Joint Detection and Channel Estimation Algorithms for QS-CDMA Signals Over Time-Varying Channels

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***Abstract*—We consider a quasi-synchronous code-division multiple access (QS-CDMA) cellular system, where the code delay uncertainty at the base station is limited to a small number of chips. For such QS-CDMA systems, the need for code acquisition is eliminated, however, the residual code tracking and channel estimation problems still have to be solved. An extended Kalman filter (EKF) is employed to track the user delays and channel coefficients. By separating data detection, based on the QR decomposition combined with the M-algorithm (QRD-M) from the delay/channel estimation process, the computational complexity can be significantly reduced as the number of users increases. Simulations show that the EKF channel estimator performance is improved when the QRD-M algorithm is used instead of the MMSE detector or decorrelator for data decisions.**

***Index Terms*—Code division multiple access, Kalman filtering, multiuser channels.**

I. Introduction

A QUASI-SYNCHRONOUS code-division multiple access (QS-CDMA) cellular system has been previously proposed in [1]–[5]. QS operation is obtained, for example, by using GPS receivers at the base station and mobiles to correct for propagation delay and obtain a universal clock [5]. In QS-CDMA systems, the need for code acquisition can be simplified as in [6], and the delay uncertainty is limited to a small number of chips. Although residual code tracking is still required, the need for full code acquisition required in an asynchronous system is eliminated.

The mobile channel is usually characterized by time-varying multipath fading and Doppler shifts, resulting in a significant degradation in the bit error rate (BER) performance. Previous work on joint channel estimation/data detection for CDMA includes several approaches. In [7], an expectation maximization (EM) algorithm is used to jointly estimate the data and channels. The multistage interference cancellation receiver for CDMA with relatively fast/slow fading channels has been proposed in [8]–[10], and the effect of channel estimation errors is analyzed in [11]. The least-mean-square (LMS) algorithm for asynchronous systems [12], and the recursive least-squares (RLS) algorithm-based joint channel estimators for QS-CDMA [13] have also been considered. However, the RLS and LMS algorithms implicitly assume a time-invariant or slowly time-varying channel, and thus may not perform well in rapid fading. Furthermore, accurate parameter estimation over a time-varying communication channel is still an important problem in QS-CDMA applications due to the residual timing error and multipath.

It is well known that the Kalman filter is the optimal minimum variance estimator for channels described by first-order Gauss–Markov processes [14]. Here we show that the extended Kalman filter (EKF) [14] can correctly model each user's delay as a single nonlinear parameter [15], such that the EKF-based channel estimators can be combined with a new joint multiuser detection algorithm. This approach is somewhat similar to [16], where the authors approximated the composite data/fading amplitudes by a Gaussian autoregressive (AR) process. However, the AR model in [16] does not account for the finite alphabet nature of the data. In contrast, the EKF approach here incorporates an M-algorithm-based multiuser detector that correctly models the binary symbols. The EKF method is also superior to the RLS algorithm for QS-CDMA channel estimation in [13], which incorporates the residual delay into a channel with length longer than the actual multipath spread. This approximation reduces system capacity by increasing signal subspace size. The EKF approach avoids this overparameterization of the channel by correctly modeling the delays as nonlinear state variables. In this paper, we propose a new joint multiuser detection and channel estimation algorithm, which enables us to use only one EKF. By using the QRD-M algorithm with the EKF to track the delays and channel coefficients, we can significantly improve the BER performance over that of the decorrelator and the minimum mean squared error (MMSE) detector in QS-CDMA systems.

The remainder of this paper is organized as follows. In Section II, the channel and signal models are described. The joint multiuser detection and channel estimation algorithm is described in Section III. Simulation results are provided in Section IV, and conclusions are summarized in Section V.

II. Spread-Spectrum Signal Models for QS-CDMA Signals Over Time-Varying Channels

We consider a discrete-time model for the received QS-CDMA signal over time-varying channels. The multipath is

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modeled by a tapped delay line (TDL) with taps, $f_{k,l}(n) \in \mathcal{C}$, spaced T_s s apart [15], [19] for a signal with bandwidth $1/2T_s$ Hz. The received signal for the nth symbol interval, $nT \leq t < (n+1)T$, using a time guard interval [3] is modeled by

$$r(t) = \sum_{k=1}^{K} b_k(n) \sum_{l=0}^{N_f-1} f_{k,l}(n) s_k(t - nT - T_k(n) - lT_s) + n(t) \tag{1}$$

where K is the number of users, N_f is the number of multipaths, and $T_k(n) \in \mathcal{R}$ is the delay of user k. The noise $n(t)$ is circular white Gaussian with spectral density $2N_0$ and the data is binary and uncorrelated, such that $E\{b_k(n) b_{k'}(n')\} = \delta_{k,k'}\delta_{n,n'}$.

A realistic and analytically tractable model for the band-limited CDMA waveform [17] is

$$s_k(t) = \sqrt{\frac{2E_b}{T}} \sum_{j=0}^{N-1} c_{k,j} \frac{1}{\pi} \cdot \left[S_i\left(2\pi \frac{(t - jT_c)}{T_c}\right) - S_i\left(2\pi \frac{(t - jT_c - T_c)}{T_c}\right)\right] \tag{2}$$

with $S_i(x) \triangleq \int_0^x (\sin y/y) dy$. In (2), E_b represents energy per bit, T is the bit duration, and $c_{k,l} \in \{+1, -1\}$ is the spreading sequence. The constant N is the number of chips per symbol, and T_c is the chip interval, with $NT_c = T$. Note that $s_k(t)$ is a binary sequence low-pass filtered to $1/T_c$ Hz [17], with approximate duration T s. The propagation delays $T_k(n)$ are uniform in the interval $[-MT_s, MT_s]$ with $MT_s \ll NT_c$ under the QS assumption. The received low-pass signal sampled at instances $(nN_s + m)T_s$, $1/T_s = 2/T_c$, $N_s = T/T_s$, is given by

$$r(nN_s + m) = \sum_{k=1}^{K} b_k(n) \sum_{l=0}^{N_f-1} f_{k,l}(n) \cdot s_k(mT_s - T_k(n) - lT_s) + n((nN_s + m)T_s) \tag{3}$$

for $m = 0, 1, \ldots, N_s - 1$. The sampled received QS-CDMA signal vector for the nth symbol interval can then be written as

$$\mathbf{r}(n) = \sum_{k=1}^{K} b_k(n) \sum_{l=0}^{N_f-1} f_{k,l}(n) \mathbf{s}_k(T_k(n) + lT_s) + \mathbf{n}(n) \tag{4}$$

where

$$\begin{aligned} \mathbf{r}(n) &\triangleq [r(nN_s), r(nN_s+1), \ldots, r((n+1)N_s - 1)]^T \\ \mathbf{n}(n) &\triangleq [n(nN_s), \ldots, n((n+1)N_s - 1)]^T \\ &\sim \mathcal{N}\left(\mathbf{n}(n); \mathbf{0}, \frac{2N_0}{T_s}\mathbf{I}_{N_s \times N_s}\right) \end{aligned} \tag{5}$$

where $\mathcal{N}(\mathbf{x}; \overline{\mathbf{x}}, \mathcal{R}_{\mathbf{x}})$ represents a circular Gaussian density with mean vector $\overline{\mathbf{x}}$ and covariance matrix $\mathcal{R}_{\mathbf{x}}$, and $\mathbf{s}_k(\tau)$ is defined by

$$\mathbf{s}_k(\tau) \triangleq [s_k(-\tau), s_k(T_s - \tau), s_k(2T_s - \tau), \ldots, s_k((N_s - 1)T_s - \tau)]^T. \tag{6}$$

Note that $\mathbf{r}(n)$, a vector of Nyquist samples, becomes an approximate sufficient statistic as the observation window becomes larger than the duration of $r(t)$ [18]. To simplify the notation, we define a signal matrix $\mathbf{S}_k(T_k(n)) \in \mathcal{R}^{N_s \times N_f}$, and a channel vector $\mathbf{f}_k(n) \in \mathcal{C}^{N_f}$ for user k as follows:

$$\begin{aligned} \mathbf{S}_k(T_k(n)) &\triangleq [\mathbf{s}_k(T_k(n)), \mathbf{s}_k(T_k(n) + T_s), \ldots, \mathbf{s}_k(T_k(n) + (N_f - 1)T_s)], \\ \mathbf{f}_k(n) &\triangleq [f_{k,0}(n), f_{k,1}(n), \ldots, f_{k,N_f-1}(n)]^T. \end{aligned} \tag{7}$$

