2.5) a)

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 2 & 5 & -7 & -5 & -2 \\ 2 & -1 & 1 & 3 & 4 \\ 5 & 2 & -4 & 2 & 6 \end{array}\right] \begin{array}{l} \\ -2R_1 \\ -2R_1 \\ -5R_1 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 0 & 3 & -5 & -3 & -4 \\ 0 & -3 & 3 & 5 & 2 \\ 0 & -3 & 1 & 7 & 1 \end{array}\right] \begin{array}{l} \\ \cdot \frac{1}{3} \\ +R_2 \\ +R_2 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 0 & 1 & -\frac{5}{3} & -1 & -\frac{4}{3} \\ 0 & 0 & -2 & 2 & -2 \\ 0 & 0 & -4 & 4 & -3 \end{array}\right] \begin{array}{l} \\ \\ \cdot -\frac{1}{2} \\ -2R_3 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 1 \\ 0 & 1 & -\frac{5}{3} & -1 & -\frac{4}{3} \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$rk(A) \neq rk(A|b) \therefore S = \emptyset$.

2.5) b)

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 1 & 1 & 0 & -3 & 0 & 6 \\ 2 & -1 & 0 & 1 & -1 & 5 \\ -1 & 2 & 0 & -2 & -1 & -1 \end{array}\right] \begin{array}{l} \\ -R_1 \\ -2R_1 \\ +R_1 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 0 & 2 & 0 & -3 & -1 & 3 \\ 0 & 1 & 0 & 1 & -3 & -1 \\ 0 & 1 & 0 & -2 & 0 & 2 \end{array}\right] \begin{array}{l} \\ \text{swap with } R_3 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & -3 & -1 \\ 0 & 2 & 0 & -3 & -1 & 3 \\ 0 & 1 & 0 & -2 & 0 & 2 \end{array}\right] \begin{array}{l} \\ \\ -2R_2 \\ -R_2 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & -5 & 5 & 5 \\ 0 & 0 & 0 & -3 & 3 & 3 \end{array}\right] \begin{array}{l} \\ \\ \cdot -\frac{1}{5} \\ \cdot -\frac{1}{3} \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right] \begin{array}{l} \\ \\ \\ -R_3 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ -R_3 \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} +R_2 \\ \\ \\ \\ \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Particular solution: $\underline{x} = [3, 0, 0, -1, 0]^T$

Solution to $A\underline{x} = \underline{b}$:

$$\left\{ \underline{x} \in \mathbb{R}^5 : \underline{x} = \lambda_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \lambda_1, \lambda_2 \in \mathbb{R} \right\}$$

General solution:

$$\left\{ \underline{x} \in \mathbb{R}^5 : \underline{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \lambda_1, \lambda_2 \in \mathbb{R} \right\}$$