Nomenclature

Sets

$w \in \mathcal{W}$:	Set of Warehouses
$t \in \mathcal{T}$:	Set of simulation periods

Decision Variables

$z_{w,t}$:	Current storage capacity of warehouse w at time t
$x_{w,t}$:	Amount to order at warehouse w at time t
$y^{\text{receive}}_{w,q,t}$:	Amount received at warehouse q from warehouse w at time t
$y^{\text{send}}_{q,w,t}$:	Amount sent from warehouse w to warehouse q at time t
$m_{w,t}$:	Amount missing from warehouse w at time t
$\delta_{w,t}$:	Degradation factor of warehouse w at day t
λ'_t :	Decision variable in dual problem of polyhedral uncertainty method.
$\mu'_{w,t}$:	Decision variable in dual problem of polyhedral uncertainty method.
λ_t :	Decision variable in dual problem of budget uncertainty method.
$\mu_{w,t}$:	Decision variable in dual problem of budget uncertainty method.
$y^0_{w,q,t}$:	Decision variable derived from linear decision rule .
$Q_{w,q,t}$:	Decision variable derived from linear decision rule .
β :	Decision variable for the linearization of the uncertainty part.
$\alpha_{w,q,t}$:	Auxiliary decision variable for the linearization of a constraint .
$\phi_{w,q,t}$:	Auxiliary decision variable for the linearization of a constraint .
$\sigma_{w,q,t}$:	Auxiliary decision variable for the linearization of a constraint .
$\eta_{w,q,1}$:	Auxiliary decision variable for the linearization of a constraint .
$\theta_{w,q,t}$:	Auxiliary decision variable for the linearization of a constraint .

Parameters

$D_{w,t}$:	District's demand for warehouse w
$P_{w,t}$:	Price of coffee from external suppliers ordered from warehouse w at time t
$\text{cost}_{tr,w,q}$:	Per unit transportation cost from warehouse w to warehouse q
$\text{cost}_{miss,w}$:	Penalty for missing district's demand
$C^{transport}_{w,q}$:	Daily transportation limit from warehouse w to warehouse q
C^{storage}_{w} :	Capacity limit for coffee storage at warehouse w
$\overline{\delta}_{w,t}$:	Average degradation factor of warehouse w at day t
$\rho_{w,t}$:	Range for deviation of degradation factor for warehouse w at day t
$\nu_{w,t}$:	worst case selection of coefficients from J_i .

Introduction

The goal of this assignment is to make the best decision for the same three warehouse problem as assignment A, by minimizing the system's cost. The prices for the coffee are known beforehand for all 5 days, therefore the decisions that must be made will not take into consideration this uncertainty. However, some of the coffee that is stored in the warehouses is lost due to degradation, thus an extra factor is introduced in this problem.

Here is the optimization problem:

$$\min_{x_{w,t}, y^{\text{send}}_{w,q,t}, y^{\text{receive}}_{w,q,t}, z_{w,t}, m_{w,t}} \left\{ \sum_{w \in \mathcal{W}} \sum_{t \in \mathcal{T}} \left(p_{w,t} x_{w,t} + \sum_{q \in \mathcal{W}} e_{w,q} y^{\text{send}}_{w,q,t} + b_w m_{w,t} \right) \right\} \tag{1}$$

subject to:

$$0 \leq z_{w,t} \leq C^{\text{storage}}_{w}, \quad \forall w \in \mathcal{W},\, t \in \mathcal{T} \tag{2}$$

$$\sum_{q \in \mathcal{W}} y^{\text{send}}_{w,q,t} \leq z_{w,t-1}, \quad \forall w \in \mathcal{W},\, t \in \mathcal{T} \setminus \{1\} \tag{3}$$

$$\sum_{q \in \mathcal{W}} y^{\text{send}}_{w,q,1} \leq z_{w,0}, \quad \forall w \in \mathcal{W} \tag{4}$$

$$y^{\text{send}}_{w,q,t} \leq C^{\text{transp}}_{w,q}, \quad \forall w, q \in \mathcal{W},\, t \in \mathcal{T} \tag{5}$$

$$z_{w,t} \leq \delta_{w,t} z_{w,t-1} + x_{w,t} + \sum_{q \in \mathcal{W}} y^{\text{receive}}_{w,q,t} - \sum_{q \in \mathcal{W}} y^{\text{send}}_{w,q,t} - D_{w,t} + m_{w,t}, \quad \forall w \in \mathcal{W},\, t \in \mathcal{T} \tag{6}$$

