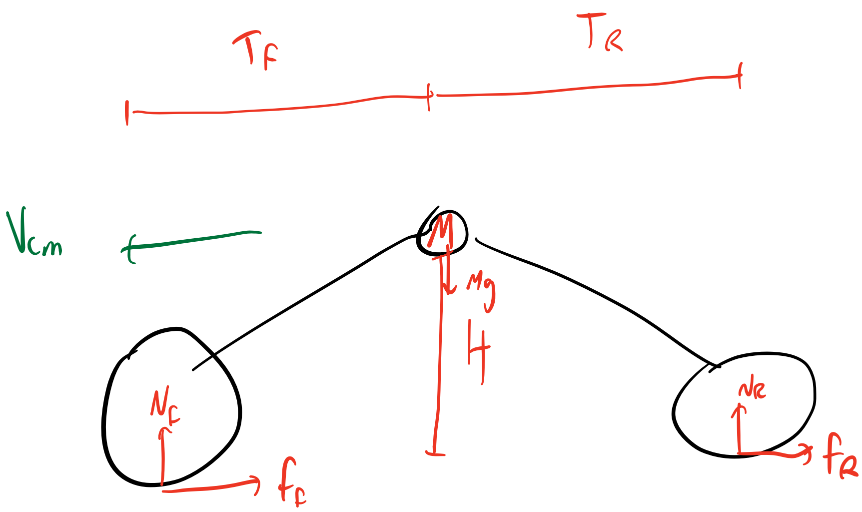
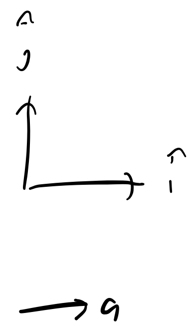
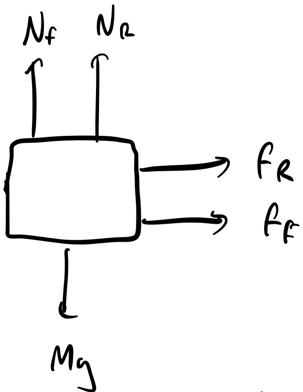
Part 1

Simplified Bicycle Model Analysis - Brake Bias

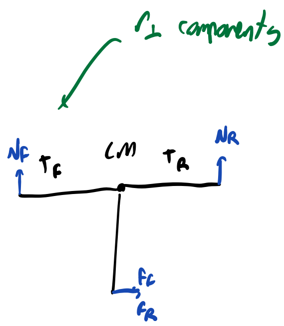




FBD, about CM



Torque, about CM



	$\sum F =$	Ma	
$\hat{i}$:	$f_F + f_R =$	$Ma\hat{i}$	(1)
$\hat{j}$	$N_F + N_R - Mg =$	$\cancel{Ma\hat{j}} \rightarrow 0$	(2)

$\sum \tau = \; r \times f = I\alpha$ (about CM)

$(T_R\hat{i} + N_R\hat{j}) + (-T_F\hat{i} \times N_F\hat{j}) +$ $(-H\hat{j} \times (F_F + F_R)\hat{i})$	$= 0$

(car is not rotating - at least hopefully not)

$$T_R N_R - T_F N_F + H(f_F + f_R) = 0 \quad (3)$$

Brake Bias:

$\frac{N_F}{N_R} = r$ ← ratio between normal forces

or, $N_F = r N_R$ (4)

Knowns:

converts N to f based on data

M, g, T_F, T_R, H, $f_L(x)$, $f_N(x)$ ← converts f to N

Unknowns:

$a\hat{i}$, f_F, f_R, N_F, N_R

Really only 3 unknowns since $f_R = 2 \cdot F_L\left(\frac{N_R}{2}\right)$ and vice versa from the FBD we can extract the following two equations:

$\hat{i}$: $F_f + F_R = Ma\hat{i}$ (1)

$\hat{j}$: $N_F + N_R = Mg$ (2)

We next define a normal force bias ratio term r, with the range of 0-1.

when $r=0$, $N_R = 0$ and $N_F = Mg$ (all weight is in front)

when $r=1$, $N_R = Mg$ and $N_F = 0$ (all weight is in rear)

Therefore,

$$N_R = M \cdot g \cdot r \quad , \quad N_F = M \cdot g \cdot (1-r) \quad (3,4)$$

Next, using the existing data we have to convert from normal forces to longitudinal forces, we get:

$$f_F = 2 \cdot f_L\left(\frac{N_F}{2}\right) \quad (5)$$

$$f_R = 2 \cdot f_L\left(\frac{N_R}{2}\right) \quad (6)$$

(Note: we are using a simplified bicycle model, so the mul/div accounts for this).

