"Balance velocities" (Bamber et al. 2000)

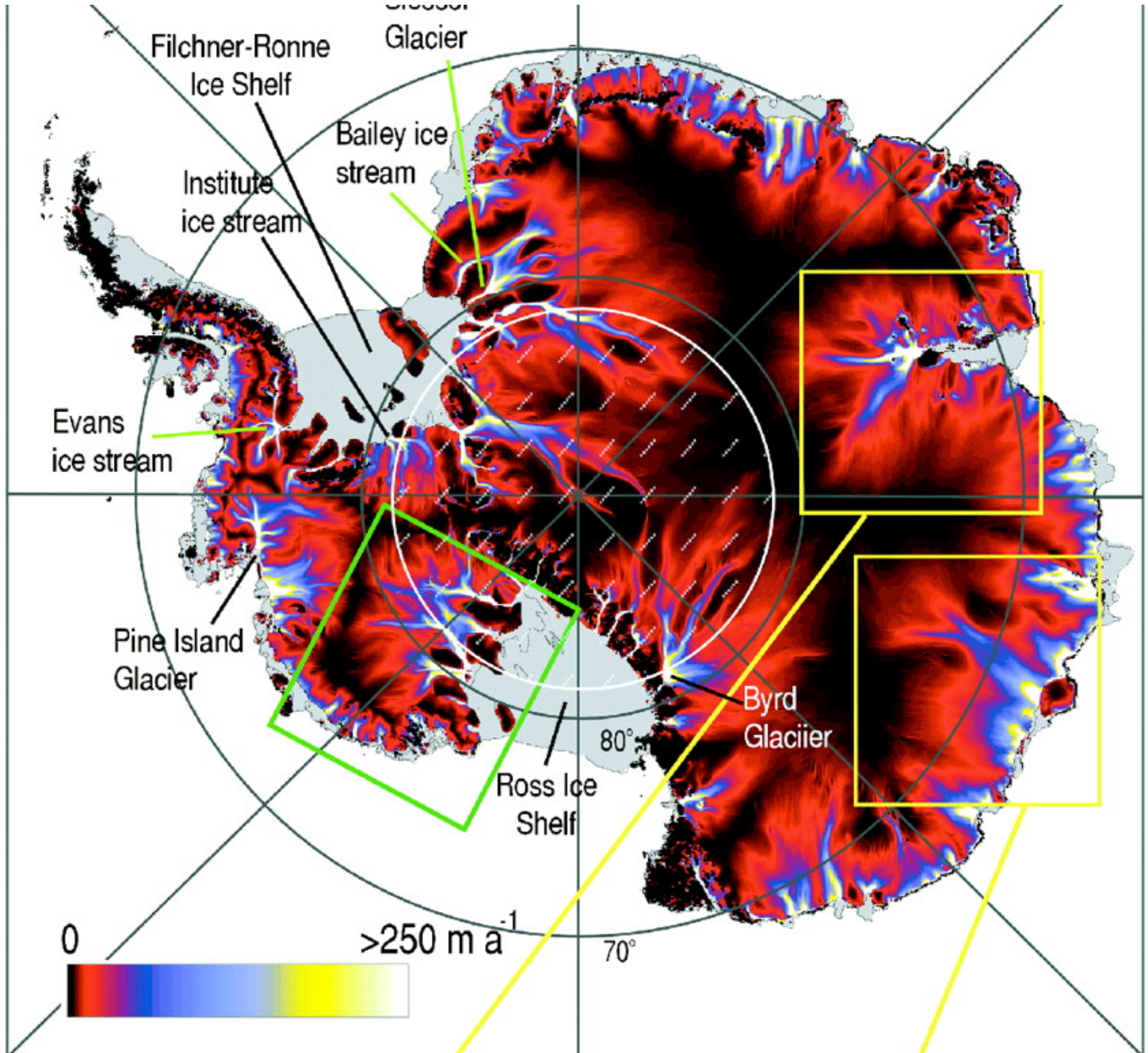
Mass conservation

$$\dot{h} = \dot{b} - \nabla \cdot \overrightarrow{q}$$

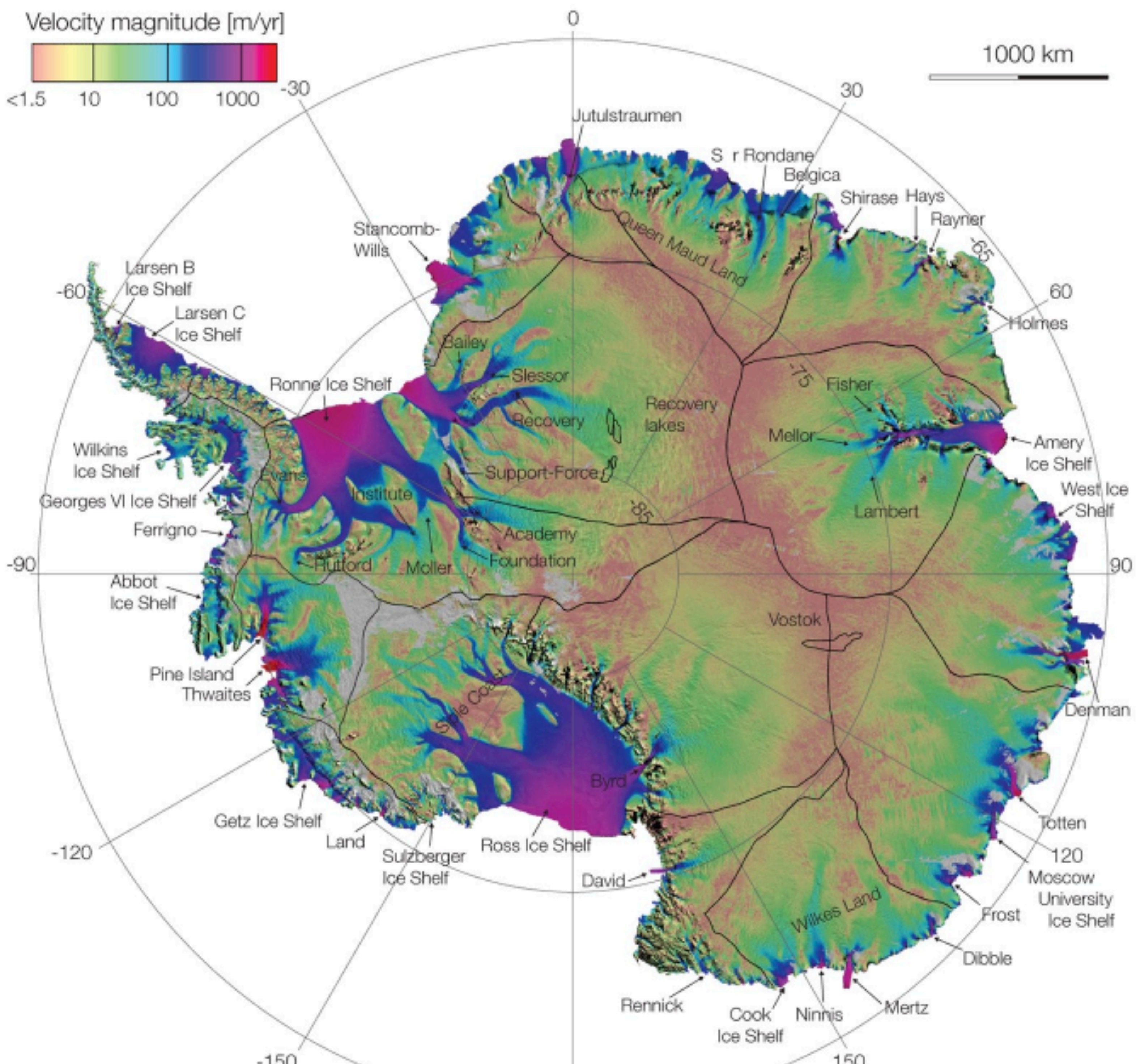
$$0 = \dot{b} - \nabla \cdot \overrightarrow{q}$$

