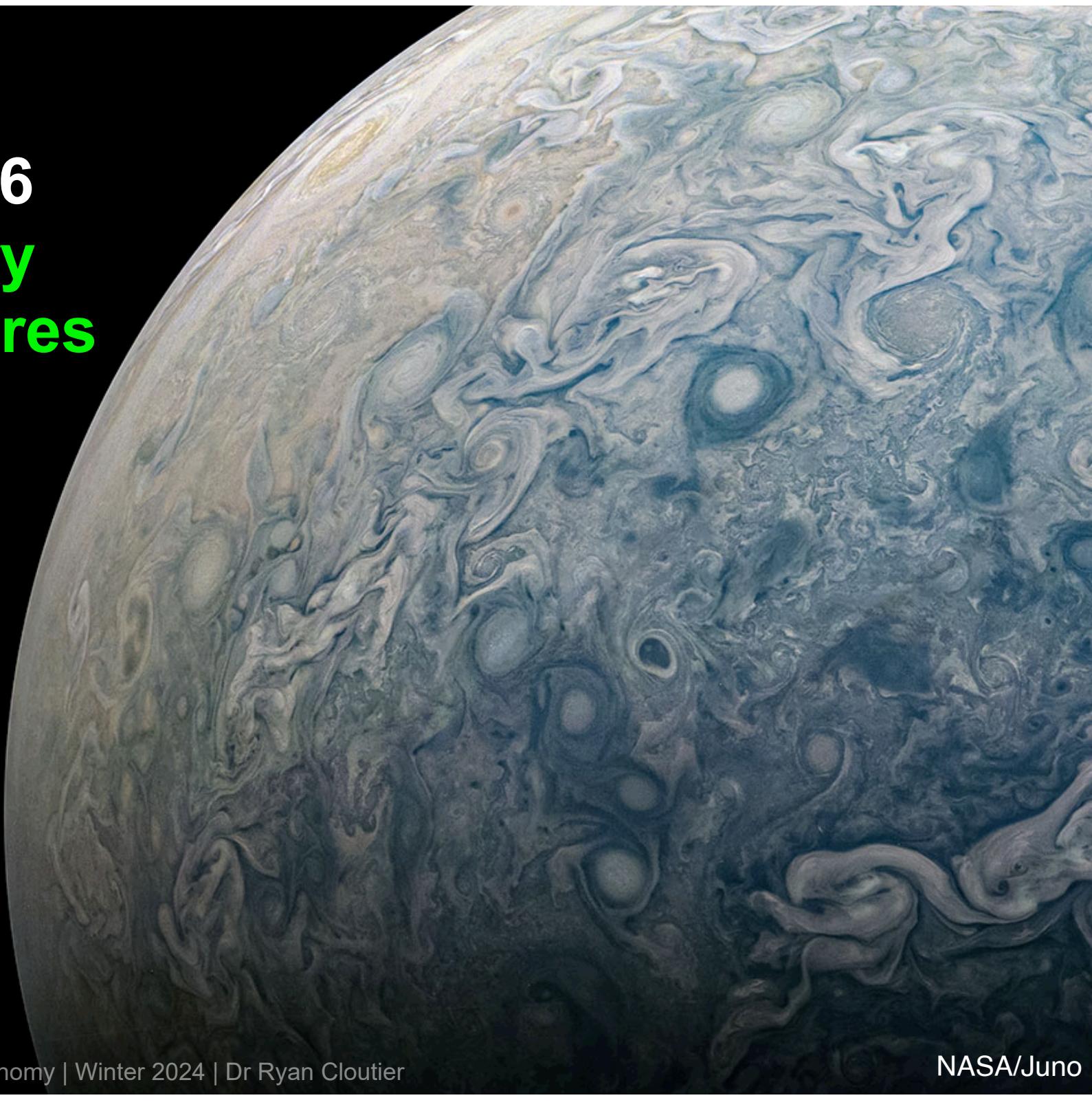


# Lecture 6

## Planetary Atmospheres



# Learning Objectives - Planetary Atmospheres

- 1) Derive the expression for hydrostatic equilibrium and use it to derive to pressure and density profiles of an Earth-like planet's atmosphere
- 2) Describe the energy sources that dictate the thermal structure of a planet's atmosphere
- 3) Understand the conditions that lead to atmospheric convection and evaluate where atmospheres are convective based on their thermal structures
- 4) List the main chemical constituents of the planetary atmospheres in the solar system and on exoplanets from transmission spectroscopy
- 5) Outline the physics behind the cloud formation process
- 6) Outline the physics behind winds in the solar system and beyond

# Atmospheres

The **gaseous outer layers** of planets and moons

Terrestrial planet atmospheres

- thin
- mass fractions of  $\lesssim 10^{-3}$
- dominated by heavy species  $\text{N}_2$ ,  $\text{CO}_2$ , and others

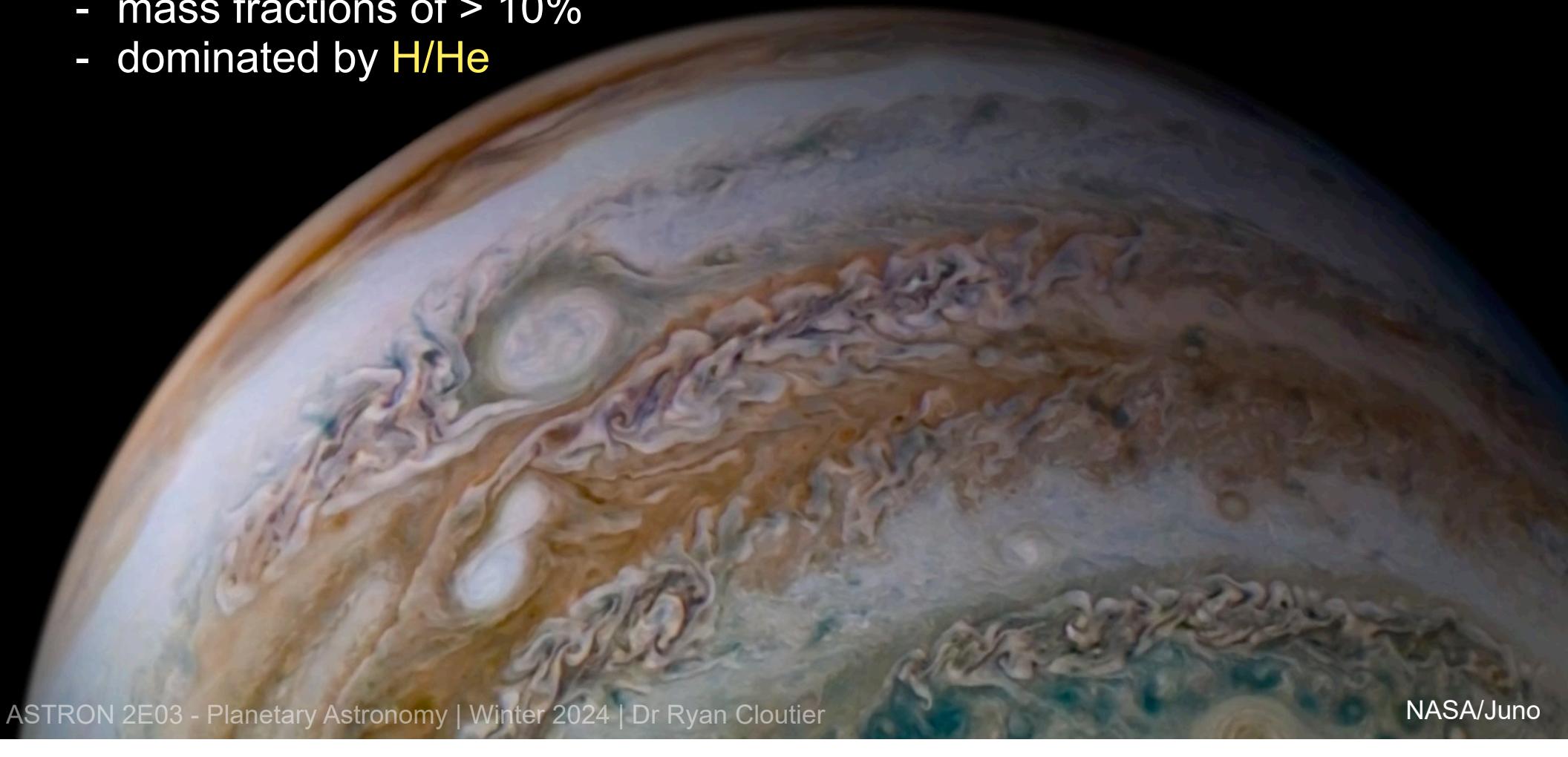


# Atmospheres

The **gaseous outer layers** of planets and moons

Giant planet atmospheres

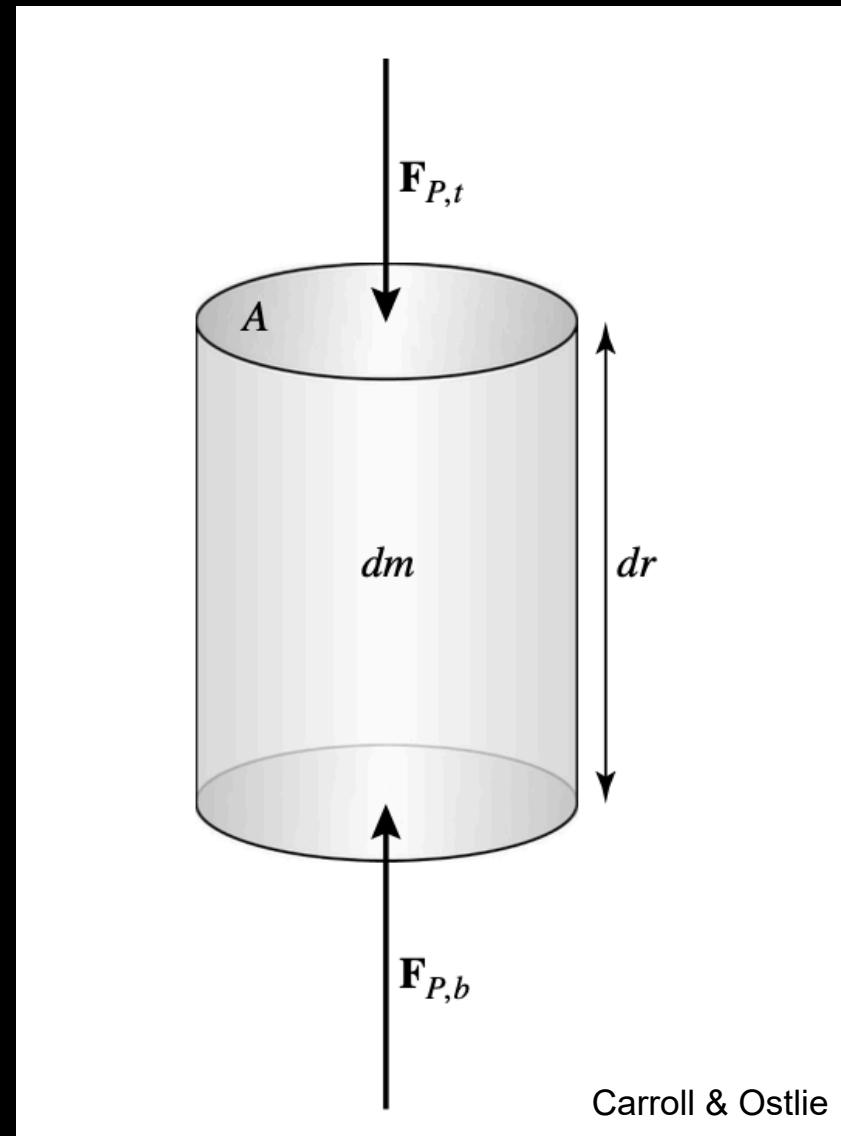
- deep
- mass fractions of > 10%
- dominated by H/He



# Hydrostatic Equilibrium (HSE)

Atmospheres are held up against the force of gravity  $F_g$  by a gas pressure gradient

$$\Delta P = \frac{F_{P,t} - F_{P,b}}{\Delta A}$$



Carroll & Ostlie

# Hydrostatic Equilibrium (HSE)

In class we'll derive the expression for HSE given below

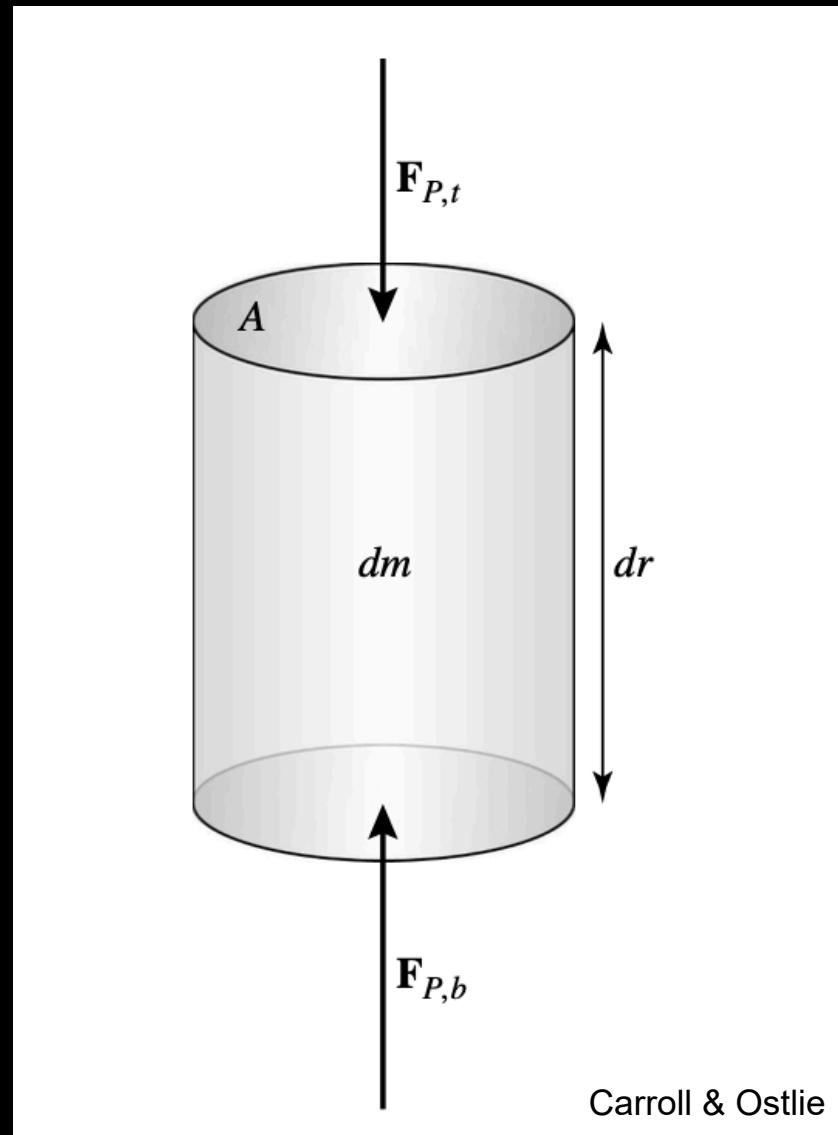
$$\frac{dP}{dr} = -\rho g$$

$P$ : fluid pressure

$r$ : vertical coordinate (i.e. height of the atmosphere)

$\rho$ : fluid density

$g$ : gravitational acceleration



Carroll & Ostlie

# Atmospheric structure

We're interested in knowing how the atmospheric pressure changes with height in the atmosphere.

To do this, we need to write down an equation of state (EOS) for the atmospheric gas

$$P(\rho, T)$$

An EOS typically has the form above.

It describes the relationship between a material's pressure, density, and temperature

# Atmospheric structure

In low to moderate pressure environments, typical of terrestrial planet atmospheres, the atmosphere is well-described as an ideal gas

$$P = \frac{\rho k_B T}{\mu m_H}$$

$P$ : fluid pressure

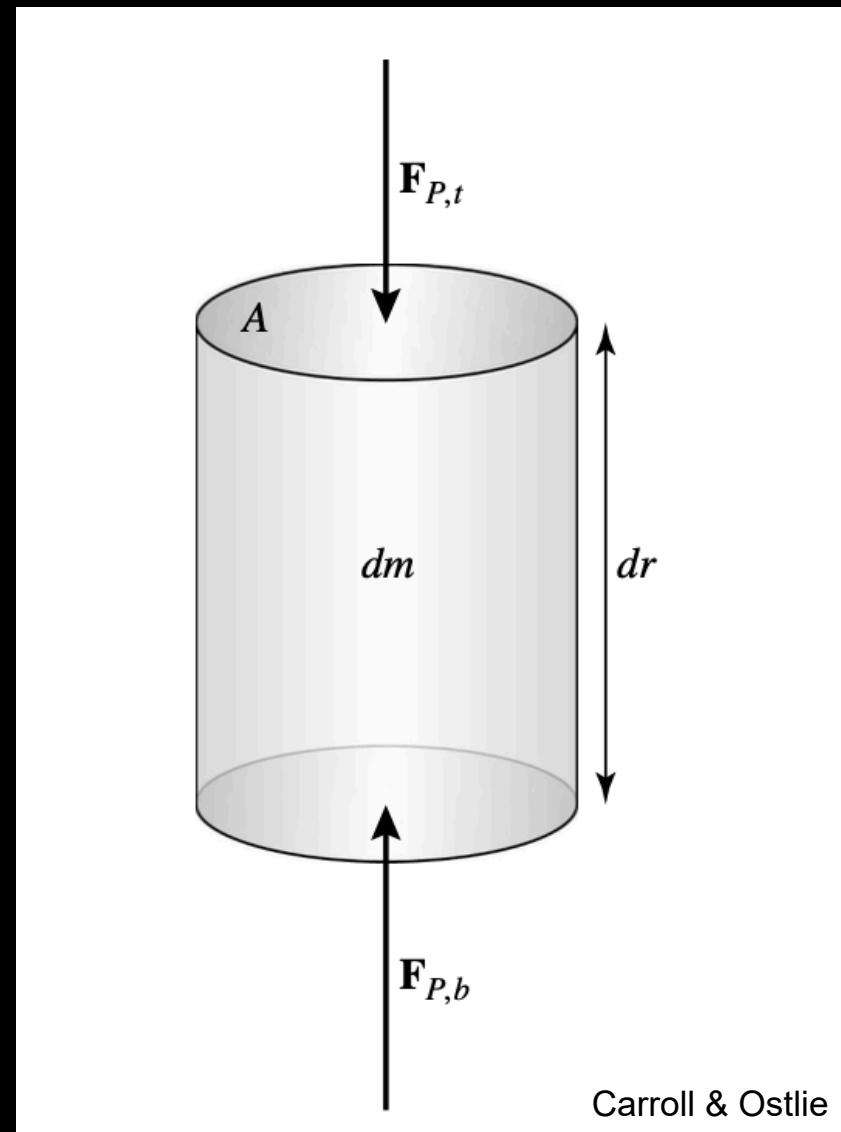
$k_B$ : Boltzmann constant

$T$ : fluid temperature

$\rho$ : fluid density

$\mu$ : mean molecular weight

$m_H$ : mass of the hydrogen atom



Carroll & Ostlie

# Mean Molecular Weight ( $\mu$ )

**Atomic mass unit** ~ the proton mass ~ the neutron mass =  $1.67 \times 10^{-27}$  kg

Gas	Composition	Mean molecular weight $\mu$
Molecular nitrogen	$N_2 = 2^*N$	$2^*14 = 28$
Molecular oxygen	$O_2 = 2^*O$	$2^*16 = 32$
Water	$H_2O = 2^*H + O$	$2^*1 + 16 = 18$
Earth atmosphere	78% $N_2$ + 22% $O_2$	$0.78^*28 + 0.22^*32 \sim 29$

