Near ML detection using Dijkstra's algorithm with bounded list size over MIMO channels

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***Abstract*— We propose Dijkstra's algorithm with bounded list size after QR decomposition for decreasing the computational complexity of near maximum-likelihood (ML) detection of signals over multiple-input-multiple-output (MIMO) channels. After that, we compare the performances of proposed algorithm, QR decomposition M-algorithm (QRM-MLD), and its improvement. When the list size is set to achieve the almost same symbol error rate (SER) as the QRM-MLD, the proposed algorithm has smaller average computational complexity.**

I. Introduction

The channel capacity of multiple-input-multiple-output (MIMO) channels linearly increases with the number of antennas [1], [2]. Maximum-likelihood (ML) detection provides the minimum error rate. However, the computational complexity of the simple ML detection algorithm grows exponentially with the number of transmit antennas. Thus, we need an efficient algorithm that achieves similar error rate to the ML detection. The QR decomposition M-algorithm (QRM-MLD) [5], [6] and sphere decoding (SD) [3] are possibly the most promising algorithms. In [10], to reduce the computational complexity, Dijkstra's algorithm is applied to SD which achieves same error rate as ML detection. Both the QRM-MLD and Dijkstra's algorithm are tree search based algorithms. Dijkstra's algorithm uses the list of unlimited size to keep detection candidates. However, the computational complexities of the QRM-MLD and Dijkstra's algorithm are still high. To reduce the computational complexity, we propose Dijkstra's algorithm with bounded list size. When proposed algorithm's list size is set to achieve the almost same symbol error rate (SER) as the QRM-MLD, the computational complexity of proposed algorithm is lower than the QRM-MLD.

This paper is organized as follows. In Section 2, we introduce the system model of MIMO channels. In Section 3, we review the QRM-MLD and its improvement, then propose Dijkstra's algorithm with bounded list size. In Section 4, we show the comparison between the computational complexity of the QRM-MLDs and proposed algorithm by computer simulations. Finally, we give the conclusion in section 5.

II. System model

We consider the uncoded system with t transmit antennas and r receive antennas, and we assume $r \geq t$. We assume that the noise at each receive antenna is the additive white Gaussian noise (AWGN). Let $\mathtt{x}$ be a $t \times 1$ vector consisting of complex envelopes of transmitted signals with the signal constellation $\mathtt{S}$, $\mathtt{H}$ an $r \times t$ fading matrix whose (k, j) entry is a complex fading coefficient between j-th transmit antenna and k-th receive antenna, $\mathtt{z}$ an $r \times 1$ complex vector whose component is noise at each receive antenna, and $\mathtt{y}$ an $r \times 1$ complex vector whose component is the received signal component at each receive antenna. The model of this channel is written as

$$\mathtt{y} = \mathtt{H}\mathtt{x} + \mathtt{z}. \tag{1}$$

We assume that the receiver knows the channel state information $\mathtt{H}$ perfectly.

In this case, the ML detection of the transmitted signal over the channel (1) can be formulated as finding

$$\hat{\mathtt{x}}_{ml} = \arg\min_{\mathtt{x} \in \mathtt{S}^t} ||\mathtt{y} - \mathtt{H}\mathtt{x}||^2. \tag{2}$$

III. Near ML detection algorithm

In this section, we propose the new near ML detection algorithm. First, to calculate (2) efficiently, we explain how to find the ML signal by tree search algorithm in Section 3.1. Then, we review near ML detection algorithms called QRM-MLD [5], [6] and its improvement [8] in Section 3.2. Finally, we propose Dijkstra's algorithm with bounded list size in Section 3.3.