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Task 1

Box uncertainty sets

In this section the problem is reformulated as a robust optimization problem. It is known that the degradation factor is $\delta_{w,t}$ is always between 0.8 and 1. The uncertainty sets are reformulated using the mean value and the range: So $\delta_{w,t} = \overline{\delta}_{w,t} + \rho_{w,t} \cdot \zeta_{w,t}, \quad |\zeta_{w,t}| \leq 1$, with $\overline{\delta}_{w,t} = 0.9$ and $\rho_{w,t} = 0.1$.

$$\delta_{w,t} = 0.9 + 0.1 \cdot \zeta_{w,t}, \quad |\zeta_{w,t}| \leq 1 \tag{1.a}$$

$k_{w,t}$ will be defined as the sum of decision variables that are deterministic, more specifically:

$$k_{w,t} = z_{w,t} - \sum_{q\in W} y^{\text{receive}}_{w,q,t} + \sum_{q\in W} y^{\text{send}}_{w,q,t} + D_{w,t} - m_{w,t} - x_{w,t}$$

$$\delta_{w,t} \cdot z_{w,t-1} \geq k_{w,t}, \forall w, t \tag{1.b}$$

therefore combining (1.a) and (1.b):

$$(0.9 + 0.1 \cdot \zeta_{w,t}) \cdot z_{w,t-1} \geq k_{w,t}, \iff 0.9 \cdot z_{w,t} + 0.1 \cdot \zeta_{w,t} \cdot z_{w,t-1} \geq k_{w,t}, \quad \forall |\zeta_{w,t}| \leq 1 \tag{1.c}$$

For the task at hand the worst case realization of uncertainty here is the minimum because of the "$\geq$" constraint.

$$0.9 \cdot z_{w,t-1} + \min_{-1\leq\zeta_{w,t}\leq 1} \left\{0.1 \cdot \zeta^{w,t} \cdot z_{w,t-1}\right\} \geq k_{w,t} \tag{1.d}$$

Since $z_{w,t-1} \geq 0$ the worst case realization of the minimum is obtained for $\zeta^{w,t} = -1$. So (1.d) can be written :

$$0.9 \cdot z_{w,t-1} - 0.1 \cdot z_{w,t-1} \geq k_{w,t} \iff 0.8 \cdot z_{w,t-1} \geq k_{w,t} \tag{1.e}$$

$$0.8 \cdot z_{w,t-1} \geq z_{w,t} - \sum_{q\in W} y^{\text{receive}}_{w,q,t} + \sum_{q\in W} y^{\text{send}}_{w,q,t} + D_{w,t} - m_{w,t} - x_{w,t} \tag{1.e}$$

Task 2

More information about the uncertainty

For this task there is no constraint for the capacity of coffee that can be transferred in between warehouses, thus constraint (6) can be replaced by :

$$\sum_{w\in W} z_{w,t} \leq \sum_{w\in W} \left(\delta_{w,t} z_{w,t-1} + x_{w,t} - D_{w,t} + m_{w,t}\right), \quad \forall t \in T.$$

