UCLA - Henry Samueli School of Engineering
Department of Computer Science
CS112 - Homework Set 2
Due date Wednesday, January 23rd

The homework must be submitted online in electronic form using the PDF format. Verify that the file is readable before submitting, broken files and late submissions will not be graded. Please indicate your complete name and UID in the headings.

A.

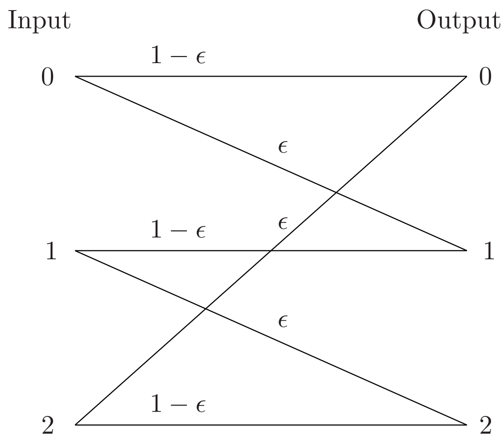
Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection:
(1) at least one of the events A, B, C occurs
(2) at most one of the events A, B, C occurs
(3) none of the events A, B, C occurs
(4) all three events A, B, C occur
(5) exactly one of the events A, B, C occurs
(6) events A and B occur, but not C
(7) either event A occurs or, if not, then B also does not occur

B.

Roll two fair dice independently. define the events:
A = $\{First\,die\,is\,1, 2\,or\,3\}$
B = $\{First\,die\,is\,2, 3\,or\,6\}$
C = $\{Sum\,of\,outcomes\,is\,9\} = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$
Are A, B, C independent? Prove your answer.

C.

A ternary communication channel is shown in the following figure.



Ternary Communication Channel

The input symbols 0, 1, and 2 occur with probabilities 1/2, 1/4, and 1/4, respectively.
(a) Find the probabilities of the output symbols, in terms of ϵ
(b) Suppose the observed output is 0. Find the probabilities that the input was 0, 1, or 2.
(c) Find the probability of error, i.e., the probability that the output symbol is different from the input symbol sent, in terms of ϵ.

D.

Suppose that you are the manager in a manufacturing plant and there are large lots of products that need to be tested for quality. It is perhaps impractical to test each one in the lot, and so we sample. In each lot of 100 items 5 are tested and the lot is rejected if any of the tested items is found defective.
(a) Find the probability of accepting a lot with 5 defective items. Repeat for 10 defective items.
(b) Recompute the probabilities in (a) if a lot is accepted and at most 1 of the tested items is found defective.

E.

Suppose you are a contestant on a game show, and you are given the choice of three doors: Behind one door is gold; behind the others, goats. Your goal is to pick the curtain behind which there is gold. You pick a door, say No. 1, and the host, who knows whats behind the doors, opens another door, say No. 3, which has a goat. He then says to you, Do you want to pick door No. 2? We will use you and contestant interchangeably in this problem.
(a) What is the sample space for this random experiment?
(b) Assume that the placement of the gold behind the curtains is random, that the contestants choice of curtains is random and independent of the gold placement, and that the game-show hosts choice of the curtain with the goat is equally likeley between the two alternatives (if the two unselected curtains contain goats). Specify the probability measure for this random experiment.
(c) Use your answer to compute the probability of winning the gold if the contestant decides to switch.

F

Suppose that you saved your paper on one of your 4 hard drives and they are now corrupted. You give all of them to a technician that can recovery the file with probability p. Given that he searches on disk 1 but cannot find your paper, what is the probability that your thesis is on disk i for i = 1, 2, 3, 4?

A.

(a) $A \cup B \cup C$
(b) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C) \cup (A^c \cap B^c \cap C^c)$
(c) $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$ (De Morgan)
(d) $(A \cap B \cap C)$
(e) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$
(f) $A \cap B^c \cap C^c$
(g) $A \cup (A^c \cap B^c)$

B.

Their intersection is not empty therefore they are not independent.
A = {First die is 1,2, or 3} = {(1,x),(2,x),(3,x)} for x = 1,2,...6
B = {First die is 2,3, or 6} = {(1,x),(2,x),(3,x)} for x = 1,2,...6
C = {Sum of outcomes is 9} = {(3,6),(4,5),(5,4),(6,3)}
Therefore:
$A \cap B$ = {(2,x),(3,x)} for x = 1,2,...6
$A \cap C$ = {(3,6)}
$B \cap C$ = {(3,6),(6,3)}

C.