A. QR decomposition

To calculate (2) efficiently, we compute a QR decomposition of $\mathtt{H}$ and obtain an upper triangular matrix $\mathtt{R}$ and a unitary matrix $\mathtt{Q}$ with $\mathtt{H} = \mathtt{QR}$. Since $\mathtt{Q}$ is unitary,

$$||\mathtt{y} - \mathtt{H}\mathtt{x}||^2 = ||\mathtt{Q}^*\mathtt{y} - \mathtt{Q}^*\mathtt{H}\mathtt{x}||^2 = ||\mathtt{Q}^*\mathtt{y} - \mathtt{R}\mathtt{x}||^2. \tag{3}$$

Let $\xi = \mathtt{Q}^*\mathtt{y} = (\xi_1, \cdots, \xi_r)^T$. The ML detection problem (2) can be reformulated as finding

$$\begin{aligned}
\hat{\mathtt{x}}_{ml} &= \arg\min_{\mathtt{x} \in \mathtt{S}^t} ||\xi - \mathtt{R}\mathtt{x}||^2 \\
&= \arg\min_{\mathtt{x} \in \mathtt{S}^t} \left\{ \sum_{j=1}^{t} |\xi_j - \sum_{i=j}^{t} \mathtt{R}_{j,i} x_i|^2 + \sum_{k=t}^{r} |\xi_k|^2 \right\} \\
&= \arg\min_{\mathtt{x} \in \mathtt{S}^t} \left\{ \sum_{j=1}^{t} |\xi_j - \sum_{i=j}^{t} \mathtt{R}_{j,i} x_i|^2 \right\}. \qquad (4)
\end{aligned}$$

The second equality above follows from the fact that the second term in the second equation is irrelevant to $\mathtt{x}$.

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To calculate (4) efficiently, we consider a weighted directed graph as follows. The decisions on x_i construct a tree where nodes at k-th depth are correspond to the candidate of x_{t-k+1} [4], and the root node is placed at depth 0. Then, the metric value, which is the weight of branch, between a node $\hat{x}_i$ that has $\hat{x}_t, \cdots, \hat{x}_{i+1} (\hat{x}_k \in \mathtt{S}, i+1 \leq k \leq t)$ as ancestor nodes from the root node and its parent node is defined by

$$m_i = |\xi_i - R_{i,i}\hat{x}_i - \sum_{j=i+1}^{t} R_{i,j}\hat{x}_j|^2.$$

The distance of each node from the root node, which is called the accumulated metric value in this paper, is equal to the sum of the metric values of branches from the root node to the node itself. The accumulated metric value from the root node to the bottom node whose depth is t is

$$\sum_{i=1}^{t} m_i = \sum_{j=1}^{t} |\xi_j - \sum_{i=j}^{t} \mathtt{R}_{j,i}\hat{x}_i|^2. \quad (5)$$

Because $\hat{\mathtt{x}}$ that makes (5) minimum is equal to $\hat{\mathtt{x}}_{ml}$ of (4), the shortest path from the root node to the bottom node corresponds to the ML signal [4].

B. QRM-MLD

The QRM-MLD [5], [6], which is a breadth-first tree search based algorithm, finds a near ML signal. The QRM-MLD keeps only M nodes at each depth with the smallest accumulated metric values [7], instead of testing all the candidate in $\mathtt{S}^t$ according to (4). At each depth, only M nodes make their child nodes. We call a node that makes its child node detection node in this paper.

An improvement to QRM-MLD proposed in [8] reduces the number of detection nodes from the original QRM-MLD. This improved QRM-MLD has threshold value at each depth. The depth i 's threshold value Δ_i is defined by

$$\Delta_i = E_{i,min} + X\phi^2, \quad (6)$$

where $E_{i,min}$ is the smallest accumulated metric value of the node at i-th depth in the nodes whose parent node is a detection node. X is a fixed constant number, and ϕ^2 is the noise variance. At each depth, select the nodes that have smaller accumulated metric value than threshold value Δ_i. If the number of selected nodes is more than M, only M nodes with smallest accumulated metric values are selected.

Note that both algorithms do not always find the ML signal. For a small or medium M values, the complexity is substantially lower than the simple ML detection algorithm. However, the final result is no longer guaranteed to be the ML signal.

C. Proposed algorithm: Dijkstra's algorithm with bounded list size

Dijkstra's algorithm is an efficient algorithm to find the shortest path from a point to a destination in a weighted graph [9]. Dijkstra's algorithm uses the list of unlimited size to keep candidate nodes. If we use Dijkstra's algorithm to find the shortest path from the root to one of nodes at the bottom depth, we can get the node with minimum $||\mathtt{y} - \mathtt{H}\hat{\mathtt{x}}||^2$ among all nodes at the bottom depth and it corresponds to the ML estimate [10]. However, this algorithm still has high computational complexity. To reduce the computational complxity, we propose a modified version of Dijkstra's algorithm whose list keeps only L nodes with the smallest accumulated metric values in the list.