# Atmospheric Structure

In class we'll show that for an atmosphere in HSE and composed of an ideal gas, the atmospheric pressure and density profiles are

$$P(r) = P_0 e^{-r/H}$$

$$\rho(r) = \rho_0 e^{-r/H}$$

where the subscript 0 indicates the values at the surface and  $H$  is the atmospheric scale height

$$H = \frac{k_B T}{\mu m_H g}$$

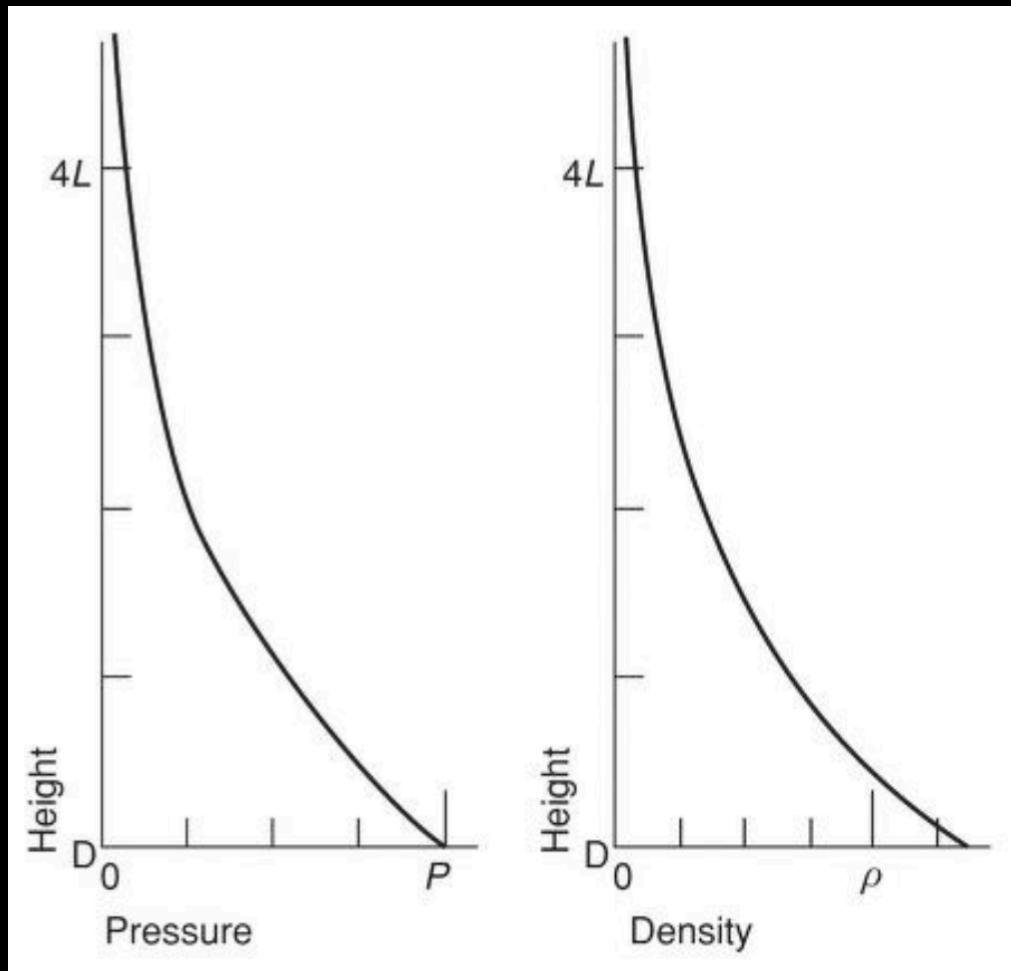
# Atmospheric Structure

$$P(r) = P_0 e^{-r/H}$$

$$\rho(r) = \rho_0 e^{-r/H}$$

where

$$H = 8.3 \text{ km} \left( \frac{T}{288 \text{ K}} \right) \left( \frac{g}{9.8 \text{ m/s}} \right)^{-1} \left( \frac{\mu}{29 m_H} \right)^{-1}$$



Carroll & Ostlie

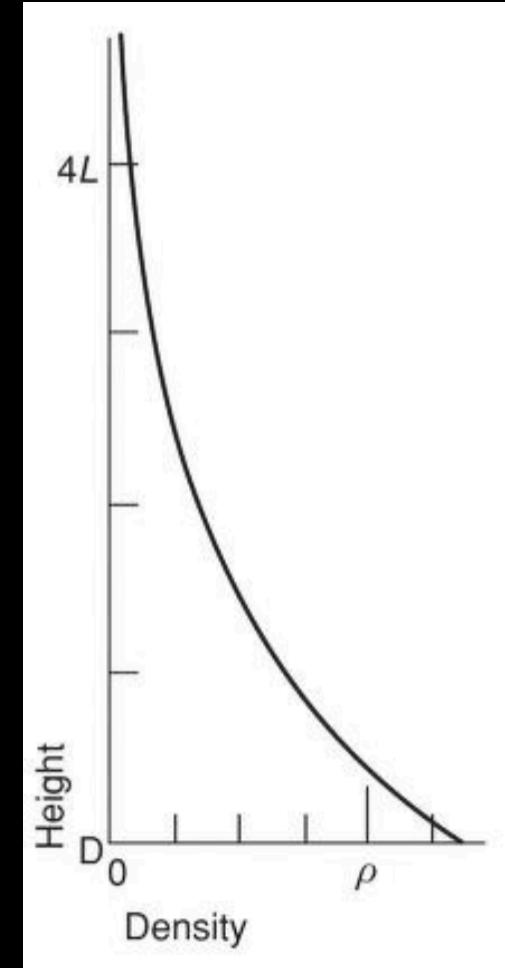
# Atmospheric Mass

Note that most of the atmosphere's mass is within **one scale height  $H$**

$$\rho(r) = \rho_0 \exp(-r/H)$$

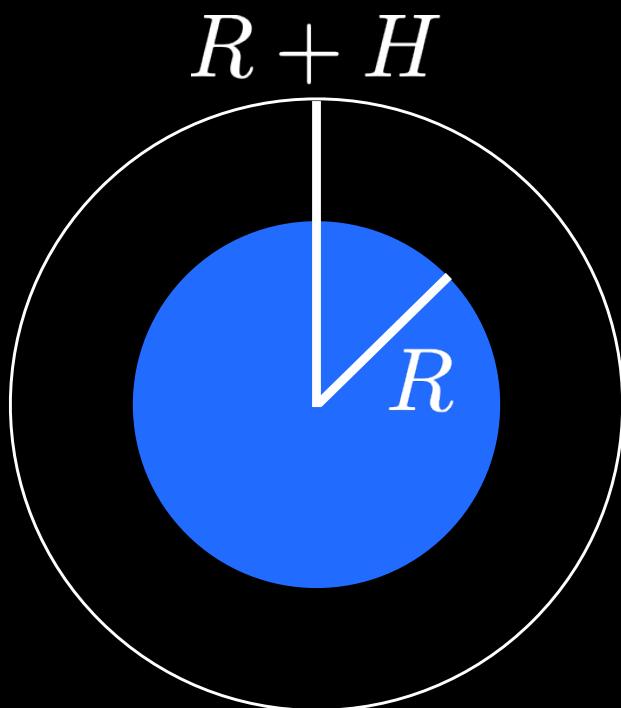
where

$$H = 8.3 \text{ km} \left( \frac{T}{288 \text{ K}} \right) \left( \frac{g}{9.8 \text{ m/s}} \right)^{-1} \left( \frac{\mu}{29 m_H} \right)^{-1}$$



Carroll & Ostlie

# Atmospheric Mass

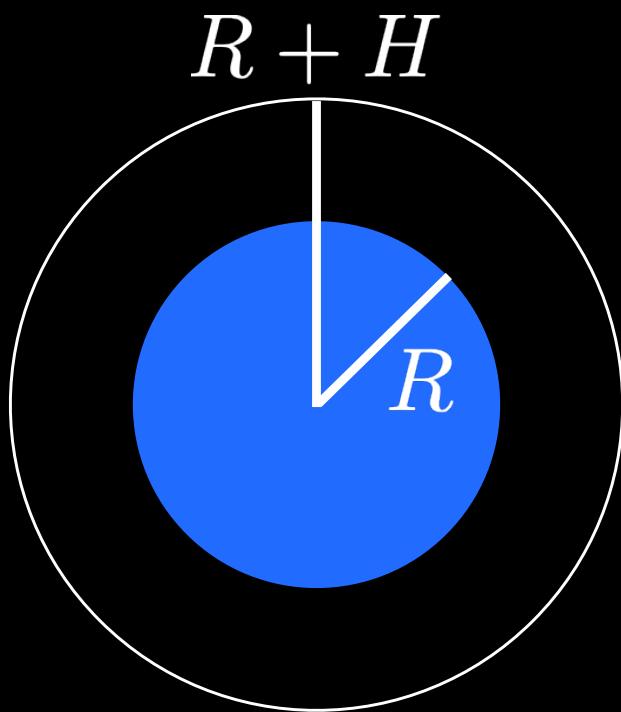


Normally, one would calculate the atmospheric mass by **integrating the atmospheric density profile**

$$\rho(r) = \rho_0 e^{-r/H}$$

from the planet's surface to  $H$ .  
(noting that  $\rho_0=1.2 \text{ kg/m}^3$ )

# Atmospheric Mass



But we can approximate the atmospheric mass by noting that most of the mass is contained within  $r < H$ , with  $\rho_0 = 1.2 \text{ kg/m}^3$ .

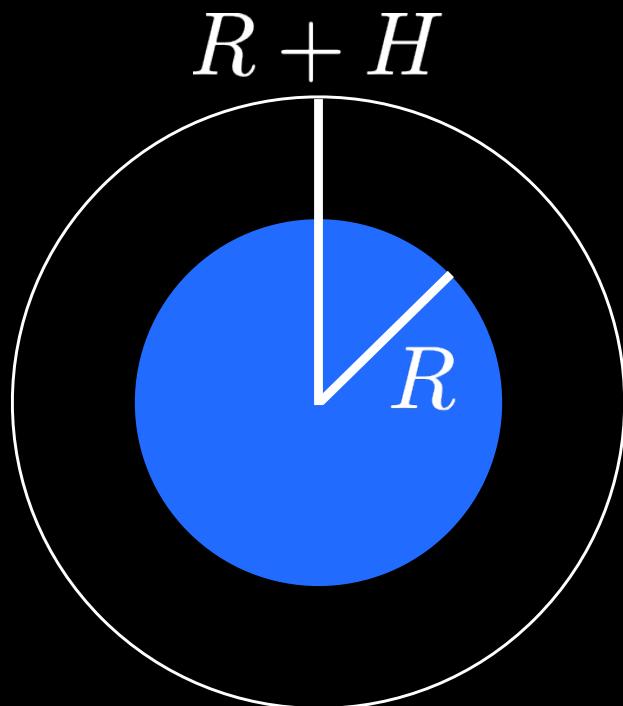
The atmospheric mass is then

$$\int_0^{M_{atm}} dM \approx \rho_0 dV$$

$$\begin{aligned} M_{atm} &\approx \rho_0 \int_R^{R+H} 4\pi r^2 dr \\ &= \frac{4\pi}{3} \rho_0 [(R+H)^3 - R^3] \\ &= 5 \times 10^{18} \text{ kg} \end{aligned}$$

Actual is  $5.1 \times 10^{18} \text{ kg}!!$

# Atmospheric Mass



On terrestrial planets for which  $H \ll R$ ,  
the atmospheric mass can be  
approximated as

$$M_{atm} = M_{atm,\oplus} \left( \frac{\rho}{\rho_\oplus} \right) \left( \frac{H}{H_\oplus} \right)$$

# Thermal Structure

Note that in deriving the pressure and density profiles

$$P(r) = P_0 e^{-r/H}$$

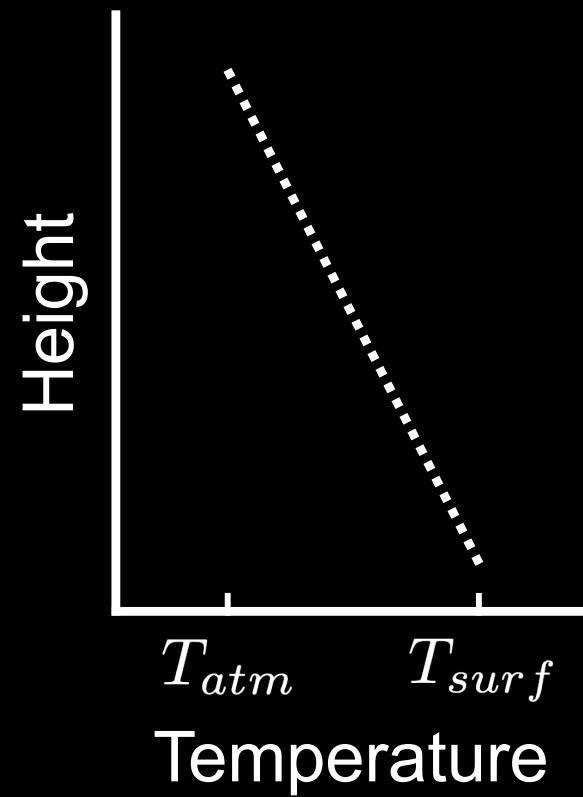
$$\rho(r) = \rho_0 e^{-r/H}$$

we implicitly assumed that the atmospheric temperature profile was isothermal

# Thermal Structure

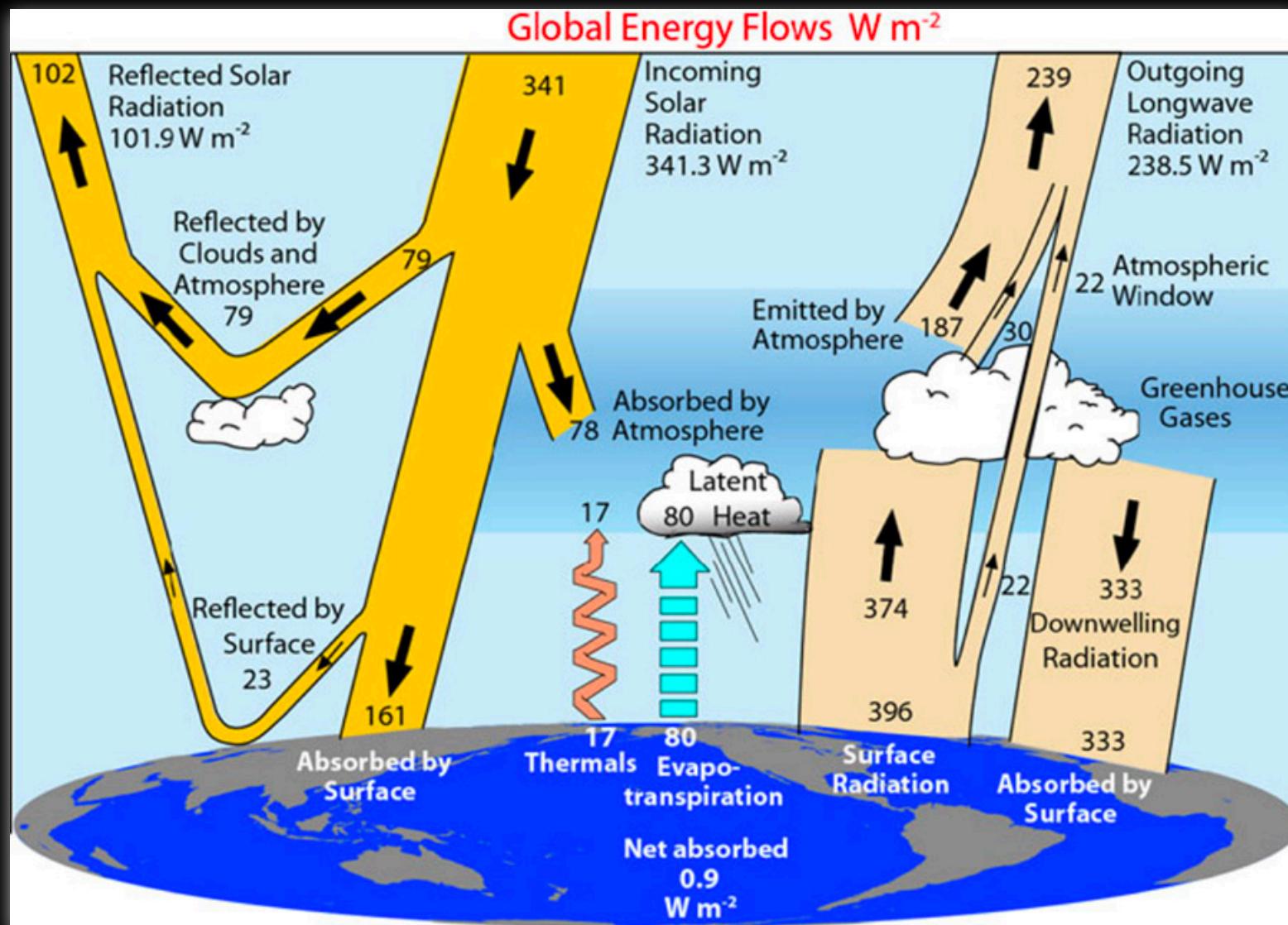
But recall from our discussion of  
that **greenhouse effect** that

$$\frac{dT}{dr} < 0$$



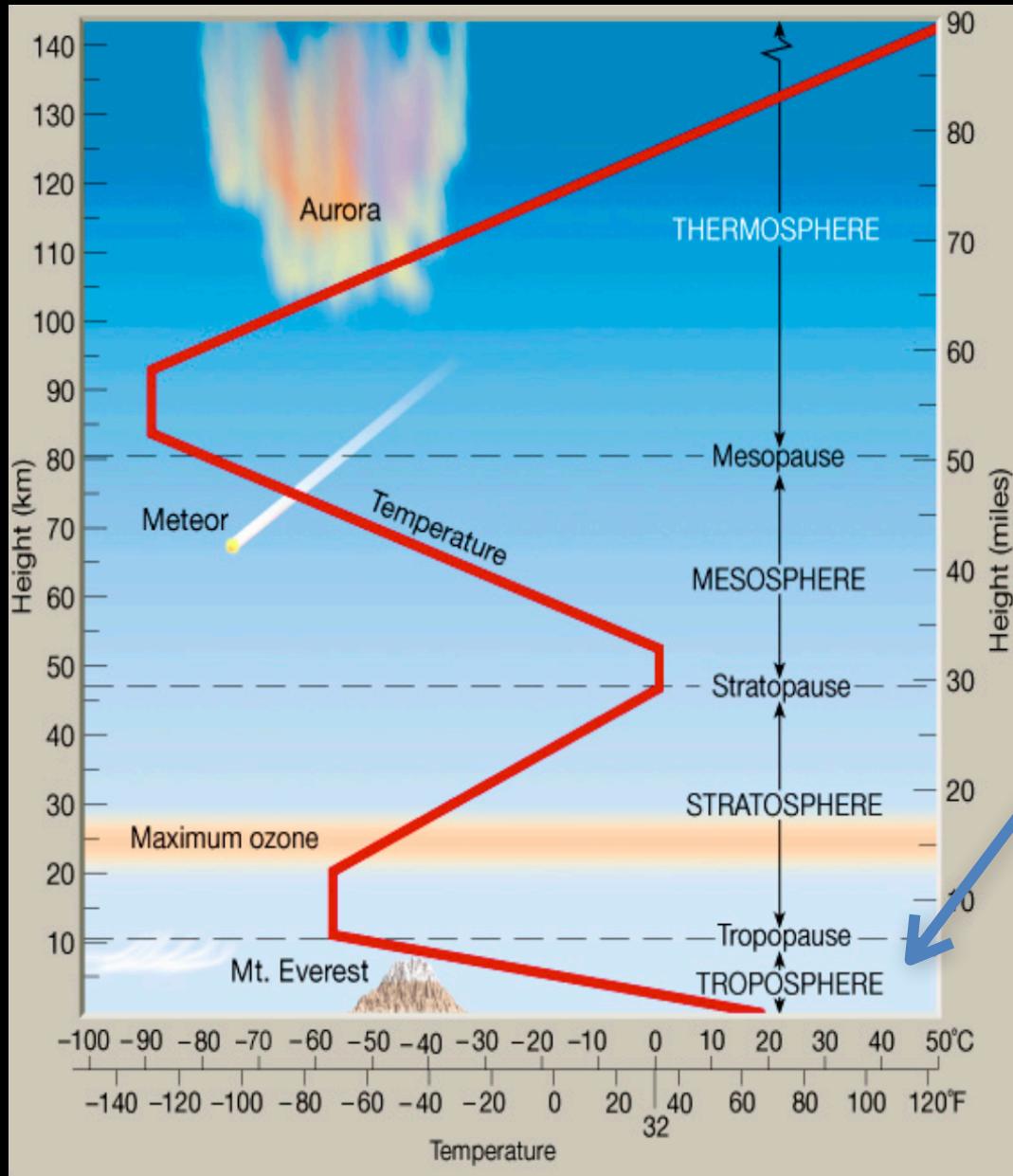
**We must consider all available energy sources**

# Radiative feedback alters the atmospheric temperature profile



Kevin Trenberth, John Fasullo and Jeff Kiehl

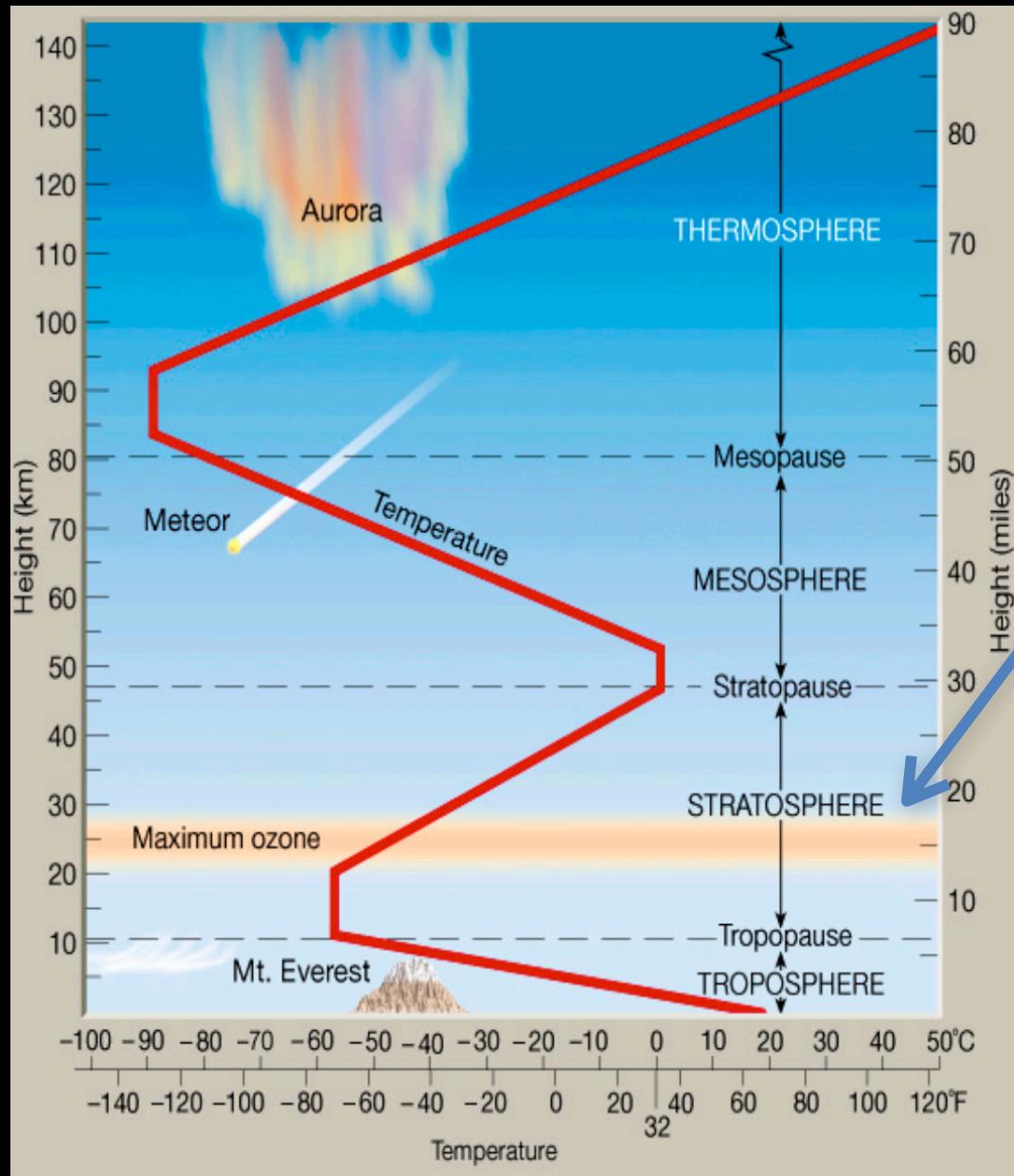
# Earth's Atmosphere



## TROPOSPHERE

- **Densest portion containing most of atmospheric mass below  $H = 8.3 \text{ km}$**
- **Rich in GH gases ( $\text{H}_2\text{O}, \text{CO}_2, \text{CH}_4$ )**
- **Energy transport is via convection**

