# A SIMPLE AND EFFICIENT CONTROL ROD CUSPING MODEL FOR THREE-DIMENSIONAL PIN-BY-PIN CORE CALCULATIONS

FISSION REACTORS

**KEYWORDS:** *control rod cusping problem, pin-by-pin core calculation, inverse spectral index weighting*

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*A new solution for the control rod cusping problem in the three-dimensional pin-by-pin core calculation is proposed in this paper. The current advanced nodal code resolves this issue by estimating the one-dimensional axial flux distribution in a partially rodded node. However, direct application of this approach to the three-dimensional pin-by-pin calculation is impractical since the leakage effect in the radial direction is significant and the one-dimensional model for axial flux distribution is no longer valid. This issue has been neither addressed nor resolved yet. In this paper, a new approach that utilizes the inverse of the spectral index obtained in the assembly calculation is used to estimate the flux distribution inside the partially rodded mesh. The proposed model was implemented in the SCOPE2 code, which is a three-dimensional pin-by-pin nodal-transport code for pressurized water reactor core calculations, and a verification calculation was carried out to confirm the validity of the proposed method. From the calculation results, oscillation in the differential worth of control rods (i.e., the cusping effect) is damped, and the proposed model can almost reproduce that obtained by the reference calculation. The additional computation time for the proposed model is negligible. Consequently, the proposed control rod cusping model is an attractive method in three-dimensional pin-by-pin calculations.*

## I. INTRODUCTION

The three-dimensional nodal method is widely used for light water reactor (LWR) analysis.[1,2] In the current advanced nodal method, the heterogeneous structure inside a fuel assembly is homogenized, and an assembly average cross-section set is assigned for each mesh. The typical mesh size in LWR analysis is ~10 cm for the radial direction and 10 to 20 cm for the axial direction.

Since the assembly average cross section is generated through a series of two-dimensional lattice physics calculations, the heterogeneous structure of the fuel assembly in the radial direction can be homogenized. However, heterogeneity in the axial direction, e.g., enrichment or burnable absorber distribution in the axial direction and control rod insertion, cannot be directly treated in the lattice physics code due to its inherent restriction. Therefore, the axially heterogeneous structure in the LWR core is generally considered in a core calculation code.

If the heterogeneous (i.e., material) boundary is consistent with the mesh boundary in a core calculation code, the axial heterogeneity can be handled in a straightforward manner, i.e., different cross-section sets are assigned to different meshes. However, misalignment between material and mesh boundaries cannot be directly handled by a core calculation code, since only one cross-section set can be assigned for a mesh. Therefore, the misalignment between material and mesh boundaries, i.e., a so-called axially heterogeneous mesh, requires proper modeling in a core calculation code.

Axially heterogeneous fuel assemblies such as axial blankets can be treated through appropriate uniform or nonuniform meshing in the core calculation code, since the material boundary can be defined by the fuel specification and does not change during calculations. Contrary to this, a control rod requires more robust modeling, since its position frequently changes during operation and the

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material boundary, i.e., control rod tips, can be located at every position in the axial direction.

There may be two major approaches to solve this issue: adaptive meshing and homogenization.

In the former approach, a mesh, which contains the material boundary inside it, is divided into two meshes. For example, if a control rod is partially inserted in a mesh, the mesh is divided into rodded and unrodded meshes. This approach is reasonably straightforward in terms of the calculation model, but it may require significant programming considerations that could make a core calculation code complex and difficult to maintain.

In the latter approach, a homogenization procedure is required for the axially heterogeneous mesh. However, since the latter is much simpler than the former in terms of programming, many core calculation codes adopt this approach as described later.

