GRAIN SIZE ANALYSIS OF SOILS

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Lecture Topics

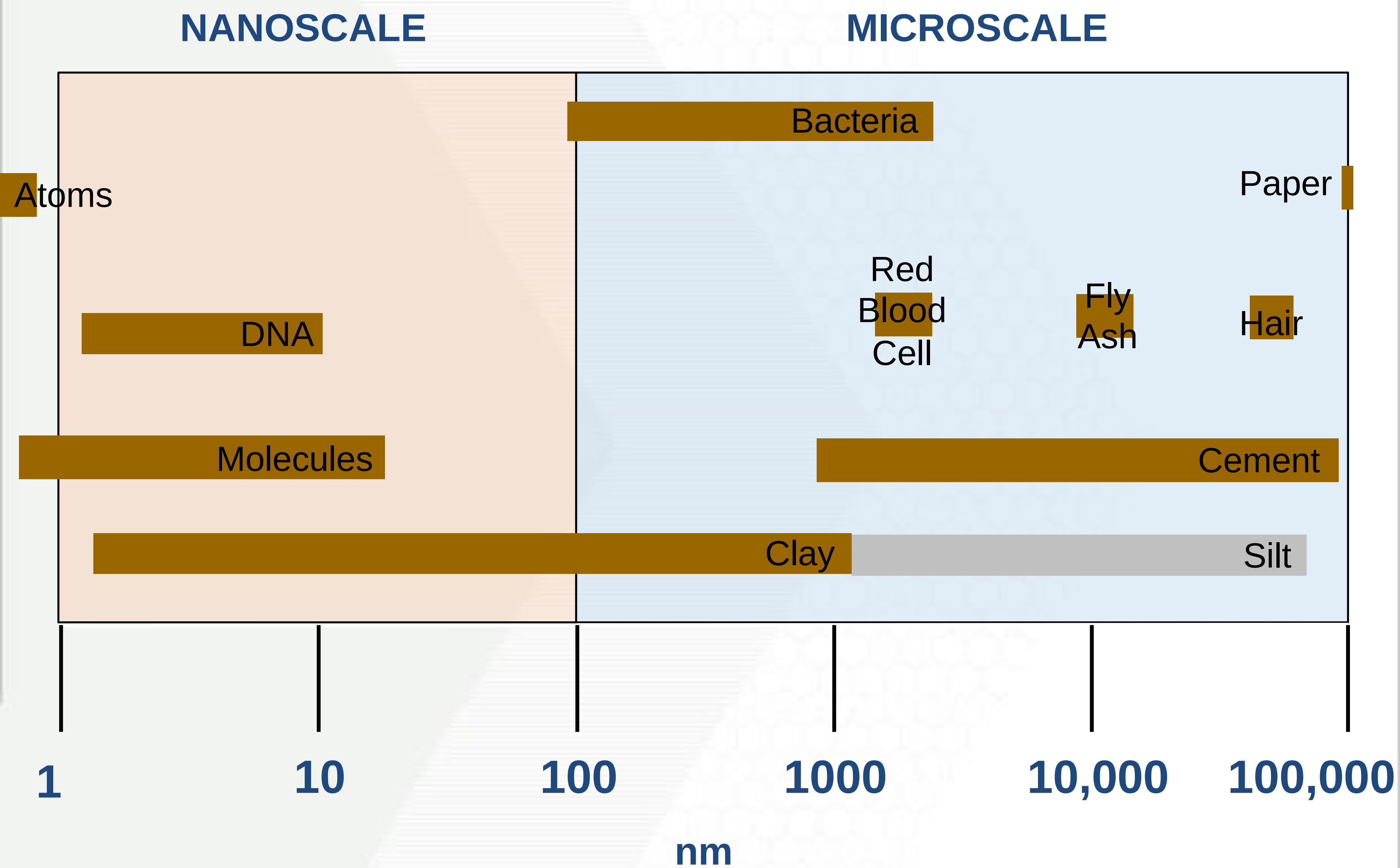
- Introduction
- Grain size distribution
- Characterization methods
  - Coarse grains: sieve analysis
  - Fine grain: Hydrometer