Using the definitions of (7), (4) can be rewritten in several forms as

$$\begin{aligned} \mathbf{r}(n) &= \sum_{k=1}^{K} b_k(n) \mathbf{S}_k(T_k(n)) \mathbf{f}_k(n) + \mathbf{n}(n) \\ &= \mathbf{S}_{f(n)} \mathbf{b}(n) + \mathbf{n}(n) \\ &= \mathbf{S}_{b(n)} \mathbf{f}(n) + \mathbf{n}(n) \end{aligned} \tag{8}$$

where

$$\begin{aligned} \mathbf{S}_{f(n)} &= [\mathbf{S}_1(T_1(n))\mathbf{f}_1(n), \mathbf{S}_2(T_2(n))\mathbf{f}_2(n), \ldots, \mathbf{S}_K(T_K(n))\mathbf{f}_K(n)] \in \mathcal{C}^{N_s \times K} \\ \mathbf{S}_{b(n)} &= [\mathbf{S}_1(T_1(n)) b_1(n), \mathbf{S}_2(T_2(n)) b_2(n), \ldots, \mathbf{S}_K(T_K(n)) b_K(n)] \in \mathcal{R}^{N_s \times KN_f} \\ \mathbf{b}(n) &= [b_1(n), b_2(n), \ldots, b_K(n)]^T \in \{+1, -1\}^K \\ \mathbf{f}(n) &= [\mathbf{f}_1^H(n), \mathbf{f}_2^H(n), \ldots, \mathbf{f}_K^H(n)]^H \in \mathcal{C}^{KN_f}. \end{aligned} \tag{9}$$

Under the uncorrelated scattering model [19], we obtain

$$E\{f_{k,l}(n) f_{k',l'}^*(n)\} = \sigma_{k,l}^2 \delta_{k,k'} \delta_{l,l'} \tag{10}$$

where $\sigma_{k,l}^2$ is the power for the lth path of the kth user. For simplicity, we will assume that the channel coefficients and user delays are first-order AR processes [15] in the sequel

$$\begin{aligned} f_{k,l}(n) &= \alpha_f f_{k,l}(n-1) + w_{k,l}(n), \\ T_k(n) &= \alpha_\tau T_k(n-1) + w'_k(n) \end{aligned} \tag{11}$$

where $w_{k,l}(n)$ and $w'_k(n)$ are zero mean white Gaussian noises with variance σ_f^2 and σ_τ^2, respectively. Note that α_f and α_τ are approximated as identical for all users and paths. However, the EKF can be readily modified to accommodate higher order channels [15], [20].

III. Joint Channel/Delay Estimation and Data Detection for QS-CDMA

A. *Review of the Optimal Channel/Delay Estimator and Multiuser Detector*

The structure of the optimal channel estimator and detector is first reviewed. Let $\mathbf{r}^n = \{\mathbf{r}(1), \ldots, \mathbf{r}(n)\}$ and $\mathbf{b}_i^n = \{\mathbf{b}_i(1), \ldots, \mathbf{b}_i(n)\}$ be the cumulative measurement and ith data sequences, respectively, where $i = 1, 2, \ldots, 2^{Kn}$. A state vector, $\mathbf{x}(n) \in \mathcal{C}^{K(N_f+1)}$, associated with all K users is defined by

$$\mathbf{x}(n) \triangleq [T_1(n), f_{1,0}(n), \ldots, f_{1,N_f-1}(n), \ldots, T_K(n), f_{K,0}(n), \ldots, f_{K,N_f-1}(n)]^T. \tag{12}$$

Recall from Section II that the received signal vector is given by

$$\mathbf{r}(n) = \sum_{k=1}^{K} b_k(n) \sum_{l=0}^{N_f-1} f_{k,l}(n) \mathbf{s}_k(T_k(n) + lT_s) + \mathbf{n}(n). \tag{13}$$

Note that, provided that the time guard interval exceeds the multipath spread, there is no ISI in the representation of (13) [3]. The optimal data detector in the maximum *a posteriori* sense is

$$\hat{\mathbf{b}}^n_{\text{opt}} = \arg \max_{\mathbf{b}^n_i \in \{+1,-1\}^{nK}} p\left(\mathbf{b}^n_i | \mathbf{r}^n\right). \tag{14}$$

Using Bayes' rule, the following recursive expression for the posterior data probability is readily obtained:

$$p\left(\mathbf{b}^n_i|\mathbf{r}^n\right) = \frac{1}{c}\, p\left(\mathbf{r}(n)|\mathbf{b}^n_i, \mathbf{r}^{n-1}\right) p\left(\mathbf{b}^{n-1}_i|\mathbf{r}^{n-1}\right) \tag{15}$$

where c is a normalization constant, and the likelihood $p(\mathbf{r}(n)|\mathbf{b}^n_i, \mathbf{r}^{n-1})$ is approximated as

$$p\left(\mathbf{r}(n)|\mathbf{b}^n_i, \mathbf{r}^{n-1}\right) \approx \mathcal{N}\left(\mathbf{r}(n); \hat{\mathbf{r}}_i(n|n-1), \mathbf{\Sigma}_i(n|n-1)\right) \tag{16}$$

where $\hat{\mathbf{r}}_i(n|n-1)$ and $\mathbf{\Sigma}_i(n|n-1)$ are computed by an EKF [14] conditioned on $\mathbf{b}^n_i$. Note that (16) would be exact if $\mathbf{r}(n)$ was linear in both the delays and channel coefficients [14]. Equation (16) is still an accurate approximation if the tracking error is less than half a chip, so that the actual signal waveform can be approximated by a linear function of the delay [21]. The estimated signal $\hat{\mathbf{r}}_i(n|n-1)$ is

$$\hat{\mathbf{r}}_i(n|n-1) = \sum_{k=1}^{K} b^i_k(n) \sum_{l=0}^{N_f-1} \hat{f}^i_{k,l}(n|n-1)\mathbf{s}_k(\hat{T}^i_k(n|n-1) + lT_s) \tag{17}$$

and the corresponding innovations covariance matrix $\mathbf{\Sigma}_i(n|n-1)$ is approximately

$$\mathbf{\Sigma}_i(n|n-1) \approx E\{[\mathbf{r}(n) - \hat{\mathbf{r}}_i(n|n-1)] \cdot [\mathbf{r}(n) - \hat{\mathbf{r}}_i(n|n-1)]^H|\mathbf{r}^{n-1}, \mathbf{b}^n_i\}. \tag{18}$$

The estimates $\hat{f}^i_{k,l}(n|n-1)$, $\hat{T}^i_k(n|n-1)$, approximate

$$\begin{aligned}
\hat{f}^i_{k,l}(n|n-1) &\approx E\{f_{k,l}(n)|\mathbf{r}^{n-1}, \mathbf{b}^n_i\}\\
\hat{T}^i_k(n|n-1) &\approx E\{T_k(n)|\mathbf{r}^{n-1}, \mathbf{b}^n_i\}\\
\hat{\mathbf{x}}_i(n|n-1) &= [\hat{T}^i_1(n|n-1), \hat{\mathbf{f}}^i_1(n|n-1)^H, \ldots\\
&\qquad \hat{T}^i_K(n|n-1), \hat{\mathbf{f}}^i_K(n|n-1)^H]^H,\\
\hat{\mathbf{f}}^i_k(n|n-1) &= [\hat{f}^i_{k,0}(n|n-1), \hat{f}^i_{k,1}(n|n-1), \ldots,\\
&\qquad \hat{f}^i_{k,N_f-1}(n|n-1)]^T.
\end{aligned} \tag{19}$$

That is, the estimates $\hat{f}^i_{k,l}(n|n-1)$ and $\hat{T}^i_k(n|n-1)$ represent EKF one-step predictions, conditioned on the cumulative data $\{\mathbf{b}^n_i\}$ and the cumulative measurement $\mathbf{r}^{n-1}$. The EKF can recursively compute $\hat{\mathbf{x}}_i(n|n-1)$ and $\mathbf{\Sigma}_i(n|n-1)$. However, since the received signal vector $\mathbf{r}(n)$ is nonlinear in the user delays, $\{T_k(n)\}$, the EKF provides only approximations for the means and covariances.

