2.13 a.

$$\begin{array}{ccc} \underline{c_1} & \underline{c_2} & \underline{c_3} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & -2 & -1 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{matrix} \\ -R_1 \\ -2R_1 \\ -R_1 \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \cdot -\frac{1}{2} \\ +\frac{1}{2}R_2 \\ \\ \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\text{rank}(A_1) = 2$

$\text{nullity}(A_1) = 3 - \text{rank}(A_1) = 3 - 2 = 1$ (rank-nullity theorem)

$\Rightarrow \dim(U_1) = 1$

$$\begin{array}{ccc} \underline{c_4} & \underline{c_5} & \underline{c_6} \end{array}$$

$$\begin{bmatrix} 3 & -3 & 0 \\ 1 & 2 & 3 \\ 7 & -5 & 2 \\ 3 & -1 & 2 \end{bmatrix} \begin{matrix} \cdot \frac{1}{3} \\ \\ \\ \\ \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 3 \\ 7 & -5 & 2 \\ 3 & -1 & 2 \end{bmatrix} \begin{matrix} \\ -R_1 \\ -7R_1 \\ -3R_1 \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix} \begin{matrix} \\ \cdot \frac{1}{3} \\ -\frac{2}{3}R_2 \\ -\frac{2}{3}R_2 \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\text{rank}(A_2) = 2$

$\text{nullity}(A_2) = 3 - \text{rank}(A_2) = 3 - 2 = 1$ (rank-nullity theorem)

$\Rightarrow \dim(U_2) = 1$

b.

$\underline{c_1} + \underline{c_2} - \underline{c_3} = \underline{0}$

$\Rightarrow$ Basis of U_1 : $\{[1, 1, -1]^T\}$

$\underline{c_3} + \underline{c_4} - \underline{c_5} = \underline{0}$

$\Rightarrow$ Basis of U_2 : $\{[1, 1, -1]^T\}$

c.

$U_1 = U_2 = \text{span}\{[1, 1, -1]^T\}$

$\Rightarrow$ Basis of $(U_1 \wedge U_2)$: $\{[1, 1, -1]^T\}$