Putting equations together, we yield an expression for deceleration of the car:

$$a\hat{i} = \frac{2}{M}\left[f_L\left(\frac{M \cdot g \cdot (1-r)}{2}\right) + f_L\left(\frac{M \cdot g \cdot r}{2}\right)\right] \quad (A)$$

One issue: the car may rotate if we apply an acceleration calculated by (A). We can solve the balance of torques for acceleration, then see where the two are equal, to find the point of maximum deceleration.

First, calculating front and rear wheel base length:

when (A) and (B) are equal, we know that the car is at its maximum deceleration (and won't rotate).

Assumption: The tires are braking at their maximum rate longitudinally of $2 \cdot f_i \left(\frac{N_R}{2}\right)$ at a normal force of N_R and won't slip.

for My21:

front-Rear split:

0.752 - 0.248

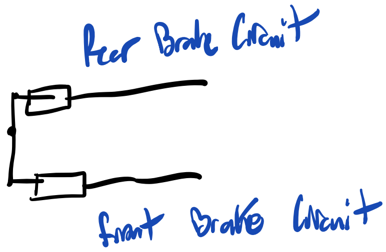
Max decel:

14.823 m/s²

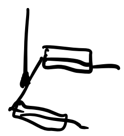
Front & Rear Loop Diagram

Bias Bar

Top View

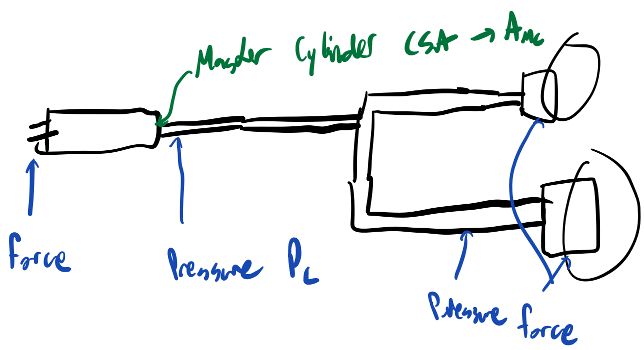


Side View



Loop

Master cylinder (from bias bar)



Requirements:

$R_p < 3.4$

$R_B \sim 0.5$

$$\frac{P_R}{P_F} = \frac{L_R}{L_F}$$

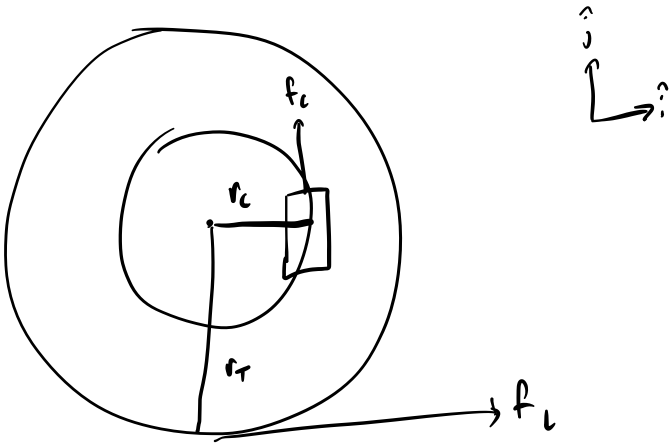
Can adjust:

A_{mc} (master cylinder area)

Part 2

Caliper Force Analysis

Tire Model



f_c = caliper force

r_c = caliper radius

r_T = tire radius

f_L = longitudinal braking force

Balance of Torques:

$r \times f =$	$\sum T = 0$
$(r_c \hat{i} \times f_c \hat{j}) + (-r_T \hat{j} \times f_L \hat{i}) = 0$	
$r_c \cdot f_c + r_T \cdot f_L = 0$	

Therefore,

$f_c = -\frac{r_T \cdot f_L}{r_c}$ => Caliper longitudinal force on brake pad

$f_c = \mu_c \cdot N_c \quad \therefore$

$N_c = -\frac{r_T \cdot f_L}{r_c \cdot \mu_c}$ => Caliper normal force on brake pad