The optimal decision can be approximated by the EKF innovations as

$$\begin{aligned}
\hat{\mathbf{b}}^n_{\text{opt}} &= \arg \max_{\mathbf{b}^n_i \in \{+1,-1\}^{nK}} p\left(\mathbf{b}^n_i|\mathbf{r}^n\right)\\
&\approx \arg \max_{\mathbf{b}^n_i \in \{+1,-1\}^{nK}} \prod_{l=1}^{n} \mathcal{N}\\
&\qquad \cdot (\mathbf{r}(l); \hat{\mathbf{r}}_i(l|l-1), \mathbf{\Sigma}_i(l|l-1))
\end{aligned} \tag{20}$$

where equiprobable data symbols $b_k(n)$ are assumed. Equation (20) means that the optimal multiuser detection and channel estimator has a complexity of $O(2^{nK})$ [17], [22], where n is the number of received signal vectors. Because of this complexity, suboptimal joint detectors based on the EKF technique have been developed. The EKF has been combined with the symbol-by-symbol detection (SBSD) algorithm in [6]. However, the SBSD algorithms for QS-CDMA are still highly complex. In this paper, by exploiting the quasisynchronous nature of the QS channel, we propose a new EKF-based channel/delay estimator for QS-CDMA systems which is computationally tractable.

B. Extended Kalman Filter for Channel/Delay Estimation

The extended Kalman filter (EKF) that we will develop works with a state vector $\mathbf{x}(n)$ consisting of both real and complex-valued channel state variables. Recall that the received signal vector sampled at the Nyquist sampling frequency, $1/T_s = 2/T_c$, with T_c the chip interval, is given by (4). Since $\mathbf{r}(n)$ is a nonlinear function in $\{T_k(n)\}$ in (4), we apply the EKF to jointly estimate the channel and delays [15], [17]. The system model and observation equations for QS-CDMA signals are of the form

$$\begin{aligned}
\mathbf{x}(n+1) &= \mathbf{F}\mathbf{x}(n) + \mathbf{G}\mathbf{w}(n)\\
\mathbf{r}(n) &= \mathbf{h}(\mathbf{x}(n)) + \mathbf{n}(n)\\
&\triangleq \mathcal{H}(\mathbf{x}(n))\mathbf{b}(n) + \mathbf{n}(n)
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\mathbf{F} &= \text{block diag}\{\mathbf{F}_1, \ldots, \mathbf{F}_K\}\\
&\in R^{K(N_f+1)\times K(N_f+1)}\\
\mathbf{G} &= \mathbf{I}\\
\mathbf{h}(\mathbf{x}(n)) &= \sum_{k=1}^{K} b_k(n) \sum_{l=0}^{N_f-1} f_{k,l}(n)\mathbf{s}_k(T_k(n) + lT_s)\\
&\in \mathcal{C}^{N_s}\\
\mathcal{H}(\mathbf{x}(n)) &= [\mathbf{h}_1(\mathbf{x}_1(n)), \mathbf{h}_2(\mathbf{x}_2(n)), \ldots, \mathbf{h}_K(\mathbf{x}_K(n))]\\
&\in \mathcal{C}^{N_s\times K}\\
\mathbf{h}_j(\mathbf{x}_j(n)) &= \sum_{l=0}^{N_f-1} \mathbf{s}_j(T_j(n) + lT_s)f_{j,l}(n)\\
&\in \mathcal{C}^{N_s}\\
\mathbf{w}(n) &= [w'_1(n), w_{1,0}(n), \ldots, w_{k,0}(n), \ldots,\\
&\qquad w'_K(n), w_{K,0}(n), \ldots, w_{K,N_f-1}(n)]^T\\
\mathbf{n}(n) &\sim \mathcal{N}(\mathbf{n}(n); \mathbf{0}, \mathcal{R}_\mathbf{n})\\
\mathbf{w}(n) &\sim \mathcal{N}(\mathbf{w}(n); \mathbf{0}, \mathcal{Q}),\\
E\{\mathbf{n}(n)\mathbf{w}^H(n)\} &= \mathbf{0}.
\end{aligned} \tag{22}$$

In the formulation of (21), the received signal vector $\mathbf{r}(n)$ is a nonlinear function in $\mathbf{x}(n)$. The one-step transition matrix for user k is $\mathbf{F}_k = \text{diag}\{\alpha_\tau, \alpha_f, \ldots, \alpha_f\}$, and the vector $\mathbf{h}(\mathbf{x}(n))$ is a function of user delays $\{T_k(n)\}$, channel coefficients $\{f_{k,l}(n)\}$, and the data vector $\mathbf{b}(n)$. The measurement noise covariance matrix and the process noise covariance are, respectively, equal to $\mathcal{R}_\mathbf{n} = 2N_0\mathbf{I}/T_s$ and $\mathcal{Q} = \text{blockdiag}\{\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_K\}$, with $\mathcal{Q}_k = \text{diag}\{\sigma_\tau^2, \sigma_f^2, \ldots, \sigma_f^2\}, \forall k$. We also assume that the matrices $\mathcal{R}_\mathbf{n}$ and $\mathcal{Q}$ are known *a priori*. Note that N_0 is typically known in a communications receiver from the RF/IF amplifier noise figure. $\mathcal{Q}$ is a function of the Doppler spread, which is known in some applications.

Now suppose that $\mathbf{b}(n)$ is approximated as $\hat{\mathbf{b}}(n)$, a decision made using previous channel estimates $\hat{\mathbf{x}}(n|n-1)$, and the second- and higher order terms in the Taylor series expansions are negligible for a linearization of the observation function $\mathbf{h}(\mathbf{x}(n))$ about the previous estimate of the state, $\hat{\mathbf{x}}(n|n-1)$. Then

$$\begin{aligned}\mathbf{h}(\mathbf{x}(n)) \approx \mathbf{h}(\hat{\mathbf{x}}(n|n-1)) + \mathbf{H}(\hat{\mathbf{x}}(n|n-1))\mathbf{x}(n)\\ - \mathbf{H}(\hat{\mathbf{x}}(n|n-1))\hat{\mathbf{x}}(n|n-1)\end{aligned} \quad (23)$$

where the Jacobian matrix, $\mathbf{H}(\hat{\mathbf{x}}(n|n-1)) \in \mathcal{C}^{N_s \times K(N_f+1)}$, is

$$\mathbf{H}(n|n-1) \triangleq \mathbf{H}(\hat{\mathbf{x}}(n|n-1)) \triangleq \left.\frac{\partial \mathbf{h}(\mathbf{x}(n))}{\partial \mathbf{x}^T(n)}\right|_{\mathbf{x}(n)=\hat{\mathbf{x}}(n|n-1)}. \quad (24)$$

Thus, the system model and observation equations we consider for the QS channel problem are of the form

$$\begin{aligned}\mathbf{x}(n+1) &= \mathbf{F}\mathbf{x}(n) + \mathbf{G}\mathbf{w}(n),\\ \mathbf{r}(n) &\approx \mathbf{h}(\hat{\mathbf{x}}(n|n-1)) + \mathbf{H}(n|n-1)\mathbf{x}(n)\\ &\quad - \mathbf{H}(n|n-1)\hat{\mathbf{x}}(n|n-1) + \mathbf{n}(n).\end{aligned} \quad (25)$$

By exploiting the quasi-synchronous nature of the QS channel, we propose a new joint multiuser detection and channel estimation algorithm which requires only one EKF. Specifically, a decision $\hat{\mathbf{b}}(n)$ at time n is made using a single predicted channel estimate $\hat{\mathbf{x}}(n|n-1)$. This data decision is then used to update a single EKF based on $\hat{\mathbf{x}}(n|n-1)$ and $\mathbf{r}(n)$. Thus

$$\begin{aligned}\hat{\mathbf{x}}(n|n-1) &\approx E\{\mathbf{x}(n)|\mathbf{r}^{n-1}, \hat{\mathbf{b}}^{n-1}\},\\ \hat{\mathbf{b}}(n)_{\text{CML}} &= \arg\min_{\mathbf{b}(n)\in\{+1,-1\}^K} \\ &\quad \cdot \|\mathbf{r}(n) - \mathcal{H}(\hat{\mathbf{x}}(n|n-1))\mathbf{b}(n)\|^2\\ \hat{\mathbf{x}}(n|n) &\approx E\{\mathbf{x}(n)|\mathbf{r}^n, \hat{\mathbf{b}}(n)_{\text{CML}}, \hat{\mathbf{b}}^{n-1}\}\\ \hat{\mathbf{x}}(n+1|n) &\approx E\{\mathbf{x}(n+1)|\mathbf{r}^n, \hat{\mathbf{b}}(n)_{\text{CML}}, \hat{\mathbf{b}}^{n-1}\}\\ \hat{\mathbf{b}}^{n-1} &\triangleq \{\hat{\mathbf{b}}(1)_{\text{CML}}, \hat{\mathbf{b}}(2)_{\text{CML}}, \ldots, \hat{\mathbf{b}}(n-1)_{\text{CML}}\}\end{aligned} \quad (26)$$

with the corresponding error covariance matrices

$$\begin{aligned}\mathbf{P}(n|n-1) &\triangleq \text{var}(\mathbf{x}(n)|\mathbf{r}^{n-1}, \hat{\mathbf{b}}^{n-1})\\ \mathbf{P}(n|n) &\triangleq \text{var}(\mathbf{x}(n)|\mathbf{r}^n, \hat{\mathbf{b}}(n)_{\text{CML}}, \hat{\mathbf{b}}^{n-1})\\ \mathbf{P}(n+1|n) &\triangleq \text{var}(\mathbf{x}(n+1)|\mathbf{r}^n, \hat{\mathbf{b}}(n)_{\text{CML}}, \hat{\mathbf{b}}^{n-1}).\end{aligned}$$