Case a

For any given day, the total degradation factor $\sum\limits_{w} \delta_{w,t}$ is never less than 2.6, and we know that $\delta_{w,t} = \overline{\delta}_{w,t} + \rho^{w,t} \cdot \zeta^{w,t}, \quad \forall |\zeta^{w,t}| \leq 1$. Since $\overline{\delta}_{w,t} = 0.9$ and $\rho^{w,t} = 0.1$:

$$\delta_{w,t} = 0.9 + 0.1 \cdot \zeta^{w,t} \tag{a.i}$$

Combining the knowledge for the degradation factor and (a.i):

$$\sum_{w\in W} \delta_{w,t} \geq 2.6, \quad \forall t \in T$$

$$\sum_{w\in W} (0.9 + 0.1 \cdot \zeta^{w,t}) \geq 2.6, \quad |\zeta^{w,t}| \leq 1$$

$$\sum_{w\in W} 0.9 + 0.1 \cdot \sum_{w\in W} \zeta^{w,t} \geq 2.6, \quad |\zeta^{w,t}| \leq 1$$

$$2.7 + 0.1 \cdot \sum_{w\in W} \zeta^{w,t} \geq 2.6$$

$$0.1 \cdot \sum_{w\in W} \zeta_{w,t} \geq -0.1$$

$$\sum_{w\in W} \zeta_{w,t} \geq -1, \quad |\zeta^{w,t}| \leq 1 \tag{a.ii}$$

As described in the slides, the above problem can be solved using a polyhedral uncertainty set $U = \{\zeta : D\zeta \leq d\}$. For the task at hand, the finite set of linear inequalities can be written as $U = \{D \cdot \sum \zeta^{w,t} \geq d\}$ with $D = [1, \ldots, 1]$ and $d = [-1, \ldots, -1]$. The polyhedral uncertainty sets will be used to bring it to a linearized form:

$$\sum_{w\in W} z_{w,t} \leq \sum_{w\in W} (\delta_{w,t} \cdot z_{w,t-1} + x_{w,t} - D_{w,t} - m_{w,t}), \quad \forall t \in T$$

Using (a.ii) as the polyhedral uncertainty set and a.i:

$$\sum_{w\in W} (0.9 + 0.1 \cdot \zeta^{w,t}) \cdot z_{w,t-1} \geq \sum_{w\in W} (z_{w,t} - x_{w,t} + D_{w,t} - m_{w,t}) \iff$$

$$0.9 \cdot \sum_{w \in W} z_{w,t-1} + 0.1 \cdot \sum_{w \in W} \zeta_{w,t} \cdot z_{w,t-1} \geq \sum_{w \in W} (z_{w,t} - x_{w,t} + D_{w,t} - m_{w,t}) \quad \forall \zeta^{w,t} \in U = \{\sum_{w,t} \zeta^{w,t} \geq -1\} \tag{a.iii}$$

Defining $h_{w,t} = \left(\sum_{w \in W} z_{w,t} - \sum x_{w,t} + \sum D_{w,t} - \sum m_{w,t} - 0.9 \cdot \sum z_{w,t-1}\right) \cdot 10$, a.iii can be written as:

$$\sum_{w \in W} \zeta_{w,t} \cdot z_{w,t-1} \geq h_{w,t}, \quad \forall \zeta_{w,t} \in U = \{\sum_{w \in W} \zeta^{w,t} \geq -1\}$$

Since this constraint must be feasible for all outcomes of uncertainty ζ in the set U:

$$\min_{\sum_{w \in W} \zeta^{w,t} \geq -1} \left\{ \sum_{w \in W} \zeta_{w,t} \cdot z_{w,t-1} \right\} \geq h_{w,t}$$

The dual problem associated with the primal problem is formulated as follows:

$$\max \quad -\lambda'_t + \sum_{w \in W} \mu'_{w,t} - \sum_{w \in W} \nu'_{w,t}$$

Subject to:

$$z_{w,t-1} = \lambda'_t - \mu'_{w,t} + \nu'_{w,t}, \quad \forall w, t$$
$$\lambda'_t \geq 0, \quad \mu'_{w,t} \geq 0, \quad \nu'_{w,t} \geq 0$$

Dual Explanation

Here is a breakdown of the steps I took to arrive at the final dual formulation:

1. Understanding the Primal Problem: - The primal problem provided a set of constraints and an objective function involving the degradation factor $\delta_{w,t}$. The key was to ensure that all constraints were properly formulated to align with a minimization problem, meaning they should all have $\geq$ inequalities.

