CS 112 HW

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A. 1. $A \cup B \cup C$

2. $(\overline{A \cup B}) \cup (\overline{A \cup C}) \cup (\overline{B \cup C})$

3. $\bar{A} \cap \bar{B} \cap \bar{C}$

4. $A \cap B \cap C$

5. $\bar{A} \cup \bar{B} \cup \bar{C}$

6. $A \cap B \cap \bar{C}$

7. $A \cup \bar{B}$

B.

2nd \ first	1	2	3	4	5	6
1	11	21	31	41	51	61
2	12	22	32	42	52	62
3	13	23	33	43	53	63
4	14	24	34	44	54	64
5	15	25	35	45	55	65
6	16	26	36	46	56	66

A B C

$P(A) = \frac{1}{2}$ $\quad P(A \cap B) = 1/4$

$P(B) = \frac{1}{2}$ $\quad P(A \cap C) = 1/36$

$P(C) = 1/9$ $\quad P(B \cap C) = 1/18$

$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{9}{36} = \frac{1}{4}$

$P(C) = 1/9$

These are <u>not</u> independent

C. a) $P(\overset{out}{0}) = \frac{1}{2}(1-\varepsilon) + \frac{1}{4}(\varepsilon) = \frac{1}{2} - \frac{\varepsilon}{2} + \frac{\varepsilon}{4} = \frac{1}{2} - \frac{1\varepsilon}{4}$

$P(\overset{out}{1}) = \frac{\varepsilon}{2} + \frac{1}{4}(1-\varepsilon) = \frac{\varepsilon}{2} + \frac{1}{4} - \frac{\varepsilon}{4} = \frac{1}{2} + \frac{\varepsilon}{4}$

$P(\overset{out}{2}) = \frac{\varepsilon}{4} + \frac{1}{4}(1-\varepsilon) = \frac{\varepsilon}{4} + \frac{1}{4} - \frac{\varepsilon}{4} = \frac{1}{4}$

b) $P(\overset{in}{0} \mid \overset{out}{0}) = \frac{P(\text{in } 0 \cap \text{out } 0)}{P(\text{out } 0)} = \frac{\frac{1}{2}(1-\varepsilon)}{\frac{1}{2} - \frac{\varepsilon}{4}} = \frac{\frac{1-\varepsilon}{2}}{\frac{2-\varepsilon}{4}} = \frac{2(1-\varepsilon)}{2-\varepsilon} = \frac{2-2\varepsilon}{2-\varepsilon}$

$P(\overset{in}{1} \mid \overset{out}{0}) = \frac{P(\text{in } 1 \cap \text{out } 0)}{P(\text{out } 0)} = 0$

$P(\overset{in}{2} \mid \overset{out}{0}) = \frac{P(\text{in } 2 \cap \text{out } 0)}{P(\text{out } 0)} = \frac{\frac{\varepsilon}{4}}{\frac{2-\varepsilon}{4}} = \frac{\varepsilon}{2-\varepsilon}$

c) $P(\overset{in}{2} \mid \overset{out}{0}) + P(\overset{in}{0} \mid \overset{out}{1}) + P(\overset{in}{1} \mid \overset{out}{2}) = \longrightarrow \frac{\varepsilon}{2-\varepsilon} + \frac{2\varepsilon}{2+\varepsilon} + \varepsilon$

$\frac{P(\text{in } 0 \cap \text{out } 1)}{P(1)} = \frac{\frac{1\varepsilon}{2}}{\frac{2+\varepsilon}{4}} = \frac{2\varepsilon}{2+\varepsilon}$

$\frac{P(\text{in } 1 \cap \text{out } 2)}{P(2)} = \frac{\varepsilon/4}{1/4} = \varepsilon$

D. a) $\binom{100}{5} \cdot .05^0 \cdot (.95)^5$

$\binom{100}{5} \cdot .90^5$

b) $\binom{100}{5} .05^1 (.95)^4 + \binom{100}{5} .05^0 (.95)^5$

$\binom{100}{5} .1 (.9)^4 + \binom{100}{5} .1^0 (.9)^5$

E. a) $S = \{ \text{no } 1, \text{no } 2, \text{no } 3 \}$

b) 0 $\quad$ 1/3 $\quad$ 2/3 $\quad$ 1 $\leftarrow \{A, B\}$, where A = your choice door, B = gold door

$\{1,1\}$	$\{1,2\}$
$\{2,2\}$	$\{1,3\}$
$\{3,3\}$	$\{2,1\}$
	$\{2,3\}$
	$\{3,1\}$
	$\{3,2\}$

c) If contestant switches, it is 2/3 chance of winning.

F. Let i = disk #

$P(\text{Found} \mid i=1) = p$, $P(\text{not found} \mid i=1) = 1-p$

$P(\text{Found} \mid i=2,3,4) = 0$, $P(\text{not found} \mid i = 2,3, \text{or } 4) = 1$

so,

$$P(i=1 \mid \text{not found}) = \frac{P(\text{not found} \mid i=1)\, P(i=1)}{P(\text{not found})} = \frac{(1-p)\cdot\frac{1}{\cancel{4}}}{\frac{4-p}{\cancel{4}}} = \frac{1-p}{4-p}$$

$$P(\text{not found}) = \frac{(1-p)+1+1+1}{4} = 1 - \frac{p}{4}$$

$$P(i=2,3,4) = 1 - \frac{1-p}{4-p}$$

$$P(i=2) = P(i=3) = P(i=4) = \frac{4-p-1+p}{4-p} \Big/ 3 = \frac{3}{4-p} \Big/ 3 = \frac{1}{4-p}$$