It is well known that MMSE detectors [23], [24] and decorrelators [25], [26] can yield approximate solutions for $\hat{\mathbf{b}}(n)$ in (26). The decorrelator is actually an unconstrained ML estimator, while the MMSE detector is the exact ML detector when the desired users data/amplitudes are circular Gaussian random variables. It is shown in the sequel that significantly better BER performance than the MMSE detector and the decorrelator can be obtained by a suitable approximation to the constrained maximum-likelihood (CML) solution (26). Specifically, a variant of the M-algorithm (or the breadth-first detection algorithm) [27]–[29], referred to here as the QRD-M algorithm, is introduced as a systematic approximation to $\hat{\mathbf{b}}(n)_{\text{CML}}$.

C. QR Decomposition

To find a practical approximation to $\hat{\mathbf{b}}(n)_{\text{CML}}$ in (26), we will apply the QRD-M algorithm. Let $\hat{\mathbf{f}}_k(n|n-1)$ be an estimate of the channel for the kth user

$$\begin{aligned}\hat{\mathbf{f}}_k(n|n-1) \triangleq [\hat{f}_{k,0}(n|n-1), \hat{f}_{k,1}(n|n-1), \ldots,\\ \hat{f}_{k,N_f-1}(n|n-1)]^T.\end{aligned} \quad (27)$$

Using (27), the corresponding estimated user power is defined as $\|\hat{\mathbf{f}}_k(n|n-1)\|^2$. Employing estimated user powers, rearrange the signal-channel matrix as follows:

$$\begin{aligned}\mathcal{H}(\hat{\mathbf{x}}(n|n-1)) = [\mathbf{h}_{(1)}(\hat{\mathbf{x}}_{(1)}(n|n-1)), \mathbf{h}_{(2)}(\hat{\mathbf{x}}_{(2)}(n|n-1)),\\ \ldots, \mathbf{h}_{(K)}(\hat{\mathbf{x}}_{(K)}(n|n-1))]\end{aligned} \quad (28)$$

such that $\|\hat{\mathbf{f}}_{(1)}(n|n-1)\|^2 \leq \|\hat{\mathbf{f}}_{(2)}(n|n-1)\|^2 \leq \cdots \leq \|\hat{\mathbf{f}}_{(K)}(n|n-1)\|^2$. Also, reorder the data vector as $\mathbf{b}(n) = [b_{(1)}(n), b_{(2)}(n), \ldots, b_{(K)}(n)]^T$.

Applying the QR decomposition (QRD) to the ML cost function, we obtain the following modified minimization problem:

$$\begin{aligned}&\hat{\mathbf{b}}(n)_{\text{CML}}\\ &= \arg\min_{\mathbf{b}(n)\in\{+1,-1\}^K} \|\mathbf{y}(n) - \mathbf{Q}^H\mathcal{H}(\hat{\mathbf{x}}(n|n-1))\mathbf{b}(n)\|^2\\ &= \arg\min_{\mathbf{b}(n)\in\{+1,-1\}^K} \{|y_K(n) - R_{K,K}b_{(K)}(n)|^2\\ &\quad + \cdots |y_2(n) - R_{2,2}b_{(2)}(n) \cdots - R_{2,K}b_{(K)}(n)|^2\\ &\quad + |y_1(n) - R_{1,1}b_{(1)}(n) \cdots - R_{1,K}b_{(K)}(n)|^2\\ &\quad + |y_{K+1}(n)|^2 + |y_{K+2}(n)|^2 + \cdots + |y_{N_s}(n)|^2\}\end{aligned} \quad (29)$$

where $\mathbf{y}(n) = \mathbf{Q}^H\mathbf{r}(n)$, $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$, and

$$\begin{aligned}\mathbf{Q}^H\mathcal{H}(\hat{\mathbf{x}}(n|n-1)) &\triangleq \mathbf{R} = \begin{bmatrix}\mathbf{R}_{K\times K}\\ \mathbf{0}_{(N_s-K)\times K}\end{bmatrix}\\ \mathbf{R}^H\mathbf{R} &= \mathcal{H}(\hat{\mathbf{x}}(n|n-1))^H\mathcal{H}(\hat{\mathbf{x}}(n|n-1)).\end{aligned} \quad (30)$$

In the derivation of (29), we use the fact that $\mathbf{R}_{K\times K}$ is an upper triangular matrix, such that we can make decisions from the strongest to the weakest user sequentially. Note that the QRD applies an orthogonal transformation to the signal-channel matrix $\mathcal{H}(\hat{\mathbf{x}}(n|n-1))$ and, thus, it is better suited to receivers which use the sampled data directly. The Householder transformation [30] is an effective way to realize (30). Note that, since the transformation is invertible, $\mathbf{y}(n)$ is still an approximate sufficient statistic. The minimization problem given in (29) can be exactly solved by applying the binary tree searching technique as shown in Fig. 1. To optimally estimate the constrained data $b_{(i)}(n), \forall i \in \{1, \ldots K\}$, we need to evaluate 2^K hypotheses. Thus, the complexity grows exponentially with the number of users. To minimize the computational complexity of the binary tree searching algorithm, we will consider an alternative QRD-M based sequential detection algorithm next.

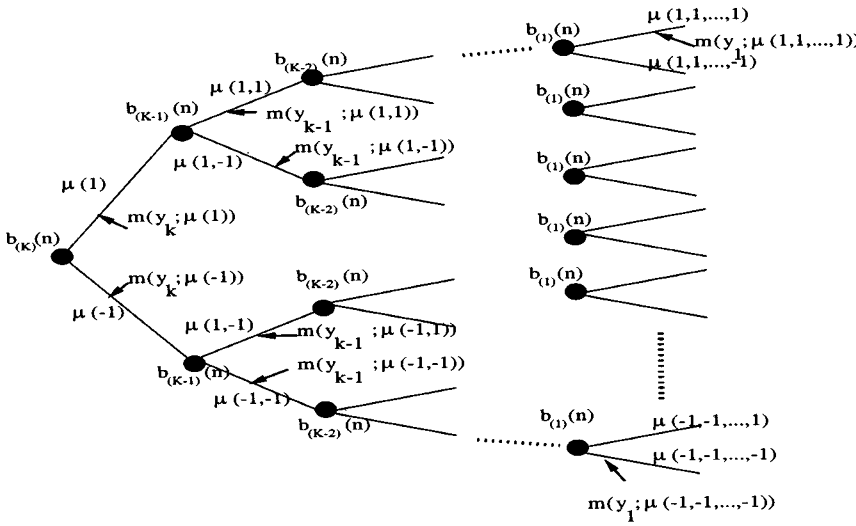


Fig. 1. Binary tree for user data detection.

