Fundamentals

SFWRENG 2CO3: Data Structures and Algorithms

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Collection types

A *collection type* is an *data type* used to manage a *collection of values*.

Typically an *abstract data type*: the implementation is hidden from the user.

Collection types are implemented via *data structures*.

Collection types: Bag

A *bag* B is a collection to which values can be added, but not removed:

- ADD(B, v) add value v to a bag B;
- EMPTY(B) return `true` if bag B holds no values;
- SIZE(B) returns the number of values in B.

In addition, one can *iterate* over the values currently in bag B.

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Remark

Note that EMPTY can be implemented via SIZE.
Not all data structures provide an efficient SIZE, however!

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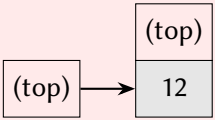

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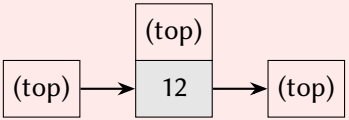

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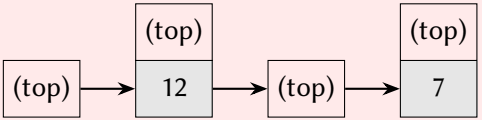

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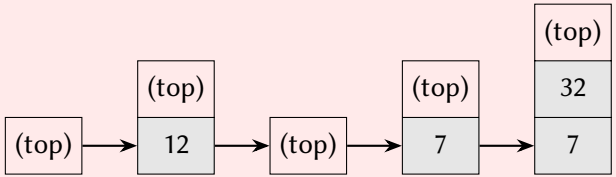

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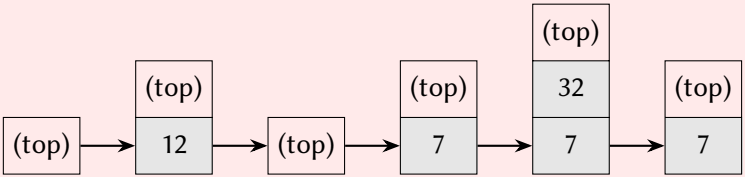

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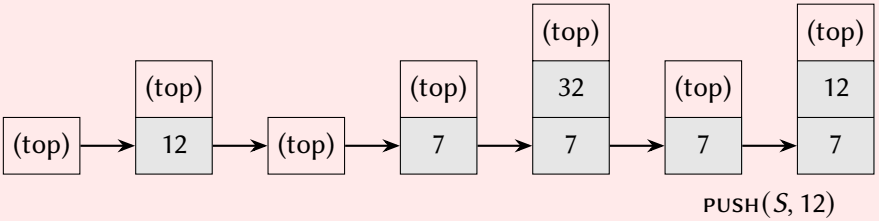

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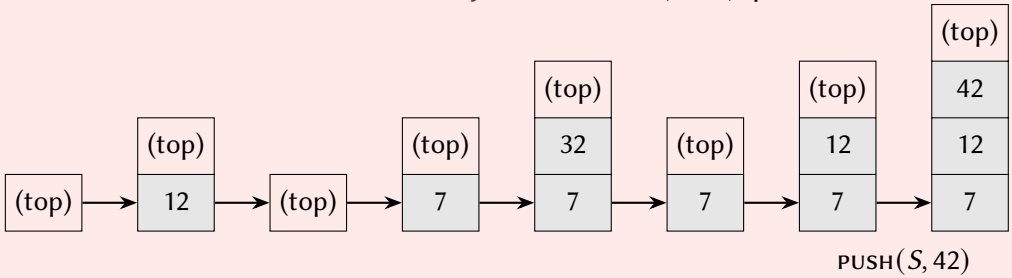

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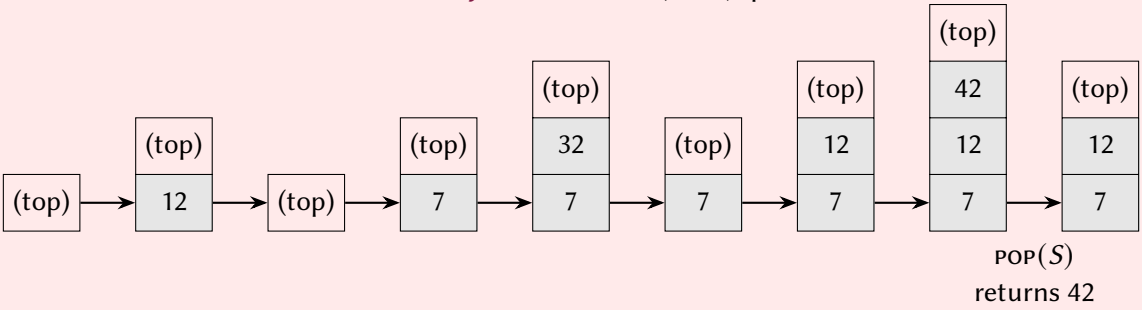

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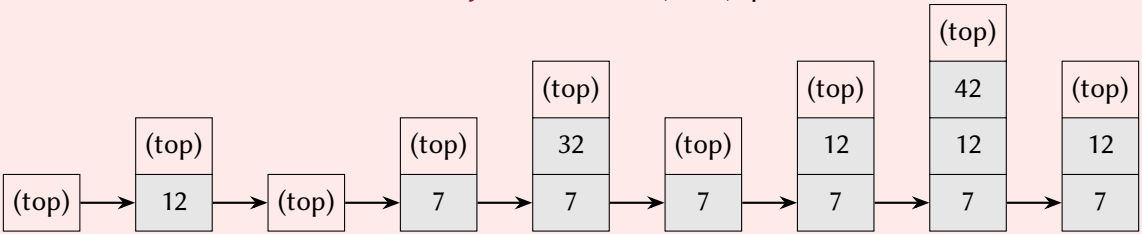

Stacks are used *everywhere*: e.g., function calls are implemented via stacks.