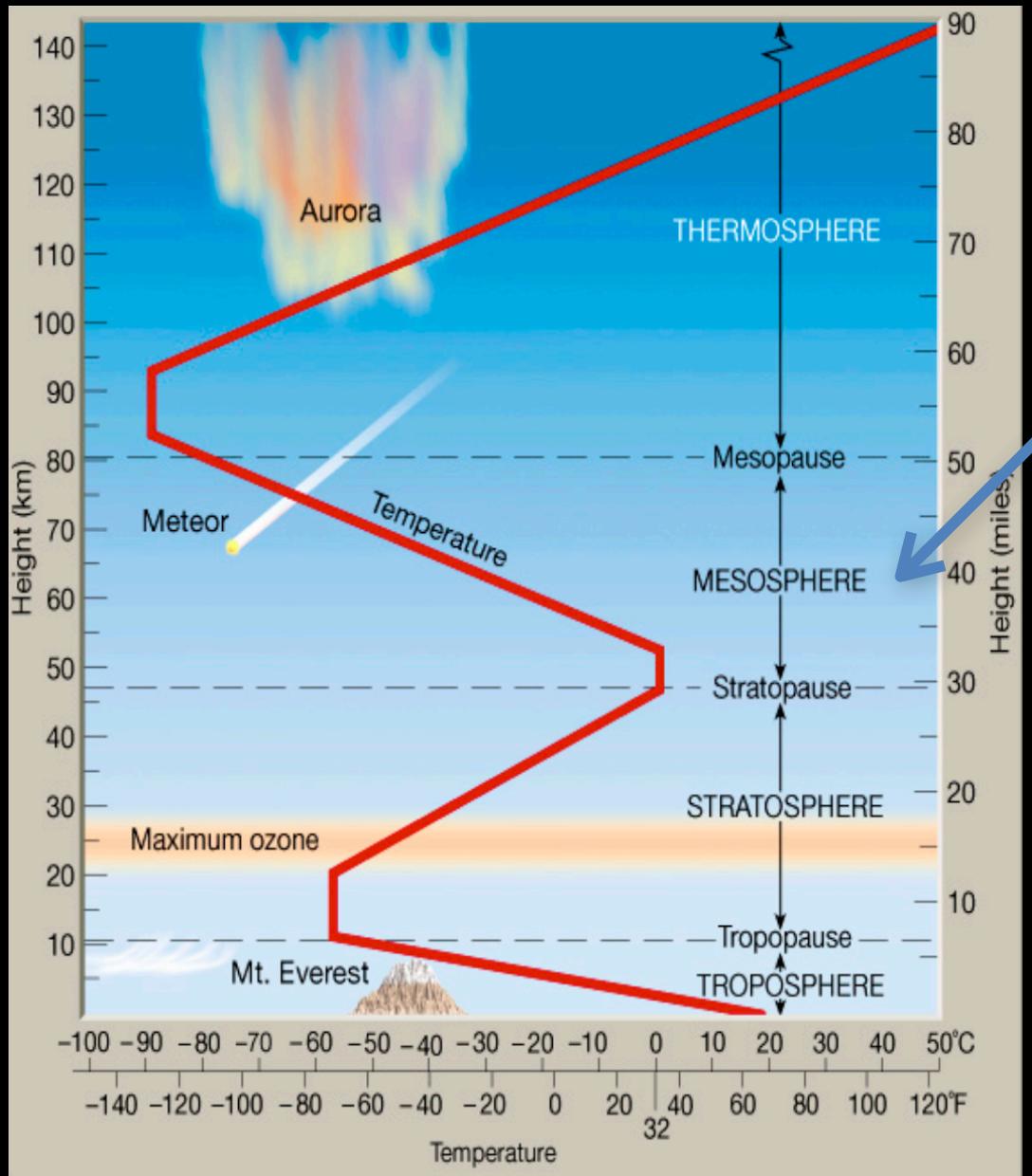
# Earth's Atmosphere



## STRATOSPHERE

- Thermal inversion produced by UV absorption of O<sub>3</sub>

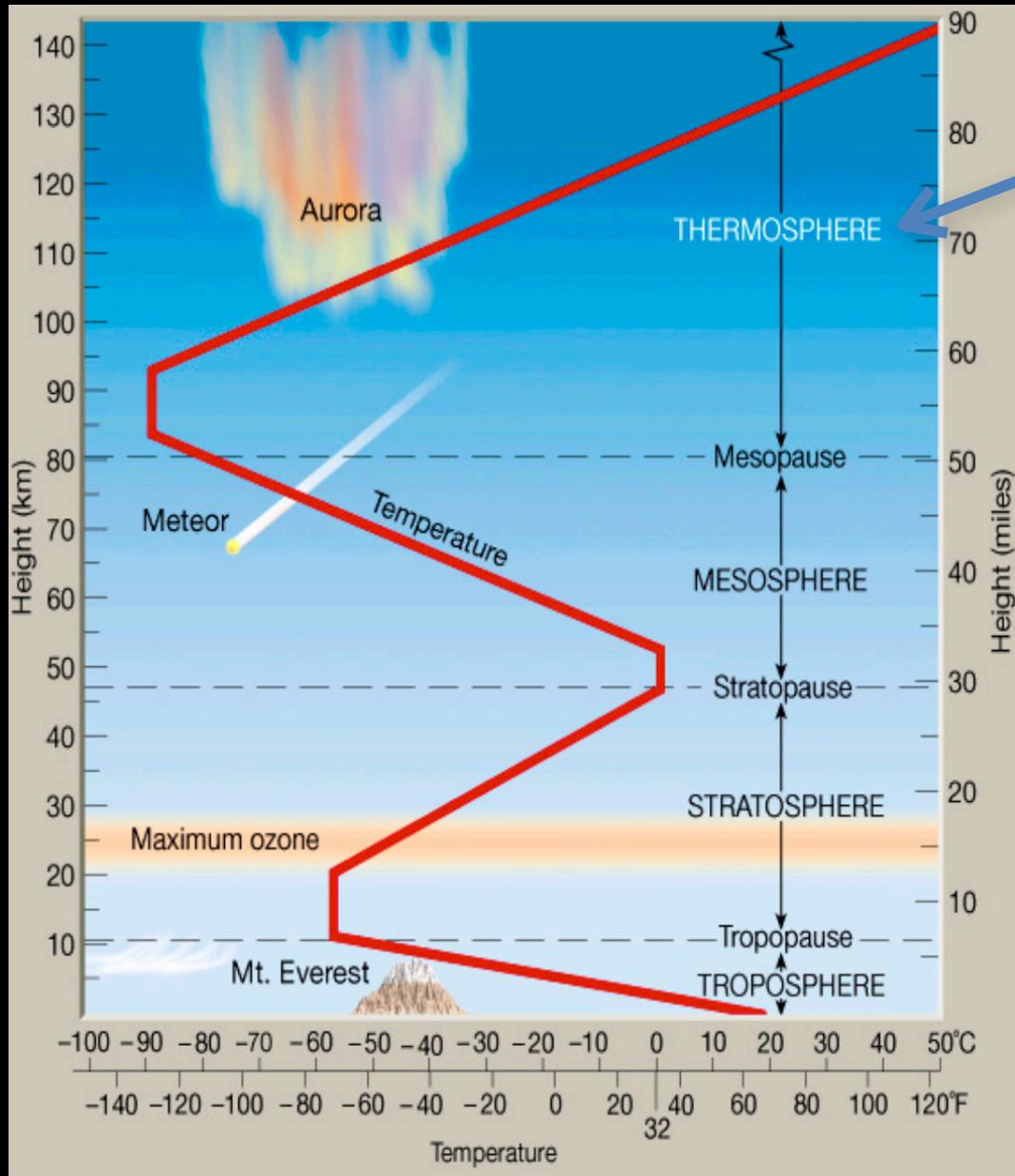
# Earth's Atmosphere



## MESOSPHERE

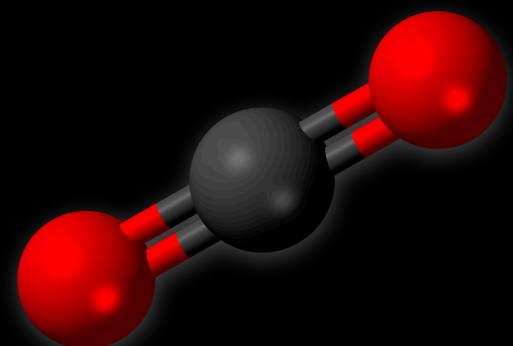
- Lower O<sub>3</sub> abundance results in **inefficient UV heating**
- GH gases do not “see” as much longwave radiation as in the troposphere, such that the **GH effect is weak**

# Earth's Atmosphere



## THERMOSPHERE

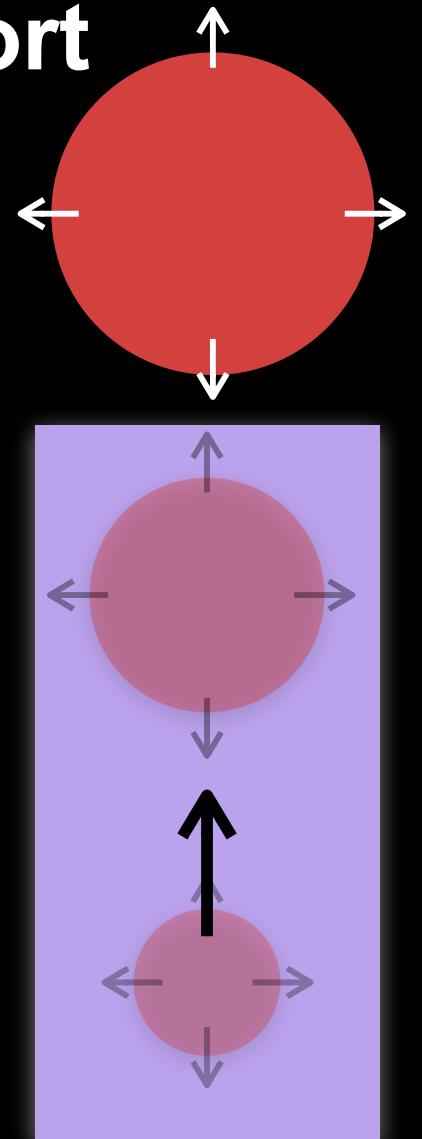
- UV heating by O<sub>2</sub>
- Auroral heating by the interactions with the solar winds
- Atmosphere density is low such that molecular cooling is inefficient



# Convective energy transport

Consider a **parcel of air** that is hotter than its surroundings

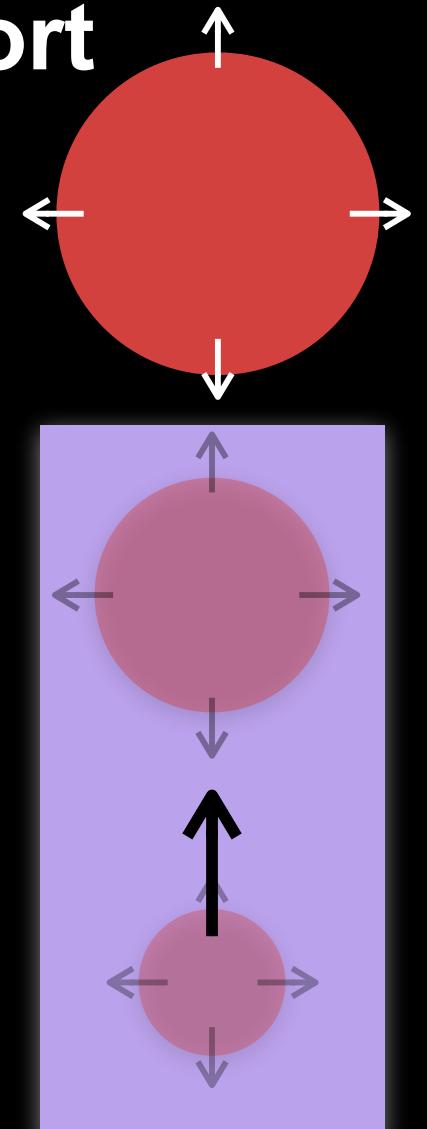
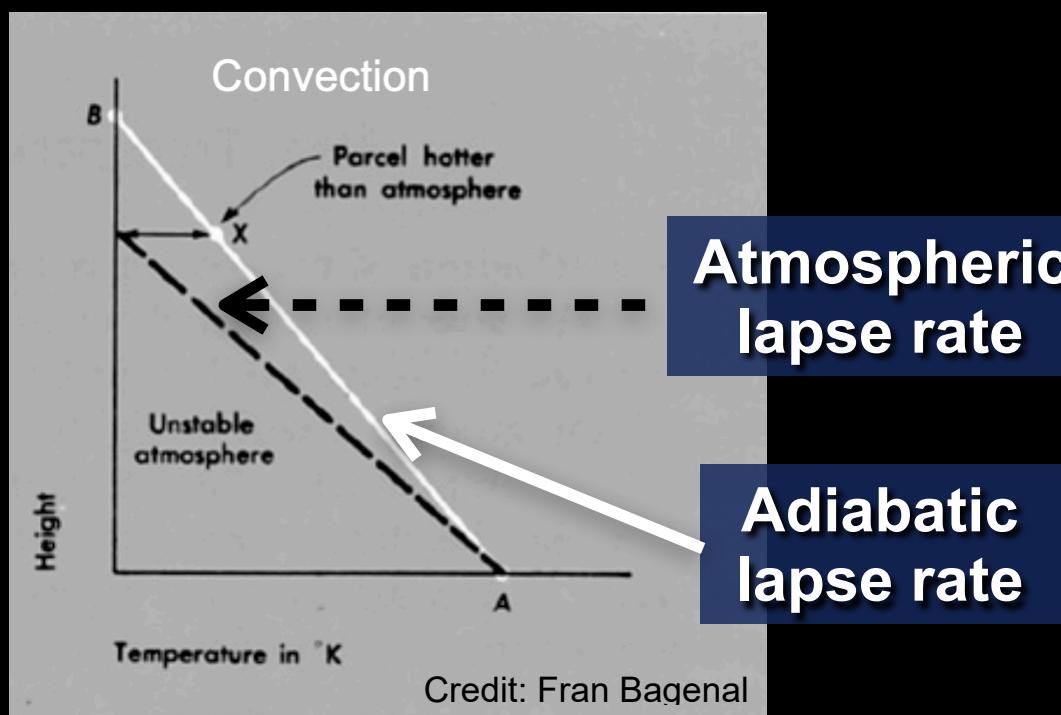
- according to the ideal gas law,  $P_{\text{parcel}} > P_{\text{surroundings}}$  so the **parcel expands** to reach a pressure balance
- its lower density causes the **parcel to rise** (buoyancy)
- because  $P$  decreases with altitude, the rising parcel continues to **expand and cool as it rises**
- if cooling is slow, then the parcel will **remain warmer than its surroundings** and continue to **rise**, carrying thermal energy with it (i.e. convective heat transport)



# Convective energy transport

An atmosphere is said to be **unstable against convection** (i.e. it is convective)

if its temperature drops rapidly with altitude compared to the change in parcel temperature



# Adiabatic lapse rate

The cooling rate of a rising parcel of gas

$$\left( \frac{dT}{dr} \right)_{\text{ad}} = - \frac{\gamma - 1}{\gamma} \left( \frac{g \mu m_H}{k_B} \right)$$

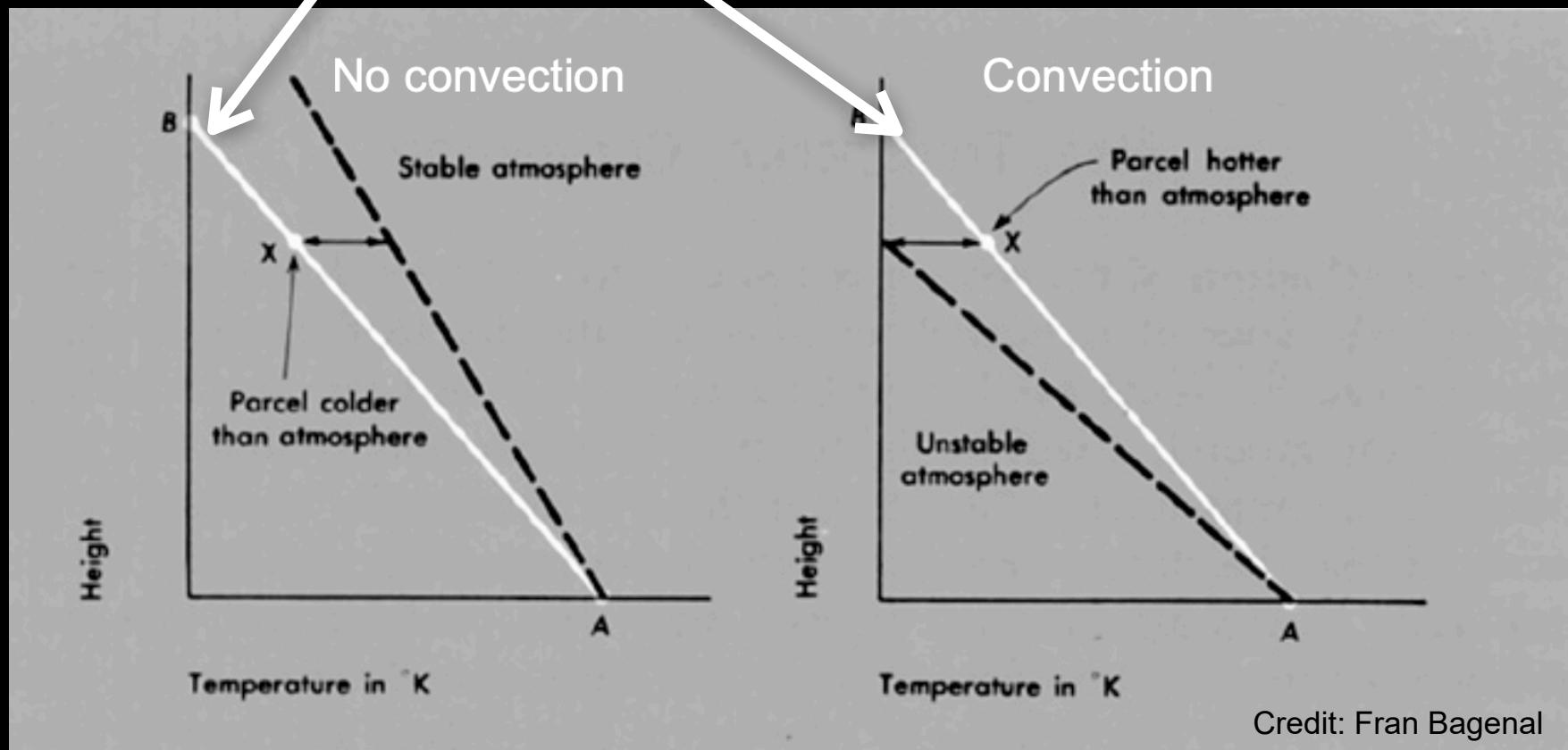
Depends on the planet's surface gravity, mean molecular weight, and the adiabatic index  $\gamma$  describing the heat required to raise the gas temperature at a fixed  $P$  and  $V$ .

