2.18) f, g are automorphisms on E.

$\Rightarrow f, g$ are linear, bijective, and map $V \to V$.

Assume $\exists \underline{x}, \underline{y} \in E : gf(\underline{x}) = gf(\underline{y}), \quad \underline{x} \neq \underline{y}$ is true.

$f(\underline{x}) \neq f(\underline{y}) \quad \because f$ is injective, $\underline{x} \neq \underline{y}$.

$g(f(\underline{x})) \neq g(f(\underline{y})) \quad \because g$ is injective, $f(\underline{x}) \neq f(\underline{y})$.

This contradicts the assumption $\exists \underline{x}, \underline{y} \in E : gf(\underline{x}) = gf(\underline{y}), \quad \underline{x} \neq \underline{y}$.

$\Rightarrow \forall \underline{x}, \underline{y} : gf(\underline{x}) = gf(\underline{y}), \quad x = y$.

$g \circ f$ is injective.

$f(\underline{0}_E) = \underline{0}_E \quad \because f$ is a linear mapping $E \to E$.

$\Rightarrow \ker(\underline{0}_E) \quad \because f$ is injective.

$gf(\underline{0}_E) = g(\underline{0}_E)$

$\qquad = \underline{0}_E \quad \because g$ is a linear mapping $E \to E$.

$g \circ f$ is injective $\because f, g$ are injective.

$\Rightarrow \ker(g \circ f) = \underline{0}_E$

$\Rightarrow \ker(f) = \ker(g \circ f) = \underline{0}_E$

$\mathrm{Im}(f) = E \quad \because f$ is a surjective mapping $E \to E$.

$\mathrm{Im}(g) = E \quad \because g$ is a surjective mapping $E \to E$.

$\mathrm{Im}(g \circ f) = \mathrm{Im}(gf(E)) = \mathrm{Im}(g(E)) = E$

$\Rightarrow \mathrm{Im}(g) = \mathrm{Im}(g \circ f) = E$

$\ker(f) \wedge \mathrm{Im}(g) = \{\underline{0}_E\} \wedge \{E\} = \{\underline{0}_E\}$