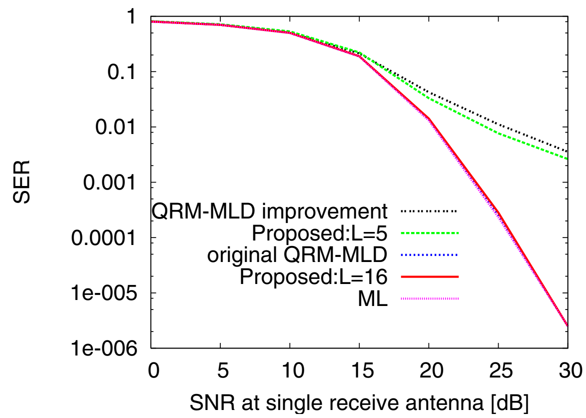

Fig. 1. (color) symbol error rate

We show Dijkstra's algorithm with bounded list size.

1) Create an empty list for nodes.
2) Insert all nodes at the first level into the list.
3) Select the node A having smallest accumulated metric value in the list and remove it from the list. If the depth of A is t, then output the node A and its ancestor nodes as the ML signal and finish this algorithm.
4) Insert all A's child nodes into the list.
5) Arrange the nodes in the list according to the accumulated metric value by the quick sort. If the list has more than L nodes, select the L nodes with the smallest accumulated metric values in the list, and discard other nodes from the list.
6) Go back to Step 3.

The node whose child nodes are inserted into the list is called detection node in this paper. Because the discarded nodes, which are decided at Step 5, and their descendant nodes are not examined, the proposed algorithm dose not examine all the candidate in $\mathtt{S}^t$ according to (4). Thus, the proposed algorithm dose not always find the ML signal.

When we use LDPC codes or turbo codes after detection, we have to compute N most likely signals [11]. Such signals can be computed by this algorithm's modification that is finished after output N signals with the smallest accumulated metric value.

IV. COMPUTER SIMULATION

In this section, we compare the computational complexity, the number of detection nodes and the number of comparison of real numbers among the proposed algorithm and the QRM-