2. Formulating the Lagrangian: - To dualize the problem, I first constructed the Lagrangian function, introducing Lagrange multipliers (e.g., $\lambda'_t, \mu'_{w,t}$) for each constraint in the primal problem.

3. Applying the KKT Conditions: - The Karush-Kuhn-Tucker (KKT) conditions were applied to the Lagrangian. These conditions included: - Stationarity: Deriving conditions from the derivative of the Lagrangian with respect to the primal variables. - Primal Feasibility: Ensuring that the primal constraints are satisfied. - Dual Feasibility: Ensuring that the dual variables (Lagrange multipliers) satisfy non-negativity constraints. - Complementary Slackness: Ensuring that each constraint is either active (binding) or the associated Lagrange multiplier is zero.

4. Deriving the Dual Problem: - Using the KKT conditions, I derived the dual problem. This involved converting the primal objective and constraints into their dual counterparts, ensuring the objective was to maximize a function since the primal was a minimization

problem. - The dual constraints were derived directly from the stationarity condition, and the bounds for the dual variables were identified from the primal's inequality constraints.

5. Rewriting and Finalizing: - The final dual problem was rewritten in a standard form, ensuring all variables and constraints were correctly defined.

Case b

For this case, the degradation factor deviates from its average value of 0.9 in at most 2 warehouses out of 3, and for the rest of the warehouses it remains 0.9. For any given day, the number of coefficients needs to be restricted using the budget of uncertainty Γ_i. Therefore, $\Gamma_i = 2, \hat{\delta}_{w,t} = 0.1, \overline{\delta}_{w,t} = 0.9$.

The worst-case formulation is:

$$\sum_{w\in W} \overline{\delta}_{w,t} \cdot z_{w,t-1} + \min_{\substack{S_i \subseteq J_i,\\ |S_i| \leq \Gamma_i}} \left\{ \sum_{w\in W} \hat{\delta}_{w,t} \cdot z_{w,t-1} \right\} \geq \sum_{w\in W} z_{w,t} - \sum_{w\in W} x_{w,t} + \sum_{w\in W} D_{w,t} - \sum_{m\in M} m_{w,t}, \quad \forall t \in T$$

Where J_i is the set of coefficients that can change due to uncertainty in constraint 6. Thus, each day we know that in 2 out of 3 warehouses, the degradation factor is going to deviate from its average value.

Since Γ_i is an integer, we can simplify:

$$(\Gamma_i - \lfloor \Gamma_i \rfloor) \cdot \hat{\delta}_{w,t} \cdot z_{w,t} = 0$$

Now, let's formulate the subproblem as a linear program (LP) for each $t \in T$:

$$\Big\downarrow \text{subproblem}$$

The minimization problem is:

$$\min_{\substack{S_i \subseteq J_i,\\ |S_i| \leq \Gamma_i}} \left\{ \sum_{w\in W} \hat{\delta}_{w,t} \cdot z_{w,t-1} \right\}$$

This can be written as:

$$\min \sum_{w\in W} \hat{\delta}_{w,t} \cdot z_{w,t-1} \cdot \nu_{w,t}$$

Subject to:

$$\sum_{w\in W} \nu_{w,t} \leq \Gamma_t, \quad \nu_{w,t} \geq 0, \quad \forall w \in W$$

$$0 \leq \nu_{w,t} \leq 1, \quad \forall w \in W$$

$$\Big\downarrow \text{dual}$$

The dual of the above problem is:

$$\max \Gamma_t \cdot \lambda_t - \sum_{w\in W} \mu_{w,t}$$

Subject to:

$$\lambda_t - \mu_{w,t} \leq \hat{\delta}_{w,t} \cdot z_{w,t-1}, \quad \forall w \in W$$

$$\lambda_t \geq 0, \quad \mu_{w,t} \geq 0$$

Incorporating the Dual Problem into the Main Objective Now, replace the subproblem into the original objective and include the constraint $z_{w,t-1} \geq 0$:

$$\min_{x_{w,t}, y^{send}_{w,q,t}, y^{receive}_{w,q,t}, z_{w,t}, m_{w,t}} \left\{ \sum_{w \in W} \sum_{t \in T} \left(p_{w,t} x_{w,t} + \sum_{q \in W} e_{w,q} y^{send}_{w,q,t} + b_w m_{w,t} \right) \right\}$$