    - Stokes' law
    - Drag
- Grain size distribution curves
- Grain size coefficients

Grain Size Distribution of Soils

- One of the first things we want to characterize about a soil is the range of particle sizes that are present
- Grain sizes can vary over approximately eight orders of magnitude

Soil	Size (mm)	Size (nm)	Size (in)
Boulder	750 mm	7.5×10^8 nm	30 in
Gravel	75 mm	7.5×10^7 nm	3 in
Sand	0.75 mm	7.5×10^5 nm	0.03 in
Silt	0.075 mm	7.5×10^4 nm	0.003 in
Clay	0.0000075 mm	7.5 nm	3×10^{-7} in



1 nm = 0.000000001 m = 1×10^{-9} m = 1 billionth of a meter

Characterization

- Range of sizes and mass of particles in each size range
- We use two different methods to mechanically characterize the soils:
  - Sieve analysis
    - Coarse grained particles
    - > 0.075 mm
  - Hydrometer
    - Fine grained particles
    - <0.075 mm

Characterization: Sieve Analysis

- Sieve Analysis
  - Shake the soil samples through series of screens {sieves}
    - progressively smaller openings
  - Measure the amount of soil retained
  - Convert to a plot of % passing versus particle size
  - Plot on log scale because range of particle sizes is so large}
  - Assumes that the particles are discrete
  - Reports number as an "equivalent" diameter
  - Particles are not perfect spheres

U.S. Standard Sieve Sizes	
Sieve No.	**Opening (mm)**
4	4.75
5	**4.00**
6	**3.35**
7	**2.80**
8	**2.36**
10	**2.00**
12	**1.70**
14	**1.40**
16	**1.18**
18	**1.00**
20	**0.850**
25	**0.710**
30	**0.600**
35	**0.500**
40	**0.425**

U.S. Standard Sieve Sizes	
Sieve No.	**Opening (mm)**
45	**0.355**
50	**0.355**
60	**0.250**
70	**0.212**
80	**0.180**
100	**0.150**
120	**0.125**
140	**0.106**
170	**0.090**
200	**0.075**
270	0.053
400	0.038

