ECE 3300
MATLAB Assignment 2

This assignment (1) demonstrates how to work with complex-valued signals and periodic signals in MATLAB, and (2) shows how to obtain energy, power, correlation, and mean-square error.

Complex Numbers and Complex-Valued Lists

`a=3+4j` sets `a` to the complex number $3+4j$. Complex numbers can be used in lists; an example is `b=[3-4j,3j,-4]`. `abs(x)` computes the magnitude, `angle(x)` finds the phase (in radians), `conj(x)` takes the complex conjugate, `real(x)` determines the real part, and `imag(x)` obtains the imaginary part (excluding the `j`). Each of these commands can be used on lists; for example, if `b` is as defined above, `abs(b)` equals `[5,3,4]`, `angle(b)` equals `[-0.9273 1.5708 3.1416]`, `conj(b)` equals `[3+4j,-3j,-4]`, `real(b)` equals `[3,0,-4]`, and `imag(b)` equals `[-4,3,0]`.

Rounding Numbers

The command `round(t)` rounds `t` to the nearest integer. If `t` is exactly between two integers, it rounds up to the higher integer. `round` can be used on lists; for example, if `t=[-2 -1.4 -0.8 -0.2 0.4 1.0 1.6 2.2]`, `round(t)=[-2 -1 -1 0 0 1 2 2]`. Similarly, the command `ceil(t)` rounds `t` up to the next higher integer, and the command `floor(t)` rounds `t` down to the next lower integer. Both of these commands can also be used on lists. For the example of `t` above, `ceil(t)=[-2 -1 0 0 1 1 2 3]` and `floor(t)=[-2 -2 -1 -1 0 1 1 2]`.

Periodic Signals

Suppose $\tilde{x}(t)$ is a periodic signal with fundamental period T_0 and fundamental cycle $x(t)$ and define $y(t) = x(t - T_0 r(t/T_0))$, where $r(\cdot)$ is the function `round`$(\cdot)$. Then we claim that $y(t)$ *equals* $\tilde{x}(t)$. To prove this result we need to show two facts: (1) $y(t)$ has fundamental cycle $x(t)$. (2) $y(t)$ is periodic with fundamental period T_0. To prove (1), we note that $-T_0/2 \le t < T_0/2$ implies that $-\frac{1}{2} \le t/T_0 < \frac{1}{2}$; simply divide each term in the first inequality by T_0 to see this. It follows that $r(t/T_0) = 0$, and thus, for $-T_0/2 \le t < T_0/2$, $y(t) = x(t - T_0 r(t/T_0)) = x(t)$. To prove (2), we first note that $r(t-1) = r(t) - 1$. (To see this, note that $r(5.8-1) = r(4.8) = 5 = 6 - 1 = r(5.8) - 1$.) It follows that

$$\begin{aligned}
y(t - T_0) &= x(t - T_0 - T_0 r(\tfrac{t-T_0}{T_0})) \\
&= x(t - T_0 - T_0(r(\tfrac{t}{T_0} - 1))) \\
&= x(t - T_0 - T_0(r(\tfrac{t}{T_0}) - 1)) \\
&= x(t - T_0 - T_0 r(\tfrac{t}{T_0}) + T_0) \\
&= x(t - T_0 r(\tfrac{t}{T_0})) \\
&= y(t)
\end{aligned}$$

Thus we conclude that $\tilde{x}(t) = x(t - T_0 r(t/T_0))$. We can write this as $\tilde{x}(t) = x(t_{\text{per}})$ where $t_{\text{per}} = t - T_0 r(t/T_0)$.

To apply this result to MATLAB, suppose we wish to implement a periodic signal $\tilde{x}(t)$ with $T_0 = 2$ and fundamental cycle $x(t) = t^2(u(t) - u(t-1))$ using a sampling time of 0.01. Further suppose we wish to represent $\tilde{x}(t)$ over ten periods, from $t = -10$ to 10. We first set `t=-10:0.01:10` and `t_per=t-2.*round(t./2)`. Next we set `x=t_per.^2.*((t_per>=0)&(t_per<1))`. The periodic signal from $t = -10$ to $t = 10$ is represented by `t` and `x`. To prevent roundoff issues, T_0 should be an integer multiple of the sampling time.

Discrete-time periodic signals are defined similarly except the the time values are integers. For example, suppose $N_0 = 7$ and $x[n] = \frac{n+3}{2}(u[n+2] - u[n-3])$, and we wish to represent $\tilde{x}[n]$ from $n = -50$ to $n = 50$. We use the following MATLAB commands: `n=-50:50`, `n_per=n-7.*round(n./7)`, and `x=(n_per+3)./2.*((n_per>=-2)&(n_per<=2))`; the signal is represented from $n = -50$ to $n = 50$ by `n` and `x`.

PROBLEM STATEMENT: PART ONE

Consider (1) the periodic signal $\tilde{x}(t)$ with fundamental period $T_0 = 8$ and fundamental cycle $x(t) = (16 - t^2)^{1/3}(u(t+4) - u(t-4))$ and (2) the non-periodic complex-valued signal $z(t) = \frac{2-jt}{1-jt} e^{2jt}(u(t+8) - u(t-8))$. In everything below use a sampling time of 0.01.

1. Plot $\tilde{x}(t)$ from $t = -12$ to $t = 12$.
2. Plot $\frac{d}{dt}\tilde{x}(t)$ from $t = -12$ to $t = 12$.
3. Plot $|z(t)|$ from $t = -10$ to $t = 10$.
4. Plot $\angle z(t)$ from $t = -10$ to $t = 10$.