$\gamma = 5/3, 7/5, 4/3$  for monoatomic, diatomic, and polyatomic gases respectively

# Adiabatic lapse rate

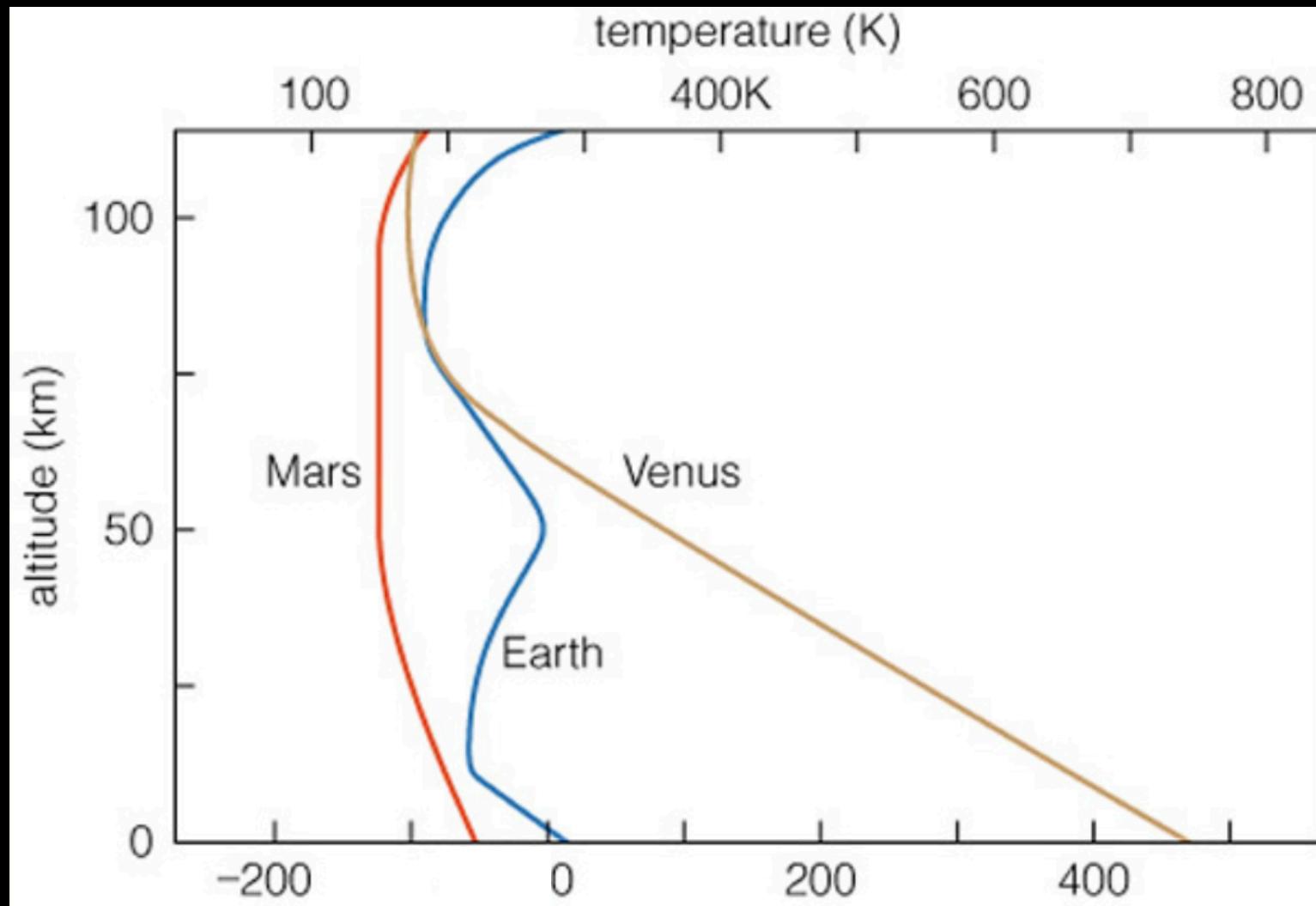
The cooling rate of a rising parcel of gas

$$\left( \frac{dT}{dr} \right)_{\text{ad}} = -\frac{\gamma - 1}{\gamma} \left( \frac{g\mu m_H}{k_B} \right)$$

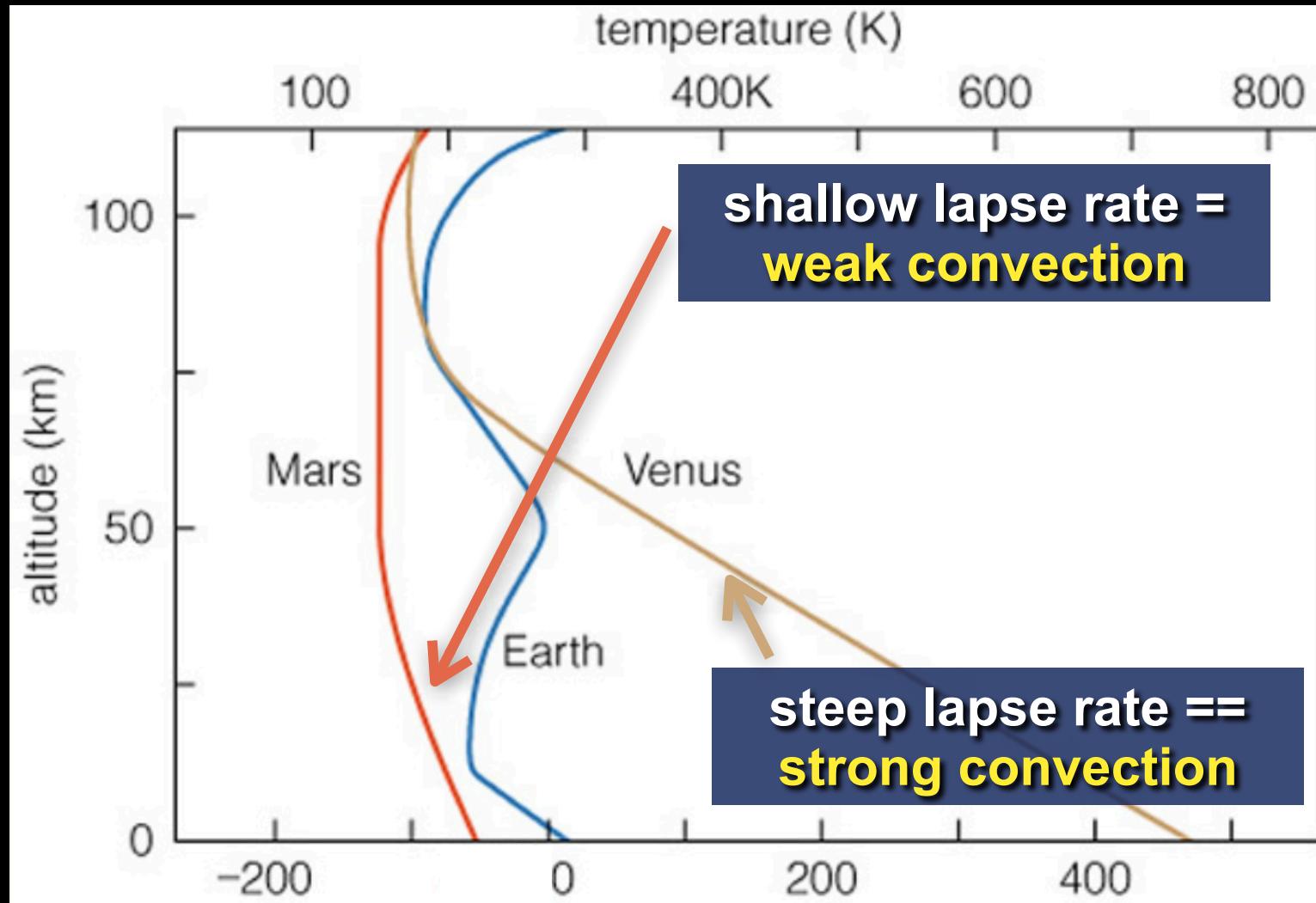


Credit: Fran Bagenal

# Comparative Atmospheres



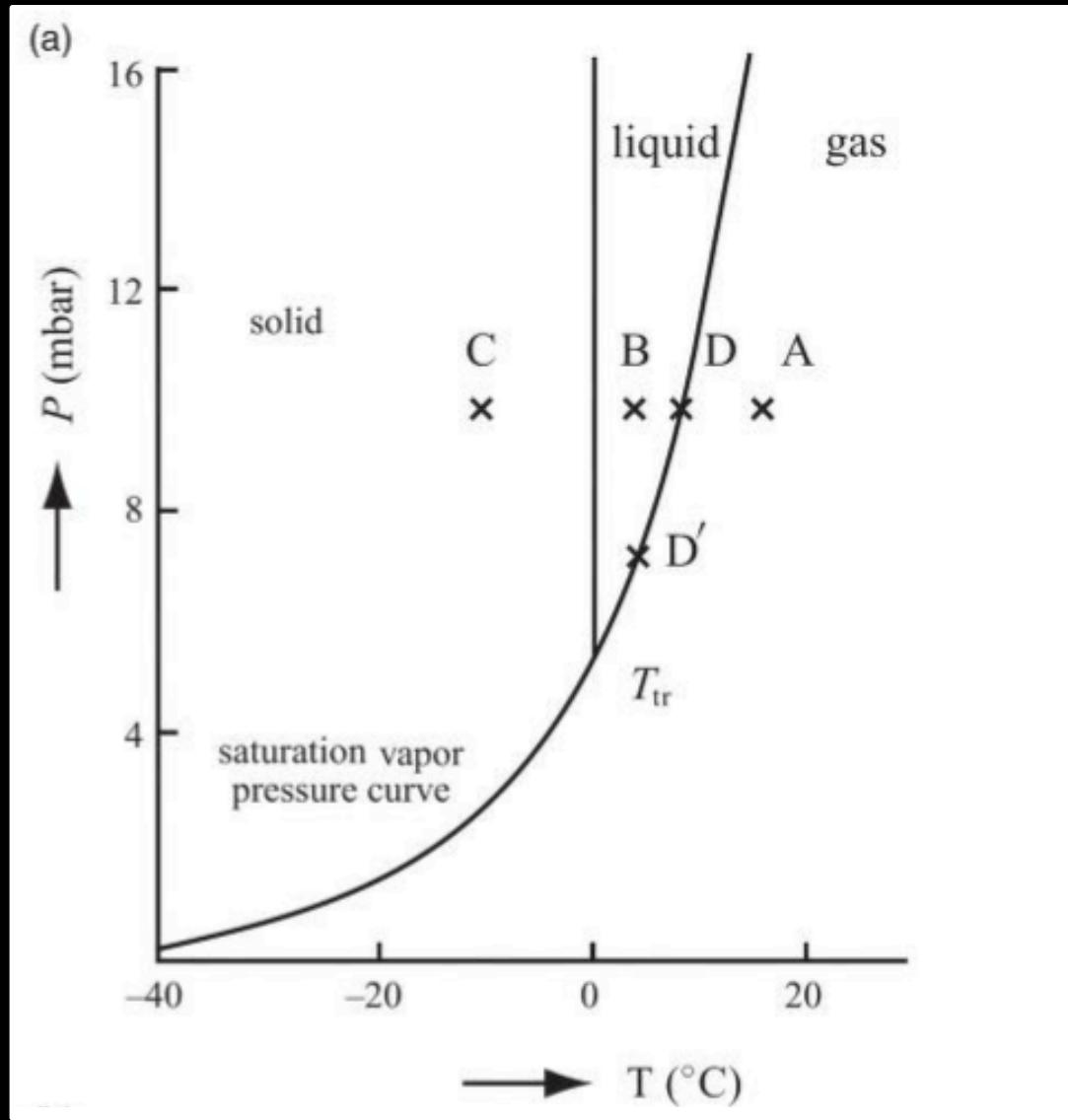
# Comparative Atmospheres



# **Cloud formation modifies the atmospheric lapse rate**



# Cloud Formation



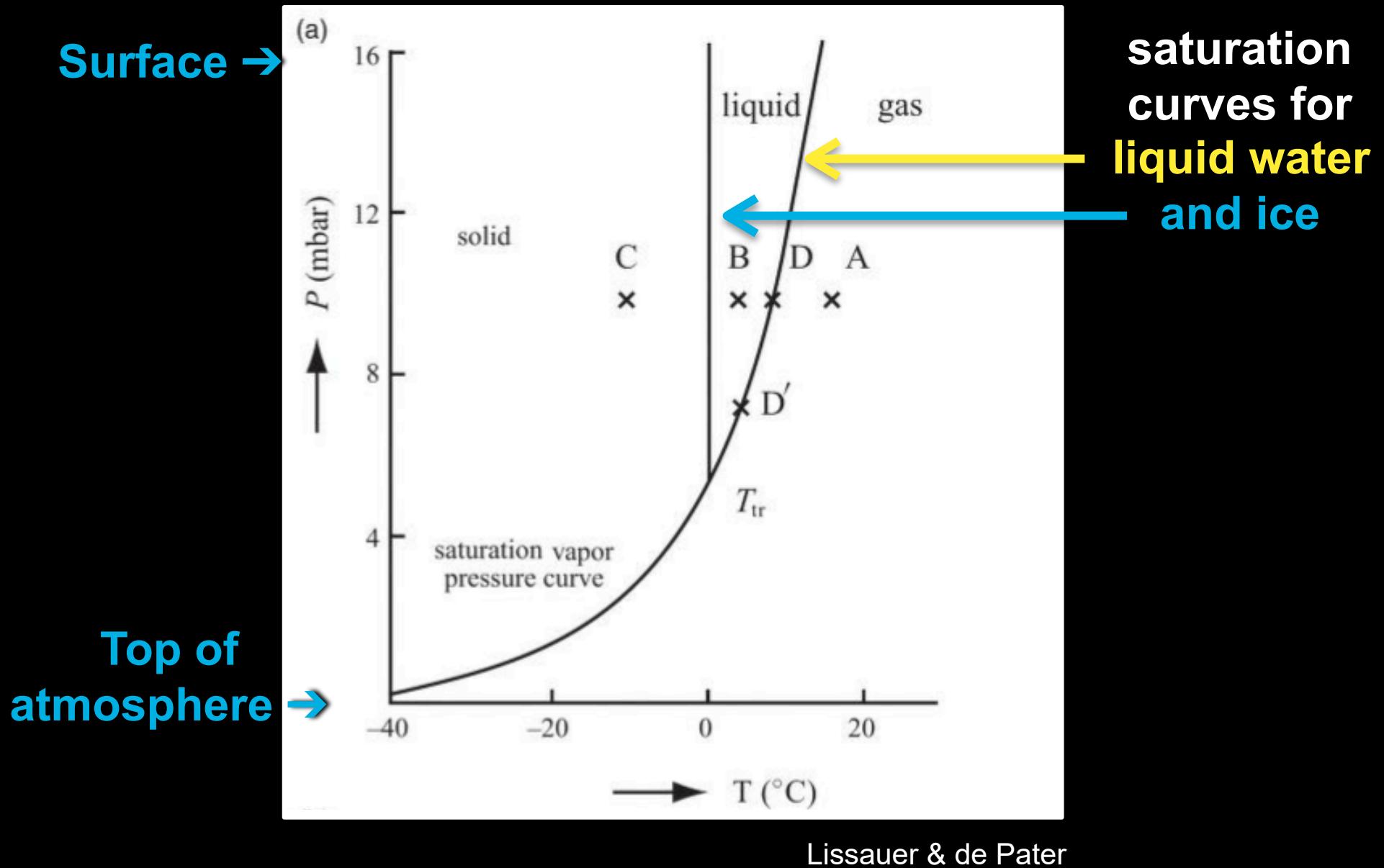
At a fixed  $P$  and  $T$ , air cannot contain more water than the **saturation vapour pressure**, which is given by the **Clausius-Clapeyron EOS**

$$P = C_L e^{-L/(R_{gas}T)}$$

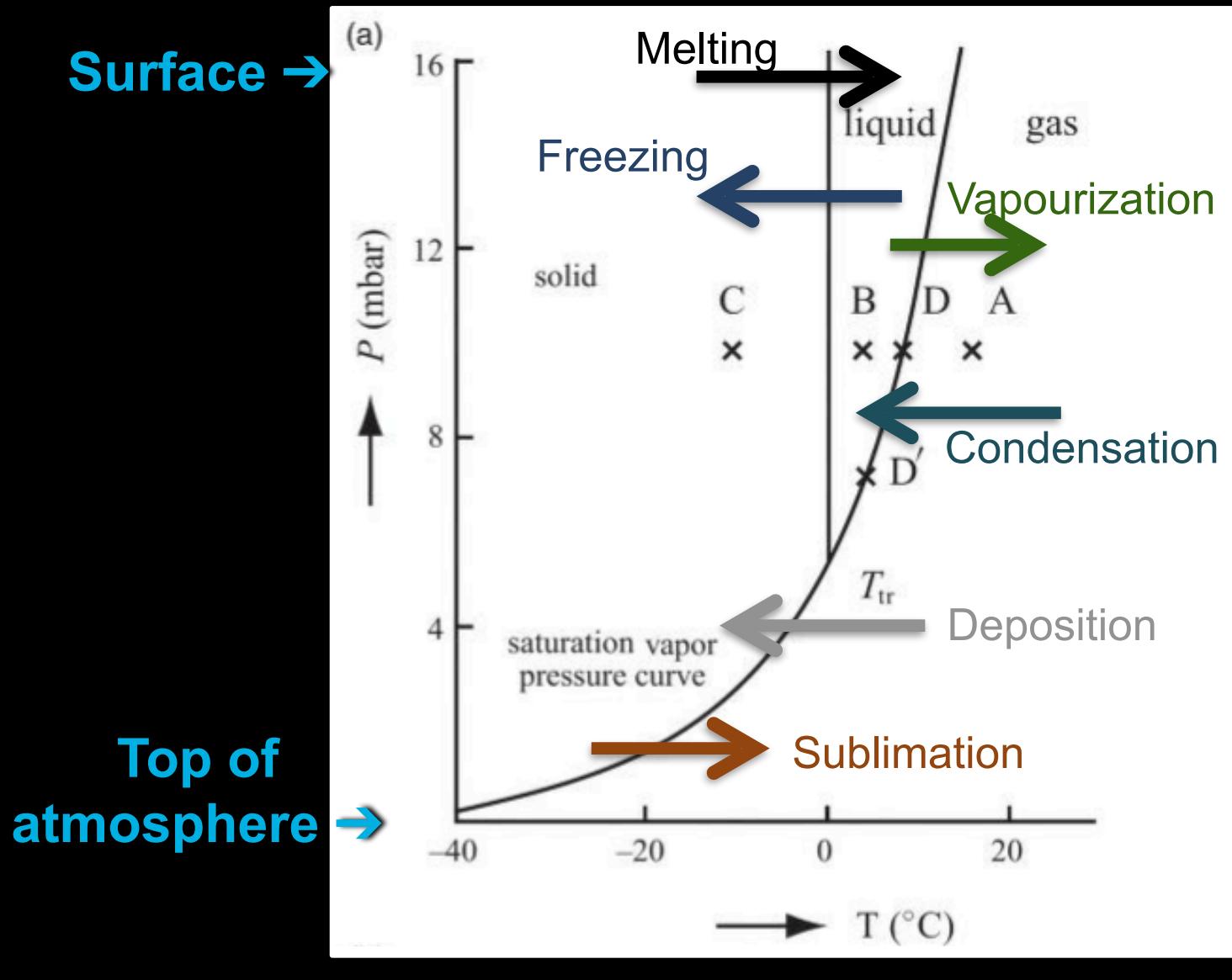
$P$ : saturation vapour pressure of water  
 $C_L$ ,  $R_{gas}$ : gas constants  
 $L$  latent heat  
 $T$ : air temperature