Gravel – larger than #4 sieve

Sand

Silt and clay – finer than #200

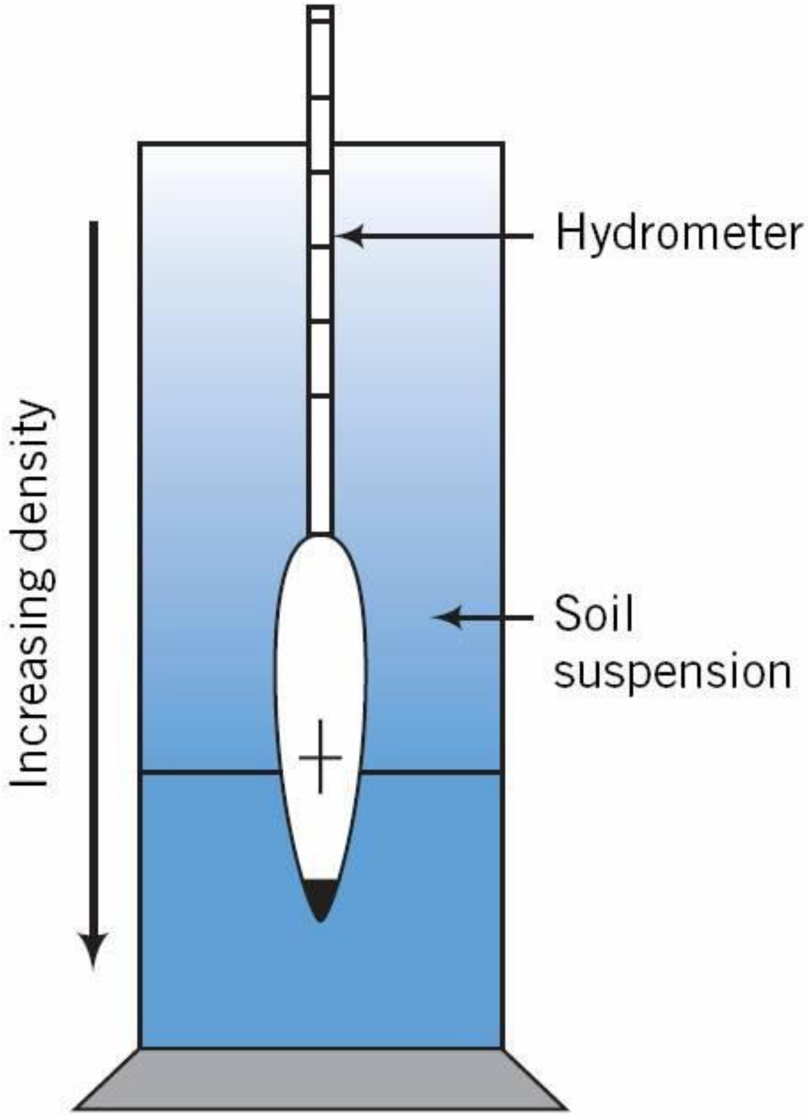
Sieve Analysis

Characterization: Hydrometer Analysis

- Used when the particle sizes are too small to separate by shaking

- Clay sized particles have significant attractive forces when dry, so:
  - Use sedimentation through water for analysis
  - Particles are placed in water settle as a function of shape, size, weight, and the viscosity of the water
  - Assume the soil particles are all spheres and they settle according to Stokes' Law

Hydrometer



Stokes' Law

- Two major forces acting on a particle suspended in water
  - Gravitational $f_g = \rho_s g V_p$
    - where ρ_s = the density of the particle
    - g = the gravitational constant
    - V_p = the volume of the particle
  - Buoyancy $f_b = \rho_w g V_p$
    - where ρ_w = the density of water
- Forces of buoyancy and gravity act in opposite directions:
  - if the density of the particle is equal to the density of water, the particle will not move relative to the water
  - If the particle is more dense than water, then the net force is greater than zero, and the particle will start to accelerate

$$F_I = f_{net} = (\rho_s - \rho_w) g V_p$$ Driving Force

Drag Force Generated by acceleration

- Once the particle starts to move, another force is generated:
  - Drag force $F_D = C_D A_C \rho_w \frac{V_s^2}{2}$
  - where C_D = the drag coefficient
  - A_c = the cross-sectional area of the particle
  - V_s = the particle settling velocity
- Note: the drag force:
  - Acts opposite to the driving force
  - Increases with the square of the velocity
- Eventually a steady "terminal" velocity will be reached
- For example, a human free fall in atmosphere
  - Roughly 200 km/hr or 120 miles/hr (belly to earth)
  - = ~10% of that with parachute (20 km/hr or 12 mph)
  - Parachute increases drag and surface area resisting fall
- Terminal velocity occurs when the driving force = the drag force

$$(\rho_s - \rho_w) g V_p = C_D A_c \rho_w \frac{V_s^2}{2}$$

Hydrometer Analysis

- Assume clay particles are falling through water at terminal velocity

$$(\rho_s - \rho_w) g V_p = C_D A_c \rho_w \frac{V_s^{\,2}}{2}$$

- And are spherically shaped:

$$\frac{V_p}{A_c} = \frac{{}^{4}\!/_{3}\,\pi \left(\frac{d}{2}\right)^3}{\pi \left(\frac{d}{2}\right)^3} = \frac{2}{3} d$$

- Substitute to describe the terminal velocity of a sphere falling in a fluid

$$V_s^{\,2} = \frac{4g(\rho_s - \rho_w) d}{3 C_D\, \rho_w}$$

Drag Coefficient

- For particles falling in water, we define the drag coefficient as a function of the Reynolds Number:

$$N_{Re} = \frac{\phi V_S \rho_W d}{\mu}$$

- For our purposes:
  - NRe<1 is laminar
  - NRe > 104 is turbulent
  - ϕ = shape factor for non-spheres (1.0 for spheres)
  - μ = dynamic viscosity
  - $\rho_W = \gamma / g$

