

# Writing Game Theory in L<sup>A</sup>T<sub>E</sub>X

Thiago Silva\*

First Version: November 22, 2015

This Version: November 14, 2020

## List of Figures and Tables

1	2x2 Matrix: Prisoner's Dilemma Normal-Form Game . . . . .	3
2	Two Matrices: Generalization of Prisoner's Dilemma . . . . .	4
3	Finding the Nash Equilibrium of Prisoner's Dilemma (Underlining Players' Best Responses) . . . . .	4
4	Prisoner's Dilemma in Extensive-form . . . . .	5
5	Finding the Subgame Perfect Nash Equilibrium in Prisoner's Dilemma (Using Double Lines) . . . . .	5
6	Finding the Nash Equilibrium (Using Dots) . . . . .	6
7	3x3 Matrix: A Game with Three Actions . . . . .	6
8	2x4 Matrix: A Game with Two Actions for $P_1$ and Four Actions for $P_2$ . . .	7
9	Bach or Stravinsky? . . . . .	7
10	The Game of Chicken (The Hawk-Dove Game) . . . . .	7
11	Matching Pennies . . . . .	7
12	Game with Mixed Strategies (Example 1) . . . . .	7
13	Game with Mixed Strategies (Example 2) . . . . .	8
14	Centipede Game . . . . .	8
15	Three Players Game: Combining Extensive Form with Matrix Form . . . . .	8
16	Alternative Three Players Game: Combining Extensive Form with Matrix Form	8
17	Veto Game in Extensive-Form (One Node for $P_1$ and Three Nodes for $P_2$ ) . .	9
18	Extensive-Form Game with Imperfect Information (Highlighting a Subgame)	9
19	Extensive-Form Game with Imperfect Information and a Large Information Set	10
20	Game Tree with Curved Information Set . . . . .	10

---

\*PhD Candidate, Department of Political Science, Texas A&M University, College Station, TX, 77843-4348, USA. E-mail: [nsthiago@tamu.edu](mailto:nsthiago@tamu.edu)

In order to present how to write game theory in L<sup>A</sup>T<sub>E</sub>X, I will use the most popular game in game theory called the Prisoner’s Dilemma (PD) as an example. The PD shows why two rational individuals might not cooperate even if cooperation is in their best interest, thus resulting in a sub-optimal outcome. The game was formalized and named by Albert Tucker.<sup>1</sup>

The Prisoner’s Dilemma (in static or normal-form game) consist of:

- **A set of players**  $N$ , and  $N = \{1, 2\}$ , where 1 stands for “Player 1” and 2 stands for “Player 2.”
- For each  $i \in N$ , a set of actions  $S_i$ —that is, **a set of actions** or a set of strategies— $S_i = \{C, D\}$ , where  $C$  stands for “Cooperate” and  $D$  stands for “Defect.” In PD  $S_1 = S_2 = \{C, D\}$ .
- For each  $i \in N$ , a **preference relation**  $\succsim_i$  over  $S_1 \times S_2$  (i.e., the Cartesian product). Instead of outcome, we usually use the term *action profiles* (or strategy profiles), i.e., a combination of actions such as (DC), (CC), and so on. Players care about their actions, because their actions lead to action profiles that have payoff utilities assigned to them. Each player has a utility function  $v_i : S_1 \times S_2 \rightarrow \mathbb{R}$ . For any collection of sets  $S_1, S_2, \dots, S_n$ , we define  $S_1 \times S_2 \times \dots \times S_n = \{(s_1, s_2, \dots, s_n) | s_1 \in S_1, \dots, s_n \in S_n\}$ . We call  $(s_1, s_2, \dots, s_n)$  the ordered “n-tuples” ordered pairs  $(s_1, s_2)$ . This is important to show that  $(1, 2) \neq (2, 1)$ . For PD game  $S_1 \times S_2 = \{CC, CD, DC, DD\}$ .

The ordering of the action profiles, from best to worst—where the first action in parentheses represents player 1’s action and the second action represents player 2’s action—is  $(D, C)$ , where player 1 defects and player 2 cooperates;  $(C, C)$ , where both 1 and 2 cooperate;  $(D, D)$ , where both 1 and 2 defect, and;  $(C, D)$ , where 1 cooperates and 2 defects.

