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A three-dimensional nodal method with Channel-wise Intrinsic Axial Mesh Adaptation



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ABSTRACT

In a conventional coarse mesh nodal method the more accurate treatment of intra-nodal axial heterogeneity requires iterative axial node re-homogenization using axial flux profiles either reconstructed from core-wise coarse mesh solution or obtained from channel-wise axial fine mesh calculation. In this paper a new nodal method formulation, using Channel-wise Intrinsic Axial Mesh Adaptation (CIAMA), is proposed to solve this problem in a more fundamental way. For a given transverse (radial) leakage, along each axial channel a rigorous sub-node heterogeneous calculation is performed with the explicit axial heterogeneity within each coarse axial node. However, the transverse leakage between the axial channels is still calculated on the basis of coarse axial nodes, using the axially averaged radial current in each coarse axial node. Since the coupling between the axial channels is through the coarse axial nodes, it is not necessary to match the boundaries of the axial sub-nodes of neighboring axial channels in order to incorporate the axial sub-node calculation as an intrinsic part of the whole core global calculation. Therefore in the CIAMA nodal method, each axial channel is allowed to have its own sub-nodes adapting to its own axial heterogeneity variation. The CIAMA method has been implemented in the commercial code EGRET, which is used to qualify CIAMA. Excellent results of modeling fuel grid and control rod movement are presented. Application of CIAMA to three-dimensional pin-by-pin core calculation is also discussed and demonstrated to work well.

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1. Introduction

Three-dimensional (3D) nodal diffusion code has become the standard tool for modeling a commercial light water reactor core. It handles the radial heterogeneity of a fuel assembly or fuel bundle via the node homogenization and the pin power reconstruction process. The axial heterogeneity is relatively less severe and can be treated in a similar way, except for the complication of the variable axial geometry caused by the movement of control rods or control blades. Particularly in cases of BWR or the advanced design of Westinghouse AP1000 PWR, the control blades or control rods constantly move during normal operation. The axial heterogeneity variation is usually treated either with relatively crude approximate models or more sophisticatedly with online axial node re-homogenization such that the global 3D coarse-mesh nodal solution and the channel-wise axial 1D fine-mesh solution are performed iteratively (Bahadir and Lindahl, 2009; Boyd et al., 2009).

In a recently published work (Lima et al., 2012), an interesting idea was proposed to retain explicitly the axial node interior heterogeneity in the conventional nodal expansion method (NEM) formulation. NEM assumes certain flux expansion base functions and determine the expansion coefficients by applying the weighted residue method to the transversely integrated 1D nodal diffusion equation. When applying the weighted residue method to the 1D diffusion equations, integration on each term in the equation over each node is performed to project out the 1D flux expansion coefficients. In the above cited paper, the axial node interior cross-section heterogeneity is explicitly represented by step functions such that the weighted residue integral over the node contains products of step functions and flux base functions, which can be analytically evaluated. This way the axial node interior heterogeneity is intrinsically contained in the NEM matrix equations for the flux expansion coefficients. Subsequently Li (2014) adopts the same method but keeps only the average value of the step function in the reaction term on the left side of the diffusion equation and moves the deviation from the average to the source term on the right side of the diffusion equation. Both papers show good results of using the new NEM formulation. This new NEM still uses coarse axial

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node and assumes the same traditional expansion base functions, which cannot correctly capture the details of the node interior axial flux shape. It is important to note that in this new NEM formulation, albeit the incorporation of the axial node interior heterogeneity, the transverse (radial) leakage between adjacent axial nodes is still based on the axially averaged radial current in each axial coarse node. However since the transverse leakage effect is secondary, this approximation is expected to be quite good.

