Q. Simplify the given expression and implement using logic gates

1. $F = A\bar{B}C + \bar{A}BC + ABC$

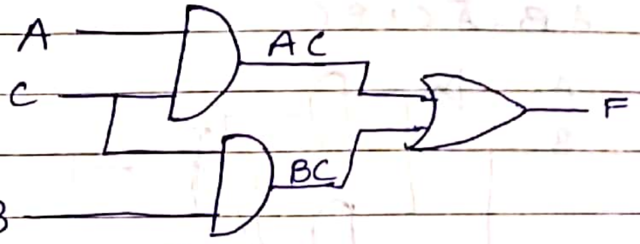
$F = AC(B+\bar{B}) + \bar{A}BC$

$F = AC + \bar{A}BC$

$F = \cancel{A}C(A + \bar{A}B)$

$F = C(A+B)$

$F = AC + BC$ = $C(A+B)$



2. $F = (X+\bar{Z})(\overline{Z+WY}) + (VZ+W\bar{X})(\overline{Y+Z})$

$F = (X+\bar{Z})(\bar{Z}(\bar{W}+\bar{Y})) + (VZ+W\bar{X})(\bar{Y}\cdot\bar{Z})$

$F = (X+\bar{Z})(\bar{Z}\bar{W}+\bar{Y}\bar{Z}) + (VZ\bar{Y}\bar{Z} + W\bar{X}\bar{Y}\bar{Z})$

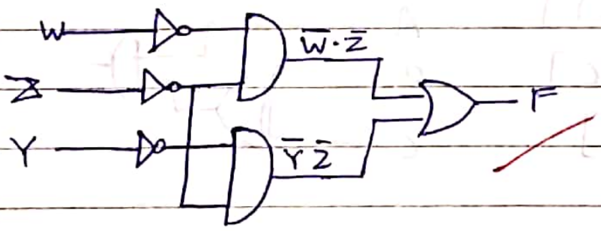
$F = X\bar{Z}\bar{W} + X\bar{Y}\bar{Z} + \bar{Z}\bar{Z}\bar{W} + \bar{Z}\bar{Z}\bar{Y} + V\bar{Y}\bar{Z}Z + W\bar{X}\bar{Y}\bar{Z}$

$F = X\bar{Z}\bar{W} + X\bar{Y}\bar{Z} + \bar{W}\bar{Z} + \bar{Y}\bar{Z} + \bar{Y}\bar{X}\cancel{\bar{Y}}\bar{Z}W$

$F = \bar{Z}\bar{W}(1+X) + \bar{Y}\bar{Z}\overset{(X+\bar{X}W)}{\cancel{(X+1+X)}} + \cancel{\bar{Y}\bar{X}\bar{Z}W}\ \bar{Y}\bar{Z}$

$F = \bar{Z}\bar{W} + X\bar{Y}\bar{Z} + W\bar{X}\bar{Y}\bar{Z} + \bar{Y}\bar{Z}$

$F = \bar{Z}\bar{W} + \bar{Y}\bar{Z}$ = $\bar{Z}\cdot(\bar{W}+\bar{Y})$ 1 AND gate and 1 OR gate.



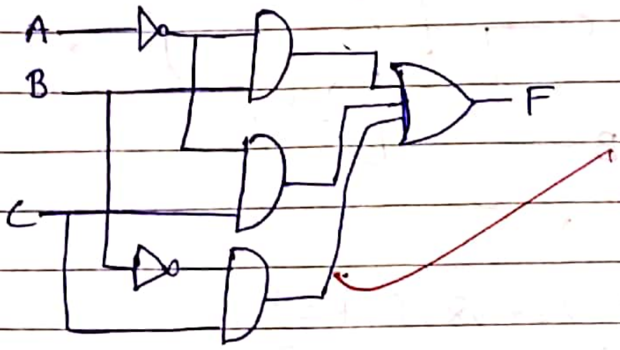
Use minimum no. of gates.

3. $\overline{AB}(B+C)+AB(\overline{B+C})$

$(\bar{A}+\bar{B})(B+C)+AB(\bar{B}\cdot\bar{C})$

$(\bar{A}B+B\bar{B}+\bar{B}C)+AB\bar{B}\bar{C}$

$\bar{A}B+\bar{A}C+\bar{B}C$



4. $A[B+C(AC+\overline{AB})]$

$A[B+\cancel{AC+}\,C(AC+\bar{A}+\bar{B})]$

$A[B\;AB+A\,C(AC+\bar{A}+\bar{B})$

$AA+$

4. $A[B+C(AC+\overline{AB})]$

$A[B+C(AC+\bar{A}+\bar{B})]$

$A[B+AC+\bar{A}C+\bar{B}C]$

$A[B+C(A+\bar{A}+\bar{B})]$

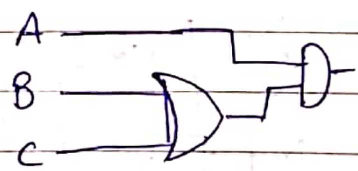
$A[B+C(1+\bar{B})]$

$A[B+C+\bar{B}C)$

$A[B+C]$

$AB+AC$

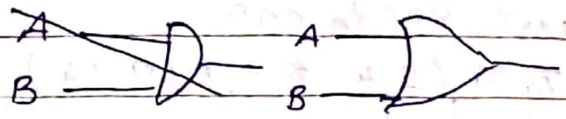
$A(B+C)$



5. $A + \bar{A}B + AB$

$A + B(A + \bar{A})$

$A + B$



6. $A\bar{B} + \bar{A}B + AB + \bar{A}\bar{B}$

$A(B + \bar{B}) + \bar{A}(B + \bar{B})$

$A + \bar{A} = \underline{1}$

26/7/19.

Tutorial 2

Q1. Convert POS to SOP

$Y = \pi M(2, 4, 5, 7, 10, 13, 14)$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

8	4	2	1	
D	C	B	A	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
				5
				6
				7
1	0	0	0	8

~~$Y = AB\bar{C} + A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C$~~

$Y = (A+B+C)(A+\bar{B}+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

$= (A\cdot A + A\bar{B} + AC + AB + B\bar{B} + BC + AC + \bar{B}C + C\cdot C)(\bar{A}\cdot\bar{A} + \bar{A}\bar{B} + \bar{A}C$
$+ \bar{A}B + B\bar{B} + BC + \bar{A}\bar{C} + \bar{B}\bar{C} + C\bar{C})$

$= (A + AB + A\bar{C} + AB + B\bar{C} + AC + \bar{B}C)(\bar{A} + \bar{A}\bar{B} + \bar{A}C + \bar{A}B$
$+ BC + \bar{A}\bar{C} + B\bar{C})$

$= (A + A(B+\bar{B}) + A(C+\bar{C}) + B\bar{C} + \bar{B}C)(\bar{A}(1 + \bar{B} + C + B + \bar{C})$
$+ BC + \bar{B}C)$

$= (A + A + A + B\bar{C} + \bar{B}C)(\bar{A} + BC)$

$= (A + B\bar{C} + \bar{B}C)(\bar{A} + C)$

$= A\bar{A} + AC + \bar{A}B\bar{C} + B\bar{C}\bar{C} + \bar{A}\bar{B}C + \bar{B}CC$

Q1. Convert POS to SOP

$Y = \pi M(2, 4, 8, 9, 10, 13, 14)$

~~A B C D~~

D	C	B	A	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	1	0

$$Y = (A+B+\bar{C}+D)(A+\bar{B}+C+D).(\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})$$
$$(\bar{A}+B+\bar{C}+D)(\bar{A}+\cancel{\bar{B}+C}+\bar{D})(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

