A New ZCT Precoded OFDM System with Pulse Shaping: PAPR Analysis

Imran Baig and Varun Jeoti, *Member, IEEE*
Electrical and Electronic Engineering Department,
Universiti Teknologi PETRONAS, Tronoh, 31750, Perak, Malaysia
imran_baig_mirza@yahoo.com
varun_jeoti@petronas.com.my

***Abstract*—This paper studies the Peak to Average Power Ratio (PAPR) improvements in various precoding based Orthogonal Frequency Division Multiplexing (OFDM) systems. In particular, the Zadoff-Chu matrix Transform (ZCT) precoder based PAPR reduction technique is analyzed. The ZCTs are obtained from Zadoff-Chu (ZC) sequences by filling ZCT kernel row-wise or alternatively column wise. Row wise filling gives rise to Constant Envelope OFDM (CE-OFDM) system with 0 dB PAPR, while column wise filling give rise 7.8 dB, at clip rate of 10^{-3} with system subcarriers $N = 64$ for QPSK modulation. However, since even CE-OFDM systems are required to operate with pulse shaping that helps in keeping out-of-band radiation low and meeting the transmission spectrum mask requirement, the PAPRs are no longer 0 dB. Therefore, in this paper, we present PAPR analysis of various precoding based OFDM systems with the popular Root Raised Cosine (RRC) pulse shaping. Simulation results show that, the ZCT Row-wise precoder based OFDM (ZCT-R-OFDM) system has lower PAPR than the ZCT Column-wise precoder based OFDM (ZCT-C-OFDM) system, the Hadamard Transform precoder based OFDM (WHT-OFDM) systems and the conventional OFDM systems.**

***Keywords*- Zadoff-Chu matrix Transform (ZCT); OFDM conventional; ZCT-C-OFDM system; ZCT-R-OFDM system; Peak to Average Power Ratio (PAPR)**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission scheme that has become the technology of choice for next generation wireless and wireline digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrow band interference and frequency selective fading [1].

OFDM thwarts Inter Symbol Interference (ISI) by inserting a Guard Interval (GI) using a Cyclic Prefix (CP) and moderates the frequency selectivity of the Multi Path (MP) channel with a simple equalizer [2]. OFDM is widely adopted in various communication standards like Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), and even in the beyond 3G Wide Area Networks (WAN) etc. However, among others, the Peak to Average Power Ratio (PAPR) is still one of the major drawbacks in the transmitted OFDM signal [3]. A large number of PAPR reduction techniques have been proposed in the literature [4]-[10]. The precoding based techniques, however, show great promise as they are simple linear techniques to implement without the need of any side information.

In this paper, a PAPR analysis of the Zadoff-Chu matrix Transform Row-wise precoder based OFDM (ZCT-R-OFDM) system with Root Raised Cosine (RRC) pulse shaping is presented and compared with other precoding based OFDM systems. The rest of the paper is organized as follows: Section II describes the basics of the OFDM system and PAPR, In Section III we present the ZCT precoded OFDM (ZCT-OFDM) system and Section IV presents simulation results and section V concludes the paper.

II. OFDM SYSTEM AND PAPR

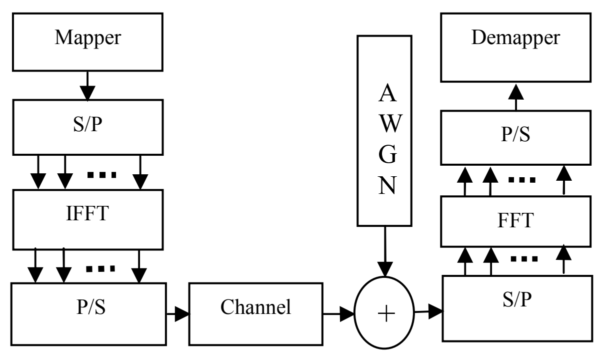

Fig. 1 A Conventional OFDM system

Fig. 1 illustrates the general block diagram of an OFDM system. Baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. We can write the complex vector of size N as $X = [X_0, X_1, X_{2...} X_{N-1}]^T$. X is then passed through the IFFT block. The complex baseband OFDM signal with N subcarriers can be written as:-

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k . e^{j2\pi\frac{n}{N}k} \text{ , } n=0, 1, 2... N\text{-}1 \quad (1)$$

Here $j=\sqrt{-1}$ and the PAPR of OFDM signal in (1) can be written as

$$PAPR = \frac{max[|x_n|^2]}{E[|x_n|^2]} \quad (2)$$

where E [.] denotes expectation.

