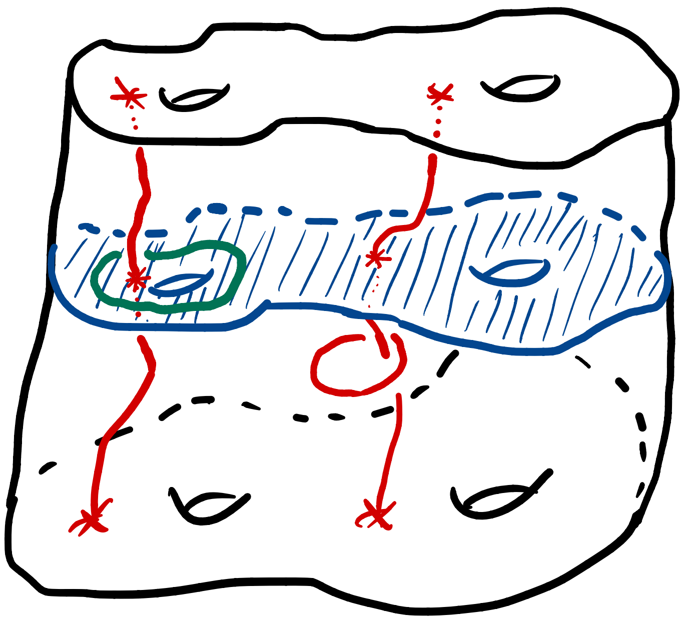
CFT/TFT correspondence beyond semisimplicity

QU-Day 3/2023, 12.09.2023, Aaron Hofer



Outline:
i) Motivation
ii) CFT's
ii) Crash course on TFTs
iii) FRS-construction
iv) Beyond semisimplicity

i) Motivation

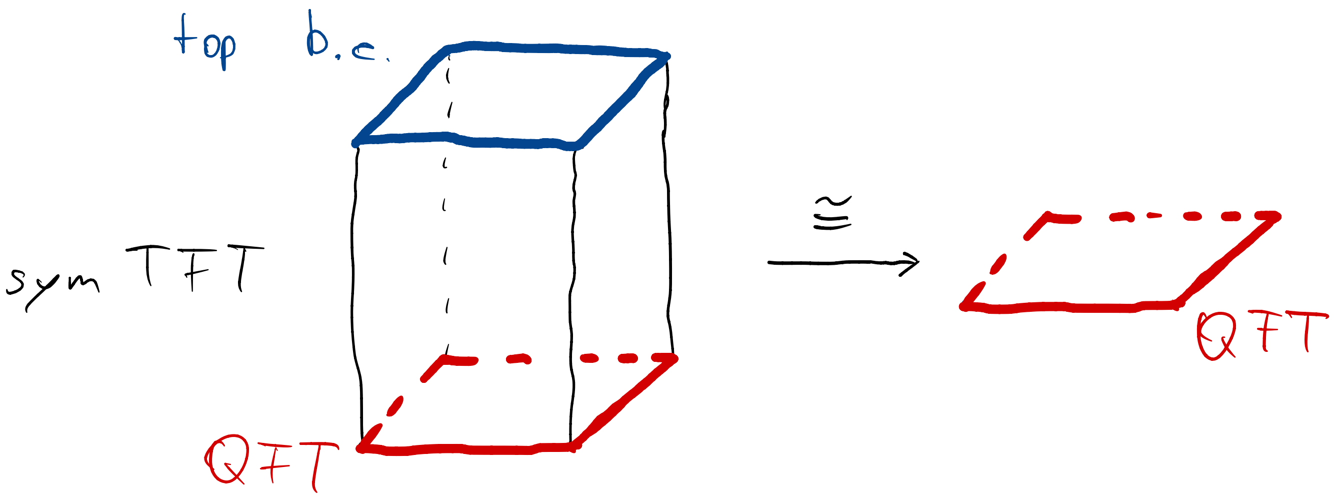
Recently a popular perspective on quantum field theories has emerged, namely that of Symmetry TFT's.

i) Motivation

Recently a popular perspective on quantum field theories has emerged, namely that of Symmetry TFT's. In this framework the symmetries of a d-dim QFT are encoded in a $(d+1)$-dim TFT with topological "Dirichlet" boundary condition:

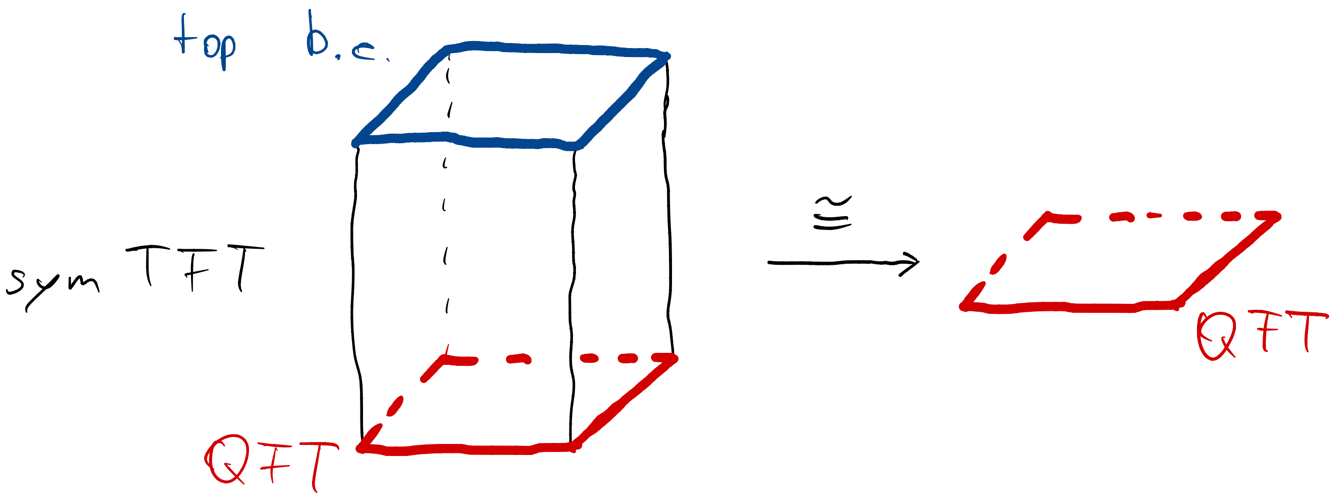
i) Motivation

Recently a popular perspective on quantum field theories has emerged, namely that of Symmetry TFT's. In this framework the symmetries of a d-dim QFT are encoded in a $(d+1)$-dim TFT with topological "Dirichlet" boundary condition:



i) Motivation

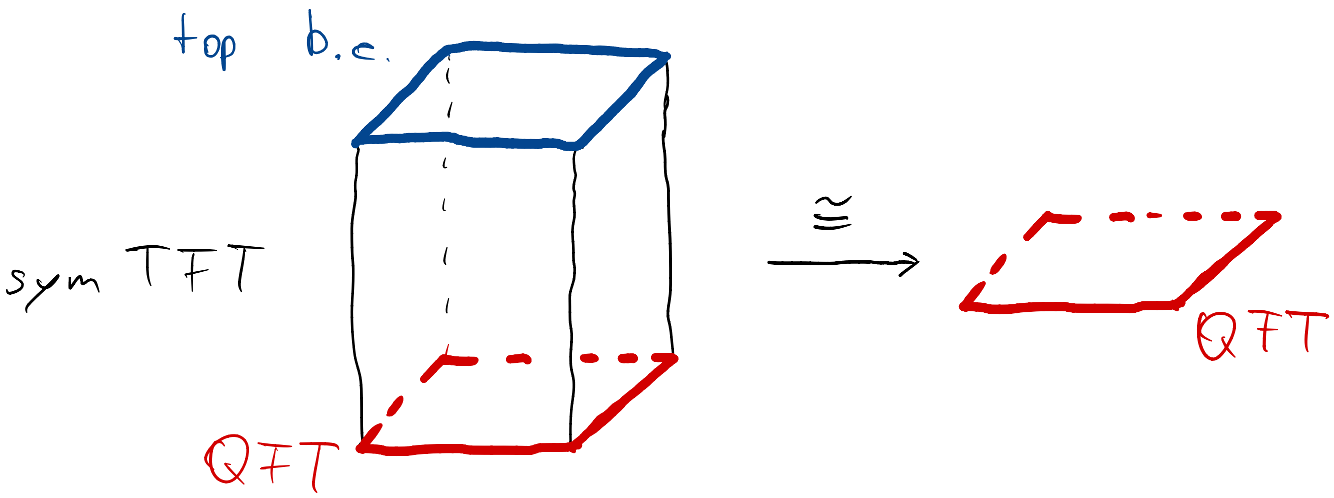
Recently a popular perspective on quantum field theories has emerged, namely that of Symmetry TFT's. In this framework the symmetries of a d-dim QFT are encoded in a (d+1)-dim TFT with topological "Dirichlet" boundary condition:



Can we understand this in a precise way?

i) Motivation

Recently a popular perspective on quantum field theories has emerged, namely that of Symmetry TFT's. In this framework the symmetries of a d-dim QFT are encoded in a $(d+1)$-dim TFT with topological "Dirichlet" boundary condition:

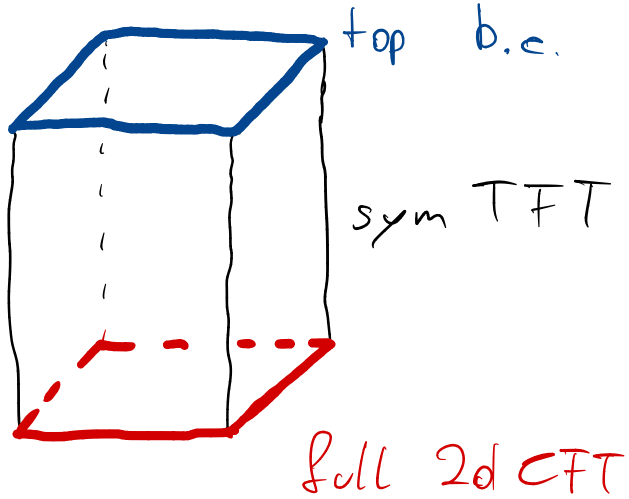


Can we understand this in a precise way?

Yes, for certain classes of 2d CFT's!

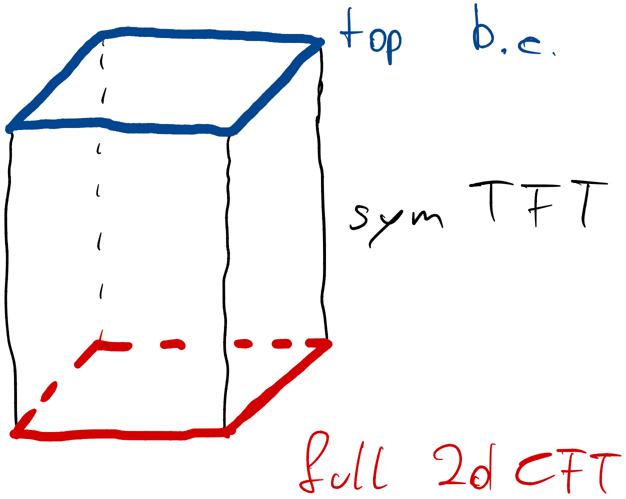
i) Motivation

CFT–TFT correspondence:



i) Motivation

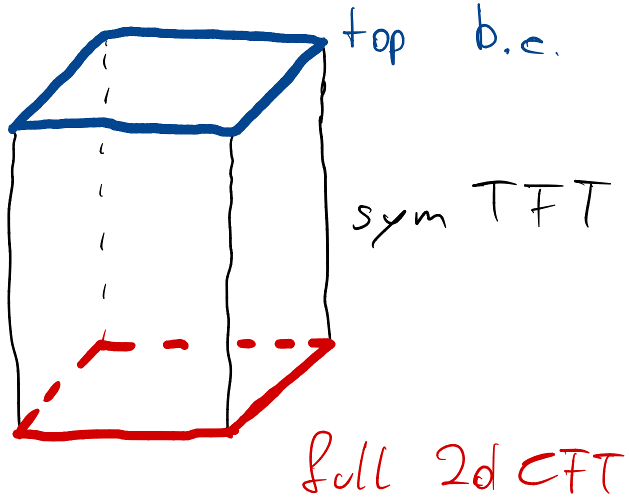
CFT–TFT correspondence:



What is this mathematically?

i) Motivation

CFT–TFT correspondence:

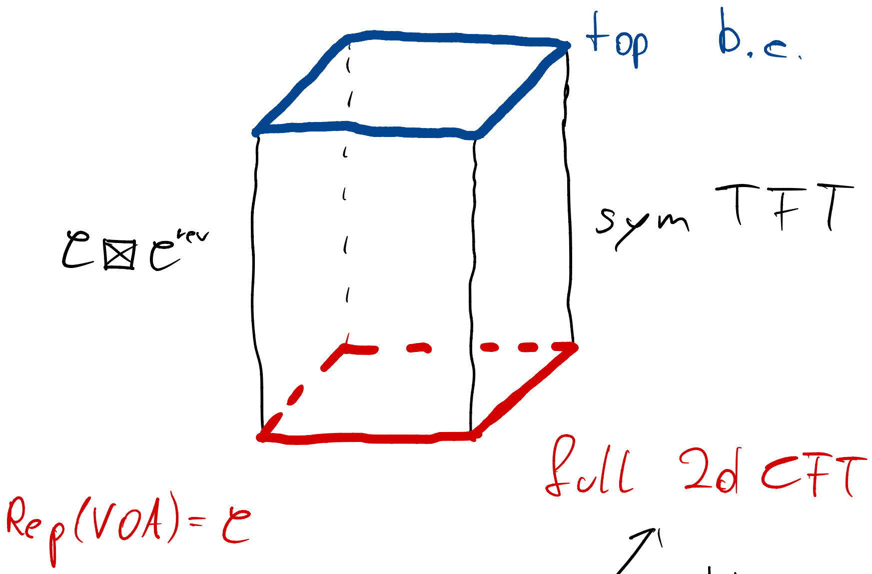


What is this mathematically?

Working Hypothesis: vertex operator algebra
+ modular functor
+ field content
+ correlators

i) Motivation

CFT–TFT correspondence:

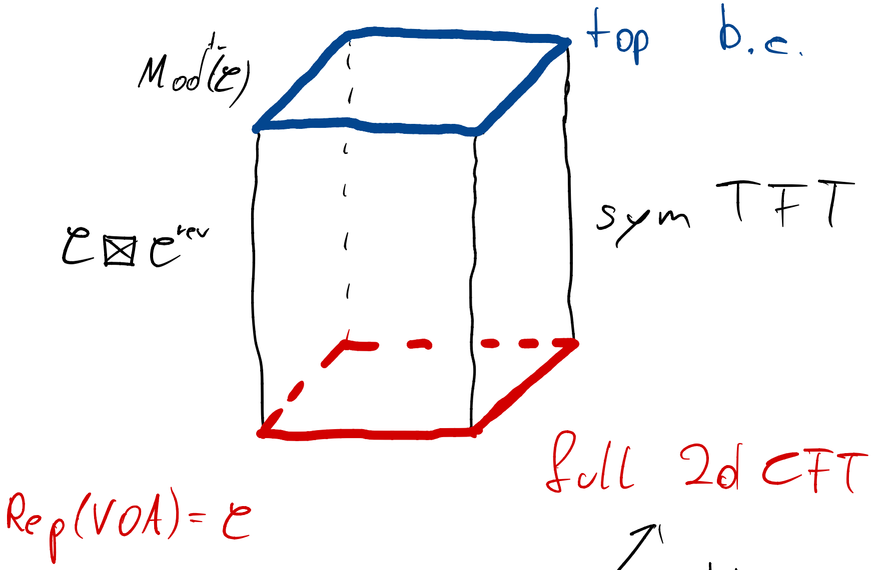


What is this mathematically?

Working Hypothesis: vertex operator algebra $\longrightarrow$ sym. TFT
+ modular functor
+ field content
+ correlators

i) Motivation

CFT–TFT correspondence:



What is this mathematically?

Working Hypothesis: vertex operator algebra $\longrightarrow$ sym. TFT

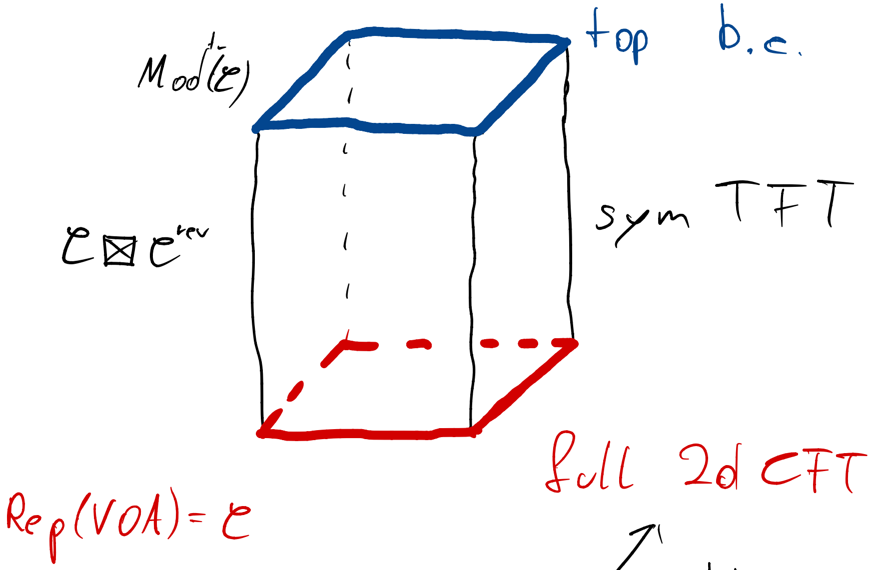
+ modular functor $\longleftarrow$ conjecture

+ field content

+ correlators $\longleftarrow$ FRS-construction + top. boundary condition

i) Motivation

CFT – TFT correspondence:



What is this mathematically?

semisimple	finite non-s.s.	derived
✓	?	??
✓	✓	??
MF ✓ Cor ✓ fields ✓	MF ✓ Cor ? fields ?	MF ✓ Cor ?? fields ??