Lissauer & de Pater

# Cloud Formation

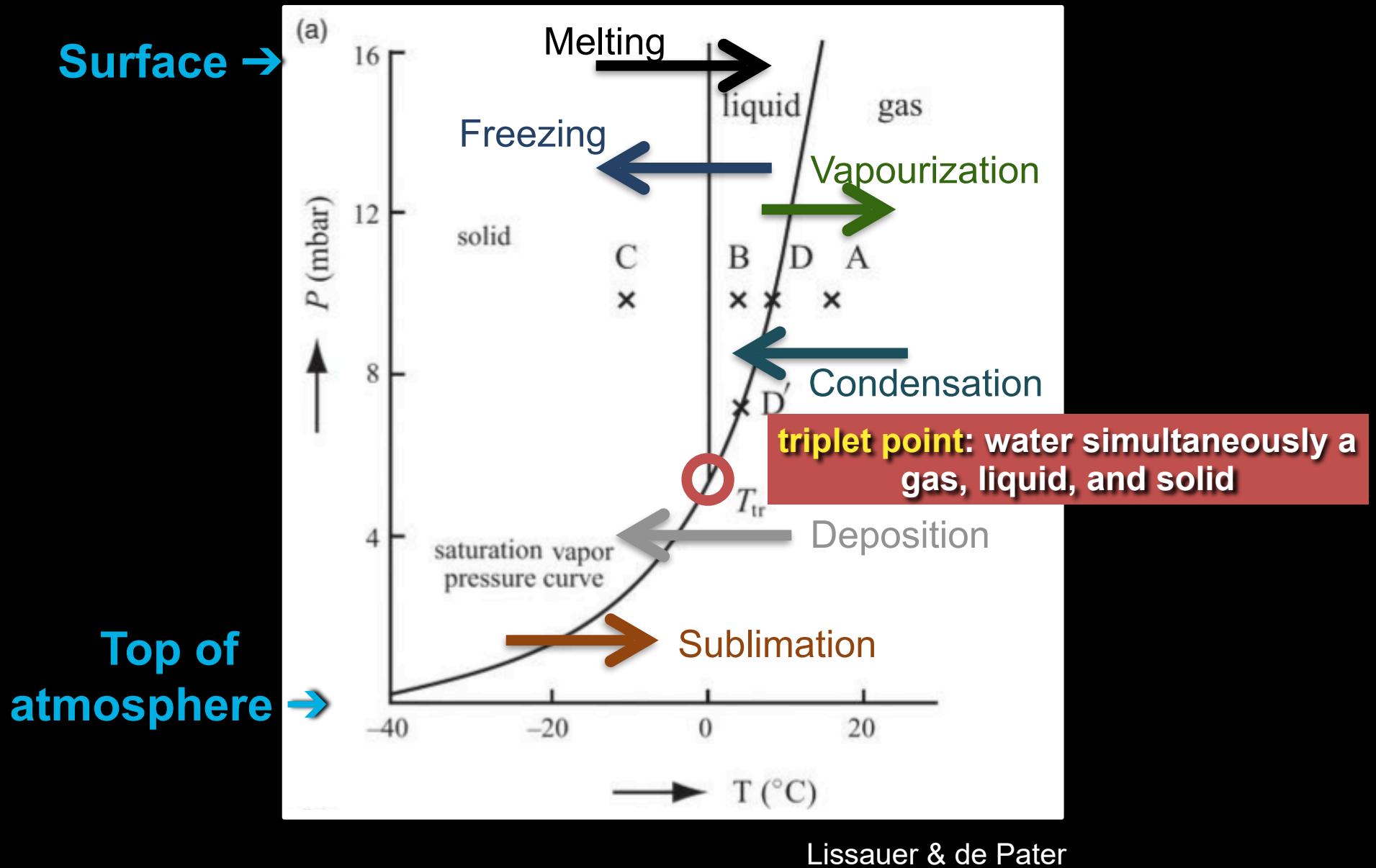


# Cloud Formation



Lissauer & de Pater

# Cloud Formation

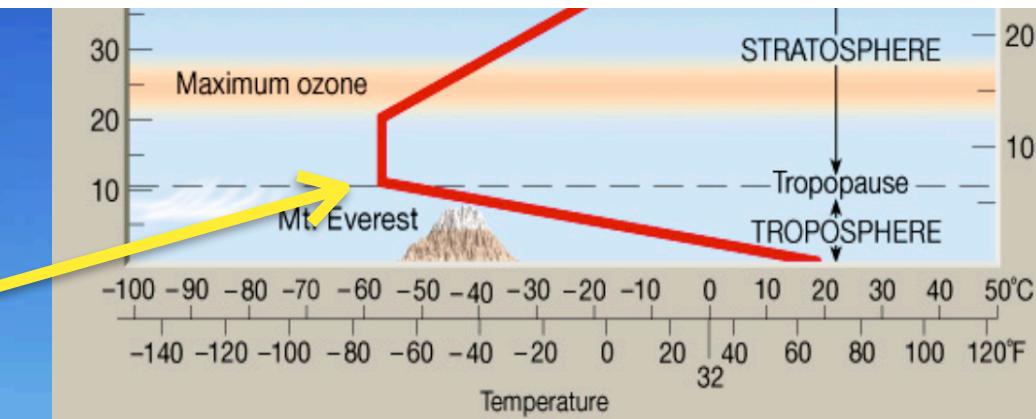


Lissauer & de Pater

The formation of **water droplets** and  
**ice crystals** make up clouds on Earth



**Condensation halts at the tropopause where the lapse rate turns over**



**Produces a “cloud anvil”**



# Clouds in the solar system

**Giant planets:**

$\text{NH}_3$ ,  $\text{H}_2\text{S}$ ,  $\text{CH}_4$

**Mars:**

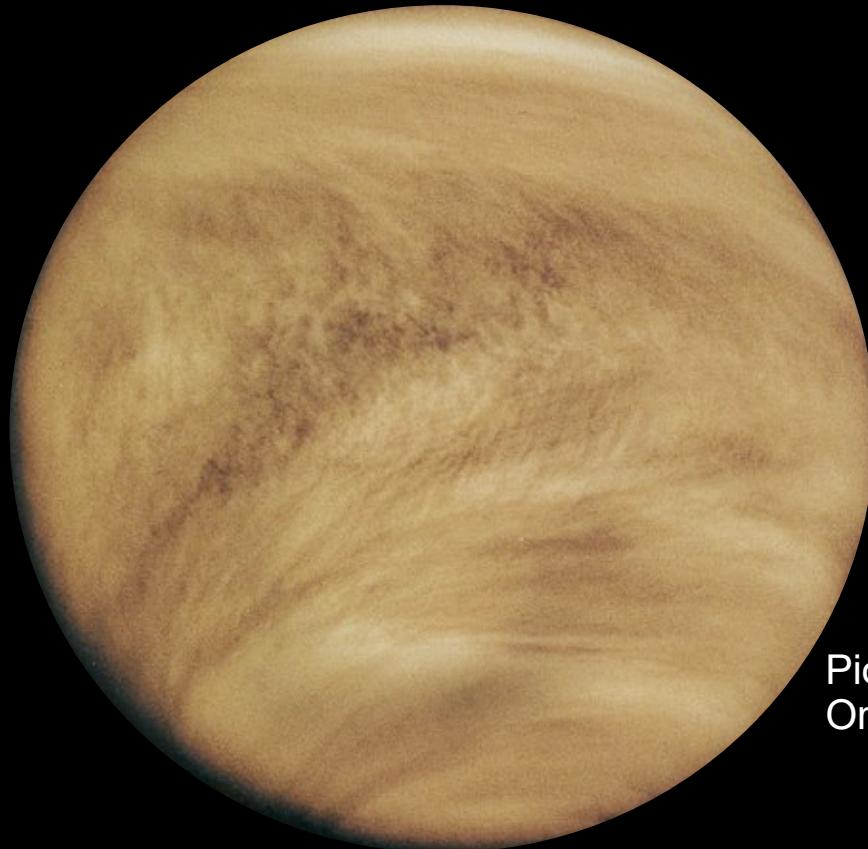
$\text{CO}_2$

**Venus:**

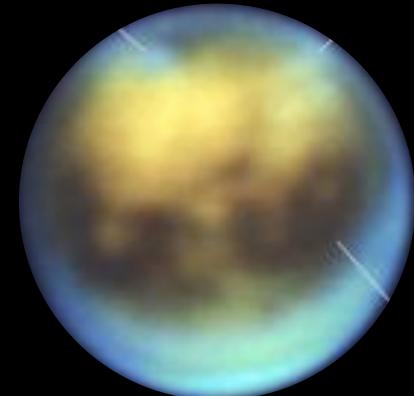
$\text{H}_2\text{SO}_4$

**Titan** (largest of Saturn's moons):

$\text{CH}_4$ ,  $\text{C}_2\text{H}_6$ , + other complex hydrocarbons



Pioneer Venus Orbiter



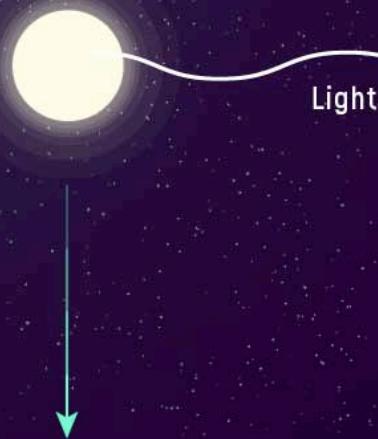
Keck/NIRC2

Can we detect clouds on exoplanets?

Yes, via transmission spectroscopy

# Absorption & Emission

Continuous light source



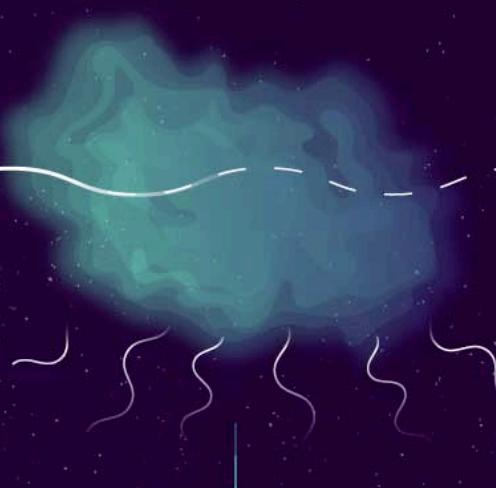
## CONTINUOUS SPECTRUM

Spectrum that contains **all wavelengths** emitted by a hot, dense, light source



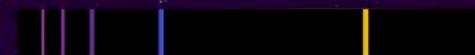
blackbody

Cloud of gas



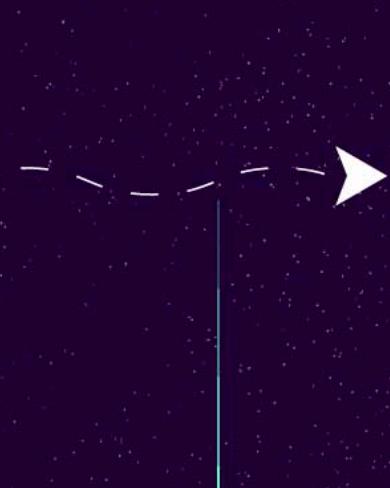
## EMISSION SPECTRUM

Shows **colored lines** of light emitted by glowing gas



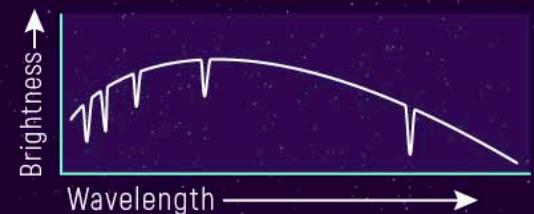
Hot gas

Cloud of gas



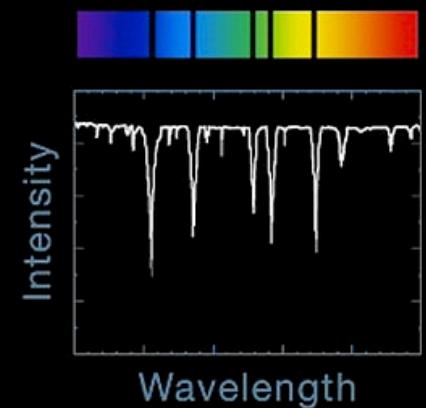
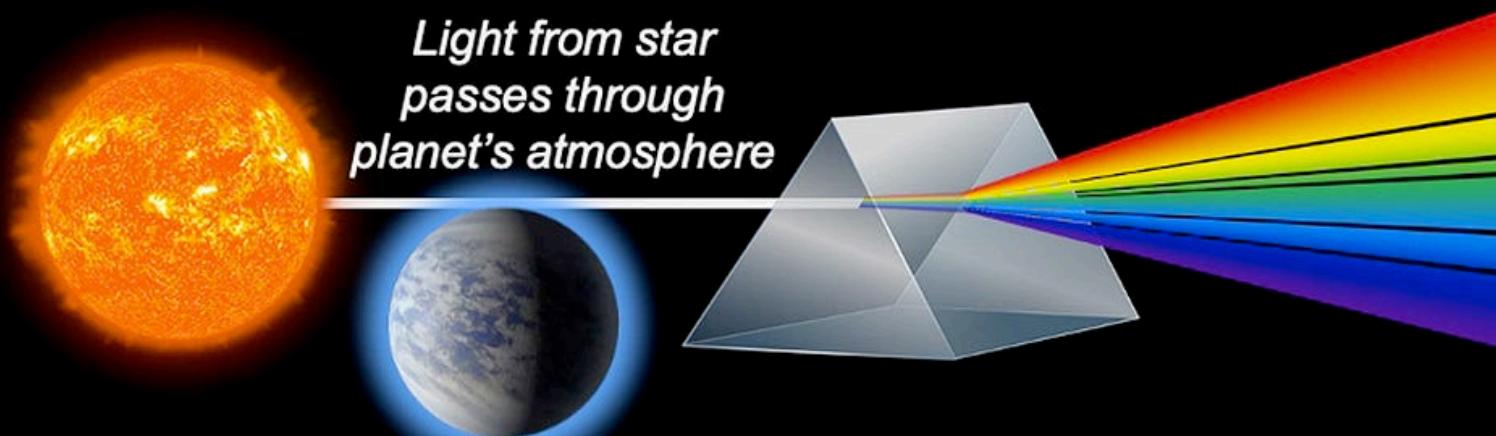
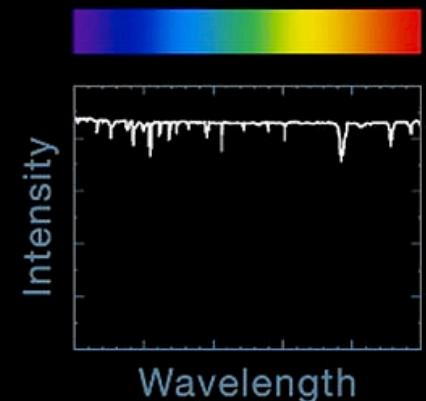
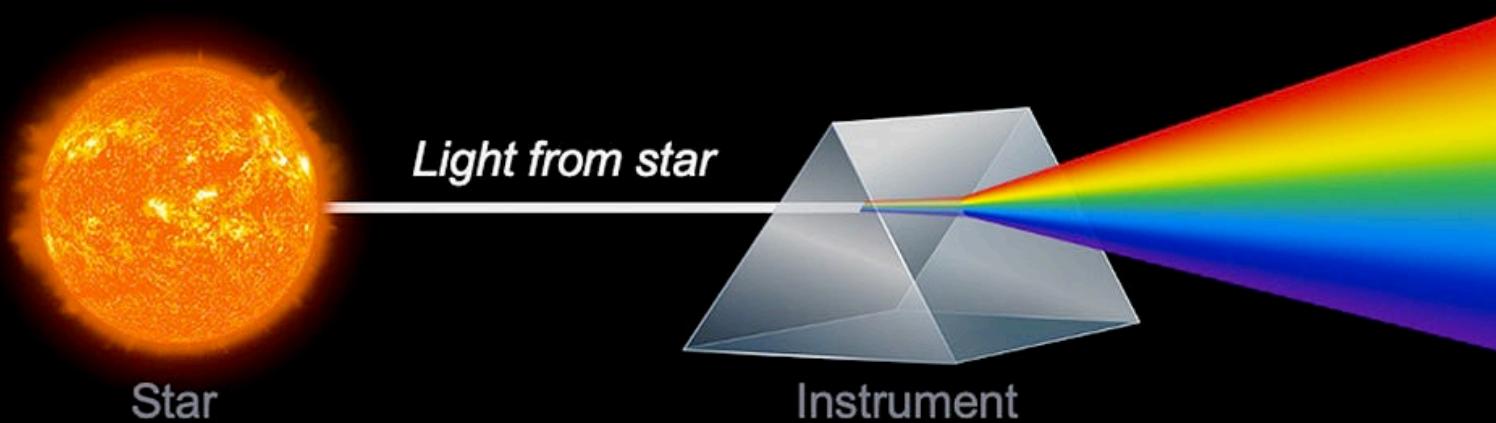
## ABSORPTION SPECTRUM

