CS 112 HW 8 pg 1

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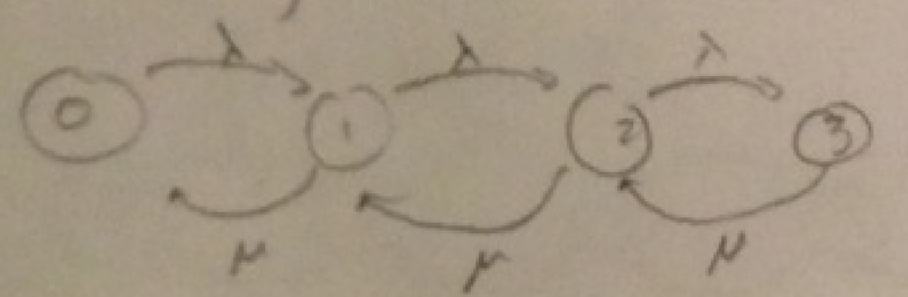
1. Following structure:



... M/M/L queue

λ = taxi arrival rate 1/min
μ = customer arrival 2/min

a) $p = \lambda/\mu = \frac{1}{2} < 1 \Rightarrow$ CTMC
$\pi_i = (1-p)p^i,\ i = 0,1,2$ where π_i defines limiting distribution
Average # of taxis waiting $= \sum_{i=0}^{\infty} i\pi_i = \frac{p}{1-p} = \frac{1/2}{1-1/2} = \boxed{1}$

b. An arriving customer will get a taxi as long as CTMC is in state $\lambda_i \geq 1$
Probability for those states is $p = 0.5$

2. a) $(1+4)P_0 = 2P_1$
$(1+2+4)P_1 = (1+4)P_0 + 2P_2$
$(1+2+4)P_2 = 4P_0 + (4+4)P_1 + 2P_3$
$(1+2+4)P_i = (1+4)P_i + 2P_{i+1} + \sum_{i=0}^{6-2} 4P_i$

b) $p = 2/5\, P_1$

5. $T = \overline{N}/\lambda$
$p = \frac{\lambda}{\mu} = 1 - P_0 = 1 - e^{-(\alpha/\mu)}$
$\lambda = \mu(1 - e^{-\alpha/\mu})$
$T = \overline{N}/\lambda = \alpha/\mu^2 (1 - e^{-\alpha/\mu})^{-1}$
$\lambda = \sum_{k=0} \lambda_k P_k$
$= \sum_{k=0}^{\infty} \frac{\alpha}{k+1} \frac{e^{-\alpha/\mu}(\alpha/\mu)^k}{k!} = \frac{\alpha e^{-\frac{\alpha}{\mu}}}{\alpha/\mu} \sum_{k=0}^{\infty} \frac{(\frac{\alpha}{\mu})^{k+1}}{(k+1)!} = \mu e^{-\alpha/\mu}(e^{\alpha/\mu} - 1) = \mu(1 - e^{-\alpha/\mu})$

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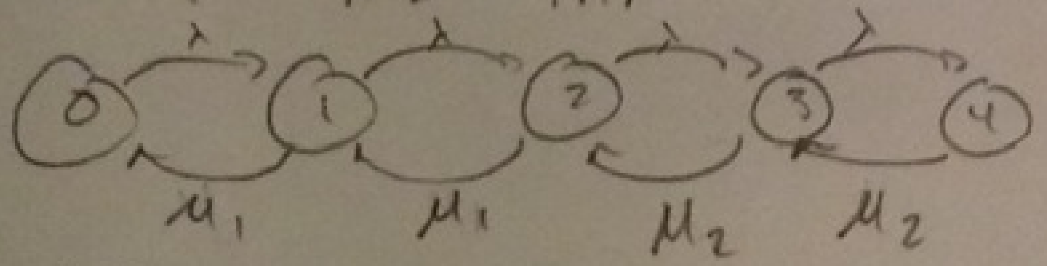
3. $\lambda = 40/hr$

$\mu_1 = 30/hr \quad \mu_2 = 60/hr$



$\lambda \pi_0 = \pi_1 \mu_1$

$\lambda \pi_0 + \pi_2 \mu_1 = \pi_1(\mu_1 + \lambda)$ $\qquad \pi_1 = .2186$

$\lambda \pi_1 + \pi_2 \mu_2 = \pi_2(\mu_1 + \lambda) \Rightarrow \quad \pi_2 = .292$

