# CS112 Homework 7
# Continuous Time Markov Chains

Homeworks are due **03/6/13**

**Q1.** Consider two machines maintained by a single repairman. Machine $i$ functions for an exponential amount of time with rate $\mu_i$ before breaking down, $i = 1, 2$. The repair times for either machine are exponential with parameter $\mu$. Can we analyze this as a birth-death process? If not, how can we analyze it?

**Q2.** Consider a birth-death system in which $\lambda_k = \lambda$ and $\mu_k = k\mu$ for $k \geq 0$. For all k find the difference differential equations for $P_k(t) = P$[k in system at time t].

**Q3.** Consider a system in which the birth rate decreases and the death rate increases as he number in system increases as follows: $\lambda_k = (K - k)\lambda$ for$k \leq K$ and 0 otherwise. And $\mu_k = k\mu$ for $k \leq K$ and 0 otherwise. For all k find the difference differential equations for $P_k(t) = P$[k in system at time t].

**Q4.** A small barbershop has room for atmost two customers (including the one being served). Potential customers arrive at a Poisson rate of three per hour, and the successive service times are independent exponential random variables with mean 0.25 hours. What is:
a) The average number of customers in the shop?
b) The proportion of potential customers that get a hair cut?
c) If the barber could work twice as fast how much more business would he do?
d) If the barber hired another barber who works just as fast as him then how much more business would the barber do?

**Q5.** A single repairperson looks after both machine 1 and 2. Each time it is repaired, machine $i$ stays up for an exponential time with rate $\lambda_i$, $i = 1, 2$. When machine $i$ fails, it requires an exponentially distributed amount of work with rate $\mu_i$ to complete its repair. The repairperson will always service machine 1 when it is down. For instance, if machine 1 fails while 2 is being repaired, then the repairperson will immediately stop work on machine 2 and start on 1. What proportion of time is machine 2 down?