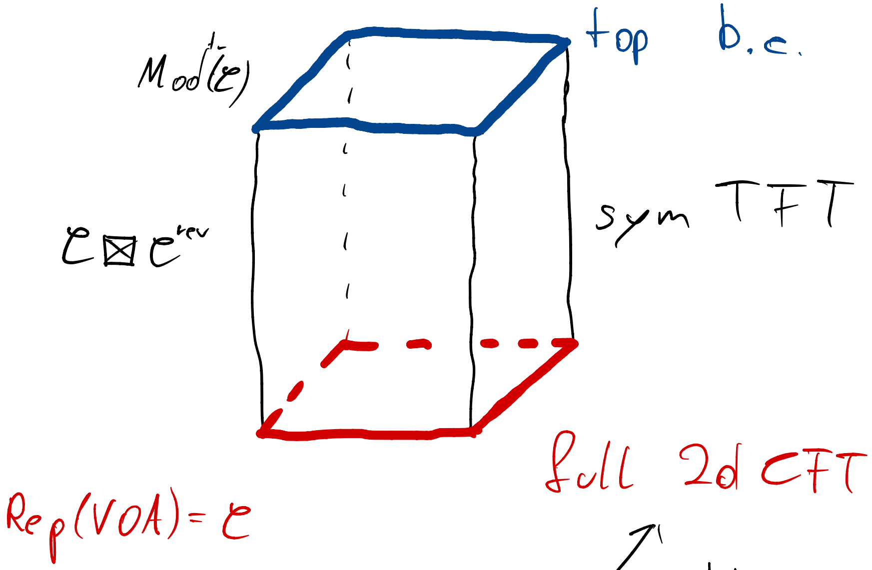
Working Hypothesis: vertex operator algebra $\longrightarrow$ sym. TFT

+ modular functor $\xleftarrow{\text{conjecture}}$ sym. TFT
+ field content
+ correlators

$\xleftarrow{\text{FRS-construction}}$ sym. TFT + top. boundary condition

i) Motivation

CFT – TFT correspondence:



What is this mathematically?

semisimple	finite non-s.s.	derived
✓	?	??
✓	✓	??
MF ✓ Cor ✓ fields ✓	MF ✓ Cor ? fields ?	MF ✓ Cor ?? fields ??

Working Hypothesis: vertex operator algebra $\longrightarrow$ sym. TFT

+ modular functor $\xleftarrow{\text{conjecture}}$ sym. TFT
+ field content
+ correlators

field content, correlators $\xleftarrow{\text{FRS-construction}}$ sym. TFT + top. boundary condition

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

2) field operators $\in$ VOA-modules which assemble into "nice" category

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

2) field operators $\in$ VOA-modules which assemble into "nice" category

3) for any Riemann surface we get a conformal block space $\Sigma \longmapsto BL(\Sigma)$

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

2) field operators $\in$ VOA-modules which assemble into "nice" category

3) for any Riemann surface we get a conformal block space $\Sigma \longmapsto BL(\Sigma)$

4) correlators of full CFT are elements of block space of complex double

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

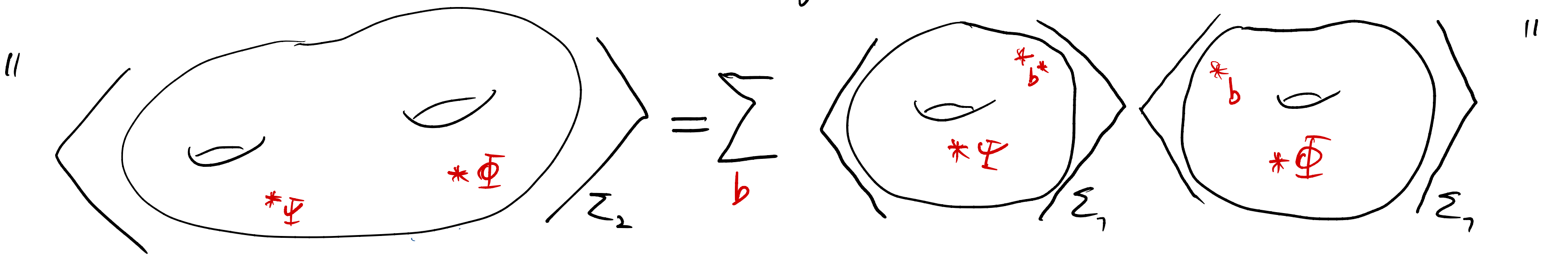
1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

2) field operators $\in$ VOA-modules which assemble into "nice" category

3) for any Riemann surface we get a conformal block space $\Sigma \longmapsto Bl(\Sigma)$

4) correlators of full CFT are elements of block space of complex double

5) nice behaviour under cutting of surfaces (factorisation)



ii) CFT's

For 2d CFT's we expect the following mathematical structure:

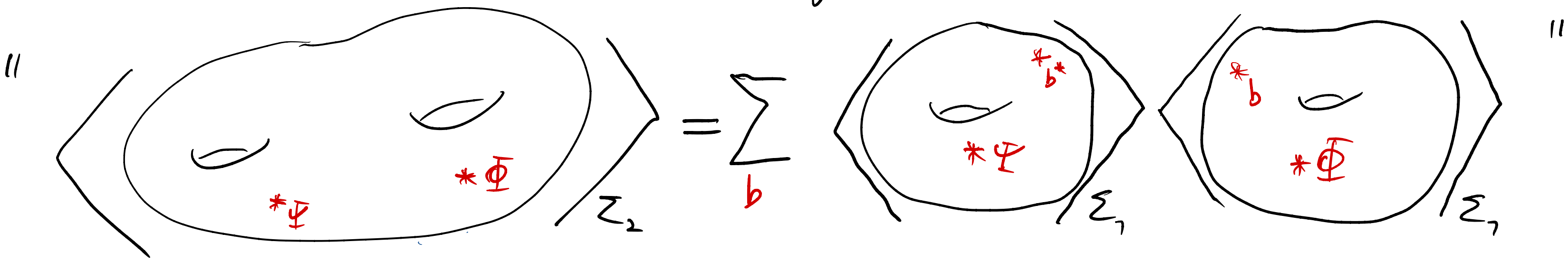
1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

