Formulation of the 3 Warehouses Problem

Sets

- $w, q \in W$: set of warehouses, where w and q belong to $\{1, 2, 3\}$
- $t \in T$: set of timeslots (daily), in $\{1, 2, \dots\}$

Parameters

- $D_{w,t}$: coffee demand for warehouse w in period t, being constant at 4 units per day
- C_w^{storage}: storage capacity limit for warehouse w
- $C_{w,q}^{\text{transp}}$: daily transportation capacity limit for what warehouse w can send to warehouse q, where if $w = q$ then the capacity is zero
- $p_{w,t}$: external coffee price for warehouse w at time t, continuous in $[0, 10]$
- $e_{w,q}$: transportation per-unit cost between warehouse w and q, where if $w = q$ then the cost is zero
- b_w: per-unit cost of missing daily demand for warehouse w

Variables

- $x_{w,t}$: continuous, amount of coffee ordered in timeslot t by warehouse w
- $z_{w,t}$: continuous, amount of coffee stored by the end of timeslot t in warehouse w
- $m_{w,t}$: continuous, amount of coffee missing when daily demand is not met in warehouse w in t
- $y_{w,q,t}^{\text{send}}$: continuous, amount of coffee sent by warehouse w to q in timeslot t
- $y_{w,q,t}^{\text{receive}}$: continuous, amount of coffee received by warehouse w from q in timeslot t

Assumptions

- The initial stock is $z_{w,0} = 2, \forall w \in W$

In this case, t can be equal to 0 as $z_{w,0}$ will represent the initial stock for warehouse w in $t = 1$. This also allows for units to be transported between warehouses in $t = 1$, so $y_{w,q,1}$ can take a value different than zero.

Objective Function

Minimize the total cost of the coffee distribution system (orders, transfers, and missed) so that the demands are met. The objective function includes all 3 costs: orders placed, transport between warehouses, and failing to meet the demand.

$$\min \sum_{w \in W} \sum_{t \in T} x_{w,t} \cdot p_{w,t} + \sum_{w \in W} \sum_{q \in W} \sum_{t \in T} y^{\text{send}}_{w,q,t} \cdot e_{w,q} + \sum_{w \in W} \sum_{t \in T} m_{w,t} \cdot b_w \quad (1)$$

Constraints

1. To always respect the truck (transport) capacity between the warehouses:

$$y^{\text{send}}_{w,q,t} \leq C^{\text{transp}}_{w,q}, \quad \forall w, q \in W, t \in T \quad (2)$$

2. To have the same quantity being received and sent between two warehouses in a specific period:

$$y^{\text{send}}_{w,q,t} = y^{\text{receive}}_{q,w,t}, \quad \forall w, q \in W, t \in T \quad (3)$$

3. To always respect the storage capacity in all warehouses:

$$z_{w,t} \leq C^{\text{storage}}_{w}, \quad \forall w \in W, t \in T \quad (4)$$

4. To ensure demand fulfillment in all warehouses in $t \in T$:

$$x_{w,t} + m_{w,t} + z_{w,t-1} + \sum_{q \in W} y^{\text{receive}}_{w,q,t} = D_{w,t} + z_{w,t} + \sum_{q \in W} y^{\text{send}}_{w,q,t}, \quad \forall w \in W, t \in T \quad (5)$$

5. The amount sent between warehouses can only be determined by the amount stored in the previous time slot:

$$\sum_{q \in W} y^{\text{send}}_{w,q,t} \leq z_{w,t-1}, \quad \forall w \in W, t \in T \quad (6)$$

6. Non-negativity for all variables:

$$x_{w,t} \geq 0,\ z_{w,t} \geq 0,\ m_{w,t} \geq 0, \quad \forall w \in W, t \in T$$

$$y^{\text{send}}_{w,q,t} \geq 0,\ y^{\text{receive}}_{w,q,t} \geq 0, \quad \forall w \in W, t \in T \quad (7)$$