CS 112 HW 3

Jonathan Nguy
603799761

1. 1.1 $P(X=1) = \binom{2}{1} \cdot \frac{1}{4} \cdot \frac{3}{4}$

$P(X=2) = \frac{1}{4^2} = \frac{1}{16}$

$P(X=3) = 0$

1.2 $P(X=1) = \frac{3}{4}$

$P(X=2) = 1/4$

1.3 $P(X=1 | \text{one is S}) = \dfrac{\frac{6}{16}}{\frac{6}{16} + \frac{1}{16}} = \frac{6}{7}$

$P(X=2 | \text{one is S}) = \dfrac{1/16}{7/16} = \frac{1}{7}$

2. 2.1 $P(X=1) = (\frac{1}{2})$

$P(X=5) = (\frac{1}{2})^4 (\frac{1}{2})$

$P(X=10) = (\frac{1}{2})^9 (\frac{1}{2})$

2.2 $\frac{1}{2}$ → only occurs when boy is the first baby

3. 3.1 $f = P(X) = \dfrac{\binom{18}{X}\binom{28}{12-X}}{\binom{46}{12}}$

3.2. $P(X=12) = \dfrac{\binom{18}{12}\binom{28}{0}}{\binom{46}{12}} = \dfrac{\binom{18}{12}}{\binom{46}{12}}$

3.3 $P(X=6) = \dfrac{\binom{18}{6}\binom{28}{6}}{\binom{46}{12}}$

4. 4.1. $\frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10} = .3$

4.2. 0 → only 1 ball is new now

5. 5.1 $P(X = \text{at least } 1) = 1 - \left(\frac{36}{40}\right)\left(\frac{35}{39}\right)\left(\frac{34}{38}\right)\left(\frac{33}{37}\right)$

5.2 $P(X = \text{at least } 2)\ 1 - \left[\left(\frac{36}{40}\right)\left(\frac{35}{39}\right)\left(\frac{34}{38}\right)\left(\frac{33}{37}\right) + \binom{4}{1}\left(\frac{4}{40}\right)\left(\frac{36}{39}\right)\left(\frac{35}{39}\right)\left(\frac{34}{39}\right)\right]$

6 6.1. $\frac{1}{n!} = P(\text{perfect})$

6.2. $P(m \text{ persons}) = \frac{1}{n!} \cdot (n-m)! = \frac{(n-m)!}{n!}$

6.3 $P(m \text{ picks hat from last } m) = \frac{m!}{n!} \cdot (n-m)!$

7. $P(\text{correct}) = P(\overset{\text{guess}}{\text{input}} \text{ flipped \& output flipped})$
$+ P(\overset{\text{guess}}{\text{input}} \text{ not flipped \& output not flipped})$
$= (\varepsilon^2 + (1-\varepsilon)^2) \leftarrow$ per bit
$= (\varepsilon^2 + (1-\varepsilon)^2)^n$, where n is # of bits

8. Assuming each person is equally likely to win (½ chance)
Using Bernoulli trials, the $P(k \text{ games})$ is $\left(\frac{1}{2}\right)^{k-1}\left(\frac{1}{2}\right)$

no streak last streak

this $= \left(\frac{1}{2}\right)^k = \frac{1}{2^k}$

9. $= P(1 - P(\text{all success})) = 1 - p^n$

10. $\lambda = 100/\text{hr}$
$t = 1\text{hr}$
$k = 1 \leftarrow$ threshold

$$P = \frac{e^{-100}\ 100}{1} = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$