Investigation: Quadratic Graphs and Factoring

Algebra 1

One of the reasons we learned to factor trinomials is that we can use factoring to find certain features of quadratic graphs. We can also use factoring to solve quadratic equations. The purpose of this investigation is to discover the connections between factoring, quadratic graphs, and solutions to quadratic equations.

I. Factors and x-intercepts.

A. Fill in the *first two columns* of this table, showing the general form and factored form of each quadratic equation.

General Form	**Factored Form**	***x*-intercepts**
$y=x^2-5x+4$	$y=(x-4)(x-1)$	(4, 0) and (1, 0)
$y=x^2+6x+8$		
	$y=(x+3)(x-3)$	
	$y=(x+2)^2$	
$y=x^2+7x-18$		
	$y=(x-3)(x-5)$	

B. Now graph each quadratic equation on Geogebra, and find its *x*-intercepts. Record them in the third column of the table. The first one has been done for you.

C. Look back at the completed table. What relationship do you see between the factored form of an equation, and its *x*-intercepts? Describe this relationship as specifically as possible:

✏ ____

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II. Use what you've learned.

A. Use what you just discovered to fill in this table, and predict the *x*-intercepts *without graphing* the equations.

General Form	**Factored Form**	***x*-intercepts**
$y=x^2+5x+6$		
$y=x^2-7x-8$		

B. Now check your answers to part D by graphing the equations and looking at the *x*-intercepts. Were your answers correct? If not, fix them. ✏ _________________

C. You can also use what you've discovered to write a quadratic equation if you know the *x*-intercepts. In the table below, write an equation in factored form for a parabola that has the given x-intercepts. Then rewrite the equation in general form.

General Form	Factored Form	x-intercepts
		(5, 0) and (–2, 0)
		(1, 0) and (–1, 0)

III. Solving quadratic equations

A. You can use what you've discovered here to solve equations that have x^2 in them. Consider the following equation:

$$x^2-5x+4=0$$

This equation has just one variable (x), so it is not a function to be graphed, but rather this is the kind of equation we would want to solve to find the value of x. Notice that this equation is almost like $y=x^2-5x+4$ (a parabola), except that instead of y we see a 0. In other words, we have set $y=0$. Answer the questions below to consider this further:

B. Where on the coordinate plane does $y = 0$? ✎ ____

C. Graph the equation $y=x^2-5x+4$. Where on this graph does $y = 0$? Give the specific coordinates. ✎ ____

D. What do we call these points? ✎ ____

E. So, what are the solutions to the equation $x^2-5x+4=0$? ✎ ____________________________

F. Check your solutions by substituting each one for x in the equation $x^2-5x+4=0$.

✎ ____

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G. Re-write the expression x^2-5x+4 in factored form. ✎ ______________________________

H. What is the relationship between the factored form in part F and the solutions to the equation in part E?

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I. Use what you just did to solve these equations. Start by writing them in factored form:

Equation	Factored Form	Solutions
$x^2-5x+4=0$	$(x-4)(x-1)=0$	$x = 4$ and $x = 1$
$x^2-7x+12=0$		
$x^2+x-6=0$		

IV. When a > 1

A. What if the leading coefficient (a) is not 1? Use Geogebra to find the x-intercepts of each of these parabolas. [Hint: If the intercepts are not whole numbers, write them as *improper fractions*.]

General Form	Factored Form	x-intercepts
$y=2x^2+7x-4$	$y=(2x-1)(x+4)$	
$y=2x^2+9x+10$	$y=(2x+5)(x+2)$	
$y=3x^2-3x+2$	$y=(3x-2)(x-1)$	
$y=8x^2+6x-9$	$y=(2x+3)(4x-3)$	

B. What relationship do you see between the factored form and the x-intercepts in this table? Describe this relationship as specifically as possible:

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