The players’ preferences can be represented according to payoff functions. First, we assign a utility function  $u$ , such as  $u_1$  for player 1, and  $u_2$  for player 2. We can express the order of

---

<sup>1</sup>See Poundstone, William. 1992 *Prisoner’s Dilemma*. New York City: Anchor Books.

players' preferences as,

For player 1:

$$u_1(D, C) > u_1(C, C) > u_1(D, D) > u_1(C, D) \quad (1)$$

For player 2:

$$u_2(C, D) > u_2(C, C) > u_2(D, D) > u_2(D, C)$$

Then, we can assign the respective payoffs,

For  $P_1$ :

$$u_1(D, C) = 3 > u_1(C, C) = 2 > u_1(D, D) = 1 > u_1(C, D) = 0 \quad (2)$$

For  $P_2$ :

$$u_2(C, D) = 3 > u_2(C, C) = 2 > u_2(D, D) = 1 > u_2(D, C) = 0$$

We can represent the game in a payoff matrix, also called “normal-form game”:

Table 1: 2x2 Matrix: Prisoner's Dilemma Normal-Form Game

		Player 2	
		C	D
Player 1	C	2, 2	0, 3
	D	3, 0	1, 1

The traditional Prisoners's Dilemma can be generalized from its original setting (see the right matrix below): If both players cooperate, they both receive the reward payoff  $R$  for cooperating. If both players defect, they both receive the punishment payoff  $P$ . If 1 defects while 2 cooperates, then 1 receives the temptation payoff  $T$ , while 2 receives the “sucker's” payoff,  $S$ . Symmetrically, if 1 cooperates while 2 defects, then 1 receives the sucker's payoff  $S$ , while 2 receives the temptation payoff  $T$ .

Figure 2: Two Matrices: Generalization of Prisoner's Dilemma

		$P_2$	
		Cooperate	Defect
$P_1$	Cooperate	2, 2	0, 3
	Defect	3, 0	1, 1

		$P_2$	
		Cooperate	Defect
$P_1$	Cooperate	$R, R$	$S, T$
	Defect	$T, S$	$P, P$

Finding the Nash equilibrium (NE) of the Prisoner's Dilemma (PD) game: The action profile  $a^*$  in a strategic game with ordinal preferences is a Nash equilibrium if, for every player  $i$  and every action  $a_i$  of player  $i$ ,  $a^*$  is at least as good according to player  $i$ 's preferences as the action profile  $(a_i, a_{-i}^*)$  in which player  $i$  chooses  $a_i$  while every other player  $-i$ , with  $i \neq -i$ , chooses  $a_{-i}^*$ . Therefore, for every player  $i$

$$u_i(a^*) \geq u_i(a_i, a_{-i}^*) \forall a_i \in A_i \quad (3)$$

where  $A_i$  is the set of actions for player  $i$ , and  $u_i$  is a payoff function that represents player  $i$ 's preferences.

Table 3: Finding the Nash Equilibrium of Prisoner's Dilemma (Underlining Players' Best Responses)

		Player 2	
		C	D
Player 1	C	2, 2	0, <u>3</u>
	D	<u>3</u> , 0	<u>1</u> , <u>1</u>

Each underlined action indicates the best response of each player according to the other player's action. Therefore,  $NE = (D, D)$ .

The PD can also be depicted in a game of extensive-form (or decision tree), such as:

Figure 4: Prisoner's Dilemma in Extensive-form

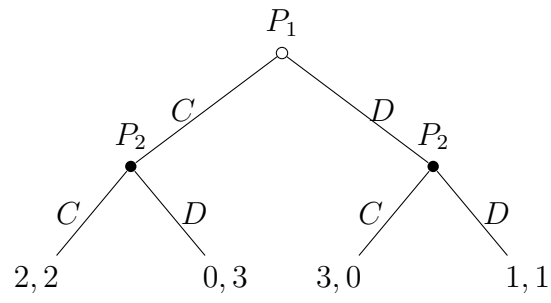
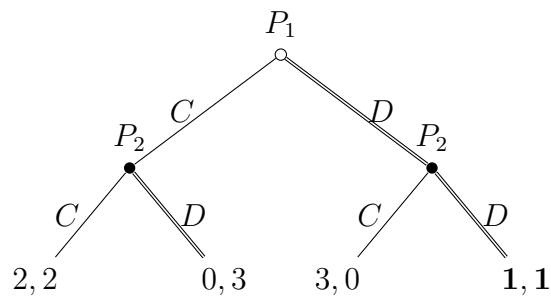