Homogenization accuracy is crucial in the latter approach. If a poor homogenization technique is used, misprediction of control rod worth could be significantly large. For example, if the simple volume weighting technique is used for homogenization, control rod worth cannot be adequately predicted. Especially, the error of predicting differential worth becomes significantly large. This issue is the so-called control rod cusping problem.[3]

Various homogenization procedures were developed for the control rod cusping problem in the three-dimensional nodal method as will be described in Sec. II. Since these methods provide satisfactory results in practical applications, the control rod cusping problem was considered to be almost resolved in the framework of the advanced nodal method, which is commonly used for LWR core calculations.[3–7]

Recently, three-dimensional fine-mesh (i.e., pin-by-pin) calculations are applied to reactor core analyses to utilize less-approximate neutronics models.[8–10] Emerging cost-effective hardware and developments in numerical methods enable such precise core calculation models to be used in practical in-core fuel management calculations.

In the three-dimensional pin-by-pin calculations, independent calculation meshes are assigned to each fuel pin inside an assembly, and homogenization of the fuel assembly is no longer necessary except for the heterogeneous structure inside a fuel cell, i.e., pellet, clad, and moderator. Although the mesh size in the radial direction is much smaller than that of the conventional advanced nodal method, that in the axial direction is similar in both methods, i.e., typically 10 to 20 cm for the axial direction. There are two reasons for coarse meshing in the axial direction even in the three-dimensional pin-by-pin calculations:

1. Since the heterogeneity in the axial direction is not very strong in a typical LWR, coarse meshing is appropriate.

2. Since the three-dimensional pin-by-pin calculation requires considerable computation resources (both memory and calculation time), fine discretization for the axial direction is impractical.

Therefore, the control rod cusping problem also exists in the three-dimensional pin-by-pin calculation. Unfortunately, computational models of the cusping problem for the conventional advanced nodal model are not directly applicable to the three-dimensional pin-by-pin calculation as will be discussed in Sec. II. This issue has not been addressed or resolved yet.

This paper provides an efficient and practical solution for the control rod cusping problem of three-dimensional fine-mesh calculations.

In Sec. II the cusping models (i.e., homogenization method of axially heterogeneous mesh) applied for the advanced nodal method so far will be briefly reviewed, and their applicability to the three-dimensional pin-by-pin calculation will be discussed. Then, a new model for the control rod cusping problem of pin-by-pin calculation is proposed. In Sec. III numerical results will be provided to demonstrate the effectiveness of the proposed model. The paper is summarized in Sec. IV.

## II. CONTROL ROD CUSPING MODEL

### II.A. Conventional Model

Several homogenization methods for control rod cusping models have been proposed so far. These are the simple volume weighting method, the approximate flux weighting method,[4] the analytical flux weighting with discontinuity factor method,[5] the bilinear weighting method,[6] and the equivalent-node method.[7] These methods almost provide satisfactory results except the simple volume weighting method. Since no flux weighting is taken into account in the simple volume weighting method, extremely large errors are observed.

In general, the above method estimates the one-dimensional flux distribution in the axial direction and utilizes it for flux-volume weighting. Since the size of a mesh in the radial direction is relatively large in the advanced nodal method, the leakage effect in the radial direction does not significantly affect the axial flux distribution inside a mesh. Therefore, a one-dimensional model is appropriate to estimate the flux distribution inside a mesh.

### II.B. Applicability of Conventional Model for Three-Dimensional Fine-Mesh Calculation

In the three-dimensional pin-by-pin calculations, the size of a mesh is ~1 cm for the radial direction and 10 to 20 cm for the axial direction. Compared to those of the advanced nodal method, the size in the radial direction is much smaller as shown in Fig. 1. Therefore, the leakage

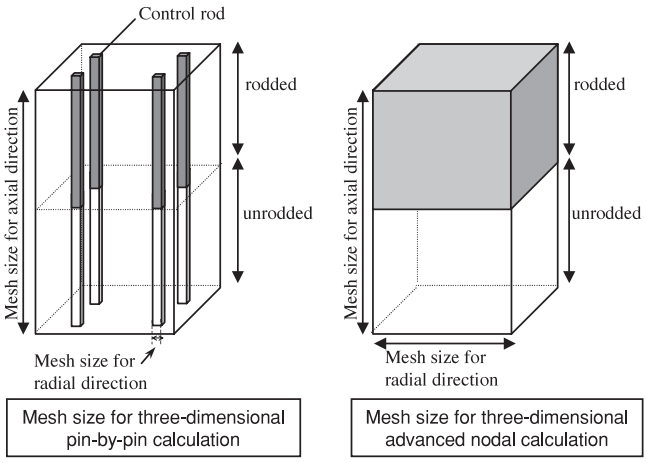


Fig. 1. Mesh size for three-dimensional pin-by-pin geometry and advanced nodal method (not to scale).

effect of the radial direction in a mesh is much larger than that of the advanced nodal method. Consequently, the one-dimensional calculation model for flux distribution is not valid in the three-dimensional pin-by-pin calculations.