Q2. Express the eqn in SOP form

$F(A,B,C,D) = \pi_M (0, 2, 5, 7)$

$\Rightarrow (A+B+C)(A+\bar{B}+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$\Rightarrow (A + A\bar{B} + AC + AB + B\bar{B} + BC + AC + \bar{B}C + C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

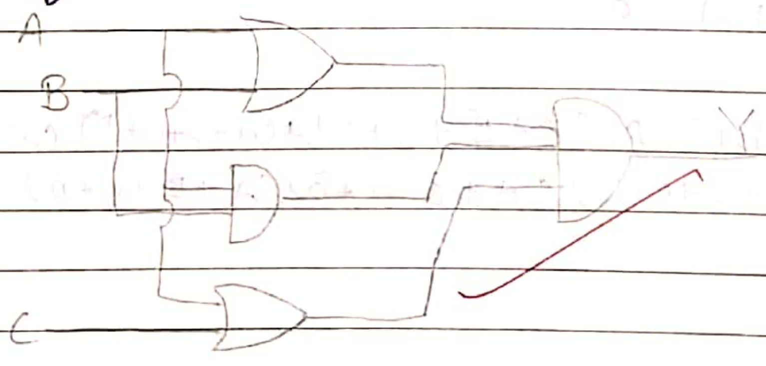
$\Rightarrow [A(1+\bar{B}+C+B) + B\bar{B} + C(1+\bar{B}+A+B)] \cdot (\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}B + B\bar{B}$
$+ B\bar{C} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{C}\bar{C}]$

$\Rightarrow (A+C)[\bar{A}(1+\bar{B}+\bar{C}+B) + \bar{C}(B+\bar{A}+\bar{A}+1)]$

$\Rightarrow (A+C)(\bar{A}+\bar{C})$

$\Rightarrow \boxed{A\bar{C} + \bar{A}C} \Rightarrow$ standard SOP $\Rightarrow \sum m(1,3,4,6)$.

Q 3. Simplify it



$Y = (A+B)(A \cdot B)(A+C)$

$= (A \cdot AB + ABB)(A+C)$

$= (AB + AB)(A+C)$

$= AB(A+C)$

$= AB + ABC$

$= \boxed{AB}$

Q4. Simplify 3 variable logic expression

$Y = \pi_M(1,3,5)$

$(A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})$

$[A \cdot A + A\bar{B} + A\bar{C} + AB + B\bar{C} + A\bar{C} + \bar{B}C + \bar{C}][\bar{A}+\bar{B}+\bar{C}]$

$[A(1+B+\bar{C}+\bar{B}) + \bar{C}(1+B+\bar{B}+A)](\bar{A}+B+\bar{C})$

$[A+\bar{C}][\bar{A}+B+\bar{C}]$

$[A\bar{A} + AB + A\bar{C} + \bar{A}\bar{C} + B\bar{C} + \bar{C}\bar{C}]$

$\bar{C}(1+B+\bar{A}+A) + AB$

$\boxed{AB+\bar{C}}$

Q5 Simplify

$$(C+\bar{C}D)(C+\bar{C}\bar{D})[(AB+\bar{A}\bar{B})(A\oplus B)]$$

$$[C+C\bar{C}\bar{D}+C\bar{C}D+\bar{C}\bar{D}D][(AB+\bar{A}\bar{B})(A\oplus B)]$$

$$C[(AB+\bar{A}\bar{B})(\bar{A}B+A\bar{B})]$$

$$C[A\bar{A}B+AB\bar{B}+\bar{A}\bar{B}B+A\bar{A}B]$$

$$\underline{0}$$

9/8/2012

PAGE No.
DATE 82

TUTORIAL-3

Q. Minimize the following boolean expression using realize it using logic gate

① $Y = \Sigma m(1, 3, 5, 9, 11, 13)$

② $Y = \Sigma m(1, 2, 9, 10, 11, 14, 15)$

③ $Y = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15)$

④ $Z = f(A, B, C, D) = \Sigma m(0, 2, 4, 7, 11, 13, 15)$

① $Y = \Sigma m(1, 3, 5, 9, 11, 13)$

pair1 → $\bar{C}D$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1: 1	3: 1	2
$\bar{A}B$ 01	4	5: 1	7	6
AB 11	12	13: 1	15	14
$A\bar{B}$ 10	8	9: 1	11: 1	10

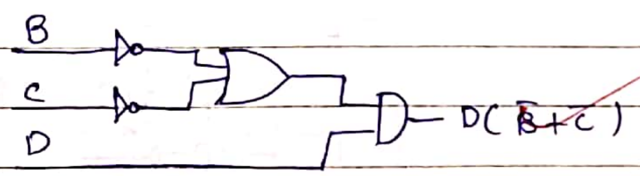
pair2 → $\bar{B}D$

$$Y = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + AB\bar{C}D + A\bar{B}\bar{C}D + A\bar{B}CD$$

$$= \bar{A}\bar{B}D[C+\bar{C}] + B\bar{C}D[A+\bar{A}] + A\bar{B}D[C+\bar{C}]$$

$$= \bar{A}\bar{B}D + B\bar{C}D + A\bar{B}D$$

$$= \bar{B}D + B\bar{C}D \Rightarrow D(\bar{B}+\bar{C})$$

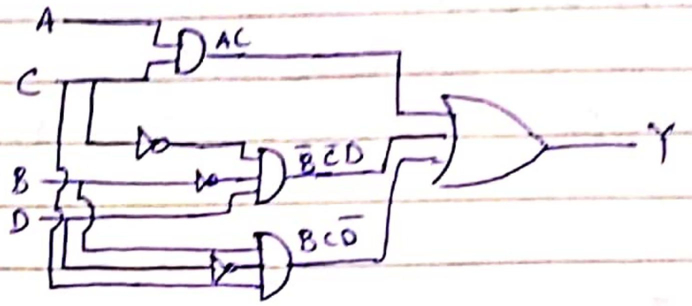


Scanned by CamScanner

② $Y = \Sigma m(1,2,9,10,11,14,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00		1		1
$\bar{A}B$ 01				
AB 11			1	1
$A\bar{B}$ 10		1	1	1

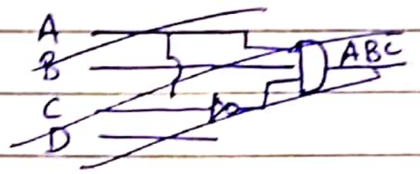
$Y = \bar{B}\bar{C}D + AC + B C\bar{D}$ $\Rightarrow AC + \bar{B}[\bar{C}D + C\bar{D}] = AC + \bar{B}[C \oplus D]$

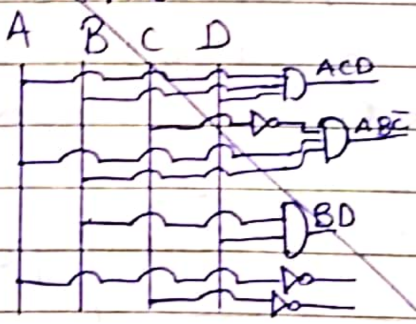


③ $Y = \Sigma m(1,5,6,7,11,12,13,15)$

AB \ CD	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

$Y = BD + AB\bar{C} + ACD + \bar{A}\bar{C}D + \bar{A}BC$

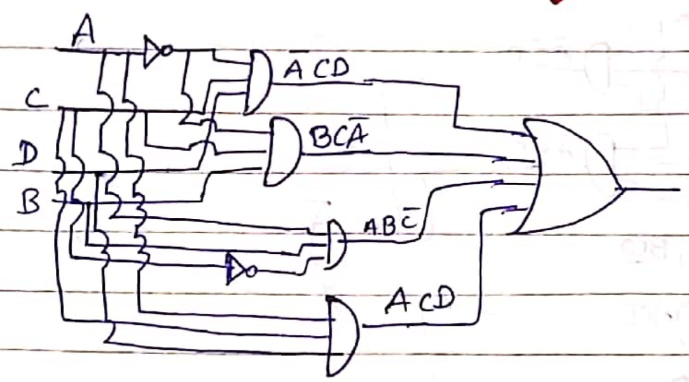




③ $Y = \Sigma_m (1, 5, 6, 7, 11, 12, 13, 15)$

AB \ CD	$\bar C \bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A \bar B$ 00		1		
$\bar A B$ 01		1	1	1
AB 11	1	1	1	
$A\bar B$ 10			1	