Figure 5: Finding the Subgame Perfect Nash Equilibrium in Prisoner's Dilemma (Using Double Lines)



## Other Examples and Game Forms

Table 6: Finding the Nash Equilibrium (Using Dots)

		Player 2	
		C	D
Player 1	C	2, 2	0, $\dot{3}$
	D	$\dot{3}$ , 0	1, 1

The small dot on top of each payoff indicates the best response of each player according to the other player's action. **Note:** Be careful to not confuse strategy profiles (or outputs) with payoff utilities (i.e.,  $(D, D) \neq (1, 1)$ ). The equilibrium (or equilibria) of a game refers to the strategy profile(s). Therefore,  $NE = (D, D)$ .

Table 7: 3x3 Matrix: A Game with Three Actions

		Player 2		
		Cooperate	Defect	Neither
Player 1	Cooperate	$R, R$	$S, T$	$T, S$
	Defect	$T, S$	$P, P$	$R, S$
	Neither	$T, S$	$P, P$	$S, S$

Table 8: 2x4 Matrix: A Game with Two Actions for  $P_1$  and Four Actions for  $P_2$

		$P_2$			
		C Unconditionally	D Unconditionally	Imitation Move	Opposite Move
$P_1$	C	$R, R$	$S, T$	$R, R$	$S, T$
	D	$T, S$	$P, P$	$P, P$	$T, S$

Table 9: Bach or Stravinsky?

		Player 2	
		Bach	Stravinsky
Player 1	Bach	3, 2	0, 0
	Stravinsky	0, 0	2, 3

Figure 10: The Game of Chicken (The Hawk-Dove Game)

		$P_2$	
		Swerve	Straight
$P_1$	Swerve	0, 0	-1, 1
	Straight	1, -1	-10, -10

		$P_2$	
		Swerve	Straight
$P_1$	Swerve	Tie, Tie	Lose, Win
	Straight	Win, Lose	Crash, Crash

Table 11: Matching Pennies

		Player 2	
		Heads	Tails
Player 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

Table 12: Game with Mixed Strategies (Example 1)

		$P_2$	
		(q) A	(1 - q) B
$P_1$	(p) A	$\alpha, \beta$	$\epsilon, \zeta$
	(1 - p) B	$\gamma, \delta$	$\eta, \theta$

Table 13: Game with Mixed Strategies (Example 2)

		$P_2$	
		$(q) D$	$(1 - q) E$
$P_1$	$(x) A$	$\iota, \kappa$	$o, \pi$
	$(y) B$	$\lambda, \mu$	$\rho, \sigma$
	$(1 - x - y) C$	$\nu, \xi$	$\tau, v$

Figure 14: Centipede Game

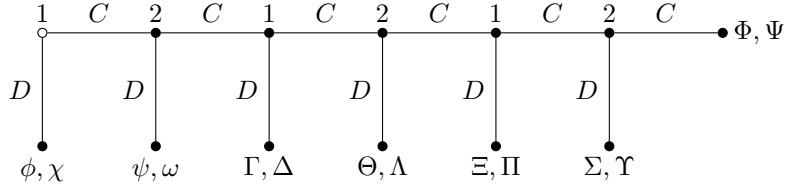


Figure 15: Three Players Game: Combining Extensive Form with Matrix Form

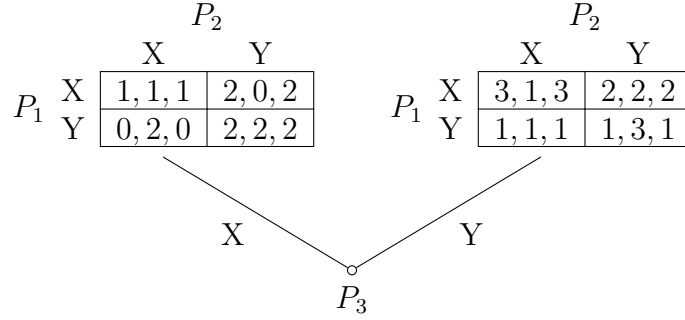


Figure 16: Alternative Three Players Game: Combining Extensive Form with Matrix Form

