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1. This isn't a birth-death process because we need more info than just how many work. We also need to know which machine is working.

We can analyze it as follows:

States:

B: both working

1: 1 works, 2 down & repaired

2: 2 works, 1 down & repaired

0_1: both down, 1 repairing

0_2: both down, 2 repairing

$V_B = \mu_1 + \mu_2$, $V_1 = \mu_1 + \mu$, $V_2 = \mu_2 + \mu$, $V_{0_1} = V_{0_2} = \mu$

$P_{B,1} = \frac{\mu_2}{\mu_1 + \mu_2} = 1 - P_{B,2}$, $P_{1,B} = \frac{\mu}{\mu_1 + \mu} = 1 - P_{1,0_2}$, $P_{2,B} = \frac{\mu}{\mu + \mu_2} = 1 - P_{2,0_1}$

$P_{0_1,1} = P_{0_2,2} = 1$

2. $\lambda_k = \begin{cases} \lambda, & k \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$\mu_k = \begin{cases} k\mu, & k \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$$\frac{\partial P_k(t)}{\partial t} = -(\lambda + k\mu)P_k(t) + \mu(k+1)P_{k+1}(t) + \lambda P_{k-1}(t)$$

3. $\lambda_k = \begin{cases} (K-k)\lambda, & k \leq K \\ 0 & \text{otherwise} \end{cases}$ $\mu_k = \begin{cases} k\mu, & k \leq K \\ 0 & \text{otherwise} \end{cases}$

For $k = 0$,

$$\frac{\partial P_0(t)}{\partial t} = \mu P_1(t) - K\lambda P_0(t)$$

For $0 < k < K$

$$\frac{\partial P_k(t)}{\partial t} = \lambda(K-k+1)P_{k-1}(t) + (k+1)\mu P_{k+1}(t) - [(K-k)\lambda + k\mu]P_k(t)$$

For $k = K$

$$\frac{\partial P_K(t)}{\partial t} = \lambda P_{K-1}(t) - K\mu P_K(t)$$

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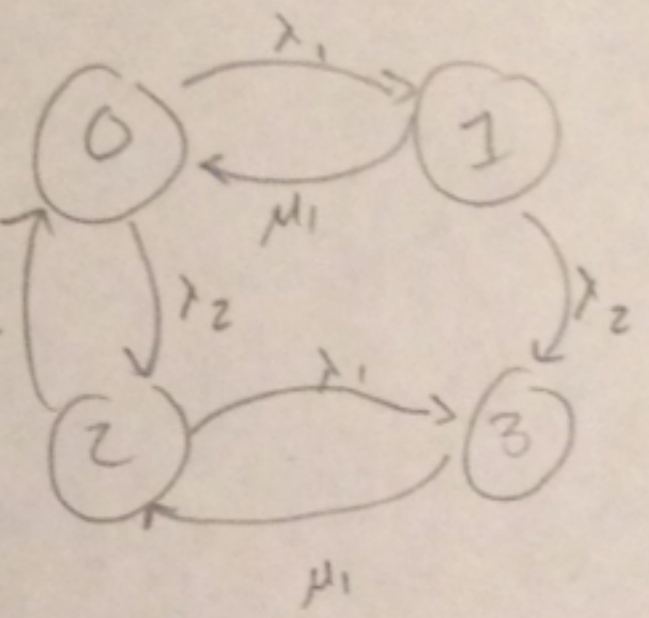
5. this has 4 states:

0: no machines down
1: machine 1 down & 2 up
2: machine 1 up & 2 down
3: both down.



$$\mu_1 P_1 + \mu_2 P_2 = (\lambda_1 + \lambda_2) P_0$$
$$\lambda_1 P_0 = (\mu_1 + \lambda_2) P_1$$
$$\lambda_2 P_0 + \mu_1 P_3 = (\mu_2 + \lambda_1) P_2$$
$$\lambda_1 P_2 + \lambda_2 P_1 = \mu_1 P_3$$

$$\Rightarrow AP = \begin{bmatrix} \lambda_1+\lambda_2 & -\mu_1 & -\mu_2 & 0 \\ -\lambda_1 & \mu_1+\lambda_2 & 0 & 0 \\ -\lambda_2 & 0 & \mu_2+\lambda_1 & -\mu_1 \\ 0 & -\lambda_2 & -\lambda_1 & \mu_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = 0$$

$\sum_{i=0}^{3} P_i = 1$, therefore the proportion of time #2 is down $= P_2 + P_3$

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4 arrival $= \lambda = 3/hr$

Service rate $= \mu = 4/hr$

$k = 2$, $\rho = \frac{\lambda}{\mu} = \frac{3}{4} = .75$

a) Average # of customers =

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{3}{4-\lambda} = 3$$

b) Proportion of potential customers that get a haircut

potential customers that enter shop

$$(L/L>0) = \frac{\mu}{\mu-\lambda} = \frac{4}{4-3} = 4$$

Since $k=2$,

potential customers $= \frac{2}{4} = \frac{1}{2}$

c) $\mu = 8/hr$

$\Rightarrow \rho = \frac{3}{8} = .375$

potential customers that enter shop:

$$(L/L>0) = \frac{\mu}{\mu-\lambda} = \frac{8}{8-3} = \frac{8}{5} = 1.6,$$

since there's room for 2, he will be able to serve everyone who enters.

d) since two barbers working at x speed is equivalent to one barber working at 2x speed, the answer is the same as c), everyone who enters is served