2.3)

<u>Closure</u>

$$M_1 M_2 = \begin{bmatrix} 1 & x_2 + x_1 & z_2 + x_1 y_2 + z_1 \\ 0 & 1 & y_2 + y_1 \\ 0 & 0 & 1 \end{bmatrix} \in G$$

$(G, \cdot)$ is closed.

<u>Associativity</u>

Standard matrix multiplication is associative $\therefore$ $(G, \cdot)$ is associative.

<u>Neutral element</u>

$(G, \cdot)$ has neutral element $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

<u>Inverse element</u>

$$\det \begin{bmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} = 1 \neq 0$$

$\Rightarrow$ The inverse element exists for all elements in $(G, \cdot)$.

<u>Comment</u>

$(G, \cdot)$ is a group.

<u>Commutativity</u>

Counterexample:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $(G, \cdot)$ is not commutative.

<u>Comment</u>

$(G, \cdot)$ is not an Abelian group.