$\lambda \pi_2 + \pi_4 \mu_2 = \pi_3(\mu_2 + \lambda)$ $\qquad \pi_3 = .194$

$\lambda \pi_3 = \pi_4 \mu_2$ $\qquad \pi_4 = .1296$

a) Proportion both attendants free $\pi_0 = .164$

b) S_1 salary of major operater

S_2 salary of assistant

$S_1 + S_2 = 100 \qquad S_1/S_2 = (1-\pi_0)(\pi_3 + \pi_4) \Rightarrow S_1 = (2.58) S_2$

$S_2 = 27.90$

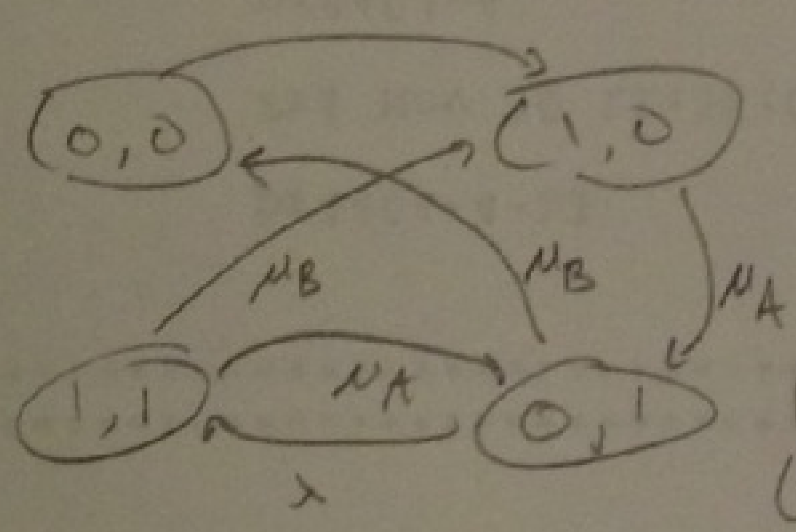
$S_1 = 72.1$

c) $W = \dfrac{1}{(\lambda \times (\pi_{(0,0)} + \pi_{(0,1)}))} = \dfrac{1}{40 \times (.164 + .2186)} = .0653$ hours

4. $\lambda = 2/hr$ $\qquad$ State def = (0/1 if server is idle/busy)

$\mu_A = 4/hr$ $\qquad$ state space = $\{(0,0),(0,1),(1,0),(1,1)\}$

$\mu_B = 4/hr$



$\pi_{(0,1)} \mu_A = \pi_{(0,0)} \lambda \Rightarrow 2\pi_{(0,1)} = \pi_{(0,0)}$

$\pi_{(0,1)} \mu_B + \pi_{(0,0)} \lambda = \pi_{(1,0)} \mu_A$

$\Rightarrow 2\pi_{(0,1)} + \pi_{(0,0)} = 2\pi(1,0)$

$(\pi_{(1,0)} + \pi_{(1,1)}) \mu_A = \pi_{(0,1)}(\mu_B + \lambda) \Rightarrow 2(\pi_{(1,0)} + \pi_{(1,1)}) = 3\pi_{(0,1)}$

$(\pi_{(0,1)} \lambda) = \pi_{(1,1)}(\mu_A + \mu_B) \Rightarrow 2\pi_{(0,1)} = 8\pi_{(1,1)} \Rightarrow \pi_{(0,1)} = 4\pi_{(1,1)}$

$\pi_{(0,1)} = .22 \qquad \pi_{(1,1)} = .055 \qquad \pi(1,0) = .275$

a.) $\pi_{(0,0)} + \pi_{(0,1)} = .66$

b) $\pi_{(0,1)} + \pi_{(1,1)} = .275$

c) $L = 0 \times \pi_{(0,0)} + 1 \times (\pi_{(1,0)} + \pi_{(0,1)}) + 2 \times \pi_{(1,1)} = .605$

d) $W = \dfrac{1}{\lambda \cdot (\pi_{(0,0)} + \pi_{(0,1)})} = .4583$