Shows **dark lines or gaps** in light after the light passes through a gas



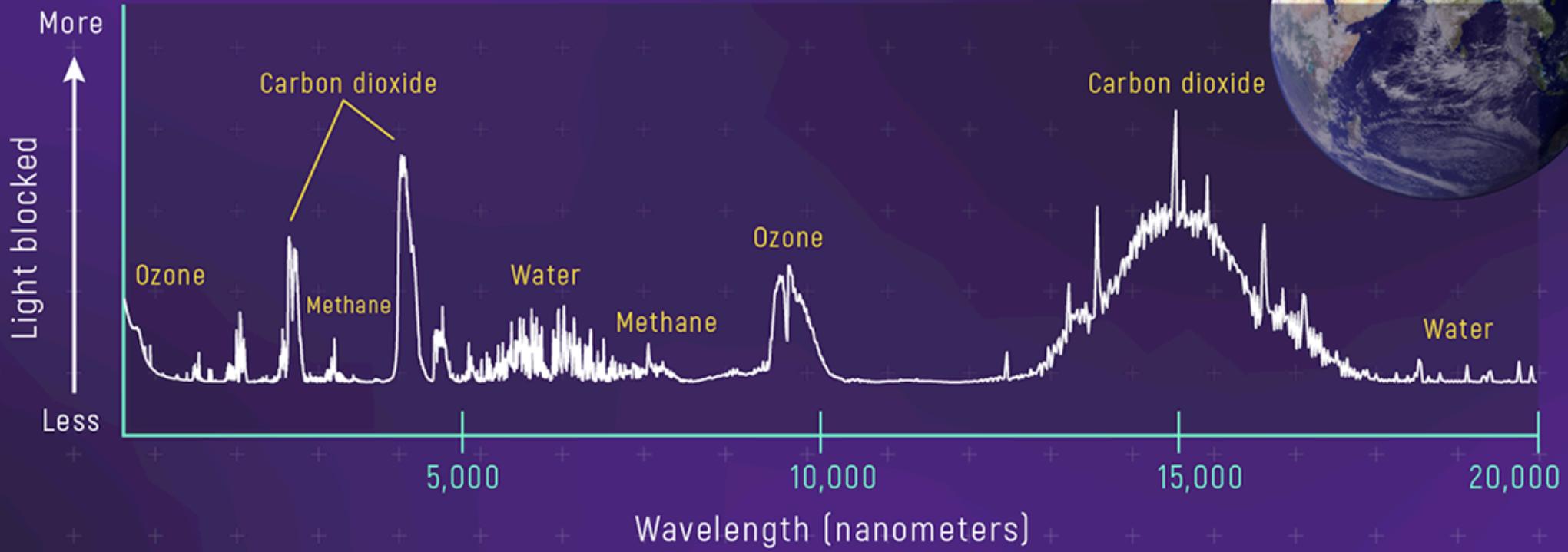
Cold gas

# Transmission Spectroscopy

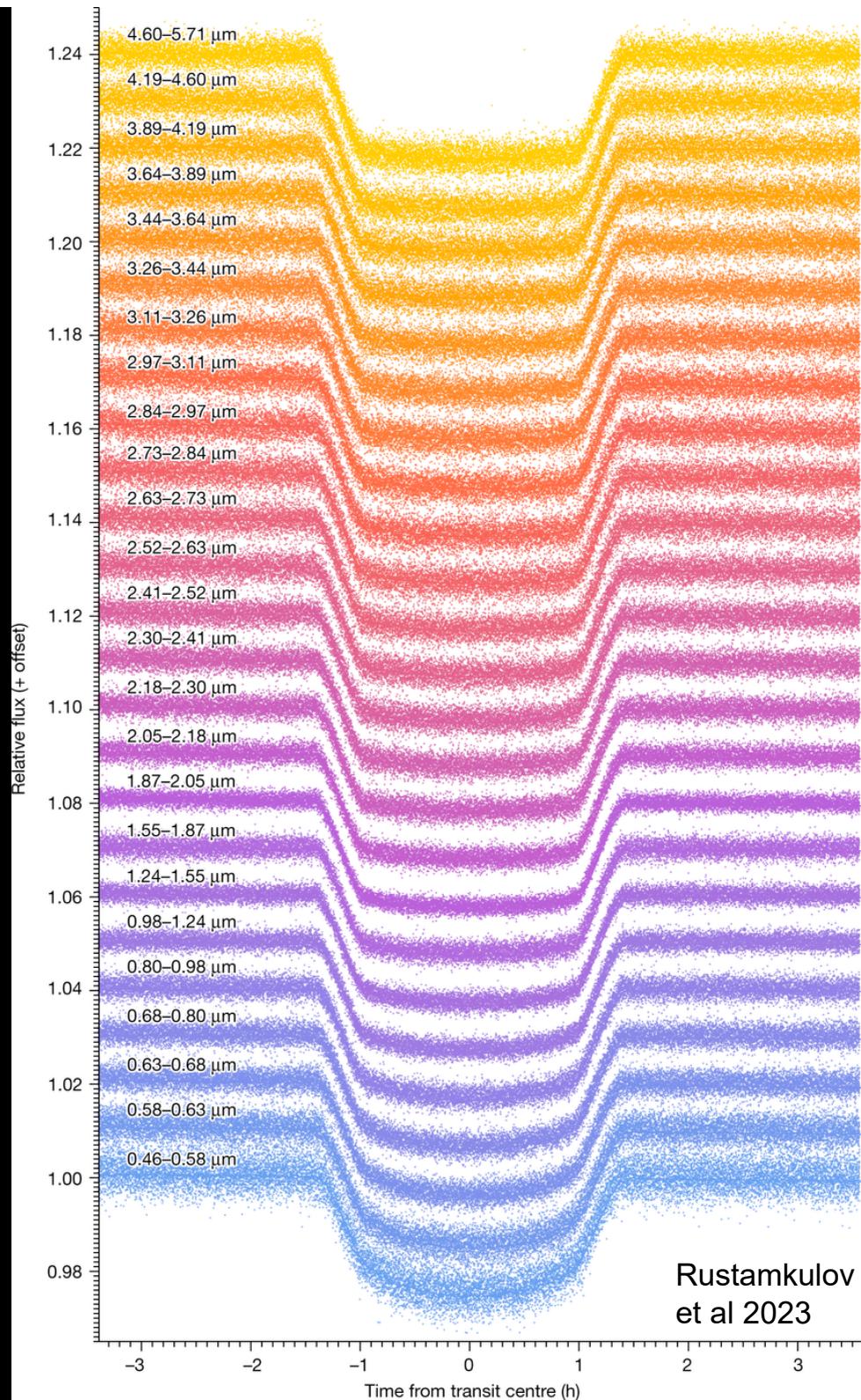


# Transmission Spectroscopy traces atmospheric composition

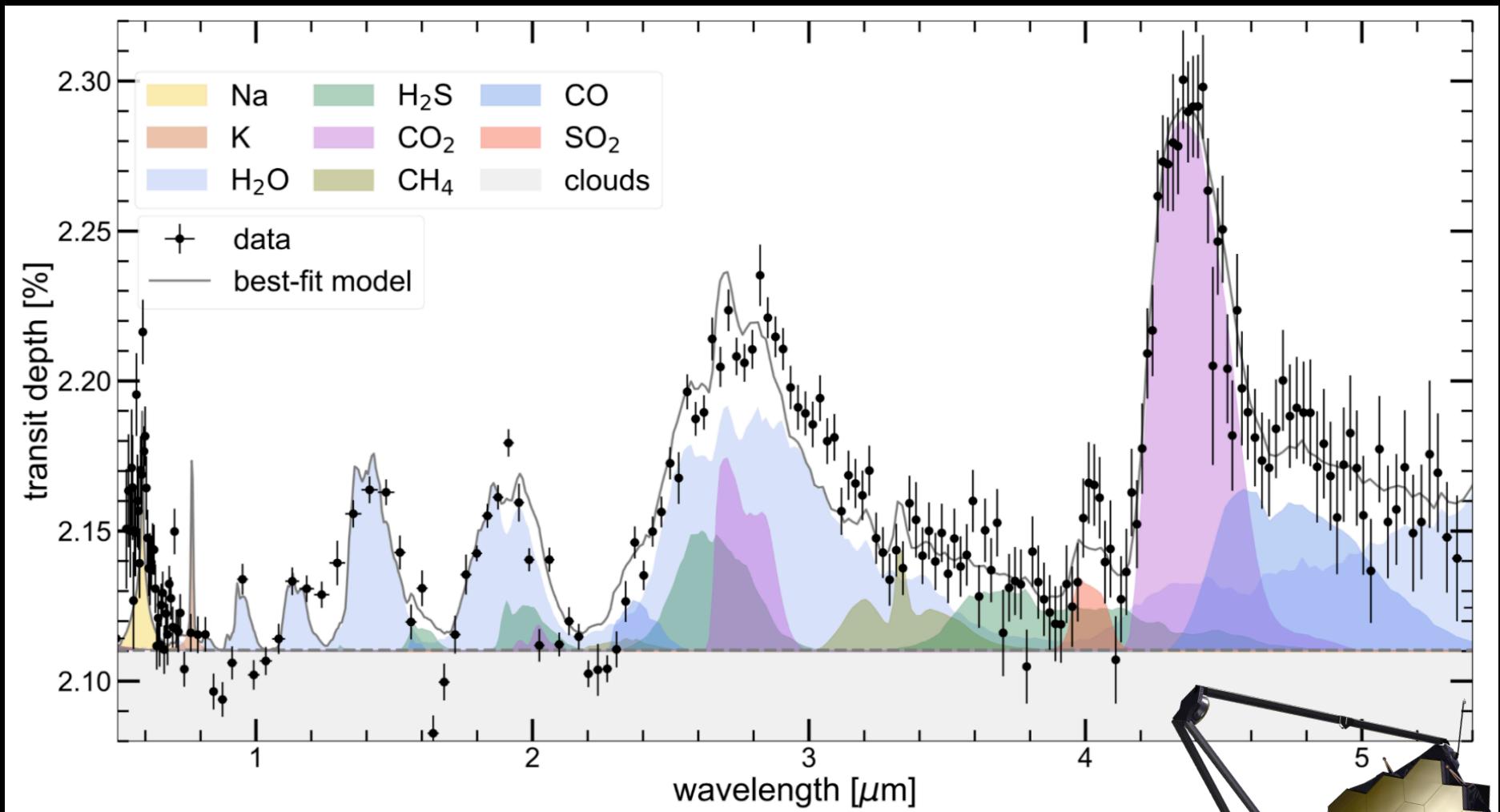
TRANSMISSION SPECTRUM OF AN EARTH-LIKE ATMOSPHERE



# Transmission Spectroscopy = transit depth as a function of wavelength



# Transmission Spectrum of the hot Jupiter WASP-39 b

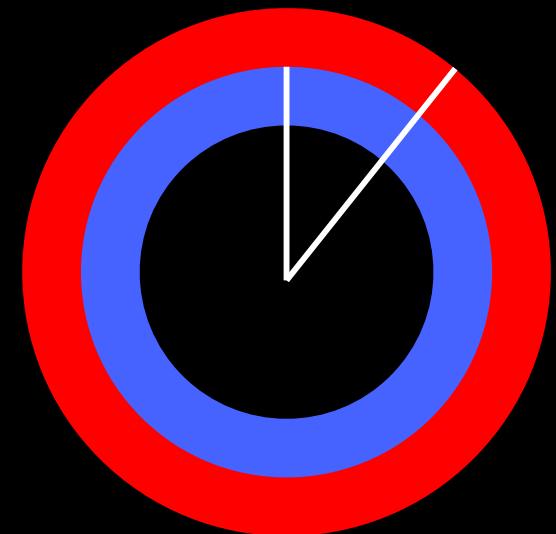


Rustamkulov et al 2023

**Absorption depth  $\Delta Z$  scales with atmospheric scale height  $H$**

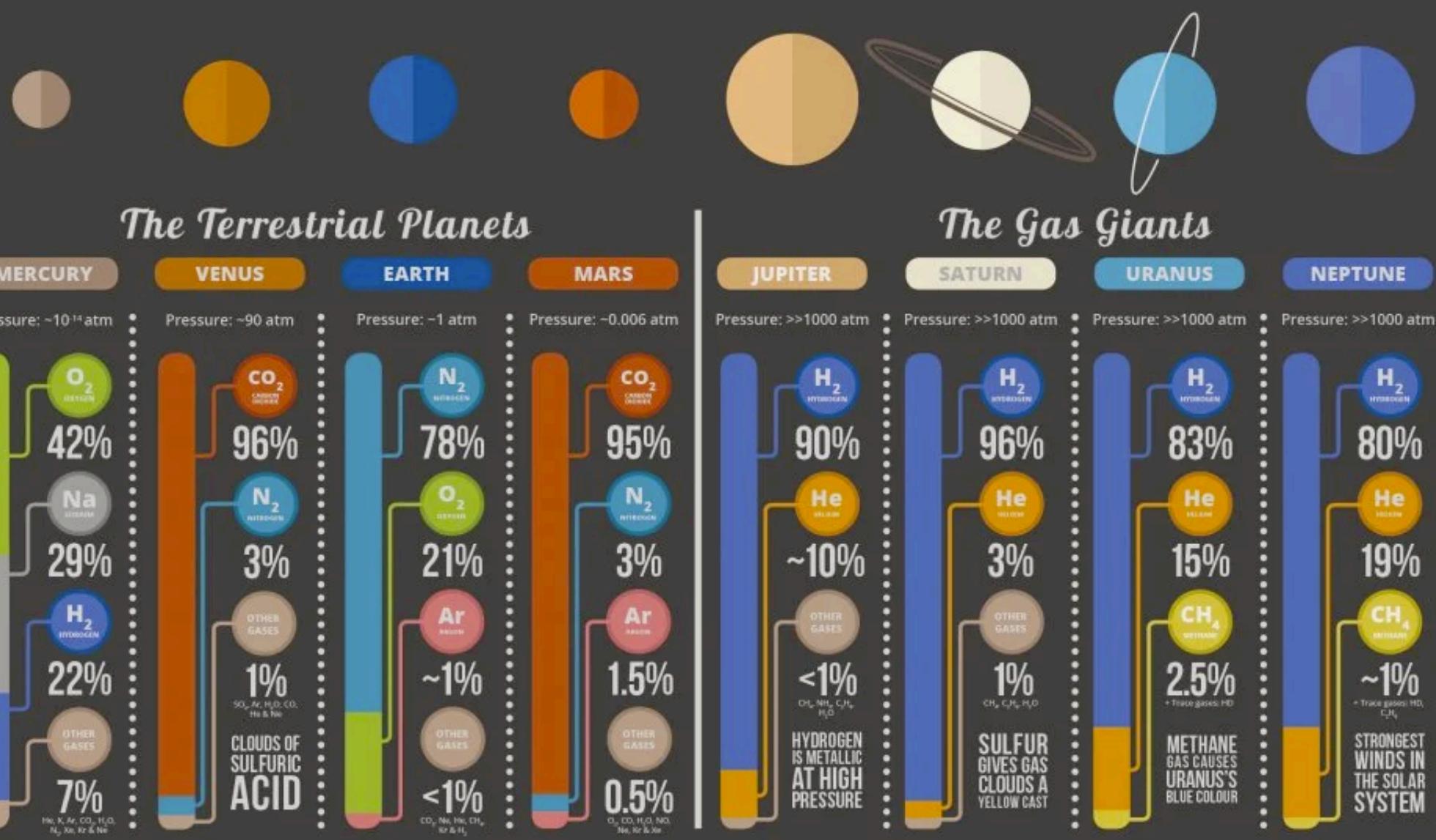
$$H = \frac{k_B T}{\mu m_H g}$$

$$\begin{aligned}\Delta Z &= \frac{\pi(R_p + nH)^2}{\pi R_\star^2} - \frac{\pi R_p^2}{\pi R_\star^2} \\ &\approx 2 \left( \frac{R_p}{R_\star} \right)^2 \left( \frac{nH}{R_p} \right)\end{aligned}$$

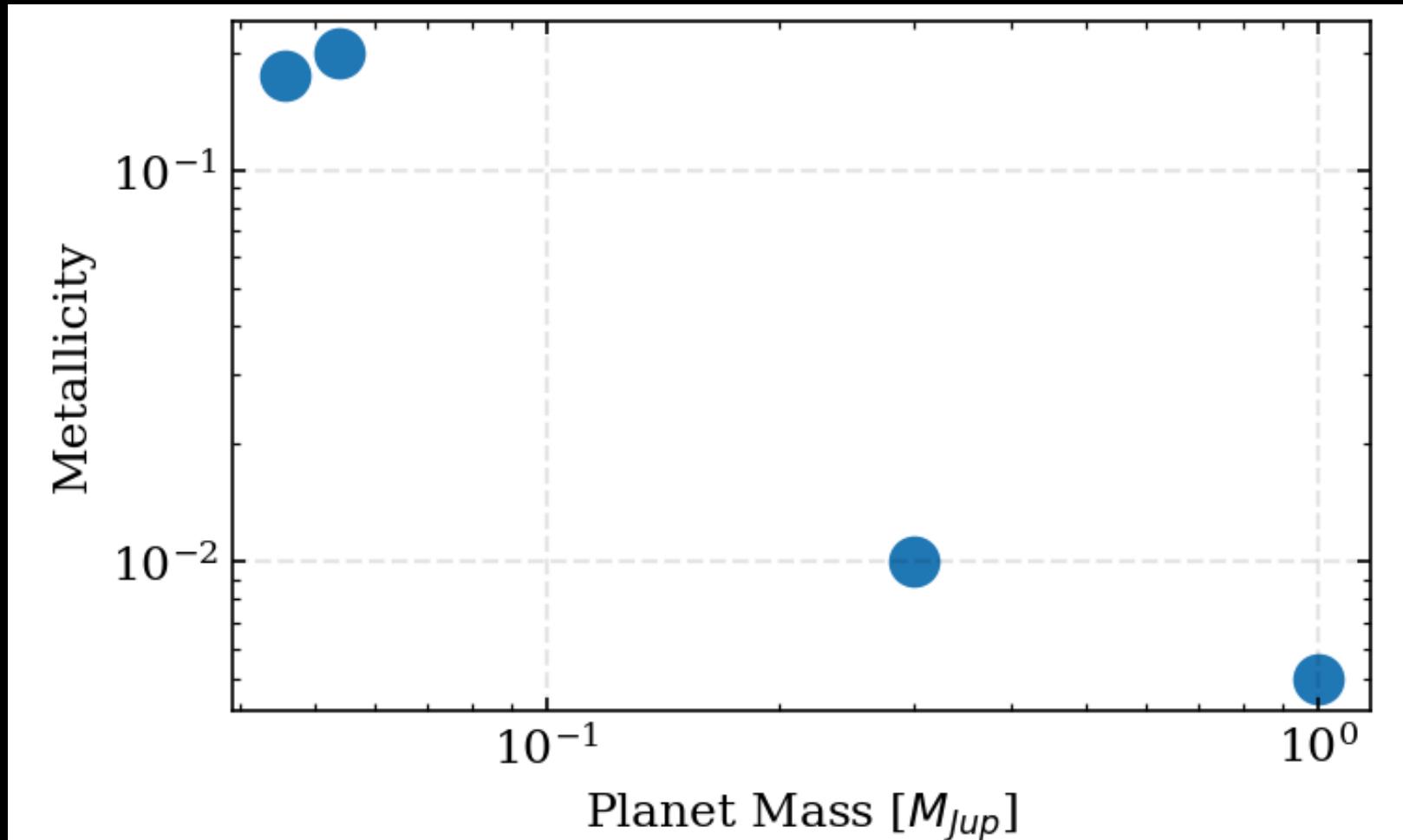


Kempton et al 2016

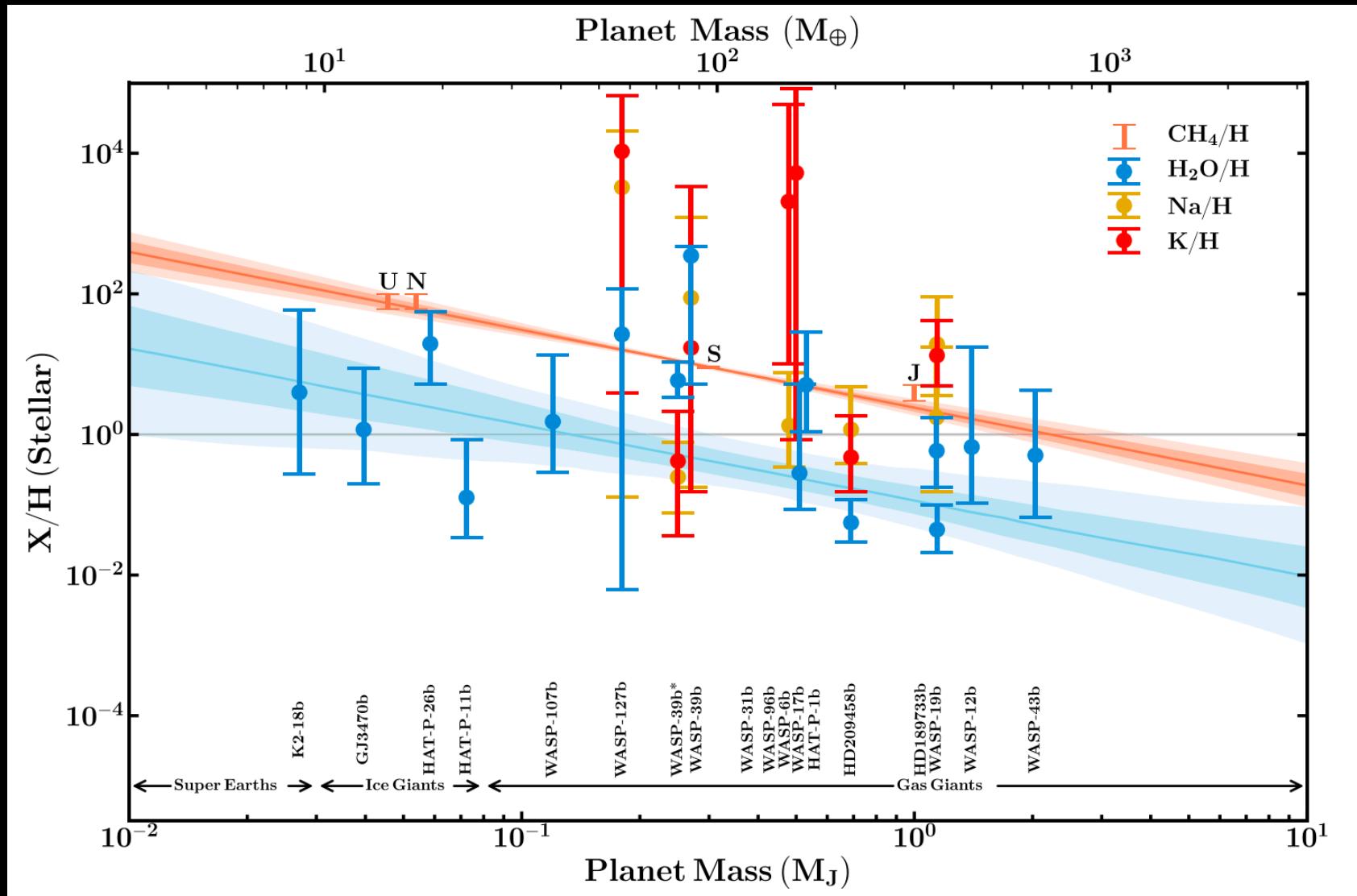
# Atmospheric Composition in the solar system



# The gas giants in the solar system exhibit a **mass-metallicity trend**



Maybe the mass-metallicity trend  
extends to exoplanets...

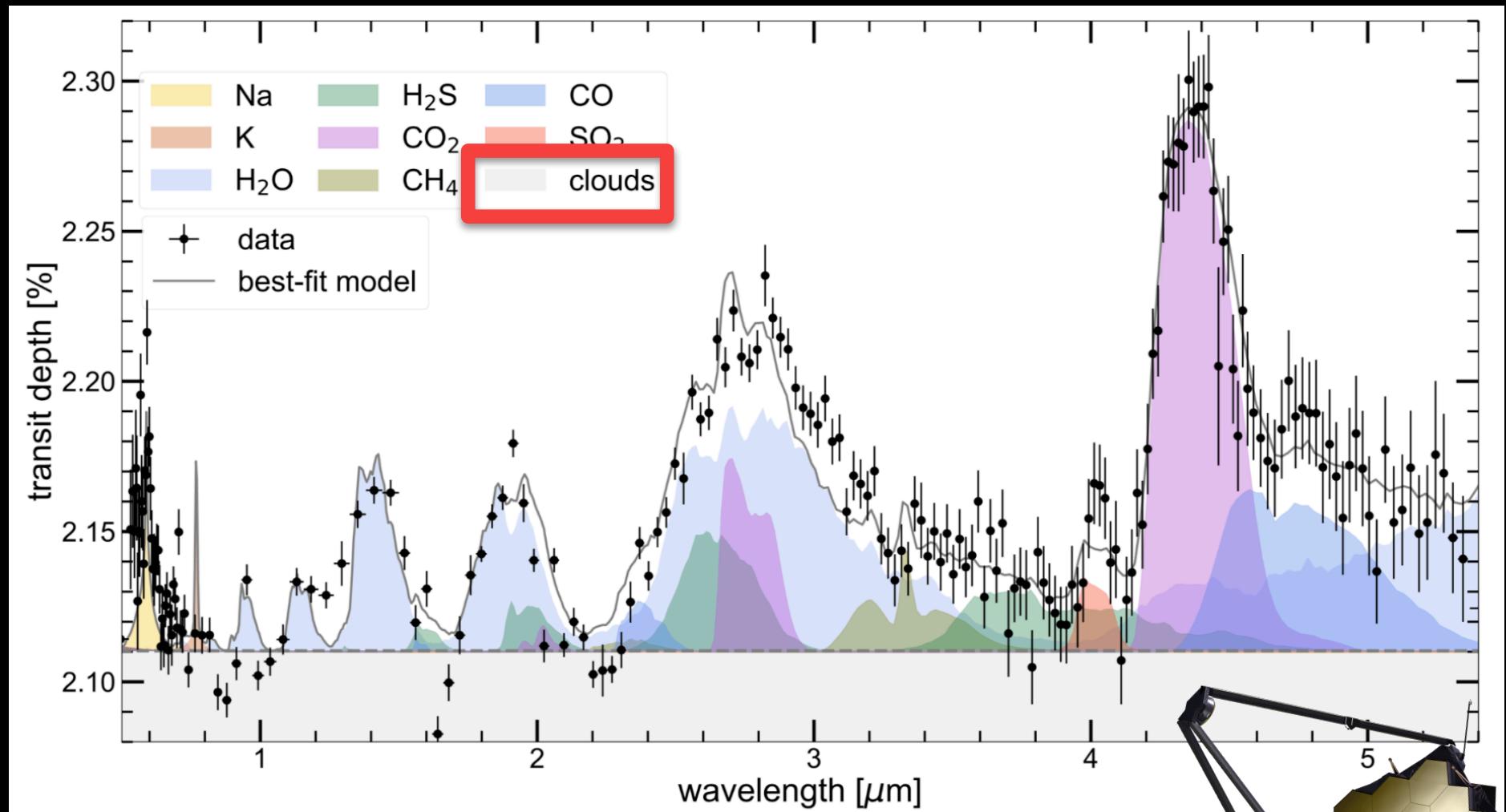


Wellbanks et al 2019

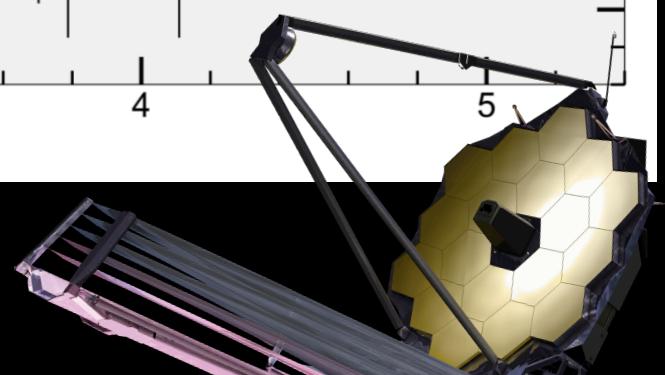
**Recall the initial question that started our discussion on transmission spectroscopy...**

