \{1^k \mid k = n^2, n >= 1\}$

Solution:

Both the given language are not regular.

(1) Refer to the example 3.45 from the book.

(2) Refer to the example 3.43 from the book.

Q.28 With the help of a suitable example, prove: 'regular sets are closed under union, concatenation, and Kleene closure'.

Solution:

Refer to the section 3.5.2.

Q.29 Explain the following applications of regular expressions:

(1) grep utility in UNIX

(2) Finding pattern in text

Solution:

(1) grep utility in UNIX: Refer to the section 3.8.3.

(2) Finding pattern in text: Refer to the section 3.8.2.

Q.30 Construct the NFA and DFA for the following languages:

(i) $L = \{x \in \{a, b, c\}^* \mid x$ contains exactly one b immediately following $c\}$

(ii) $L = \{x \in \{0, 1\}^* \mid x$ starts with 1 and $|x|$ is divisible by 3$\}$

(iii) $L = \{x \in \{a, b\}^* \mid x$ contains any number of a's followed by at least one $b\}$

Solution:

The regular expressions denoting the languages mentioned are,

(i) $r = (a + b + cb)^*$

(ii) $r = 1\ (0+1)\ (0+1)[(0+1)\ (0+1)\ (0+1)]^*$

(iii) $r = a^*bb^*$

For NFA/DFA construction refer to the section 3.4.2.

Q.31 Let $\Sigma = \{0, 1\}$. Construct regular expressions for each of the following:

(a) $L_1 = \{W = \Sigma^* \mid W$ has at least one pair of consecutive zeros$\}$

(b) $L_2 = \{W \in \Sigma^* \mid W$ has no pair of consecutive zeros$\}$

(c) $L_3 = \{W \in \Sigma^* \mid W$ starts with either '01' or '10'$\}$

(d) $L_4 = \{W \in \Sigma^* \mid W$ consists of even number of 0's followed by odd number of 1's$\}$

Solution:

(a) $r = [\ (1 + 0\ )^*\ (00)\ (1 + 0\ )^*]^+$

(b) $r = (0 + \in)\ (1+10)^*$

(c) $r = (01 + 10)\ (1+0)^*$

(d) $r = (00)^*\ 1\ (11)^*)$

Q.32 Construct a regular expression for the following DFA:

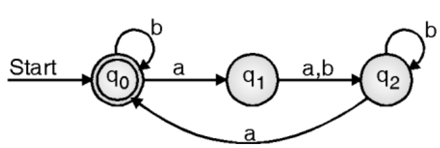


Figure 3.41: Example DFA

Solution:

The state equations for the given DFA are:

$q_0 = q_0\,b + q_2\,a + \in$

$q_1 = q_0\,a$

$q_2 = q_1\,a + q_1\,b + q_2\,b$

Substituting for q_1 in q_2,

$q_2 = q_0\,aa + q_0\,ab + q_2\,b$

$= q_0\,a\,(a + b) + q_2\,b$

$q_2 = q_0\,a\,(a + b)b^*$ ... using Arden's Theorem

Substituting for q_2 in q_0,

$q_0 = q_0\,b + q_0\,a\,(a + b)b^*\,a + \in$

$= q_0\,(b + a\,(a + b)b^*\,a\,) + \in$

$q_0 = \in (b + a\,(a + b)b^*\,a\,)^*$ ... using Arden's Theorem

Hence, $q_0 = (b + a\,(a + b)b^*\,a\,)^*$

q_0 being the only final state for the DFA, regular expression is,

$(b + a\,(a + b)b^*\,a\,)^*$

Q.33 Let $L = \{0^n \mid n$ is a prime number$\}$; show that L is not regular.

Solution:

Length of every string in L is a prime number.

Step 1: Let us assume that the language L is a regular language. Let n be the constant of the pumping lemma.

Step 2: Let us choose a sufficiently large string z such that $z = 0^l$, for some large $l > 0$; the length of z is given by: $|z| = l \geq n$.

Since we assumed that L is a regular language and from the language definition it is an infinite language, we can now apply pumping lemma.. This means that we should be able to write z as: $z = uvw$.

Step 3: As per pumping lemma, every string 'uv^iw', for all $i \geq 0$, is in L. Likewise, $|v| \geq 1$, which means that v cannot be empty and must contain one or more symbols.

Let us consider the case when v contains a single symbol:

In this case, $z = uvw = 0^l$, which means that the number of 0's in z is a prime number. As per pumping lemma, we would expect 'uv^2w' also to be a member of L; however, this cannot be possible, as v contains only a single symbol, and adding one to the prime number length would not always yield perfect prime length. Thus, pumping v would yield strings with non-prime lengths. Thus, 'uv^2w' is not a member of L. This contradicts our assumption that L is regular.

Let us now consider the case when v contains perfect prime number of 0's. A sample v could be written as: '000' (three 0's), or '00000' (five 0's), and so on. When we try to pump v multiple times, such as, for example, v^2 = 000000 (six 0's), or v^2 = 0000000000 (10 0's), and so on, we find that the length does not remain a perfect prime, and we get a string which is against the language definition, which is '0^i'. Thus, we can say that 'uv^2w' is not a member of L. This contradicts our assumption that L is regular.

