CS 112 HW 4

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1.1 $6/11$

1.2 $4, 5 \Rightarrow \frac{2}{6} = 1/3$

1.3 $1/11$

2.1 $P[x \le 5] = \int_0^5 \frac{1}{15} e^{-x/15} dx = -e^{-x/15} \Big|_0^5 = -e^{-5/15} + 1 = -e^{-1/3} + 1 = .2835$

2.2 $P[5\ x > 5] = \int_5^\infty \frac{1}{15} e^{-x/15} dx = -e^{-x/15} \Big|_5^\infty = 0 + e^{-1/3} = (e^{-1/3})^5 = e^{-5/3}$

2.3 $P[1\ x < 5] = -e^{-1/3} + 1$

one lasts less than 5 yrs.

$P[\text{at least 1 lasts} > 5 \text{ yrs}] = 1 - (-e^{-1/3} + 1)^5$

2.4 $P[\text{at least 5 of 10 last more than 5}] = \sum_{i=5}^{10} \binom{10}{i} (e^{-1/3})^i (1 - e^{-1/3})^{10-i}$

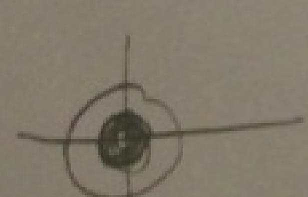
3.1 $10, 11, 12, 13, 14 = 5, \ 5/11$

3.2 $15, 16, 17, 18, 19, 20 = 6 \quad 3/6 = 1/2$

3.3. $1/11 \Rightarrow$ 16 minutes

4.1 area: $P[\text{size} < \frac{1}{4}\pi, r = \frac{1}{2}] = \frac{1}{2}$

4.2



$r < \frac{1}{2}$ $P[\overline{OB} \text{ circle has area } < \frac{1}{4}\pi] = \frac{\frac{1}{4}\pi}{\pi} = \frac{1}{4}$

5.1 $\lambda = 10$ usrs/min $= \frac{1}{6}$ users/sec

$P[\text{no visists in last 10 seconds}] = P[X \ge 10] = e^{-\frac{1}{6}(10)} = e^{-5/3}$

5.2 3rd user w/in 12 seconds ← memoryless property

$= 1 - e^{-\lambda t} = 1 - e^{-\frac{1}{6}(12)} = 1 - e^{-2}$

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6.1 $P[\text{min lifecycle}] = \lambda_k e^{-\lambda_k t}$ where $\lambda_k = \max[\lambda_1, \lambda_2, \dots \lambda_n]$

6.2 $P[j \text{ is failure}] = \dfrac{\prod_{i=1}^{n} (1-\lambda_i e^{\lambda_i t})}{\lambda_j e^{\lambda_i t}} (1-\lambda_j e^{\lambda_j t})$

7.1 $\lambda = 10/6$, $t = \min \Rightarrow * 60 = 600/6 = 100/hr$

$P[X<1] = 1 - e^{-\lambda t} = 1 - e^{-100}$

9.1 $F_{X,Y}(x,y) = \int_0^1 \int_0^1 c(x^2+y^2)\,dy\,dx = 1$

$= c\int_0^1 \int_0^1 (x^2+y^2)\,dy\,dx = c\int_0^1 \left(x^2y + \frac{y^3}{3}\Big|_0^1\right) dx$

$= c\int_0^1 \left[x^2 + \frac{1}{3}\right] dx = c\left[\frac{x^3}{3} + \frac{1}{3}x\right]\Big|_0^1 = c\left(\frac{1}{3}+\frac{1}{3}\right) = \frac{2c}{3} = 1$

$\Rightarrow c = \frac{3}{2}$

9.2 $P[.5<X<2, -1<Y<.5] = \int_{.5}^{1}\int_0^{.5} \frac{3}{2}(x^2+y^2)\,dy\,dx$

$= \int_{.5}^{1} \left(\frac{3}{2}x^2y + \frac{y^3}{2}\right)\Big|_0^{.5} dx = \int_{.5}^{1}\left[\frac{3}{4}x^2 + \frac{1}{16}\right] dx =$

$= \left(\frac{\cancel{3}}{4}\frac{x^3}{\cancel{3}} + \frac{1}{16}x\right|_{.5}^{1} = \frac{1}{4} + \cancel{\frac{1}{16} - \left(\frac{1}{32} + \frac{1}{32}\right)} = \frac{1}{4}$

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8. $f(x,y) = \dots$

$F_X(x) = \int_{-\infty}^{x}\int_{-\infty}^{\infty} u e^{-u(y+1)}\,dy\,du$

$= \int_0^x -e^{-uy}e^{-u}\Big|_0^{\infty} du$

$-0e^{-u} + e^{-u}$

$\int_0^t e^{-u}\,du = -e^{-u}\Big|_0^x$

$F_X(x) = 1 - e^{-x}$

$f(x) = e^{-x}$

$F_Y(y) = \int_{-\infty}^{\infty} x e^{-x(y+1)}\,dx$

$-e^{-xy}e^{-x}\Big|_0^{\infty}$

$(0e^{-x}) + e^{-x}$

e^{-x}

$f_Y(y) = \frac{1}{(y+1)^2}$

$f_{X,Y}(x,y) \neq (e^{-x})\left(\frac{1}{(y+1)^2}\right)$ not independent

$F_X(x) = P[X \leq x] = 1 - P[X > x]$

$= 1 - P[\text{no stars within } x \text{ distance}]$

$= 1 - P[N = 0 \text{ in sphere of radius } x \text{ centered @ origin}]$

$= 1 - e^{-\rho \frac{4}{3}\pi x^3}$

pdf $f_X(x) = 4\pi\rho x^2 e^{-\rho\frac{4}{3}\pi x^3}$ //

$\quad\quad \| \; \frac{\partial F_X(x)}{\partial x}$