Subject to:

$$\sum_{w} \overline{\delta}_{w,t} \cdot z_{w,t-1} + \Gamma_t \cdot \lambda_t - \sum_{w} \mu_{w,t} \geq \sum_{w \in W} z_{w,t} - \sum_{w \in W} x_{w,t} + \sum_{w \in W} D_{w,t} - \sum_{m \in M} m_{w,t}, \quad \forall t \in T$$

$$\lambda_t - \mu_{w,t} \leq \hat{\delta}_{w,t} \cdot z_{w,t-1}, \quad \forall t \in T, \forall w \in W$$

$$\lambda_t \geq 0, \quad \mu_{w,t} \geq 0$$

Deterministic Constraints Finally, add the rest of the deterministic constraints:

$$\begin{aligned}
&0 \leq z_{w,t} \leq C^{storage}_{w}, \quad \forall w \in W, t \in T \\
&\sum_{q \in W} y^{send}_{w,q,t} \leq z_{w,t-1}, \quad \forall w \in W, t \in T \setminus \{1\} \\
&\sum_{q \in W} y^{send}_{w,q,1} \leq z_{w,0}, \quad \forall w \in W \\
&y^{send}_{w,q,t} \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T
\end{aligned}$$

Final Note:

The reformulated problem will be more restricted as they have more constraints than the previous part. However, there is more information about the uncertainty. Therefore the new solution will exhibit worst results (higher costs) on average but since we will have more information about the uncertainty it will exhibit better results when the worst case is realized.

Task 4

Internal redistribution

We use the linear decision rule

$$y_{w,q,t}^{send} = y_{w,q,t}^{0} + Q_{w,q,t} \cdot \zeta_{w,t}$$

with $w \in W, q \in W, t \in T$, $y_{w,q,t}^{0} \in R$, $Q_{w,q,t} \in R$ and $-1 \leq \ \zeta_{w,t} \leq 1$.

Transformation of constraint (3)

$$\sum_{q \in W} y_{w,q,t}^{send} \leq z_{w,t-1}, \quad \forall w \in W, t \in T \setminus \{1\} \quad (3)$$

Insert linear decision rule and also we are going to include that the transportation can be decide ad-hoc:

$$\sum_{q \in W} (y_{w,q,t}^{0} + Q_{w,q,t} \cdot \zeta_{w,t}) \leq z_{w,t-1}, \quad \forall w \in W, t \in T, -1 \leq \ \zeta_{w,t} \leq 1$$

Formulate worst-case:

$$\sum_{q \in W} y_{w,q,t}^{0} + \max_{-1 \leq \ \zeta_{w,t} \leq 1} \sum_{q \in W} Q_{w,q,t} \cdot \zeta_{w,t} \leq z_{w,t-1}, \quad \forall w \in W, t \in T, -1 \leq \ \zeta_{w,t} \leq 1$$

Reformulate to absolute value (due to box uncertainty) and positive sign due to $\leq$ constraint:

$$\sum_{q \in W} y_{w,q,t}^{0} + \sum_{q \in W} |Q_{w,q,t}| \leq z_{w,t-1}, \quad \forall w \in W, t \in T$$

Linearize absolute value using new variable $\alpha_{w,q,t} \geq 0, \quad \forall w \in W, t \in T, q \in W$:

$$\sum_{q \in W} y_{w,q,t}^{0} + \sum_{q \in W} \alpha_{w,q,t} \leq z_{w,t-1}, \quad \forall w \in W, t \in T$$

And

$$-\alpha_{w,q,t} \leq \ Q_{w,q,t} \leq \alpha_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

Transformation of constraint (4)

$$\sum_{q \in W} y_{w,q,1}^{send} \leq z_{w,1}, \quad \forall w \in W \quad (4)$$