Inspired by the new NEM formulation described in the above, we hereby present a new 3D nodal method formulation that solves this axial heterogeneity problem in a more fundamental way. Under the assumption of a given transverse (radial) leakage, along each axial channel a rigorous sub-node heterogeneous calculation is performed with the explicit axial heterogeneity within each coarse axial node. However, the transverse (radial) leakage between the axial channels is still calculated on the basis of coarse axial nodes, using the axially averaged radial current in each coarse axial node. Since the coupling between the axial channels is through the coarse axial nodes, it is not necessary to match the boundaries of axial sub-nodes of neighboring axial channels in order to incorporate the axial sub-node calculation as an intrinsic part of the 3D global calculation. Therefore each axial channel is allowed to have its own sub-nodes adapting to its own axial heterogeneity variation. In this new 3D nodal method each coarse node is radially homogeneous but may be axially heterogeneous with sub-nodes adapting to its exact axial geometry. The transversely integrated 1D nodal equations are solved in the radial directions using coarse nodes, while in the axial direction using channel dependent sub-nodes. We call this new 3D nodal method the Channel-wise Intrinsic Axial Mesh Adaptation (CIAMA) method.

In Section 2 the detailed formulation of CIAMA will be given. The CIAMA nodal method has been implemented in the commercial code EGRET of Shanghai NuStar Nuclear Power Technology Co., Ltd. (Zhang et al., 2014a), which is used to obtain the qualification results in this paper. Section 3 presents the qualification results of CIAMA for the two cases of fuel grid modeling and the cusping effect of partially inserted control rod. Section 4 discusses the CIAMA application to 3D pin-by-pin calculation and its qualification for a BWR mini-core problem. Section 5 concludes the paper.

2. Implementation of CIAMA

The CIAMA method is implemented in the EGRET code within the general framework of the semi-analytic nodal method (SANM) for solving the multi-group neutron diffusion equation (Zimin et al., 1998). Without reproducing the details of SANM that can be found in the quoted reference, only a brief review summary will be given here for the completeness of the discussion. For each energy group, the transversely-integrated 1D nodal flux is approximately expanded in low-order Legendre polynomials plus hyperbolic functions, which are the "analytic" solutions to the homogeneous part of the "fixed source" diffusion equation. With the tentatively given (and iteratively updated) parabolic profile of transverse leakage, all the unknown group-wise nodal flux expansion coefficients can be expressed in terms of the node surface net currents via the weighted residue method,

$$a_n = C(J^R \pm J^L) + \sum_{i=1}^2 (A_i q_i + B_i l_i), \tag{1}$$

where a_n is the nth term nodal expansion coefficient, q_i the ith order neutron source expansion coefficient, l_i the ith order coefficient of transverse leakage profile, J^R and J^L net currents on the right and left surfaces of the node, A, B and C are coefficients related to the nodal macroscopic cross sections and mesh size.

Substituting Eq. (1) back into the semi-analytic expansion expression of transversely-integrated 1D nodal flux, one may straightforwardly relate nodal surface average fluxes to the nodal surface average net currents. Then, by applying the following continuity condition of nodal surface fluxes,

$$f_k^R \phi_k^R = f_{k+1}^L \phi_{k+1}^L, \tag{2}$$

a matrix equation can be derived with the nodal surface net currents in one coordinate direction as the only unknowns,

$$E_k J_k^L - (F_k + F_{k+1}) J_k^R + E_{k+1} J_{k+1}^R = G_k + H_{k+1},$$
(3)

where node k and k+1 are two neighboring nodes, E and F are nodal coefficients, which only depend on nodal nuclear and geometric parameters, G and H are source related terms, including neutron source and the extra transverse leakage source.

Similar algebraic equations can also be derived for the other two coordinate directions and these coupled equations are solved by alternatively sweeping in three coordinate directions. All the interface currents along the same coordinate direction are solved simultaneously by inverting the tri-diagonal sub-matrix coupling these currents. The obtained surface currents on any three adjacent surfaces are used to update the parabolic profiles of the transverse leakage in the other two coordinate directions. After the alternate-direction neutron current sweeping process is terminated, the nodal average group flux is updated and the source terms contributed by this group are also updated. Starting from the highest energy group, the 3D multi-group eigenvalue problem is solved by employing the conventional source iteration method.

Adopting the alternate-direction current sweeping method makes the implementation of CIAMA straightforward. The only theoretical issue left is how to construct the parabolic profile of transverse leakage in the axial direction for the sub-nodes. Since the transverse leakage coupling is still on the basis of coarsemesh nodes, the parabolic transverse (radial) leakage profile is constructed as usual from the average radial surface currents on the three adjacent axial coarse-mesh nodes. This parabolic leakage profile is then shared by all the sub-nodes within the same coarse-mesh node, as illustrated in Fig. 1. When solving the 1D equation in a radial direction, the sub-nodes within a coarse node are collapsed to a single node via the transverse (axial) integration, represented by one set of homogenized cross sections that are calculated using the axial sub-node flux as the weighting function.

