

2.10) a)

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -3 \\ 3 & -2 & 8 \end{bmatrix} \text{ swap with } R_2$$

$$\rightsquigarrow \begin{bmatrix} -1 & 1 & -3 \\ 2 & 1 & 3 \\ 3 & -2 & 8 \end{bmatrix} \begin{array}{l} \cdot -1 \\ +2R_1 \\ +3R_1 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -3 \\ 0 & 1 & -1 \end{bmatrix} \begin{array}{l} \\ \cdot \frac{1}{3} \\ -\frac{1}{3}R_2 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

There exists a non-pivot column \therefore the vectors are dependent.

b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ swap with } R_3$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{array}{l} \\ -2R_1 \\ -R_1 \\ -R_1 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{array}{l} \\ \\ -R_2 \\ -R_2 \\ -R_2 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \\ \\ -R_3 \\ -R_3 \end{array}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

All the columns are pivot columns \therefore the vectors are independent.