We may need to interpolate data when…

- Comparing datasets that are defined in different grid system in space and/or in time
- Data points are sparse and/or irregular, and it needs to be placed on regular grid before data analysis/interpretation
- There is a data gap (missing data) that needs to be filled
- Preparing inputs (e.g. boundary conditions) for a model that are continuous, smooth and without gaps

1

Interpolation as a linear operator

- Mathematically, $f_{est}(x) = \Sigma_n w_n f(n)$

2

Interpolation as a linear operator

- Mathematically, $\boxed{f_{est}(x)} = \Sigma_n w_n \boxed{f(n)}$

Output, e.g. estimated temperature on **regular grid**

Input, e.g. observed temperature on **irregular grid**

3

Interpolation as a linear operator

- Mathematically, $\boxed{f_{est}(x)} = \Sigma_n w_n \boxed{f(n)}$

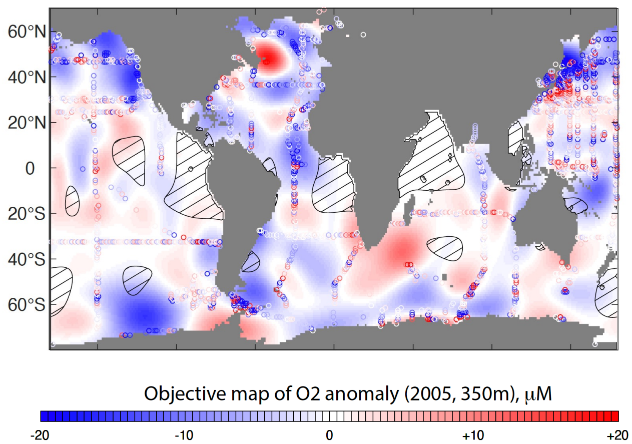
Output, e.g. estimated temperature on **regular grid**

Input, e.g. observed temperature on **irregular grid**

$$\begin{bmatrix} f_{est,1} \\ f_{est,2} \\ \vdots \\ f_{est,m} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{2,1} & \cdots & a_{1,n} \\ a_{1,2} & a_{2,2} & \cdots & a_{2,n} \\ & & \vdots & \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

4

Objective mapping (optimal interpolation)



Objective map of O2 anomaly (2005, 350m), μM

Objective mapping is a least square fit to the irregular, noisy observations, commonly used in meteorology and oceanography

$$f_{est}(x) = \Sigma_n w_n f(n)$$

5

Two dimensional interpolation

- Input/output are in 2 dimensions: f(x,y)
- Example: Input is a temperature on 2° x 2° longitude-latitude grid and the desired output is on 1° x 1° grid.
- Bilinear interpolation → this is what I use for the first try. First, linearly interpolate in one direction, and then linearly interpolate again in the other direction
  - Other options would be nearest neighbor and cubic spline

6

Smoothing

- Real data often contains noise.
- For example, atmospheric data contains high-frequency weather events on top of slowly varying climate signals. Ocean data can contain the effects of tides, waves, eddies, seasonal cycles etc…
- Sometimes, we want to remove the high-frequency “noises” so that we can focus on the slowly varying climate signals.
- Smoothing (filtering) is almost necessary step in data analysis

7

FFT filter

- Fourier series → decomposes the data into linear combination of sine and cosine waves
- Fast Fourier Transform (FFT) can be used to separate the data into different frequency domain, and ”truncate” the data to chop off certain frequency components

In [10]:
```
Yhat = np.fft.fft(Y)    # get Fourier coefficients
N    = np.size(Y)
freq = np.fft.fftfreq(N) # get frequency axis
```

In [11]:
```
pd=1/freq       # calculate the period
mask= (pd < 5) # logical index with period < 5 yr
Yhat[mask]=0
Ylf=np.fft.ifft(Yhat)
```

8