2.8) a)

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 4 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \cdot \frac{1}{2} \\ \\ -2R_1 \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right] \begin{matrix} \\ -3R_1 \\ \\ \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & -\frac{3}{2} & 1 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right] \begin{matrix} \\ \cdot -2 \\ \\ \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ +R_2 \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$$

$$\operatorname{rk} \begin{bmatrix} 1 & \frac{3}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \neq \operatorname{rk} \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 2 & \frac{1}{2} \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\therefore$ A is singular.

b)

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ -R_1 \\ -R_1 \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ -R_2 \\ -R_2 \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ \text{swap with } R_4 \\ \\ \end{matrix}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & -2 & 1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \cdot -1 \\ -2R_3 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 \end{array}\right] \begin{array}{l} -R_3 \\ -R_3 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & -2 \end{bmatrix}$$