$$q(x) = \int_0^x \dot{b}(x) dx'$$

$$u(x) = \frac{1}{h(x)} \int_0^x \dot{b}(x) dx'$$



Ice velocity measurements without assuming balanced flow

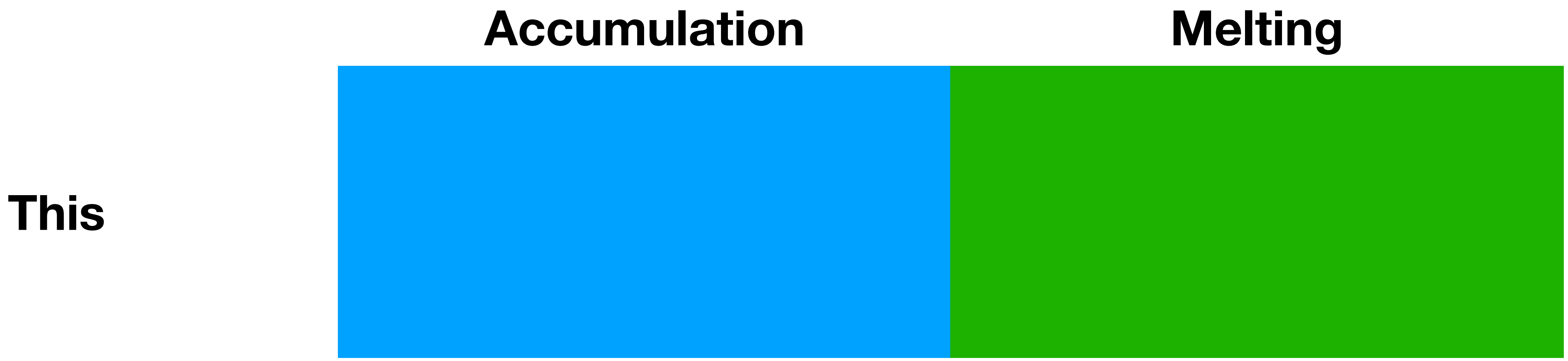


Rignot et al. 2011

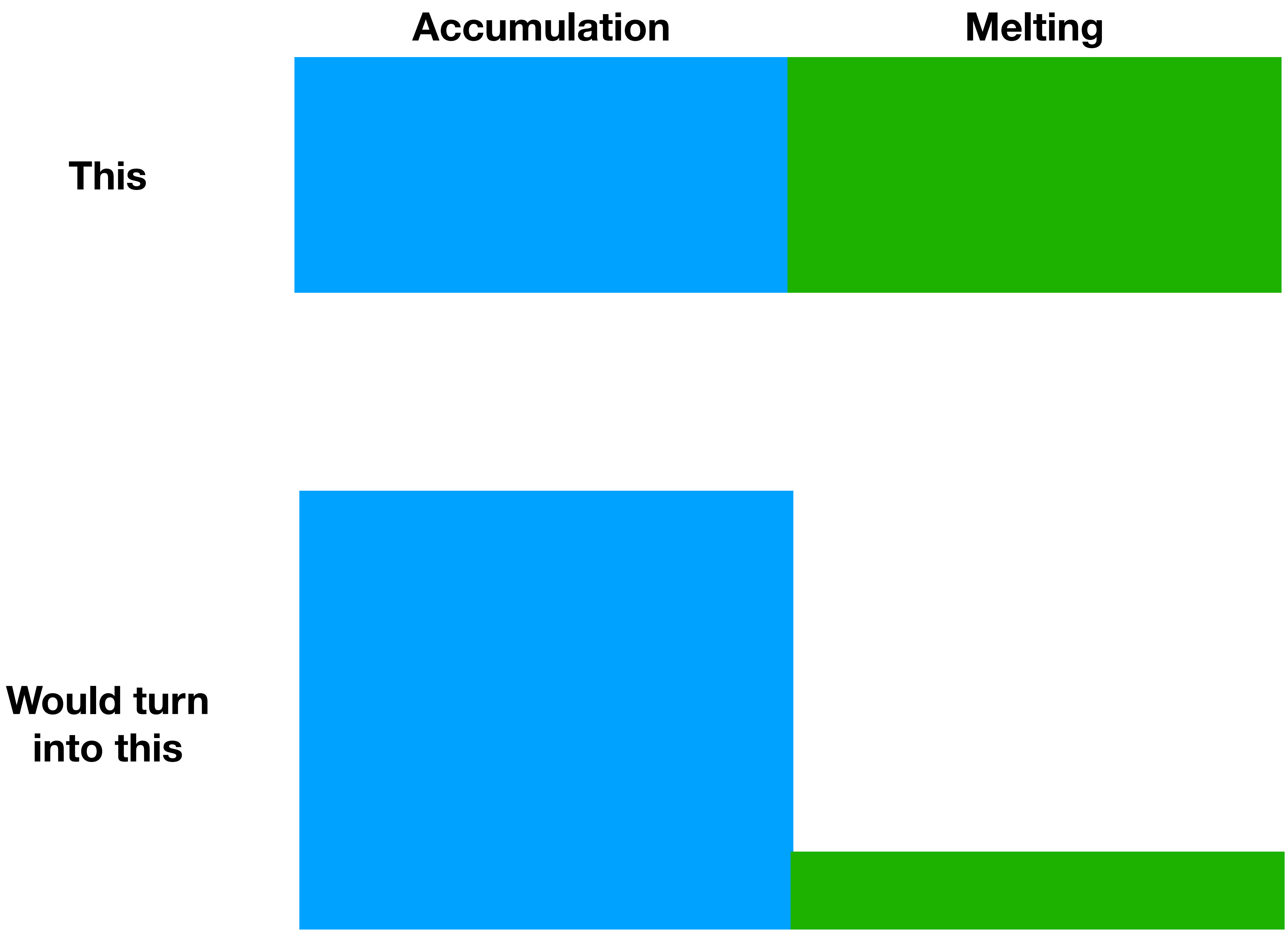
Ice Flow

EAS 8403

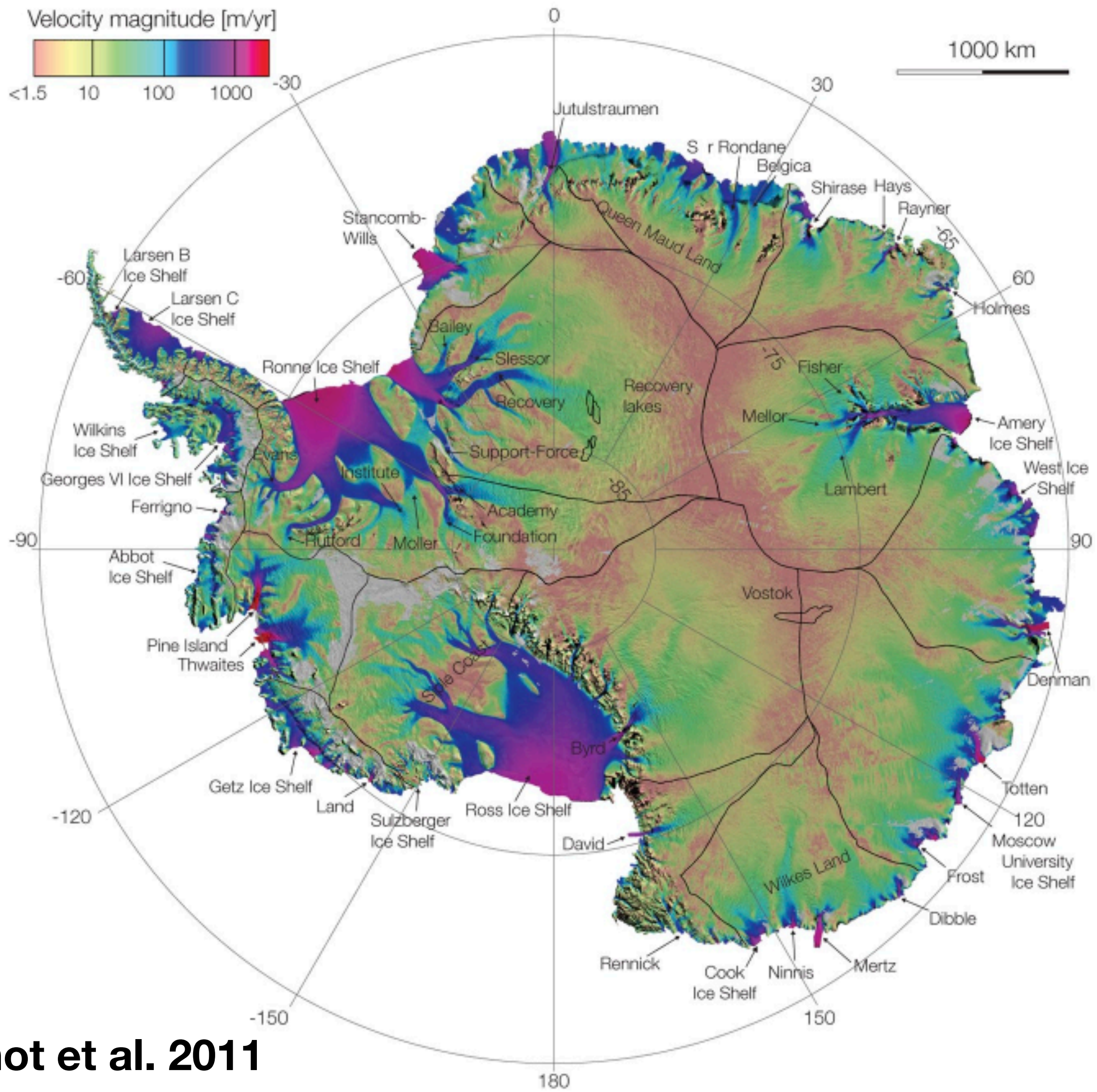
If ice didn’t move



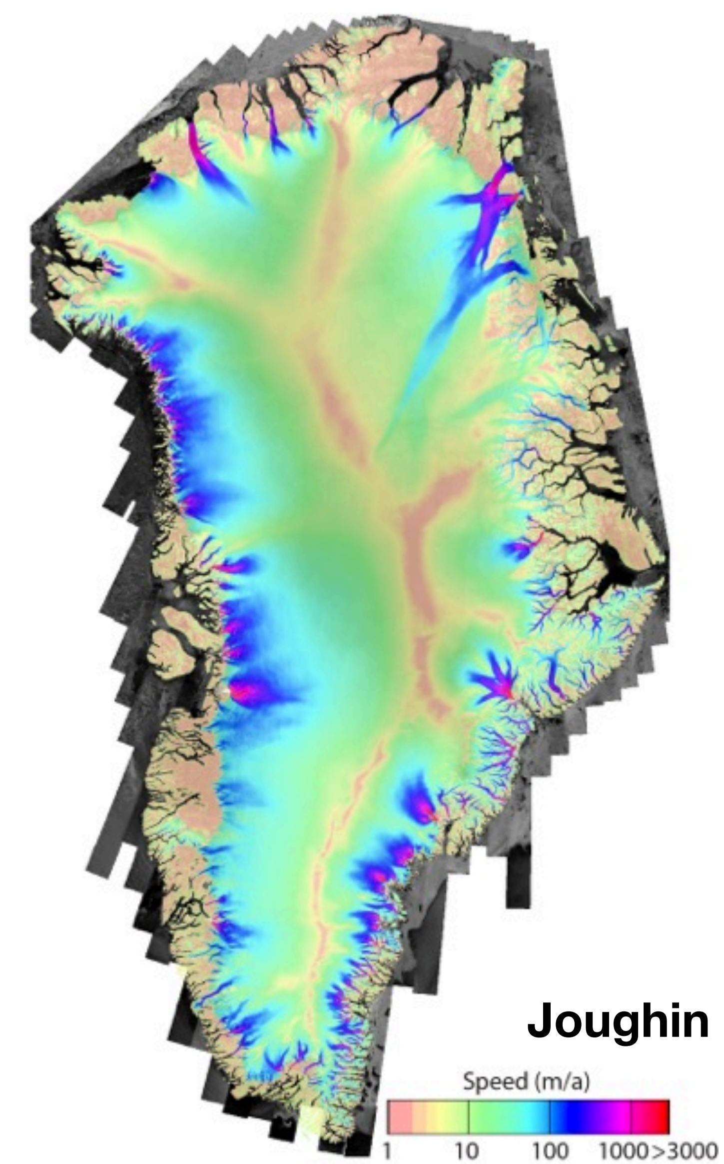
If ice didn’t move



Ice flow redistributes mass from accumulation region to ablation region



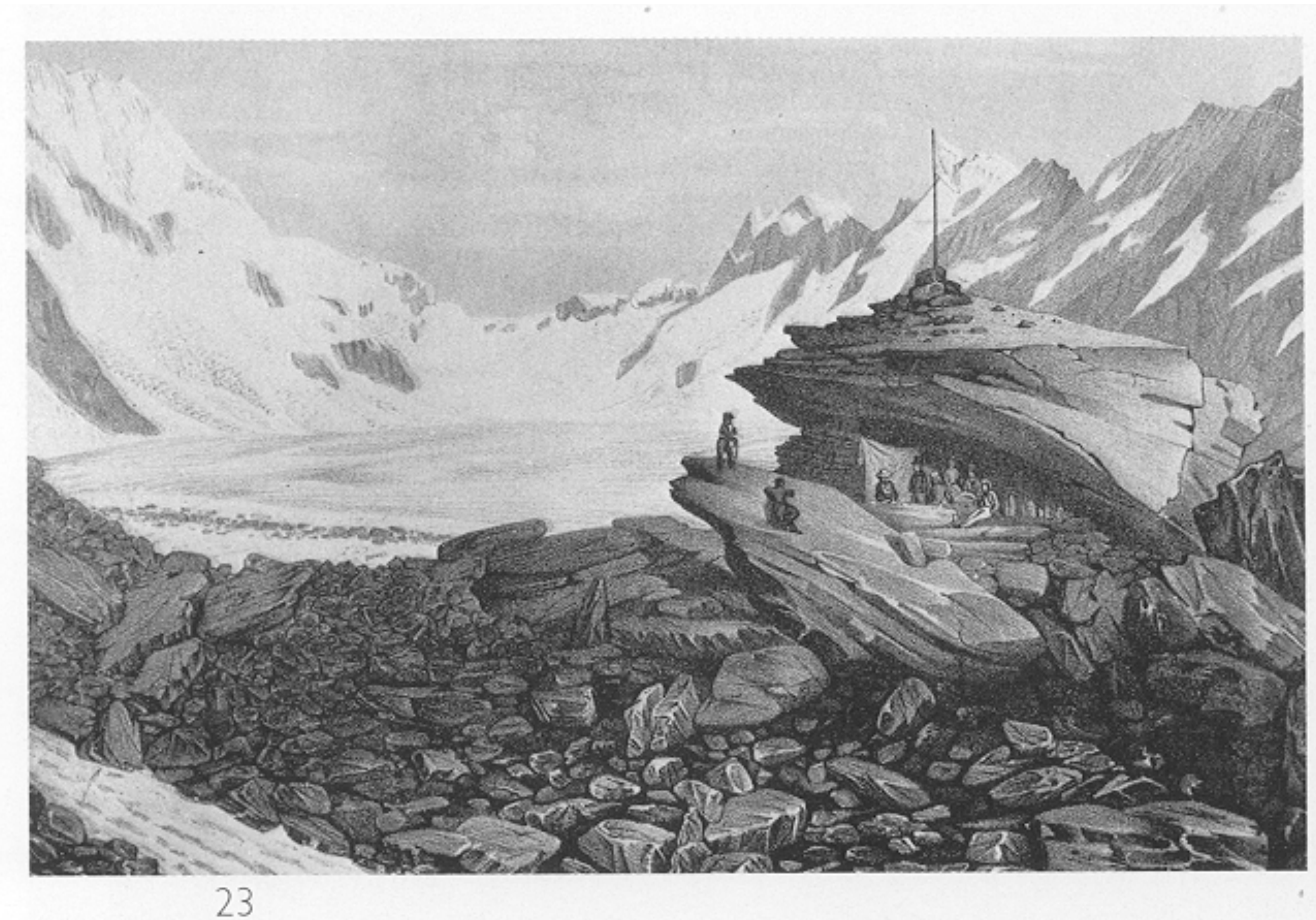
Rignot et al. 2011



Joughin et al. 2010

Accumulation and ablation are important for overall mass balance, but their importance is mediated by ice flow

Glaciers Move!



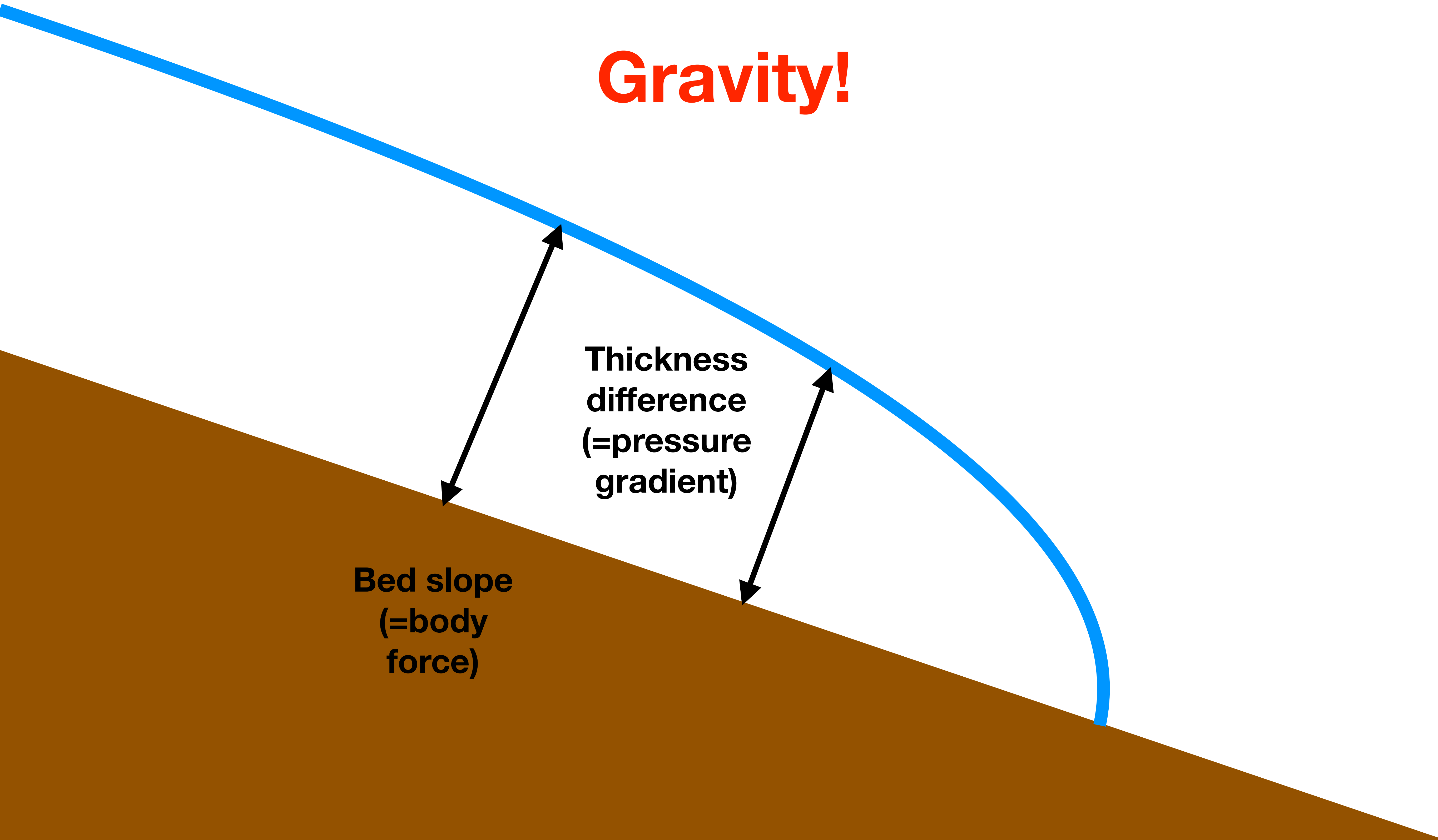
Unteraaregletscher

- It was noticed that this rock moved down the valley.
- Agassiz was a pioneer of early measurements starting in the mid 1800’s.
- For a longtime there was heated debate about how the solid ice moved.

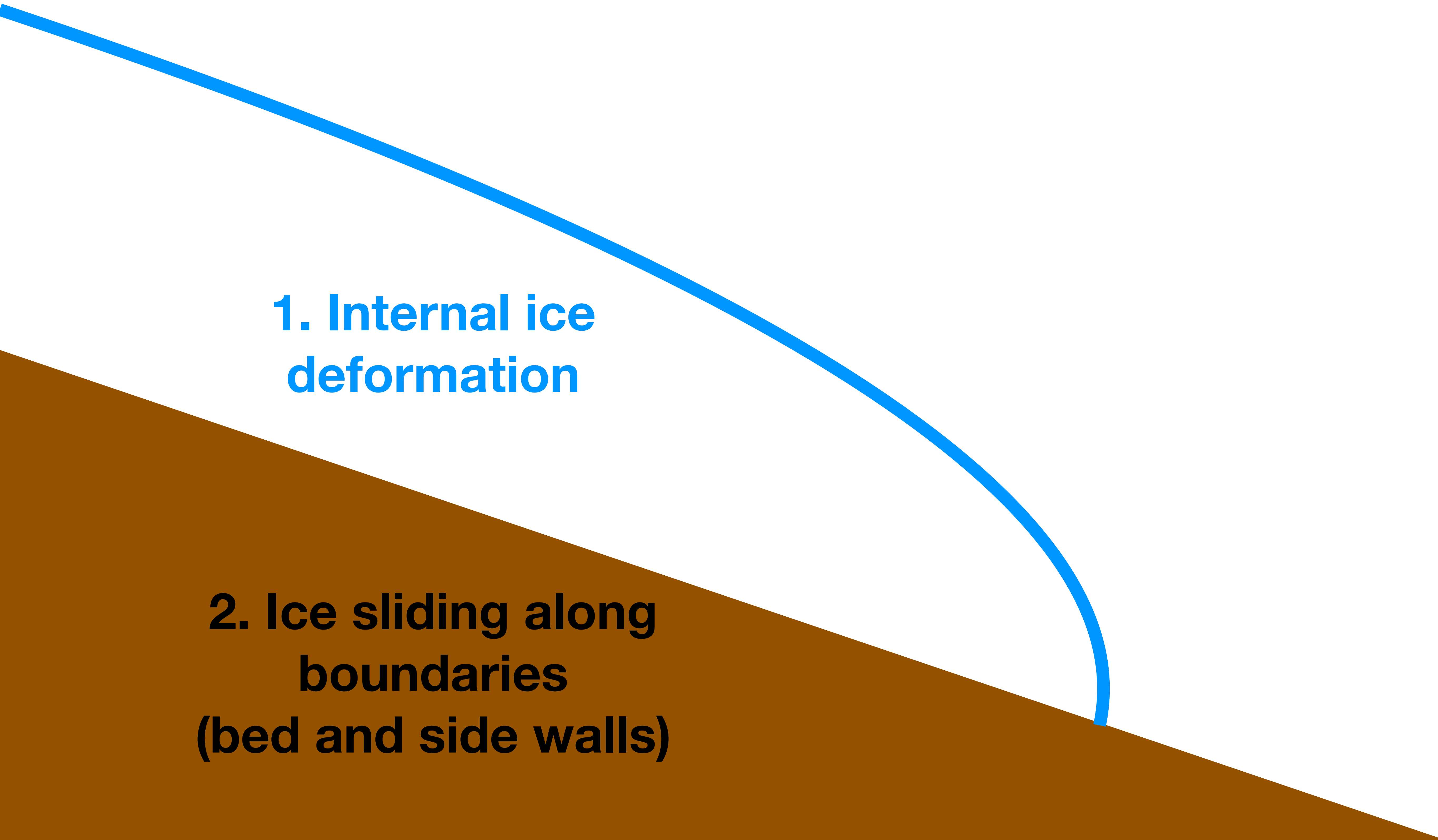
Slide from C. Raymond lectures

Elephant Foot Glacier, Antarctica

Why does ice move/flow?