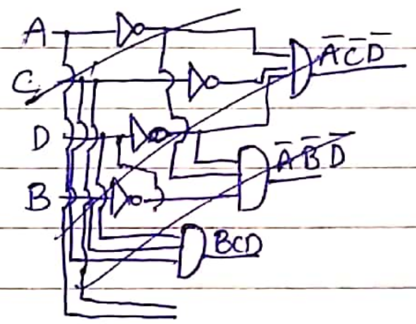
$Y = \bar A \bar C D + \bar A B C + A B \bar C + A C D$

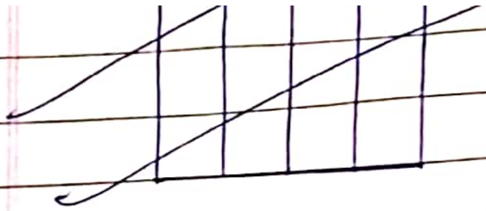


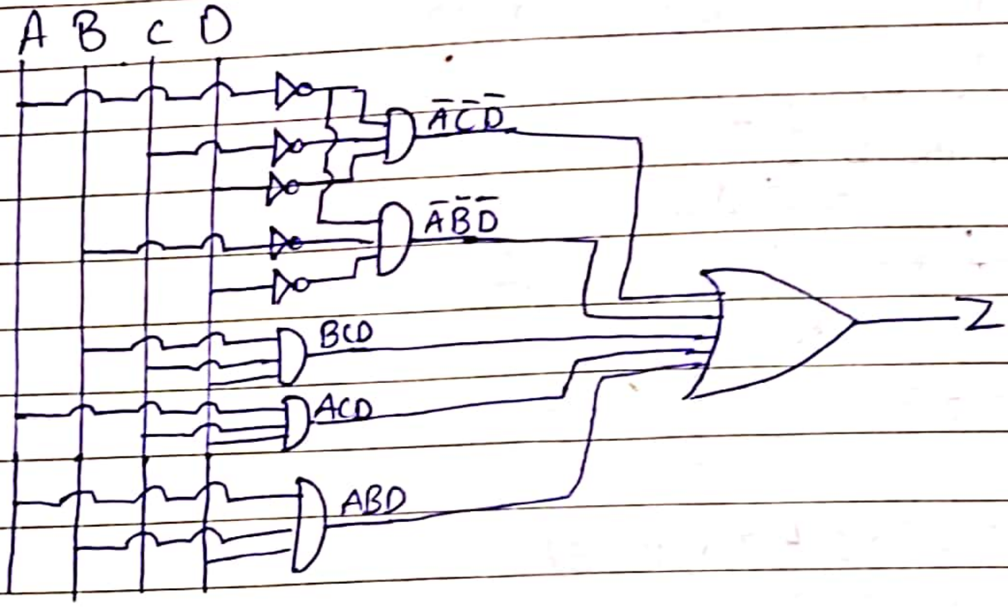
④ $Z = \Sigma_m (0, 2, 4, 7, 11, 13, 15)$

AB \ CD	$\bar C \bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A \bar B$ 00	1			1
$\bar A B$ 01	1		1	
AB 11		1	1	
$A\bar B$ 10			1	

$Z = \bar A \bar C \bar D + \bar A \bar B \bar D + BCD + ACD + ABD$







9/8/2019

PAGE No.
DATE 14/8/19

TUTORIAL-4

Q1. Simplify Using K-Map And Implement Using Logic Gates.

① $Y = \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + A\bar{C} + \bar{B}$

② $Y = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$
$+ A\bar{B}\bar{C}D + A\bar{B}C$

③ $F(P,Q,R,S) = \sum m(2,3,6,7,8,9,10,11)$

④ $F(P,Q,R,S) = \prod M(3,4,5,6,7,10,11,15)$

⑤ $f(A,B,C,D) = \sum m(2,4,5,6,7,8,9,10,11,12)$

⑥ $F(A,B,C,D) = \sum m(1,3,4,6,9,11,12,14)$

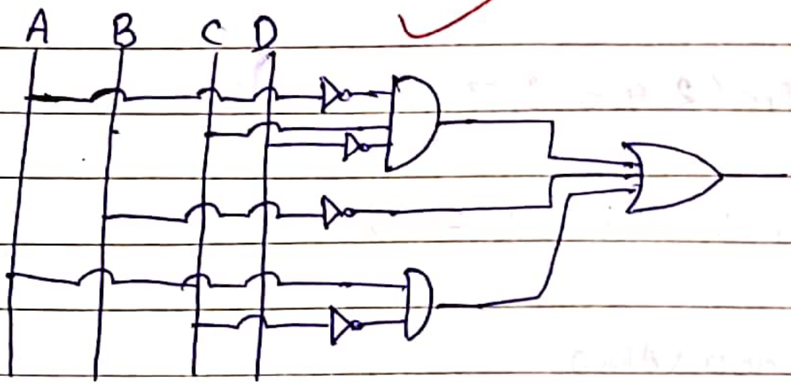
⑦ Given: $A + \bar{B}\bar{C} + AB\bar{D} + ABCD$
① Design K-MAP
② Express the given Expression is std SOP
③ Express the given Expression in std POS
④ Implement simplified Eqn using Logic Gates

⑧ Given: $Y = (A+B+\bar{C}+\bar{D})(\bar{A}+C+\bar{D})(\bar{B}+C)(\bar{B}+\bar{C})(A+\bar{B})(\bar{B}+\bar{D})$
① Design K-MAP and simplify
② Express in std POS
③ Express in std SOP
④ Impliment simplified eqn using logic gates

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01				1
AB 11	1	1		
$A\bar{B}$ 10	1	1	1	1