Part a

$$P(Y=0) = \frac{1}{2}(1-\epsilon) + \frac{1}{4}(0) + \frac{1}{4}(\epsilon) = \frac{1}{2} - \frac{1}{4}\epsilon$$

$$P(Y=1) = \frac{1}{4}(1-\epsilon) + \frac{1}{2}(0) + \frac{1}{4}(\epsilon)) = \frac{1}{4}$$

$$P(Y=2) = \frac{1}{4}(1-\epsilon) + \frac{1}{4}(0) + \frac{1}{2}(\epsilon) = \frac{1}{4} + \frac{1}{4}\epsilon$$

Part b

$$P(X=0|Y=0) = \frac{P(Y=0|X=0)P(X=0)}{P(Y=0)} = \frac{(1-\epsilon)(\frac{1}{2})}{\frac{1}{2}-\frac{1}{4}\epsilon}$$

$$P(X=1|Y=0) = \frac{P(Y=0|X=1)P(X=1)}{P(Y=0)} = \frac{0}{\frac{1}{2} - \frac{1}{4}\epsilon}$$

$$P(X=2|Y=0) = \frac{P(Y=0|X=2)P(X=2)}{P(Y=0)} = \frac{\epsilon\frac{1}{4}}{\frac{1}{2} - \frac{1}{4}\epsilon}$$

Part c

$$P(\epsilon) = P(X=x, Y \neq x) = \sum_X P(Y \neq x|X=x)P(X=x) = \frac{1}{2}\epsilon + \frac{1}{4}\epsilon + \frac{1}{2}\epsilon = \epsilon$$

D.

Part a

Given N the number of elements in the lot, and B the number of broken element in the lot. The probability of accepting a lot with 5 broken pieces is

$$P(A) = \prod_{i=0}^{i<5} \frac{N-B-i}{N}$$

$$P(A) = 0.695286 \;\; for \;\; B = 5, N = 100$$

$$P(A) = 0.125571 \;\; for \;\; B = 10, N = 100$$

Part b

The number of broken pieces can be calculated as follows

$$P(K=k) = f(k, N, m, n) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$$

Where N is total number of pieces in the lot, n is the dimension of the sample, and m is the number of broken pieces in the lot.
For a sample of 5 pieces:

$$P(K <= 1) = P(K=0) + P(K=1) = \frac{\binom{5}{0}\binom{100-5}{5-0}}{\binom{100}{5}} + \frac{\binom{5}{1}\binom{100-5}{5-1}}{\binom{100}{5}}$$

For a sample of 10 pieces:

$$P(K <= 1) = P(K = 0) + P(K = 1) = \frac{\binom{5}{0}\binom{100-5}{10-0}}{\binom{100}{10}} + \frac{\binom{5}{1}\binom{100-5}{10-1}}{\binom{100}{10}}$$

E.

Part a

There are three doors and three possible choices, respectively door 1, 2 or 3. The abstract space Ω of the possible outcomes is therefore any possible triplet [x,y,z] with $x, y, z \in \{1, 2, 3\}$.

Part b

Given [x,y,z] we consider:
x = {The winning door}
y = {The door chosen by the contestant}
z = {The door proposed by the host}

$$P(\{[x, y, z]\}) = \frac{1}{3} * \frac{1}{3} * \frac{1}{2} = \frac{1}{18} \ \ with \ \ x, y, z \in \{1, 2, 3\} \ \ y = x \ \ and \ \ z \neq x, y$$

$$P(\{[x, y, z]\}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9} \ \ with \ \ x, y, z \in \{1, 2, 3\} \ \ y \neq x \ \ and \ \ z \neq x, y$$

$$P(\{[x, y, z]\}) = 0 \ \ with \ \ x, y, z \in \{1, 2, 3\} \ \ and \ \ z = x, y$$

Part c

Pr{Winning changing the game}= $P([1, 2, 3] \cup [2, 1, 3] \cup [3, 2, 1 \cup [2, 3, 1] \cup [1, 3, 2] \cup [3, 1, 2]) = 6 * \frac{1}{9} = 66.6$

F.

Let A be the event that the technician picks the first disk and finds nothing, and let B_i be the event that your paper is on disk i. Note that B_1, B_2, B_3, and B_4 partition the sample space so, applying Bayes' rule, we have:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_i P(A|B_i)P(B_i)}$$

$$P(B_i|A) = \frac{P(A|B_i)\frac{1}{4}}{(1-p)+1+1+1}$$

So we can write:

$$P(B_i|A) = \begin{cases} \frac{1-p}{4-p} & i = 1 \\ \frac{1}{4-p} & i = 2, 3, 4 \end{cases}$$