See

prelab.

The transfer function of a DC electric motor with respect to angular velocity is:

$$G_\omega(s) = \frac{\Omega(s)}{V(s)} = \frac{A}{\tau s + 1}$$

Where

- A and τ are positive, real-valued constants
- $V(s)$ and $\Omega(s)$ are voltage and angular velocity as function of s in the Laplace domain.

Note that $\Omega(s) := \mathcal{L}\{\omega(t)\}$

Q1.

We will now develop a formula for the motor DC gain constant A in terms of a step input change ΔV and step output change $\Delta \omega$

⊙ problem a.

Using the final value theorem, find an expression for the steady state value of $\omega(t)$ when a step input of amplitude V_x is applied.

Given that the final value theorem, the steady state value of $f(t)$ is given by:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

When a step input $V(s) = \frac{V_x}{s}$ is applied, the output in the Laplace domain is given by:

$$\Omega(s) = G_\omega(s) \cdot V(s) = \frac{A}{\tau s + 1} \cdot \frac{V_x}{s}$$

$$\lim_{t \to \infty} \omega(t) = \lim_{s \to 0} s\Omega(s) = \lim_{s \to \infty} \left(\frac{A \cdot V_x}{\tau s + 1} \right)$$

Since $\tau > 0$ when $s \to 0$:

$$\lim_{t \to \inf} \omega(t) = \frac{A \cdot V_x}{\tau \cdot 0 + 1} = A \cdot V_x$$

> The steady-state value of $\omega(t)$ under a step amplitude of V_x is directly proportional to the product of DC gain and step input V_x

⊙ problem b.

Give the expression for $\omega(t)$ in response to a step input V_x at time $t = 0$. Assume a non-zero initial condition for $\omega(t)$, i.e., $\omega(t) = \omega_0$.

The zero-state response to step input V_x is given by the inverse Laplace transform of $G_\omega(s) \cdot V(s)$:

$$\omega_{zs}(t) = AV_x \cdot (1 - e^{-\frac{t}{\tau}})$$

The zero-input response to initial condition ω_0 can be modeled as the response to an impulse input $\omega_0 \delta(t)$. The Laplace transform of the impulse response is:

$$\Omega_{zi}(s) = \frac{A \cdot \omega_0}{\tau s + 1}$$

Which the zero-input response is given by: $\omega_{zi}(t) = \omega_0 \cdot e^{-\frac{t}{\tau}}$

The total response $\omega(t)$ is the sum of the zero-state and zero-input responses (due to linearity):

$$\omega(t) = \omega_{zs}(t) + \omega_{zi}(t) = AV_x \cdot (1 - e^{-\frac{t}{\tau}}) + \omega_0 \cdot e^{-\frac{t}{\tau}}$$

⊘ **problem c.**

For $\omega(t)$ in computed in part b, what is the $\lim_{t\to\infty} \omega(t)$? How does it compare to result in part a?

$$\lim_{t\to\infty} \omega(t) = \lim_{t\to\infty} \left(AV_x \cdot (1 - e^{-\frac{t}{\tau}}) + \omega_0 \cdot e^{-\frac{t}{\tau}} \right)$$

Since $e^{-\frac{t}{\tau}} \to 0$ as $t \to \infty$, the steady-state value of $\lim_{t\to\infty} \omega(t)$ is:

$$\lim_{t\to\infty} \omega(t) = AV_x(1 - 0) + 0 = AV_x$$

> We see that the steady-state value of $\omega(t)$ is the same. The initial condition ω_0 does not affect the steady-state value of $\omega(t)$, only influence transient response.

⊘ problem d.

Now assume that you run the motor with an initial step input of V_{min} until time t_0. At time t_0, assume that the system has reached steady state and the step input is changed to V_{max} at time t_0. In other words, the system input will take the form

$$v(t) = \begin{cases} v_{\text{min}} & \text{if } 0 \le t < t_0 \\ v_{\text{max}} & \text{if } t \ge t_0 \end{cases}$$

where $t_0 \gg \tau$ and V_{max} and V_{min} may be non-zero.

Use the results above to show that:

$$A = \frac{\Delta\omega}{\Delta V}$$

where $\Delta V = V_{\max} - V_{\min}$ and $\Delta\omega = \omega_{ss} - \omega_0$ where ω_0 is the steady-state response to a constant input $V_{\min}$ and ω_{ss} is the steady-state response to the input $V_{\max}$

For input $V_{\min}$, the steady-state response is $\omega_0 = A \cdot V_{\min}$

Similarly, for input $V_{\max}$, the steady-state response is $\omega_{ss} = A \cdot V_{\max}$

Thus, the change in steady-state response is

$$\Delta\omega = \omega_{ss} - \omega = A \cdot V_{\max} - A \cdot V_{\min} = A \cdot \Delta V$$

Thus $A = \frac{\Delta\omega}{\Delta V}$

Q2.

Using the formula derived in Q1, use the following graphs to calculate A for this system.

Given $A = \frac{\Delta\omega}{\Delta V}$, from the graph $V_{\min} = 1$ and $V_{\max} = 5$ and $\omega_0 = 5$ and $\omega_{ss} = 25$, $A = 5$

Q3.

For a first order system, the time it takes a step response to reach 63.2% of its steady state value ($t_1 - t_0$ in Fig. 1) is the response's time constant τ. i.e., at time t_1, $\omega(t_1) = 0.632\Delta\omega + \omega_0$. Find the time constant τ for the above system.

$$\omega(t_1) = 0.632\Delta\omega + \omega_0 = 0.632 \cdot (25 - 5) + 5 = 17.64$$

From the graph, $\tau \approx 0.8$ sec

Q4.

Using A and τ calculated in Q2 and Q3, find the transfer function in terms of s

$$G_\omega(s) = \frac{\Omega(s)}{V(s)} = \frac{A}{\tau s + 1} = \frac{3}{0.05s + 1}$$

Q5.

The system quickly rises to a steady-state value without any oscillation, which suggests a first-order system. The transfer function:

$$G(s) = \frac{K}{\tau s + 1}$$

Q6.

Deriving a transfer function experimentally and then use simulation software to design is preferable in situations where experimenting directly with the plant poses high risks, incurs excessive costs, requires downtown. For example, a chemical processing plant, probably we don't want to experiment with the actual system, since it could lead to dangerous chemical reactions, waste of materials. Using simulation engineers can safely and cost-effectively design and test control strategies before implementing in real system.

Conversely, deriving transfer function experimentally and then using simulation software to design is not preferable in situations

where the plant is simple and safe to experiment with, and the cost of experimenting is low. For instance, a small educational laboratory setup with low-cost components and minimal hazardous concerns would mean it might be more practical and education to design and calibrate the controller directly through experimentation and observe behaviours in real-time.

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lab.

○ Check to change the TCP address of the model URI from `QUARC > Settings > Preferences > Model`

$$G(w) = \frac{1.867468}{0.027947 * s + 1}$$