For example, let us consider a quarter assembly node of a pressurized water reactor (PWR) whose height is 20 cm and in which a control rod cluster is partially inserted into a midplane of the node. In this case, the size of the control rod cell is ~1 cm for the radial direction and 20 cm for the axial direction, and the composition of the top half of the cell is control rod and that of the bottom half is guide thimble with coolant. If the conventional homogenization method is applied, the cell is solved in one-dimensional slab geometry; half of the slab is control rod and the rest is guide thimble and coolant. Since the flux distribution in such slab geometry will be significantly different from the actual three-dimensional one, the conventional one-dimensional model cannot be directly applied for three-dimensional pin-by-pin calculations.

A straightforward extension of the conventional homogenization model is application of the three-dimensional calculation instead of the one-dimensional one. For example, the three-dimensional multicell calculation (e.g., $3 \times 3$ or $5 \times 5$ cell) with the control rod at the center position will provide an appropriate flux distribution inside the axially heterogeneous control rod cell. However, such calculation requires considerable additional computation time and a dedicated flux calculation module.

Some approximate two-dimensional models, e.g., cylindrical approximation with buffer fuel, could reduce the computational burden, but additional computation time and a dedicated calculation module are still necessary.

The above problem has not been addressed or resolved yet. A new homogenization procedure to solve this issue will be proposed in Sec. II.C.

### II.C. Proposed Model

In the three-dimensional model, the effect of radial leakage on the axial flux distribution is larger than that of axial leakage due to the size of a mesh. If the axial leakage is not very large, the spectrum in a cell will be similar to that obtained by the assembly calculations. This will be true especially for the inner part of the fuel assembly, since the spectrum interaction effect between adjacent fuel assemblies rapidly reduces as a function of distance from the periphery of the assembly.

The above insight is the underlying concept of the newly proposed method in this paper. A spectral index can be used for flux weighting factor for axial homogenization. The spectral index obtained by the assembly calculation is the ratio of the fast flux to the thermal flux and is easily obtained in the assembly calculation. It is usually tabulated as one of the fuel properties and is often used to correct the cross-section change due to the spectrum interaction between assemblies. If the fast flux distribution can be estimated, the thermal flux distribution can be derived by multiplying by the inverse of the spectral index.

Since variation of the fast flux in the LWR core is much smoother than that of thermal flux, the fast fluxes inside a partially rodded mesh (i.e., fast flux in rodded and unrodded regions) can be estimated by the approximate flux weighting method[4] as follows:

$$\bar{\phi}^{fast,core}_{rod\text{-}in,k} = \frac{\Delta z_{k-1}\bar{\phi}^{fast,core}_{k-1} + f_{ins}\Delta z_k \bar{\phi}^{fast,core}_{k}}{\Delta z_{k-1} + f_{ins}\Delta z_k}$$

and

$$\bar{\phi}^{fast,core}_{rod\text{-}out,k} = \frac{\Delta z_{k+1}\bar{\phi}^{fast,core}_{k+1} + (1-f_{ins})\Delta z_k \bar{\phi}^{fast,core}_{k}}{\Delta z_{k+1} + (1-f_{ins})\Delta z_k} \quad , \tag{1}$$

where

$\bar{\phi}^{fast,core}_{rod\text{-}in,k}$ = estimated average fast flux of rodded region in partially rodded mesh ($k$)

$\bar{\phi}^{fast,core}_{rod\text{-}out,k}$ = estimated average fast flux of unrodded region in partially rodded mesh ($k$)

$\Delta z_{k-1}, \Delta z_k, \Delta z_{k+1}$ = mesh width of rodded ($k-1$), partially rodded ($k$), and unrodded ($k+1$) meshes

$\bar{\phi}^{fast,core}_{k-1}, \bar{\phi}^{fast,core}_{k}, \bar{\phi}^{fast,core}_{k+1}$ = average flux of rodded ($k-1$), partially rodded ($k$), and unrodded ($k+1$) meshes

$f_{ins}$ = fraction of rodded region in the partially rodded mesh.