$N_c = A_c \cdot P_c \quad \therefore$

Radius of tire

Longitudinal force

$$P_L = \frac{-r_T \cdot F_L}{r_C \cdot \mu_C \cdot A_C}$$ => Line pressure required at caliper

Caliper CSA

Coeff. of friction of caliper

Radius of caliper

$$P_L = \frac{R_F}{A_{MC}}$$

Reaction force of master cylinder against bias bar

CSA of master cylinder

Pressure front and rear, derived:

Tire radius

Front longitudinal force

$$P_F = \frac{r_T \cdot f_F}{r_C \cdot \mu_C \cdot (2 \cdot \pi \cdot r_p^2)}$$

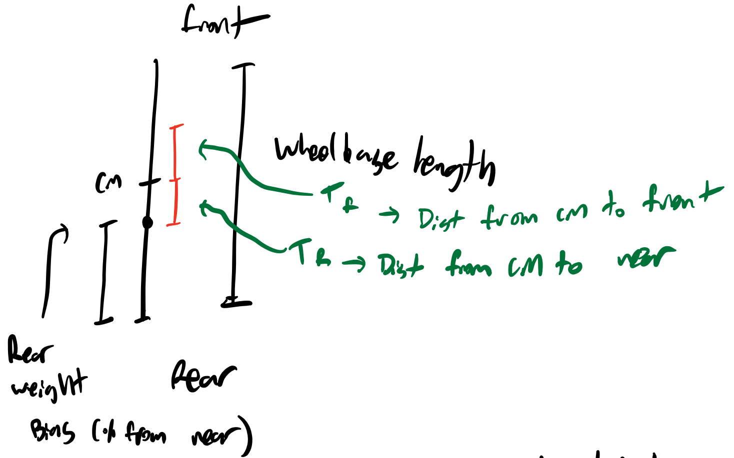
Radius of caliper piston

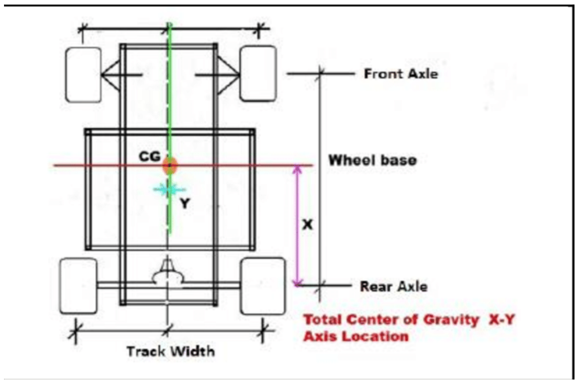
Radius of calpr

CoF of caliper

Caliper piston cont

$$P_R = \frac{r_T \cdot f_R}{r_C \cdot \mu_C \cdot (2 \cdot \pi \cdot r_p^2)}$$





Therefore,

wheelbase length, total

Rear weight Bias

$$T_F = W_{tot} \cdot (1 - W_{RWB})$$

$$T_R = W_{tot} \cdot W_{RWB}$$

Then, solving balance of torques:

$$f_F + f_R = M a_{\hat{i}}$$

Height of CM

$$T_R N_R - T_F N_F + H(f_F + f_R) = 0$$

$$T_R N_R - T_F N_F + H(M a_{\hat{i}}) = 0$$

$$N_R T_R - N_F T_F + H M a_{\hat{i}} = I \alpha = 0$$

$$a_{\hat{i}} = \frac{N_R T_R - N_F T_F}{-HM}$$

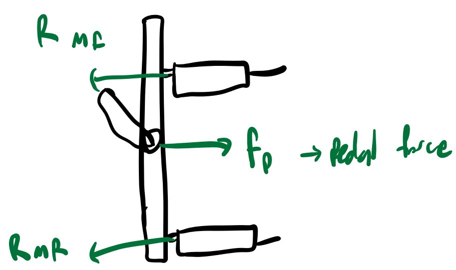
Putting everything together,

$$a_{\hat{i}} = \frac{\left[(M \cdot g \cdot r) \cdot (W_{tot} \cdot W_{RWB})\right] - \left[(M \cdot g \cdot (1-r)) \cdot (W_{tot} \cdot (1 - W_{RWB}))\right]}{-HM} \qquad (B)$$

