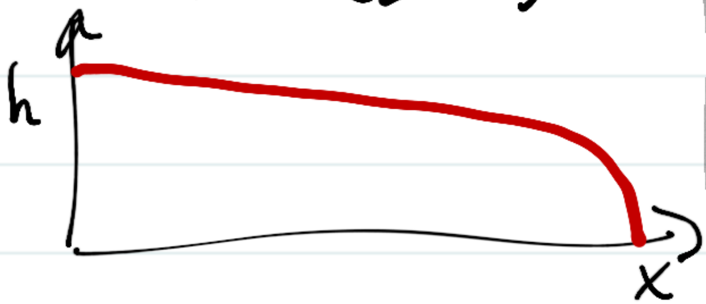
A variation on Oerlemans 1981, 2003

Two ingredients

① Ice Thickness

$$h(x) = \sigma (L-x)^{\frac{1}{2}}$$



② SMB vary w/elev

$$a = a_0 + \beta h$$

(Integrate SMB over h)

$$A_{total} = \int_0^L \left[a_0 + \beta\sigma (L-x)^{\frac{1}{2}} \right] dx$$

$$= a_0 L + \frac{2}{3}\beta\sigma L^{3/2}$$

$$V = \int_0^L \sigma (L-x)^{\frac{1}{2}} dx$$

$$= \frac{2}{3}\sigma L^{3/2}$$

$$\dot{V} = \sigma L^{\frac{1}{2}} \dot{L}$$

(Calc total ice volume)

$$\sigma L^{\frac{1}{2}} \dot{L} = a_0 L + \frac{2}{3}\beta\sigma L^{3/2}$$

$$\boxed{\dot{L} = \frac{a_0}{\sigma} L^{\frac{1}{2}} + \frac{2}{3}\beta L}$$

Fixed Points (i.e. potential steady-states)

$\dot{L} = 0$

$$0 = L_*^{\frac{1}{2}}\left[\frac{a_0}{\sigma} + \frac{2}{3}\beta L_*^{\frac{1}{2}}\right]$$

$$\boxed{L_* = 0} \qquad \boxed{L_* = \left(\frac{3}{2}\frac{a_0}{\sigma\beta}\right)^2}$$

Linear Stability Analysis

Do this part first

Purpose: to determine whether a system near a particular state will evolve towards a nearby stable configuration or quickly move towards another configuration.

Consider $\frac{dx}{dt} = f(x)$

Fixed points are where $f(x_*) = 0$

→ Local linear stability of a system state is captured by $\frac{\partial f}{\partial x}$

→ Evaluate at fixed points to determine if these system states are stable ($\frac{\partial f}{\partial x} < 0$) or unstable ($\frac{\partial f}{\partial x} > 0$) or neutral ($\frac{\partial f}{\partial x} = 0$)

In our ice sheet model:

$$f(L) = \frac{a_0}{\sigma} L^{\frac{1}{2}} + \frac{2}{3}\beta L$$

$$\frac{\partial f}{\partial L} = \frac{1}{2}\frac{a_0}{\sigma} L^{-\frac{1}{2}} + \frac{2}{3}\beta$$

$$\left.\frac{\partial f}{\partial L}\right|_{L_*=0} \to -\infty \quad \underline{\text{Stable}}$$

$$\left.\frac{\partial f}{\partial L}\right|_{L_*=\left(\frac{3}{2}\frac{a_0}{\sigma\beta}\right)^2} = \frac{1}{2}\frac{a_0}{\sigma}\left(\frac{3}{2}\frac{a_0}{\sigma\beta}\right)^{2 \cdot -\frac{1}{2}} + \frac{2}{3}\beta$$

$$= \beta > 0 \quad \underline{\text{Unstable}}$$