Then, the average thermal fluxes for the rodded ($\bar{\phi}^{thermal,core}_{rod\text{-}in,k}$) and the unrodded ($\bar{\phi}^{thermal,core}_{rod\text{-}out,k}$) regions in the partially rodded mesh can be estimated as follows:

$$\bar{\phi}^{thermal,core}_{rod\text{-}in,k} = (\bar{\phi}^{thermal,assembly}_{rod\text{-}in} / \bar{\phi}^{fast,assembly}_{rod\text{-}in}) \cdot \bar{\phi}^{fast,core}_{rod\text{-}in,k}$$

and

$$\bar{\phi}^{thermal,core}_{rod\text{-}out,k} = (\bar{\phi}^{thermal,assembly}_{rod\text{-}out} / \bar{\phi}^{fast,assembly}_{rod\text{-}out}) \cdot \bar{\phi}^{fast,core}_{rod\text{-}out,k} \ , \quad (2)$$

where

$\phi^{fast,assembly}_{rod\text{-}in}, \phi^{thermal,assembly}_{rod\text{-}in}$ = fast and thermal flux in the rod cell obtained by the assembly calculation

$\phi^{fast,assembly}_{rod\text{-}out}, \phi^{thermal,assembly}_{rod\text{-}out}$ = fast and thermal flux in the unrodded (guide thimble) cell obtained by the assembly calculation.

Using Eqs. (1) and (2), the average cross section of the partially rodded mesh is obtained as follows:

$$\Sigma_{ave} = \frac{\Sigma_{Rod\text{-}in} f_{ins} \bar{\phi}_{Rod\text{-}in} + \Sigma_{Rod\text{-}out}(1 - f_{ins}) \bar{\phi}_{Rod\text{-}out}}{f_{ins} \bar{\phi}_{Rod\text{-}in} + (1 - f_{ins}) \bar{\phi}_{Rod\text{-}out}} \ , \quad (3)$$

where

$\Sigma_{ave}$ = average cross section

$\Sigma_{rod\text{-}in}$ = cross section for rodded part (control rod)

$\Sigma_{rod\text{-}out}$ = cross section for unrodded part (guide thimble)

$\bar{\phi}_{Rod\text{-}in}$ = average flux in rodded part

$\bar{\phi}_{Rod\text{-}out}$ = average flux in unrodded part.

Since the inverse of spectral index (ISI) ($\phi^{thermal}/\phi^{fast}$) is used in Eq. (2), the proposed method is called the inverse spectral index weighting or the ISI weighting method.

The ISI method for the control rod cusping problem is summarized as follows:

1. calculates the spectral index ($\phi_{fast}/\phi_{thermal}$) during the assembly calculation and tabulates them in the cross-section table used in the core calculation
2. sets the control rod position in the core calculation
3. detects a partially rodded mesh and estimates the volume fraction of rodded and unrodded regions
4. estimates average fast flux for rodded or unrodded region using Eq. (1)
5. estimates average thermal flux for rodded or unrodded region using Eq. (2)
6. obtains the average cross section using Eq. (3)
7. performs the core calculation using the average cross section.

The fast flux in Eq. (1) is affected by the homogenized cross section of the partially rodded mesh, which is obtained by Eq. (3). Therefore, some nonlinear iteration should be necessary to obtain the converged solution. Fortunately, since nonlinear effects such as the moderator temperature and the buildup of Xe are usually treated in the common core calculation code as the nonlinear feedback iteration, the above nonlinear iteration of the ISI weighting can be treated in the same framework.