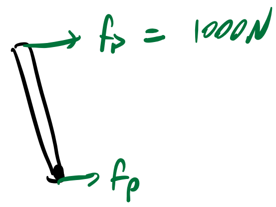
Part 3

Bias Bar Analysis

Top View



Pedal



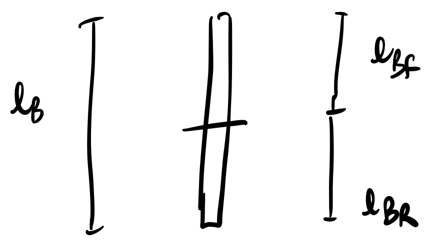
$$f_{ID} \cdot R_p = \underbrace{R_{MF} + R_{MR}}_{\text{Master cylinder forces}} \qquad (1)$$

↑ Driver pedal force

↑ Pedal ratio

Defining Bias Bar Ratio

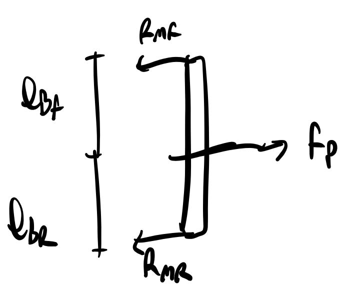
Bias Bar, Top View



$$l_{BF} = l_B \cdot R_B$$

$$l_{BR} = l_B \cdot (1 - R_B)$$

Balance of Torques



$$r \times F = \sum T = 0$$

$$(l_{BF}\,\hat{i} \times -R_{mF}\,\hat{j}) + (l_{BR}\,\hat{i} \times -R_{mR}\,\hat{j}) = 0$$

$$(l_{BF} \cdot R_{mF}) - (l_{BR} \cdot R_{mR}) = 0$$

Therefore,

$$l_B \cdot R_B \cdot R_{mF} = l_B \cdot (1 - R_B) \cdot R_{mR} \quad (2)$$

Assumption:

$$\frac{P_R}{P_F} = \frac{L_R}{L_F} \Rightarrow \quad P_R \cdot A_c = N_R$$

$$N_R \cdot \mu = f_c$$

$$f_c = \frac{r_R}{r_T} = L$$

Line pressure

$$R_{mF} = P_F \cdot A_{mc,F} \leftarrow \text{Master cylinder area}$$

$$R_{mR} = P_R \cdot A_{mc,R}$$

Reaction force of master cylinder against bar

∴ Using (2) to calculate Bias Bar Ratio

$$\cancel{l_B} \cdot R_B \cdot P_F \cdot A_{mc,F} = \cancel{l_B} \cdot (1 - R_B) \cdot P_R \cdot A_{mc,R}$$

$$\frac{R_B}{1 - R_B} = \frac{P_R \cdot A_{mc,R}}{P_F \cdot A_{mc,F}} \Rightarrow \text{Define constant } C \text{ as } \left[\frac{P_R \cdot A_{mc,R}}{P_F \cdot A_{mc,F}}\right]$$

$\therefore$

$$\frac{R_B}{1-R_B} = C \rightarrow R_B = C(1-R_B)$$

$$R_B = C - C \cdot R_B$$

$$R_B + CR_B = C$$

$$R_B(1+C) = C$$

$$R_B = \frac{C}{1+C}$$

Rear Pressure

Master cylinder area, rear

Front pressure

Master cylinder area, front

$$R_B = \frac{\dfrac{P_R \cdot A_{mc,R}}{P_F \cdot A_{mc,F}}}{\left[1 + \dfrac{P_R \cdot A_{mc,R}}{P_F \cdot A_{mc,F}}\right]}$$

Bias bar ratio

$\therefore$ Using (1) to calculate Pedal Ratio

$$F_{ID} \cdot R_P = (A_{mc,F} \cdot P_F) + (A_{mc,R} \cdot P_R)$$

1200 N

Front pressure

Rear pressure

Master cylinder area, rear

$$R_P = \frac{(A_{mc,F} \cdot P_F) + (A_{mc,R} \cdot P_R)}{\ldots}$$

F_{ID} ← Ideal driver force

Pedal ratio

Master cylinder area, front