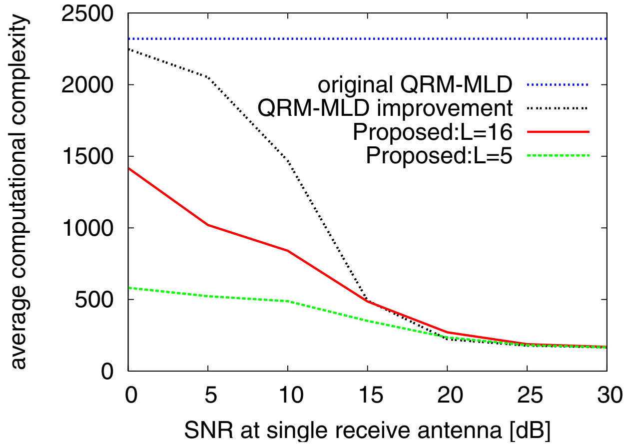

Fig. 2. (color) average computational complexity

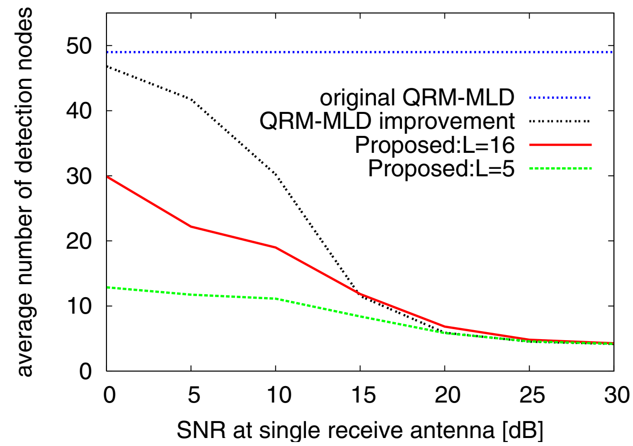

Fig. 4. (color) average number of detection nodes

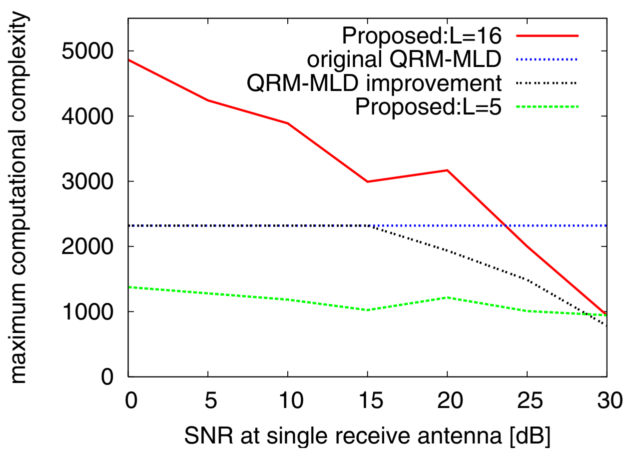

Fig. 3. (color) maximum computational complexity

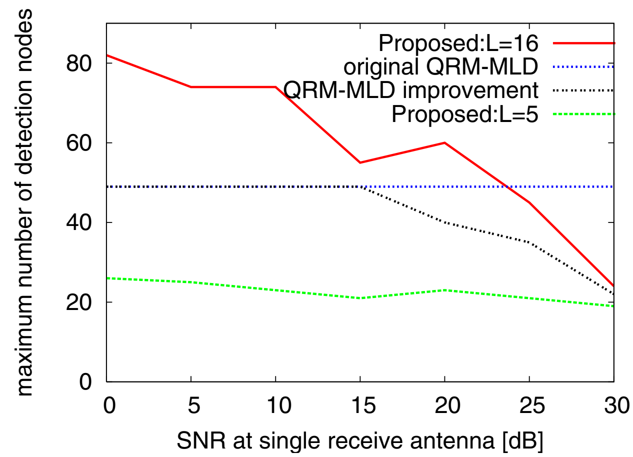

Fig. 5. (color) maximum number of detection nodes

MLDs. Throughout the simulations, we consider the following system model.

- The number of transmit antennas $t = 4$, and the number of receive antennas $r = 4$.
- The signal constellation at each transmit antenna is 16-QAM and all signals are drawn according to the uniform i.i.d. distribution.
- The fading coefficients obey the $CN(0, 1)$ distribution, and the receiver knows it perfectly.
- The noise at each recieve antenna obeys the $CN(0, \phi)$ distribution. ϕ is caluculated by $\phi^2 = tE_s \times 10^{(-SNR/10)}$, where E_s is the average symbol energy.
- We transmit 100000 signals, which is 400000 symbols, and every 100 signals, change the fading matrix.

If $M = 16$ is used and the signal constellation is 16-QAM, QRM-MLD has symbol error rate (SER) near to the ML detection [7]. So, we use $M = 16$. In QRM-MLD's improvement, we use $X = 2$ in (6) as used in [8]. In order for the proposed algorithm to have the similar SER to QRM-MLD and its improvement, we use two versions of proposed algorithm whose list sizes are $L = 16$ and $L = 5$. Figure 1 shows that the proposed algorithm with $L = 16$, the original QRM-MLD and the ML algorithm have almost the same SER throughout this simulations. The proposed algorithm with $L = 5$ and QRM-MLD's improvement also have similar SER throughout this simulations.

We count the number of multiplication and division of complex number as the computational complexity. Since the part of QR decomposition is the common part of all compared algorithm, we do not include that part in comparison of complexity.

In QRM-MLDs, we use the quick sort to arrange the nodes and decide M nodes with the smallest accumulated metric value at each depth.

Because the QRM-MLD keeps M nodes at each depth, the number of detection nodes and the computational complexity are completely determined by M. However, in the proposed algorithm and QRM-MLD's improvement, the number of detection nodes and the computational complexity are not fixed.