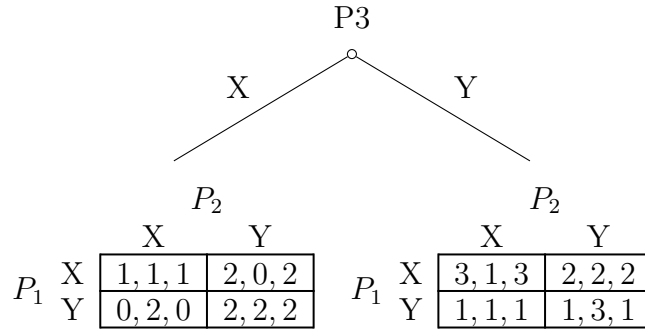
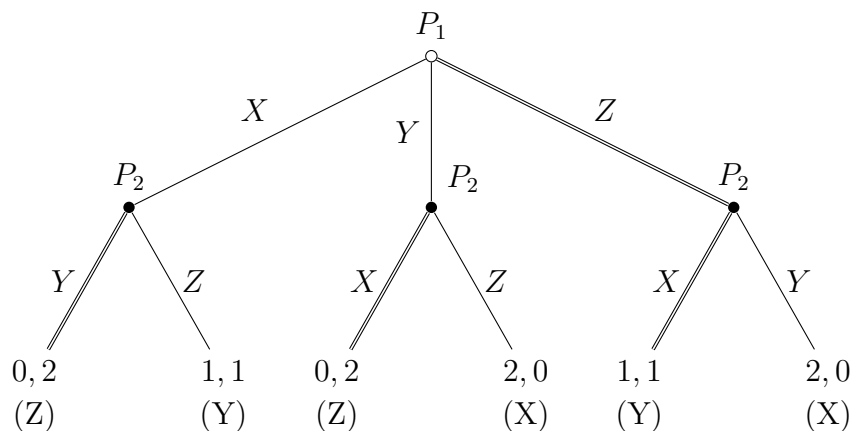




Figure 17: Veto Game in Extensive-Form (One Node for  $P_1$  and Three Nodes for  $P_2$ )



Based on exercise 163.2 of Osborne's book:  
 Osborne, Martin J. 2004. *An Introduction to Game Theory*. Oxford: Oxford University Press.

Figure 18: Extensive-Form Game with Imperfect Information (Highlighting a Subgame)