III. SYSTEM MODEL

A. Zadoff-Chu (ZC) Sequences and Zadoff-Chu matrix Transform (ZCT)

Zadoff-Chu (ZC) sequences are class of poly phase sequences having optimum correlation properties. ZC sequences have an ideal periodic autocorrelation and constant magnitude. According to [11], ZC sequences of length N can be defined as:-

$$a_n = \begin{cases} e^{\frac{j2\pi r}{N}\left(\frac{k^2}{2}+qk\right)} & for\ N\ Even \\ e^{\frac{j2\pi r}{N}\left(\frac{k(k+1)}{2}+qk\right)} & for\ N\ Odd \end{cases} . \quad (3)$$

where k = 0, 1, 2… N-1, q is any integer, r is any integer relatively prime to N and $j=\sqrt{-1}$. The kernel of the ZCT is defined in (4), is obtained by reshaping the ZC sequence by $k = mL + l$ as hereunder:-

$$A = \begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0(L-1)} \\ a_{10} & a_{11} & \cdots & a_{1(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(L-1)0} & a_{(L-1)1} & \cdots & a_{(L-1)(L-1)} \end{bmatrix} \quad (4)$$

Here m is the row variable and l the column variable. In other words, the $N= L^2$ point long ZC sequence fills the kernel of the matrix row-wise. As shown in [8], the PAPR reduces to 0 dB in this case. However, if the kernel of the matrix is filled column wise, where $k = m + lL$, the PAPR does not reduce to 0 dB, instead it reduces to 7.8 dB [4].

B. ZCT precoded OFDM system

Fig. 2 shows the ZCT precoded OFDM (ZCT-OFDM) system. In this system, the kernel of the ZCT acts as a row-wise precoding matrix A of dimension $N = L \times L$ and it is applied to constellations symbols before the IFFT to reduce the PAPR. In the ZCT precoded OFDM system, the baseband modulated data is passed through S/P converter which generates a complex vector of size L that can be written as $X = [X_0, X_1, \ldots X_{L-1}]^T$.

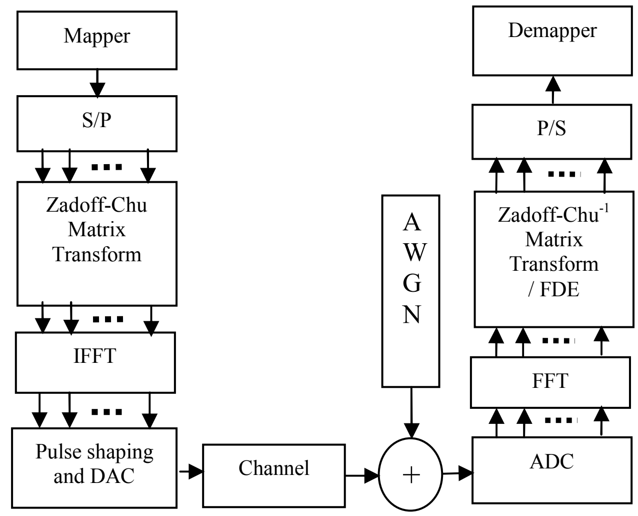

Fig. 2 A ZCT precoded OFDM system

Then ZCT precoding is applied to this complex vector which transforms this complex vector into new vector of same length L. This new vector, of length L transformed by ZCT precoding can be written as $Y=AX= [Y_0, Y_1, Y_{2\ldots} Y_{L-1}]^T$, where A is a precoder matrix of size $N = L \times L$ and Y_m can be written as:-

$$Y_m = \sum_{l=0}^{L-1} a_{m,l} . X_l \qquad m = 0,1, \ldots L-1 \quad (5)$$

$a_{m,l}$ means m^{th} row and l^{th} column of precoder matrix. Expanding Equation (5), using row wise sequence reshaping $k = mL + l$ and putting q=0, r=1 in Equation (3) we get:-