$$Y = \bar{A}\bar{B} + \bar{A}C\bar{D} + A\bar{C} + A\bar{B}$$

$$= \bar{B} + \bar{A}C\bar{D} + A\bar{C}$$

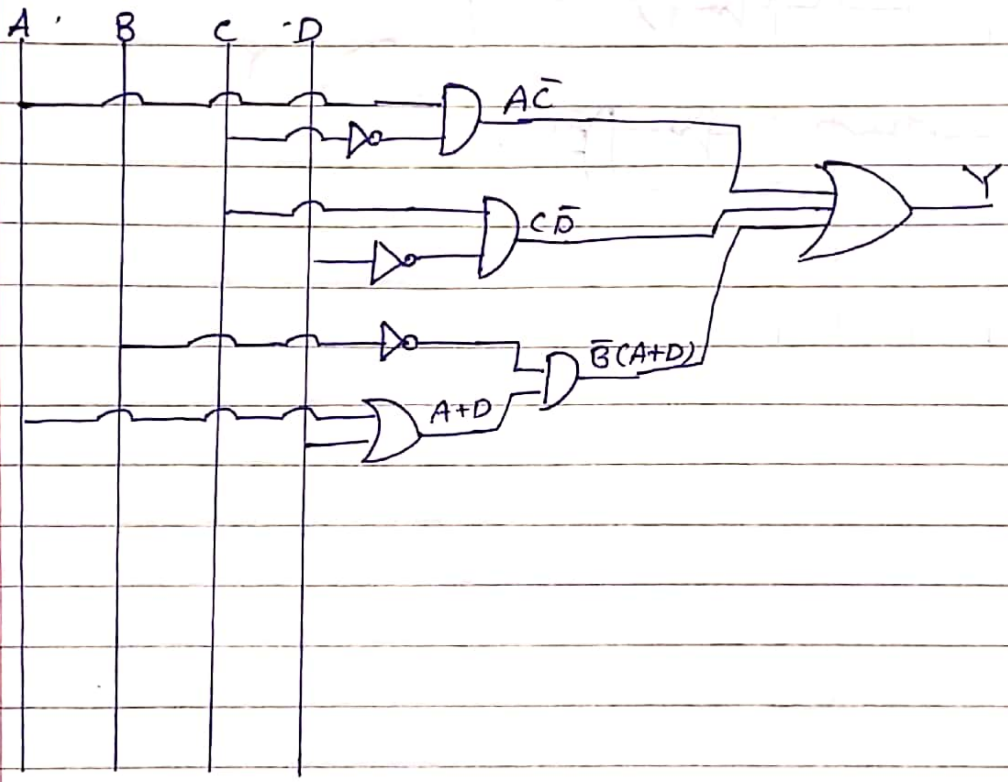


PAGE No.
DATE / /

(2) $Y = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + AB\bar{C}D + ABC\bar{D} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}D$
$+ A\bar{B}C$

AB \ CD	00	01	11	10
00		1	1	1
01				1
11	1	1		1
10	1	1	1	1

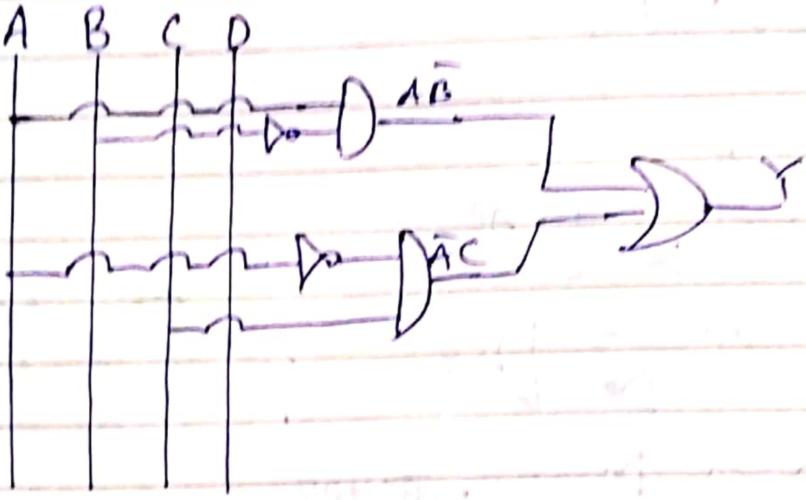
$Y = A\bar{C} + A\bar{B} + C\bar{D} + \bar{B}D$
$= A\bar{C} + C\bar{D} + \bar{B}(A+D)$



3 $F(A,B,C,D) = \Sigma m(2,3,6,7,8,9,10,11)$

AB \ CD	00	01	11	10
00	0	1	3 1	2 1
01	4	5	7 1	6 1
11	12	13	15	14
10	8 1	9 1	11 1	10 1

$Y = A\bar{B} + \bar{A}C$



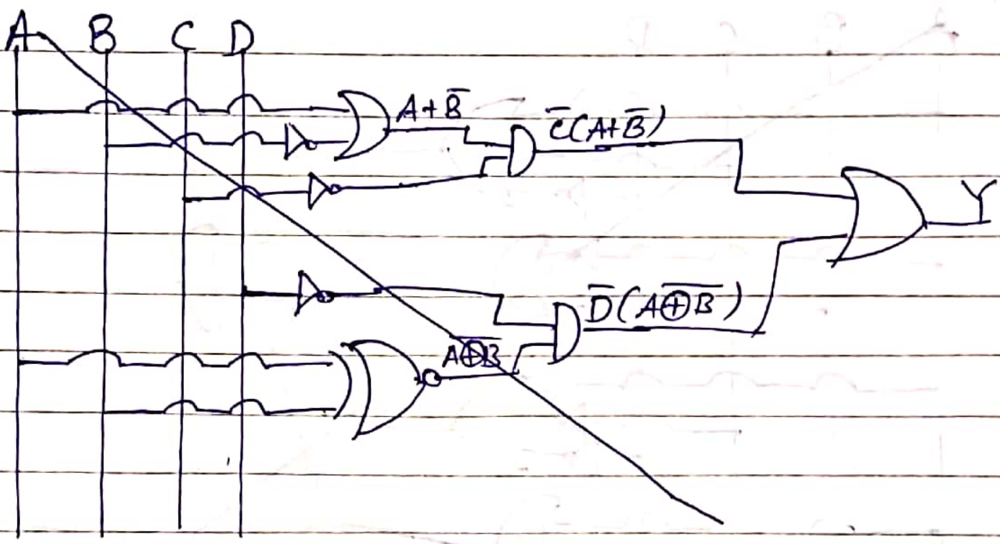
4. $F(A,B,C,D) = \Pi M(3,4,5,6,7,11,10,15)$

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	1	0	1
10	1	1	0	0

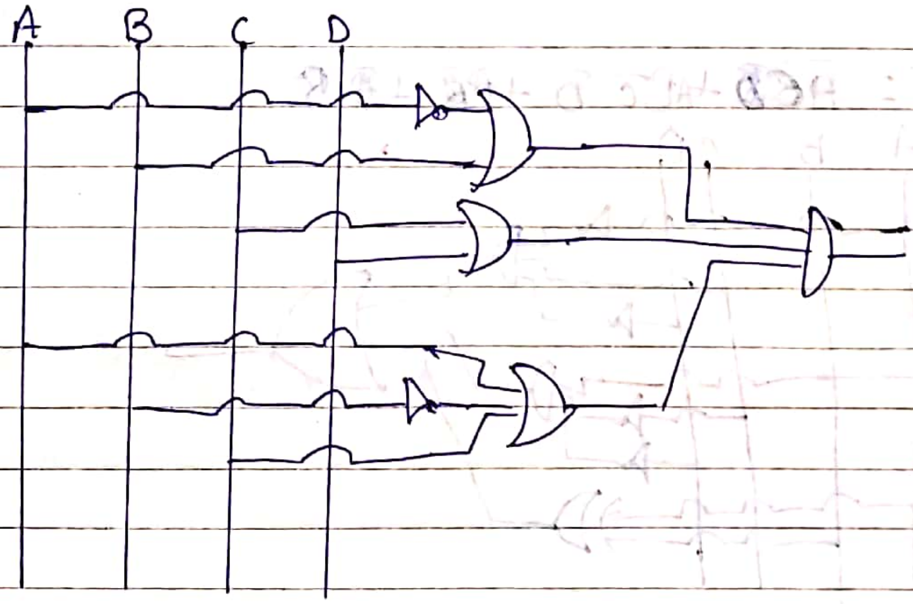
$Y = A\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + AB\bar{D}$

