

PS: Coding Basics

1. The sum \mathcal{S}_k is given by

$$\mathcal{S}_k = \sum_{n=0}^k \frac{(-1)^n}{(2n+1)^7} \quad .$$

- (a) Write a function that calculates the value of \mathcal{S}_k (as a function of k). Output \mathcal{S}_k for $k = 10$, $k = 100$, $k = 1000$ in a for loop.
- (b) Write a new function that calculates the value of \mathcal{S}_k (as a function of k) with only vector operations (i.e. vectorize the code from part a). Output \mathcal{S}_k for $k = 10$, $k = 100$, $k = 1000$.
- (c) One can show that

$$\mathcal{S}_\infty = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^7} = \frac{61\pi^7}{184320} \quad .$$

Write a script with a while loop to determine the *smallest* value of k which fulfills

$$|\mathcal{S}_k - \mathcal{S}_\infty| < 10^{-4} \quad .$$

2. Write a script that solves the following linear system of equations using matrix operations:

$$\begin{aligned} 2x + y - 4z &= -5 \\ 3x - y + 9z &= 5 \\ 5x + 2y + 2z &= -1 \end{aligned} \quad .$$

Give the solution (x, y, z) .