Using stacks to evaluate expressions in postfix notation

Postfix notation is a notation in which *operators* follow their *operands*.

Example

- "1 2 +" is equivalent to $1 + 2$.
- "1 2 3 + -" is equivalent to $1 - (2 + 3)$.
- "1 2 + 3 -" is equivalent to $(1 + 2) - 3$.
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Expressions in postfix notation are very easy to evaluate using a stack.

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Algorithm EvaluatePN(e):

Input: e is in posftix notation.

1: S := an empty stack.
2: **for each** term $t \in e$ **do**
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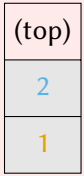

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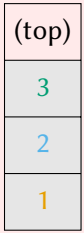

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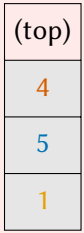

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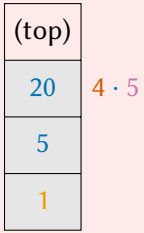

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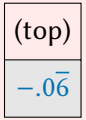

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Dijkstra's Two-Stack Algorithm (book) evaluates expressions in *infix* (normal) notation: this by building a postfix notation and simultaneously applying EvaluatePN.

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Enqueue(Q, v)	add value v to the top of queue Q;
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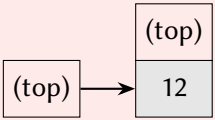

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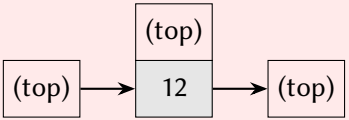

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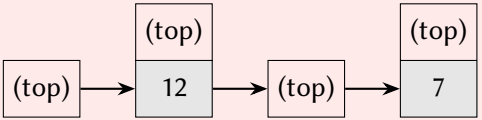

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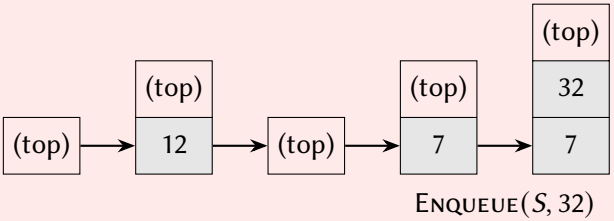

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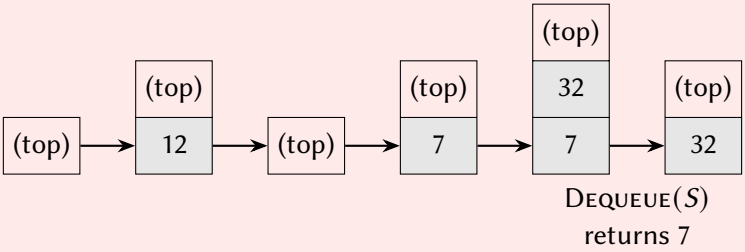

Collection types: Queue

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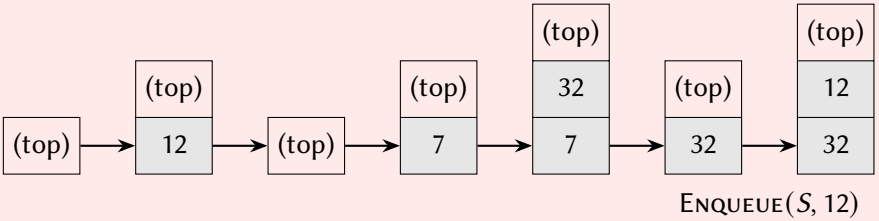

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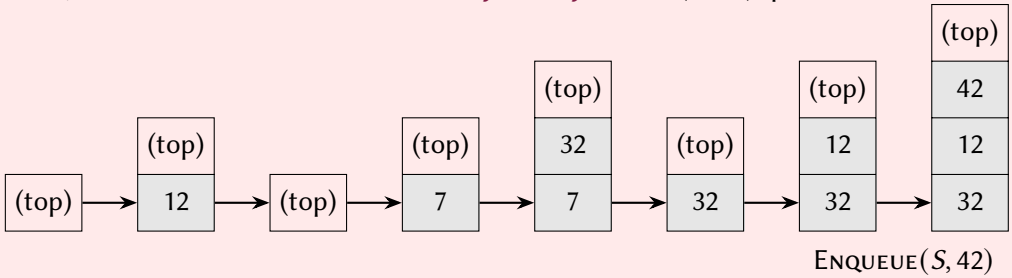

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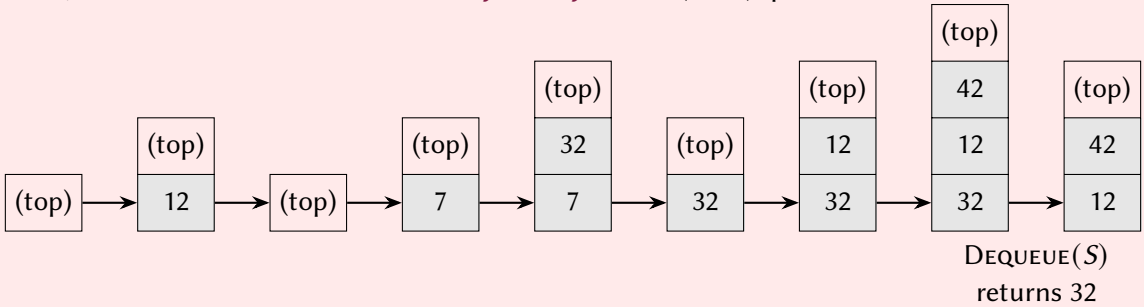

