

2.17)

$$A_{\Phi} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow{\text{swap with } R_2} \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & -3 & 0 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow{\begin{matrix} -3R_1 \\ -R_1 \\ -2R_1 \end{matrix}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & -4 & -1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{\begin{matrix} \cdot -1 \\ -4R_2 \\ +R_2 \end{matrix}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 7 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow{\begin{matrix} \cdot \frac{1}{7} \\ +\frac{3}{7}R_3 \end{matrix}}$$

$$\xrightarrow{\sim} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{rk}(A_{\Phi}) = 3$$

$$\ker(\Phi) = [0, 0, 0]^T \quad \because \text{all the column vectors of } A_{\Phi} \text{ are linearly independent.}$$

$$\text{Im}(\Phi) = \left\{ \begin{bmatrix} (3x_1 + 2x_2 + x_3), (x_1 + x_2 + x_3), (x_1 - 3x_2), (2x_1 + 3x_2 + x_3) \end{bmatrix}^T \mid x_1, x_2, x_3 \in \mathbb{R} \right\}$$