CS112 - Homework #6

Due Date: Monday, 02/25/2013

1. An organization has N employees where N is a large number. Each employee has one of three possible job classifications and changes classifications (independently) according to a Markov Chain with transition probabilities

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

   What percentage of employees are in each classification?

2. On any given day Gary is either cheerful (C), so-so (S) or glum (G). If he is cheerful today, then he will be C, S or G tomorrow with probabilities 0.5, 0.4 and 0.1, respectively. If he is feeling so-so today, then he will be C, S or G tomorrow with probabilities 0.3, 0.4 and 0.3. If he is glum today, then he will be C, S or G tomorrow with probabilities 0.2, 0.3, 0.5.

   (a) Find the transition probability matrix $\mathbf{P}$.

   (b) In the long run, what proportion of time is Gary in each of the three 'moods'?

3. Suppose that whether or not it rains today depends on previous weather conditions through the last two days. That is, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.

(a) Define appropriate states in order to make the above model a Markov Chain.

(b) Find the transition probability matrix **P** for the states defined in part (a).

4. Suppose that coin 1 has probability 0.7 of coming up heads, and coin 2 has probability 0.6 of coming up heads. If the coin flipped today comes up heads, then we select coin 1 to flip tomorrow, and if it comes up tails, then we select coin 2 to flip tomorrow. If the coin initially flipped is equally likely to be coin 1 or coin 2, then what is the probability that the coin flipped on the third day after the initial flip is coin 1?

5. Three out of every four trucks on the road are followed by a car, while one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

6. Consider the homogenous Markov Chain whose state diagram is given in figure 1

   (1) Find P, the probability transition matrix.

   (2) Under what conditions if any will the chain be irreducible and aperiodic.

   (3) Solve for the equilibrium probability vector π.

   (4) What is the mean recurrence time for state E_2.

   (5) For what values of α and p will we have $\pi_1 = \pi_2 = \pi_3$? (Give a physical interpretation of this case.

7. For the discrete time Markov Chains in Figure 2:

   (a) Which states are transient?

   (b) Which states are absorbing?

   (c) Do the stationary state probabilities exist? in the case of F and G, does the answer depend on the exact values of the parameters p and q?

8. Each morning an individual leaves his house and goes for a run. He is equally likely to leave either from his front or back door. Upon leaving the house, he chooses a pair of running shoes (or goes running barefoot if there are no shoes at the door from which he departed). On his return, he is equally likely to enter, and leave his running shoes, either by the front or back door. If he owns a total of k pairs of running shoes, what proportion of the time does he run barefooted?

