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CS 112 HW 1

A. Laplace:

1) $f(x) = e^{ax}$

$F(s) = \int_0^\infty e^{ax} e^{-sx} dx = \int_0^\infty e^{-x(s-a)} dx = -\frac{1}{s-a} e^{-x(s-a)} \Big|_0^\infty = 0 - \left(-\frac{1}{s-a}\right) = \frac{1}{s-a}$

2) $f(x) = x^2$

$F(s) = \int_0^\infty x^2 e^{-sx} dx = \frac{x^2}{s} e^{-sx} - \frac{2x}{s^2} e^{-sx} - \frac{2}{s^3} e^{-sx} \Big|_0^\infty = 0 - 0 - 0 - 0 + 0 + \frac{2}{s^3} = \frac{2}{s^3}$

x^2	$+$	e^{-sx}
$2x$	$-$	$-\frac{1}{s}e^{-sx}$
2	$+$	$\frac{1}{s^2}e^{-sx}$
0		$-\frac{1}{s^3}e^{-sx}$

3. $f(x) = 4x^2 - 3x + 7$

$F(s) = \int_0^\infty (4x^2 - 3x + 7) e^{-sx} dx = \frac{-4x^2}{s} e^{-sx} + \frac{8x}{s^2} e^{-sx} - \frac{8}{s^3} e^{-sx} + \frac{3x}{s} e^{-sx} + \frac{3}{s^2} e^{-sx} - \frac{8}{s^3} e^{-sx}$

$4x^2$	$+$	e^{-sx}
$8x$	$-$	$-\frac{1}{s}e^{-sx}$
8	$-$	$\frac{1}{s^2}e^{-sx}$
0	$+$	$-\frac{1}{s^3}e^{-sx}$

$-3x$	$+$	e^{-sx}
-3	$-$	$-\frac{1}{s}e^{-sx}$
0	$+$	$1/s^2 e^{-sx}$

$= 0 - \left(-\frac{8}{s^3} + \frac{3}{s^2} - \frac{7}{s}\right)$

$= \frac{8}{s^3} - \frac{3}{s^2} + \frac{7}{s}$

4. $f(x) = (x-1)^2$

$F(s) = \int_0^\infty (x-1)^2 e^{-sx} dx = -\frac{(x-1)^2}{s} e^{-sx} - \frac{2(x-1)}{s^2} e^{-sx} - \frac{2}{s^3} e^{-sx} - 0 - 0 - 0$

$= -\left(-\frac{1}{s} + 2\frac{1}{s^2} - \frac{2}{s^3}\right) = \frac{1}{s} - \frac{2}{s^2} + \frac{2}{s^3}$

$(x-1)^2$	e^{-sx}
$-2(x-1)$	$-\frac{1}{s}e^{-sx}$
2	$+\frac{1}{s^2}e^{-sx}$
	$-\frac{1}{s^3}e^{-sx}$

B. $\sum_{i=1}^{n} n^3 = n^4$

C. $F^*(s) = \dfrac{s+3}{(s-2)(s+1)} = \dfrac{A}{s-2} + \dfrac{B}{s+1} \Rightarrow s+3 = A(s+1) + B(s-2)$

$s = -1 \Rightarrow 2 = B(-3),\ B = -\frac{2}{3}$

$s = 2 \Rightarrow 5 = A(3),\ B = \frac{5}{3}$

$\Rightarrow \dfrac{-2}{3(s-2)} + \dfrac{5}{3(s+1)}$

$F^*(s) = \dfrac{1}{(s^2+1)(s^2+2s+1)} = \dfrac{As+B}{(s^2+1)} + \dfrac{C}{(s+1)} + \dfrac{D}{(s+1)^2} \Rightarrow 1 = (As+B)(s+1)^2 + C(s^2+1)(s+1) + D(s^2+1)$

$1 = A(s^3+2s^2+s) + B(s^2+2s+1)$
$+ C(s^3+s^2+s+1) + D(s^2+1)$

$s = -1 \Rightarrow 1 = 0 + 0 + D(2) \Rightarrow D = \frac{1}{2}$

$0 = A + C \qquad A = -C$

$0 = 2A + B + C + D \qquad 0 = 2B \Rightarrow B = 0$

$0 = A + 2B + C \qquad C = 1/2$

$1 = B + C + D \qquad A = -1/2$

$= \dfrac{-s}{2(s^2+1)} + \dfrac{1}{2(s+1)} + \dfrac{1}{2(s+1)^2}$

$b = 1/2 \quad a = 1 \quad c = -1/2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

D. $F^{-1}\left\{\dfrac{s+3}{(s-2)(s+1)}\right\} = -\frac{2}{3}e^{2t} - \frac{2}{3}e^{-t}$

$F^{-1}\left\{\dfrac{1}{(s^2+1)(s^2+2s+1)}\right\} = -\frac{1}{2}\sin t + \frac{1}{2}xe^{-t}$

c_2

E. $x(t) = \dfrac{dx(t)}{dt} + 2\dfrac{d^2x(t)}{dt^2}$

$1 = \lambda + 2\lambda^2$

$\lambda^2 + \frac{1}{2}\lambda - \frac{1}{2}$

$x(t) = c_1 e^{\frac{1}{2}t} + c_2 e^{t}$

$x(0) = c_1 + c_2 = 1$

$x'(0) = \frac{1}{2}c_1 + c_2 = \frac{1}{2}$

$\rightarrow x(t) = e^{\frac{1}{2}t}$

$c_2 = 1 - c_1$

$\frac{1}{2}c_1 + 1 - c_1 = \frac{1}{2}$

$-\frac{1}{2}c_1 = +\frac{1}{2} \quad c_1 = -1$