2) field operators $\in$ VOA-modules which assemble into "nice" category

3) for any Riemann surface we get a conformal block space $\Sigma \mapsto Bl(\Sigma)$

4) correlators of full CFT are elements of block space of complex double

5) nice behaviour under cutting of surfaces (factorisation)



6) correlators should be invariant under mapping class group actions (related to single valuedness)

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

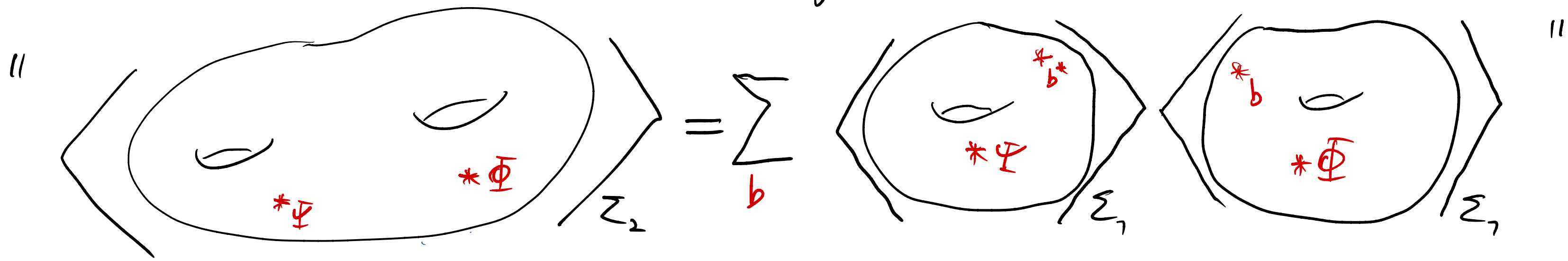
1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

2) field operators $\in$ VOA-modules which assemble into "nice" category

3) for any Riemann surface we get a conformal block space $\Sigma \longmapsto BL(\Sigma)$

4) correlators of full CFT are elements of block space of complex double

5) nice behaviour under cutting of surfaces (factorisation)



6) correlators should be invariant under mapping class group actions (related to single valuednes)

This can be packaged neatly using (higher) categorical language and TFTs $\longrightarrow$ Modular functors + Correlators + field maps

iii) Crash course on TFTs

Def A topological field theory is a symmetric monoidal functor

$$Z : \mathrm{Bord}_{d,d-1} \longrightarrow \mathrm{Vect}$$

iii) Crash course on TFTs

<u>Def</u> A topological field theory is a symmetric monoidal functor

$$\mathcal{Z}: \mathrm{Bord}_{d,d-1} \longrightarrow \mathrm{Vect}$$

closed $(d-1)$-manifold $\quad \Sigma \longmapsto \mathcal{Z}(\Sigma)$ "state space"

iii) Crash course on TFTs

<u>Def</u> A topological field theory is a symmetric monoidal functor

$$\mathcal{Z} : \mathrm{Bord}_{d,d-1} \longrightarrow \mathrm{Vect}$$

closed $(d-1)$-manifold $\quad \Sigma \longmapsto \mathcal{Z}(\Sigma)$ "state space"

bordism $\quad \Sigma \xrightarrow{M} \Sigma' \longmapsto \mathcal{Z}(\Sigma) \xrightarrow{\mathcal{Z}(M)} \mathcal{Z}(\Sigma')$ "time evolution"

+ compatability with gluing, $\otimes_{\mathbb{C}}, \ldots$

iii) Crash course on TFTs

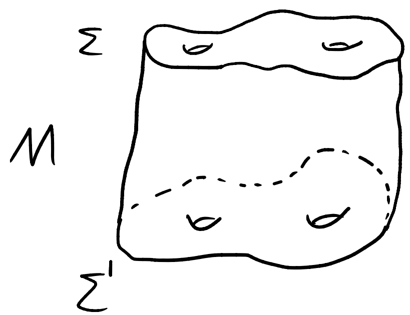
Def A topological field theory is a symmetric monoidal functor

$$Z : \mathrm{Bord}_{d,d-1} \longrightarrow \mathrm{Vect}$$

closed $(d-1)$-manifold $\quad \Sigma \longmapsto Z(\Sigma)$ "state space"

bordism $\quad \Sigma \xrightarrow{M} \Sigma' \longmapsto Z(\Sigma) \xrightarrow{Z(M)} Z(\Sigma')$ "time evolution"

+ compatibility with gluing, $\otimes_{\mathbb{C}}, \ldots$



$$\begin{array}{c} Z(\Sigma) \\ \big\downarrow Z(M) \\ Z(\Sigma') \end{array}$$

iii) Crash course on TFTs

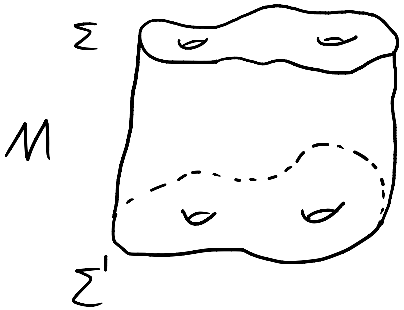
Def A topological field theory is a symmetric monoidal functor

$$\mathcal{Z} : \mathrm{Bord}_{d,d-1} \longrightarrow \mathrm{Vect}$$

closed $(d-1)$-manifold $\quad \Sigma \longmapsto \mathcal{Z}(\Sigma)$ "state space"

bordism $\quad \Sigma \xrightarrow{M} \Sigma' \longmapsto \mathcal{Z}(\Sigma) \xrightarrow{\mathcal{Z}(M)} \mathcal{Z}(\Sigma')$ "time evolution"

