UCLA - Henry Samueli School of Engineering
Department of Computer Science
CS112 - Homework Set 5
Due date Wednesday, January 30th

The homework must be submitted online in electronic form using the PDF format. Verify that the file is readable before submitting, broken files and late submissions will not be graded. Comment your solutions step by step. Please indicate your complete name and UID in the headings.

1

A given program has an execution time that is uniformly distributed between 10 and 20 seconds. The number of interrupts that occur during execution is a Poisson random variable with parameter λt where t is the program execution time. The probability distribution of the number of interrupts is therefore $P(X = k) = \frac{(\lambda t)ke^{-\lambda t}}{k!}$.

1.1

What is $E[N|T = t]$, where N is the number of interrupts the program experiences, and T is the running time of the program.

1.2

Find the expected number of interrupts the program experiences during a randomly selected execution.

2

A datagram subnet allows routers to drop packets whenever they need to. The probability of a router discarding a packet is p. Consider the following network:

$$Source - -SourceRouter - -DestinationRouter - -Destination$$

If either of the routers discards a packet, the source host eventually times out and tries again. If both host-router and router-router lines are counted as hops, determine the mean number of hops a packet makes per transmission.

3

In a particular computer storage system, information is retrieved by entering the system at a random point and systematically checking through all the items in the system at a uniform speed. Suppose it takes a seconds to start the search, and it takes b seconds to run through theentire system. Consider the random variable T that denotes the required time to reach any given item of information.

3.1

Find the pmf of T.

3.2

Show that the mean time to reach a given item is given by $a + b/2$

3.3

What is the variance of T

4

The cost of producing M special transistors is of the form $C = a + bM + CM^2$ where a,b and c are positive constants. M is distributed as a Poisson, find the expected value of C.

5

A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom.

5.1

Assuming that the prisoner will always select doors 1, 2, and 3 with probabilities 0.5, 0.3, 0.2, what is the expected number of days until he reaches freedom?

5.2

Assuming that the prisoner is always equally likely to choose among those doors that he has not used, (e.g. if the prisoner initially tries door 1, then when he returns to the cell, he will now select only from doors 2 and 3), what is the expected number of days until he reaches freedom?

1

1.1

$E[N|T=t]=\lambda t$, since for fixed running time, the number of interrupts is a Poisson random variable with mean λt

1.2

$$E[N]=\int_{10}^{20} E[N|T=t]f_T(t)dt$$

$$E[N]=\int_{10}^{20} \lambda t\frac{1}{10}dt$$

$$=\frac{1}{20}|_{10}^{20}=15\lambda$$

2

The pmf is equal to:

$$p_H(H)=\begin{cases} p & \text{for} \quad h=1 \\ p(1-p) & \text{for} \quad h=2 \\ (1-p)^2 & \text{for} \quad h=3 \end{cases}$$

Therefore

$$E[H]=p+2p(1-p)+3(1-p)^2=p^2-3p+3$$

3

The required time will have a uniform distribution from time a to time $a+b$

3.1

$$p_T(t)=\begin{cases} \frac{1}{b+1} & \text{for} \quad t=1 \\ 0 & \quad otherwise \end{cases}$$

3.2

$$E[T] = \sum_{t=a}^{a+b} tP[T = t] = \frac{a + (a+1) + ... + (a+b)}{b+1} =$$

$$\frac{[(a + a + ...a) + (1 + 2 + ... + b)]}{b+1} = \frac{(b+1)a + \frac{b(b+1)}{2}}{b+1}$$

$$= a + \frac{b}{2}$$

4

4.1

From the linearity property of the average we can write:

$$E[C] = E[a + bM + cM^2] =$$

$$= E[a] + bE[M] + cE[M^2]$$

a is a constant whereas $E[M] + cE[M^2]$ are the first and second moment of the Poisson Distribution. In order to calculate them we use the z-transform $G_M(z) = e^{-n(1-z)}$. then we know

$$E[M] = \frac{dG_M(z)}{dz}|_1 = n$$

$$E[M^2] - E[M] = \frac{d^2G_M(z)}{dz^2}|_1 = n^2$$

so

$$E[M^2] = n^2 + n$$

5

Let X denote the number of door chosen, and let N be the total number of days spent in jail.

5.1

Conditioning on X we get,

$$\sum_{i=1}^{3} E[N|X=i]P[X=i]$$

The process restarts each time the prisoner returns to his cell. Therefore,

$$E[N|X=1] = 2 + E[N]$$

$$E[N|X=2] = 3 + E[N]$$

$$E[N|X=3] = 0$$

Thus
$E[N] = 0.5(2 + E[N]) + 0.3(3 + E[N]) + 0.2 \cdot 0 = 9.5$ days

5.2

Let N_i denote the number of additional days the prisoner spends after having initially chosen door i.

$$E[N] = \frac{1}{3}(2 + E[N_1]) + \frac{1}{3}(3 + E[N]) + \frac{1}{3} \cdot 0$$

and since

$$E[N_1] = \frac{1}{2}(3) + \frac{1}{2}(0) = \frac{3}{2}$$

$$E[N_2] = \frac{1}{2}(2) + \frac{1}{2}(0) = 1$$

$$E[N] = \frac{5}{2}$$