

2.7)

$$\left[\begin{array}{ccc|c} 6 & 4 & 3 & 12x_1 \\ 6 & 0 & 9 & 12x_2 \\ 0 & 8 & 0 & 12x_3 \end{array} \right] \cdot \frac{1}{6}$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{2} & 2x_1 \\ 0 & -4 & 6 & 12x_2 - 12x_1 \\ 0 & 8 & 0 & 12x_3 \end{array} \right] \cdot -\frac{1}{4} + 2R_2$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{2} & 2x_1 \\ 0 & 1 & -\frac{3}{2} & 3x_1 - 3x_2 \\ 0 & 0 & 12 & 12x_3 + 12x_1 - 12x_2 \end{array} \right] \cdot \frac{1}{12}$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{2} & 2x_1 \\ 0 & 1 & -\frac{3}{2} & 3x_1 - 3x_2 \\ 0 & 0 & 1 & x_3 + x_2 - x_1 \end{array} \right] - \frac{1}{2}R_3 + \frac{3}{2}R_3$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & 0 & \frac{5}{2}x_1 - \frac{1}{2}x_3 - \frac{1}{2}x_2 \\ 0 & 1 & 0 & \frac{3}{2}x_1 - \frac{3}{2}x_2 + \frac{1}{2}x_3 \\ 0 & 0 & 1 & x_3 + x_2 - x_1 \end{array} \right] - \frac{2}{3}R_2$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{3}{2}x_1 + \frac{1}{2}x_2 - \frac{3}{2}x_3 \\ 0 & 1 & 0 & \frac{3}{2}x_1 - \frac{3}{2}x_2 + \frac{1}{2}x_3 \\ 0 & 0 & 1 & x_3 + x_2 - x_1 \end{array} \right]$$

$$x_1 = \frac{3}{2}x_1 + \frac{1}{2}x_2 - \frac{3}{2}x_3 \Rightarrow \frac{1}{2}x_1 + \frac{1}{2}x_2 - \frac{3}{2}x_3 = 0 \quad (1)$$

$$x_2 = \frac{3}{2}x_1 - \frac{3}{2}x_2 + \frac{1}{2}x_3 \Rightarrow \frac{1}{2}x_1 - \frac{5}{2}x_2 + \frac{1}{2}x_3 = 0 \quad (2)$$

$$x_3 = x_3 + x_2 - x_1 \Rightarrow x_2 = x_1 \quad (3)$$

$$(1) \frac{1}{2}x_1 + \frac{1}{2}x_1 - \frac{3}{2}x_3 = 0 \Rightarrow x_1 = \frac{3}{2}x_3$$

$$(2) \frac{3}{2}x_1 - \frac{5}{2}x_1 + \frac{1}{2}x_3 = 0 \Rightarrow x_1 = \frac{3}{2}x_3$$

Row equations are consistent $\therefore A\underline{x} = 12\underline{x}$ has solutions.

$$x_1 : x_2 : x_3 = 1 : 1 : \frac{2}{3}$$

$$x_1 + x_2 + x_3 = 1$$

$$\Rightarrow x_1 + x_1 + \frac{2}{3}x_1 = 1$$

$$\Rightarrow \frac{8}{3}x_1 = 1$$

$$\Rightarrow x_1 = \frac{3}{8}, x_2 = \frac{3}{8}, x_3 = \frac{1}{4}$$

$$\Rightarrow \underline{x} = \left[\frac{3}{8}, \frac{3}{8}, \frac{1}{4} \right]^T$$