

$$= \frac{-a^2 + a^2 + a - a}{a + 1}$$

$$= 0$$

\Rightarrow The inverse element exists for all elements in $(\mathbb{R} \setminus \{-1\}, *)$.

Commutativity

$$a * b = ab + a + b$$

$$\begin{aligned} b * a &= ba + b + a \\ &= ab + a + b \end{aligned}$$

$$\Rightarrow a * b = b * a$$

$(\mathbb{R} \setminus \{-1\}, *)$ is associative.

Conclusion

$(\mathbb{R} \setminus \{-1\}, *)$ is an Abelian group.

2.1) b)

$$3 * x * x = 15$$

$$\Rightarrow (3x + 3 + x) * x = 15$$

$$\Rightarrow (4x + 3) * x = 15$$

$$\Rightarrow (4x^2 + 3x) + (4x + 3) + x = 15$$

$$\Rightarrow 4x^2 + 8x - 12 = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$