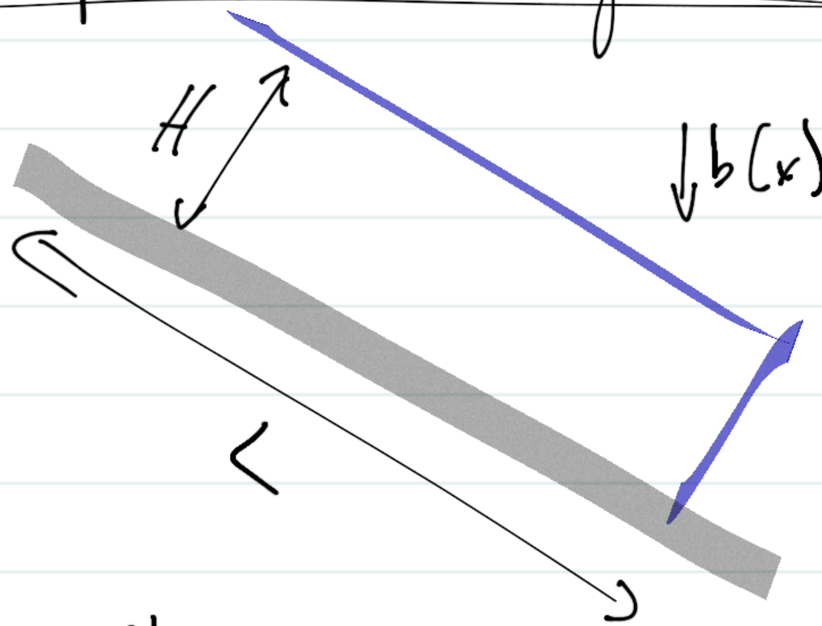
A simple mountain glacier model



All ice flow occurs in the interior of the box

$$\frac{dV}{dt} = \int_0^L b(x)\,dx \qquad V = HL$$

$$\frac{d(HL)}{dt} = L\frac{dH}{dt} + H\frac{dL}{dt}$$

A reasonable assumption: all volume changes occur through length changes $\left(\frac{dH}{dt} = 0\right)$

$$H\frac{dL}{dt} = \int_0^L b(x)\,dx$$

Now, the key for a <u>linear response analysis</u>: consider a steady-state length ($\bar{L}$) and small perturbations (L')

$$L = \bar{L} + L'$$

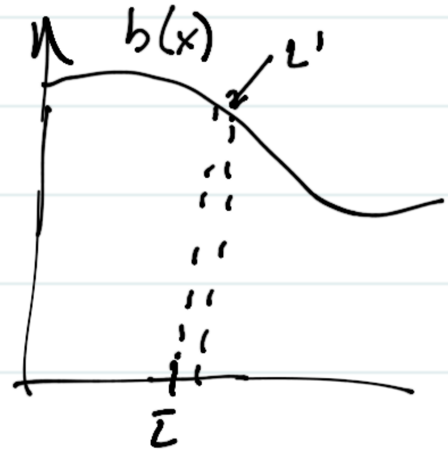
$$H \frac{d}{dt}(\bar{L} + L') = \int_0^{\bar{L}+L'} b(x)\,dx$$

By definition, the LHS balances RHS at steady-state:

$$\boxed{H \frac{d\bar{L}}{dt}} + H \frac{dL'}{dt} = \boxed{\int_0^{\bar{L}} b(x)\,dx} + \int_{\bar{L}}^{\bar{L}+L'} b(x)\,dx$$

These terms are <u>zero</u>

In the limit that L' small
$b(\bar{L}+L') \approx b(\bar{L})$



What is left:

$$H \frac{dL'}{dt} = \int_{\bar{L}}^{\bar{L}+L'} b(x)\,dx$$

$$H \frac{dL'}{dt} = L' b(\bar{L})$$

$$\frac{dL'}{dt} = \frac{b(\bar{L})}{H} L'$$

A typical mtn glacier:

$H \sim 100\,m$ $\bar{b}(\bar{L}) = -1 \frac{m}{yr}$

$\tau \sim 100\,yr$

Ask for solution from class

$$L'(t) = L'(t=0)\, e^{\frac{b(\bar{L})}{H} t}$$

*This is just like the linear stability analysis

Where the e-folding timescale is

In general $\bar{b}(\bar{L}) < 0$

$$\boxed{\tau = \frac{-H}{b(\bar{L})}}$$

1. Thicker glaciers slower response
2. More neg. SMB faster response

We can go farther and consider a small perturbation in SMB profile, $b(x)$

$$b(x) = \bar{b}(x) + b'$$

$$H\frac{d\bar{L}}{dt} + H\frac{dL'}{dt} = \int_0^{\bar{L}+L'} [\bar{b}(x) + b'] dx$$

$$H\frac{d\bar{L}}{dt} + H\frac{dL'}{dt} = \int_0^{\bar{L}+L'} \bar{b}(x)\,dx + \int_0^{\bar{L}+L'} b'\,dx$$

$$\cancel{H\frac{d\bar{L}}{dt}} + H\frac{dL'}{dt} = \cancel{\int_0^{\bar{L}} \bar{b}(x)\,dx} + \int_{\bar{L}}^{\bar{L}+L'} \bar{b}(x)\,dx + \int_0^{\bar{L}+L'} b'\,dx$$

Again, the steady-state terms go to zero (by definition)

$$H\frac{dL'}{dt} = L'\bar{b}(\bar{L}) + b'\bar{L} + \cancel{b'L'}$$

two small terms multiplied are negigable ($L' \ll \bar{L}$)

Once the glacier reaches a new steady-state $\frac{dL'}{dt} = 0$ and . . .

$$L'\bar{b}(\bar{L}) + b'\bar{L} = 0$$

Put another way

$$\frac{L'}{\bar{L}} = -\frac{b'}{\bar{b}(\bar{L})}$$

$$\boxed{L' = -\frac{\bar{L}}{\bar{b}(\bar{L})}\,b'}$$

Sensitivity of glacier length to a change in SMB

In general $\bar{b}(\bar{L}) < 0$