2.8) a)

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 4 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \cdot \tfrac{1}{2} \quad -2R_1$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right] -3R_1$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & -\frac{3}{2} & 1 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right] \cdot -2$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right] +R_2$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$$

$rk(A) \neq 3 \quad \therefore \quad A$ is singular.

b)

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ -R_1 \\ -R_1 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ -R_2 \\ -R_2 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 1 \end{array}\right] \text{swap with } R_4$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & -2 & 1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \cdot -1 \\ -2R_3 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 \end{array}\right] \begin{array}{l} \\ \\ -R_3 \\ -R_3 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & -2 \end{bmatrix}$$