Investigation 8B: Gradients of Tangent Lines

Math Studies 2

Yesterday we discussed the idea of a tangent line, and connected it to a roller coaster. A tangent line shows the direction and steepness of the roller coaster at a certain point on a graph of a function. In this activity, you will measure the gradients of tangent lines and look for patterns in the gradients of tangent lines at different places on the same function.

You will need a computer with GeoGebra installed on it. You will NOT need access to the internet for this activity (except at the end to turn in your work).

I. Open GeoGebra

II. Set up a sketch

A. In the Input Bar at the bottom, type

f(x)=x^2 – 3x – 5

B. You should see a parabola. Right-click the parabola, choose Object Properties..., and set the color and style of the parabola so it stands out.

C. Plot point P on the parabola by typing

P=Point[f]

D. Now draw the tangent to *f* at P by typing

T=Tangent[P, f]

E. Set the color and style of line T so it stands out.

F. Drag P around to see how the tangent line changes.

G. Measure the gradient of the tangent line by typing

m=Slope[T]

H. Drag P again and see how the gradient changes.

III. Investigate a Quadratic Function

A. Go to Options | Point Capturing, and choose "Fixed to Grid."

B. Drag P again, and notice that now P "jumps" so that its *x*-coordinate is always an integer.

C. Use your sketch to fill in the table at right. Drag P to each *x*-coordinate listed, and write down the gradient of the tangent T at each point.

D. Do you notice any patterns in the table you recorded above? Write them here:

x-coordinate of P	gradient of tangent at P
–2	
–1	
0	
1	
2	
3	
4	
5	

E. Look back at your table of gradients. Think of the first column as x and the second column as y. Write an equation relating x and y. [Hint: This is a linear relationship!] Show work here:

IV. Investigate a Cubic Function

A. Go back to your GeoGebra sketch and change the function by typing the following:

f(x) = 0.1x^3 – 0.8x^2 + 5

B. This will change the graph to a "Cubic Function" (one with an exponent of 3).

C. Drag the point P around this graph, and fill in the table at right.

x-coordinate of P	gradient of tangent to $f(x)$ at P
–3	
–2	
–1	
0	
1	
2	
3	
4	
5	
6	
7	

D. Do you notice any patterns in this table? Write them here:

E. Is this a linear relationship? How do you know it is or isn't?

F. Open a new GeoGebra window (Ctrl + N), and go to View | Spreadsheet View.

G. Go to Options | Labeling and choose "No New Objects."

H. In columns A and B of the spreadsheet, type in your data from the table above (leave out the headers of each column—just type the numbers).

I. Use the mouse to highlight the two columns of numbers you just typed. Right-click the highlighted cells and choose "Create List of Points."

J. You should see a scatter plot of the points appear. What shape do these points appear to make?

K. You can tell GeoGebra to find an equation for the data you just plotted. Do this by typing the following:

FitPoly[list1,2]

L. The equation of this fit line should appear on the left-hand side of the screen as $f(x)$. Write the equation here:

V. Generalize

A. In both parts above, you graphed a function $f(x)$, then found another function that calculates the gradient of the original $f(x)$ at different points. Record these results in the table below. Then pick at least four more polynomials of your choice (don't try to get too fancy here!) and use GeoGebra to find their corresponding "gradient functions."

function $f(x)$	gradient function
$f(x)=x^2-3x-5$	
$f(x)=0.1x^3-0.8x^2+5$	

B. Look at the table you have made. What patterns do you see? Can you state a general rule for finding the "gradient function" of a polynomial $f(x)$? Write your rule here.

VI. Turn it in!

A. Save BOTH of your GeoGebra files and upload them to ManageBAC under the assignment "Investigation 8B: Gradients of Tangent Lines."

B. Turn in this handout.

C. Work on Handout 8A from yesterday. Turn in Handout 8A.

VII. Homework: Write a learning log entry. The objective for today was:

Given a function f(x), be able to state a rule for finding a new function that calculates the gradient of the tangent to the graph of f(x) at any point.