Insert linear decision rule:

$$\sum_{q \in W} (y_{w,q,1}^{0} + Q_{w,q,1} \cdot \zeta_{w,1}) \leq z_{w,1}, \quad \forall w \in W, -1 \leq \ \zeta_{w,1} \leq 1$$

Formulate worst-case:

$$\sum_{q \in W} y^0_{w,q,1} + \max_{-1 \leq \ \zeta_{w,1} \leq 1} \sum_{q \in W} Q_{w,q,1} \cdot \zeta_{w,1} \leq z_{w,1}, \quad \forall w \in W, -1 \leq \ \zeta_{w,1} \leq 1$$

Reformulate to absolute value (due to box uncertainty) and positive sign due to $\leq$ constraint:

$$\sum_{q \in W} y^0_{w,q,1} + \sum_{q \in W} |Q_{w,q,1}| \leq z_{w,1}, \quad \forall w \in W$$

Linearize absolute value using new variable $\eta_{w,q,1} \geq 0, \quad \forall w \in W, q \in W$:

$$\sum_{q \in W} y^0_{w,q,1} + \sum_{q \in W} \eta_{w,q,1} \leq z_{w,1}, \quad \forall w \in W$$

And

$$-\eta_{w,q,1} \leq \ Q_{w,q,1} \leq \eta_{w,q,1}, \quad \forall w \in W, q \in W$$

Transformation of constraint (5)

$$y^{send}_{w,q,t} \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T \quad (5)$$

$$y^0_{w,q,t} + Q_{w,q,t} \cdot \zeta_{w,t} \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T, -1 \leq \ \zeta_{w,t} \leq 1$$

Formulate worst case

$$y^0_{w,q,t} + \max_{-1 \leq \ \zeta_{w,t} \leq 1} Q_{w,q,t} \cdot \zeta_{w,t} \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T, -1 \leq \ \zeta_{w,t} \leq 1$$

Reformulate using absolute value and a positive sign because of the $\leq$ constraint

$$y^0_{w,q,t} + |Q_{w,q,t}| \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T$$

Linearize absolute value using new variable $\theta_{w,q,t} \geq 0, \quad \forall w \in W, t \in T, q \in W$:

$$y^0_{w,q,t} + \theta_{w,q,t} \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T$$

And

$$-\theta_{w,q,t} \leq \ Q_{w,q,t} \leq \theta_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

we have to transform also that $y^{send}_{w,q,t} \geq 0$

So no we have

$$y^{send}_{w,q,t} \geq 0, \quad \forall w, q \in W, t \in T$$

$$y^{0}_{w,q,t} + Q_{w,q,t} \cdot \zeta_{w,t} \geq 0, \quad \forall w, q \in W, t \in T, -1 \leq \ \zeta_{w,t} \leq 1$$

Formulate worst case

$$y^{0}_{w,q,t} + \min_{-1 \leq \ \zeta_{w,t} \leq 1} Q_{w,q,t} \cdot \zeta_{w,t} \geq 0, \quad \forall w, q \in W, t \in T, -1 \leq \ \zeta_{w,t} \leq 1$$

Reformulate using absolute value and a negative sign because of the $\geq$ constraint

$$y^{0}_{w,q,t} - |Q_{w,q,t}| \geq 0, \quad \forall w, q \in W, t \in T$$

Linearize absolute value using new variable $\pi_{w,q,t} \geq 0, \quad \forall w \in W, t \in T, q \in W$:

$$y^{0}_{w,q,t} - \pi_{w,q,t} \geq 0, \quad \forall w, q \in W, t \in T$$

And

$$-\pi_{w,q,t} \leq \ Q_{w,q,t} \leq \pi_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