It should be emphasized that in the CIAMA method, when solving the 1D nodal equation in the axial direction, all the sub-nodes or coarse nodes are treated equally. Each sub-node has its own base functions to expand its 1D axial flux. This is quite different

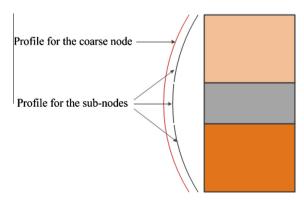


Fig. 1. Transverse leakage profile for an axial coarse node and its sub-nodes.

from the earlier work by Lima et al. or Li, where although using a step function to represent the axial heterogeneity within a coarse node they used one single set of base functions to expand the 1D flux over the whole coarse node.

It is also worth noting that when CIAMA is implemented in EGRET, each sub-node has its own thermal-hydraulic feedback and, if applicable, its own fuel burn-up as well. This feature enables EGRET to faithfully represent the fine variation of neutron flux in the axial direction and to produce detailed axial power information directly from the 3D global calculation. The use of CIAMA provides EGRET with the following unique advantages:

- Handling the axial heterogeneity issue at the fundamental level of nodal equation formulation to eliminate the need of constructing a separate auxiliary 1D fine-mesh heterogeneous model.
- Producing more accurate axial power distribution.
- No need to introduce axial discontinuity factors.
- Avoiding 3D/1D coupled iterations and the risk of convergence problem.
- Simpler methodology and easier to implement.

3. Qualification of CIAMA

To qualify the CIAMA method, the EGRET code is used to model two problems, the fuel grid problem and the partially inserted control rod problem. The two cases are respectively presented in the following two sub-sections.

3.1. Fuel grid model

When the coarse-mesh nodal method (typically with nodes of 10–20 cm axial height) is applied to practical core analysis, there must exist nodes containing fuel grid (approximately of 3 cm height for PWR). The presence of fuel grid causes significant axial heterogeneity in the core nodal model. For such an axial node as illustrated in Fig. 2, EGRET adopts three sub-nodes within the coarse node to explicitly represent three different material slices. With this fuel grid model each sub-node is axially homogeneous such that the original axial heterogeneity no longer poses any problem.

To demonstrate the performance of the EGRET fuel grid model, Fig. 3 shows an example of the normalized axial power distribution for a BOC HZP state of a commercial PWR core. The continuous solid curve shows the reference solution, along which the errors in the CIAMA method solution and the volume-weighted homogenization method solution are also shown. It is obvious that the local effect caused by fuel grids is well-reflected in the EGRET solution using the CIAMA method.

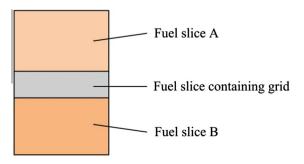


Fig. 2. One axial coarse node containing a fuel grid.

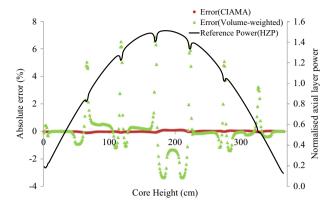


Fig. 3. Axial grid effect prediction for a commercial PWR core.

3.2. Control rod model

Similar to the case of fuel grid, a coarse axial node containing partially inserted control rod induces the same kind of node homogenization problem. If not properly modeled, the presence of partially inserted control rod causes errors in axial power distribution and the non-physical "cusping" effect on differential control rod worth, and consequently on the transient power variation as well during a rod movement transient. Various methods have been developed to resolve this problem in the general framework of advanced nodal method (Dall'osso, 2002; Lozano and Argoés, 2008; Bahadir and Lindahl, 2009; Downar et al., 2011), however mainly based on the idea of axial coarse node re-homogenization or 3D/1D coupled iteration.