$$Y_m = \sum_{l=0}^{L-1} (e^{j\frac{\pi(mL+l)^2}{L^2}}). X_l$$

$$= e^{j\pi m^2} \sum_{l=0}^{L-1} ((e^{j\frac{\pi l^2}{L^2}}. X_l). \ e^{j\frac{2\pi ml}{L}}) \quad (6)$$

where $m = 0, 1, 2 \ldots L-1$. Equation (6) represents the ZCT precoded constellations symbols. The complex baseband ZCT-R-OFDM signal with L subcarriers can be written as:-

$$x_n = \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} Y_m . e^{j2\pi\frac{n}{L}m} \text{ , } n=0,1,2,\ldots,L\text{-}1 \quad (7)$$

The complex passband transmit signal, $x(t)$ of ZCT-R-OFDM after Root Raised Cosine (RRC) pulse shaping and D/A of x_n can be written as:-

$$x(t) = e^{j\omega_c t} \sum_{n=0}^{L-1} x_n . r(t - n\check{T}) \quad (8)$$

where ω_c is carrier frequency, $r(t)$ is baseband pulse, $\breve{T} = \left(\frac{M}{N}\right).T$ is compressed symbol duration in seconds after IFFT. The RRC pulse shaping filter can be defined as:-

$$r(t) = \frac{sin\left(\frac{\pi t}{\breve{T}}(1-\alpha)\right)+4\alpha\frac{t}{\breve{T}}.cos\left(\frac{\pi t}{\breve{T}}(1+\alpha)\right)}{\frac{\pi t}{\breve{T}}.\left(1-\frac{16\alpha^2 t^2}{\breve{T}^2}\right)} \quad (9)$$

$0 \leq \propto \leq 1$, where $\propto$ is rolloff factor. The PAPR of ZCT-R-OFDM signal in (8) with pulse shaping can be written as:-

$$PAPR = \frac{\max\limits_{0\leq t\leq N\breve{T}}[|x(t)|^2]}{\frac{1}{N\breve{T}}\int_0^{N\breve{T}}[|x(t)|^2]dt} \quad (10)$$

The PAPR of ZCT-R-OFDM signal in (7) without pulse shaping can be written as:-

$$PAPR = \frac{\max\limits_{n=0,1,\dots,N-1}[|x_n|^2]}{\frac{1}{M}\sum_{n=0}^{N-1}[|x_n|^2]} \quad (11)$$

It should be pointed out that the orthogonality of the symbols after introducing precoding is maintained, as the precoding matrix is cyclic auto-orthogonal [6].

IV. SIMULATION RESULTS

Extensive simulations in MATLAB(R) have been carried out for the PAPR analysis of ZCT-R-OFDM system with RRC pulse shaping. To show the PAPR analysis of the ZCT-R-OFDM system, the data is generated randomly then modulated by QPSK. All the simulations have been performed based on 10^5 random OFDM blocks. Simulation parameters that we use are given in the following Table. 1 as under:-

TABLE 1
SYSTEM PARAMETERS

Oversampling Factor	4
System Subcarriers	64, 512
Precoding	WHT and ZCT
Modulation	QPSK
Pulse Shaping	Root Raised Cosine (RRC)
Roll Off Factor of RRC	$\alpha = 0.22$
CCDF Clip Rate	10^{-3}

We evaluate the PAPR statistically by using Complementary Cumulative Distribution Function (CCDF). The CCDF of the PAPR for OFDM signals is used to express the probability of exceeding a given threshold $PAPR_0$ ($CCDF = Prob\,(PAPR > PAPR_0)$.