Similarly, if we consider that v contains any number of 0's, then on pumping it we will get into a situation where the string has non-prime length, which is against the language definition. For example, if v contains 2 zeros and if we pump it say 2 times, we will get the string "0000" which does not have a perfect prime length.

Hence, the language $L = \{0^n \mid n \textit{ is a prime number}\}$ is non-regular.

Q.34 Prove or disprove the following for regular expressions r, s and t.

(a) $(rs + r)^* r = r (sr + r)^*$

(b) $s (rs + s)^* r = rr^* s (rr^* s)^*$

(c) $(r + s)^* = r^* + s^*$

(d) $(r^* s^*)^* = (r + s)^*$

Solution:

(a) Let $r_1 = (rs + r)^* r$, hence $L(r_1)$ = {r, rsr, rr, rrr, rrrr, rsrr, rsrsr, rsrsrr, ... }

Let $r_2 = r (sr + r)^*$, hence $L(r_2)$ = {r, rsr, rr, rsrr, rsrsr, rsrsrr, rrr, rrrr, ... }

As the language denoted by r_1 and r_2 is same, we can say that r1= r2. Thus, (rs + r)* r = r (sr + r)*

(b) Let $r_1 = s (rs + s)^* r$, hence $L(r_1)$ = {sr, srsr, srsrsr, srsrsr, ssr,sssr,ssssr,srssr, ... }

Let $r_2 = rr^* s (rr^* s)^*$, hence $L(r_2)$ = {rs, rrs, rrsrrs, rsrs,rrrs,rrrsrrs, ... }

As the languages denoted by r_1 and r_2 are not same. Hence, $s (rs + s)^* r \neq rr^* s (rr^* s)^*$

(c) Let $r_1 = (r + s)^*$, hence, $L(r_1)$ = {r, s, rr, rs, sr, ss, rrr, sss, rss, ... }

Let $r_2 = r^* + s^*$, hence, $L(r_2)$ = {r, s, rr, ss, rrr, sss, rrrr, ... }

The strings like 'rs', 'sr' 'rss' and so on are not part of $L(r_2)$. Hence, $(r + s)^* \neq r^* + s^*$

(d) Let $r_1 = (r^* s^*)^*$, hence, $L(r_1)$ = {r, s, rs, rsrs, rrr, ss, sss, rrs, rrrs, rsss, rsssrss, ... }

Let $r_2 = (r + s)^*$, hence, $L(r_2)$ = {r, s, rr, rrr, ss, sss, ssss, rrs, rsr, rrrs, rssrss, ... }

As the language denoted by r_1 and r_2 is same, we can say that r1= r2. Thus, $(r^* s^*)^* = (r + s)^*$

Q.35 State whether each of the following statements is true of false. Justify your answer. Assume that all languages are defined over the alphabet {0, 1}.

(a) If ($L1 \subseteq L2$) and ($L1$ is not regular), then $L2$ is not regular

(b) If ($L1 \subseteq L2$) and ($L2$ is not regular), then $L1$ is not regular

(c) If $L1$ and $L2$ are not regular, then ($L1 \cup L2$) is not regular

Solution:

(i) If ($L1 \subseteq L2$) and $L1$ is not regular, then $L2$ is not regular.

The statement is not always true, that means, it is false. Let us consider the example of following languages $L1$ and $L2$,

Let, $L1$ = $\{0^n1^n \mid n >= 0\}$

= $\{\in, 01, 0011, 000111, \ldots\}$

$L1$ here is not a regular language; $L1$ actually is a CFL.

Let, $L2$ = 0^*1^*

= $\{\in, 0, 1, 00, 01, 10, 11, 000, \ldots, 0011, \ldots\}$

$L2$ is a regular language as we know.

Thus, even though ($L1 \subseteq L2$) and $L1$ is not regular, $L2$ is regular.

Hence, the statement is false.

(ii) If ($L1 \subseteq L2$) and $L2$ is not regular, then $L1$ is not regular.

The statement is false.

Let us consider the example of following languages $L1$ and $L2$,

Let, $L2$ = {set of all palindrome strings over {0, 1}}

$= \{\in, 0, 1, 00, 11, 000, 010, 101, 111, 0000, \ldots\}$

$L2$ here is not a regular language; $L2$ actually is a CFL.

Let, $L1 = \{\in, 0, 1, 00, 11, 000, 111, 0000, \ldots\}$

$L1$ thus contains strings consisting of all 0's or 1's or an empty string. $L1$ is actually a regular language and we can denote it using a regular expression, $r = 0^* + 1^*$.

Thus, even though ($L1 \subseteq L2$) and $L2$ is not regular, $L1$ is regular.

Hence, the statement is false.

(iii) If $L1$ and $L2$ are not regular, then ($L1 \cup L2$) is not regular.

The statement is true. As we know most of the languages are closed under union. For example, if we take union of two CFLs the result is also a CFL.

Q.36 Use pumping lemma to check whether the language, $L = \{ ww \mid w \in \{0, 1\}^* \}$ is regular or not.

Solution:

Refer to the example 3.45 from the book.