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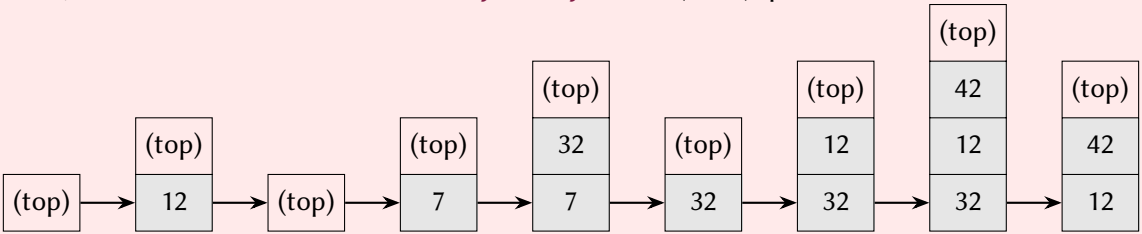

Queues are used used *everywhere*: e.g., communication buffers
(for network packages, for tasks exchanged between producer-consumer threads, ...).

Data structures: Fixed-size ring buffers

Ring Buffer: a data structure that can hold up-to-N values.

$entries[0 \ldots N)$	An array that can hold N values.
start	The position in *entries* of the *first* value in the buffer.
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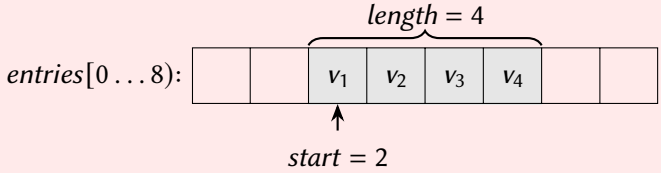

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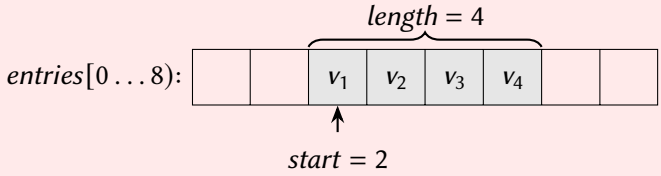

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Input: R is a non-full ring buffer ($R.length \neq N$).

1: **if** $R.start + R.length < N$ **then**
2: $\quad R.entries[R.start + R.length] := v$.
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4: $\quad R.entries[R.start + R.length - N] := v$.
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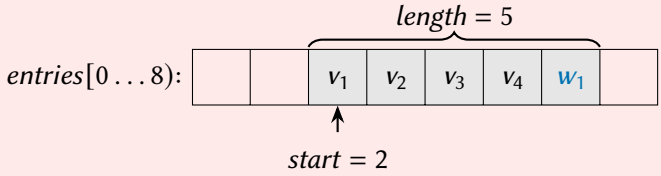

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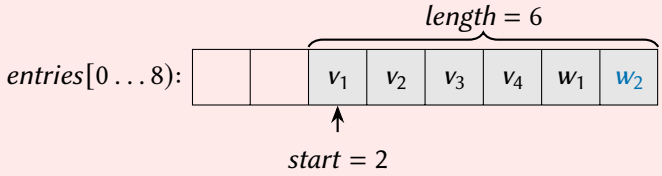

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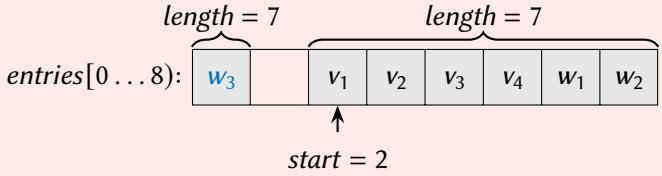

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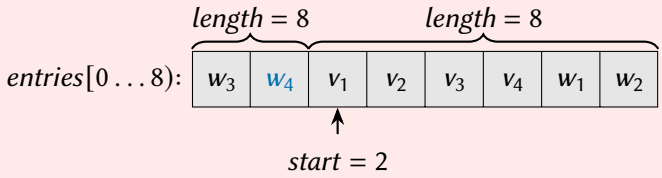

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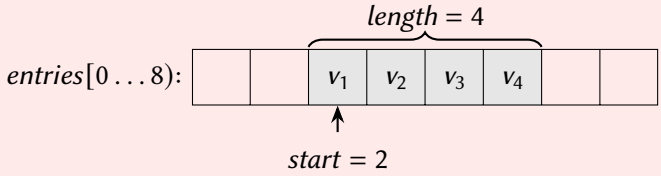

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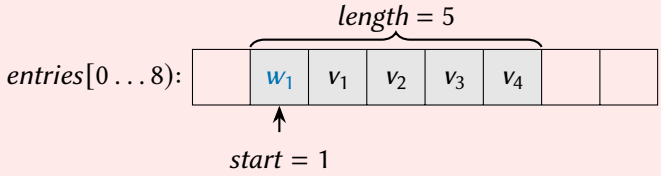

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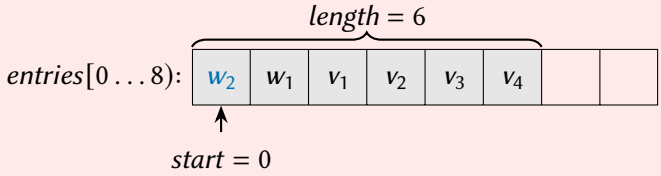

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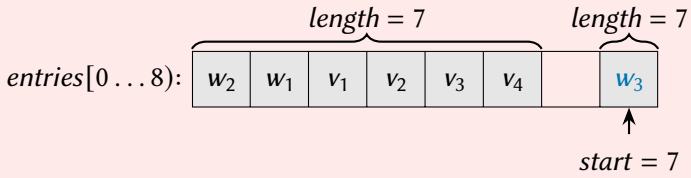

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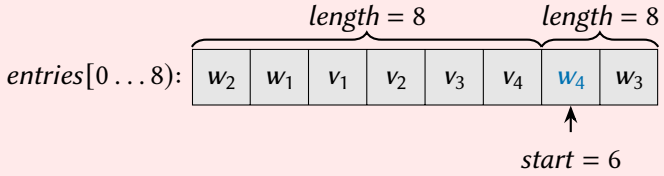

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PopFront(R) undoes PushFront.