According to Figures 2, 4 and 6, the propose algorithm with $L = 16$ reduece the average computational complexity,

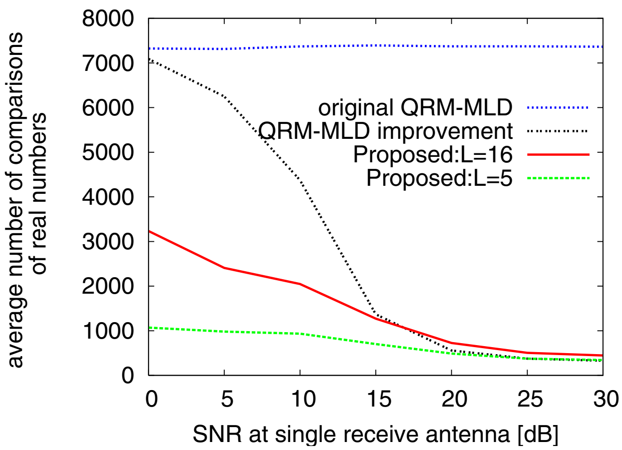

Fig. 6. (color) average number of comparison of real numbers

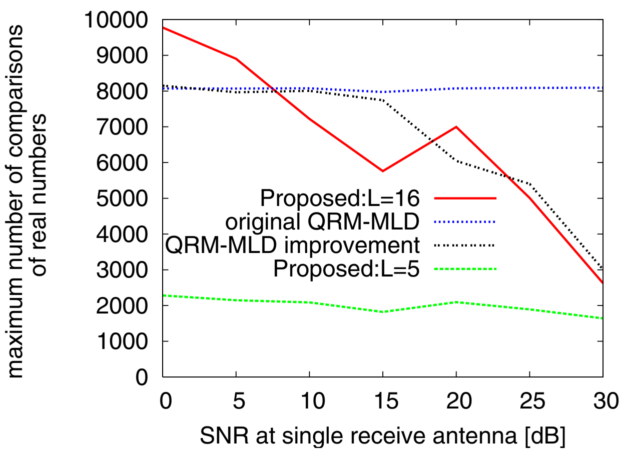

Fig. 7. (color) maximum number of comparison of real numbers

average number of detection nodes and average number of comparison of real numbers from original QRM-MLD. Moreover, in the case of high SNR, although the proposed algorithm with $L = 16$ has much smaller SER than QRM-MLD's improvement according to Figure 1, the average computational complexity of proposed algorithm with $L = 16$ is almost the same as QRM-MLD's improvement. In the case of low SNR, the average computational complexity, average number of detection nodes and average number of comparison of real numbers of the proposed algorithm with $L = 5$ are lower than QRM-MLD's improvement. In the case of high SNR, the average computational complexity, average number of detection nodes and average number of comparison of real numbers of proposed algorithm with $L = 5$ are almost same as QRM-MLD's improvement while the proposed algorithm has smaller SER according to Figure 1. According to Figures 3, 5 and 7, in the case of low SNR, maximum computational complexity, maximum number of detection nodes and the maximum number of comparison of real numbers of the proposed algorithm with $L = 16$ are higher than QRM-MLDs. However, because the average computational complexity, the average number of detection nodes and the average number of comparison of real number of the proposed algorithm with $L = 16$ are lower than QRM-MLDs, we find that the proposed algorithm rarely gets high computational complexity, large number of detection nodes or large number of comparison of real numbers.

V. Conclusion

In this paper, we propose a near ML detection algorithm. When the list size is adjusted so that the proposed algorithm has the almost same symbol error rate (SER) as the original QRM-MLD, the average of the computational complexity and the number of detection nodes are reduced. When the list size is adjusted so that the proposed algorithm has the almost same symbol error rate (SER) as the QRM-MLD's improvement, in the case of low SNR, both the average computational complexity and average number of detection nodes are reduced and in the case of high SNR, the computational complexity and average number of detection nodes of proposed algorithm is almost same as QRM-MLD's improvement while SER of the proposed algorithm becomes smaller than QRM-MLD's improvement.

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