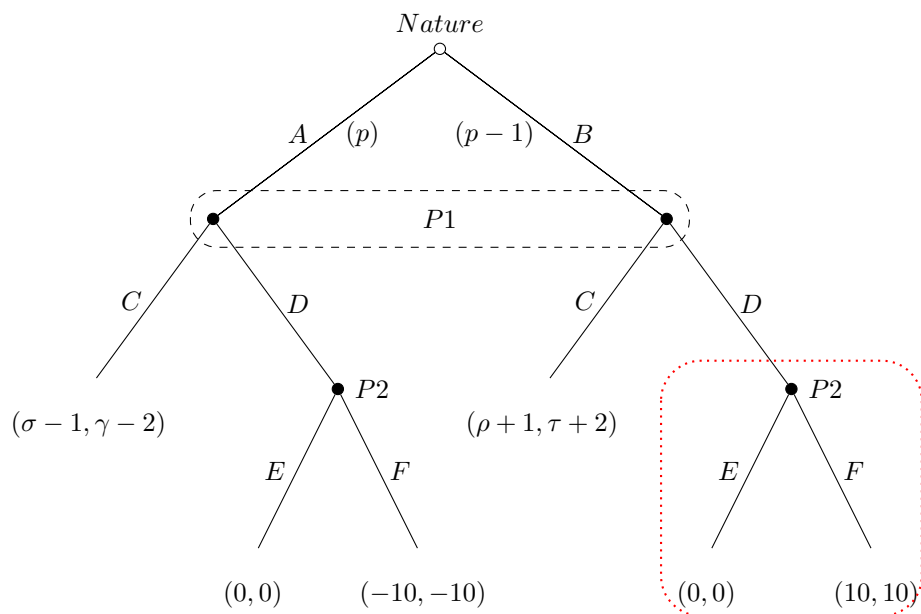
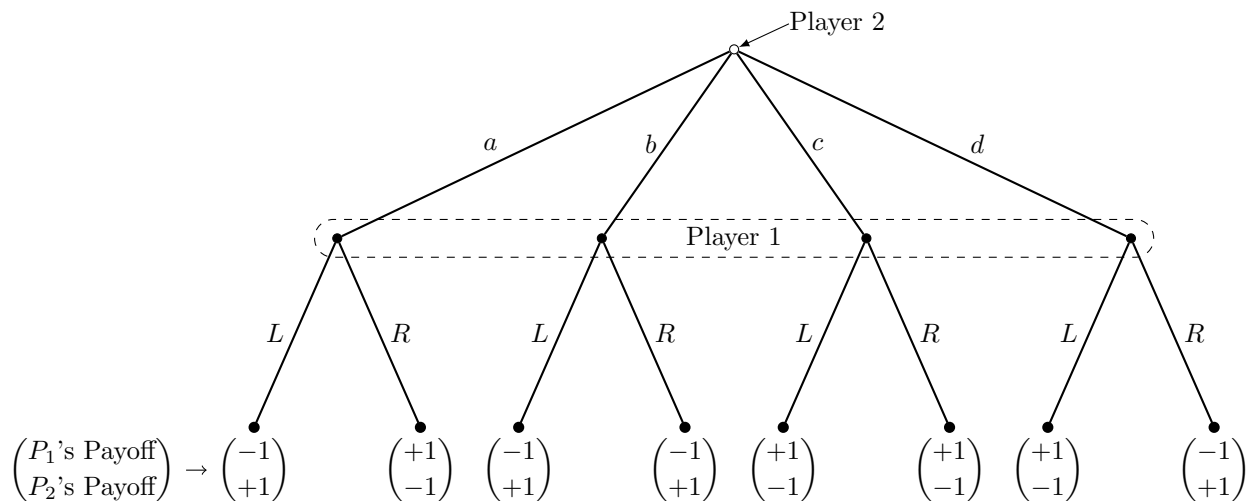
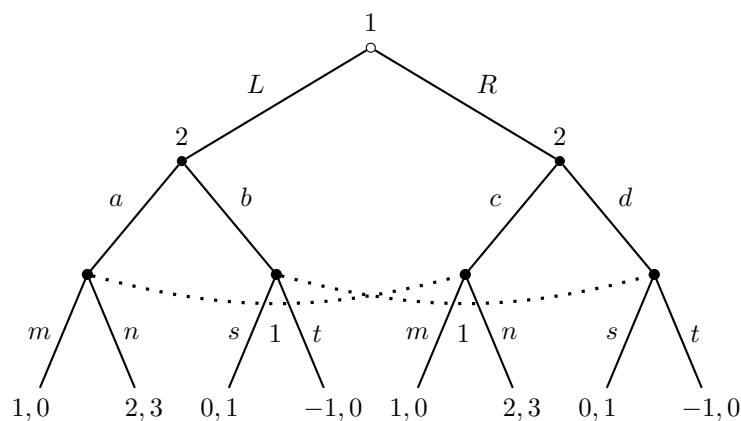


Figure 19: Extensive-Form Game with Imperfect Information and a Large Information Set



Note: This is Figure 7.D.2 in Mas-Colell, Whinston, and Green's book on microeconomic theory (1995), as replicated by Haiyun Chen (Simon Fraser University).

Figure 20: Game Tree with Curved Information Set



Note: This is Figure 6 in Osborne's "Manual for egameps.sty."