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- Provides all stack operations in $\Theta(1)$.
- Provides all queue operations in $\Theta(1)$.
- Support *random access* efficiently.
- Can be used to implement a *double-ended queue*.

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- Provides all stack operations in $\Theta(1)$.
- Provides all queue operations in $\Theta(1)$.
- Support *random access* efficiently.
- Can be used to implement a *double-ended queue*.
- *Drawback*: can hold at-most N values.

Singly linked lists

Linked Lists: a data structure that can hold a sequence of values, each stored in a *list node*.

first Pointer to the *first list node*.

last Pointer to the *last list node* (optional, for adding values to the end).

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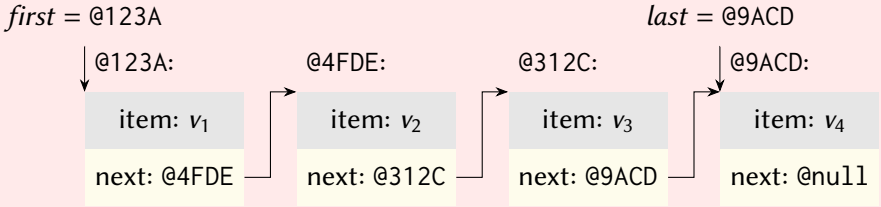

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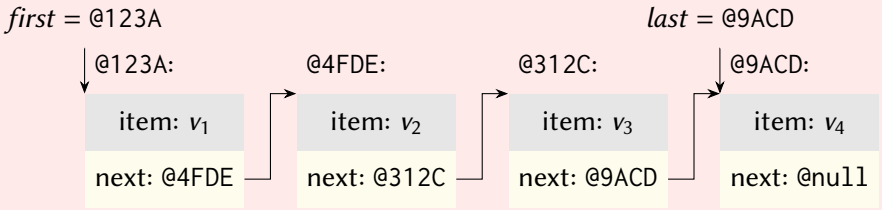

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Input: L is a linked list.

1: Create new list node n for value v.
2: $n.next$:= $L.first$.
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4: **if** $L.last$ = @null **then** List L was empty
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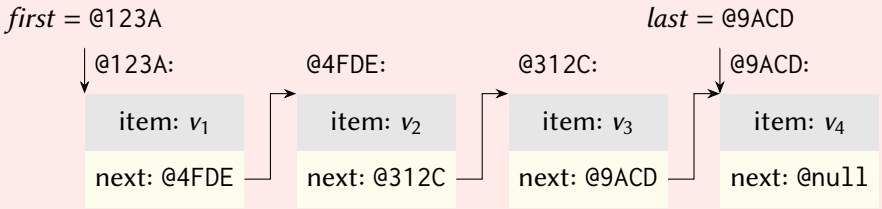

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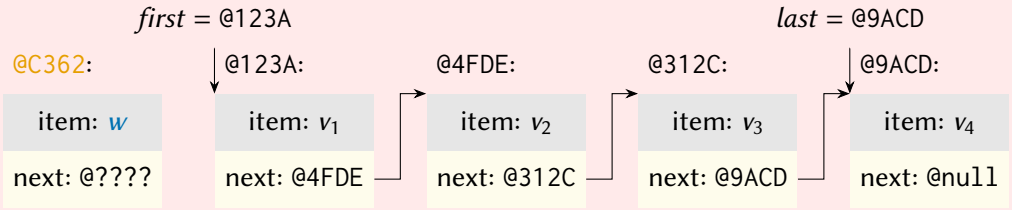

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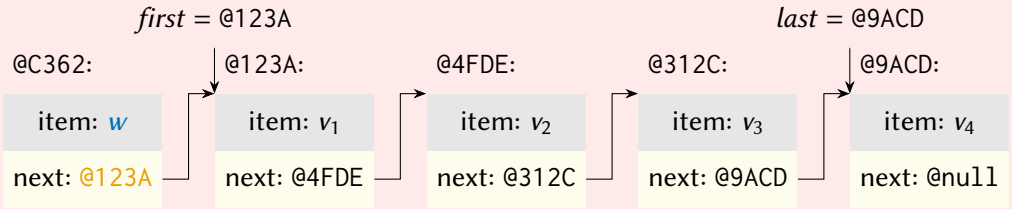

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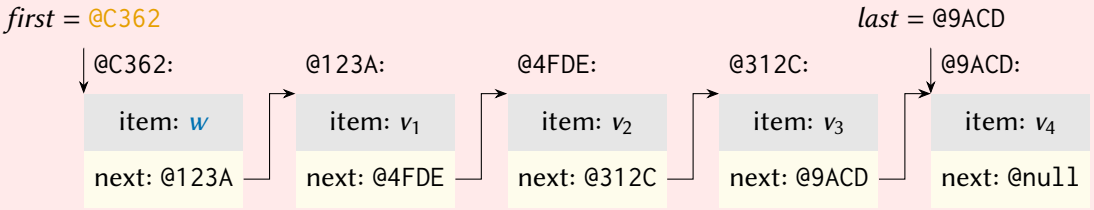