Energy of Discrete-Time Signals

Suppose the signal $x[n]$ is defined over the range $a \le n \le c$. Then the energy is given by $E_x = \sum_{n=a}^{c} |x[n]|^2$. If $x[n]$ is represented in MATLAB via `n=a:c` and `x`, the energy can be determined via `sum(abs(x).^2)`.

Correlation and Mean-Square Error of Discrete-Time Signals

Suppose the signals $x[n]$ and $y[n]$ are both defined over the range $a \le n \le c$. Then the correlation is given by $R_{x,y} = \sum_{n=a}^{c} x[n]y^*[n]$ and the mean-square error is given by $\text{MSE}_{x,y} = \sum_{n=a}^{c} |x[n] - y[n]|^2$. If $x[n]$ and $y[n]$ are represented in MATLAB via `n=a:c`, `x`, and `y`, the correlation can be determined via `sum(x.*conj(y))`, and the mean-square error can be obtained via `sum(abs(x-y).^2)`.

Energy of Continuous-Time Signals

Suppose the signal $x(t)$ is defined over the range $a \le t \le c$. Then the energy is given by $E_x = \int_a^c |x(t)|^2 dt$. We can approximate the integral as the sum of the areas of a bunch of rectangles; the ith such rectangle has width b and height $|x(a + (i-1)b)|^2$. Because the area of a rectangle is the product of its width and height, it follows that, if $c = a + kb$,

$$E_x = \int_a^c |x(t)|^2 dt = \int_a^{a+kb} |x(t)|^2 dt \approx \sum_{i=1}^{k} |x(a + (i-1)b)|^2 b$$

Thus, if the sampling time is `b`, and if $x(t)$ is represented in MATLAB by `t=a:b:c` and `x`, then E_x can be approximated by `sum(abs(x).^2).*b`.

Correlation and Mean-Square Error of Continuous-Time Signals

Suppose the signals $x(t)$ and $y(t)$ are defined over the range $a \le t \le c$. Then the correlation and mean-square error are given by $R_{x,y} = \int_a^c x(t)y^*(t)dt$ and $\text{MSE}_{x,y} = \int_a^c |x(t) - y(t)|^2 dt$. Approximating the integrals as the sum of the areas of rectangles, we have

$$R_{x,y} = \int_a^c x(t)y^*(t)dt = \int_a^{a+kb} x(t)y^*(t)dt \approx \sum_{i=1}^{k} x(a + (i-1)b)y^*(a + (i-1)b)b$$

and

$$\text{MSE}_{x,y} = \int_a^c |x(t) - y(t)|^2 dt = \int_a^{a+kb} |x(t) - y(t)|^2 dt \approx \sum_{i=1}^{k} |x(a + (i-1)b) - y(a + (i-1)b)|^2 b$$

Thus, if the sampling time is `b`, and if $x(t)$ and $y(t)$ are represented in MATLAB by `t=a:b:c`, `x`, and `y`, then $R_{x,y}$ can be approximated by `sum(x.*conj(y)).*b` and $\mathrm{MSE}_{x,y}$ can be approximated by `sum(abs(x-y).^2).*b`.

Periodic Signals

For periodic signals, the power, correlation, and mean-square error can be obtained from the energy, correlation, and mean-square error of the signals' corresponding fundamental cycles by simply dividing the quantity by the fundamental period.

The following table summarizes the results assuming the starting signal is implemented via `n=a:c` (if discrete time) or `t=a:b:c` (if continuous time) and `x`.

Quantity	Equation	t or n	MATLAB
E_x	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$	`n=a:c`	`sum(abs(x).^2)`
E_x	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	`t=a:b:c`	`sum(abs(x).^2).*b`
$R_{x,y}$	$\sum_{n=-\infty}^{\infty} x[n]y^*[n]$	`n=a:c`	`sum(x.*conj(y))`
$R_{x,y}$	$\int_{-\infty}^{\infty} x(t)y^*(t)dt$	`t=a:b:c`	`sum(x.*conj(y)).*b`
$\mathrm{MSE}_{x,y}$	$\sum_{n=-\infty}^{\infty} \lvert x[n]-y[n]\rvert^2$	`n=a:c`	`sum(abs(x-y).^2)`
$\mathrm{MSE}_{x,y}$	$\int_{-\infty}^{\infty} \lvert x(t)-y(t)\rvert^2 dt$	`t=a:b:c`	`sum(abs(x-y).^2).*b`

PROBLEM STATEMENT: PART TWO

Suppose $x(t) = 2\sin(\pi t)(u(t)-u(t-1))+t^{2/3}(u(t-2)-u(t-3))$ and $y(t) = (t-2)^2(u(t)-u(t-4))$. In everything below use a sampling time of 0.01. For each part, print out the value of the indicated quantities, including the name of each quantity with the corresponding value.

1. Determine E_x, E_y, $R_{x,y}$, and $\rho_{x,y}$.
2. Let $z(t) = y(t) - \frac{R_{x,y}}{E_x}x(t)$. Determine $R_{x,z}$ and $\rho_{x,z}$.
3. Replace $x(t)$ and $y(t)$ with different signals of your choice, and use the definition in part (ii) to obtain the corresponding $z(t)$. Again calculate $\rho_{x,y}$ and $\rho_{x,z}$. (The formula for $z(t)$ is a part of a process called the Gram-Schmidt orthogonalization procedure.)