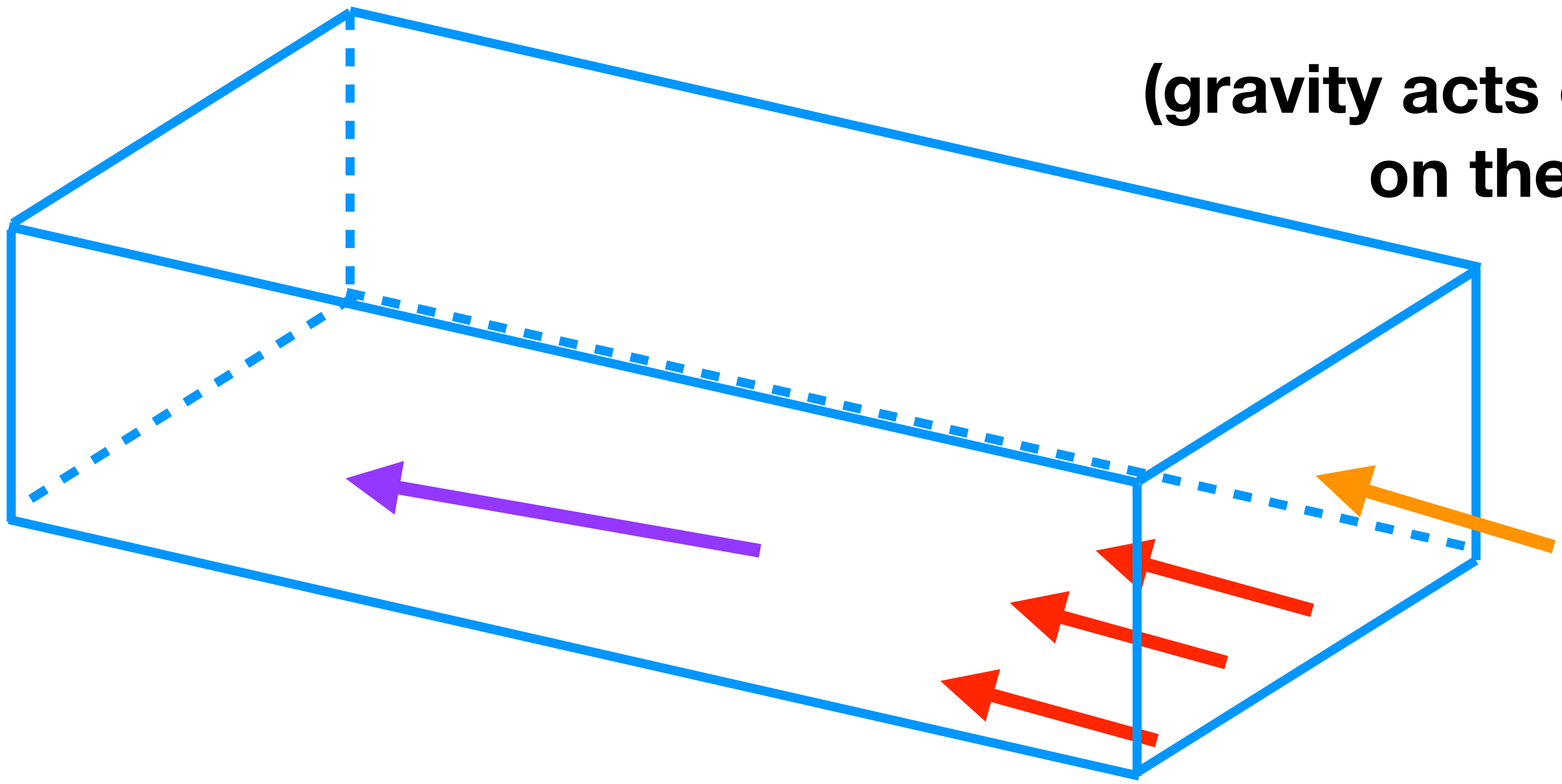
How is flow accommodated?



How can ice flow speed be determined?

...BALANCE

How can ice flow speed be determined?



(gravity acts everywhere on the ice)

Driving force (gravity) =

Internal ice stresses +
bottom friction +
side friction +
~~surface friction~~ +
front/back force

How can ice flow speed be determined?

This is a sketch about how ice flow can be predicted, but we will proceed by determining:

1. **How internal stress within the ice are determined**
2. **How to think about flow in continuum materials (aka continuum mechanics)**
3. **How flow of incompressible materials is generally determined (Cauchy Momentum Eqn)**
4. **How we can apply these general rules to the specific case of ice flow on Earth (Stokes flow and approximations therein)**

Stress vs. Strain

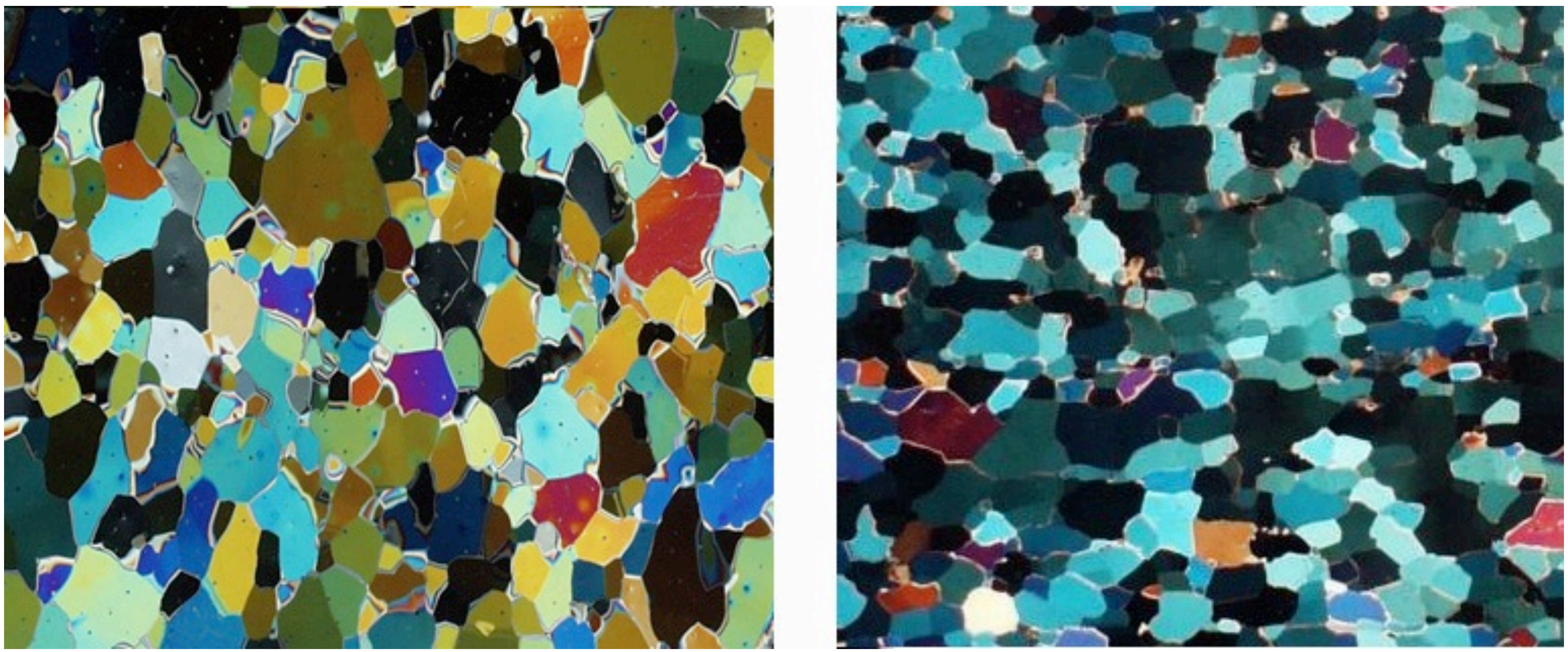
Stress: the <u>force applied</u> to a material per unit area (N/m^2)

Strain: the <u>deformation in response</u> to an applied stress (material length change per length - unitless)

A quick primer on continuum mechanics

- To the board!

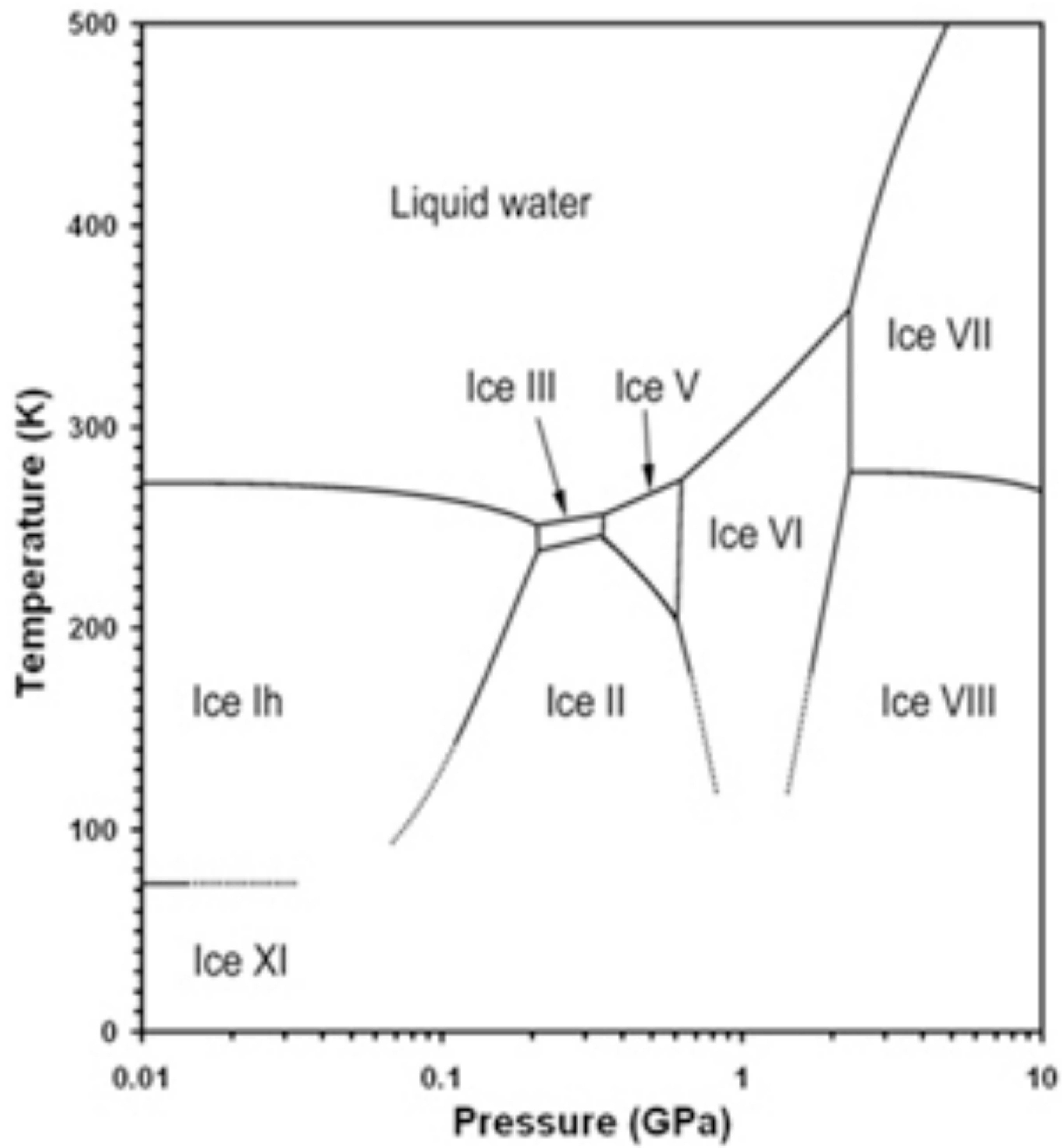
Glacial ice is crystalline



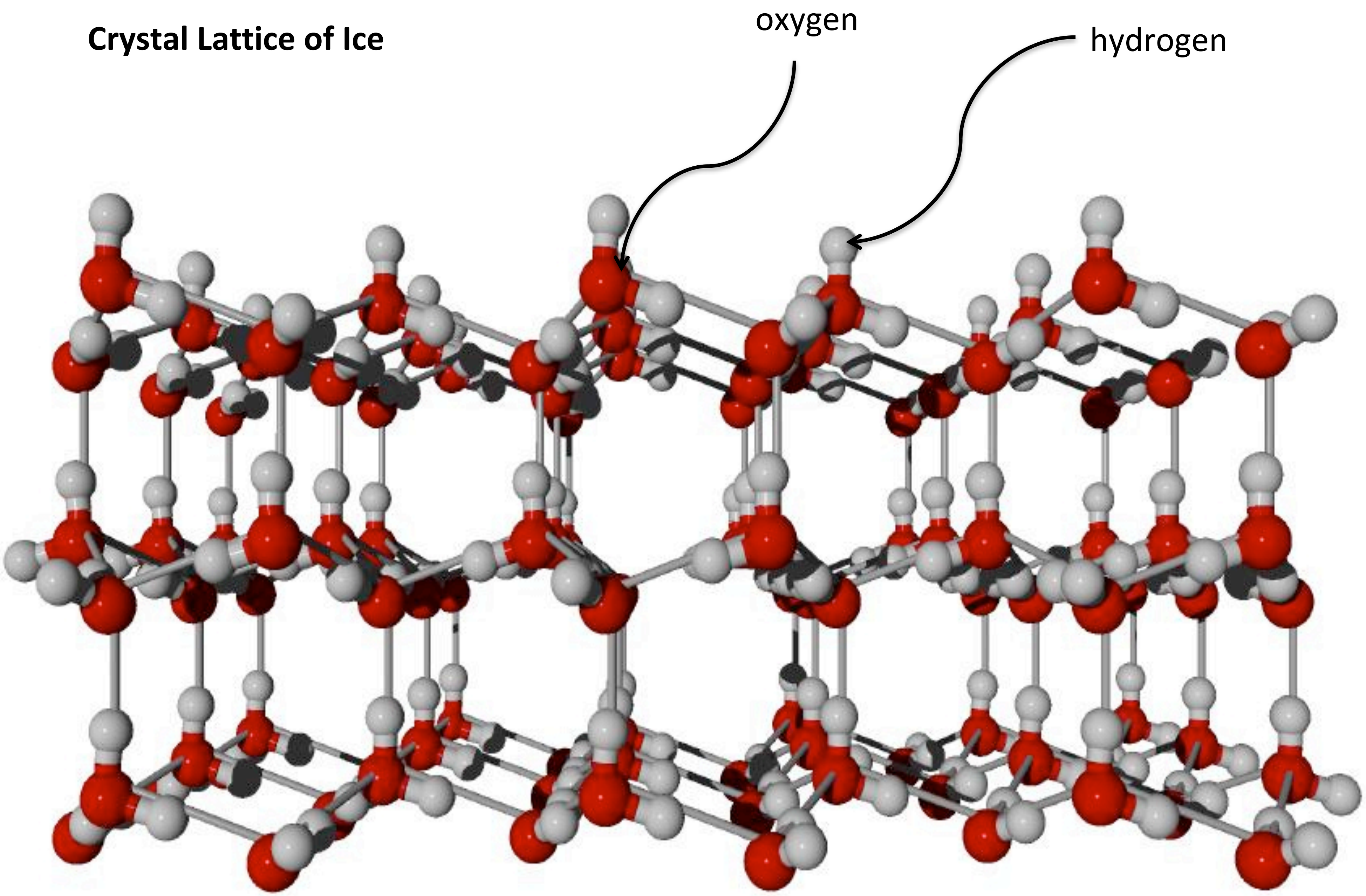
Polarized light microscopy of ice core thin sections

Crystal structure of ice

Estimation question: what type of ice are you likely to find on earth (based on the phase diagram to the right?)