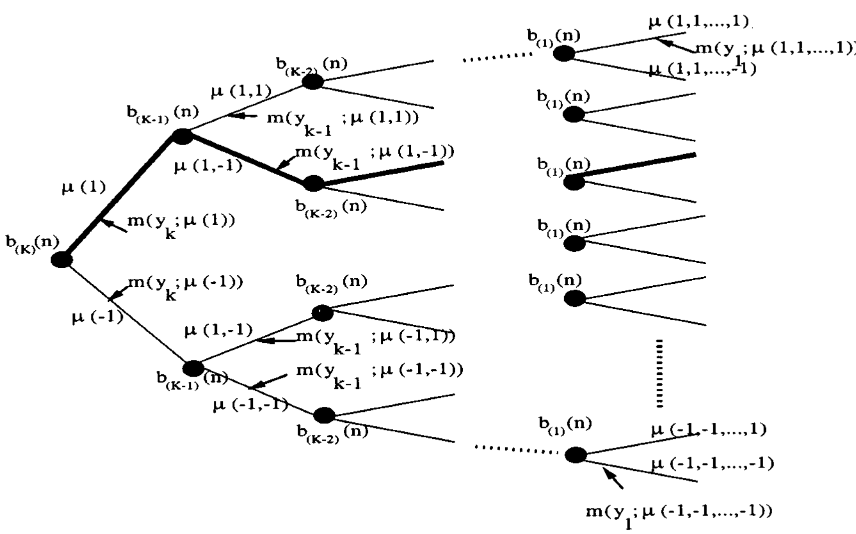


Fig. 2. M-algorithm with $M = 1$.

D. Sequential Detection Algorithm (QRD-M Algorithm)

The constrained one-shot ML estimator given in (29) can be equivalently rewritten as

$$\begin{aligned}\hat{\mathbf{b}}_{\mathrm{CML}}(n) &= \arg \min_{\mathbf{b}(n)\in\{+1,\,-1\}^K} \|\mathbf{y}(n) - \mathbf{Q}^H \mathcal{H}(\hat{\mathbf{x}}(n|n-1))\mathbf{b}(n)\|^2 \\ &= \arg \min_{\mathbf{b}(n)\in\{+1,\,-1\}^K} \sum_{k=1}^{K} m_{(k)}(y_k(n);\, \mu_{(k)}(n)) \qquad (31)\end{aligned}$$

where $\mu_{(k)}(n) \triangleq \{b_{(k)}(n), b_{(k+1)}(n), \ldots, b_{(K)}(n)\}$ represents the state for the (k)th strongest user, and the corresponding metric $m_{(k)}(y_k(n);\, \mu_{(k)}(n))$ is given by

$$\begin{aligned}m_{(k)}&\left(y_k(n);\, \mu_{(k)}(n)\right) \\ &= \left\| y_k(n) - R_{k,\,k} b_{(k)}(n) - \sum_{l=k+1}^{K} R_{k,\,l} b_{(l)}(n) \right\|^2 . \qquad (32)\end{aligned}$$

Note that the state $\mu_{(k)}(n)$ and the metric $m_{(k)}(y_k(n);\, \mu_{(k)}(n))$ are determined by the (k)th strongest user and all remaining stronger users. To approximately minimize (32), we shall apply the M-algorithm. The M-algorithm extends each survivor path and computes at most M accumulated distance metrics at each level of the tree. In the following, we demonstrate the use of the QRD-M algorithm.

Example: Let

$$\{m^1_{(k)}(y_k(n);\, \mu^1_{(k)}(n)),\, m^2_{(k)}(y_k(n);\, \mu^2_{(k)}(n)),\, m^M_{(k)}(y_k(n);\, \mu^M_{(k)}(n))\}$$

and $\{\mu^1_{(k)}(n), \mu^2_{(k)}(n), \ldots, \mu^M_{(k)}(n)\}$ be the M-minimum accumulated metrics, and the corresponding states, respectively, for the (k)th strongest user. For the $(k-1)$th strongest user,

TABLE I
QRD-M ALGORITHM TO MINIMIZE THE CONSTRAINED ML COST FUNCTION

(1) Sort users according to the estimated powers such that

$||\hat{\mathbf{f}}_{(1)}(n|n-1)||^2 \leq ||\hat{\mathbf{f}}_{(2)}(n|n-1)||^2 \leq \ldots \leq ||\hat{\mathbf{f}}_{(K)}(n|n-1)||^2$.

(2) Apply the QRD to the signal-channel matrix $\mathcal{H}(\hat{\mathbf{x}}(n|n-1))$:

$\mathbf{y}(n) = \mathbf{Q}^H\mathbf{r}(n),\ \ \mathbf{R} = \mathbf{Q}^H\mathcal{H}(\hat{\mathbf{x}}(n|n-1))$.

(3) For users $k = K, K-1, \ldots, 1$:

For survivor states $\mu^l_{(k+1)}(n)$, $l = 1, \ldots, M$:

(a) Find metrics $m_{(k)}\left(y_k(n); \mu_{(k)}(n)\right)$ corresponding to

$b_{(k)}(n), \ldots, b_{(K)}(n) \in \{+1, -1\}$ and $\mu^l_{(k+1)}(n)$.

(b) Keep the M smallest accumulated metrics and the M corresponding states $\mu^l_{(k)}(n)$.

(4) $k = k - 1$.

(5) If $k = 0$ select $\hat{\mathbf{b}}_{QRD-M}(n) = \mu_{(1)}(n)$ corresponding to smallest metric and exit.

Else if $k \neq 0$ goto (3).

TABLE II
QRD-M ALGORITHM FOR THE CASE $M = 1$, CORRESPONDING TO AN ADAPTIVE SIC ALGORITHM

(1) Compute the estimate of the strongest user data $\hat{b}_{(K)}(n)$

$\hat{b}_{(K)}(n) = \arg \min_{b_{(K)}(n)\in\{+1,-1\}} ||y_K(n) - R_{K,K}b_{(K)}(n)||^2$

$\Rightarrow \hat{b}_{(K)}(n) = sgn(Re\{y_K(n)/R_{K,K}\})$.

(2) Recursively decide weaker user data employing detected data for the stronger users

$\hat{b}_{(K-1)}(n) \approx \arg \min_{b_{(K-1)}(n)\in\{+1,-1\}} ||y_{K-1}(n) - R_{K-1,K-1}b_{(K-1)}(n) - R_{K-1,K}\hat{b}_{(K)}(n)||^2$

$\Rightarrow \hat{b}_{(K-1)}(n) = sgn(Re\{(y_{K-1}(n) - R_{K-1,K}\hat{b}_{(K)}(n))/R_{K-1,K-1}\})$.

(3) Compute the decision of the weakest user data $\hat{b}_{(1)}(n)$

$\hat{b}_{(1)}(n) \approx \arg \min_{b_{(1)}(n)\in\{+1,-1\}} ||y_1(n) - R_{1,1}b_{(1)}(n) - \sum_{l=2}^{K} R_{1,l}\hat{b}_{(l)}(n)||^2$

$\Rightarrow \hat{b}_{(1)}(n) = sgn(Re\{(y_1(n) - \sum_{l=2}^{K} R_{1,l}\hat{b}_{(l)}(n))/R_{1,1}\})$.

first compute the following accumulated metrics over the data hypothesis $b_{(k-1)} \in \{+1, -1\}$ with states $\{\mu^j_{(k)}(n)\}$:

$$\begin{aligned}
&m^{1+}_{(k-1)}(y_{k-1}(n);\, b_{(k-1)} = 1,\, \mu^1_{(k)}(n))\\
&m^{1-}_{(k-1)}(y_{k-1}(n);\, b_{(k-1)} = -1,\, \mu^1_{(k)}(n))\\
&m^{2+}_{(k-1)}(y_{k-1}(n);\, b_{(k-1)} = 1,\, \mu^2_{(k)}(n))\\
&m^{2-}_{(k-1)}(y_{k-1}(n);\, b_{(k-1)} = -1,\, \mu^2_{(k)}(n))\\
&\ldots\\
&m^{M+}_{(k-1)}(y_{k-1}(n);\, b_{(k-1)} = 1,\, \mu^M_{(k)}(n))\\
&m^{M-}_{(k-1)}(y_{k-1}(n);\, b_{(k-1)} = -1,\, \mu^M_{(k)}(n)).
\end{aligned} \tag{33}$$

At the next step, sort the metrics, $\{m^{1+}_{(k-1)}, m^{1-}_{(k-1)}, m^{2+}_{(k-1)}, m^{2-}_{(k-1)}, \ldots, m^{M+}_{(k-1)}, m^{M-}_{(k-1)}\}$, and retain only the M-smallest metrics, denoted by

$$\{m^1_{(k-1)}(y_{k-1}(n);\, \mu^1_{(k-1)}(n)),\, m^2_{(k-1)}(y_{k-1}(n);\, \mu^2_{(k-1)}(n)),\, m^M_{(k-1)}(y_{k-1}(n);\, \mu^M_{(k-1)}(n))\}$$

and their corresponding paths $\{\mu^1_{(k-1)}(n),\ \mu^2_{(k-1)}(n), \ldots, \mu^M_{(k-1)}(n)\}$. This algorithm corresponds to a suboptimal binary tree search, where only M paths are retained at each level of the tree as in Fig. 2. Note that $M = 2^K$ corresponds to the ML solution (full tree search). In Table I, we summarize the QRD-M algorithm for arbitrary M. When $M = 1$, it is readily shown that the QRD-M algorithm is equivalent to an adaptive successive interference canceler (SIC) similar to [31], [32]. For (29), Table II describes an adaptive SIC algorithm, where we compute the QR decomposition so that detection is in the order of decreasing estimated user power.