- Assume laminar flow

$$C_D = \frac{24}{N_{\mathrm{Re}}}$$

http://www.wright- brothers.org/Adventure_Wing/Han gar/1901_Wind_Tunnel/1901_WindTunnel.htm

Derivation of Stokes' Law

- Assuming laminar flow, spherical, non-aggregating particles, combine:

$$V_s^2 = \frac{4g(\rho_s - \rho_w)d}{3C_D\rho_w}$$

$$C_D = \frac{24}{N_{\text{Re}}}$$

$$N_{Re} = \frac{\phi V_S \rho_w d}{\mu}$$

- Which yields Stokes' Law:

$$V_s = \frac{g(\rho_s - \rho_w)d^2}{18\mu}$$

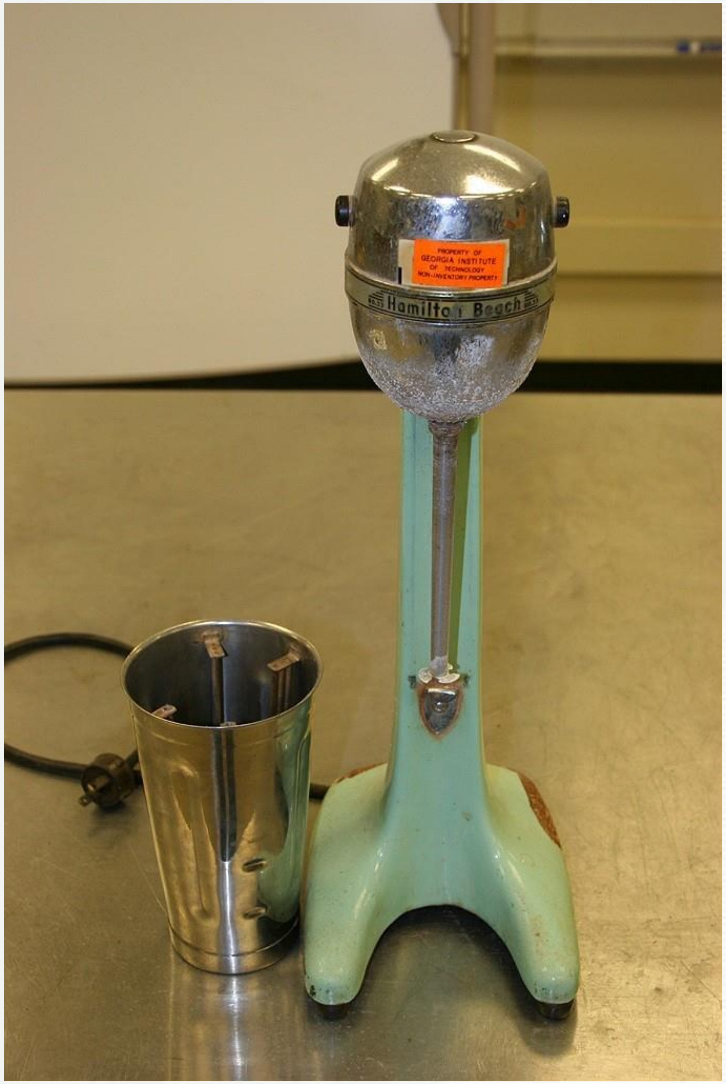
Particle Settling Velocity

- From Stokes' Law, we see particle settling velocity is a function of:
  - Particle density
  - Fluid densities
  - Particle diameter
  - Fluid viscosity