Equation (1) is derived for two-group calculations commonly used for LWR core calculations. For the multigroup calculations, Eq. (2) can be easily extended to the multigroup form as follows:

$$\phi^{g,core}_{rod\text{-}in} = (\phi^{g,assembly}_{rod\text{-}in} / \phi^{1,assembly}_{rod\text{-}in}) \phi^{1,core}_{rod\text{-}in}$$

and

$$\phi^{g,core}_{rod\text{-}out} = (\phi^{g,assembly}_{rod\text{-}out} / \phi^{1,assembly}_{rod\text{-}out}) \phi^{1,core}_{rod\text{-}out} \ , \quad (4)$$

where $g$ indicates the number of energy groups.

Note that although Eq. (4) can be used for multigroup calculations, Eq. (2) has sufficient accuracy in practical applications as will be described later.

## III. NUMERICAL RESULTS

The newly proposed method to mitigate the control rod cusping problem has been implemented in the SCOPE2 code, which is a three-dimensional pin-by-pin nodal transport code for PWR core analyses.[8–10] The SCOPE2 code utilizes the semianalytic nodal method[11] with the simplified P3 theory.[12] Calculation of control rod worth was carried out for a three-loop PWR initial core, in which 157 fuel assemblies are loaded. The material of the control rod is Ag-In-Cd, and the control rod is inserted into fuel assemblies that have a $17 \times 17$ fuel pin arrangement.

The number of energy groups in the SCOPE2 calculation is nine; those of six and three are used for fast and thermal groups, respectively. Since the spectral index used in the SCOPE2 code is evaluated in two groups (i.e., fast flux/thermal flux), the same weighting factor is applied for the three thermal energy groups to homogenize partially rodded meshes. The same weighting factor obtained by Eq. (1) was applied for the six fast groups.

The number of axial meshes for the fuel region was 24, and 1 mesh was assigned for both the bottom and top reflectors. The same mesh width for the axial direction is used in the fuel region, i.e., the fuel region is equally

divided into 24 meshes for the axial direction. For the radial direction, an independent calculation mesh is assigned to each cell. The calculation was performed at the hot-zero power condition, and the control bank of group D was inserted into the core in every 0.5 mesh (i.e., bank D position in the mesh basis unit is 24, 23.5, 23, . . . , 1, 0.5, 0). Note that the group D bank has eight control rod clusters. Note also that since the core is equally divided into 24 meshes, 1 mesh is equivalent to ~10 steps.

Four different calculations were carried out to investigate the effectiveness of the newly proposed model. The first one is the reference (REF) case, in which the number of axial meshes in the fuel region is increased to 48 to avoid the control rod cusping effect. In the REF case, no axial homogenization is necessary since the control rod tip is consistent with the mesh boundary. The second one is the volume weighting (VOL) method, in which simple volume weighting is used to homogenize partially rodded meshes. The third one is the approximate flux weighting (APR) method proposed by Gehin[4] and used in some conventional advanced nodal codes. The fourth one is the ISI weighting method, which is newly proposed in this paper.

Figures 2 and 3 show the integral and differential reactivity effects of bank D, respectively. Figure 4 shows an enlargement of the integral reactivity effect between 50 and 90 steps. Note that fine control rod movement was used to obtain the integral reactivity effect in Fig. 4, i.e., the control rod was inserted in every 0.2 mesh.

In the VOL method, a significant control rod cusping effect is observed. When the APR method is applied, the cusping behavior of the differential reactivity effect reduces, but a large oscillation still remains. In contrast to these methods, the differential reactivity effect obtained by the ISI method is smooth and almost reproduces that of the reference solution.

Although the axial homogenization procedure requires some additional computation, execution times for the three cases (except the REF case, whose node division is finer than that of the other cases) were almost the same. Namely, the computation burden incurred by the proposed method is considered to be quite small.

From the above results, the newly proposed model for the control rod cusping problem, i.e., the ISI weighting method, effectively mitigates the oscillation behavior of the differential worth of the control rod and is easy to implement.