After some manipulations for linearized observations [14], we obtain the following EKF update and prediction state error covariance conditioned on the cumulative linearized observation and data hypothesis:

$$\begin{aligned}\hat{\mathbf{x}}(n|n) =& \hat{\mathbf{x}}(n|n-1) + \mathbf{P}(n|n)\mathbf{H}^H(n|n-1)\mathbf{R}^{-1}\\ &\cdot \left(\mathbf{r}(n) - \mathcal{H}(\hat{\mathbf{x}}(n|n-1))\hat{\mathbf{b}}_{\mathrm{QRD-M}}(n)\right)\end{aligned}$$

TABLE III
CHANNEL/DELAY ESTIMATION ALGORITHM BASED ON THE EKF COMBINED WITH THE QRD-M ALGORITHM (QRD-M-EKF) ALGORITHM

(1) Given $\hat{\mathbf{x}}(n|n-1)$ and $\mathbf{P}(n|n-1)$,

For the next $\mathbf{r}(n)$:

(2) Compute the M-algorithm data decision, $\hat{\mathbf{b}}_{QRD-M}(n)$,

which approximates the constrained ML decision

$$\hat{\mathbf{b}}(n)_{QRD-M} \approx \arg \min_{\mathbf{b}(n)\in\{+1,-1\}^K} ||\mathbf{y}(n) - \mathbf{Q}^H \mathcal{H}(\hat{\mathbf{x}}(n|n-1))\mathbf{b}(n)||^2, \ \mathbf{y}(n) = \mathbf{Q}^H \mathbf{r}(n).$$

(3) Apply the EKF.

$$\mathbf{P}^{-1}(n|n) = \frac{1}{2N_0/T_s}\mathbf{H}^H(n|n-1)\mathbf{H}(n|n-1) + \mathbf{P}^{-1}(n|n-1).$$

$$\hat{\mathbf{x}}(n|n) = \hat{\mathbf{x}}(n|n-1) + \frac{1}{2N_0/T_s}\mathbf{P}(n|n)\mathbf{H}^H(n|n-1)\Big(\mathbf{r}(n) - \mathcal{H}(\hat{\mathbf{x}}(n|n-1))\hat{\mathbf{b}}_{QRD-M}(n)\Big).$$

$$\hat{\mathbf{x}}(n+1|n) = \mathbf{F}\hat{\mathbf{x}}(n|n) .$$

$$\mathbf{P}(n+1|n) = \mathbf{F}\mathbf{P}(n|n)\mathbf{F}^H + \mathcal{Q}.$$

(4) $n = n+1$. Go to (1).

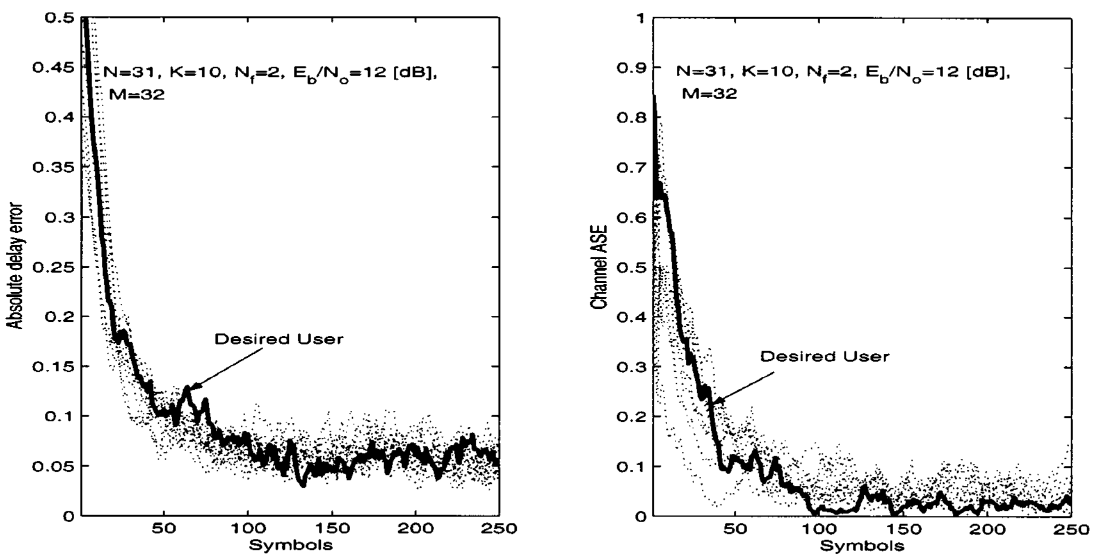


Fig. 3. Delay and channel estimates: time-invariant channel, $K = 10$, combined with the QRD-M algorithm, equal power users.

$$\begin{aligned}\mathbf{P}^{-1}(n|n) &= \mathbf{H}^H(n|n-1)\mathbf{R}^{-1}\mathbf{H}(n|n-1) \\ &\quad + \mathbf{P}^{-1}(n|n-1), \\ \hat{\mathbf{x}}(n+1|n) &= \mathbf{F}\hat{\mathbf{x}}(n|n) \\ \mathbf{P}(n+1|n) &= \mathbf{F}\mathbf{P}(n|n)\mathbf{F}^H + \mathcal{Q}. \end{aligned} \tag{34}$$

If $p(\mathbf{x}(n)|\mathbf{r}^{n-1}, \hat{\mathbf{b}}^{n-1})$ is approximated as

$$p\left(\mathbf{x}(n)|\mathbf{r}^{n-1}, \hat{\mathbf{b}}^{n-1}\right) \approx \mathcal{N}\left(\mathbf{x}(n); \hat{\mathbf{x}}(n|n-1), \mathbf{P}(n|n-1)\right) \tag{35}$$

then we can readily show that, when the linearization approximation (23) is exact, we have

$$\begin{aligned} &p\left(\mathbf{x}(n)|\mathbf{r}^{n}, \hat{\mathbf{b}}^{n}\right) = \mathcal{N}\left(\mathbf{x}(n); \hat{\mathbf{x}}(n|n), \mathbf{P}(n|n)\right) \\ &p\left(\mathbf{x}(n+1)|\mathbf{r}^{n}, \hat{\mathbf{b}}^{n}\right) \\ &\quad = \mathcal{N}\left(\mathbf{x}(n+1); \hat{\mathbf{x}}(n+1|n), \mathbf{P}(n+1|n)\right). \end{aligned} \tag{36}$$

Thus, we can jointly estimate the data, delays, and channels. In Table III, we summarize the delay-channel estimation algorithm based on the EKF combined with the QRD-M algorithm (QRD-M-EKF algorithm). For the Decorrelator-EKF and MMSE-EKF algorithms, step (2) in Table III is respectively substituted by [23]–[26],

$$\begin{aligned}&\hat{\mathbf{b}}(n)_{\mathrm{Decorr}} \\ &\quad = \mathrm{sgn}\left(\mathrm{Re}\{(\mathbf{S}^H_{\hat{f}(n|n-1)}\mathbf{S}_{\hat{f}(n|n-1)})^{-1}\mathbf{S}^H_{\hat{f}(n|n-1)}\mathbf{r}(n)\}\right)\end{aligned} \tag{37}$$

and

$$\hat{\mathbf{b}}(n)_{\mathrm{MMSE}} = \mathrm{sgn}\left(\mathrm{Re}\{\mathbf{L}^H(n)\mathbf{r}(n)\}\right) \tag{38}$$

where

$$\mathbf{L}(n) = \mathbf{S}_{\hat{f}(n|n-1)}\left(\mathbf{S}^H_{\hat{f}(n|n-1)}\mathbf{S}_{\hat{f}(n|n-1)} + \frac{2N_0}{T_s}\mathbf{I}\right)^{-1}$$