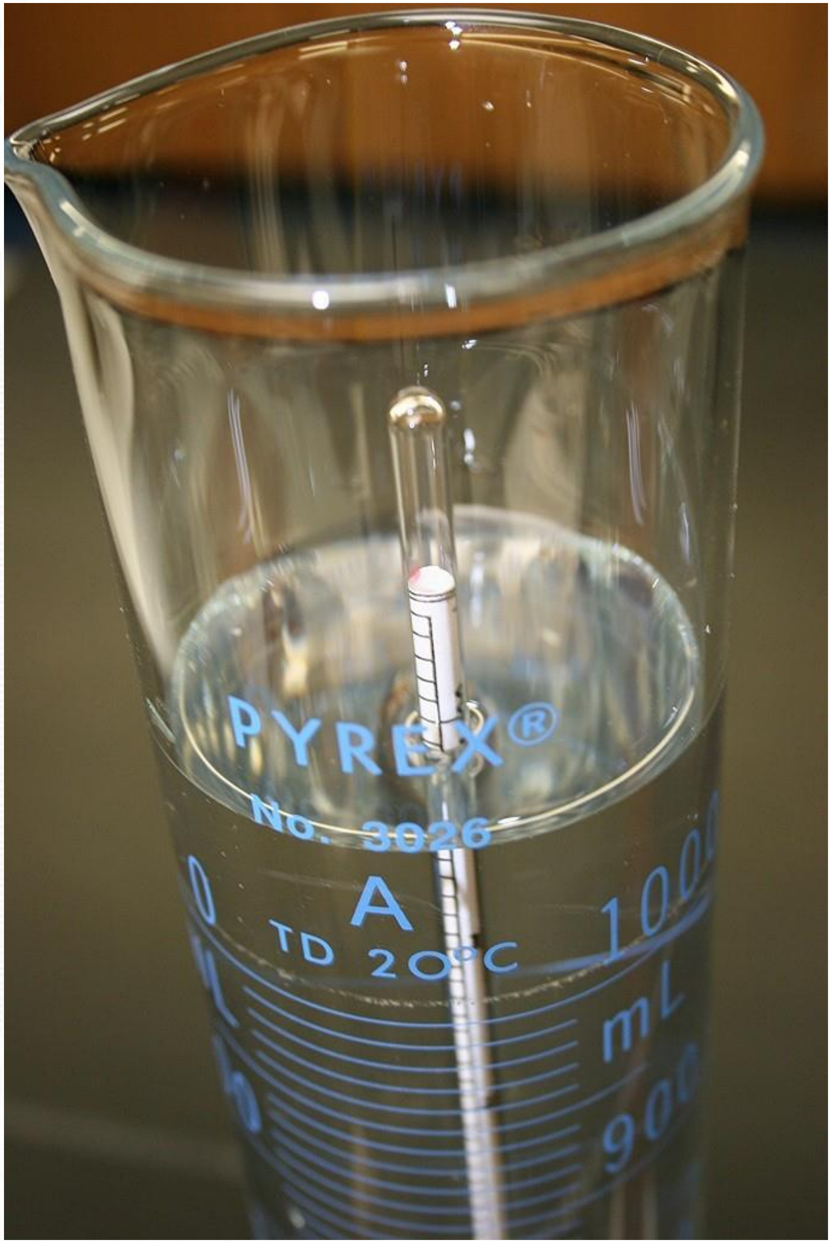
Hydrometer Test

- Suspend fine grain soil in water
- Add dispersant
- Measure density of suspension as a function of time
- Convert density to settling velocity
- Convert settling velocity to particle size