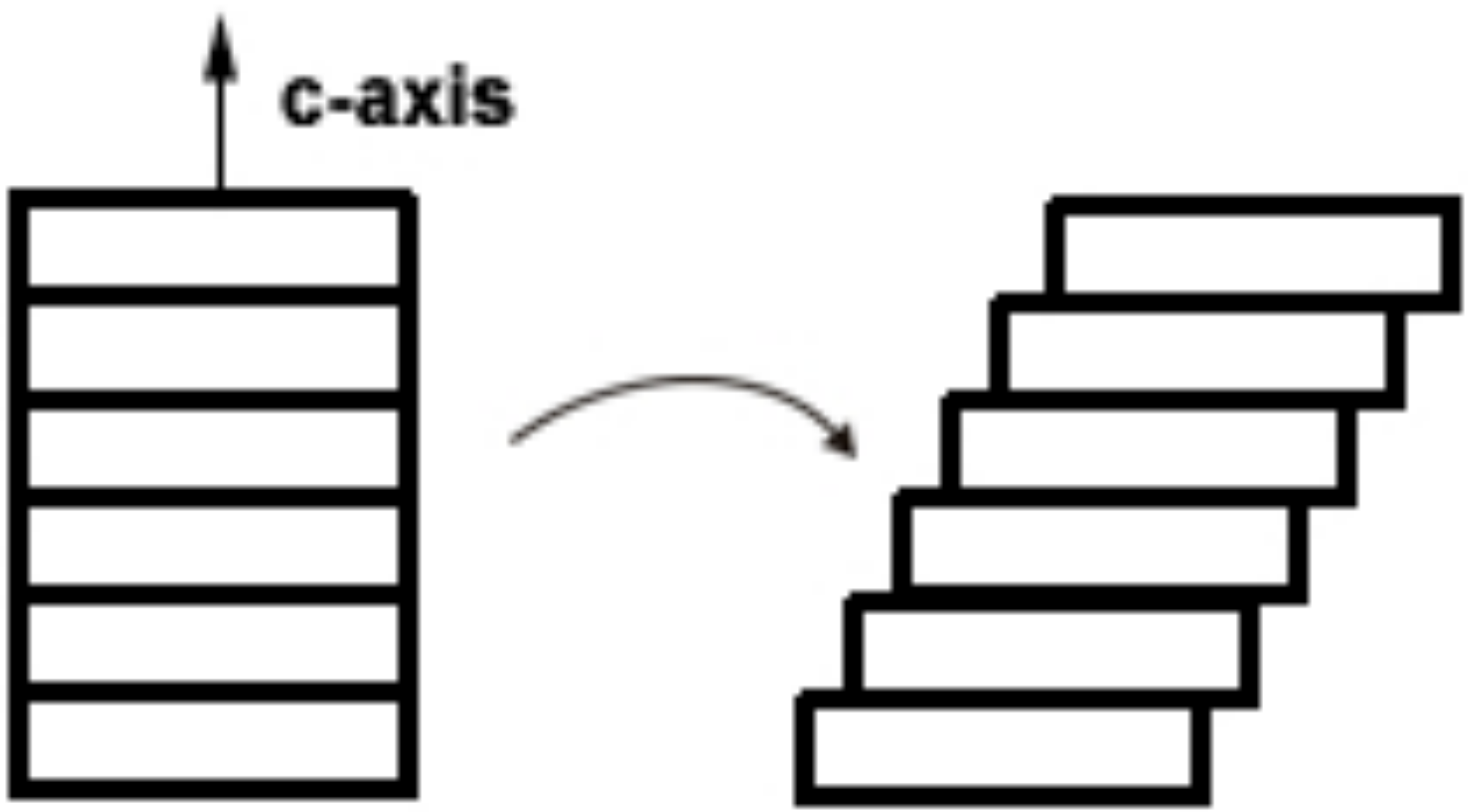
Ice 1h - h is for hexagonal



Angle between the bonds in the crystal lattice is close to a tetrahedral angle and accounts for the unusually low density of the crystal lattice.

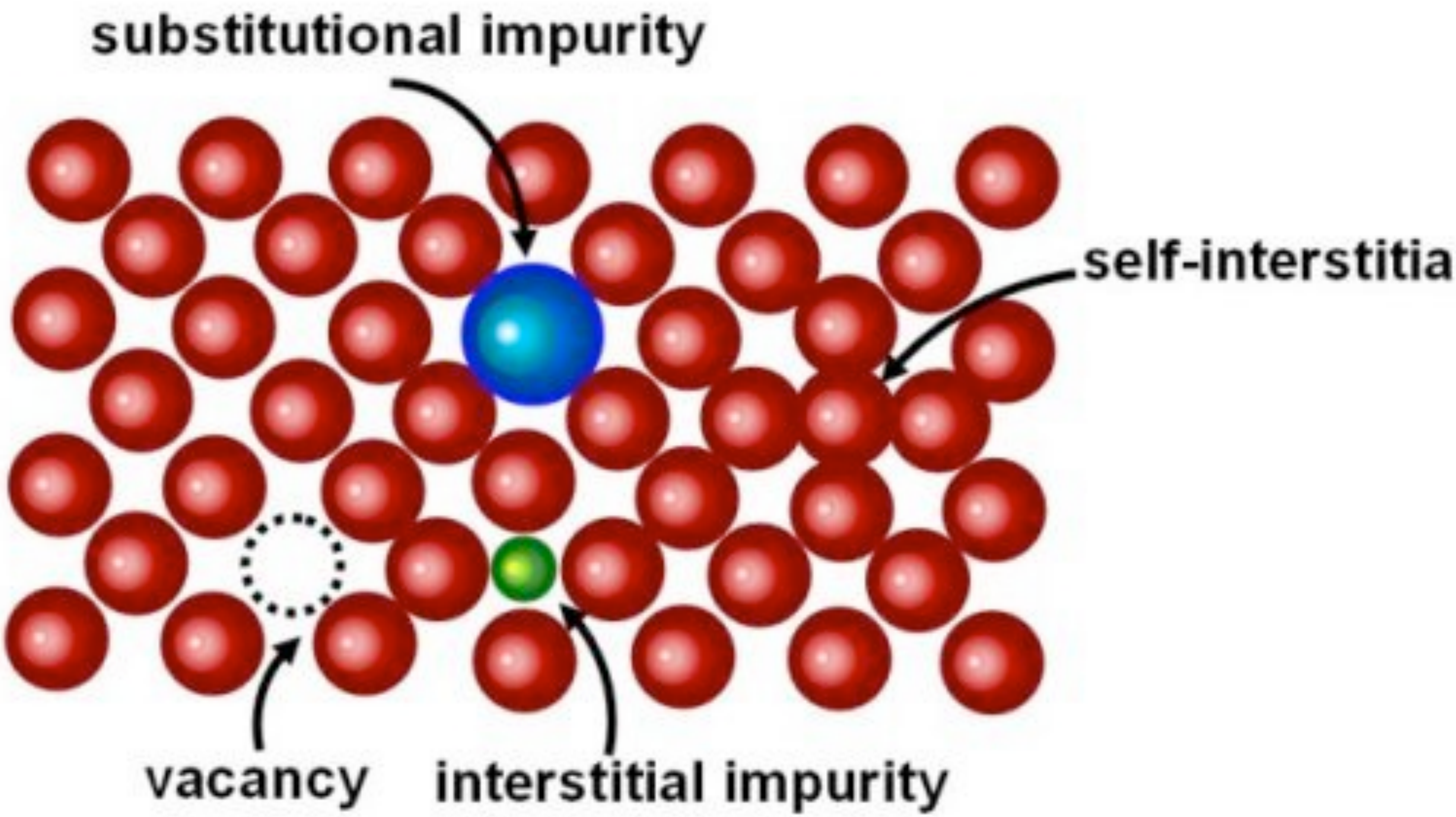
Slide from UT (G. Catania)

The deformation of a material along crystal planes (basal glide) is often referred to as “creep”

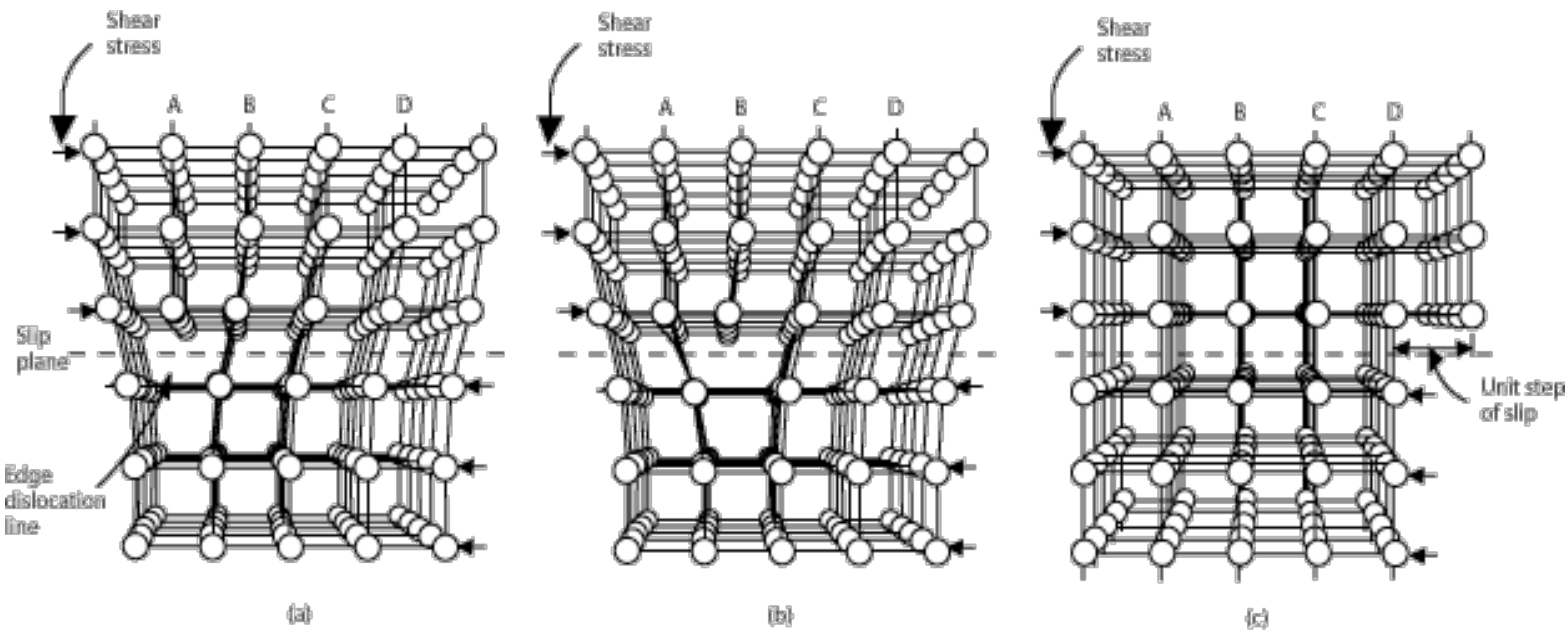


C-axis refers to the vector normal to the planar arrangement of crystals (“basal plane”)

