To the coaches out there so passionate about their craft—I hope these concepts help you persevere through all the challenges you face. It's a tough profession.

To the basketball fans, the scientists, the teachers, the parents, and the football players who are passionate about what they do—I hope that you found my work thought-provoking enough to inspire you in your craft.

Appendix 1

Calculation of Individual Offensive Ratings and Floor Percentages

Chapter 13 describes the concepts behind distributing credit to teammates cooperating to produce points and scoring possessions for their teams. Specifically, it provided the concepts and formulas for individual offensive ratings and floor percentages. This appendix gets into a little more of the details of why those formulas are constructed the way they are.

To refresh your memory: individual offensive ratings and floor percentages are constructed from the following three statistics:

- Individual scoring possessions, which represents the credit an individual gets for the times his/her team scores at least one point
- Individual possessions, which represents the credit an individual gets for the times his/her team ends a possession and gives it back to the opponents
- Individual points produced, which represents the credit an individual receives for the points that his/her team generates on the offensive end

Each of these statistics incorporates the difficulty concept, which generally gives more credit to the player performing the more difficult role in meeting a team objective. Difficulty is often defined by the probability of achieving that role, with the lower probability meaning greater difficulty and more credit.

Those are the generalities. Here are the details.

Individual Scoring Possessions

A team scores through field goals and free throws—nothing else. On field goals, individuals either score them, assist on them, or get offensive rebounds that sometimes leads to them. On free throws, no assists are ever awarded (something that seems a little unfair), so credit needs to be split only between the foul shooter and players who get offensive rebounds that lead to foul shots.

For field goals, the most critical splitting of credit is that between the shooter and the assistant. How difficult is a shot for an individual? His or her field goal percentage is an indication of that. How difficult is the assist to that player? Without measured stats, the best we can say is that the difficulty of the assist is probably proportional to the ease of a shot. Thus, for splitting the credit of a player's field goals between him and all the players who assisted those field goals, I use this equation:

$$\text{FG Part} = \text{FGM} \times \left(1 - \frac{1}{2} \times \frac{\text{PTS} - \text{FTM}}{2 \times \text{FGA}} \times q_{AST}\right)$$

This equation gives a player credit for his field goals (FGM) but subtracts credit proportional to his effective field goal percentage $\left(\frac{\text{PTS} - \text{FTM}}{2 \times \text{FGA}}\right)$ and the percentage of his shots that are assisted on (q_{AST}). The factor of one-half is used to split credit between the two players—the assistant, who has no measure of failure, and the shooter, whose missed field goals are a measure of failure.[1]

So what percentage of a player's shots are assisted on? What is q_{AST}? (This paragraph is for readers who want to earn lots of credit for accomplishing a difficult task.) Obviously, guards who record more assists than big men probably have fewer of their shots assisted on. Aside from that bit of obvious wisdom, there isn't a lot to go on. I developed a rather brutally complicated formula that I use to estimate the percentage of a player's shots that are assisted, denoted as q. You can tell how complicated the formula is by the fact that the following formula is the *simplified* version:

$$q = \frac{\text{MIN}}{\text{TMMIN}/5} q_5 + \left(1 - \frac{\text{MIN}}{\text{TMMIN}/5}\right) q_{12}$$

where

$$q_5 = 1.14 \times \frac{\text{TMAST} - \text{AST}}{\text{TMFGM}}$$

$$q_{12} = \frac{\frac{\text{TMAST}}{\text{TMMIN}} \times \text{MIN} \times 5 - \text{AST}}{\frac{\text{TMFGM}}{\text{TMMIN}} \times \text{MIN} \times 5 - \text{FGM}}$$

The equation for q is made up of two approximations, q_5 and q_{12}, each weighted by how many minutes a player is on the floor. If a player is on the floor a lot, $q5$ is more appropriate because it was developed based on the probability of a player having a field goal assisted on, assuming that the same five players play together (hence the number five on q_5) throughout a game.[2] If a player is on the floor fairly infrequently, q_{12} works better (the twelve meaning it's good for the twelfth man) because it was based upon an assumption of fairly even distribution of assists per minute.

Whew. OK. That was the painful paragraph. Now, stepping back from the details for a reality check: You may ask why a player with a good effective field goal percentage should get more credit taken away from his shots than a bad shooter. It goes back to the example early in the chapter. It is often more difficult for a passer to get a ball to a good shooter than to a poor one. So a high-percentage shooter should thank his assistants relatively more, giving more credit back to them.