With the implementation of CIAMA in EGRET, it can easily resolve the control rod cusping problem. Unlike the case of fuel grid being a stationary component of fuel assembly, control rods are moving components. Two node-meshing schemes are therefore needed for such a case. One set is a stationary 3D coarse node scheme that covers the whole solution domain. While the other is an adaptive sub-node scheme that moves along with the control rods and applies only to the axial coarse nodes containing partially inserted control rod. The adaptive sub-node scheme divides any partially rodded axial coarse node into a rodded sub-node and an unrodded sub-node. Since the number of coarse nodes containing partially-inserted control rod is very small compared to the total number of coarse nodes in a 3D core, CIAMA introduces very little extra computational cost for modeling the rod movement. The programming effort for the method implementation is also minimal.

To qualify the EGRET control rod model, we examine in details the axial power distribution calculation and the differential rod worth calculation in the test problem of control rod insertion into the core of the IAEA 3D benchmark problem (Shober, 1977). This is a PWR core with 157 assemblies. The active core height is 340 cm, and its quarter core radial layout is shown in Fig. 4. A bank of nine control rod clusters is fully inserted in this core. To study the partially inserted rod modeling issue, an additional bank of four control rod clusters is inserted to various depth in the core as shown in Fig. 4.

The reference solution is obtained by the EGRET calculation using the same set of axially uneven meshes in all the axial channels across the whole core regardless if a channel has control rod or not. In this test problem the depth of the rod insertion is increased from 20 cm to 80 cm. In this region the axial meshes are set to 2 cm height and the rod movement is by 2 cm each time so that there is never misalignment between control rod tip and mesh interface. Outside this region the axial meshes are coarse and set to 20 cm height. The radial nodal meshes are uniformly set to squares of

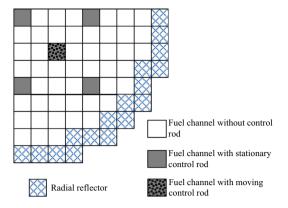


Fig. 4. Quarter core radial layout of IAEA 3D benchmark problem.

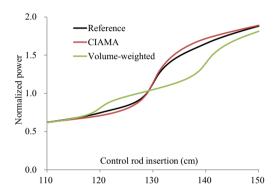


Fig. 5. Local axial power inside a rodded channel for the IAEA test problem.

 20×20 cm width. In addition to the reference solution calculation, two approximate calculations using all uniform coarse axial nodes of 20 cm height are also performed. One result is obtained by homogenizing the partially rodded nodes via simple volume averaging of the cross-sections. The other result is obtained by using the CIAMA method to accommodate the partially inserted rod by sub-nodes division only in the partially rodded nodes of the rod containing channels.

To demonstrate the local effect of a partially rodded node the axial power distribution inside a rodded channel is shown in Fig. 5, where the specific case of control rod insertion to 130 cm depth is chosen as an example. The 20 cm axial node between 120 cm and 140 cm from the top of the core contains the tip of the partially inserted control rod. The axial power distribution in this rodded channel is shown in Fig. 5 for the axial section between 110 cm and 150 cm from the top of the core, which covers the partially rodded node and half of the two adjacent axial nodes. Compared to the reference solution, the solution using the volume weighted homogenization method gives large local power error. On the other hand the CIAMA solution gives much smaller error in the local power. For the unrodded axial channel adjacent to this rodded channel the local power error is found to be generally small for both methods, which is expected and not explicitly shown here.

Fig. 6 shows the results for differential rod worth obtained with the reference solution versus the ones obtained with the two approximate solutions using coarse nodes. As expected, the reference differential rod worth varies smoothly without any cusping. The result of using volume weighting homogenization gives huge errors, with severe cusping in the differential rod worth. The CIAMA result greatly reduces the cusping errors and compares very well to the reference solution. It should be pointed out that the differential rod worth in this test problem is excessively large compared to a typical case of a commercial PWR core at normal

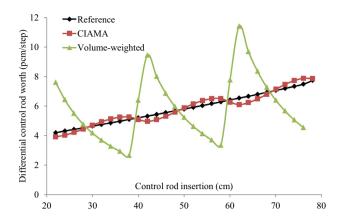


Fig. 6. Reduction of the cusping effect by using CIAMA for the IAEA test problem.