Transformation of constraint (6)

$$z_{w,t} \leq \delta_{w,t} z_{w,t-1} + x_{w,t} + \sum_{q \in W} y^{receive}_{w,q,t} - \sum_{q \in W} y^{send}_{w,q,t} - D_{w,t} + m_{w,t}, \quad \forall w \in W, t \in T$$

$$\delta_{w,t} z_{w,t-1} - \sum_{q \in W} y^{send}_{w,q,t} \geq z_{w,t} - \sum_{q \in W} y^{receive}_{w,q,t} + D_{w,t} - m_{w,t} - x_{w,t}, \quad \forall w \in W, t \in T$$

$$0.9 \cdot z_{w,t} + 0.1 \cdot \zeta_{w,t} \cdot z_{w,t-1} - \sum_{q \in \mathcal{W}} (y^{0}_{w,q,t} + Q_{w,q,t} \cdot \zeta_{w,t}) \geq z_{w,t} - \sum_{q \in \mathcal{W}} y^{\text{receive}}_{w,q,t} + D_{w,t} - m_{w,t} - x_{w,t},$$
$$\forall w \in \mathcal{W}, \ t \in \mathcal{T}, \ -1 \leq \zeta_{w,t} \leq 1$$

We define $k_{w,t} = z_{w,t} - \sum_{q \in W} y^{receive}_{w,q,t} + D_{w,t} - m_{w,t} - x_{w,t} - 0.9 \cdot z_{w,t}$

Formulate worst case:

$$\min_{-1\le\ \zeta_{w,t}\le 1} 0.1\cdot\zeta_{w,t}\cdot z_{w,t-1}-\sum_{q\in W} y^0_{w,q,t}-\min_{-1\le\ \zeta_{w,t}\le 1}\sum_{q\in W} Q_{w,q,t}\cdot\zeta_{w,t}\ge k_{w,t},\quad \forall w\in W, t\in T$$

Reformulate using absolute value and a negative sign because of the $\ge$ constraint

$$-0.1\cdot z_{w,t-1}-\sum_{q\in W} y^0_{w,q,t}+\sum_{q\in W}|Q_{w,q,t}|\ge k_{w,t},\quad \forall w\in W, t\in T$$

Linearize absolute value using new variable $\phi_{w,q,t}\ge 0,\quad \forall w\in W, t\in T, q\in W$:

$$-0.1\cdot z_{w,t-1}-\sum_{q\in W} y^0_{w,q,t}+\sum_{q\in W}\phi_{w,q,t}\ge z_{w,t}-\sum_{q\in W} y^{receive}_{w,q,t}+D_{w,t}-m_{w,t}-x_{w,t}-0.9\cdot z_{w,t},\quad \forall w\in W, t\in T$$

And

$$-\phi_{w,q,t}\le\ Q_{w,q,t}\le\phi_{w,q,t},\quad \forall w\in W, t\in T, q\in W$$

Transformation of the objective function

$$\min_{x_{w,t},y^{send}_{w,q,t},y^{receive}_{w,q,t},z_{w,t},m_{w,t}}\left\{\sum_{w\in W}\sum_{t\in T}\left(p_{w,t}x_{w,t}+\sum_{q\in W} e_{w,q}y^{send}_{w,q,t}+b_w m_{w,t}\right)\right\}$$

We will reformulate the uncertain part as constraint and addition variable $\beta\in R$

$$\min_{x_{w,t},y^{send}_{w,q,t},y^{receive}_{w,q,t},z_{w,t},m_{w,t}}\left\{\sum_{w\in W}\sum_{t\in T}(p_{w,t}x_{w,t}+b_w m_{w,t})+\beta\right\}$$

s.t

$$\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot y^{send}_{w,q,t}\le\beta\rightarrow$$

$$\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot y^0_{w,q,t}+\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot Q_{w,q,t}\cdot\zeta_{w,q,t}\le\beta,\forall-1\le\ \zeta_{w,t}\le 1\rightarrow$$

Formulate worst case:

$$\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot y^0_{w,q,t}+\max_{-1\le\ \zeta_{w,t}\le 1}\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot Q_{w,q,t}\cdot\zeta_{w,q,t}\le\beta,\forall-1\le\ \zeta_{w,t}\le 1\rightarrow$$

Reformulate using absolute value and a positive sign because of the $\leq$ constraint

$$\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot y^0_{w,q,t} + \sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot |Q_{w,q,t}| \leq \beta \rightarrow$$

