

CS112 Homework 8

Queueing preliminaries

Mar 06, 2013

Homeworks are due **03/13/13**

Q1. Consider a taxi station where taxis and customers arrive in a poisson process with respective rates of one and two per minute. A taxi will wait no matter how many taxis are waiting. But if an arriving customer does not find a taxi he leaves. Find

- a) the average number of taxis waiting
- b) the proportion of arriving customers that get taxis.

Q2. In a population growth model, suppose that immigration is allowed only when the population is 0, 1, 2, 3, or 4 (births can occur when the population size is 1, 2,..., unlimited). Assume a birth rate per individual = 1, death rate per individual = 2, immigration rate = 4 (when allowed),

- (a) set up "flow balance" equations for the p_i , using the "circle one more state each time" shortcut helpful in analyzing birth-death models.
- (b) calculate p_0 .
- (c) calculate the equilibrium fraction-of-time during which immigration is allowed.

Q3. Customers arrive at a single service facility at a Poisson rate of 40 per hour. When two or fewer customers are present, a single attendant operates at the facility and service time for each customer is exponentially distributed with a mean value of two minutes. however, when there are more than two customers present, the attendant is joined by an assistant and the mean service time reduces to one minute. Assuming a system capacity of four customers:

- a) What proportion of time are both attendants free?
- b) Each man is to receive a salary proportional to the amount of time they work. If they have to split 100 dollars between them how should this money be split?
- c) What is the average amount of time a customer spends in the system and what is the average waiting time? *Hint: Use Little's Result*

Q4. Consider a sequential-service system consisting of two servers, A and B. Arriving customers will enter this system only if server A is free. If a customer does enter, he is immediately serviced by server A. When service by A is completed he goes to server B if B is free, or if B is busy, he leaves the system. Upon completion of service at server B, the customer departs. Assuming that the Poisson arrival rate is two customers per hour and that A and B serve customers at a rate of 4 per hour and 2 per hour respectively,

- a) What proportion of customers enter the system?
- b) What proportion of entering customers receive service from B?
- c) What is the average number of customers in the system?
- d) What is the average amount of time an entering customer spends in the system?

Q5. Derive the probability density function (pdf) of the waiting time for an M/M/1 queue in terms of the system parameters λ and μ .

[Hint: Try to find the transform of the waiting time $W^*(s)$ first. To find the transform, condition on the number of customers an arrival finds in the system and then uncondition. i.e. Find the conditional $W^*(s)$ given the arrival finds k customers in the system, then uncondition.]