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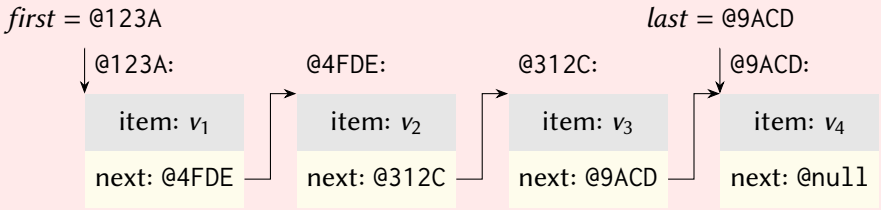

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Input: L is a linked list with node m.

1: Create new list node n for value v.
2: $n.next$:= $m.next$.
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4: **if** $L.last = m$ **then** m was the last node in L
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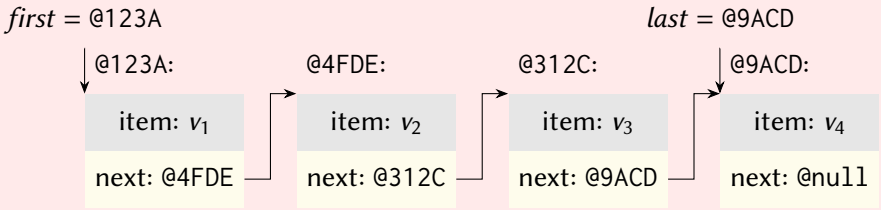

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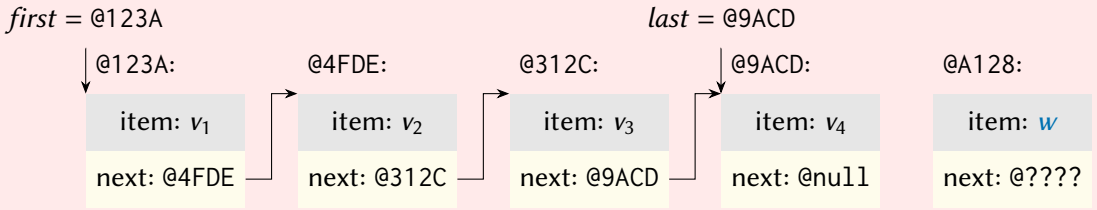

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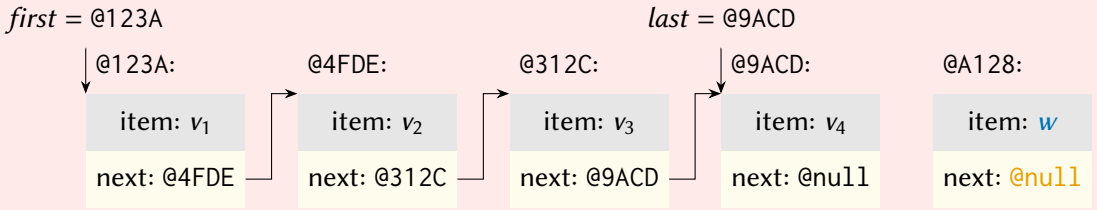

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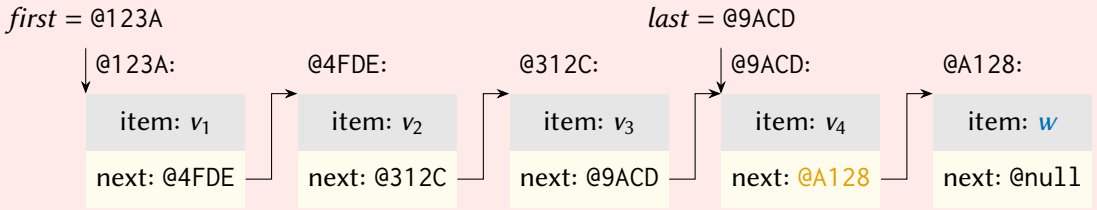

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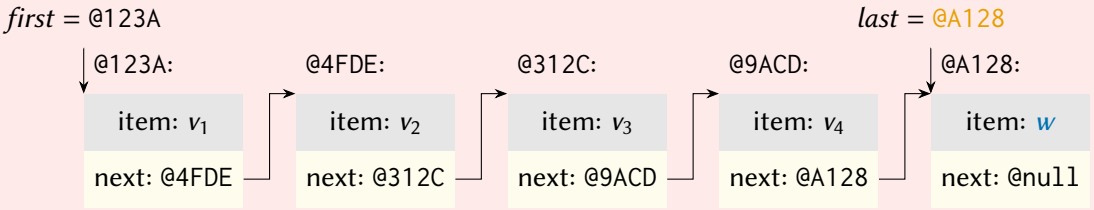

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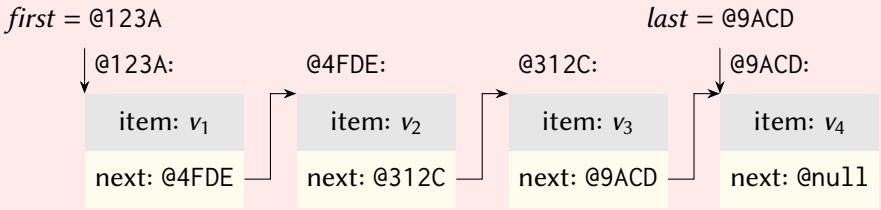

