

PS: Ordinary Differential Equations

1. Consider the ordinary differential equation

$$\frac{dx}{dt} = rx - x^3 \tag{1}$$

- (a) Use a numerical ODE solver to solve this problem with $r = 4$ in the above equation and an initial condition $x(t = 0) = 1$ over the time interval $t = [0, 10]$. Plot the numerical solution $x(t)$ over this time interval.
- (b) Use the same method from above, but with the initial condition $x(t = 0) = 3$. Plot the numerical solution.
- (c) Use the same method from above, but with the initial condition $x(t = 0) = -1$. Plot the numerical solution.
- (d) Now use a numerical ODE solver to solve the problem with $r = -4$ in the above equation and an initial condition $x(t = 0) = 1$ and also $x(t = 0) = -1$ over the time interval $t = [0, 10]$. Plot the two numerical solutions $x(t)$ over this time interval on the same plot.
- (e) Discuss the different solutions that occur for different initial conditions, and what happen when r changes from positive to negative.
- (f) Use a symbolic solver to solve for the exact solution to the ODE above, and plot the solution on a plot with a numerical solution for $r = 4$ and $x(t = 0) = 1$.