Linearize absolute value using new variable $\sigma_{w,q,t} \geq 0, \quad \forall w \in W, t \in T, q \in W$:

$$\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot y^0_{w,q,t} + \sum_{w\in W}\sum_{t\in T}\sum_{q\in W} \sigma_{w,q,t} \leq \beta \rightarrow$$

And

$$-\sigma_{w,q,t} \leq e_{w,q}\cdot Q_{w,q,t} \leq \sigma_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

Full adjustable robust model

$$\min_{x_{w,t}, y^{send}_{w,q,t}, y^{receive}_{w,q,t}, z_{w,t}, m_{w,t}} \left\{ \sum_{w\in W}\sum_{t\in T} (p_{w,t}x_{w,t} + b_w m_{w,t}) + \beta \right\}$$

s.t

$$\sum_{w\in W}\sum_{t\in T}\sum_{q\in W} e_{w,q}\cdot y^0_{w,q,t} + \sum_{w\in W}\sum_{t\in T}\sum_{q\in W} \sigma_{w,q,t} \leq \beta$$

$$-0.1\cdot z_{w,t-1} - \sum_{q\in W} y^0_{w,q,t} + \sum_{q\in W} \phi_{w,q,t} \geq z_{w,t} - \sum_{q\in W} y^{receive}_{w,q,t} + D_{w,t} - m_{w,t} - x_{w,t} - 0.9\cdot z_{w,t}, \quad \forall w \in W, t \in T$$

$$y^0_{w,q,t} - \pi_{w,q,t} \geq 0, \quad \forall w, q \in W, t \in T$$

$$y^0_{w,q,t} + \theta_{w,q,t} \leq C^{transp}_{w,q}, \quad \forall w, q \in W, t \in T$$

$$\sum_{q\in W} y^0_{w,q,t} + \sum_{q\in W} \alpha_{w,q,t} \leq z_{w,t-1}, \quad \forall w \in W, t \in T$$

$$\sum_{q\in W} y^0_{w,q,1} + \sum_{q\in W} \eta_{w,q,1} \leq z_{w,1}, \quad \forall w \in W$$

$$-\alpha_{w,q,t} \leq \ Q_{w,q,t} \leq \alpha_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

$$-\eta_{w,q,1} \leq \ Q_{w,q,1} \leq \eta_{w,q,1}, \quad \forall w \in W, q \in W$$

$$-\phi_{w,q,t} \leq \; Q_{w,q,t} \leq \phi_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

$$-\theta_{w,q,t} \leq \; Q_{w,q,t} \leq \theta_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

$$-\pi_{w,q,t} \leq \; Q_{w,q,t} \leq \pi_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

$$-\sigma_{w,q,t} \leq e_{w,q} \cdot Q_{w,q,t} \leq \sigma_{w,q,t}, \quad \forall w \in W, t \in T, q \in W$$

$$\sigma_{w,q,t} \geq 0, \quad \forall w \in W, t \in T, q \in W$$

$$\alpha_{w,q,t} \geq 0, \quad \forall w \in W, t \in T, q \in W$$

$$\beta \in R, \quad \forall w \in W, t \in T, q \in W$$

$$Q_{w,q,t} \in R, \quad \forall w \in W, t \in T, q \in W$$

$$y^0_{w,q,t} \in R, \quad \forall w \in W, t \in T, q \in W$$

The objective value obtained from solving the Adjustable Robust Optimization (ARO) problem with linear decision rules represents the optimal performance of our Internal Redistribution system under the worst-case scenario of the uncertainty set δ, while allowing for adjustments based on the realized uncertainties. Unlike classical robust optimization of task 1, which considers a static worst-case scenario, ARO provides the flexibility to adapt decision variables linearly to changes in uncertainty. This flexibility would typically result in a better objective value, reflecting the best possible outcome under the most adverse conditions, given the constraints of the linear decision rules. Thus, the objective value signifies how well our system could perform while dynamically responding to uncertainty within the predefined linear framework.