## IV. SUMMARY

A new method to mitigate the control rod cusping effect for a three-dimensional pin-by-pin core calculation was proposed. Because of the complex mesh structure of three-dimensional pin-by-pin calculations, the conventional control rod cusping model, which is based on a one-dimensional flux distribution calculation, cannot be directly applied. Therefore, a new approach, which

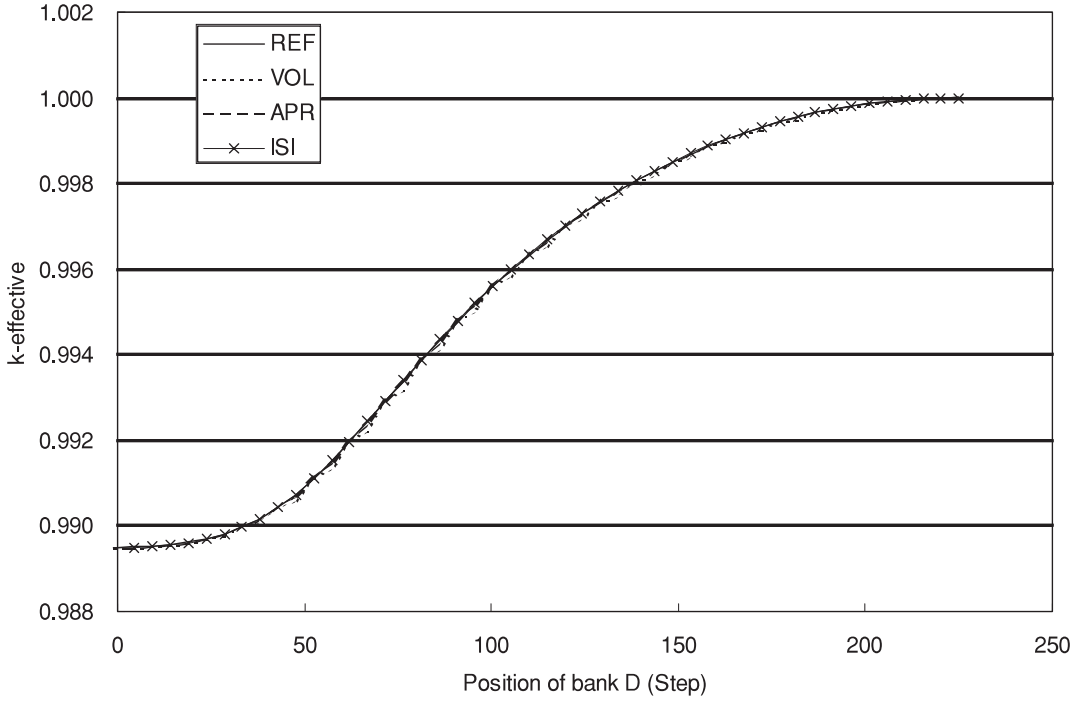


Fig. 2. Integral reactivity effect of control bank D.

