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Polynomial Graph Exploration Algebra II

Part I: In this activity, you will explore polynomial functions to find out how their coefficients affect their graphs. First, you need to open up the Excel file, "Polynomial Graph Exploration." (If it asks you about macros, click the button that says, "Enable Macros." If it does not, you might need to change the security settings: Ask Mr. Dunigan AtLee about this.) You should see a graph at the right with some "sliders" to the left. Each slider controls one of the coefficients of the polynomial graph. Below are a table of values and the window settings. You should *write* your answers to the questions below on a separate sheet of graph paper.

A. Quadratic Functions

You know that a 2nd-degree polynomial is called *quadratic*, and you are familiar with its standard form, $f(x) = ax^2 + bx + c$. Try graphing some different quadratic functions by moving the bottom three sliders (leave the top three set at 0).

1. Experiment with different values of a (the x^2 term).
   a. What does the graph look like if $a > 0$? If $a < 0$?
   b. Describe what happens as $|a|$ gets bigger or smaller.
2. Experiment with different values of c. Describe how the value of c affects the graph.
3. Experiment with different values of b (the x-term). As you move the slider for b, describe how the graph moves.
4. Draw on your paper the graph of one quadratic function, and write its equation.

B. Cubic Functions

A 3rd-degree polynomial is called *cubic*. Move the x^3 slider and see what a cubic polynomial's graph looks like.

1. How would you describe the shape of a cubic polynomial's graph?
2. Experiment with the *leading coefficient* of a cubic polynomial and describe the difference between a graph with a negative leading coefficient and a graph with a positive one.
3. Experiment with the x^2 term. Describe how you think this term affects the graph.
4. Experiment with the x term. Describe how you think this term affects the graph.
5. Draw on your paper the graph of one cubic function, and write its equation.

C. Quartic Functions

A 4th-degree polynomial is called *quartic*. Move the x^4 slider to see what these look like.

1. How would you describe the shape of a quartic polynomial's graph? (Experiment with all the sliders before you answer this question—polynomials can be more complicated than they first appear!)
2. Graph the function $f(x) = x^4 - 2x^3 - 12x^2 + 6x - 15$.
   a. Draw a copy of this graph on your paper, and label it with its equation.
   b. Notice that the graph starts out going downward, then moves upward, and so on. In total, how many times does the graph change directions?

D. Quintic Functions

A 5th-degree polynomial is called *quintic*. Move the x^5 slider and enjoy the view.

1. What does a quintic polynomial's graph look like? (Again, experiment before you answer.)
2. Graph the function $f(x) = 2x^5 + 7x^4 - 5x^3 - 15x^2 + 6x + 5$. (You may have to adjust the window settings to see the graph well.)
   a. Copy and label this graph on your paper.
   b. How many times does this graph change directions?
   c. How many x-intercepts does this graph have?
3. Look back at the quadratic and cubic polynomials you graphed.
   a. How many times does a quadratic polynomial change direction?
   b. How many times does a cubic polynomial change direction?
   c. Write a conjecture about how the degree of a polynomial affects the number of direction-changes in its graph.
4. Again look back at the polynomials you graphed.
   a. What is the highest number of x-intercepts a quadratic graph can have?
   b. What is the highest number of x-intercepts a cubic graph can have?
   c. What is the highest number of x-intercepts a quartic graph can have?
   d. Write a conjecture about how the degree of a polynomial affects the number of x-intercepts it has.

Part II: Zeros and Extrema

In this part, you are going to explore two important elements of a polynomial and its graphs: *zeros* and *local extrema*.

E. You know that the zeros of a polynomial $P(x)$ are the values of x when $P(x) = 0$. You can use a graph to estimate the zeros.

1. Use Excel to graph the function $f(x) = -2x^5 + 5x^4 + 6x^3 - 4x^2 + 1x - 13$.
   a. Copy and label the graph on your paper.
   b. In how many places does the graph cross the x-axis?
   c. Use the graph to estimate the x-coordinate at each of the points where the graph crosses the x-axis. These numbers are the *zeros* of $f(x)$.
   d. Use the zeros you found in part c to write this equation in factored form.

F. You noticed in Part I that graphs of polynomials are often "lumpy." The tops and bottoms of these lumps are important because they are the places where the graph changes directions (they are related to a calculus concept called a *derivative*). These points are called *local extrema.* ("Extrema" is plural; the singular is "extremum.") There are two kinds of extrema: minima and maxima.

1. Use Excel to graph the function $f(x) = 1x^4 + 2x^3 - 11x^2 + 3x + 1$ (notice there is no x^5 term).
   a. Copy and label the graph on your paper.
   b. On your graph, find and mark with an × the two points that are at the bottom of the "valleys" in this graph. These are the *local minima* of this graph. Estimate their coordinates and label them.
   c. On your graph, find and mark with a circle the one point that is at the top of a "hill." This point is the *local maximum* of this graph. Estimate its coordinates and label it.