Hydrometer Test



Hydrometer Test



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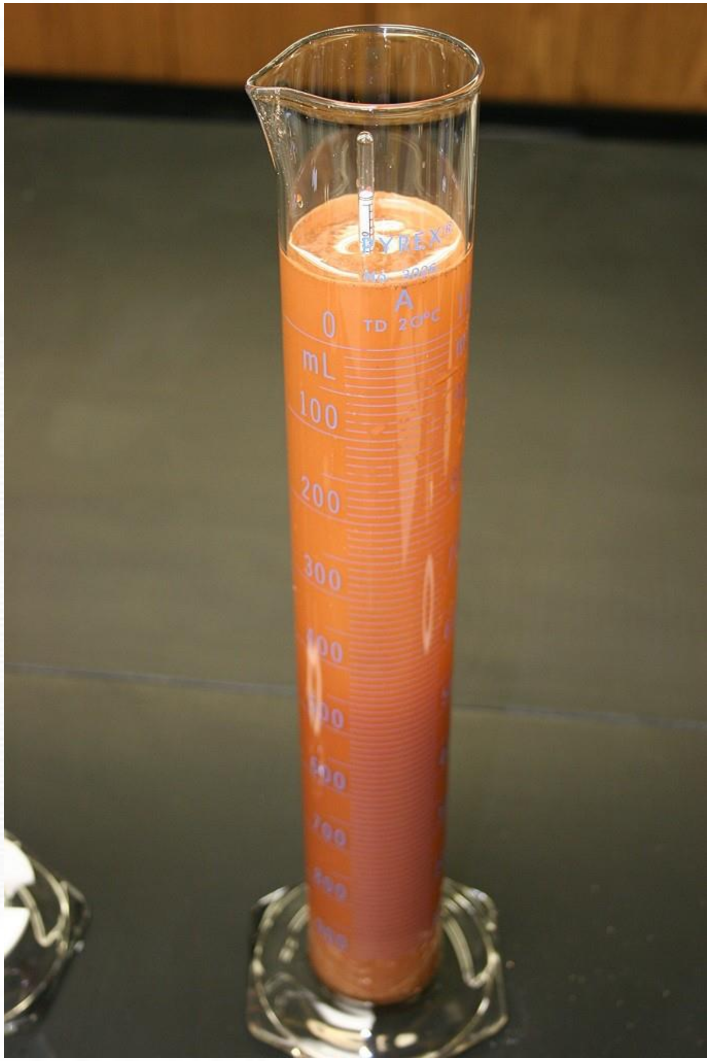