If you take away credit from a shooter for his assisted field goals, you also give credit to him when he is making the assist. This is the assist part, which is

$$\text{AST Part} = \frac{1}{2} \times \frac{(\text{TMPTS} - \text{TMFTM}) - (\text{PTS} - \text{FTM})}{2 \times (\text{TMFGA} - \text{FGA})} \times \text{AST}$$

The factor of one-half is used to account for the same lack of a statistic to measure failure of assists. Also, if you're taking away credit from shooters based on their effective field goal percentage, you give credit to assistants based on the effective field goal percentage that they create.

Before we start talking about offensive rebounds, let's consider free throws, which are easy. A player's scoring possessions off of foul shots is identical to the team formula:

$$\text{FT Part} = \left[1 - (1 - \text{FT\%})^2\right] \times 0.4 \times \text{FTA}$$

The part in brackets is just the fraction of free throw possessions on which the player makes at least one foul shot.

The basics of a player's scoring possessions consist simply of adding the field goal part and the free throw part together. Offensive rebounds, however, work by starting a new play that can lead to some of these field goals or free throws. So I take that total and subtract partial credit, giving it to offensive rebounders. The credit that goes to offensive rebounders is based on the relative difficulty of their task (the team's offensive rebounding percentage, TMOR%) to scoring on a play (the team play percentage, TMPlay%). Because an offensive rebound does not guarantee a score, the relative difficulty gets adjusted to reflect the chance that a score will happen off of that offensive rebound—the TMPlay% again. The credit given to a player getting an offensive rebound is then his offensive rebounds (OR) times the weight on offensive rebounds due to its relative difficulty (TMOR weight) and the TmPlay%:

$$\text{OR part} = \text{OR} \times \text{TMOR weight} \times \text{TMPlay\%}$$

So, when a team scores off an offensive rebound, what is the relative difficulty of an offensive rebound (where the TMOR% is 0.3) versus the difficulty of scoring on a play (where the TMPlay% is 0.45)? I get at the difficulty by looking back at the Ultimatum Game, which suggests that teammates *compete* somewhat for credit. That *competition* between roles suggests that you can pit the two roles—offensive rebounding and scoring—against one another and assign a relative weight to them based on how often they "beat" each other. The absolute difficulty that offensive rebounders overcome is 70 percent (1—TMOR%), and the difficulty that the scorers overcome is 55 percent (1—TMPlay%). Making the analogy of a 70 percent team competing against a 55 percent team, the winning percentage of that 70 percent team is going to be (borrowing from Bill James)

$$\frac{0.70 \times (1-0.55)}{0.70 \times (1-0.55) + (1-0.70) \times 0.55} = 0.66$$

The weight on an offensive rebound in this case is 0.66, and the weight on scoring would be 0.34. The formula for the offensive rebounding weight is then

$$\text{TMOREB weight} = \frac{(1-\text{TMOR\%}) \times \text{TMPlay\%}}{(1-\text{TMOR\%}) \times \text{TMPlay\%} + \text{TMOR\%} \times (1-\text{TMPlay\%})}$$

The range on this offensive rebound weight for NBA teams in 2002 was from 0.57 for Golden State, which shot poorly but got lots of rebounds, to 0.70 for Detroit, which shot reasonably well but got few offensive rebounds despite the presence of league rebound leader Ben Wallace. What this implies is that an offensive rebound was about 20 percent more valuable for Detroit than it was for Golden State. Because of how poorly the Warriors finished plays, they basically needed six offensive rebounds to help their offense as much as five Pistons offensive rebounds helped their offense.

That is the credit given to each offensive rebound, meaning that the credit taken away from the field goal and free throw producers is the sum of all credits given to offensive rebounders. This makes the final formula for scoring possessions:

$$\begin{aligned}\text{Scoring Possessions} &= (\text{FG Part} + \text{AST Part} + \text{FT Part}) \\ &\quad \times \left(1 - \frac{\text{TMOR}}{\text{TMScPoss}} \times \text{TMOR weight} \times \text{TMPlay\%}\right) + \text{OR part}\end{aligned}$$

Some of you will remember that offensive rebounds do increase a team's offensive rating about 15 percent. It may then be tempting to adjust the offensive rebound component for this. However, John Maxwell, who did the study, also reported that nearly all of this improvement in efficiency was in the offensive rebounder's own ability to score, not in helping his teammates. Because of this, the improvement already shows up in the offensive rebounder's increased field goals or free throws without a simultaneous increase in his individual possessions.