In real glacial ice, defects allow bonds to break more easily



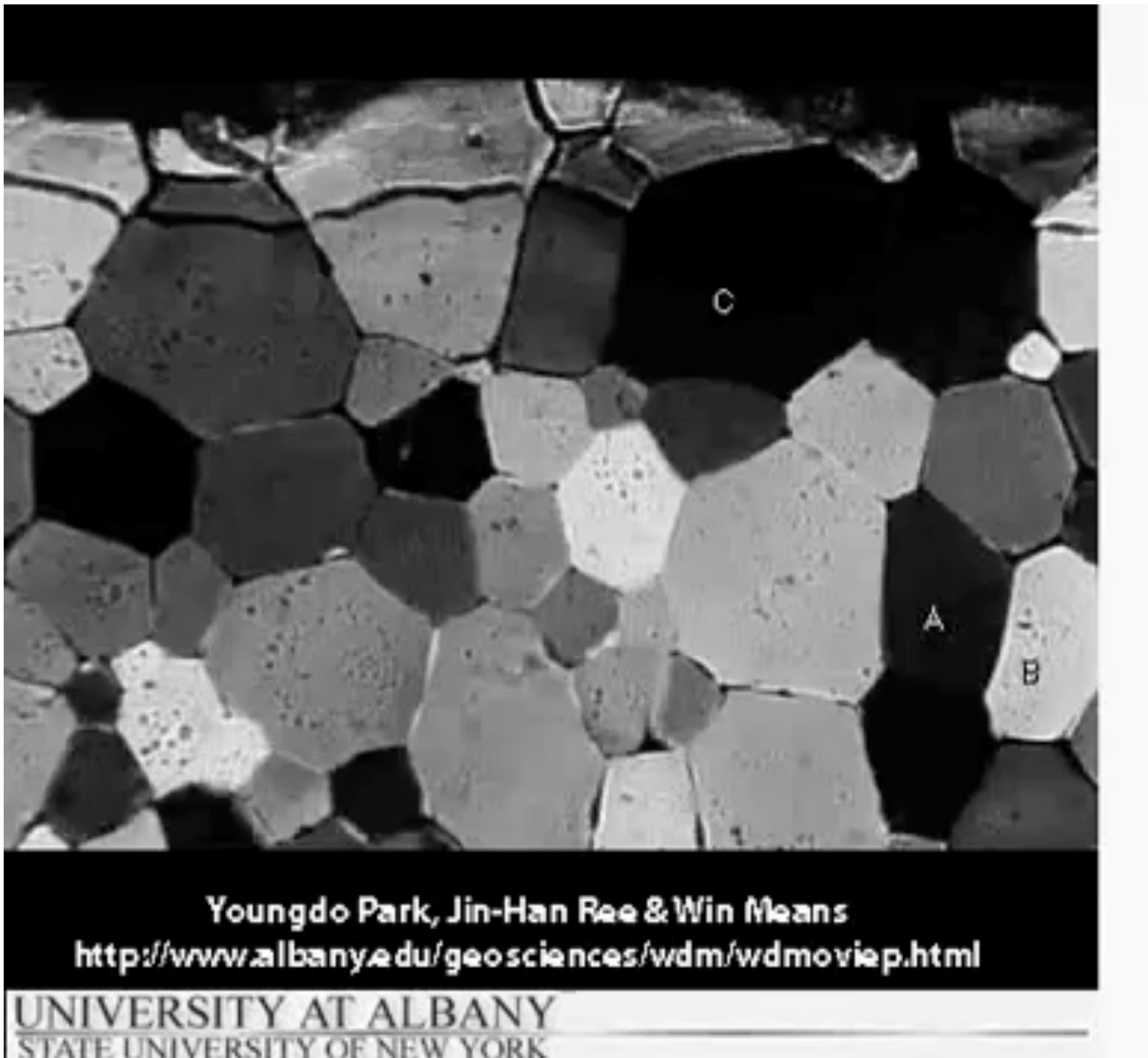
Dislocation of crystal bonds



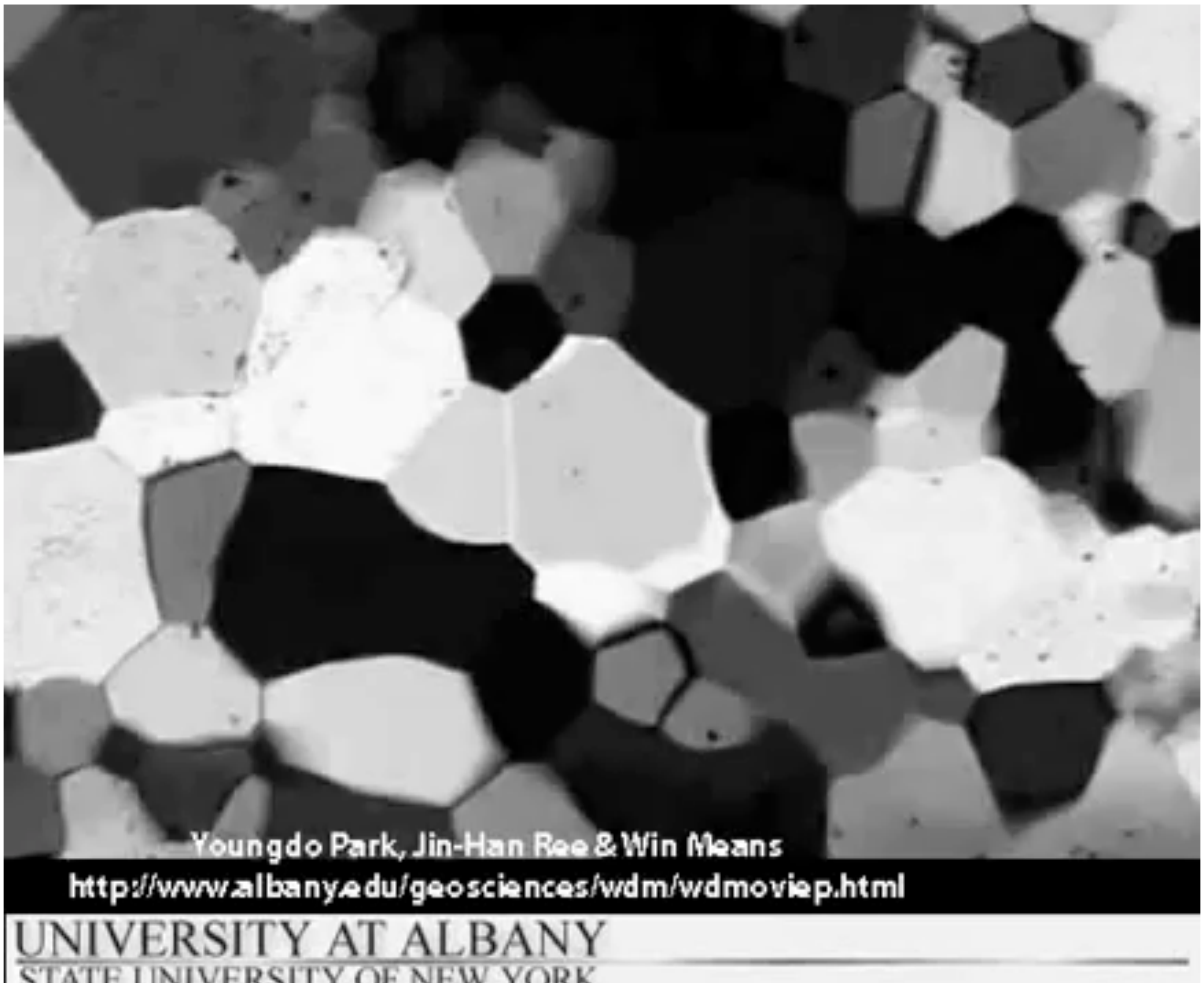
- Need to break only a few bonds at a time
- We recover energy when bonds connect at new sites

- Dislocation has moved
- Dislocations act as catalyst for easy deformation

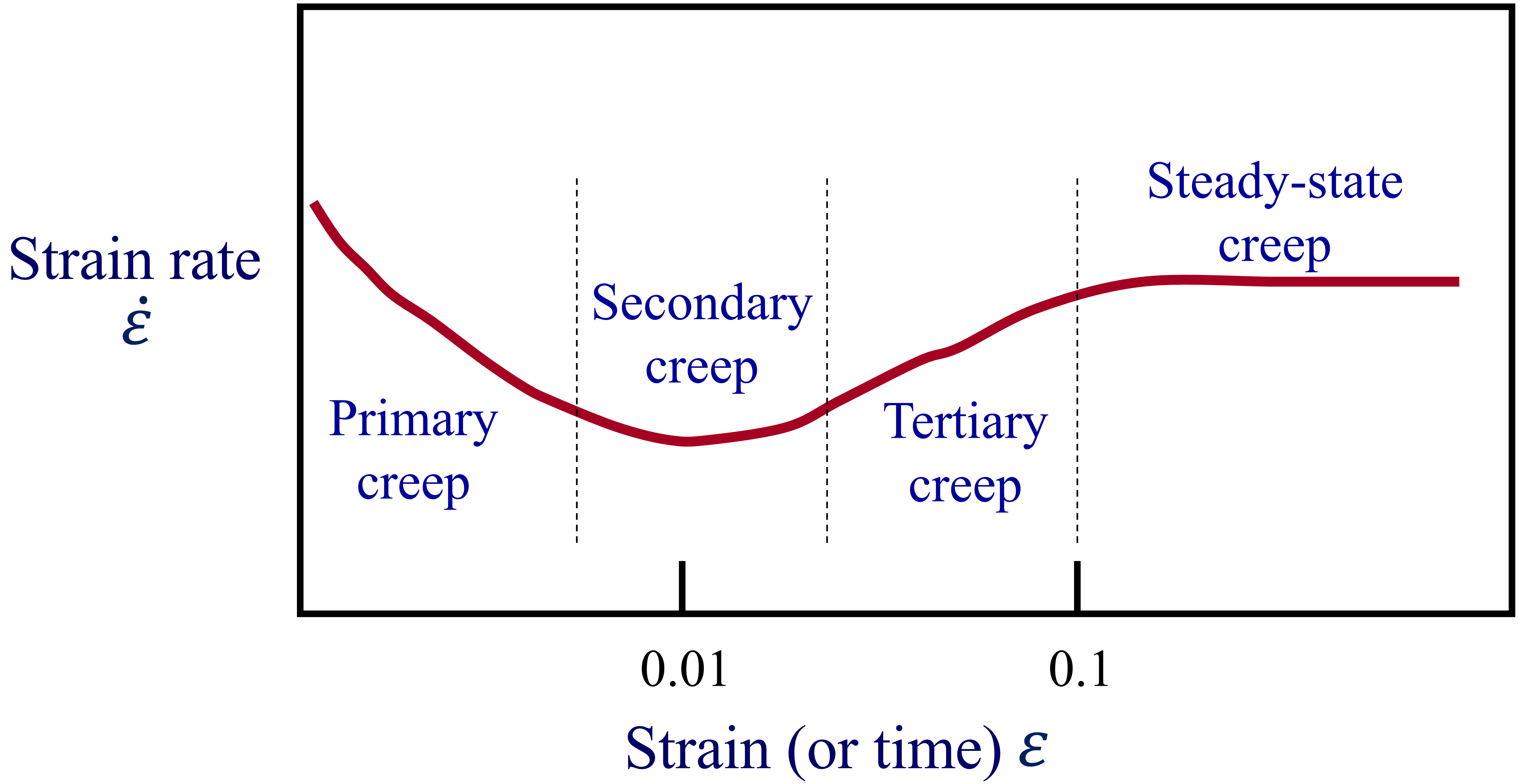
Grain boundary sliding



Grain boundary migration



Creep Behavior of Ice



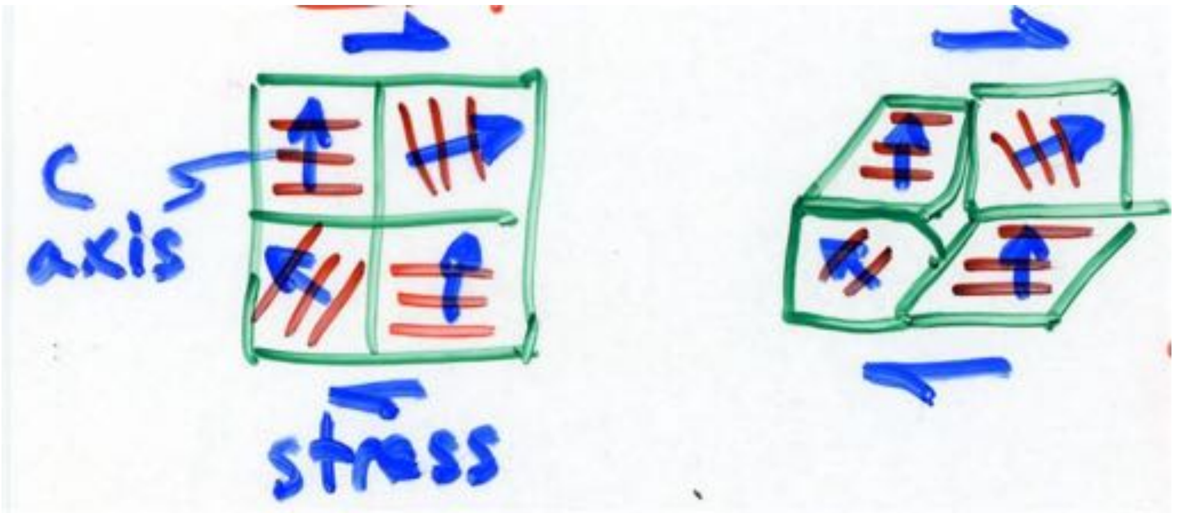
Slide from UW ES431 lectures

Primary Creep

Some crystals are hard and others are soft due to the orientation of their basal planes relative to the applied stress

- Soft grains deform easily at first
- After some deformation has occurred, hard grains start to stop deformation of soft grains

Ice initially gets harder with time

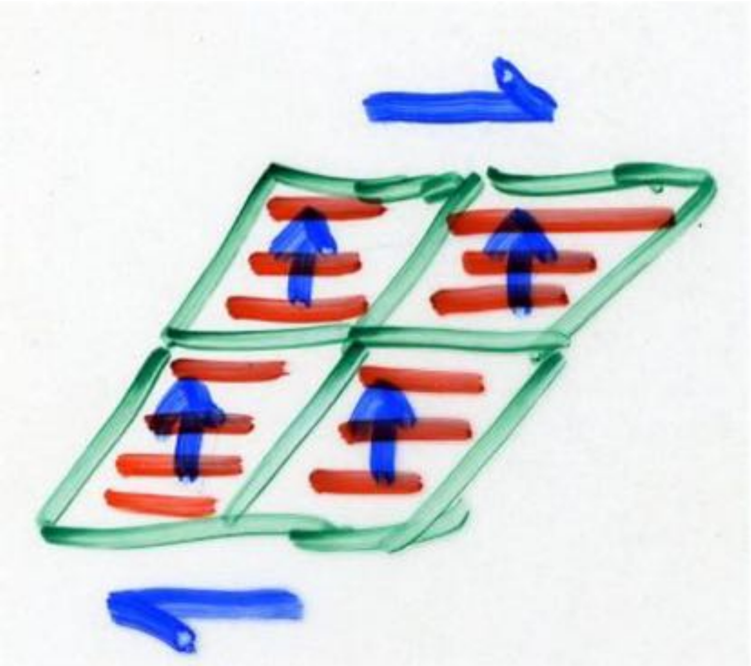


Slide from UW ES431 lectures

Secondary to Tertiary Creep

- Hard crystals get bent elastically, acquiring energy like a stretched spring
- This is an energetically unfavorable state
- They lose mass to neighbors that are able to creep without picking up strain energy

As hard grains shrink and disappear, ice becomes softer again



Most glacier flow occurs through secondary creep

Slide from UW ES431 lectures

Some rheology definitions

- The <u>rheology</u> of a material is all the properties of that material related to its flow

- One aspect of rheology is the <u>constitutive relation</u> for a material - any relationship between two quantities that describes the material response to an external stimulus

- For materials that flow, the constitutive relation typically refers to a relationship between applied stress and the material strain rate in response.

- <u>Constitutive relations are empirical</u> (though related to micro- and macroscopic behavior of the material)

Elastic materials

Physics 101 Question: What is Hooke's Law (i.e. force on a spring)?

Elastic materials

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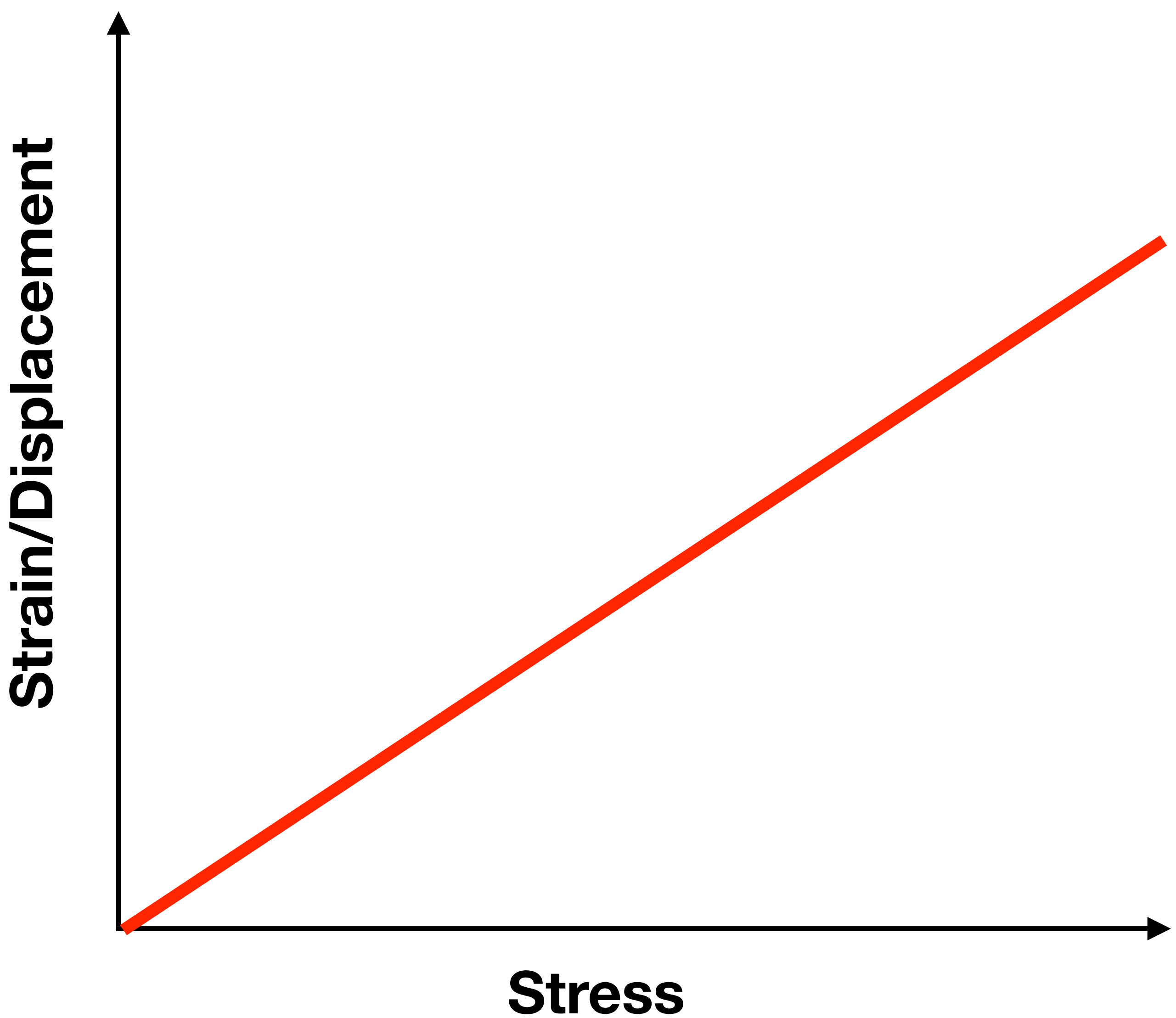
$$F = kx$$