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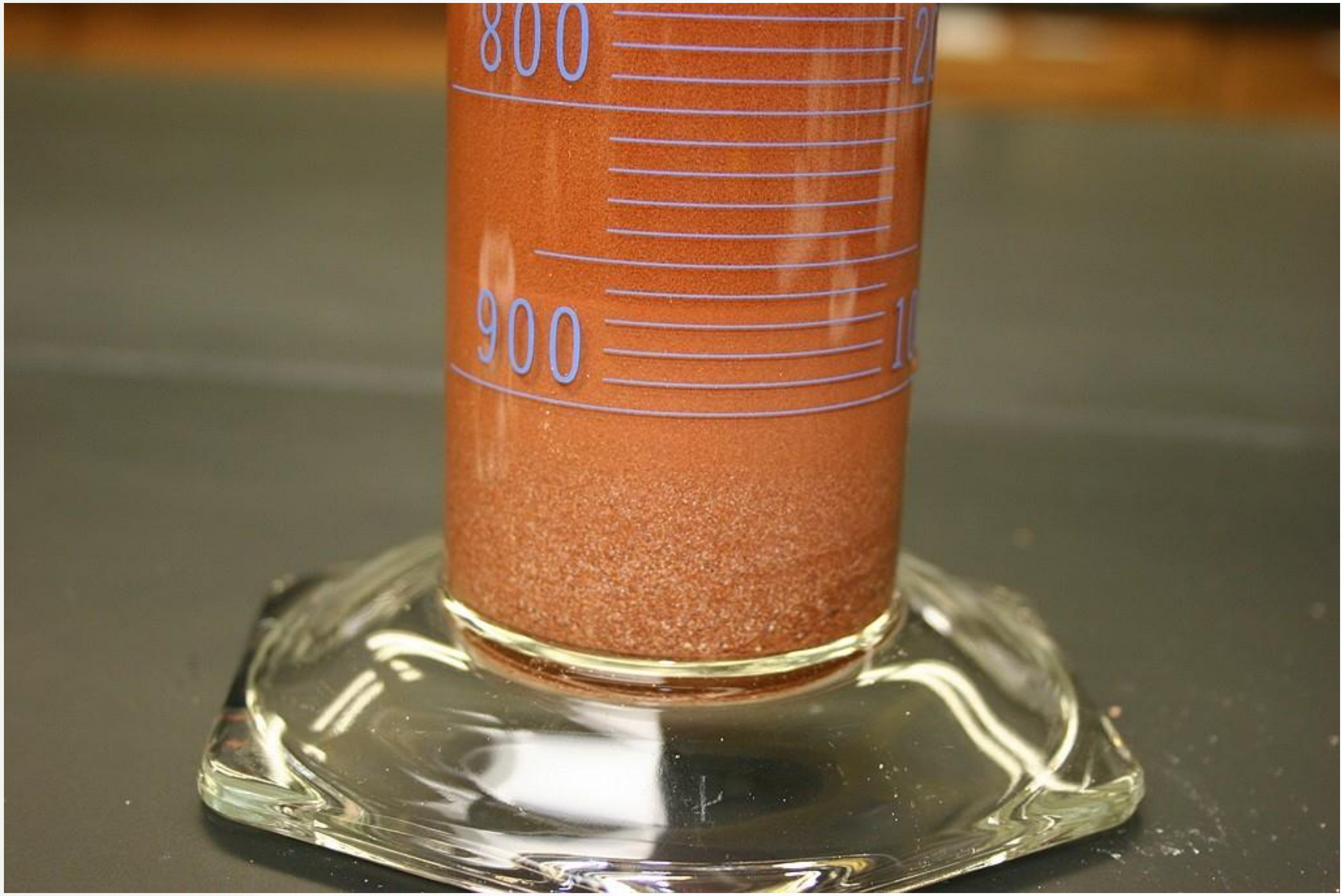
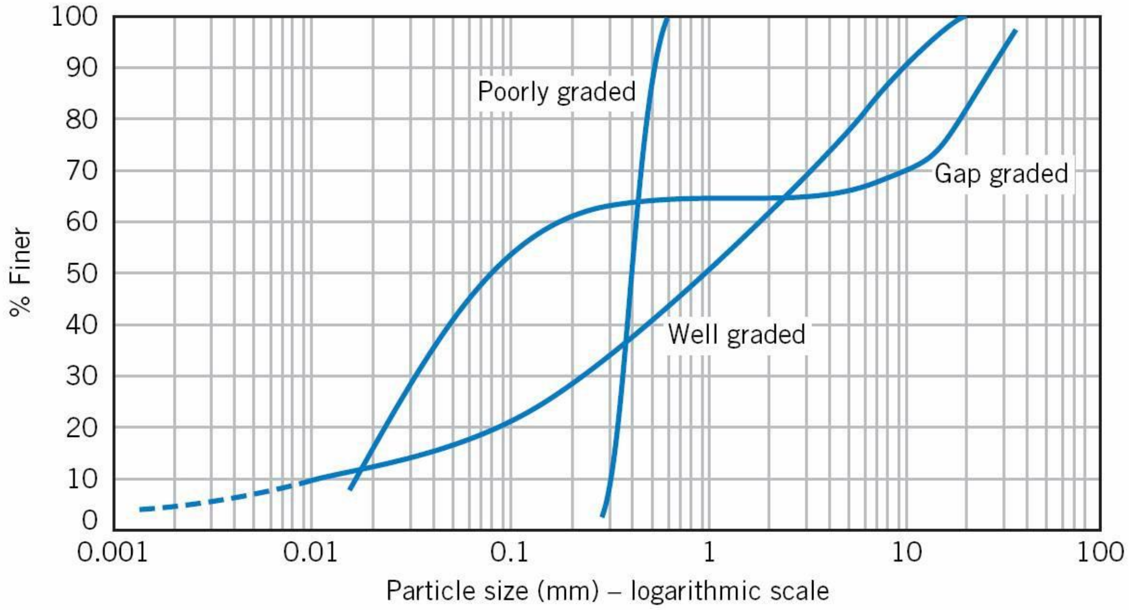