$= \bar{C}(A+\bar{B}) + \bar{D}(AB+\bar{A}\bar{B})$

$= \bar{C}(A+\bar{B}) + \bar{D}(\overline{A\oplus B})$



$Y = (\bar{A}+B)(C+D)(A+\bar{B}+C)$



5. $F(A,B,C,D) = \Sigma m(2,4,5,6,7,8,9,10,11,12)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	1 (2)
$\bar{A}B$ 01	1 (4)	1 (5)	1 (7)	1 (6)
AB 11	1 (12)	13	15	14
$A\bar{B}$ 10	1 (8)	1 (9)	1 (11)	1 (10)

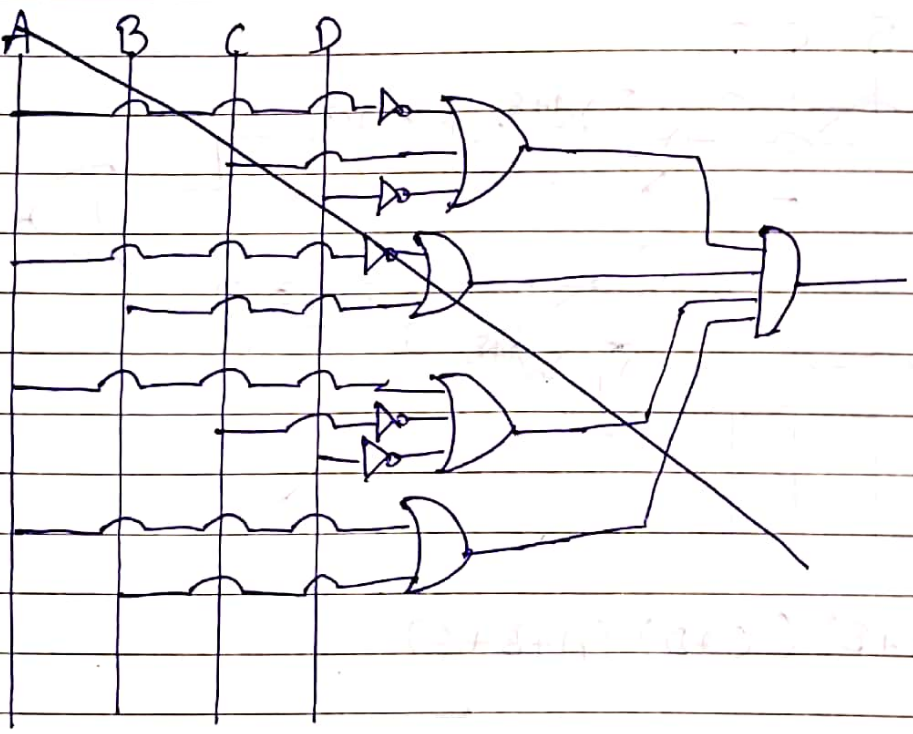
$Y = (A+C+\bar{D})\cdot(\bar{A}+B)\cdot(A+\bar{C}+\bar{D})\cdot(A+\bar{B})$

$= (\bar{A}\bar{A}+\bar{A}C+\bar{A}\bar{D}+\bar{A}B+BC+B\bar{D})$

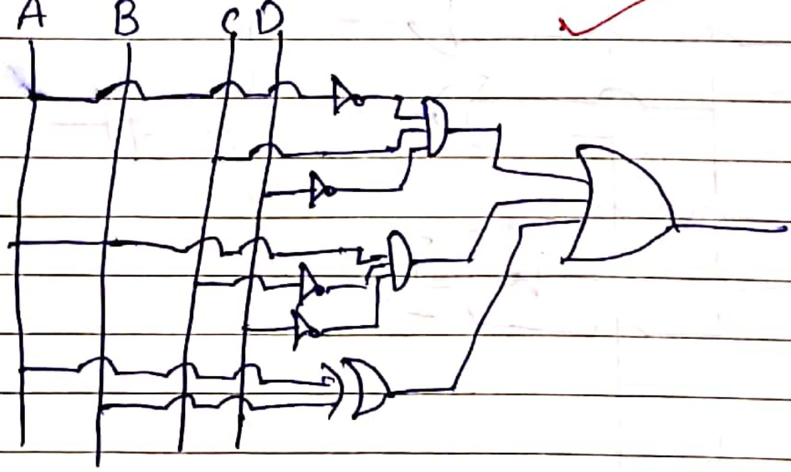
$(AA+A\bar{B}+A\bar{C}+\bar{B}\bar{C}+A\bar{D}+\bar{B}\bar{D})$

$= (\bar{A}+\bar{A}C+\bar{A}D+\bar{A}B+BC+B\bar{D})$

$(A+A\bar{B}+A\bar{C}+\bar{B}\bar{C}+A\bar{D}+\bar{B}\bar{D})$



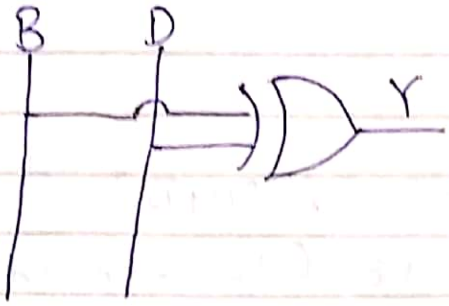
$Y = \bar{A}C\bar{D} + B\bar{C}\bar{D} + A\bar{B} + \bar{A}B$



6. $F(A,B,C,D) = \Sigma m(1,3,4,6,9,11,12,14)$

AB \ CD	00	01	11	10
00	(0)	1 (1)	1 (3)	(2)
01	1 (4)	(5)	(7)	1 (6)
11	1 (12)	(13)	(15)	1 (14)
10	(8)	1 (9)	1 (11)	(10)

$Y = B\bar{D} + \bar{B}D = B \oplus D$



7. $A + \bar{B}\bar{C} + AB\bar{D} + ABCD$

(1)

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	1	0	1	1
$A\bar{B}$	1	1	0	0

Simp. $= \bar{A} + \bar{B}\bar{C}D + ABC + A\bar{C}\bar{D}$

(2) $A + \bar{B}\bar{C} + AB\bar{D} + ABCD$

$\Rightarrow A(B+\bar{B})(C+\bar{C})(D+\bar{D}) + AB\bar{D}(C+\bar{C}) + ABCD$

$\Rightarrow (AB+A\bar{B})(CD+C\bar{D}+\bar{C}D+\bar{C}\bar{D}) + ABC\bar{D} + AB\bar{C}\bar{D} + ABCD$

$\Rightarrow ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + ABC\bar{D}$
$+ AB\bar{C}\bar{D} + ABCD$

$\Rightarrow ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}.$

③ ~~$A+B+\bar{C}+D$~~

7. $A+\bar{B}\bar{C}+AB\bar{D}+ABCD$

①

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	0	0	0	0
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

$Y = A + \bar{B}\bar{C}$

② $Y = A + \bar{B}\bar{C}$

$= A(B+\bar{B})(C+\bar{C})(D+\bar{D}) + \bar{B}\bar{C}(A+\bar{A})(D+\bar{D})$

$= (AB + A\bar{B})(CD + C\bar{D} + \bar{C}D + \bar{C}\bar{D}) + \bar{B}\bar{C}(AD + A\bar{D} + \bar{A}D + \bar{A}\bar{D})$

$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}$

$+ A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

③ ~~$Y = (\bar{A}+B+C)(\bar{A}+C)$~~

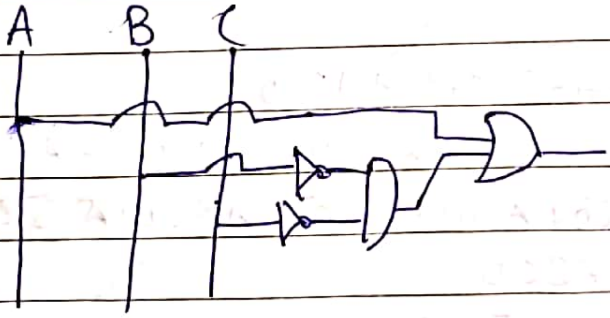
~~$= (\bar{A}+B+C+D\cdot\bar{D})(\bar{A}+C+B\cdot\bar{B}+D\cdot\bar{D})$~~

