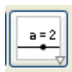
Discovering Vertex Form

Algebra 1

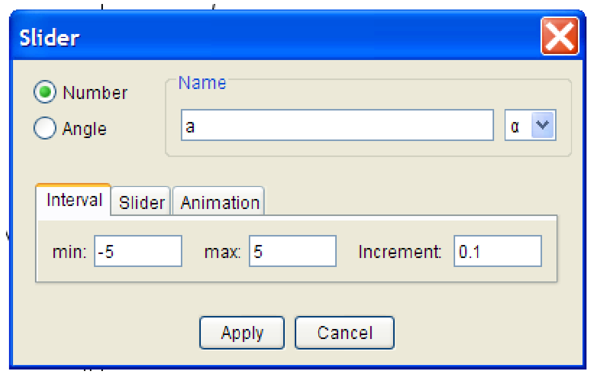
A quadratic equation is any equation with x^2 as its highest power of x. The standard way to write a quadratic function is $f(x)=ax^2+bx+c$. In this investigation, you will explore how the values of the coefficients a, b, and c affect the graph of a quadratic equation, and how the vertex, y-intercept, and x-intercepts are related to these coefficients.

I. Create a Quadratic Graph

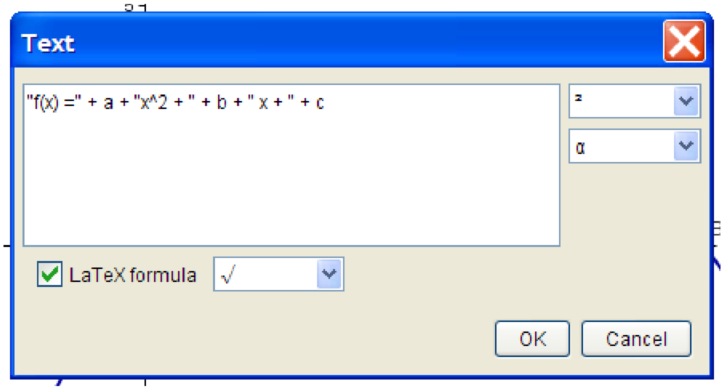
A. Open GeoGebra

B. Use the slider tool 

to create three sliders named a, b, and c. These will be the coefficients of the quadratic equation we will graph. You can accept the default settings for each slider (-5 to 5, and a size of 100).



C. In the "Input Bar" at the bottom, type the quadratic equation f(x)=a*x^2+b*x+c. This will automatically graph the equation, using the values of the sliders as the coefficients.



D. Right-click the parabola, and choose Properties.... In the "Style" tab, change the line thickness to 4, so the parabola stands out. While you're at it, give the parabola some color, too.

E. Make a text box (you get the text tool by clicking the arrow on the slider tool) that gives the equation of the parabola, by typing exactly what you see here.

F. Now you're ready to explore...

II. Finding the Vertex.

A. Imagine a vertical line through the vertex of the parabola. If you fold the parabola on this line, it is symmetrical. Therefore we call this vertical line the *axis of symmetry* of the parabola. These steps will help you discover the x-coordinate of the axis of symmetry:

B. Label the y-intercept as A.

C. Construct a horizontal line that goes through the y-intercept. [Hint: Type the equation $y = c$ in the Input box.] Right-click this line and use Properties... to make it a dashed line.

D. Construct the (other) intersection of this horizontal line, and the parabola. Call this point B.

E. The two points (A and B) represent the two solutions of the equation $ax^2+bx+c=c$. Explain why.

F. Let's solve this equation for x.

  i. First, write the equation:
  $ax^2+bx+c=c$

  ii. Subtract c from both sides:

  iii. "Factor out" x from the left side.

  iv. If the product of two factors equals 0, then one of the factors equals 0. Let's assume the factor $ax+b$ equals 0.

v. Solve $ax+b=0$, for x.
vi. The other solution of $ax^2+bx+c=c$ is when $x = 0$.
vii. Use the text tool to label both A and B with their x-coordinates. ($x = 0$ and $x=-\frac{b}{a}$)

G. Where is the axis of symmetry compared to these two points?
H. How do you find the number half-way between two numbers?
I. Find the average of $x = 0$ and $x=-\frac{b}{a}$.
J. Plot this expression as a vertical line by typing "*x* = -*b*/(2**a*)" in the Input bar. Does this appear to be the axis of symmetry?
K. Drag the sliders to be sure that the axis of symmetry stays with the parabola.
L. The axis of symmetry is also the x-coordinate of the vertex. Move the sliders to create some random parabola, and then write down the x-coordinate of its vertex.
M. Use your answer to step L, and the equation of the parabola, to find the y-coordinate of the vertex.
N. Plot the vertex by typing its coordinates in the Input bar. Were the coordinates correct?

III. Vertex Form

A. Save your current Geogebra file, and then open a new blank sketch.
B. Again, create three sliders, but call them a, h, and k.
C. The vertex form of a parabola is the equation $f(x)=a(x-h)^2+k$. Graph this equation by typing "f(x) = a(x-h)^2 + k" in the input bar.
D. Drag the sliders to see how each parameter affects the graph.
E. Why is this form called vertex form? What, specifically, do the variables h and k measure?

IV. Use What You Learned

A. Solve the following problems neatly on a separate sheet of paper.
1. Each quadratic function below is written in vertex form. What are the coordinates of each vertex?
   a. $y = (x - 2)^2 + 3$ **b.** $y = 0.5(x + 4)^2 - 2$ **c.** $y = 4 - 2(x - 5)^2$
2. The function $h(x)=-4.9(x-0.4)^2+2.5$ describes the height of a softball thrown by a pitcher, where $h(x)$ is height in meters and x is time in seconds.
   (a) How high does the ball go?
   (b) How long does it take the ball to reach its maximum height?
   (c) At what height did the pitcher release the ball when x was 0 seconds?
   (d) Find the x-values for which $h(x) = 2$, and describe their real-world meanings.
   (e) What domain and range values make sense in this situation?
   (f) Sketch a graph of this situation. Your graph should be neatly labeled and should indicated clearly which points answer each of the questions above.