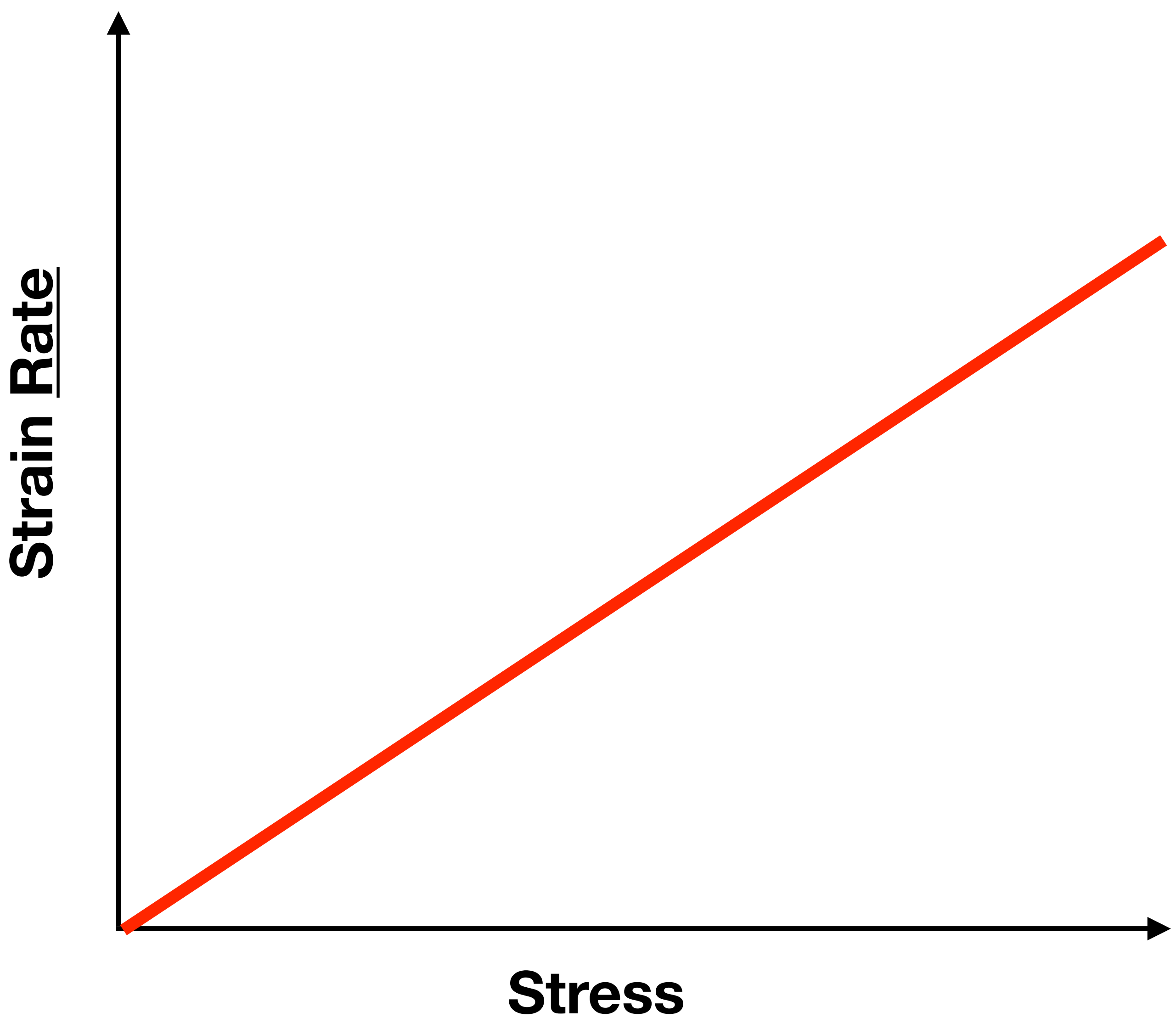


Strain is proportional to applied stress

$$\sigma = E\epsilon$$

Elastic modulus

Viscous materials (Newtonian)



Strain rate is proportional to applied stress

$$\sigma = \mu\dot{\epsilon}$$

Viscosity

Viscoelastic materials

Applied stress is proportional to strain <u>and</u> strain rate

(in one of many ways, depending on the viscoelastic model)

Viscoelastic materials

Applied stress is proportional to strain and strain rate

(in one of many ways, depending on the viscoelastic model)

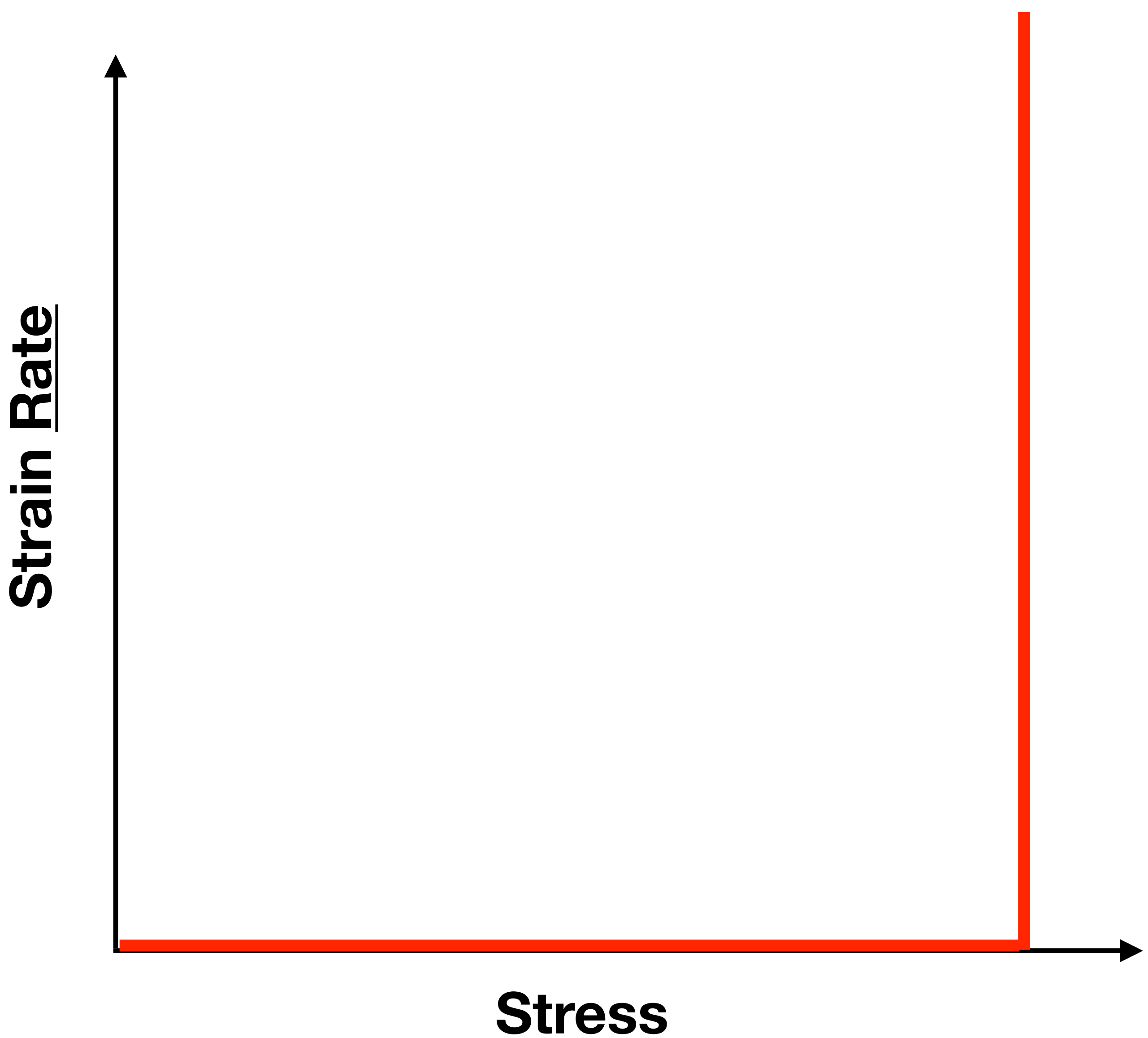
Maxwell Model

$$\frac{1}{\mu}\sigma + \frac{1}{E}\dot{\sigma} = \dot{\epsilon}$$

In the Maxwell model, both elastic and viscous deformations occur at the same time, but the response to a newly applied stress will initially be elastic, and then slowly the viscous response will take over

Maxwell time: $\dfrac{\mu}{E}$

Plastic materials

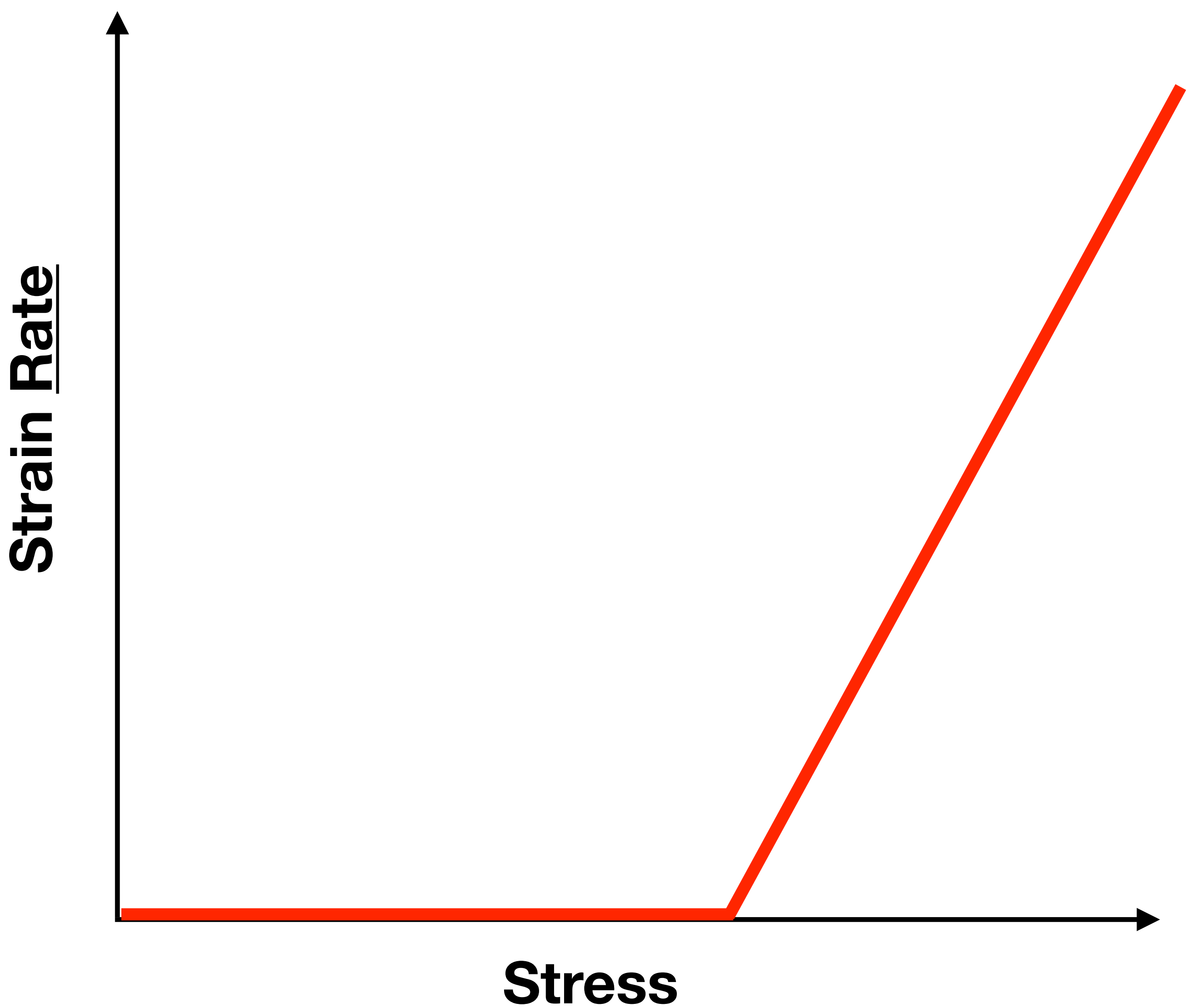


Strain rate is zero until a critical yield stress

$$\sigma = \max[\sigma, \sigma_c]$$

(Need other information to determine deformation during failure)

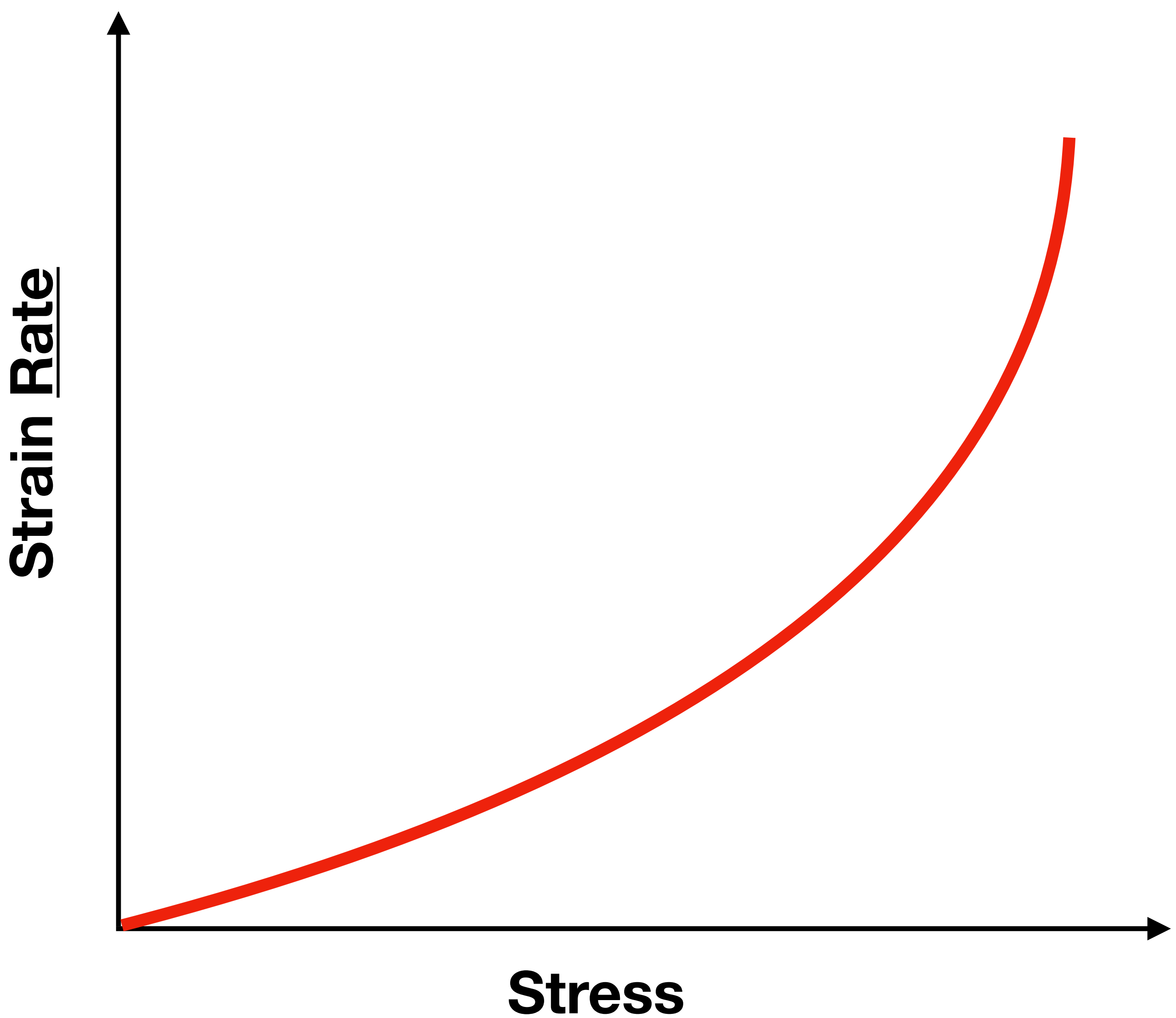
Example: Bingham Plastic



Strain rate is zero until a critical yield stress and then acts like a Viscous fluid

$$\sigma < \sigma_c \rightarrow \dot{\epsilon} = 0$$
$$\sigma \geq \sigma_c \rightarrow \dot{\epsilon} = \mu^{-1}(\tau - \tau_0)$$

Viscous materials (Non-Newtonian)