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G. With a graphing calculator, you don't have to estimate to find the zeros and extrema of a graph. When you have finished the above activities, read the notes below and on the next page to learn how to do this.

1. Graph the function $f(x) = 1x^4 + 2x^3 - 11x^2 + 3x + 1$ (same equation as in part F) on your calculator.

   a. Use your calculator to find the zeros.

   b. Use your calculator to find the two local minima.

   c. Use your calculator to find the one local maximum.

H. Your homework is Handout 6A.

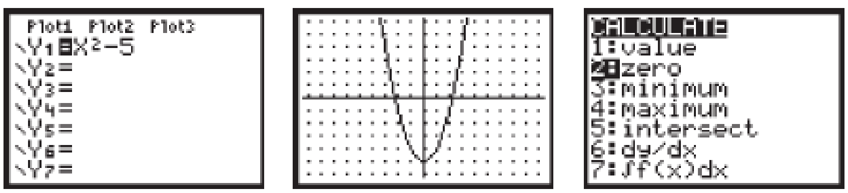
Zero Finding

Your calculator can find the zero(s) of a function.

a. Enter the equation into the Y= screen.

b. Find a window that shows the zero you want to determine and display the graph.

c. Select 2nd [CALC] 2:zero.



[−9.4, 9.4, 1, −6.2, 6.2, 1]

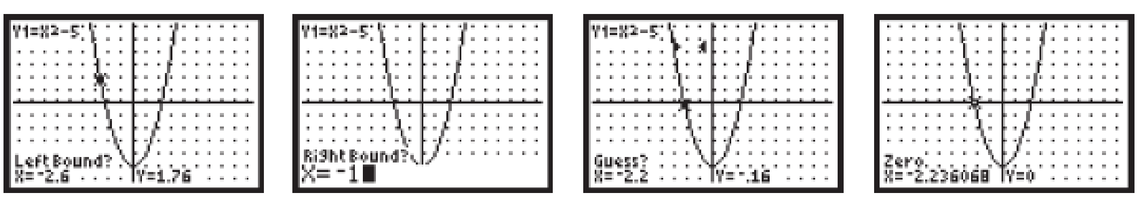
d. The calculator prompts you to enter left and right bounds around the zero and a guess. You can do this by arrowing left or right and pressing ENTER, or by typing a number.

Left Bound = an x-value that is to the left of the zero.

Right Bound = an x-value that is to the right of the zero.

Guess = an x-value that is very near the zero.

e. The calculator locates a zero between the left and right bounds, if one exists.



[−9.4, 9.4, 1, −6.2, 6.2, 1]

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Maximums and Minimums

A similar process allows you to find the coordinates of a maximum or minimum without tracing. For example, follow these steps to find the minimum of $y = (x - 3)^2 + 4$:

a. Display the graph of the function.

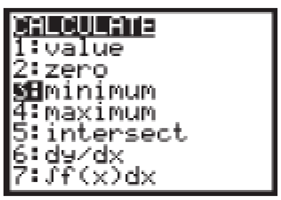
b. Press $\boxed{\text{2nd}}$ [CALC] 3:minimum.

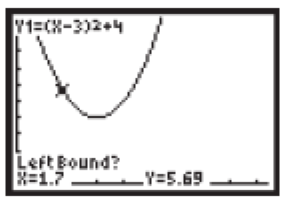
c. The prompt calls for a Left Bound?. Move the spider to the left of the minimum and press $\boxed{\text{ENTER}}$. Note: If the curve has several extreme values, you must confine yourself to the vicinity of the maximum or minimum you want.

d. The prompt calls for a Right Bound?. Move the spider to the right of the minimum and press $\boxed{\text{ENTER}}$.

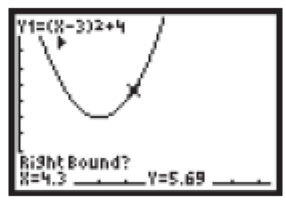
e. Finally, the prompt asks for a Guess?. Move the spider between the two bounds and press $\boxed{\text{ENTER}}$.

f. The screen shows the coordinates of the minimum between the specified bounds.





[0, 9.4, 1, 0, 10, 1]



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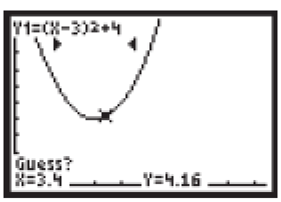

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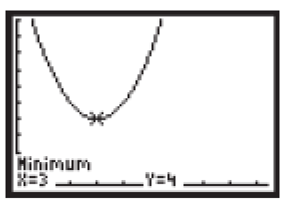

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Selecting 4:maximum results in the maximum between the specified bounds.