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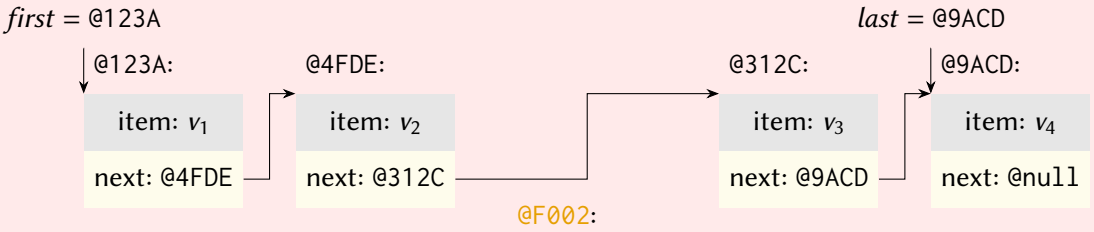

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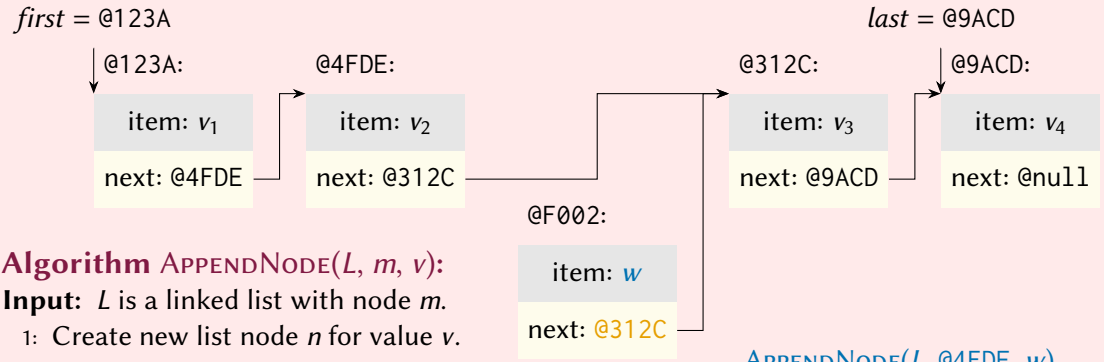

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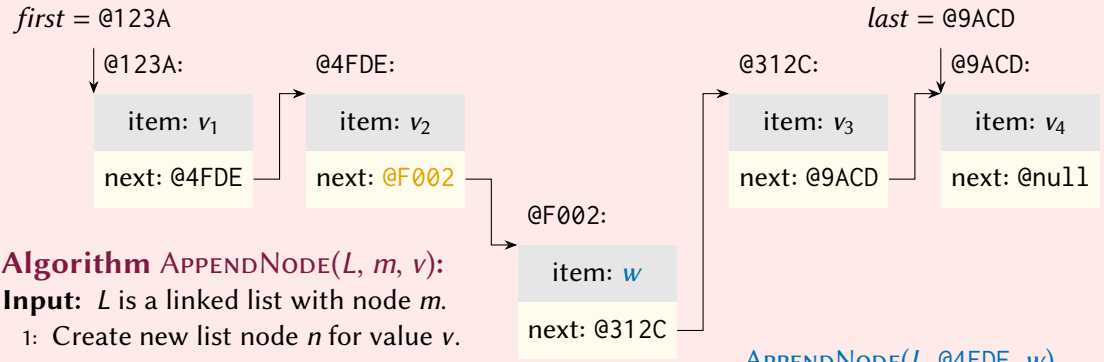

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- *first* Pointer to the *first list node*.
- *last* Pointer to the *last list node* (optional, for adding values to the end).

Each value in a linked list is held by a *list node*:

- *item* The value held by the list node.
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Removing elements from the front or after a given node is similar:

- PopFront(L) undoes PushFront.
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Make sure to free the memory associated with nodes.

In C++: use either `std::unique_ptr` or `std::shared_ptr` to free nodes *for* you.

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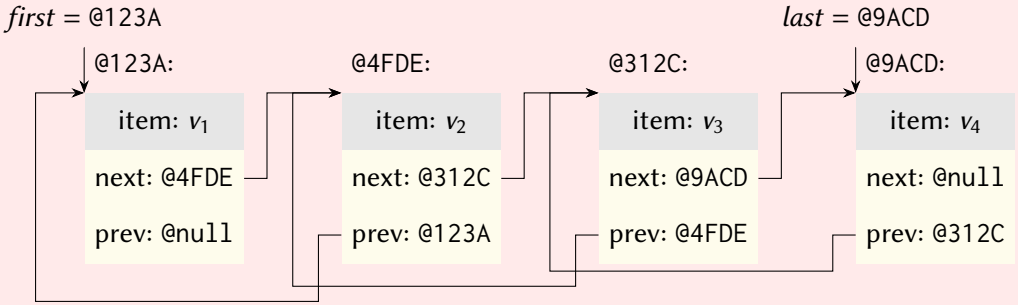

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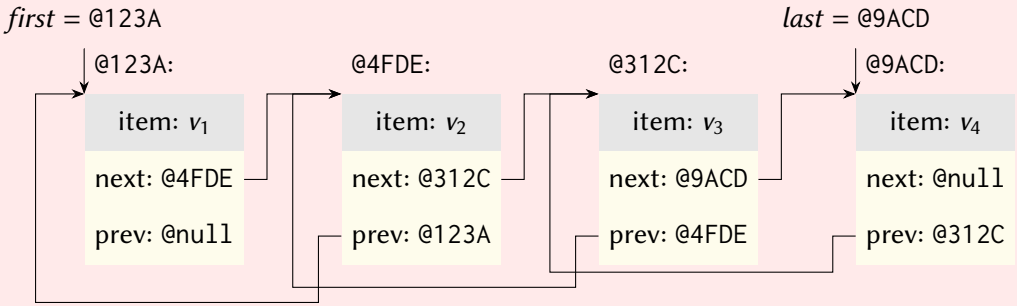