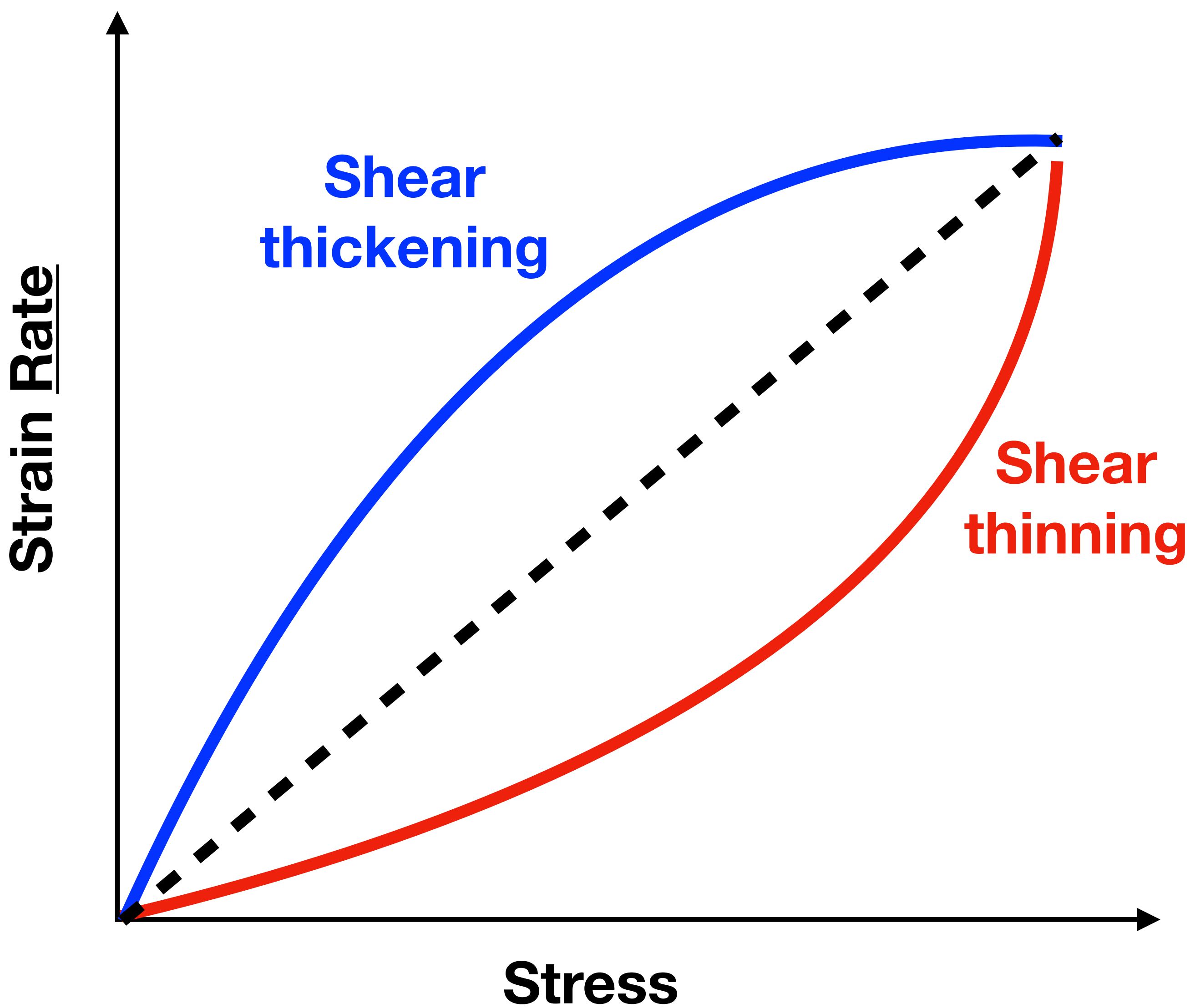


Strain rate is nonlinearly proportional to applied stress

$$\sigma^n = \mu\dot{\epsilon}$$

Viscosity

Viscous materials (Non-Newtonian)



$$\sigma^n = \mu\dot{\epsilon}$$

n<1: Shear thickening

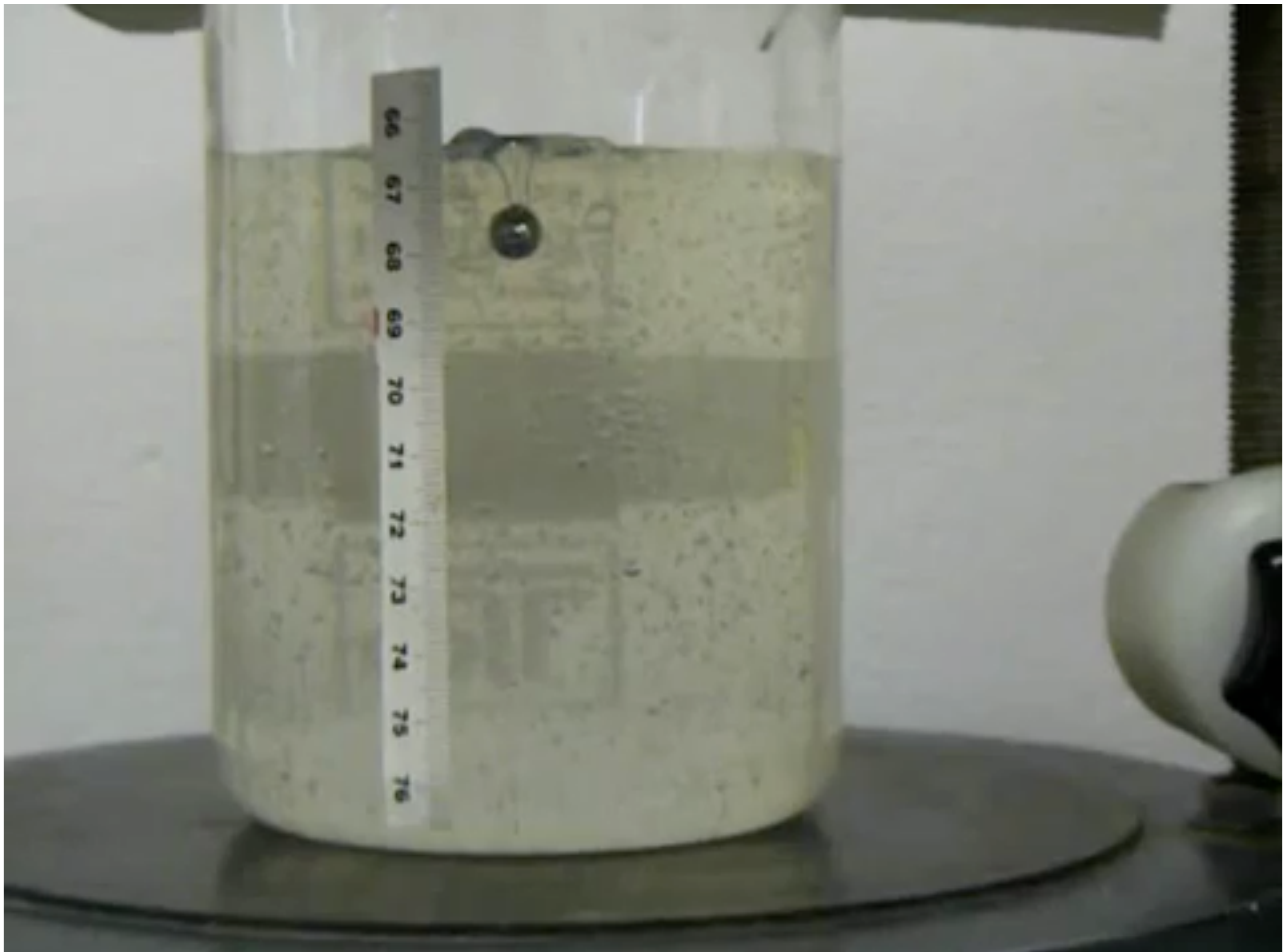
n>1: Shear thinning

The "effective" viscosity changes as a function of the strain rate.

In the limit that n→∞, a non-Newtonian fluid becomes plastic.

Shear Thickening: oobleck - soft when unstressed, hard when stressed

Shear Thinning: gel - hard when unstressed, soft when stressed

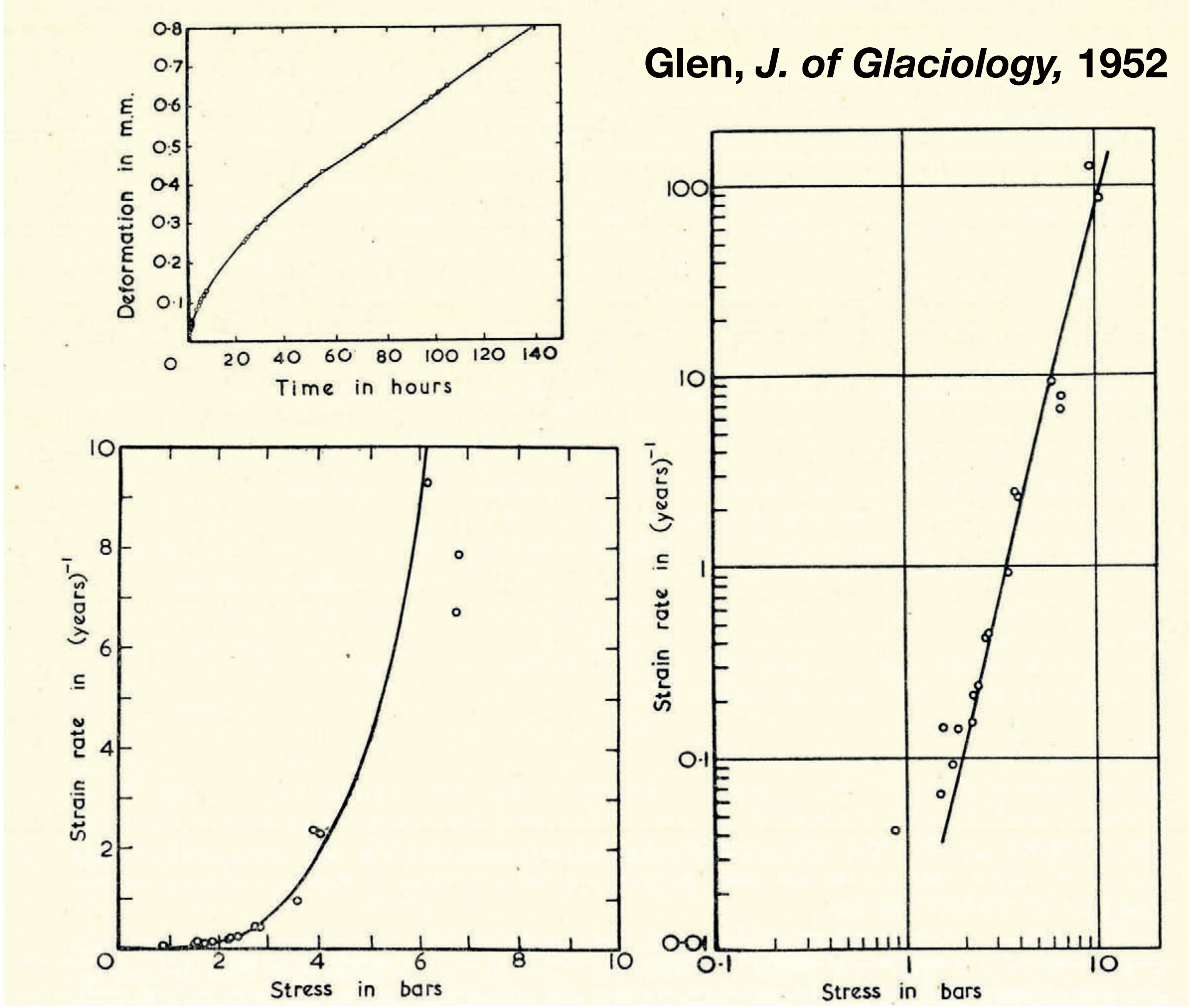


Jupyter notebook on material responses to cyclical forcing

- Split into groups of 3 and answer these questions (5 minutes):
  - As the elastic modulus (E) and viscosity (μ) of ice change how does the delay between applied stress and the strain response change?
  - Is there a combination of E and μ that describes this delay?

Glen’s flow “law” for glacier ice

(i.e. the empirically-derived constitutive relation measured for glacier ice)



$$\dot{\epsilon} = k\tau^n$$

Experiments show that n=3

Nye's generalization of Glen's Law (i.e. the modern Nye-Glen flow law)

$$\dot{\epsilon}_{ij} = A\tau_E^{n-1}\tau_{ij}$$

$$\tau_E = \sqrt{\frac{1}{2}\tau_{xx}^2 + \frac{1}{2}\tau_{yy}^2 + \frac{1}{2}\tau_{zz}^2 + \tau_{xy}^2 + \tau_{xz}^2 + \frac{1}{2}\tau_{yz}^2}$$

Second invariant of the deviatoric stress tensor
(an eigenvalue of the stress tensor)

n not always 3

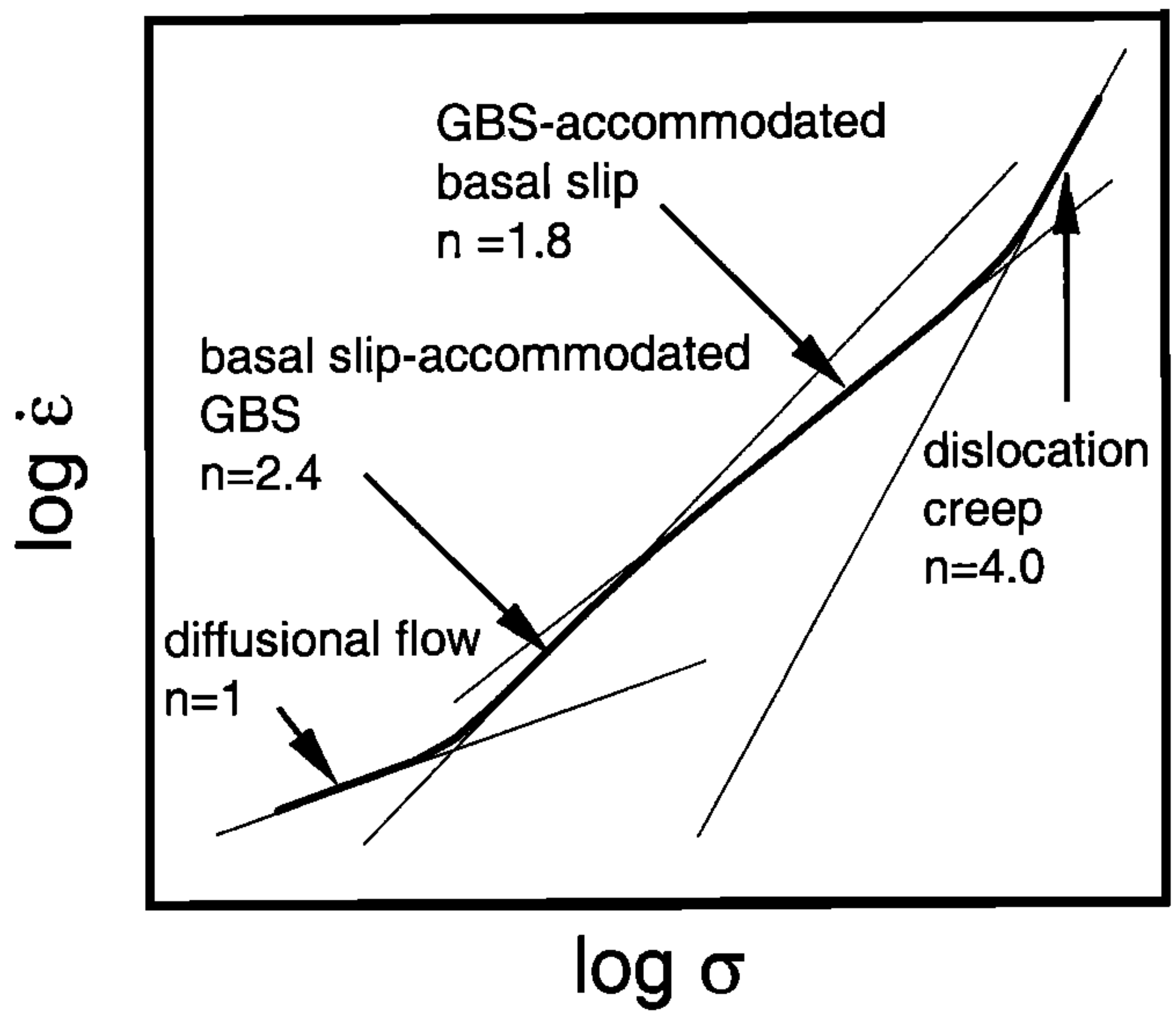


Figure 5. Schematic diagram depicting the relative contributions of each of the four creep mechanisms for ice as a function of stress.

