

2.19) a)

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{-R_1, -R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-\frac{1}{2}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\operatorname{rk}(A_{\Phi}) = 3$$

$$\Rightarrow \operatorname{Im}(\Phi) = \mathbb{R}^3$$

$$\ker(\Phi) = \{[0, 0, 0]^T\}$$

b)

$$\tilde{A}_{\Phi} = S^{-1} A_{\Phi} S$$

$$S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{swap with } R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{array} \right] \xrightarrow{-R_1, -R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array} \right] \xrightarrow{-R_2}$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array} \right]$$

$$\Rightarrow S^{-1} = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\tilde{A}_{\Phi} = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 & 1 \\ -3 & -5 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$