$Y = \pi M(2, 3, 4, 5, 6, 7)$

$= (\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+B+\bar{C}+D)'$

$(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+D)$

④



PAGE No.
DATE / /

8. $Y = (A+B+\bar{C}+\bar{D})(\bar{A}+C+\bar{D})(\bar{B}+C)(\bar{B}+\bar{C})(A+\bar{B})(\bar{B}+\bar{D})$

1)

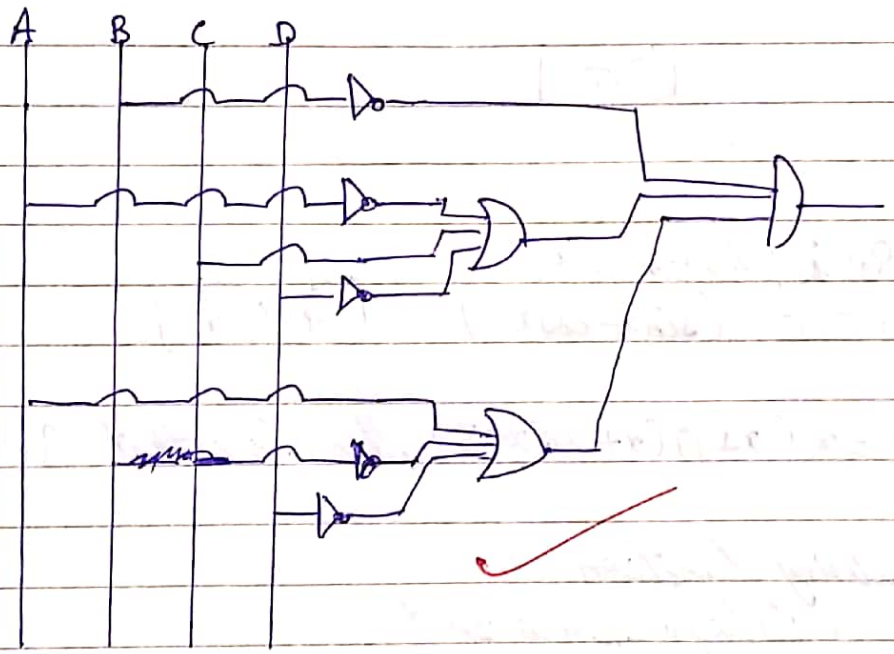
AB \ CD	$\bar{C}+\bar{D}$	$\bar{C}+D$	$C+D$	$C+\bar{D}$
$\bar{A}+\bar{B}$	0	0	0	0
$\bar{A}+B$				0
$A+B$	0			
$A+\bar{B}$	0	0	0	0

$Y = (\bar{B} \cancel{+} \cdot (\bar{A}+C+\bar{D})(A+\bar{C}+\bar{D})$

2) $Y = (\bar{A}+\bar{B}+\bar{C}+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})$
$(\bar{A}+B+C+\bar{D})(A+B+\bar{C}+\bar{D})(A+\bar{B}+\bar{C}+\bar{D})(A+\bar{B}+\bar{C}+D)$
$(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$

3) $Y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD + ABC\bar{D}$ ~~+ABC~~

4)



A Pamji
16/8/19.

Scanned by CamScanner

DATE 4/9/2019

Tutorial - 5

Q1. Is Rolle's Theorem applicable to following function :-

1. $\log\left[\frac{x^2+6}{5x}\right]$ $(2,3)$

2. $1-\sqrt[3]{(x-1)^2}$ $(0,2)$

3. $f(x) = \begin{cases} x^2-2 & ; -1 \le x \le 0 \\ x-2 & ; 0 \le x \le 1 \end{cases}$

4. $\cos^2 x$ $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

5. $(x-1)(x-3)e^{-x}$ $[1, 3]$

6. $|\cos x|$ $[0, \pi]$

Q2. Verify Rolle's Theorem for
$f(x) = e^{-x}(\sin x - \cos x)$ $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Q3. If $f(x) = x(x+1)(x+2)(x+3)$ then show that $f'(x)$ has real roots

Q4. By considering function
$f(x) = (x-2)\log x$, show that
$x \log x = 2 - x$ has root between 1 and 2.

PAGE No.
DATE / /

Q1.

1. $\log\left[\frac{n^2+6}{5n}\right]$ $[2,3)$ Since, log func. its cont. and differentiable.

$f(2) = \log\left[\frac{4+6}{10}\right] = \log 1$

$f(3) = \log\left[\frac{9+6}{15}\right] = \log 1$

$\therefore f(2) = f(3)$

Hence, rolles' theorem is applicable.

2. $1-\sqrt[3]{(n-1)^2}$ $(0,2)$

$f(0) = 1-\sqrt[3]{(0-1)^2} = 1-\sqrt[3]{(-1)^2} = 1-1 = 0$

$f(2) = 1-\sqrt[3]{(2-1)^2} = 1-3\sqrt{(1)^2} = 1-1 = 0$

$\therefore f(0) = f(2)$

Hence, rolle's theorem is applicable.

3. $f(x) = \begin{cases} x^2-2 & ; \quad -1 \le x < 0 \\ x-2 & ; \quad 0 \le x \le 1 \end{cases}$

CASE-1: $f(x) = x^2-2$, $-1 \le x \le 0$

$f(-1) = (-1)^2-2 = -1$

$f(0) = (0)^2-2 = -2$

CASE-2 : $f(x) = x-2$, $0 \le x \le 1$

$f(0) = 0-2 = -2$

$f(1) = 1-2 = -1$

$\therefore f(-1) = f(1)$

Hence, rolle's theorem is applicable.

PAGE NO	
DATE	/ /

Q1

(1) $f(x) = \log\left[\frac{x^2+6}{5x}\right]$ $(2,3)$

Since, log func. hence continuous.
Since continuous hence differentiable.

$f(2) = \log\left[\frac{4+6}{10}\right] = \log 1$

$f(3) = \log\left[\frac{9+6}{15}\right] = \log 1$

$\therefore f(2) = f(3)$

If $f(2)$ and $f(3)$ are equal hence there exist a point c between $(2,3)$ such that
$f'(c) = 0$

$\frac{d}{dx}\left[\log(x^2+6) - \log(5x)\right]$

$\frac{1}{x^2+6}(2x) - \frac{1}{5x} 5$

$\frac{2c}{c^2+6} - \frac{5}{5c} = 0$

$\frac{2c}{c^2+6} = \frac{1}{c}$

$2c^2 = c^2+6$

$c^2 = 6$

$c = \sqrt{6}$

$\therefore c = 2.44$

Hence, Rolle's Theorem is applicable.

(2) ~~$1-\sqrt[2]{(n-1)^2}$~~ $f(n) = 1-\sqrt[3]{(n-1)^2}$ $(0,2)$

Since polynomial function, its continuous
Since continuous its differentiable.

$f(0) = 1-\sqrt[3]{(-1)^2} = 0$ $f(2) = 1-\sqrt[3]{(1)^2} = 0$

$\therefore f(0) = f(2)$

If $f(0) = f(2)$ then there exist a point c between $(2,3)$ such that $f'(c) = 0$

$$\frac{d}{dn}\left(1-(n-1)^{2/3}\right) = -\frac{2}{3}(n-1)^{-1/3}$$

Now, $-\frac{2}{3}(c-1)^{-1/3} = 0$

$$\frac{1}{\sqrt[3]{c-1}} = 0$$

Since, c is getting an undefined value ~~its~~
Hence, rolle's theorem is not applicable.