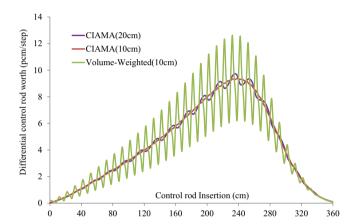


Fig. 7. Sensitivity of rod cusping to axial node size for a commercial PWR core.

operating conditions, which is to be shown later in Fig. 7. At the same elevation location, the differential rod worth in this test problem is three times of that in Fig. 7. The cause of this exaggeration is due to the different rod configuration in this IAEA benchmark problem, where another bank of nine control rod clusters are fully inserted to the core.

The error in the CIAMA result is mainly coming from using the coarse node transverse leakage profile for its sub-nodes, as shown in Fig. 1. Reducing the axial nodal size should certainly further eliminate the cusping effect. Fig. 7 shows such a study for an operating commercial PWR core with 121 assemblies at the BOC HZP condition. One bank of control rod is inserted for calculating the differential rod worth. There are three differential rod worth curves in Fig. 7. One curve is for the case of using volume weighted homogenized axial nodes of 10 cm, and the other two are obtained via the CIAMA method in the EGRET code using axial nodes of 20 cm and 10 cm respectively (Zhang et al., 2014b). To focus on examining the rod cusping effect, we intentionally smeared the fuel grid when building the core model. That is the reason why there is no grid-caused local depression on the curves. Compared to the case of volume weighted homogenization, the CIAMA result of using 20 cm axial coarse nodes shows a greatly reduced cusping with the maximum error in differential rod worth about 2.5%. Reducing the axial coarse node size to 10 cm almost eliminates the cusping in the CIAMA solution.

4. Application of CIAMA to 3D pin-by-pin calculation

In recent years considerable work has been done to develop the so-called next generation method (NGM) for light water reactor core analysis in order to overcome the various deficiencies in

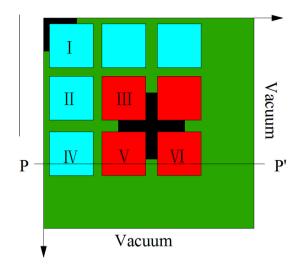


Fig. 8. Quarter core radial configuration of the mini-core.

today's method. NGM abandons assembly homogenization and performs 3D pin-by-pin whole core calculation (Tatsumi and Yamamoto, 2002; Joo et al., 2004; Jung et al., 2013). When developing these radially pin-by-pin calculation methods, coarse meshes are still used in the axial direction. Therefore the axial node homogenization issue existing in today's method still exists (Yamamoto, 2004). To demonstrate the application of CIAMA to resolving the axial heterogeneity issue in NGM, we consider the 3D BWR mini-core benchmark problem that was proposed earlier by the authors (Lu et al., 2014) for NGM pin-by-pin calculation.

Figs. 8 and 9 show the radial and axial mini-core configuration of the 3D BWR benchmark problem, the detailed specification of which is given in the reference (Lu et al., 2014). There are two crucified control blades inserted in the core, the central one inserted to 291 cm height and the outer one to 110 cm height. Other than the axial control blade insertion, the core has complicated axial heterogeneity due to the axial configuration and composition variation in the fuel bundles. The fuel rods in the bundles have axially varying enrichment and contain a number of part length fuel rods of different heights as well. The axial water density variation is also very large. As shown in Fig. 9, fuel composition variation plus the

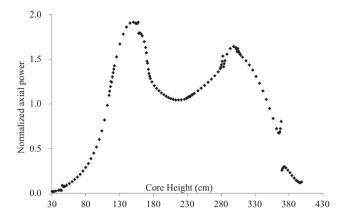


Fig. 10. Reference normalized axial power distribution of the mini-core.

water density variation divides the core into eleven different axially homogeneous sectors. Furthermore two of the eleven sectors contain partially inserted control blades, making it in total thirteen different 2D radial core slices. Each of the BWR fuel bundles also has such a severe radial heterogeneity that the core has to be modeled in pin-by-pin explicitly using homogenized pin-cell cross sections generated from 2D full core transport calculation on each of the thirteen axial slices. Each homogeneous pin cell in the core has its own axially varying cross sections.

