Investigation 4A: Introduction to Sinusoidal Functions

Math Studies 1

21st of	Guatemala	Chelsea	Fairbanks
Sept.	5:51	6:22	7:30
Oct.	5:54	6:56	9:02
Nov.	6:06	7:34	9:46
Dec.	6:22	8:02	10:58
Jan.	6:31	8:00	10:11
Feb.	6:24	7:25	8:30
Mar.	6:05	6:37	6:46
Apr.	5:44	5:45	5:52
May	5:33	5:09	4:06
Jun.	5:34	5:00	2:59
Jul.	5:43	5:18	4:15
Aug.	5:49	5:50	5:59
Sept.	5:51	6:23	7:33
Oct.	5:55	6:57	9:04
Nov.	6:06	7:35	9:49
Dec.	6:23	8:02	10:58
Jan.	6:31	7:59	10:09
Feb.	6:23	7:24	8:27
Mar.	6:05	6:37	6:46
Apr.	5:44	5:46	5:53
May	5:32	5:09	4:07
Jun.	5:34	5:00	2:59
Jul.	5:42	5:18	4:14
Aug.	5:48	5:50	5:58
Sept.	5:50	6:23	7:32
Oct.	5:54	6:56	9:04

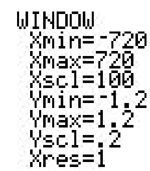
You have probably learned about algebraic functions such as linear, quadratic, and exponential ones. If so, you have discovered that these functions can be used to model diverse real-life situations, including projectile motion, time-and-distance relationships, investments or loans, and a host of other situations. However, these functions are limited when it comes to relationships like the one shown in the table at right.

Unless you live near the equator, you are probably familiar with the phenomenon recorded in this table: Because of the tilt of the earth on its axis, the length of daylight each day changes. In autumn, the days are getting shorter (and sunrise is later), until they reach a minimum in the middle of winter. Then the days start getting longer (and the sun rises earlier) until midsummer, when they start getting shorter again. The cycle repeats year after year. The table shows the time of sunrise in three cities—Guatemala City, Guatemala; Chelsea, Michigan; and Fairbanks, Alaska—starting in September, 2003.

Why can't we use elementary functions to model these data? Well, you can see that in this table, the y-values show a repeating pattern. The linear and exponential functions certainly don't do this—they are always increasing or always decreasing. And although quadratic functions move up and back down, they don't have that repeating feature we see here.

We are going to need a new function to model this data. The most common function for modeling repeating patterns is called the *sine* function. The sine of x is usually written as "sin x." This activity will introduce you to the sine function and some of its characteristics:

1. Graph $y = \sin x$ on a graphing calculator. (Make sure it is in **degree** mode—more about this later.) Use the window shown at right.



   a. What are the domain and range of this function?

   b. What is the maximum value of sin x? The minimum value?

   c. The sine function's graph looks like a wave—we often call this shape a "sine wave." Trace to estimate the distance from the beginning of one wave to the beginning of the next. What is significant about this number?

   d. Use your graph or a calculator to find these values of the sine function:

      i. sin 0

      ii. sin 100

      iii. sin -45

      iv. sin 270

2. Use a graphing calculator or graphing software to create a scatter plot of the data for Fairbanks.

   a. This graph also has that "sine wave" shape—we call it a *sinusoidal* relationship. What is the distance from the beginning of one wave to the beginning of the next in this graph? What are the minimum and maximum values of y here?

   b. Graph $y = \sin x$ on the same set of axes. Does the sine function fit this data? What needs to change?

   c. Experiment with values of a, b, and c in the equation $y = a\sin(bx) + c$ to find an equation that fits the data. How are the values of a, b, and c related to your answers in part a?

   d. Use your equation or graph to predict the time of sunrise on June 15 this year. What about December 1?

Exercises

1. Repeat step 2 of the investigation, using the data for Chelsea.
2. Repeat step 2 of the investigation, using the data for Guatemala City.
3. Graph the data (or just the equations) for the three cities, all on the same set of axes. What characteristics of the "sine wave" are the same for all three cities? What characteristics are different? Use what you know about the location of these cities to explain the reason for the differences.
4. What other quantities could you measure—like the time of sunrise—that might show the kind of repeating pattern we saw in the investigation? Make a list of as many things as you can think of.