PAGE No.
DATE / /

③ $f(x) = \begin{cases} x^2 - 2 & -1 \leq x \leq 0 \\ x - 2 & 0 \leq x \leq 1 \end{cases}$

Since, polynomial func. its continuous
~~the~~ Since continuous its differentiable.

$f(0) = 0 - 2 = -2$ $\quad f(-1) = -1$ } For $x^2 - 2$

Since $f(0) \neq f(-1)$

Rolle's theorem is not applicable

Also, $f(0) = -2$ $\quad f(1) = -1$ } For $x - 2$

$f(0) \neq f(1)$

④ $f(x) = \cos^2 x$ $\quad [-\frac{\pi}{4}, \frac{\pi}{4}]$

Since cosine function its continuous
Since continuous its differentiable

$f(-\frac{\pi}{4}) = \frac{1}{2}$ $\quad f(\frac{\pi}{4}) = \frac{1}{2}$

Since $f(-\frac{\pi}{4}) = f(\frac{\pi}{4})$ there exist a point c between $[-\frac{\pi}{4}, \frac{\pi}{4}]$ such that $f'(C) = 0$

$f'(x) = 2\cos x(-\sin x)$

$f'(c) = 2\cos c(-\sin c) = 0$

$\cos c = 0 \quad \sin c = 0$

$c = \frac{\pi}{2} \quad c = 0$

But ranges is $[-\frac{\pi}{4}, \frac{\pi}{4}]$

Hence, rolle's theorem is not applicable.

③ $f(x)=(x-1)(x-3)e^{-x}$ $[1,3]$

Since exponential its continuous

Since continuous its differentiable

$f(1)=0$ $f(3)=0$

Since $f(1)=f(3)$ there exist a point c between $[1,3]$ such that $f'(c)=0$

$$(x-3)e^{-x}+(x-1)e^{-x}-(x-1)(x-3)e^{-x}$$

$$(c-3)e^{-x}+(c-1)e^{-x}-(c-1)(c-3)e^{-c}=0$$

$$(c-1)+(c-3)-(c-1)(c-3)=0$$

$$c-1+c-3-(c^2-3c-c+3)=0$$

$$2c-4-c^2+4c-3=0$$

$$c^2-6c+7=0$$

$$\cancel{c^2-7c+c+7=0}$$

$$c=4.41, 1.58$$

Hence rolle's theorem is applicable.

⑥ $f(x)=|\cos x|$ $[0,\pi]$

Since cosine function its continuous, hence differentiable

$f(0)=1$ $f(\pi)=1$

Since $f(0)=f(\pi)$ there exist a point c such that $f'(c)=0$

$$\sin c=0$$

$$c=\frac{\pi}{2}$$

But since in $[0,\pi]$ $\cos x$ has $-ve$ value

Hence, Rolle's theorem is not applicable.

Q2. $f(x) = e^{-x}(\sin x - \cos x)$ $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Since trigonometric func. its continuous and hence differentiable.

$f(\frac{\pi}{4}) = 0$ $f(\frac{5\pi}{4}) = 0$

Since $f(\frac{\pi}{4}) = f(\frac{5\pi}{4})$ there exist a point c such that $f'(c) = 0$

$\cancel{e^{-x}} f'(x) = e^{-x}(\cos x + \sin x) - e^{-x}(\sin x - \cos x)$

$f'(c) = e^{-c}(\cos c + \sin c) - e^{-c}(\sin c - \cos c) = 0$

$\cos c + \sin c - \sin c + \cos c = 0$

$2\cos c = 0$

$\boxed{c = \frac{\pi}{2}}$

Hence Rolle's theorem is applicable.

PAGE No.
DATE / /

$f(x) = x(x+1)(x+2)(x+3)$

$f'(x) = (x+1)(x+2)(x+3) + x(x+2)(x+3) + x(x+1)(x+3) + x(x+1)(x+2)$

$= (x+1)(x^2+3x+2x+6) + x(x^2+3x+2x+6) + x(x^2+x+3x+3)$
$+ x(x^2+2x+x+2)$

$= (x^3+1)(x^2+5x+6) + x(x^2+5x+6) + x(x^2+4x+3) + x(x^2+3x+2)$

$= x^3+5x^2+6x+x^2+5x+6+x^3+5x^2+6x+x^3+4x^2+3x+x^3+3x^2+2x$

$= 4x^3+18x^2+22x+6$

$4c^3+18c^2+22c+6=0$

$2c^3+9c^2+11c+3=0$

$c = 2.62, 0.382, 1.5$

Since, all roots are real the hence proved.

$f(x) = (x-2)\log x$

$f(x) = x\log x - 2\log x$

$x\log x = f(x) + 2\log x$

Now, $x\log x = 2-x$

$f(x) = 2-x-2\log x$

$f'(x) = -1 - \frac{2}{x}$

$f'(c) = -1 - \frac{2}{x} = 0$

$= -c-2=0$

$\Rightarrow c=-2$

Q4 $f(x) = (x-2)\log x$

$x \log x = 2 - x$ f_2

$x \log x + x = 2$

$x(\log x + 1) = 2$

$x \log 10x = \log 10^2$

$(10x)^x = (100)^1$

Now,

at $x=1$ $(10)^1 = 100$ $10 < 100$

at $x=2$ $(20)^2 = (100)$ $400 > 100$

Hence, roots lie between 1 and 2.

A Baij 13/09/19

Q1. Examine the validity of the condition and conclusion of LMVT for following function:

1. $y = x^{2/3}$ [-2,2] 2. $y = x + \frac{1}{x}$ [1/2, 3] 3. $y = 2x^2 - 7x + 10$ [2,5]
4. ~~tx~~ $y = (x-1)(x-2)$ [0,4]

Q2 Show that for the curve $y = x^2 + 2K_1x + K_2$, the chord joining the points $x = a$ & $x = b$ is parallel to the tangent at $x = (a+b)/2$

Q3 Show that the chord joining the points $x=2$, $x=3$ on curve $y = x^3$ is parallel to the tangent to the curve at $x = \sqrt{\frac{19}{3}}$

Q4. At what point is the ~~targest~~ tangent to curve $y = x^n$ parallel to chord joining $(0,0)$ and (a, a^n)

ANSWER

Q1.

1. $y = x^{2/3}$ $[-2, 2]$
LMVT can't be applied as exponential term should be either integer.