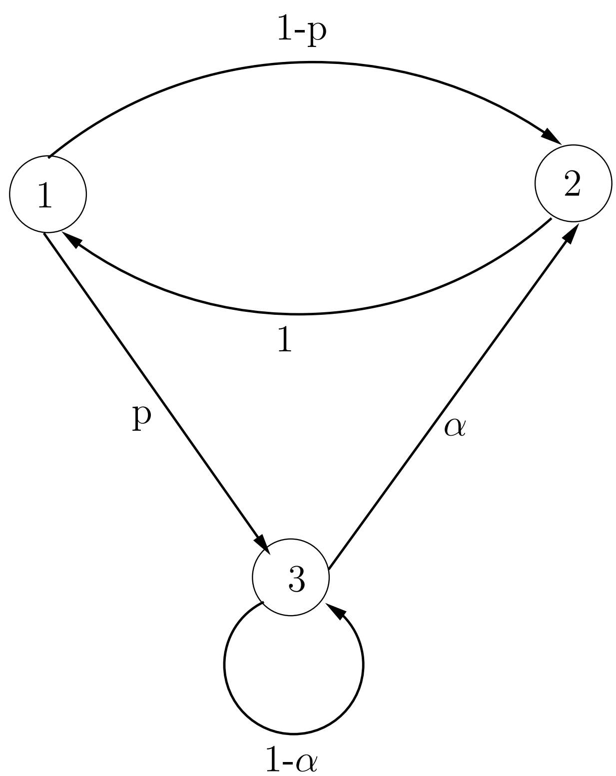

Figure 1: Markov Chain for Problem 6

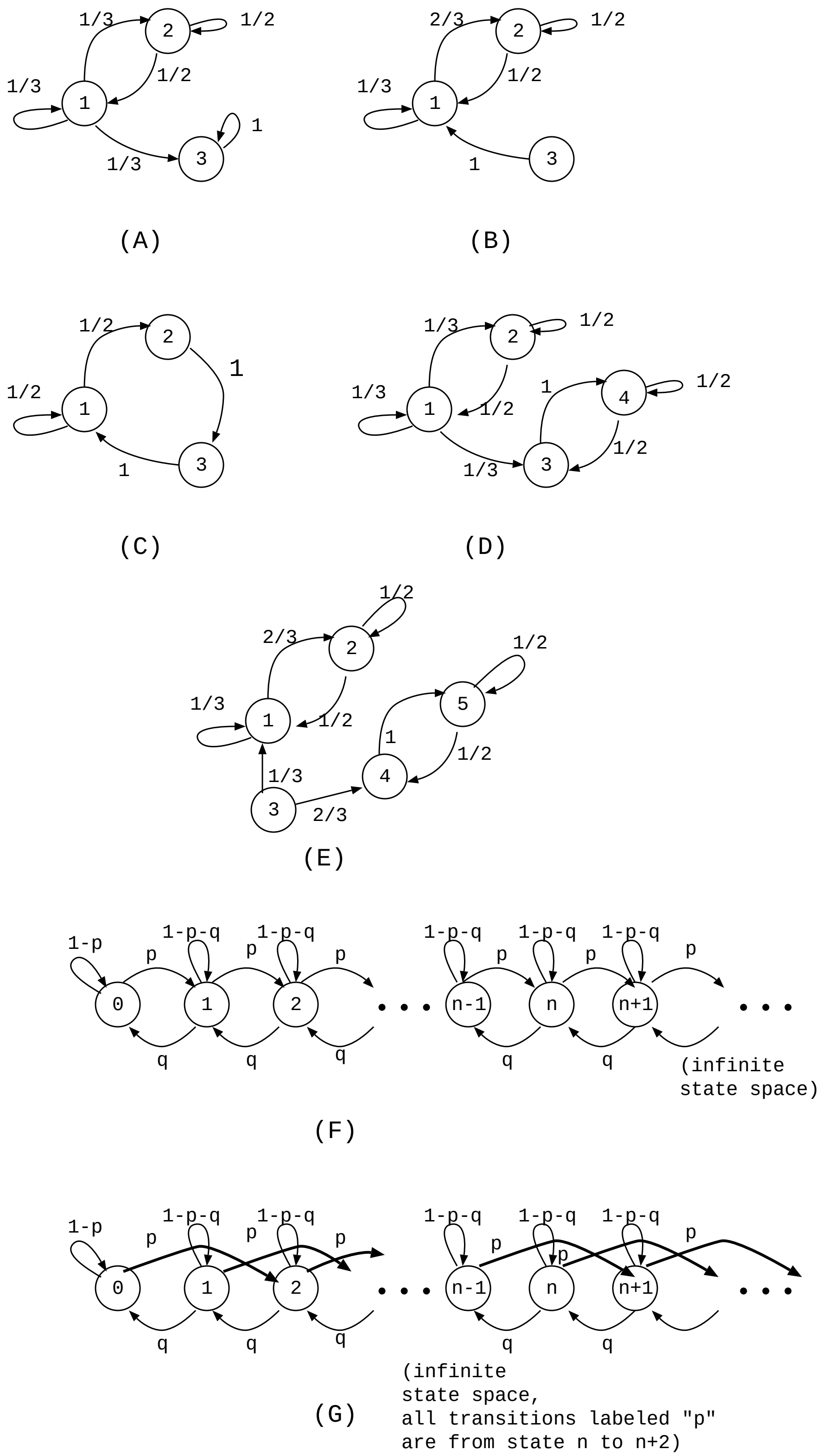

Figure 2: Discrete Time Markov Chains for Problem 7.