The reference solution of the problem is obtained by EGRET 3D pin by pin 7-group nodal diffusion calculation using homogeneous radial pin cells and homogeneous axial nodes of variable sizes up to a maximum height of 10 cm. The axial nodes are so chosen that they all lie inside anyone of the thirteen axially homogeneous sectors. Fig. 10 demonstrates the normalized axial power distribution of the reference solution in fine meshes. The fine mesh distribution comes from the pin-cell axial flux profile reconstruction inside each of the axial nodes, which uses the node interior axial flux expansion coefficients obtained intrinsically in the 3D nodal calculation and is therefore as rigorous as the 3D nodal calculation itself.

In order to assess the applicability of CIAMA to 3D pin-by-pin calculations, two additional calculations are performed. One adopts the crude volume-weighting technique to homogenize the

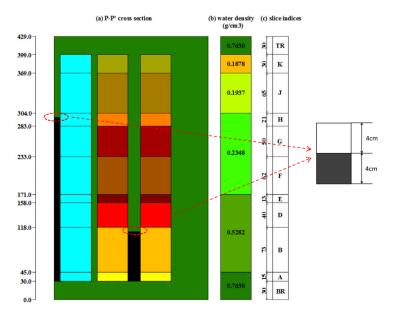


Fig. 9. Axial core configuration of the mini-core.

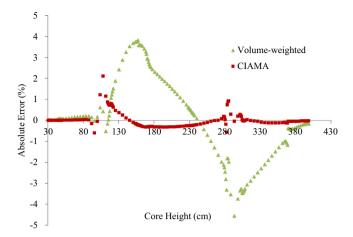


Fig. 11. Absolute error (%) in normalized axial power distribution with different methods.

partially rodded pin-cell axial nodes, while the other uses the proposed CIAMA method. The partially rodded axial mesh configuration near the control blade tips is explicitly depicted in Fig. 9, where the coarse mesh size is 8 cm with half of it containing control blade insertion. Fig. 11 shows the absolute error of the axial power distribution when compared against the reference solution. The CIAMA result is much better than that of volume weighted homogenization. This benchmark problem is very challenging and poses a severe test to the CIAMA method in particular. The radial leakage is very large for this mini-core with vacuum boundary condition. Therefore the axial flux solution is very sensitive to the profile of the large transverse (radial) leakage in the partially rodded nodes: while the transverse leakage profile is exactly the approximation in the CIAMA method. Furthermore, the radial leakage increases greatly from the bottom to the top of the core as the water density decreases dramatically. This is why the solution of the volume weighted homogenization method shows a large axial tilt. The CIAMA method gets rid of the tilt, although still gives error near the tip of the partially inserted control blades. The outer control blades are inserted to 110 cm height and affect sixteen bundles in this core of thirty-six bundles, while the center control blade is inserted to 291 cm height and affects only four bundles. At the elevation of the tip of the outer control blades there is still 2% error in the axial power. But at all the other locations the errors are all less than 1% even in the immediate neighborhood of the control blade tips. This demonstrates the great potential for CIAMA in NGM application.

5. Conclusions

The proposed CIAMA method resolves the axial heterogeneity problem in a more fundamental way, avoiding the axial node rehomogenization used in conventional nodal methods that requires iterations between 3D coarse node calculation and 1D sub-node calculation. The CIAMA method allows each individual axial channel to have (or not have) its own sub-nodes adapting to its own axial heterogeneity variation, and incorporates the axial sub-node calculation as an intrinsic part of the whole core global calculation. The only approximation in CIAMA is in the profile of radial leakage

that couples the axial channels. The profile is still based on coarse axial nodes despite allowing sub-nodes inside the coarse nodes.

The CIAMA method has been implemented in the commercial code EGRET, which is used to qualify CIAMA. Results of modeling fuel grid and control rod movement show excellent performance of CIAMA. The presence of fuel grid can be accurately captured in axial power distribution. For a commercial PWR core the maximum rod cusping error in differential rod worth can be reduced to 2.5% when using 20 cm axial nodes. If the axial node size is reduced to 10 cm, then the cusping effect is almost all eliminated. CIAMA can also be applied to 3D pin-by-pin core calculation where axial coarse mesh is still needed for practical feasibility. Applying CIAMA to pin-by-pin calculation for the very challenging 3D BWR mini-core benchmark problem very effectively reduces the error from using coarse axial nodes containing partially inserted control blades, demonstrating the good potential of using CIAMA in the development of next generation methods.

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