+ compatability with gluing, $\otimes_{\mathbb{C}}, \ldots$



$$\overset{\mathcal{Z}}{\longmapsto}$$

$$\begin{array}{c} \mathcal{Z}(\Sigma) \\ \Big\downarrow \mathcal{Z}(M) \\ \mathcal{Z}(\Sigma') \end{array}$$

Def A modular functor is a symmetric monoidal 2-functor

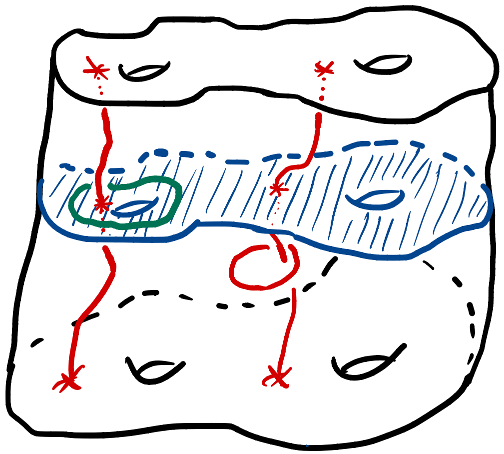
$$\mathcal{BL} : \mathrm{Bord}_{2+\varepsilon,2,1} \longrightarrow \mathrm{Prof}^{f}_{\mathbb{K}} .$$

iii) Crash course on TFTs

There is a notion of TFT with defects, where submanifolds of various codimension are decorated using extra data.

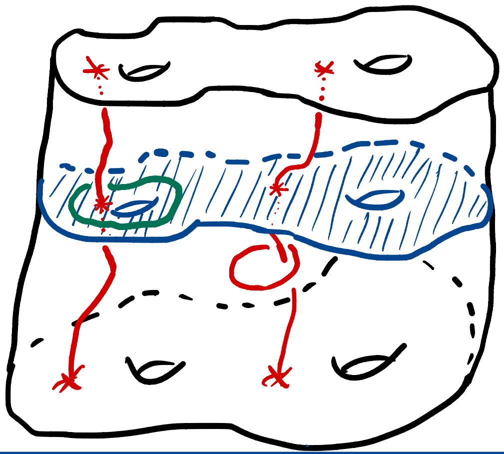
iii) Crash course on TFTs

There is a notion of TFT with defects, where submanifolds of various codimension are decorated using extra data.



iii) Crash course on TFTs

There is a notion of TFT with defects, where submanifolds of various codimension are decorated using extra data.



Thm [Reshetikhin-Turaev, Carqueville-Schaumann-Runkel, Koppen-Mulevičius-Schweigert-Runkel]

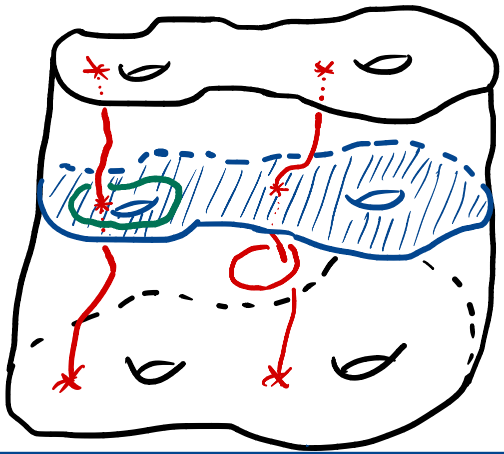
Let $\mathcal{C}$ be a modular fusion cat. There exists a TFT with defects

$$\mathcal{Z}_{\mathcal{C}} : \mathrm{Bord}_{3,2}(\mathcal{C}) \longrightarrow \mathrm{Vect}$$

constructed from $\mathcal{C}$. For us $\mathcal{C} \simeq \mathrm{Rep}(VOA)$!

iii) Crash course on TFTs

There is a notion of TFT with defects, where submanifolds of various codimension are decorated using extra data.



Thm] [Reshetikhin-Turaev, Carqueville-Schaumann-Runkel, Koppen-Mulevičius-Schweigert-Runkel]

Let $\mathcal{C}$ be a modular fusion cat. There exists a TFT with defects

$$\mathcal{Z}_{\mathcal{C}} : \mathrm{Bord}_{3,2}(\mathcal{C}) \longrightarrow \mathrm{Vect}$$

constructed from $\mathcal{C}$. For us $\mathcal{C} \simeq \mathrm{Rep}(VOA)$!

Rem] $\mathcal{Z}_{\mathcal{C}}$ can be used to construct a modular functor with

$$\mathcal{Z}_{\mathcal{C}}(\Sigma) \cong \mathcal{B}l_{\mathcal{C}}(\Sigma)$$

iv) FRS-construction

<u>Main idea:</u> Let Σ be a closed oriented surface with point defects, then

$Cor(\Sigma) \in Bl_{\mathcal{C}}(\hat{\Sigma}) \cong Z_{\mathcal{C}}(\hat{\Sigma})$ with $\hat{\Sigma} = \Sigma \sqcup -\Sigma$.

