1) CS 112 HW 5

Jonathan Nguy
603799761

1.1. $E[N|T=t]$, N is # of interrupts

$E(X)=\sum xP(X=x)$

$E(X)=\sum_{k\geq 0} k\frac{1}{k!}(\lambda t)^k e^{-\lambda t}$ (Poisson)

$E(X)=\lambda t e^{-\lambda t}\sum_{k\geq 1}\frac{1}{(k-1)!}(\lambda t)^{k-1} = \lambda t e^{-\lambda t}\sum_{j\geq 0}\frac{(\lambda t)^j}{j!}$ $(j=k-1)$

$=\lambda t e^{-\lambda t}e^{\lambda t}=\lambda t$

So, $E[N|T=t]=\lambda t$

1.2. expected #

$P(T=t)=\frac{t}{20-10}=\frac{t}{10}$ (uniform distribution

unconditioning on 1.1

$E[N]=\sum_t E[N|T]\,P_T(t)=\sum_1^{10}\lambda t\cdot\frac{t}{10}$

2. both host-router & router-router are hops,

Each packet take 1, 2 or 3 hops

For 1, it's p to drop

2, it's $(1-p)(p)$

3, its $(1-p)(1-p)$.

so it is $1\cdot p+2\cdot(1-p)(p)+3\cdot(1-p)(1-p)=p^2-3p+3$ for the mean.

3. a. assume discrete w/ time required following uniform dist. from a to a+b

$$P_T(t)=\begin{cases}\frac{1}{b+1} & t=a, a+1, \dots, a+b\\ 0 & \text{elsewhere.}\end{cases}$$

b. $E[T]=\sum_{t=a}^{a+b} tP[T=t]=\frac{1}{b+1}[a+(a+1)+\dots+(a+b)]=\frac{1}{b+1}[(a+a\dots+a)+(1+2+\dots+b)]$

$=\cancel{\frac{1}{b+1}}\left[\cancel{(b+1)}a+\frac{b\cancel{(b+1)}}{2}\right]=a+\frac{b}{2}$

c. $Var[T]=E[T^2]-(E[T])^2=\frac{1}{b+1}\left[\sum_{t=1}^{a+b}t^2-\sum_{t=1}^{a-1}\right]-\left(a+\frac{b}{2}\right)^2=\frac{b(b+2)}{12}$

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4. $E[C] = E[a + bM + cM^2]$

$= E[a] + E[bM] + E[cM^2]$

$= a + bE[M] + cE[M^2]$

1st moment 2nd moment

Z-transform of M as Poisson

$P_M(m): G(z) = e^{-n(1-z)}$

$\overline{m} = \frac{\partial G_M(z)}{\partial z}\Big|_{z=1} = n$

$\overline{m^2} - \overline{m} = \frac{\partial^2 G_M(z)}{\partial z}\Big|_{z=1} = n^2 \Rightarrow \overline{m^2} = n^2 + 1$

5. a) Conditioning on X, $E[N] = \sum_{i=1}^{3} E[N|X=i]P[X=i]$

It resets when the prisoner returns.

$E[N|X=1] = 2 + E[N]$

$E[N|X=2] = 3 + E[N]$

$E[X=3] = 0$

$E[N] = \frac{1}{2}(2 + E[N]) + .3(3 + E[N]) + .2 \times 0 \Rightarrow E[N] = 9.5$ days

b) Say N_i is # of additional days the prisoner spends after choosing door i

$E[N] = \frac{1}{3}(2 + E[N_1]) + \frac{1}{3}(3 + E[N_2]) + \frac{1}{3} \cdot 0$

but $E[N_1] = \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 0 = \frac{3}{2}$

$E[N_2] = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 1$

$E[N] = 2.5$ days.