Investigation: The Ambiguous Case of the Law of Sines
a.k.a. Better Cover Your SSA!

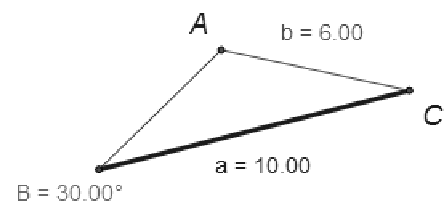
The Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ OR $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

You know that if you have two sides and an angle of a triangle, the Law of Sines can give you a side opposite one of the angles. The two angles and one side could come in several different configurations:

- **SAS**—the known Angle is between (included by) the two known Sides
- **SSA**—the known Angle is *not* between the two known Sides

In this investigation, we will discover something funny about the latter case, SSA: the Law of Sines does not always work in this case!

I. Solve this problem.
   A. In the triangle at right, use the Law of Sines to find the measure of angle A.



   B. Check with a neighbor to see that your answers agree before you continue.

II. Open the Sketchpad file.
   A. Go to the class website.
   B. Under “Algebra 2 Files,” click the link for “SSA.gsp.”

III. Understand the sketch.
   A. This sketch is a dynamic construction of the SSA triangle situation: where you know two SSides and one Angle. On the right is a diagram that shows the Angle and the two SSides.

   B. In the upper left is an “angle slider.” Drag the point labeled “drag,” and see how the angle measurement changes, and how the angle of the triangle adjusts to match it.

   C. Below the angle slider is a slider for side *a*. Adjust this slider and see how side *a* changes.

   D. There is also a slider for side *b*. Adjust this slider and see what happens: You do not always see side *b*. Instead there is a circle whose radius length equals *b*. The circle adjusts size, and tries to find a point (or points!) where it intersects the opposite side of the triangle.

IV. Solve a triangle.

A. You are going to set up the triangle you solved in Part I.

i. Push the button that sets angle B to 30°.

ii. Adjust side a so that it equals 10 cm.

iii. Set side b equal to 6 cm.

B. How many possible positions are there for side b? ✐ __________________

C. You will see two possible measurements for angle A. Does one of them agree with the answer you got in Part I? Which one? ✐ ____________________________________

V. Explore further.

A. Leave B = 30° and $a = 10$, but change b to 5 cm. Now how many possible triangles are there? ✐ ______________

B. Again, leave B = 30° and $a = 10$, but now set b to 4 cm. How many possible triangles are there in this case? ✐ ______________

C. Try one more value: set $b = 12$ cm. How many possible triangles does this create? ✐ ______________

D. Experiment with values of b and then complete the following statements:

i. ✐ There will be two triangles whenever _____ $< b <$ ______.

ii. ✐ There will be no triangles whenever $b <$ ______.

iii. ✐ There will be exactly one triangle whenever $b =$ ______ or $b \geq$ ______.

E. Let's try to find how the two angle measurement for A are related.
   i. Notice that the two possible measures for angle A are labeled on the left-hand side, as A_1 and A_2.

   ii. Go to Measure | Calculate..., and click on the measurement A_1, then click "+" and then click on the measurement A_2. You should see " $A_1 + A_2$" in the calculator. Click OK.

   iii. You will see a new measurement that adds these two angles. Try adjusting all three sliders. What do you notice about the two measurements for angle A?
   ✏ ____

VI. Conclusion

A. The main idea of this investigation is the following:

✏ **When you have a triangle with SSA information given (two ________________ and a non-included ______________), and you want to find a missing angle, there are** [sometimes/always/never] **two possible angles, but when this is true, the two angles are always _________________________.**