CS 112 HW6 pg 1. Jonathan Nguy

1. $\pi_0 = .7\pi_0 + .2\pi_1 + .1\pi_2$
$\pi_1 = .2\pi_0 + .6\pi_1 + .4\pi_2$
$\pi_2 = .1\pi_0 + .2\pi_1 + .5\pi_2$

$\underline{\pi} = \underline{\pi} P$, $\pi_0 + \pi_1 + \pi_2 = 1$

$\pi_0 = 6/17$
$\pi_1 = 7/17$
$\pi_2 = 4/17$

$\pi = (6/17, 7/17, 4/17)$

2.

$$P = \begin{array}{c|c} & \begin{matrix} C & S & G \end{matrix} \\ \begin{matrix} C \\ S \\ G \end{matrix} & \begin{bmatrix} .5 & .4 & .1 \\ .3 & .4 & .3 \\ .2 & .3 & .5 \end{bmatrix} \end{array}$$

b) $\underline{\pi} = \underline{\pi} P$

$\underline{\pi} = [.33, .37, .3]$

3. a) 4 states:
1. rains yesterday & today
2. rained today & not yesterday
3. rained yesterday, not today
4. no rain yesterday or today

b)

$$P = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} .7 & 0 & .3 & 0 \\ .5 & 0 & .5 & 0 \\ 0 & .4 & 0 & .6 \\ 0 & .2 & 0 & .8 \end{bmatrix} \end{array}$$

4. Let $X_n$ be the coin flipped on $n^{th}$ day,

$P = \begin{bmatrix} .7 & .3 \\ .6 & .4 \end{bmatrix}$. $P^3 = \begin{bmatrix} .667 & .333 \\ .666 & .334 \end{bmatrix}$

If initial was $\pi = (.5\ .5)$

$\pi P^3 = [.5\ .5] \begin{bmatrix} .667 & .333 \\ .666 & .334 \end{bmatrix} = [.6665\ .3335]$

Answer is $\underline{.6665.}$

5. Let C = car, T = truck

$$P = \begin{array}{c c} & \begin{array}{cc} C & T \end{array} \\ \begin{array}{c} C \\ T \end{array} & \begin{bmatrix} .8 & .2 \\ .75 & .25 \end{bmatrix} \end{array}$$

$\underline{\pi} = \underline{\pi} P$, $\sum_i \pi_i = 1$, so $\underline{\pi} = \left[\frac{15}{19}, \frac{4}{19}\right]$, so 4 out of every 19 are trucks

6. a)

$$P = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1-p & p \\ 1 & 0 & 0 \\ 0 & \alpha & 1-\alpha \end{bmatrix} \end{array}$$

b. If $p=0$, state 3 is unreachable from 1 & 2.
If $\alpha = 0$, states 1 & 2 are unreachable from 3.
If $p=1$ & $q=1$, then the Markov chain has a period of 3.
In other cases, Markov Chain is irreducible & aperiodic

c. $\underline{\pi} = \underline{\pi} P$

$\pi_1 = \pi_2$

$\pi_2 = (1-p)\pi_1 + \alpha \pi_3$

$\pi_3 = p\pi_1 + (1-\alpha)\pi_3$

$\sum_i \pi_i = 1$

$\underline{\pi} = \left[\frac{\alpha}{p+2\alpha}, \frac{\alpha}{p+2\alpha}, \frac{p}{p+2\alpha}\right]$

d. $E_2 = \frac{1}{\pi_2} = \frac{p}{\alpha} + 2$

e. when $p = q$, $\pi_1 = \pi_2 = \pi_3$

7. (a) A. 1
B. -
C. -
D. 1, 2
E. 3
F. -
G.

C.

(b) A. 3
B. -
C. -
D. -
E. -
F.
G. -

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8. let $X_n$ be the # of shoes at the front door.

$X_n \in \{0,1,\dots,k\}$ is a Markov chain


$\frac{1}{2}\pi_0 = \frac{1}{2}\pi_1 \Rightarrow \pi_0 = \pi_1$

$\frac{1}{2}\pi_1 = \frac{1}{2}\pi_2 \Rightarrow \pi_1 = \pi_2$

$\vdots \qquad \vdots$

$\frac{1}{2}\pi_{k-1} = \frac{1}{2}\pi_k \Rightarrow \pi_{k-1}\ \pi_k$

So $\pi_0 = \pi_1 = \dots = \pi_k$

$\pi_0 + \pi_1 + \dots + \pi_k = 1$

$\pi_0 = \pi_1 = \dots = \frac{1}{k+1}$

So proportion of time he runs barefooted $= \frac{1}{k+1}$