1 Capacitated Fixed-Charge Location Problem

In this document, you will find the detailed models for the Capacitated Facility Location Problem (CFLP) and its stochastic version. This includes the formulation of both models, the results obtained from the analysis, and the conclusions drawn from these results. The data utilized in this document were derived from the analysis conducted during the preparatory phase, ensuring a comprehensive approach to optimizing facility locations under various demand scenarios.

2 Model

In CFLP, we are considering capacity restrictions using the maximum demand that a facility can serve. The objective function minimizing the total transportation costs and building the facilities:

$$\text{Min} \qquad \sum_{j=1}^{J} f_j \cdot x_j + \sum_{i=1}^{I} \sum_{j=1}^{J} h_i \cdot c_{ij} \cdot y_{ij} \tag{1}$$

Subject to:

$$\sum_{j=1}^{J} y_{ij} = 1 \qquad \forall i \in \{1, I\} \tag{2}$$

$$y_{ij} \leq x_j \qquad \forall i \in \{1, I\}, \forall j \in \{1, J\} \tag{3}$$

$$\sum_{i=1}^{I} h_i \cdot y_{ij} \leq v_j \quad \forall j \in \{1, J\} \tag{4}$$

$$x_j \in \{0, 1\} \qquad \forall j \in \{1, J\} \tag{5}$$

$$y_{ij} \geq 0 \qquad \forall j \in \{1, J\} \tag{6}$$

The minimum objective value for the CFLP was 449,381.78. The figure for the allocations is plotted in figure 1, and the facilities and customers allocation were as follows:

- Facility 1: 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 60, 61, 62, 63, 64, 65, 66, 67.
- Facility 2: 24, 25, 26, 27, 28, 29, 30, 31.
- Facility 3: 2, 3, 4, 37, 38, 39, 55, 56, 57, 59.
- Facility 4: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17.
- Facility 8: 19, 20, 21, 22, 23, 32, 33, 34, 35, 36.

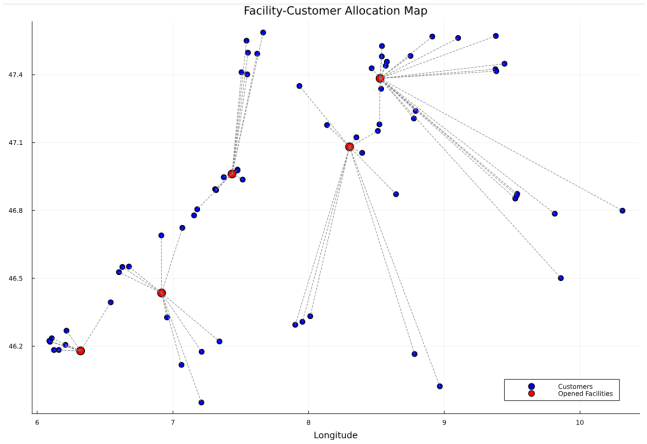

Figure 1: CFLP Customers and Facilities Allocation

3 CFLP Model vs. Company A

Company A open facilities are 1, 2, and 3. In comparison, the CFLP model suggested to open the potential facilities 4 and 8 in addition to the existing ones. The result of opening facilities 4 and 8 will result in lower transportation costs.

4 Stochastic CFLP Formulation

In the stochastic version we consider one more index 's'. This index depicts the realization of the uncertainty. Thus, the stochastic CFLP is given below:

$$\text{Minimize} \quad \sum_{j=1}^{J} f_j \cdot x_j + \sum_{s=1}^{S} q_s \cdot \left(\sum_{i=1}^{I} \sum_{j=1}^{J} h_{is} \cdot c_{ij} \cdot y_{ijs} \right) \tag{7}$$

Subject to:

$$\sum_{j=1}^{J} y_{ijs} = 1 \quad \forall i \in \{1, \ldots, I\}, \forall s \in \{1, \ldots, S\} \tag{8}$$

$$y_{ijs} \leq x_j \quad \forall i \in \{1, \ldots, I\}, \forall j \in \{1, \ldots, J\}, \forall s \in \{1, \ldots, S\} \tag{9}$$

$$\sum_{i=1}^{I} h_{is} \cdot y_{ijs} \leq v_{js} \quad \forall j \in \{1, \ldots, J\}, \forall s \in \{1, \ldots, S\} \tag{10}$$

$$x_j \in \{0, 1\} \quad \forall j \in \{1, \ldots, J\} \tag{11}$$

$$y_{ijs} \geq 0 \quad \forall i \in \{1, \ldots, I\}, \forall j \in \{1, \ldots, J\}, \forall s \in \{1, \ldots, S\} \tag{12}$$

5 Stochastic CFLP Model

The minimum objective for this version of the model comes out to be 454509.56, which slightly higher than the deterministic one. Interestingly enough the same facilities open in this case.

- Total demand served by facility 1 across all scenarios: 770.33
- Total demand served by facility 2 across all scenarios: 257.53
- Total demand served by facility 3 across all scenarios: 308.16
- Total demand served by facility 4 across all scenarios: 410.93
- Total demand served by facility 8 across all scenarios: 263.02

6 Stochastic CFLP vs. Deterministic CFLP

As discussed previously, the Stochastic CFLP shows to open the same facilities. The main difference between these two models is the amount of demand that is served by each facility. In other words, in the deterministic model, Facilities 1,2,3,4,8 served 234, 181, 159, 240, 121 visits respectively. It is apparent that the numbers in the stochastic version are quite larger and this is due to the fact that it takes into account a lot more information about the problem at hand.

7 Conclusion: How To Improve Facility Design?

Analysis from both deterministic and stochastic models indicates a strategic advantage in retaining the current facilities operational due to the prohibitive costs associated with their closure. It is then observed that out of the seven potential new locations, opening only two of them will meet the demand, while minimizing the overall cost. It is up to management to decide whether the additional investment of 5127$ is justified to accommodate the uncertainties intrinsic to the problem's stochastic elements.