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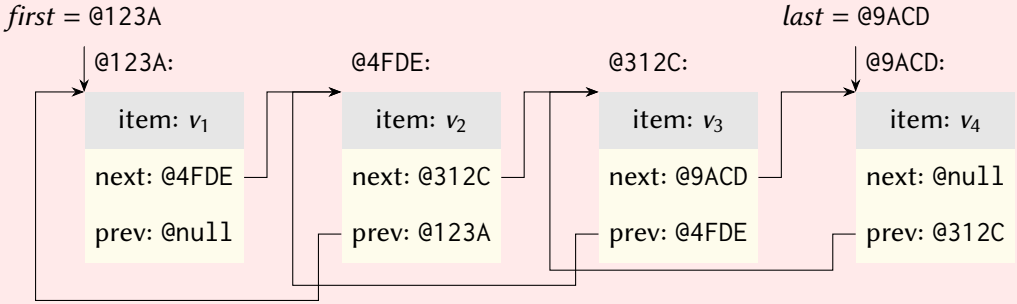

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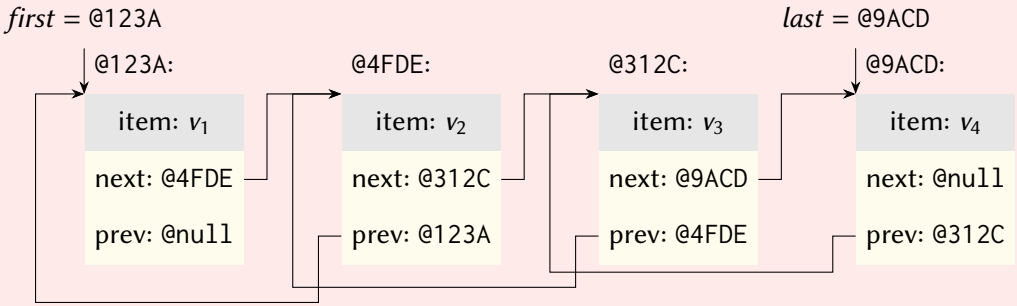

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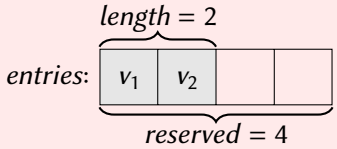

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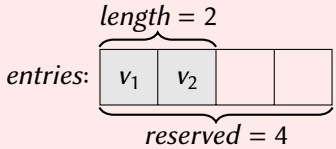

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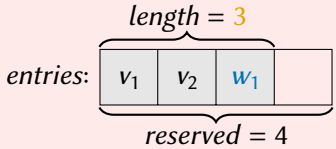

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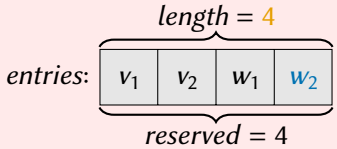

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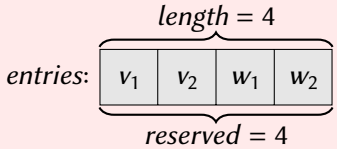

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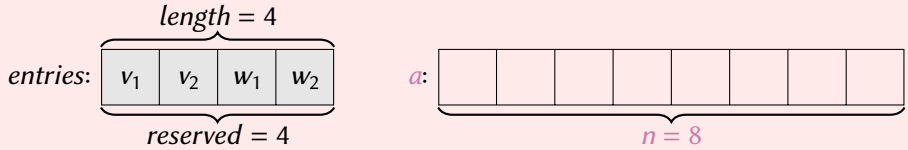

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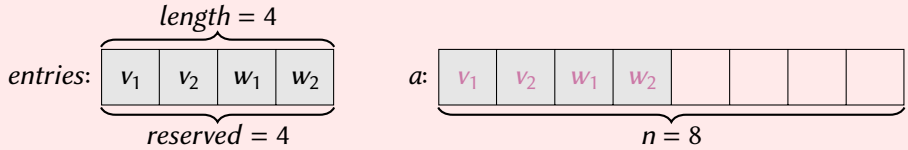

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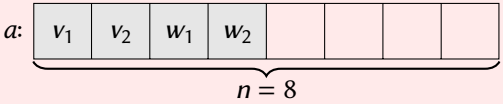

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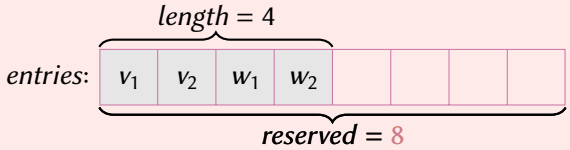

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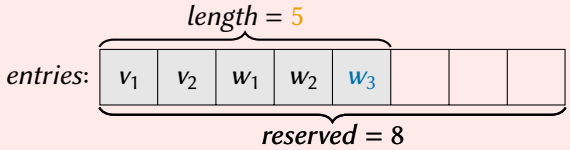

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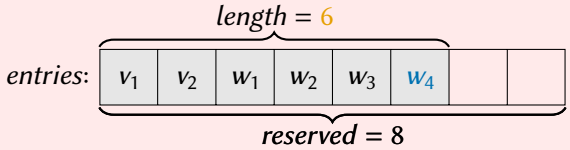

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PushBack(D, v) is either $\Theta(1)$ or $\Theta(D.reserved)$ (if InternalResize is called).