<u>Q</u>: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $Z_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

iv) FRS-construction

Main idea: Let Σ be a closed oriented surface with point defects, then

$$\mathrm{Cor}(\Sigma) \in \mathrm{Bl}_{\mathcal{C}}(\hat{\Sigma}) \cong Z_{\mathcal{C}}(\hat{\Sigma}) \quad \text{with} \quad \hat{\Sigma} = \Sigma \sqcup -\Sigma .$$

Q: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $Z_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

[FRS]: Yes! But we need surface defects $M \in \mathrm{Mod}^{\mathrm{fr}}(\mathcal{C})$ as extra input.

iv) FRS-construction

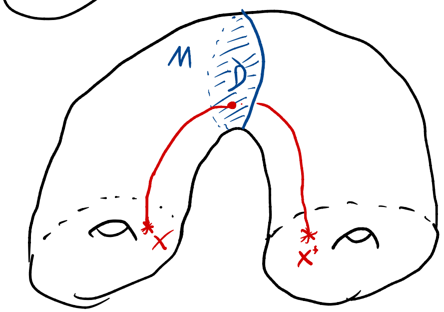
Main idea: Let Σ be a closed oriented surface with point defects, then

$$Cor(\Sigma) \in Bl_{\mathcal{C}}(\hat{\Sigma}) \cong \mathcal{Z}_{\mathcal{C}}(\hat{\Sigma}) \quad \text{with} \quad \hat{\Sigma} = \Sigma \sqcup -\Sigma .$$

Q: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $\mathcal{Z}_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

[FRS]: Yes! But we need surface defects $M \in Mod^{tr}(\mathcal{C})$ as extra input.

e.g. $\Sigma =$  :



$$\xrightarrow{\text{Apply } \mathcal{Z}} \quad Cor^{M}_{\Sigma} \in \mathcal{Z}(\Sigma) \otimes \mathcal{Z}(\Sigma)^*$$

iv) FRS-construction

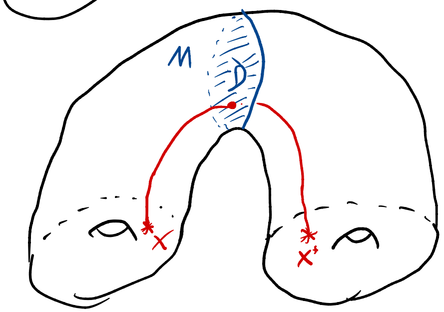
Main idea: Let Σ be a closed oriented surface with point defects, then

$$\mathrm{Cor}(\Sigma) \in Bl_{\mathcal{C}}(\hat{\Sigma}) \cong \mathcal{Z}_{\mathcal{C}}(\hat{\Sigma}) \quad \text{with} \quad \hat{\Sigma} = \Sigma \sqcup -\Sigma.$$

Q: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $\mathcal{Z}_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

[FRS]: Yes! But we need surface defects $M \in \mathrm{Mod}^{fr}(\mathcal{C})$ as extra input.

e.g. $\Sigma =$  :



$$\xrightarrow{\text{Apply } \mathcal{Z}} \quad \mathrm{Cor}^M_\Sigma \in \mathcal{Z}(\Sigma) \otimes \mathcal{Z}(\Sigma)^*$$

$M_\Sigma = \Sigma \times I$ with surface defect M at $\Sigma \times \{0\}$.

Thm [Fuchs-Runkel-Schweigert]

Defining Cor^M_Σ as above gives consistent correlators for any Σ.

iv) FRS-construction

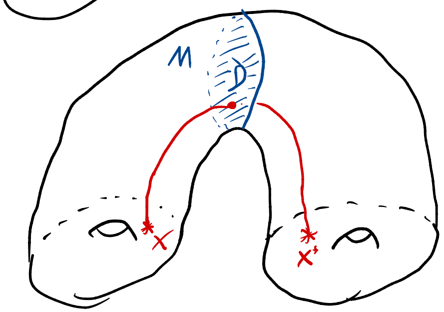
Main idea: Let Σ be a closed oriented surface with point defects, then

$$\mathrm{Cor}(\Sigma) \in \mathcal{Bl}_{\mathcal{C}}(\hat{\Sigma}) \cong \mathcal{Z}_{\mathcal{C}}(\hat{\Sigma}) \quad \text{with} \quad \hat{\Sigma} = \Sigma \sqcup -\Sigma .$$

Q: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $\mathcal{Z}_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

[FRS]: Yes! But we need surface defects $M \in \mathrm{Mod}^{fr}(\mathcal{C})$ as extra input.

e.g. $\Sigma =$  :



Apply $\mathcal{Z}$ $\longrightarrow$ $\mathrm{Cor}^{M}_{\Sigma} \in \mathcal{Z}(\Sigma) \otimes \mathcal{Z}(\Sigma)^*$

$M_\Sigma = \Sigma \times I$ with surface defect M at $\Sigma \times \{0\}$.

Thm [Fuchs-Runkel-Schweigert]
Defining $\mathrm{Cor}^{M}_{\Sigma}$ as above gives consistent correlators for any Σ.

Rmk For surfaces with boundaries more care is needed.

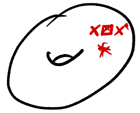
iv) FRS-construction

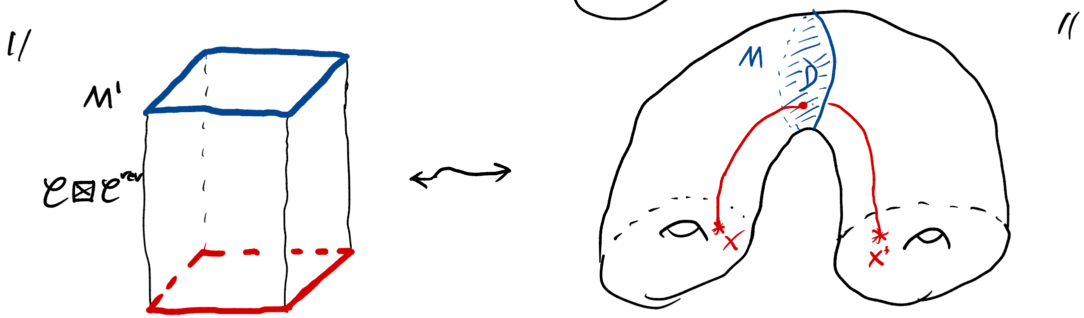
Main idea: Let Σ be a closed oriented surface with point defects, then

$Cor(\Sigma) \in Bl_{\mathcal{C}}(\hat{\Sigma}) \cong \mathcal{Z}_{\mathcal{C}}(\hat{\Sigma})$ with $\hat{\Sigma} = \Sigma \sqcup -\Sigma$.

Q: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $\mathcal{Z}_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

[FRS]: Yes! But we need surface defects $M \in Mod^{fr}(\mathcal{C})$ as extra input.

e.g. $\Sigma =$  :



$\xrightarrow{\text{Apply } \mathcal{Z}} Cor^M_\Sigma \in \mathcal{Z}(\Sigma) \otimes \mathcal{Z}(\Sigma)^*$

$M_\Sigma = \Sigma \times I$ with surface defect M at $\Sigma \times \{0\}$.