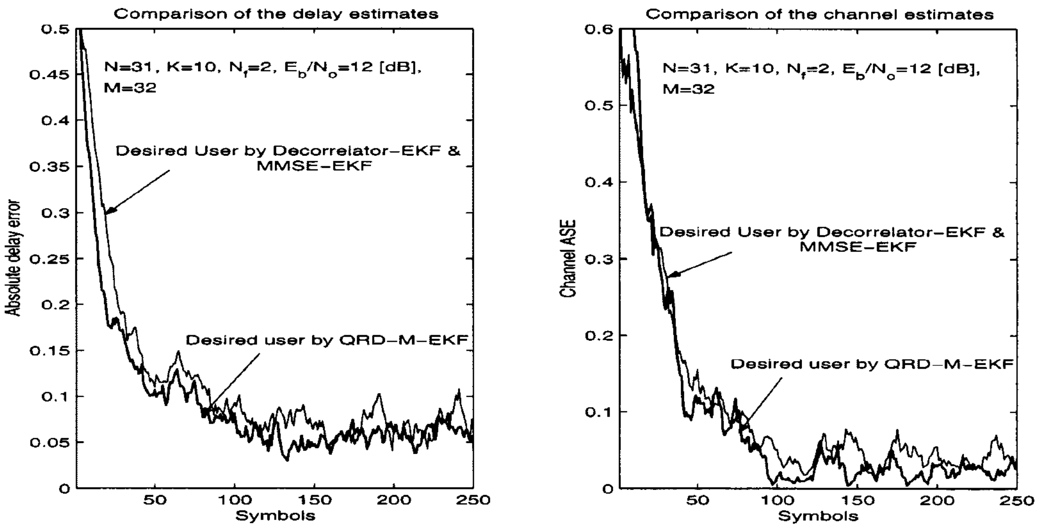
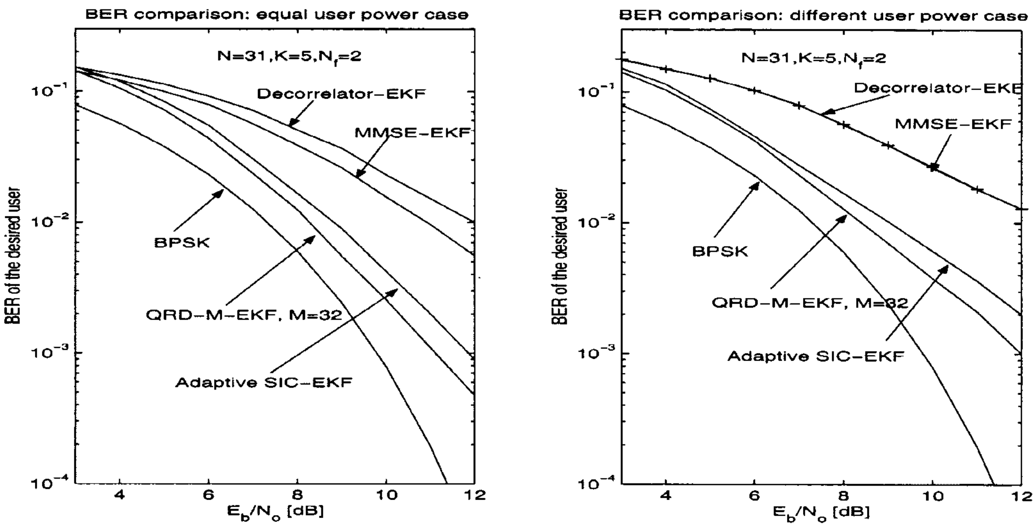


Fig. 4. Delay and channel estimate comparisons: $K = 10$, time-invariant channel, equal power users.



(a) (b)

Fig. 5. BER comparison: $K = 5, N_f = 2$ time-invariant channel.

$$
\begin{aligned}
&\in \mathcal{C}^{N_s \times K} \\
\mathbf{S}_{\hat{f}(n|n-1)} &= [\mathbf{S}_1(\hat{T}_1(n|n-1))\hat{\mathbf{f}}_1(n|n-1), \ldots, \\
&\qquad \mathbf{S}_K(\hat{T}_K(n|n-1))\hat{\mathbf{f}}_K(n|n-1)] \\
&\in \mathcal{C}^{N_s \times K}. \qquad (39)
\end{aligned}
$$

IV. Simulation Results

In our simulations, we assume the following:

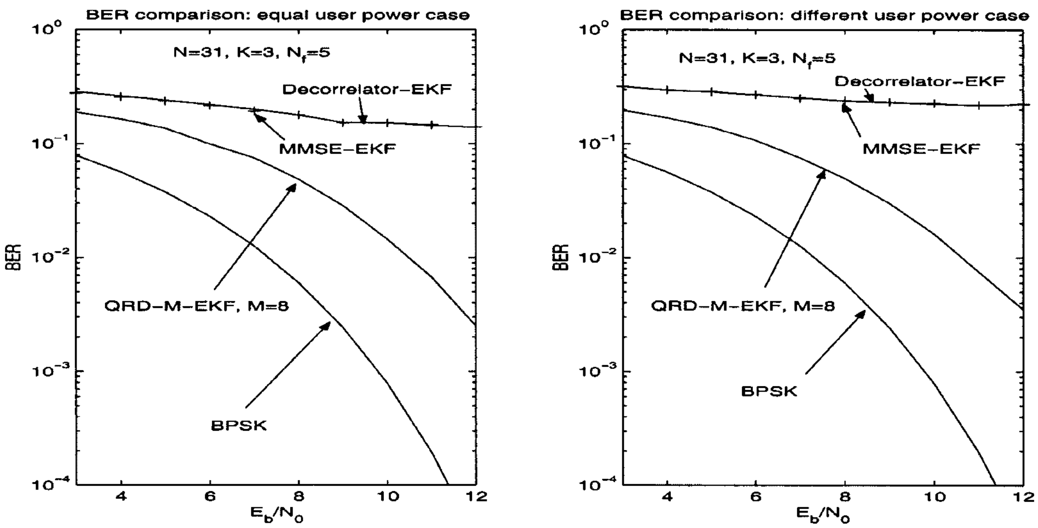
- Gold sequences with processing gain, $N = 31$, are used for the spreading sequences.
- User delays, $T_k(n)$, are uniformly chosen over the interval $[-MT_s, MT_s]$, $M = 2$.

For each iteration n, the absolute delay error given by $D \triangleq |T_k(n) - \hat{T}_k(n|n)|$ in chips and the averaged squared error (ASE) defined by

$$
\mathrm{ASE} \triangleq \frac{1}{N_f} \sum_{l=0}^{N_f-1} \left| f_{k,l}(n) - \hat{f}_{k,l}(n|n) \right|^2 / |\hat{f}_{k,l}(n|n)|^2 \quad (40)
$$

for user k are used to quantify the performance of the delay and channel estimates, respectively. We simulated our algorithm in two cases: 1) equal user powers and 2) strong near–far conditions.

First Scenario (Equal User Powers Case): Fig. 3 is the simulation result for the joint detection algorithm based on the QRD-M algorithm (QRD-M-EKF) given in Table III. The number of users, K, is assumed to be $K = 10$, and $N_f = 2$. This figure shows that the delay and channel estimates using the QRD-M-EKF algorithm converges rapidly. We can also see that all users have the same performance in the delay and channel estimates with the same convergence speed. Compared to the MMSE-EKF/Decorrelator-EKF based estimates, Fig. 4 shows that, in general, the QRD-M-EKF outperforms the MMSE-EKF and Decorrelator-EKF in terms of channel and delay ASE for $N_f = 2$. The BER performance of the QRD-M-EKF, Decorrelator-EKF and MMSE-EKF is compared in Fig. 5(a) for the equal-power user case. The results in Fig. 5(a) demonstrate an SNR gain of approximately 3.8/2.6 dB at a BER of 10^{-2}



(a) (b)

Fig. 6. BER comparison: $K = 3, N_f = 5$, time-invariant channel.