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Amortized complexity is not average case complexity!

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To support PopBack efficiently: do not *shrink* too soon:
either *never shrink* or shrink *if requested* or when $4 \cdot D.length < D.reserved$.

Dynamic arrays

Dynamic Array: a data structure that can hold an *array* of values that is resizes upon need.

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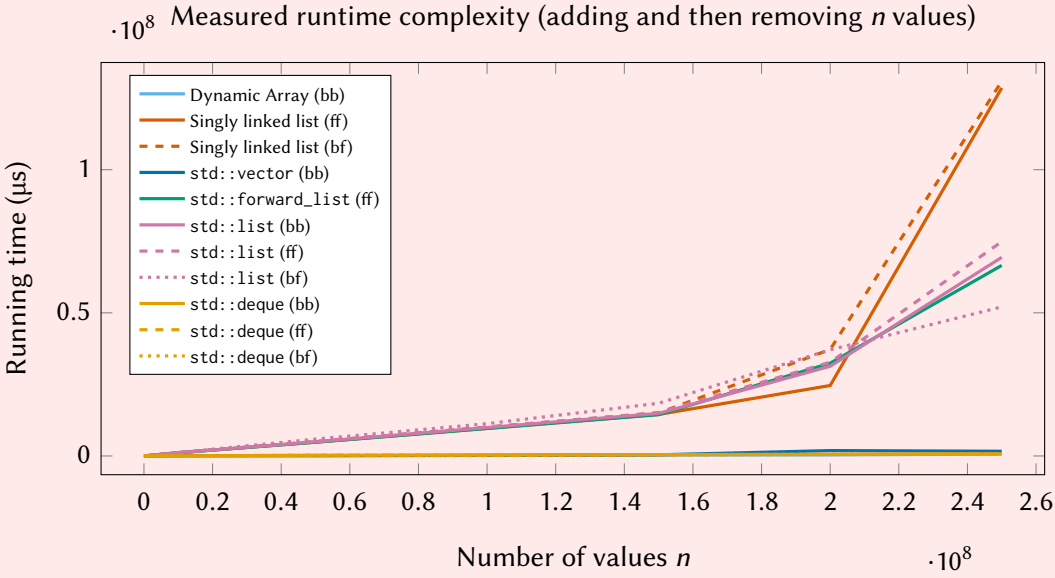
- Support *random access* efficiently.
- *Drawback*: sometimes expensive *resizes*, no efficient queue operations.

A summary of elementary containers

Data structure	Supports ADT: Queue	Stack	Dequeue	Random Access	Memory Usage
Ring Buffer	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$	Always NT
Singly linked list	$\Theta(1)$	$\Theta(1)$			$T + P + M$ per value
Doubly linked list	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$		$T + 2P + M$ per value
Dynamic array		$\Theta(1)$ (amortized)		$\Theta(1)$	$\leq M + 2T$ $\leq 2M + 3T$ (during resize)

T is the size of a value, P is the size of a pointer, M is the overhead per memory allocation.

Comparing common containers



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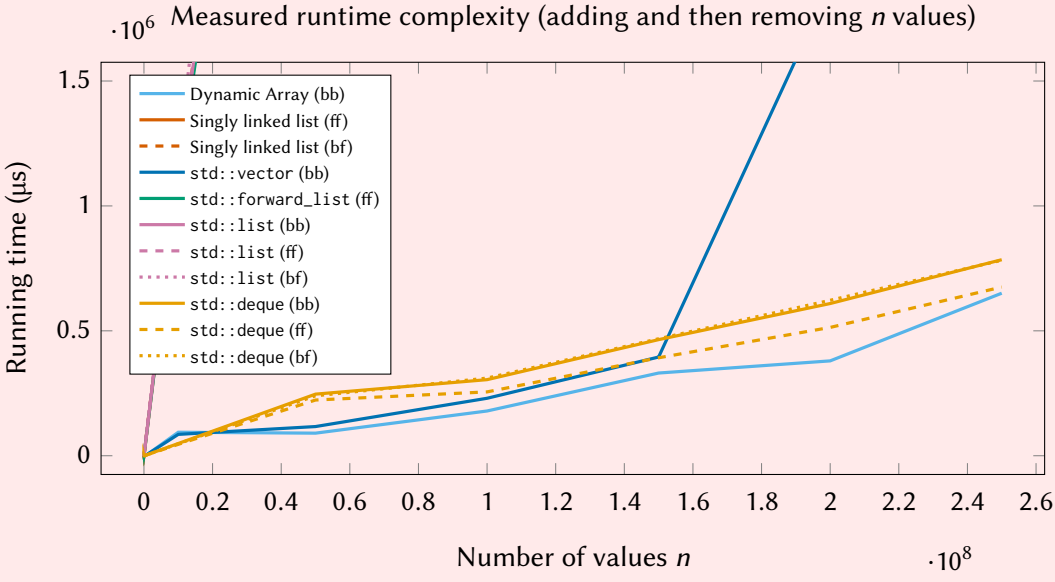

Elementary containers in practice

Data Collection or Structure	C++	Java
Ring Buffer		`java.util.ArrayDeque`
Singly Linked List	`std::forward_list`	
Doubly Linked List	`std::list`	`java.util.LinkedList`
Dynamic Array	`std::vector`	`java.util.ArrayList`
Other	`std::deque`	
Stack	`std::stack`	Use `ArrayDeque` or `ArrayList`
Queue	`std::queue`	Use `ArrayDeque`

Java provides `java.util.Vector` and `Stack` and `Queue` on top of `Vector`. These are ancient and their usage is *not recommended*. Use `ArrayList` or `ArrayDeque` instead!