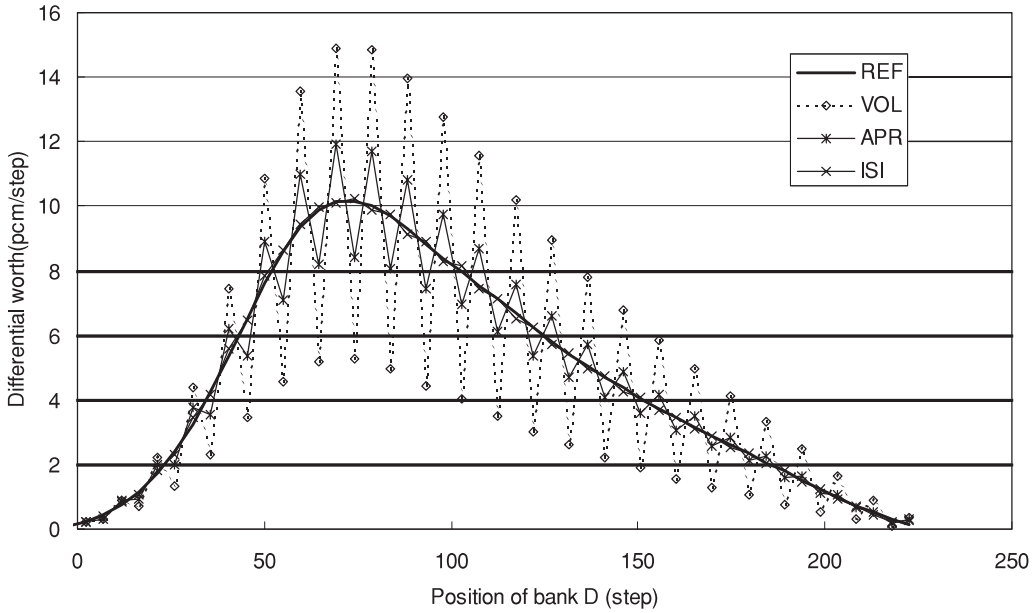


Fig. 3. Differential reactivity effect of control bank D.

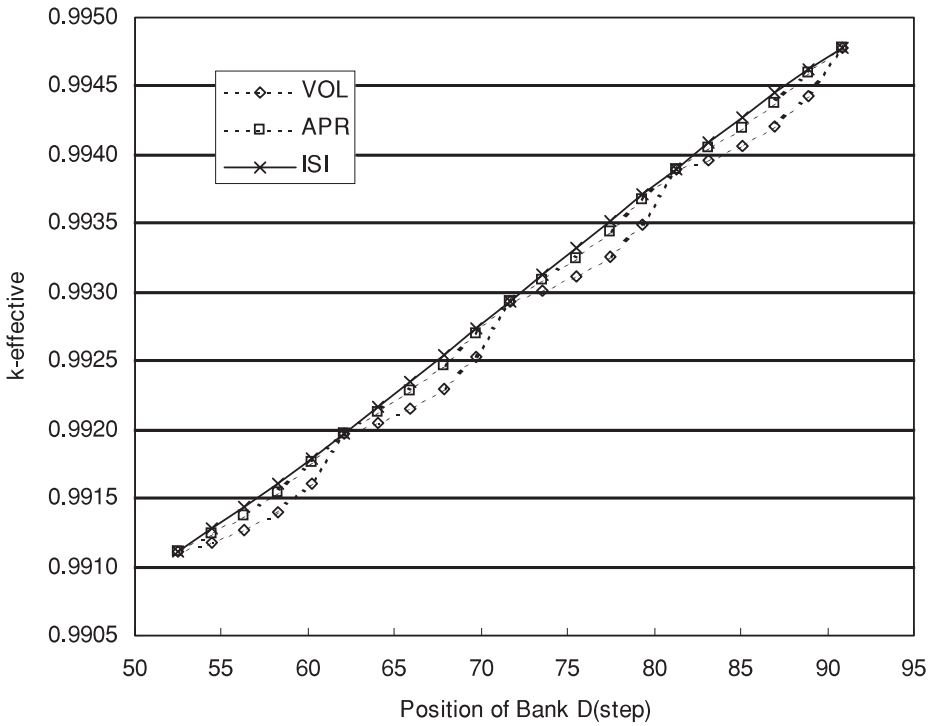


Fig. 4. Integral reactivity effect of control bank D (50 to 90 steps).

utilizes the inverse of the spectral index obtained in the assembly calculation, was proposed and tested through control rod worth calculations in a three-loop PWR core.

The proposed method, the so-called ISI weighting method, was implemented in the SCOPE2 code, which is a three-dimensional pin-by-pin nodal-transport code for PWR core analysis, and test calculations were carried out using the SCOPE2 code.

The calculation results showed that the ISI weighting method can significantly reduce the control rod

cusping effect and can reproduce integral and differential bank worth values that are similar to the reference calculation and are free from the control rod cusping effect by adapting fine axial meshing.

Consequently, the ISI weighting method newly proposed in this paper is an attractive candidate for handling the control rod cusping effect of three-dimensional pin-by-pin calculations and is a promising method for core calculations in the future.

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