2. $y = x + \frac{1}{x}$ $[\frac{1}{2}, 3]$
Since, the given function is continuos since polynomial so differential between $[\frac{1}{2}, 3]$

$f(3) = 10/3 \qquad f(\frac{1}{2}) = 5/2$

$\because f(3) \neq f(\frac{1}{2})$ there exist a point c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$1 - \frac{1}{c^2} = \frac{\frac{10}{3} - \frac{5}{2}}{3 - \frac{1}{2}}$$

$$1 - \frac{1}{c^2} = \frac{1}{3}$$

$$3c^2 - 3 = c^2$$

$$2c^2 = 3$$

$$\therefore \boxed{c = \sqrt{\frac{3}{2}}}$$

3. $y = 2x^2 - 7x + 10 \quad [2,5]$

Since the given function is $f(x)$ is polynomial, so its continuous over $[2,5]$ and differentiable over $(2,5)$

$f(2) = 2(2)^2 - 7(2) + 10 \qquad f(5) = 2(5)^2 - 7(5) + 10$

$= 8 - 14 + 10 = 4 \qquad = 25$

$\because f(5) \neq f(2)$ there exist a point c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$4c - 7 = \frac{25 - 4}{5 - 2}$$

$$4c - 7 = \frac{\cancel{21}}{3}$$

$$4c - 7 = 7$$

$$4c = 14$$

$$c = 7/2$$

4. $y = (x-1)(x-2)$ $[0,4]$

$y = x^2 - 2x - x + 2$

$y = x^2 - 3x + 2$

Since, the given function $f(x)$ is polynomial, so its continuous over $[0,4]$ and differentiable over $(0,4)$

$f(4) = (4)^2 - 3(4)+2$ $f(0) = (0)^2 - 3(0)+2$

$= \cancel{8}6$ $= 2$

$\because f(4) \neq f(0)$ there exist a point c such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$2c - 3 = \cancel{\frac{8-2}{}}$ $\cancel{\frac{8-2}{4-0}}$ $\frac{6-2}{4-0}$

$2c-3 = \cancel{\frac{6\ 3}{4\ 2}}$ $\cancel{\frac{4}{4}}$ $\therefore 2c-3 = \frac{4}{4}$

$\cancel{2c = \frac{3}{2} + 3}$ $2c - 3 = 1$

$2c = 4$

$\cancel{2c = \frac{9}{2}}$ $\boxed{c = 2}$

$\cancel{\boxed{c = \frac{9}{4}}}$

Q2 $y = x^2 + 2K_1x + K_2$ $\quad x = a, x = b$

$f(x) = x^2 + 2K_1x + K_2$

$f'(x) = 2x + 2K_1$

$= 2\left(\frac{a+b}{2}\right) + 2K_1$ $\quad \left[\because x = \frac{a+b}{2} \text{ Given}\right]$

$= a + b + 2K_1$

$f(b) = b^2 + 2K_1b + K_2$ $\quad f(a) = a^2 + 2K_1a + K_2$

Now,

$f'(x) = \frac{f(b) - f(a)}{b-a}$

$a+b+2K_1 = \frac{b^2 + 2K_1b + \cancel{K_2} - a^2 - 2K_1a - \cancel{K_2}}{b-a}$

$a+b+2K_1 = \frac{b^2 - a^2 + 2K_1b - 2K_1a}{b-a}$

$a+b+2K_1 = \frac{\cancel{(b-a)}(b+a+2K_1)}{\cancel{b-a}}$

$\therefore a+b+2K_1 = a+b+2K_1$

Hence, Proved

PAGE No.
DATE / /

$y = x^3$ $\quad\quad\quad x = 2 \quad x = 3$

$f(x) = x^3$
$f'(x) = 3x^2$
$= 3\left(\frac{19}{3}\right) \quad\quad \left[\because x = \sqrt{\frac{19}{3}}\right]$

$f'(x) = 19$

$f(3) = 27 \quad\quad\quad f(2) = 8$

Now,
$f'(x) = \frac{f(b) - f(a)}{b-a}$

$19 = \frac{27-8}{3-2}$

$\therefore 19 = 19$

Hence, Proved.

Q4. $f(x) = x^n$ $\quad\quad a = 0$ $\quad\quad b = a$

$f'(x) = nx^{n-1}$ $\quad\quad f(a) = 0$ $\quad\quad f(b) = a^n$

Now,

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$

$$nx^{n-1} = \frac{a^n}{a}$$

$$x^{n-1} = \frac{a^{n-1}}{n}$$

$$x = \frac{a}{(n)^{1/n-1}}$$

$$\boxed{x = a\,n^{(n-1)}}$$

15/04/19

DATE 18/9/19

Tutorial ->

Q1. Expand $\log(\cos x)$ about $\pi/3$.

Q2. Express $7+(x+2)+3(x+2)^3+(x+2)^4-(x+2)^5$ in ascending powers of x.

Q3. Prove That:

$$\frac{1}{1-x} = \frac{1}{3} + \frac{x+2}{3^2} + \frac{(x+2)^2}{3^3} + \frac{(x+2)^3}{3^4} + \dots$$

Q4. Expand $\tan^{-1}x$ in power of $\left(x-\frac{\pi}{4}\right)$

Q5. Show that: $\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \dots$

Q6. Prove that: $a^x = 1 + x\log a + \frac{x^2}{\lfloor 2}(\log a)^2 + \dots$

Q7. Show that: $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$

Q8. Prove that: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

Q5. $f(x) = \cos x \quad f(0) = 1$

$f'(x) = -\sin x \quad f'(0) = 0$

$f''(x) = -\cos x \quad f''(0) = -1$

$f'''(x) = \sin x \quad f'''(0) = 0$

$f''''(x) = \cos x \quad f''''(0) = 1$

$$f(x) = f(0) + \frac{x}{\lfloor 1} f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \frac{x^4}{\lfloor 4} f''''(0)$$

$$= 1 + 0 - \frac{x^2}{\lfloor 2} + 0 + \frac{x^4}{\lfloor 4} + 0 + \ldots$$

Q6. $f(x) = a^x \quad f(0) = 1$

$f'(x) = x \log a \quad f'(0) = \log a$

$f''(x) = (\log a)^2 \quad f''(0) = (\log a)^2$

$f'''(x) = (\log a)^3 \quad f'''(0) = (\log a)^3$

$$f(x) = f(0) + \frac{x}{\lfloor 1} f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \ldots$$

$$= 1 + x \log a + \frac{x^2}{\lfloor 2} (\log a)^2 + \frac{x^3}{\lfloor 3} (\log a)^3 + \ldots$$

Q7.

Q8.

Page No.
Date / /

Q7. $f(x) = (1+x)^{-1}$ $\quad f(0) = (1)^{-1}$

$f'(x) = -1(1+x)^{-2}$ $\quad f'(0) = -(1)^{-2}$

$f''(x) = 2(1+x)^{-3}$ $\quad f''(0) = 2(1)^{-3}$

$$f(x) = f(0) + \frac{x}{\underline{|1}} f'(0) + \frac{x^2}{\underline{|2}} f''(0) + \frac{x^3}{\underline{|3}} f'''(0) + \cdots$$

$$= \underline{\underline{1 - x + x^2 \mp x^3 + \cdots}}$$

Q8. $f(x) = (1-x)^{-1}$ $\quad f(0) = (1)^{-1}$

$f'(x) = (1-x)^{-2}$ $\quad f'(0) = (1)^{-2}$

$f''(x) = 2(1-x)^{-3}$ $\quad f''(0) = 2(1)^{-3}$

$f'''(x) = 6(1-x)^{-4}$ $\quad f'''(0) = 6(1)^{-4}$

$$f(x) = f(0) + \frac{x}{\underline{|1}} f'(0) + \frac{x^2}{\underline{|2}} f''(0) + \frac{x^3}{\underline{|3}} f'''(0) + \cdots$$

$$= \underline{\underline{1 + x + x^2 + x^3 + \cdots}}$$

21/09/19

Q4. $f(x) = \log(\cos x)$ about $\pi/3$

$x = \pi/3 = a \qquad h = x - \pi/3$

$f(x) = \log(\cos x) \qquad f(\pi/3) = \log(1/2) = -0.30$

$f'(x) = \frac{-\sin x}{\cos x} = -\tan x \qquad f'(\frac{\pi}{3}) = -\sqrt{3}$

$f''(x) = -\sec^2 x \qquad f''(\frac{\pi}{3}) = -4$

$f(x) = f(0) + \frac{x f'(0)}{\lfloor 1} + \frac{x^2}{\lfloor 2} f''(0) + \cdots$

$= -0.30 \cancel{\mp x} - x\sqrt{3} - 2x^2 + \cdots$

Banji 21/09/19

Q2.