Thm [Fuchs-Runkel-Schweigert]
Defining Cor^M_Σ as above gives consistent correlators for any Σ.

Rmk For surfaces with boundaries more care is needed.

v) beyond semisimplicity

Why are non-semisimple theories interesting?

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple /derived.

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple/derived.

in mathematics:

- Many 2d TFT's are non-semisimple

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple / derived.

in mathematics:

- Many 2d TFT's are non-semisimple
- There are constructions of non-semisimple CFT's, can we understand them from a 3d perspective?

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple /derived.

in mathematics:

- Many 2d TFT's are non-semisimple
- There are constructions of non-semisimple CFT's, can we understand them from a 3d perspective?
- Possibly stronger invariants

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple /derived.

in mathematics:

- Many 2d TFT's are non-semisimple
- There are constructions of non-semisimple CFT's, can we understand them from a 3d perspective?
- Possibly stronger invariants
- Topological interpretation of algebraic structure

v) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple/derived.

in mathematics:

- Many 2d TFT's are non-semisimple
- There are constructions of non-semisimple CFT's, can we understand them from a 3d perspective?
- Possibly stronger invariants
- Topological interpretation of algebraic structure
- Step towards derived TFTs

v) beyond semisimplicity

Thm [De Renzi–Gainutdinov–Geer–Patureau-Mirand–Runkel]
Let $\mathcal{C}$ be a finite modular tensor category then there exists a (non-compact) TFT with line defects

$$Z_{\mathcal{C}} : \mathrm{Bord}_{3,2}^{n.c}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\mathbb{K}}.$$

v) beyond semisimplicity

Thm] [De-Renzi-Gainutdinov-Geer-Patureau-Mirand-Runkel]
Let $\mathcal{C}$ be a finite modular tensor category then there exists a (non-compact) TFT with line defects

$$Z_{\mathcal{C}} : \mathrm{Bord}_{3,2}^{n.c}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\mathbb{K}}.$$

Thm] [H-Runkel; DGGPR]
The above TFT gives a modular functor which is isomorphic to Lyubashenko's.

v) beyond semisimplicity

Thm [De Renzi–Gainutdinov–Geer–Patureau-Mirand–Runkel]
Let $\mathcal{C}$ be a finite modular tensor category then there exists a (non-compact) TFT with line defects
$$\mathcal{Z}_{\mathcal{C}} : \mathrm{Bord}^{n.c}_{3,2}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\mathbb{K}}.$$

Thm [H–Runkel; DGGPR]
The above TFT gives a modular functor which is isomorphic to Lyubashenko's.

Can we apply the FRS-construction to non-semisimple theories?
In principle yes, however there are two technical difficulties:

v) beyond semisimplicity

Thm [De Renzi–Gainutdinov–Geer–Patureau-Mirand–Runkel]
Let $\mathcal{C}$ be a finite modular tensor category then there exists a (non-compact) TFT with line defects
$$Z_{\mathcal{C}} : \mathrm{Bord}^{n.c}_{3,2}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\mathbb{K}}.$$

Thm [H-Runkel; DGGPR]
The above TFT gives a modular functor which is isomorphic to Lyubashenko's.

Can we apply the FRS-construction to non-semisimple theories?
In principle yes, however there are two technical difficulties:

i) The non-semisimple TQFT's of [DGGPR] are without surface defects.
What is the correct algebraic datum?

v) beyond semisimplicity

Thm [De Renzi–Gainutdinov–Geer–Patureau-Mirand–Runkel]
Let $\mathcal{C}$ be a finite modular tensor category then there exists a (non-compact) TFT with line defects

$$Z_{\mathcal{C}} : \mathrm{Bord}^{n.c}_{3,2}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\mathbb{K}}.$$

Thm [H–Runkel; DGGPR]
The above TFT gives a modular functor which is isomorphic to Lyubashenko's.

Can we apply the FRS-construction to non-semisimple theories?
In principle yes, however there are two technical difficulties:

i) The non-semisimple TQFT's of [DGGPR] are without surface defects.
What is the correct algebraic datum?

ii) The operation of doubling a surface with point defects becomes very subtle for non-semisimple labeling data.
Possible solution using a reformulation with 2-categorical language.

Thank
you!

Def An open-closed **modular functor** is a symmetric monoidal 2-functor

$$\mathcal{B}\ell : \mathrm{Bord}^{or,oc}_{2+\varepsilon,2,1} \longrightarrow \mathrm{Prof}^{lex}_{\mathbb{K}}$$

0:	intervalls & circles	finite $\mathbb{K}$-linear ab. cats
1:	surfaces	left exact profunctors $\mathcal{A} \nrightarrow \mathcal{B} \Leftrightarrow \mathcal{A}^{op} \boxtimes \mathcal{B} \xrightarrow{lex} \mathrm{vect}$
2:	diffeo's / isotopy	natural trafo's
$\otimes$:	gluing	convolution (coend $(\mathcal{G} \otimes \mathcal{F})(-,\sim) := \oint^{B \in \mathcal{B}} \mathcal{G}(B,\sim) \otimes_{\mathbb{K}} \mathcal{F}(-,B)$)
$\circ$:	composition	composition
$\sqcup$:	disjoint union	Deligne $\boxtimes$

Conj. Def A **consistent system of correlators** is a symmetric monoidal oplax natural transformation

constant profunctor to $\mathbb{K}$

$$\begin{tikzcd} \mathrm{Bord}^{or,oc}_{2+\varepsilon,2,1} \arrow[r, bend left=50, "\Delta_{\mathbb{K}}"{name=U}] \arrow[r, bend right=50, "\mathcal{B}\ell"'{name=D}] & \mathrm{Prof}^{lex}_{\mathbb{K}} \arrow[from=U, to=D, phantom, "\Downarrow \mathrm{Cor}"] \end{tikzcd}$$