Goldsby & Kohlstedt 2001

What is A?

A is called the rate-factor or softness factor, which captures many things about the ice viscosity related to ice material properties.

E is the so-called "enhancement factor" for everything else that isn't temperature (i.e. things that aren't easily modeled)

$$\dot{\epsilon}_{ij} = EA\tau_E^{n-1}\tau_{ij}$$

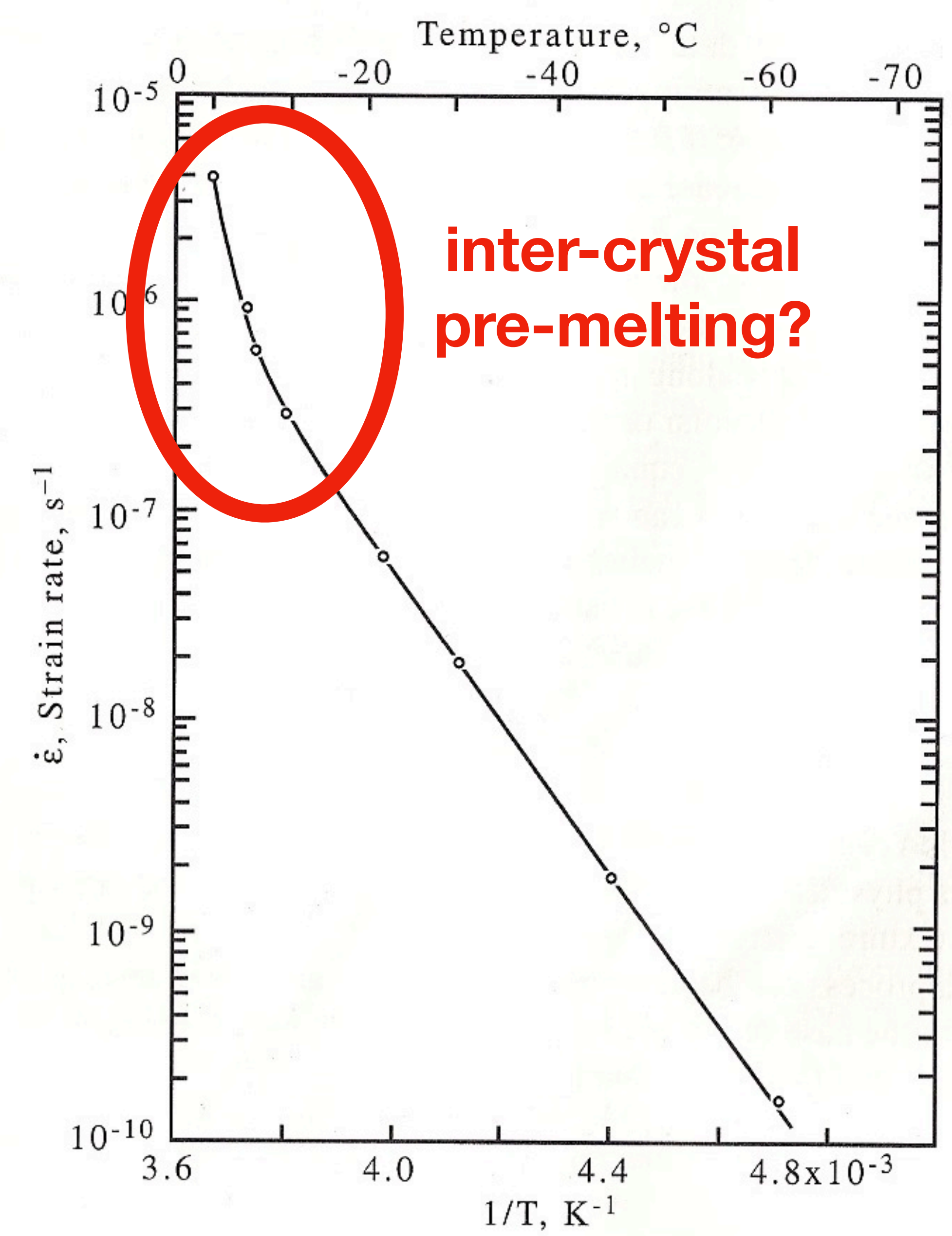
A = A(temp)

E = E(fabric, microfractures, ice crystals, impurities, etc.)

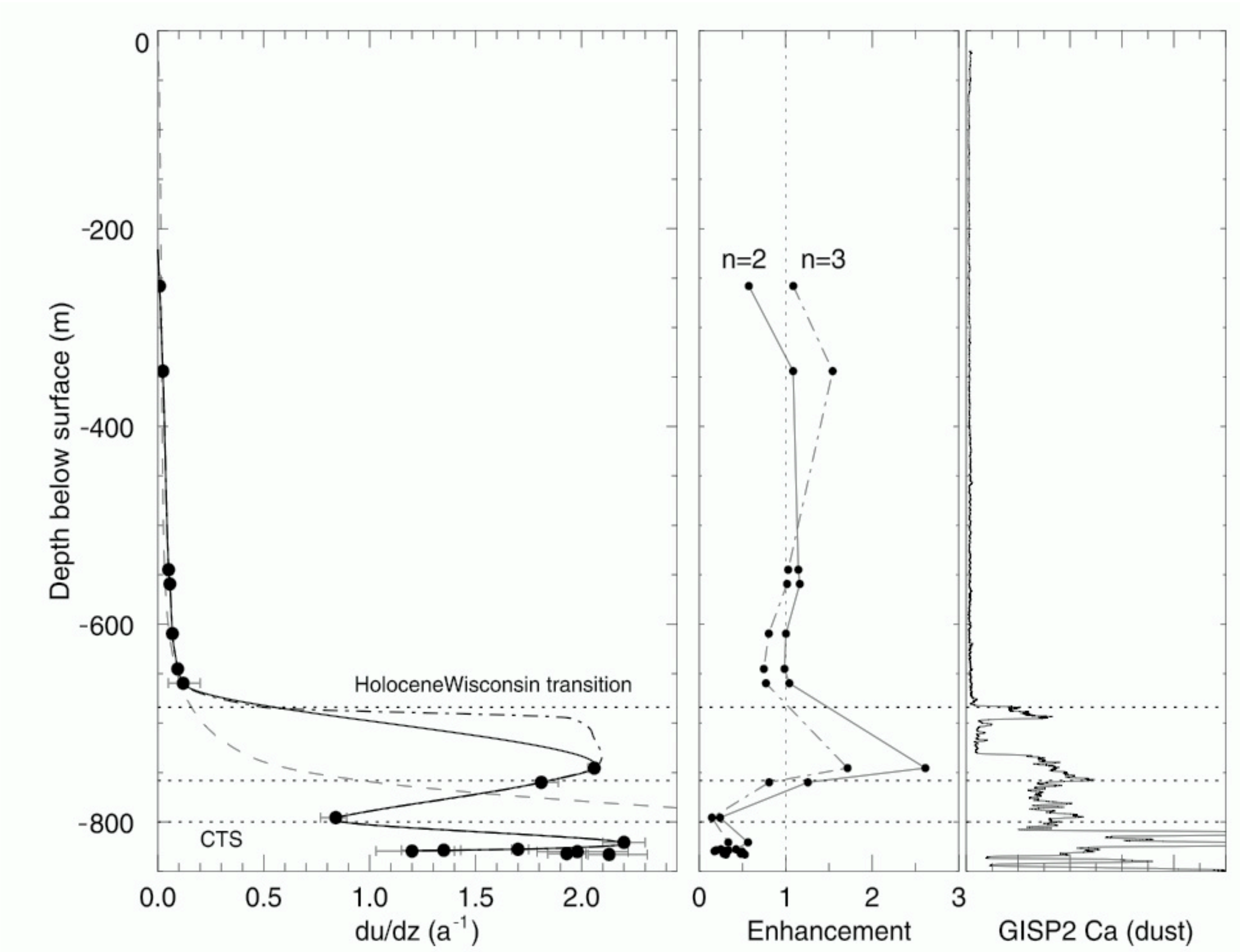
What is A?

$$A(T) = A_0 \exp(-\frac{Q}{RT})$$

The most well-understood dependence is on temperature - meaning that modeling ice flow is a coupled problem of ice mechanics <u>and</u> thermodynamics



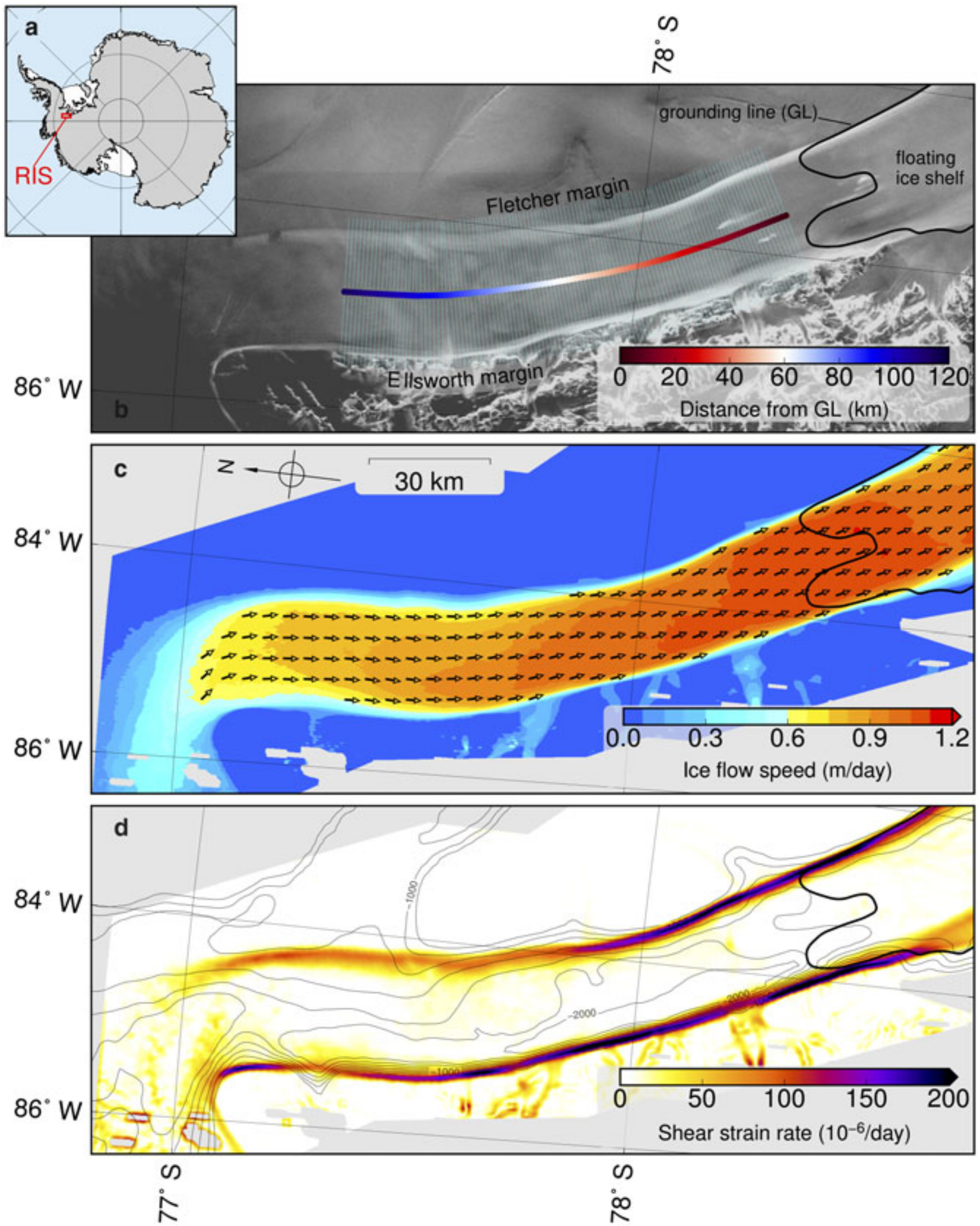
Flow enhancement and crystal size

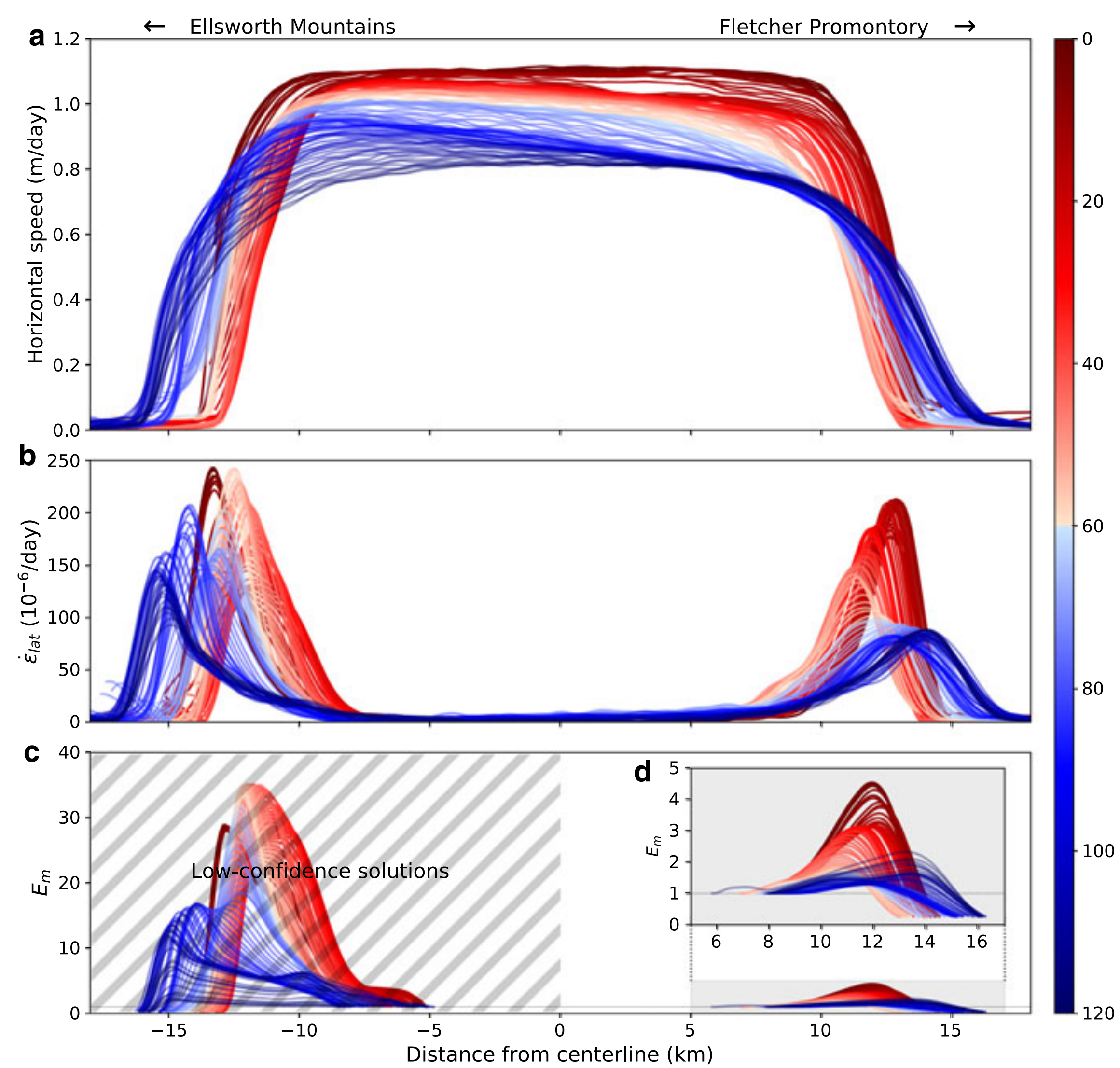


During the Wisconsin-period (65-25 kyr before present), high dust content in the atmosphere deposited on NH ice sheets and caused ice grains to be smaller (smaller crystals due to more nucleation points)

Luthi et al. 2002 - ice core from Greenland

Flow enhancement and crystal “fabric”





Water, fractures and single-orientation fabric cause significant enhancement in shear margins of ice streams

Minchew et al. 2018