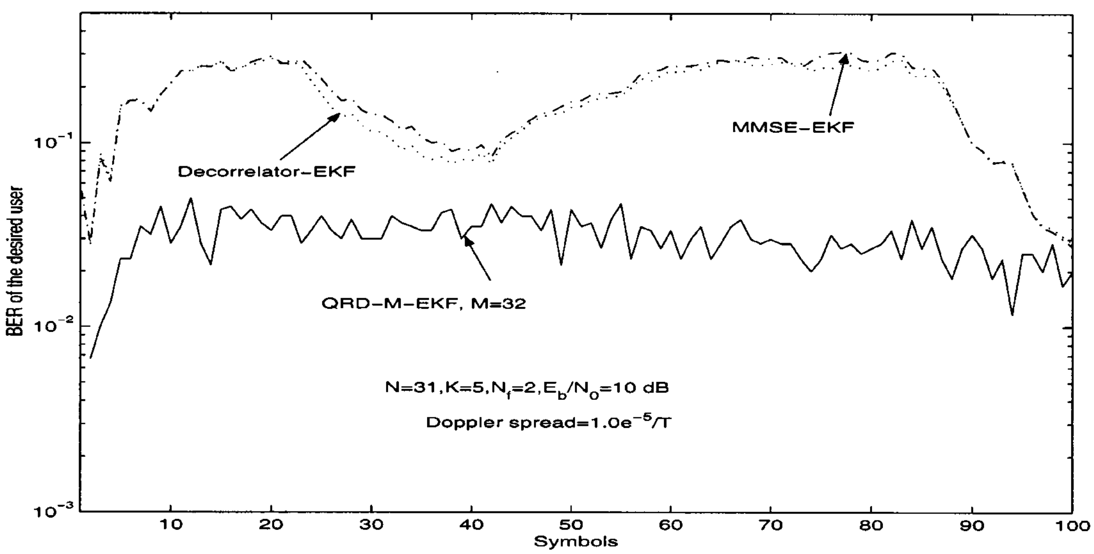


Fig. 7. BER comparison: $K = 5$, time-varying channel (Doppler spread $= 10^{-5}/T$).

for the QRD-M-EKF compared to the decorrelator/MMSE detector. Also, as M tends to 2^K, the QRD-M-EKF performs better than the adaptive SIC-EKF (QRD-M, $M = 1$). Fig. 6(a) compares BERs for the various EKF algorithms, but with a longer multipath spread of $N_f = 5$. Again, the QRD-M-EKF performance is clearly superior to the decorrelator and MMSE detector-based alternatives. Also note that Fig. 6 shows a BER error floor in the case of the longer multipath channel $N_f = 5$ for the Decorrelator-EKF and MMSE-EKF.

The performance of the EKF algorithms for a time-varying channel is shown in Fig. 7, with a Doppler spread of $10^{-5}/T$. As in the time-invariant case, the QRD-M-EKF significantly outperforms the Decorrelator-EKF and MMSE-EKF. However, for a larger Doppler spread, EKF divergence occurs more frequently. Better performance for large Doppler spreads would require consideration of more sophisticated nonlinear filtering algorithms, such as the particle filter [33], [34]. However, this topic is outside of the scope of this paper.

Second Scenario (Strong Near–Far Condition): For the same conditions as in the first scenario, Fig. 8 shows that, in a strong near-far condition with $J/S = 5$ dB, the desired (weak) user has the worst performance and convergence speed as expected. However, the delay and channel estimates still converge rapidly. Fig. 9 indicates that the channel estimation performance using the QRD-M-EKF algorithm is superior to that obtained from the Decorrelator-EKF/MMSE-EKF.

In Fig. 5(b), we plot a BER comparison between results obtained using the MMSE-EKF, Decorrelator-EKF, and adaptive SIC-EKF for the unequal power case. Simulation results show that the MMSE-EKF has the same BER performance as the Decorrelator-EKF, reflecting the fact that the MMSE detector is equivalent to the decorrelator when $J/S \to \infty$ even for a finite N_0 [35]. As in the equal user power case, for a strong near-far condition and a longer QS-channel, the QRD-M-EKF is superior to the Decorrelator-EKF, MMSE-EKF, and adaptive SIC-EKF in terms of BER performance. The superiority of

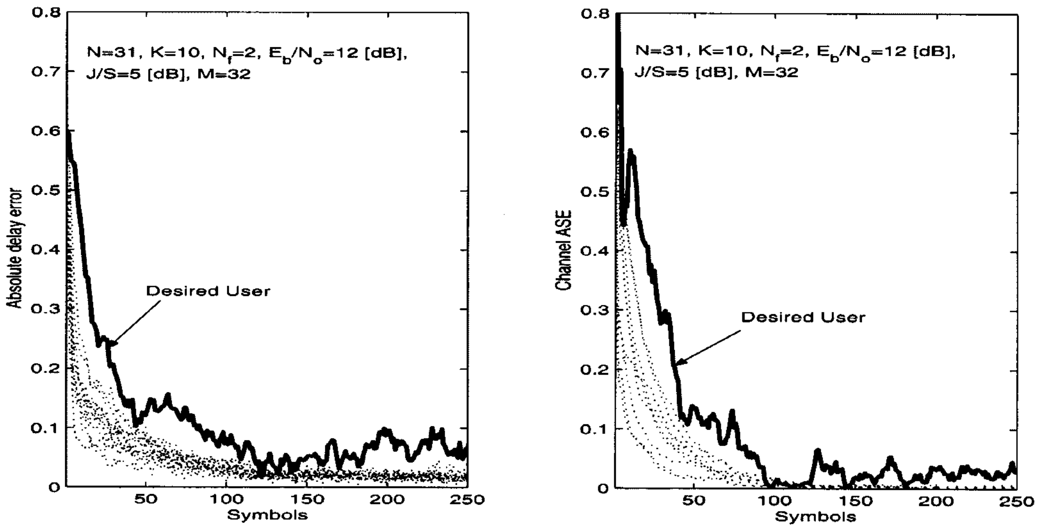


Fig. 8. Delay and channel estimates: time-invariant channel, $K = 10$, combined with the QRD-M algorithm, different power users.

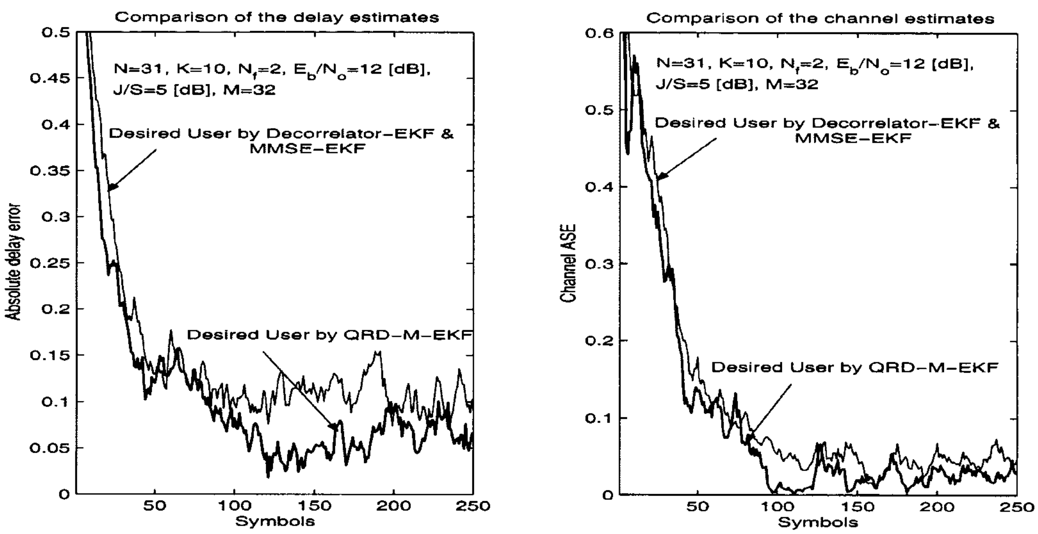


Fig. 9. Delay and channel estimate comparisons: $K = 10$, time-invariant channel, different user powers.

the QRD-M-EKF for unequal power users is also illustrated in Fig. 6(b) for the length $N_f = 5$ channel.

V. Conclusion

We presented a multiuser detection algorithm for QS-CDMA signals that uses the QRD-M algorithm to detect user data, with the EKF algorithm employed to track the user delays and channel coefficients. In general, the QRD-M multiuser detector coupled with the EKF algorithm provides a lower overall BER and channel ASE than EKF channel/delay estimators driven by MMSE and decorrelator-based decisions. It was furthermore shown that the $M = 1$ version of the QRD-M-EKF algorithm corresponds to an adaptive successive interference canceler. In the simulations presented, the QRD-M algorithm with even a small number of paths ($M = 8$ for $K = 5$ users, compared with $M = 32$ for a full search) outperformed the MMSE and decorrelator detectors. Hence, the QRD-M-EKF algorithm may provide a complete solution to the problems of residual delay tracking and channel estimation in QS-CDMA systems.

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