Workforce Scheduling

Decision Variables

x_s : Integer variable representing the number of operators assigned to shift s.
a_s : Binary variable that is 1 if shift s is activated, and 0 otherwise.

In this version we consider the fact that the matrix already accounts for both types of shifts. Thus, we only need one decision variable x_s, representing the number of operators assigned to shift. We also need to note that since break times are embedded in the matrix, no additional constraints are needed to account for them.

Objective Function

Minimize the total cost:

$$\text{Minimize} \quad \sum_{s=1}^{S} (x_s \cdot c_s + a_s \cdot f)$$

Constraints

Demand Coverage:

$$\sum_{s=1}^{S} \text{Cover}_{st} \cdot x_s \geq d_t \quad \forall t \in \{1, \ldots, 24\}$$

Maximum Number of Shifts:

$$\sum_{s=1}^{S} a_s \leq 12$$

Operator Capacity:

$$\sum_{s=1}^{S} x_s \leq 165$$

Shift Activation:

$$x_s \leq M \cdot a_s \quad \forall s \in \{1, \ldots, S\}$$

Where $\mathrm{M} = 165$ (maximum number of operators that could be assigned to any shift).

Part B: Minimizing Maximum Over-Coverage

The goal now is to revise the model to minimize the maximum overcoverage across all hours, while ensuring that the total cost does not exceed the optimal objective value from the previous model.

New Decision Variables

$$\begin{aligned}
x_s &: \text{Integer variable representing the number of operators assigned to shift } s. \\
a_s &: \text{Binary variable that is 1 if shift } s \text{ is activated, and 0 otherwise.} \\
\text{Over}_t &: \text{Continuous variable representing the overcoverage at hour } t. \\
\text{MaxOver} &: \text{Continuous variable representing the maximum overcoverage across all hours } t.
\end{aligned}$$

Objective Function

Minimize the maximum overcoverage (minimize the worst-case overcoverage):

$$\text{Minimize} \quad \text{MaxOver}$$

Constraints

Demand Coverage with Overcoverage:

$$\sum_{s=1}^{S} \text{Cover}_{st} \cdot x_s - d_t = \text{Over}_t \quad \forall t \in \{1, \dots, 24\}$$

Maximum Overcoverage:

$$\text{Over}_t \leq \text{MaxOver} \quad \forall t \in \{1, \dots, 24\}$$

Total Cost Constraint:

$$\sum_{s=1}^{S} (x_s \cdot c_s + a_s \cdot f) \leq Z^*$$

Other Constraints:

$$\begin{aligned}
&\sum_{s=1}^{S} a_s \leq 12 \quad \text{(Maximum Number of Shifts)} \\
&\sum_{s=1}^{S} x_s \leq 165 \quad \text{(Operator Capacity)} \\
&x_s \leq M \cdot a_s \quad \forall s \in \{1, \dots, S\} \quad \text{(Shift Activation)}
\end{aligned}$$

Where $\text{M} = 165$ (maximum number of operators that could be assigned to any shift).

Constraint Explanations

- **Demand Coverage with Overcoverage**: This constraint ensures that Over_t captures any excess coverage beyond the demand d_t.
- **Maximum Overcoverage**: Ensures that MaxOver is at least as large as any overcoverage Over_t.
- **Total Cost Constraint**: Ensures that the total cost does not exceed the optimal objective value Z^*.

"Spiced-Up" Complex Case

In the real world we don't have precise data. So, imagine a case where it is not possible to integrate breaks into the coverage matrix. Now,we'll model breaks and shift assignments explicitly with additional decision variables. Keep in mind we still want a MIP, so we must keep it linear.

New Decision Variables

x_s^{8hr} : Integer variable representing the number of operators assigned to 8-hour shift s.

x_s^{3hr} : Integer variable representing the number of operators assigned to 3-hour shift s.

a_s : Binary variable that is 1 if shift s is activated, and 0 otherwise.

b_s^4 : Binary variable that is 1 if the break in 8-hour shift s occurs after 4 hours, and 0 otherwise.

b_s^5 : Binary variable that is 1 if the break in 8-hour shift s occurs after 5 hours, and 0 otherwise.

Objective Function

Minimize the total cost:

$$\text{Minimize} \quad \sum_{s=1}^{S} \left(x_s^{8hr} \cdot c_s + x_s^{3hr} \cdot c_s + a_s \cdot f\right)$$

Constraints

Demand Coverage:

$$\sum_{s=1}^{S} \left(\text{Cover}_{st}^{8hr}(b_s^4, b_s^5) \cdot x_s^{8hr} + \text{Cover}_{st}^{3hr} \cdot x_s^{3hr}\right) \geq d_t \quad \forall t \in \{1, \dots, 24\}$$

No Overlap Constraint:

$$\sum_{s=1}^{S} \left(\text{Cover}_{st}^{8hr}(b_s^4, b_s^5) \cdot x_s^{8hr} + \text{Cover}_{st}^{3hr} \cdot x_s^{3hr}\right) \leq 165 \quad \forall t \in \{1, \dots, 24\}$$

Break Assignment for 8-hour Shifts:

$$b_s^4 + b_s^5 = 1 \quad \forall s \text{ that is an 8-hour shift}$$

Break Time Impact on Coverage:

$$\text{Cover}_{st}^{8hr}(b_s^4, b_s^5) = \begin{cases} 0 & \text{if } t \text{ is the break hour} \\ 1 & \text{otherwise} \end{cases}$$

Maximum Number of Shifts:

$$\sum_{s=1}^{S} a_s \leq 12$$

Operator Capacity:

$$\sum_{s=1}^{S} \left(x_s^{8hr} + x_s^{3hr}\right) \leq 165$$

Shift Activation:

$$x_s^{8hr} + x_s^{3hr} \leq M \cdot a_s \quad \forall s \in \{1, \ldots, S\}$$

Where $\mathrm{M} = 165$ (maximum number of operators that could be assigned to any shift).

Constraint Explanations

- **Demand Coverage**: Ensure that the demand for each hour t is met by the operators assigned to the shifts covering that hour. Here the $\mathrm{Cover}_{st}^{8hr}(b_s^4, b_s^5)$ will be adjusted based on whether b_4 or b_5 is activated.
- **No Overlap**: This constraint ensures that the total number of operators working at any given hour across all shifts does not exceed the available pool of operators (165 in this case). This is crucial for preventing scheduling conflicts where too many operators might be assigned to overlapping shifts during certain hours. Note that since we are using aggregate variables for the number of operators assigned to each shift, overlap in the traditional sense (where an individual operator cannot be on two shifts at the same time) is implicitly handled.
- **Break Assignment for 8-hour Shifts**: Ensure that each 8-hour shift has exactly one break, either after 4 or 5 hours.
- **Break Time Impact on Coverage**: Adjust the coverage for 8-hour shifts based on the break time. This means that for each hour t, the coverage $\mathrm{Cover}_{st}^{8hr}$ will be set to 0 during the break hour defined by b_4 or b_5
- **Operator Capacity**: Ensure that the total number of operators assigned does not exceed 165.
- **Shift Activation**: Ensure that a shift is considered active if and only if it has operators assigned.