**Can we detect clouds on exoplanets?**

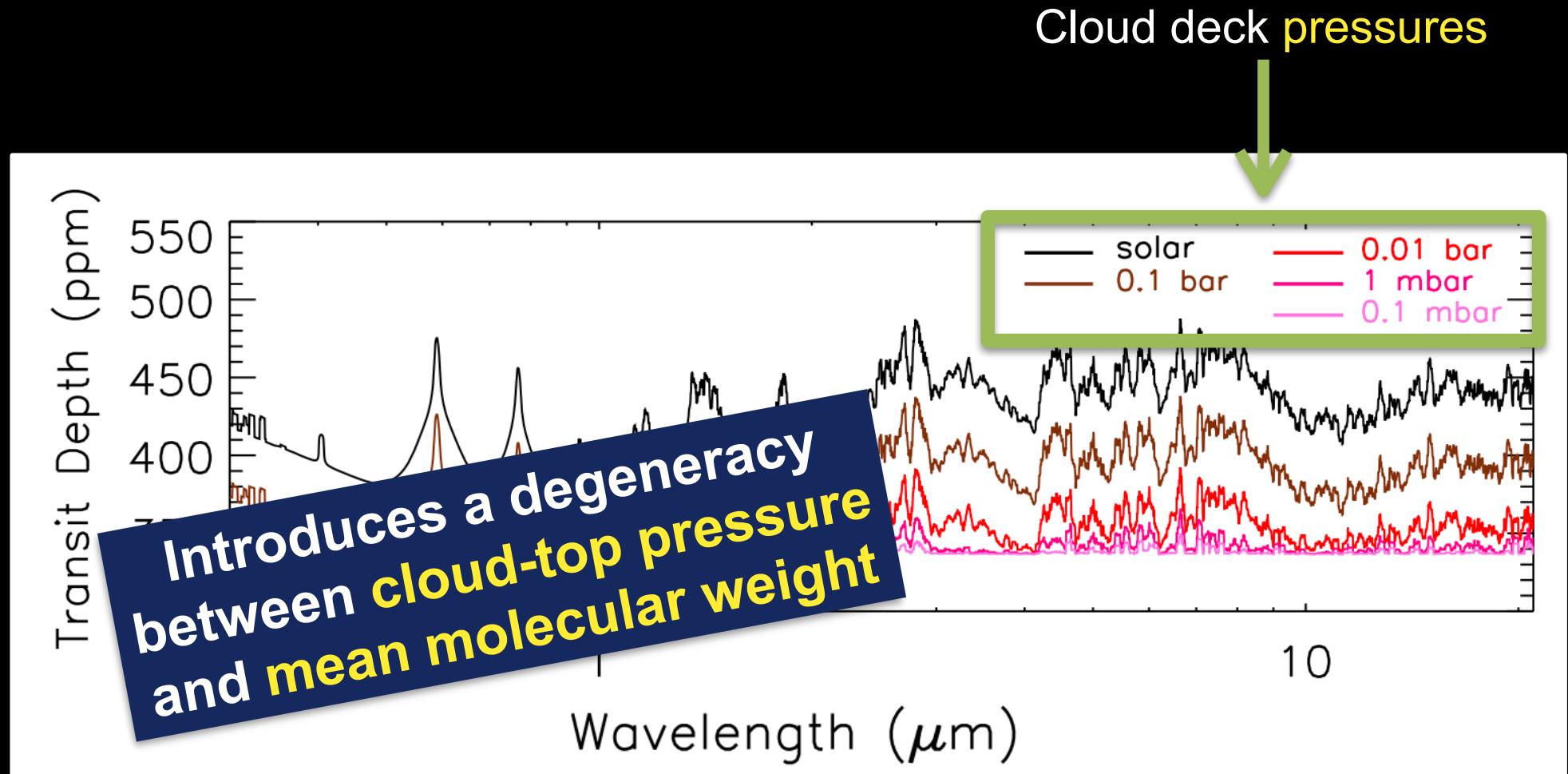
# Clouds **suppress** atmospheric chemical signatures



Rustamkulov et al 2023

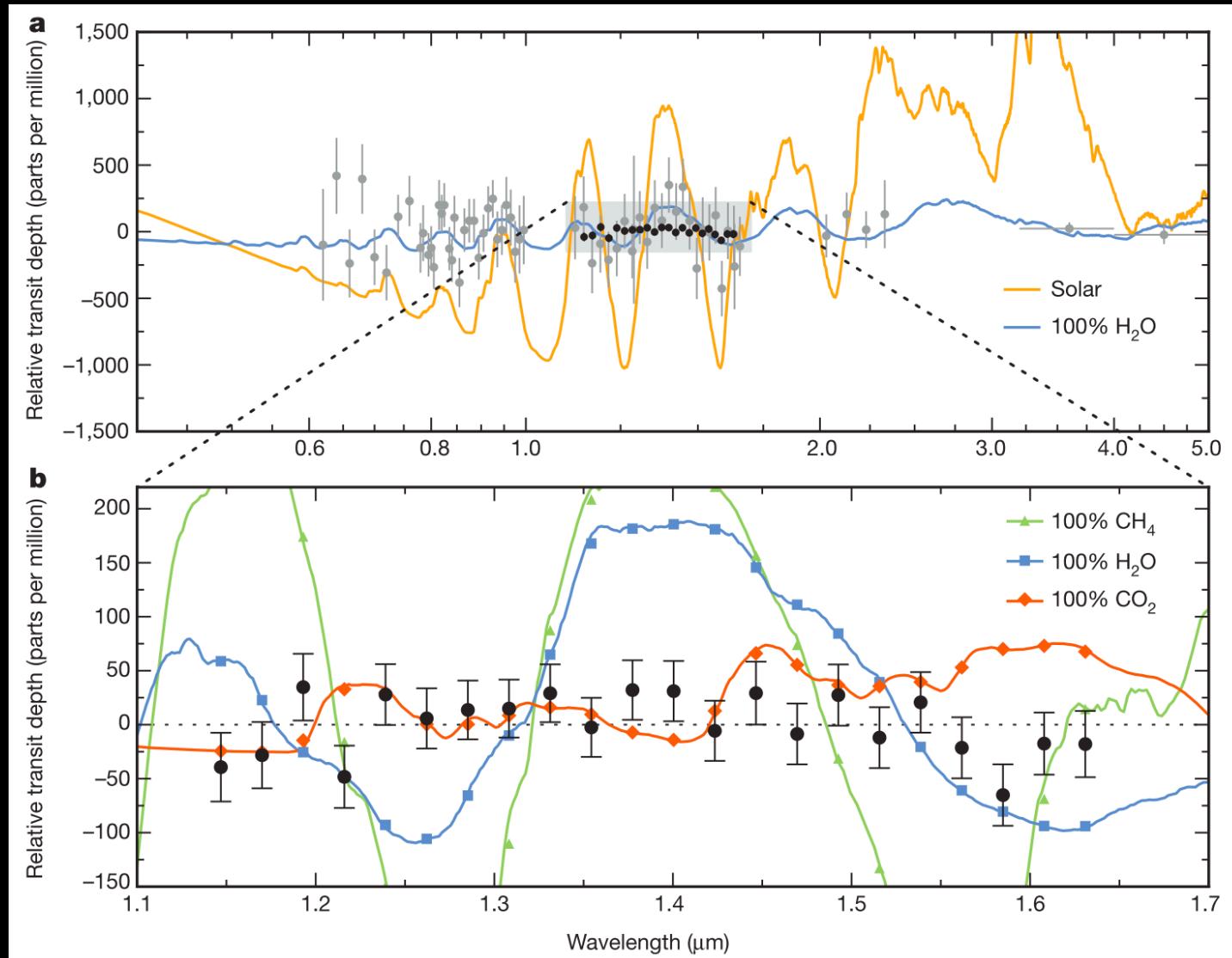


# Clouds **suppress** atmospheric chemical signatures



Kempton et al 2016

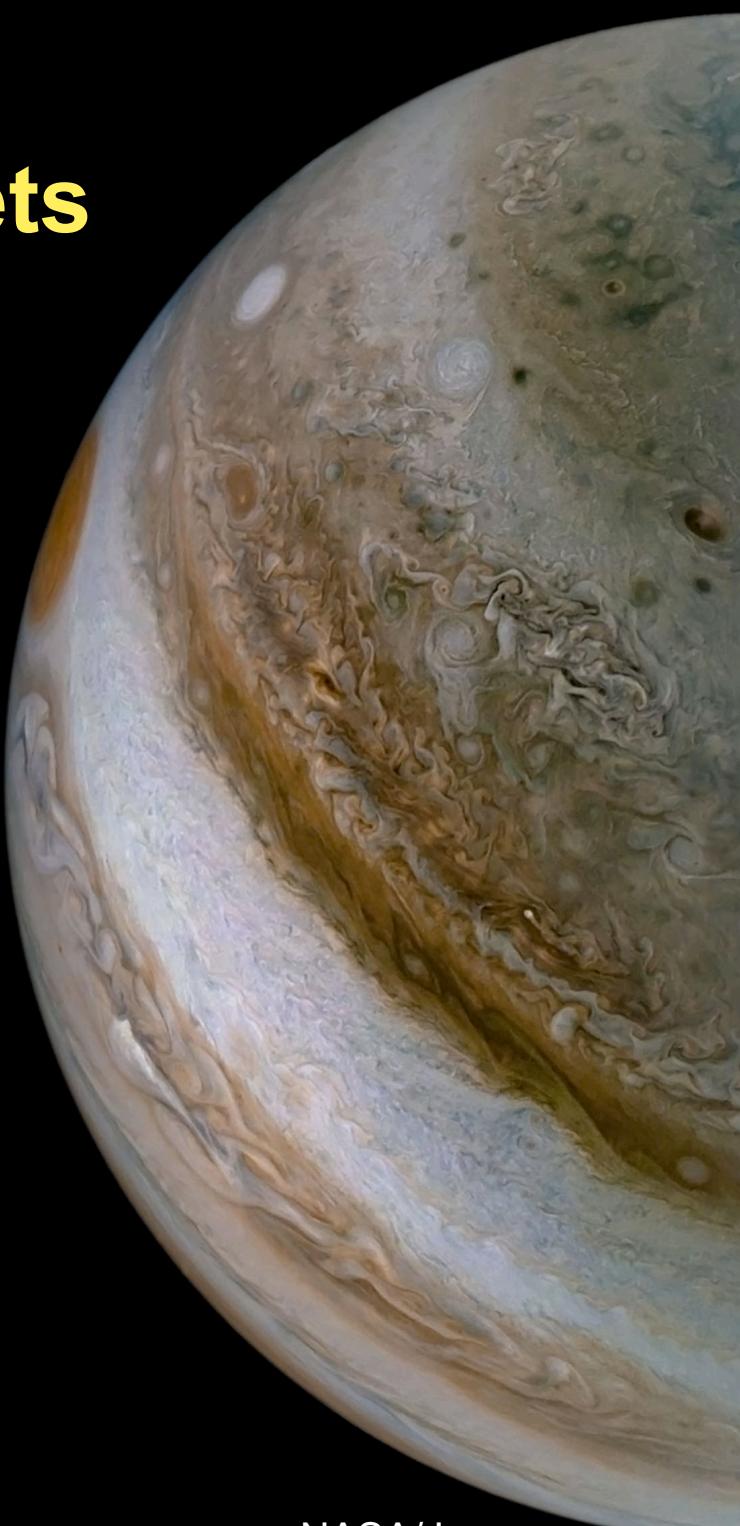
# Clouds on the sub-Neptune exoplanet GJ 1214 b



Kreidberg et al 2014

# Clouds/winds on Giant Planets

- non-water species (e.g. NH<sub>3</sub>, H<sub>2</sub>S, CH<sub>4</sub>)
- Forms **zones** and **belts**. **Why?**

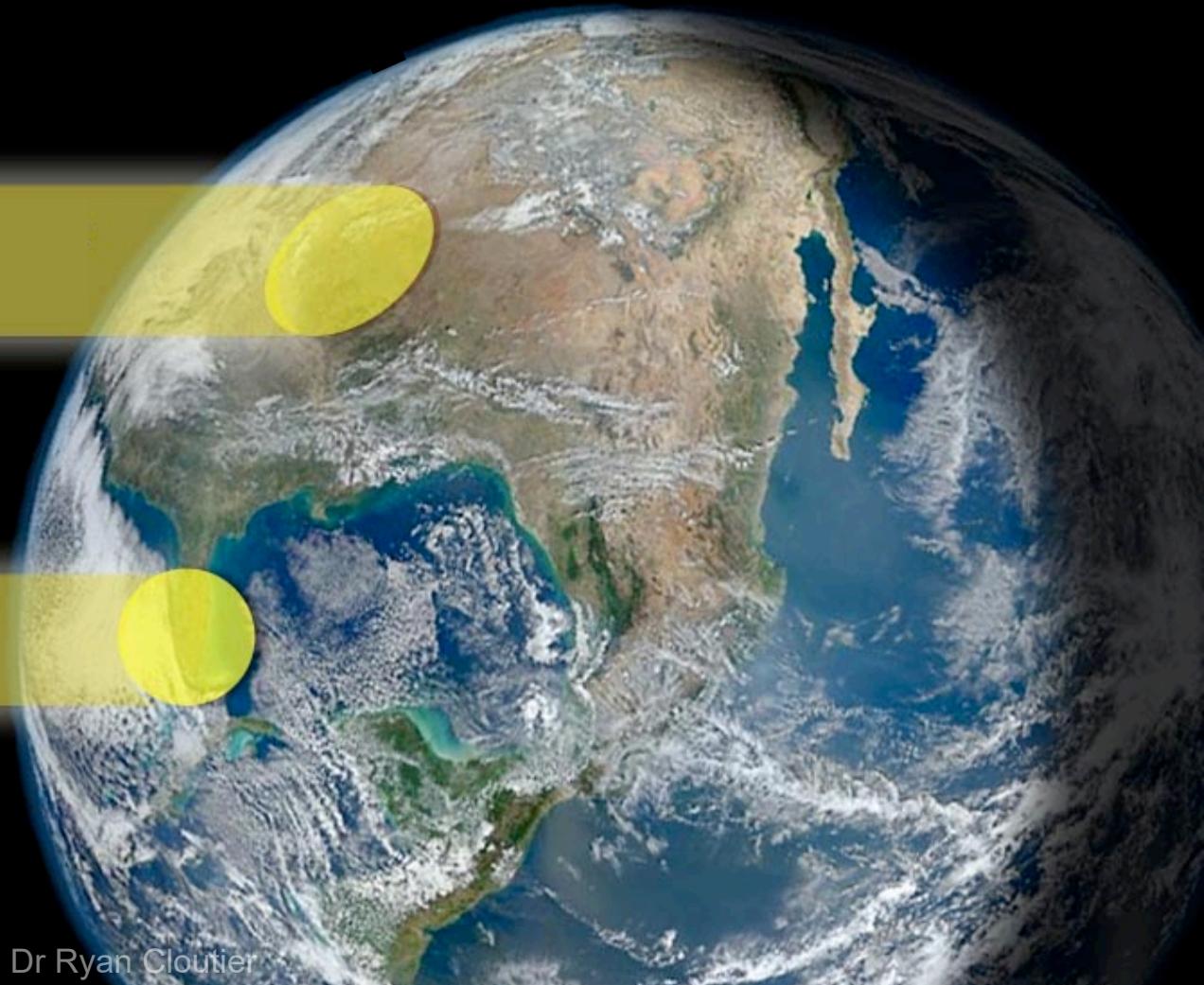


# **Incident flux on a surface patch depends on the incident angle**

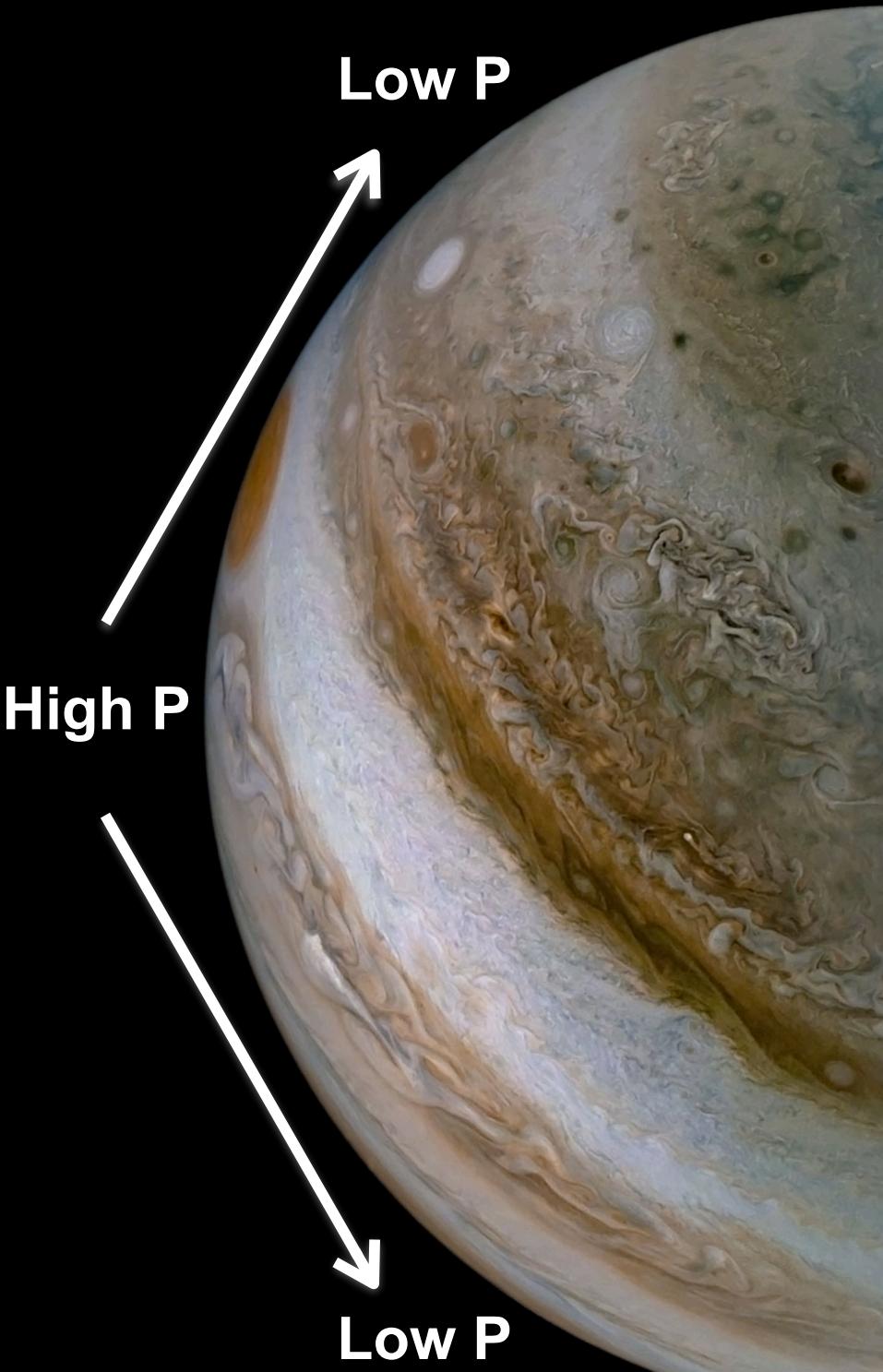
Stronger heating at the equator  
compared to high latitudes