Grain Size Distribution

- Determine % retained on each screen in the sieve analysis test; convert to % finer
- Determine % finer from hydrometer test
- Plot the % finer versus the particle diameter on a log scale.
- Often plot sieve and hydrometer on the same graph

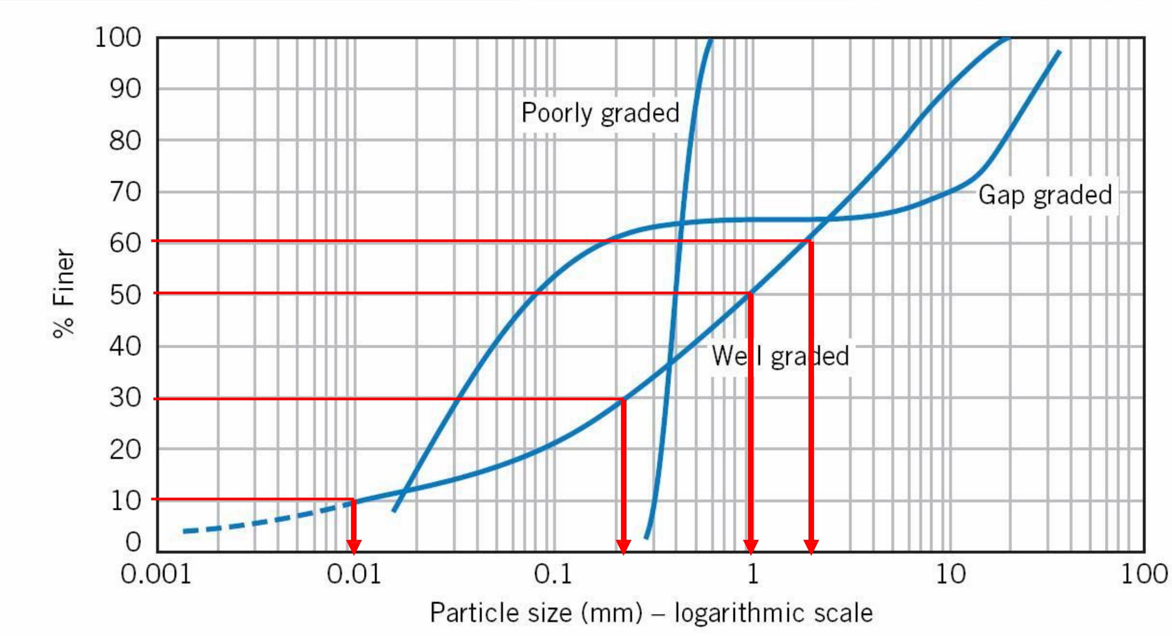
Grain Size Distribution Curves

- Poorly graded or uniform - most of the particles are one size
- Well graded - wide range of particle sizes; no concentration of particles in one size; no gaps in the grain size
- Gap graded - two predominant sizes – many large and small particles but few intermediate sizes



Characterization From Particle Size

- From the grain size distribution curve:
  - Effective size: D10 - particle diameter at which 10 % of the soil is finer
    - important in fluid flow problems
  - D_{50} - median grain size (50% finer)
  - D_{30} D_{60} used to quantify distribution of sizes



GDS Coefficients

- Uniformity coefficient
  - $C_u = D_{60}/D_{10}$
  - Measure of the spread in the range of grain sizes
  - If $C_u = 1$, only one grain size
  - As Cu increases, range of particle sizes increases
  - Cu > 4 → large particles form soil structure, small particles fill the voids
- Coefficient of curvature
  - $C_c = (D_{30})^2/(D_{60}xD_{10})$
  - Measure of the spread in the range of grain sizes
  - C_c between 1 and 3 indicates a well graded soil if $C_u > 4$ for gravels and > 6 for sands
- Also, can determine the % of gravel, sand, silt, and clay from grain size distribution plots