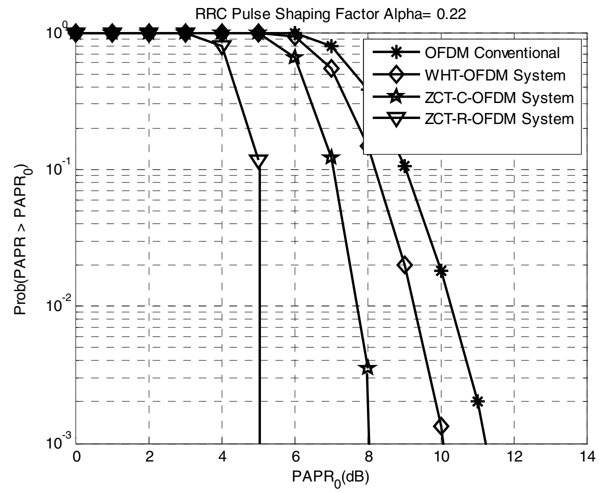

Fig. 3 CCDF Comparison of PAPR of ZCT-R-OFDM system, with ZCT-C-OFDM system, WHT-OFDM system and OFDM conventional, with RRC Pulse Shaping, using *N*=64 for QPSK

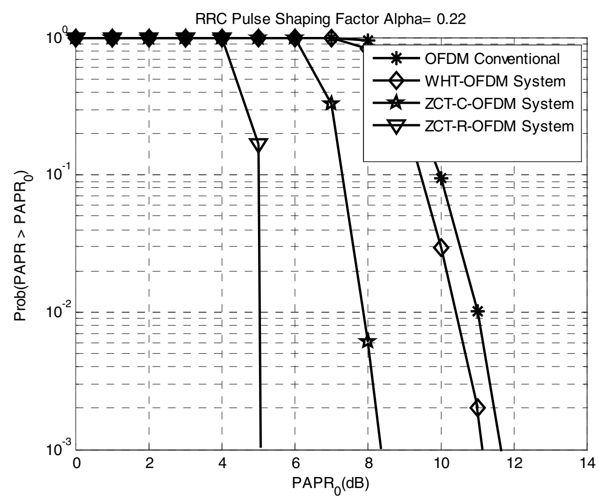

Fig. 4 CCDF Comparison of PAPR of ZCT-R-OFDM system, with ZCT-C-OFDM system, WHT-OFDM system and OFDM conventional, with RRC Pulse Shaping, using *N*=512 for QPSK

We also compared our simulation results with ZCT-C-OFDM system, WHT precoded OFDM systems and conventional OFDM systems.

Fig. 3 shows the CCDF comparison of the PAPR for the ZCT-R-OFDM system, ZCT-C-OFDM system, WHT precoded OFDM systems and conventional OFDM systems, with RRC pulse shaping. At clip rate of 10^{-3}, with the system subcarriers $N = 64$, the PAPR is 11.2dB, 10dB, 8dB and 5dB respectively, for the conventional OFDM systems, the WHT precoded OFDM systems, the ZCT-C-OFDM system and ZCT-R-OFDM system, using RRC pulse shaping (roll off factor $\alpha = 0.22$) for QPSK modulation.

Fig. 4 shows the CCDF comparison of the PAPR for the ZCT-R-OFDM system, ZCT-C-OFDM system, WHT precoded OFDM systems and conventional OFDM systems, with RRC pulse shaping. At clip rate of 10^{-3}, with the system subcarriers $N = 512$, the PAPR is 11.6dB, 11dB, 8dB and 5dB respectively, for the conventional OFDM systems, the WHT precoded OFDM systems, the ZCT-C-OFDM system and ZCT-R-OFDM system, using RRC pulse shaping (roll off factor $\alpha = 0.22$) for QPSK modulation.

V. CONCLUSION

In this paper, we present a PAPR analysis of the ZCT-R-OFDM system with RRC pulse shaping. It is obvious from the Fig. 3 and Fig. 4 that, the ZCT-R-OFDM system has lower PAPR than the ZCT-C-OFDM system, the WHT precoded OFDM systems and the conventional OFDM systems. Hence, it is concluded that the ZCT-R-OFDM system is more favourable than the ZCT-C-OFDM system, the WHT-OFDM system and the Conventional OFDM systems. Another noticeable fact is that, pulse shaping increase the PAPR of the ZCT-R-OFDM system from 0 dB to 5 dB. The ZCT-R-OFDM system does not require any power increase, complex optimization and side information to be sent for the receiver. Additionally, this system also take advantage of the frequency variations of the communication channel and can also offer substantial performance gain in fading multipath channels.

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