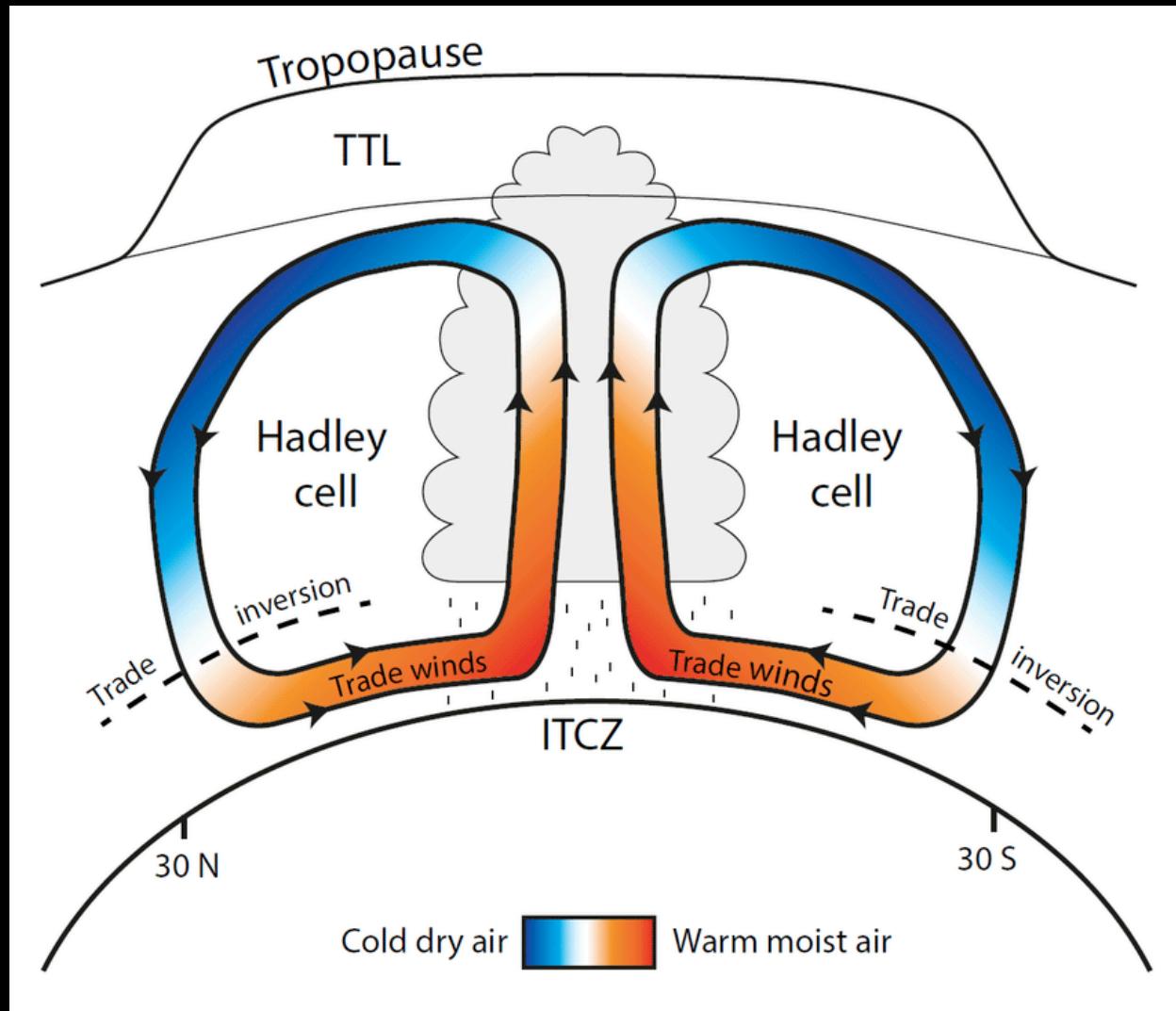
Pressure gradient  
that drives winds



- Hot equatorial air rises and **flows toward low pressure regions** (i.e. high latitudes)
- Air cools, subsides, and **flows back to the equator**

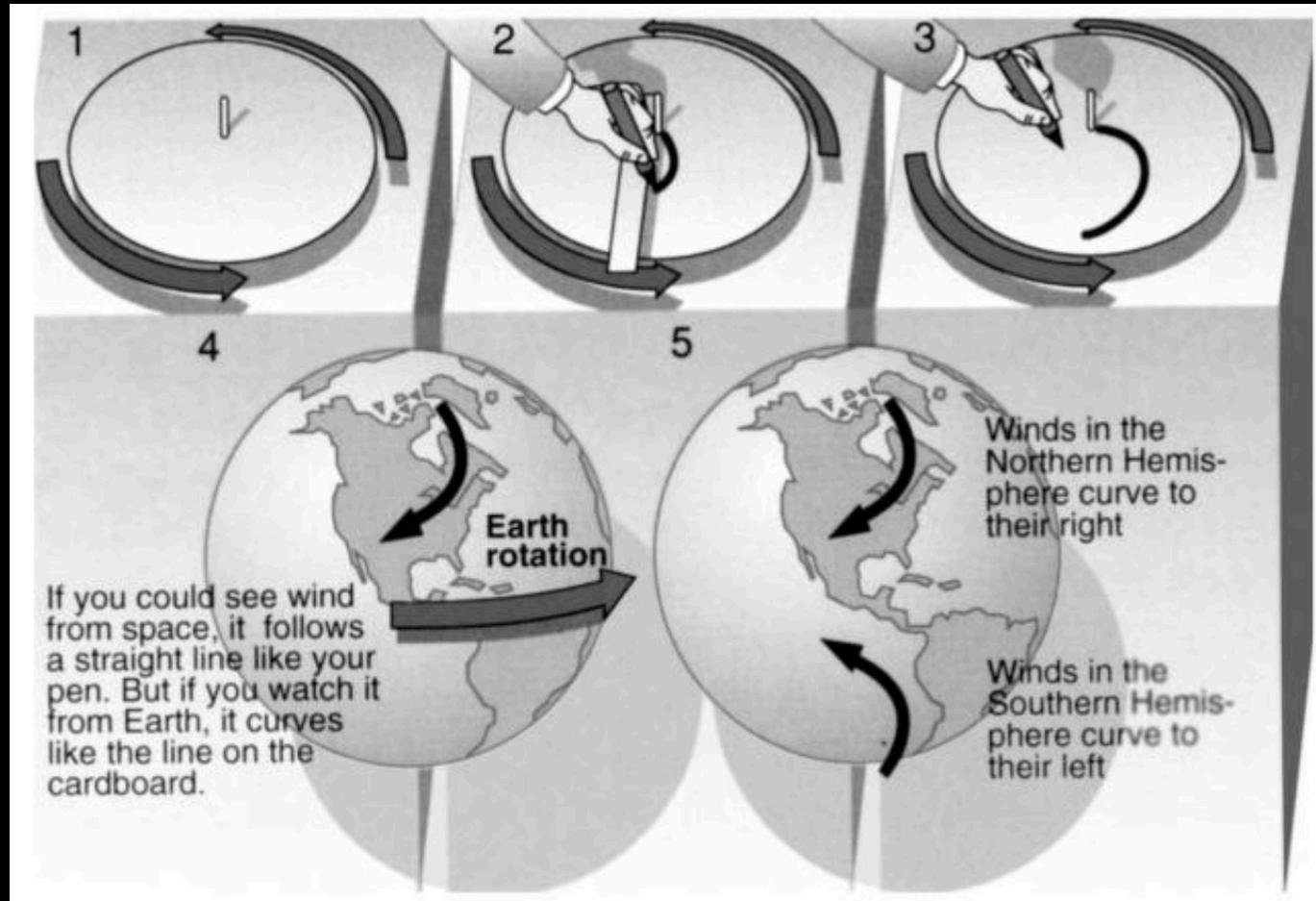


- Produces **wind cells**
- e.g. **Hadley cells** on Earth
- But what about the effect of **rotation**?



Credit: Alina Fiehn

# Coriolis Effect: winds do not follow a straight trajectory

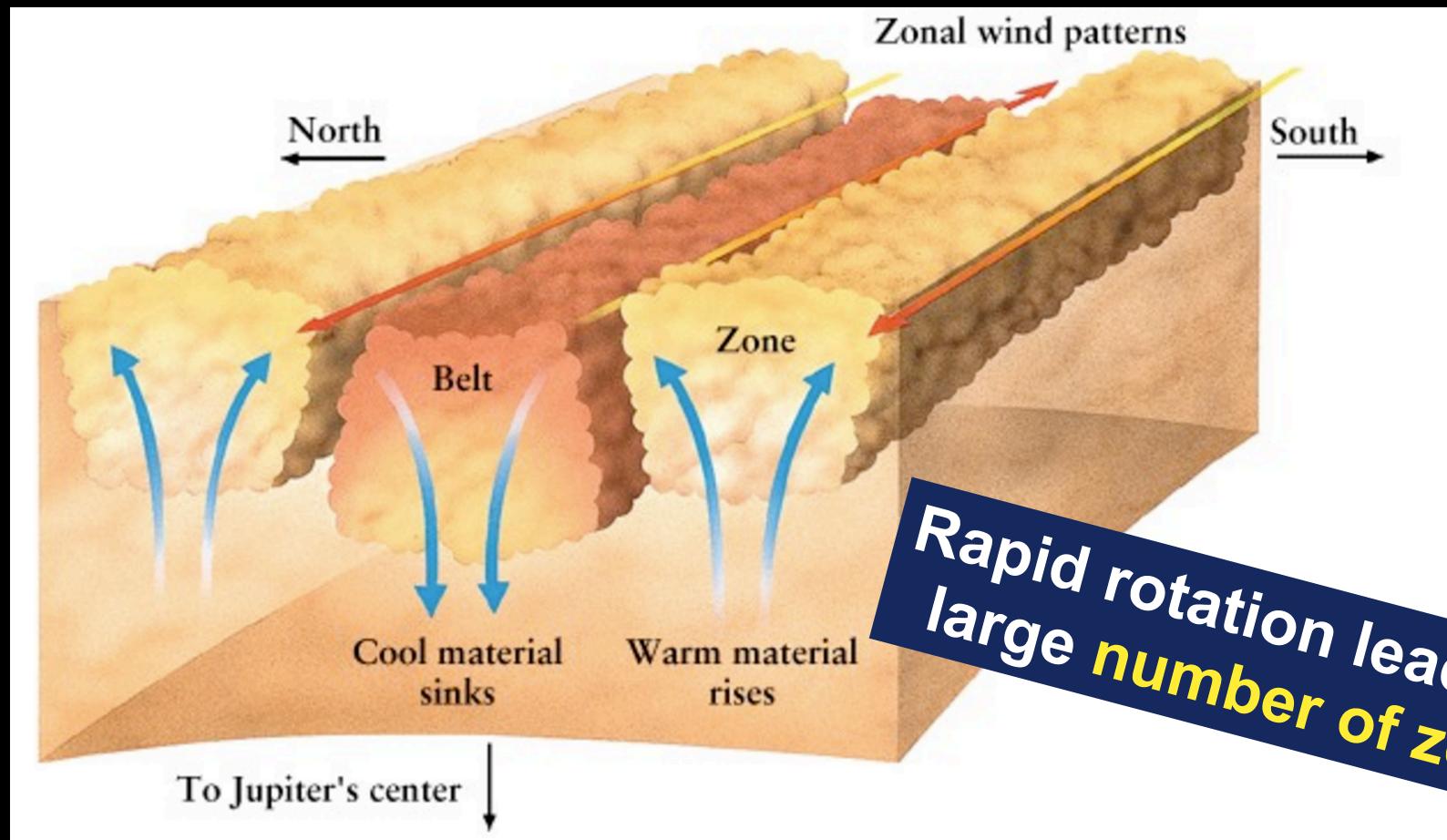


Lissauer & de Pater

On a rotating planet, winds traveling from the poles will be deflected to the **westward** (if the planet's rotation is prograde)

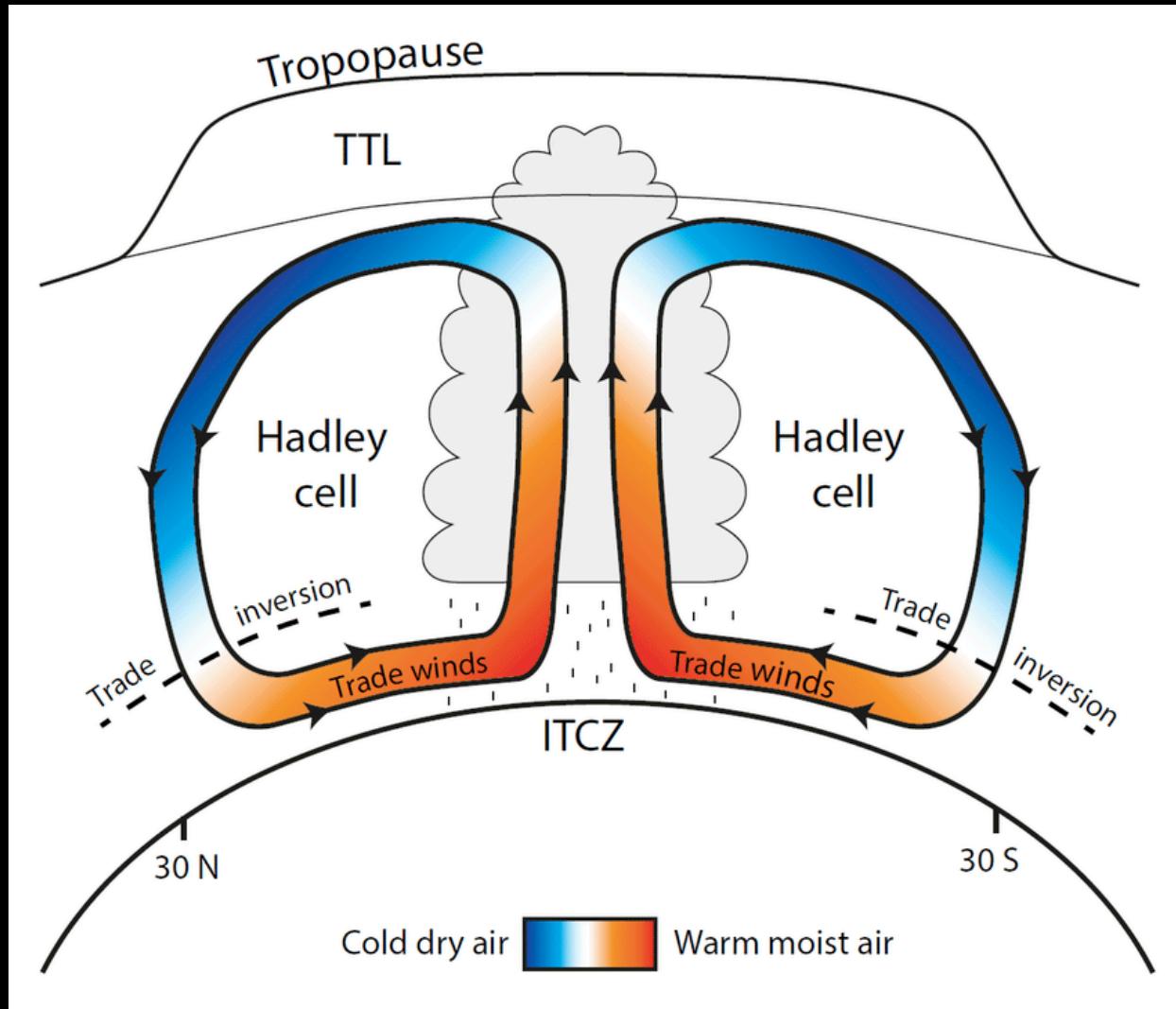
# Winds on Giant Planets

Pressure gradient + rapid rotation → zonal winds



# Winds on Terrestrial Planets

- Earth ( $P_{rot} = 24$  hrs) has **three wind cells per hemisphere**
- Venus ( $P_{rot} = 2802$  hrs) has **one large wind cells per hemisphere**

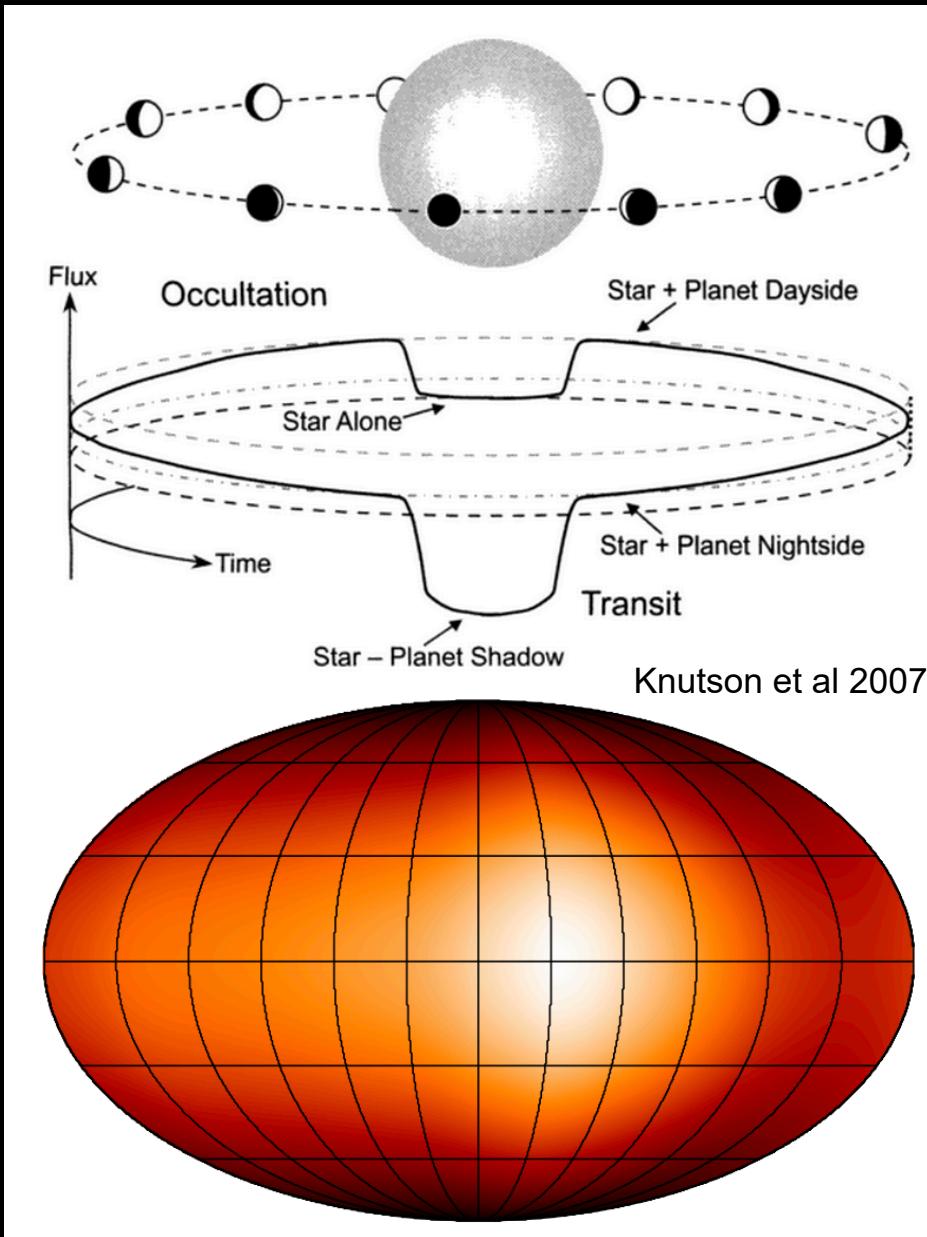


Credit: Alina Fiehn

# Winds on Exoplanets

For the majority of exoplanets, planetary rotation rates are not observable, but there are a couple of techniques that work for certain exoplanets

# Winds on tidally-locked exoplanets



Recall that tidally-locked planets have permanent (i.e. hot) daysides

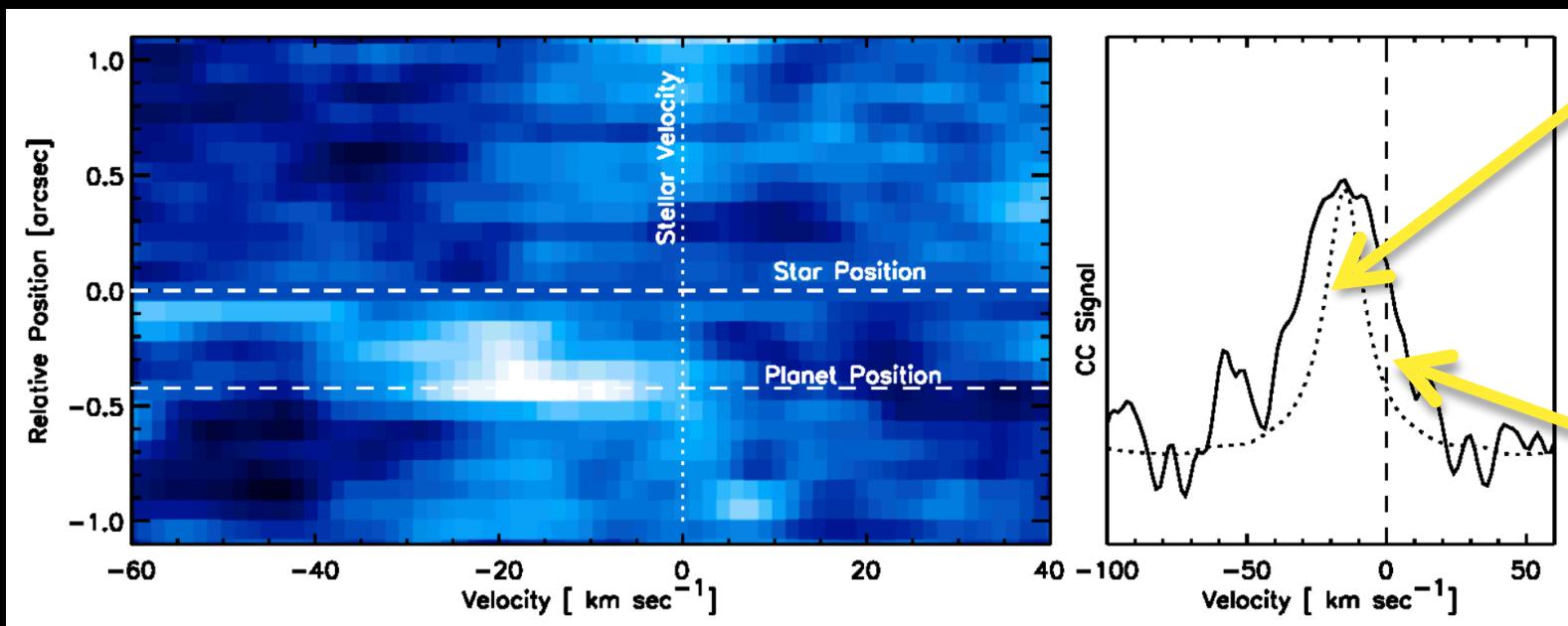
Observations around the secondary eclipse reveal a hot-spot offset



Atmospheric circulation

# Winds on directly-imaged exoplanets

**Rotation broadens spectroscopic signals**



Snellen et al 2014