PS: Coding Basics

1. The sum $\mathcal{S}_k$ is given by

$$\mathcal{S}_k = \sum_{n=0}^{k} \frac{(-1)^n}{(2n+1)^7} \quad .$$

   (a) Write a function that calculates the value of $\mathcal{S}_k$ (as a function of k). Output S_k for $k = 10$, $k = 100$, $k = 1000$ in a for loop.

   (b) Write a new function that calculates the value of $\mathcal{S}_k$ (as a function of k) with only vector operations (i.e. vectorize the code from part a). Output S_k for $k = 10$, $k = 100$, $k = 1000$.

   (c) Add a capability to your script to write both, the index k and the value of S_k in a comma-separated file *drl.csv*.

   (d) One can show that

$$\mathcal{S}_\infty = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^7} = \frac{61\pi^7}{184320} \quad .$$

   Write a script with a while loop to determine the *smallest* value of k which fulfills

$$|\mathcal{S}_k - \mathcal{S}_\infty| < 10^{-4} \quad .$$

2. Write a script that solves the following linear system of equations using matrix operations:

$$\begin{aligned} 2x + y - 4z &= -5 \\ 3x - y + 9z &= 5 \\ 5x + 2y + 2z &= -1 \quad . \end{aligned}$$

   Give the solution (x, y, z).

3. Consider the 3-D surface

$$z(x, y) = \sin(\pi x + 2\pi y) + e^{-5y^2}$$

   (a) Create a mesh for x and y over the intervals $x = [0, 1]$ and $y = [0, 3]$ with 101 grid points in x and 301 grid points in y.

   (b) Create a new array calculating values of z on this mesh.

   (c) Find every mesh point where $0.5 < z(x, y) < 1$ within the domain $x = [0, 1]$ and $y = [0, 1]$ on the mesh created in part (a).

   (d) Calculate the maximum value of a new function $w(x, y)$ within the part of the mesh where 0.5<z(x,y)<1, where

$$w(x, y) = \cos(\pi x + \pi y) + e^{-2y^2}$$