Individual Total Possessions

The scoring possession formula was the hard one. The formulas for total possessions and points produced are relatively easy. If they have hard parts, they're the same parts that are hard in the scoring possession formula.

An individual racks up a total possession when he ends the team possession, which can happen through one of four ways:

1. A scoring possession,
2. A missed field goal that is rebounded by the defense,
3. A missed foul shot that is rebounded by the defense, or
4. A turnover

So an individual's total possessions will be

$$\text{Possessions} = \text{Scoring Possessions} + \text{Missed FG Part} + \text{Missed FT Part} + \text{TOV}$$

You know scoring possessions from the previous section. The league records individual turnovers (since 1978), so the fourth term is particularly easy. The second and third terms are the only ones that require any explanation.

The possessions that end due to a player's missed field goals are just those field goals he misses multiplied by the percentage of shots not rebounded by the offense:

$$\text{Missed FG Part} = (\text{FGA} - \text{FGM}) \times (1 - 1.07 \times \text{TMOR\%})$$

The factor of 1.07 comes from the technical version of the team possessions formula, where it makes the estimates a bit more accurate. There is no adjustment for players who anecdotally have a higher percentage of their missed field goals rebounded by their own team. Supposedly, Allen Iverson and Dominique Wilkins, who took a lot of their team's shots and missed a fair amount, had role players who knew to go to the offensive glass and rebounded a higher percentage of their misses than might be expected. I don't know if it's even true, so I don't assume it is.

For the possessions that end on a missed foul shot, I assume simply that none are rebounded by the offense and the formula is

$$\text{Missed FT Part} = (1 - \text{FT\%})^2 \times 0.4 \times \text{FTA}$$

Changing this assumption doesn't make a big difference. It might help Shaq a little, but not many other players.

And that's it for the individual possession formula.

Individual Points Produced

The formula for points produced is very similar to the one for scoring possessions, but it accounts for the number of points generated on each scoring possession. It has the same structure:

$$\text{Points Produced} = (\text{FG Part} + \text{AST Part} + \text{FT Part}) \times \left(1 - \frac{\text{TMOR}}{\text{TMScPoss}} \times \text{TMOR weight} \times \text{TMPlay\%}\right) + \text{OR part}$$

but this time the parts have slightly different looks to them.

The field goal part now accounts for three-point shots made by the player:

$$\text{FG Part} = 2 \times \left(\text{FGM} + \frac{1}{2} \times \text{FG3M}\right) \times \left(1 - \frac{1}{2} \times \frac{\text{PTS} - \text{FTM}}{2 \times \text{FGA}} \times q_{AST}\right)$$

All that changes from the scoring possession formula is replacing FGM by the points generated by all field goals, which is

$$2 \times \left(\text{FGM} + \frac{1}{2} \times \text{FG3M}\right)$$

The assist part is also modified only for how many points each assist creates:

$$\text{AST Part} = 2 \times \frac{\text{TMFGM} - \text{FGM} + \frac{1}{2}(\text{TMFG3M} - \text{FG3M})}{(\text{TMFGM} - \text{FGM})} \times \frac{1}{2} \times \frac{(\text{TMPTS} - \text{TMFTM}) - (\text{PTS} - \text{FTM})}{2 \times (\text{TMFGA} - \text{FGA})} \times \text{AST}$$

The first part of this equation is an estimate of how many points were created per assists to two-point shooters and to three-point shooters.

The free throw part of the points produced formula is trivial: FT Part = FTM. At this point, the NBA doesn't give assists on two-shot fouls, so no credit from free throws gets taken and given to assistants. You could argue that some free throws occur after an assisted basket on a three-point play and therefore some credit should be taken away. It is a legitimate argument, but such assists are probably not a big factor.

The very last thing is the offensive rebound part. Again, the only difference from the scoring possession formula is a multiplication by the expected number of points on a scoring possession. For simplicity (Don't laugh! I know that almost nothing in this appendix has been "simple"), I assume that the expected number of points scored on a scoring possession is the same on all offensive rebounds. The offensive rebound part is then:

$$\text{OR part} = \text{OR} \times \text{TMOR weight} \times \text{TMPlay\%} \times \frac{\text{TMPTS}}{\text{TMFGM} + [1 - (1 - \text{TMFT\%})^2] \times 0.4 \times \text{TMFTA}}$$

That's it. These are the offensive formulas I rely on for individuals, formulas that in turn rely upon the difficulty concept for distributing credit in cooperative actions. Over the fifteen years or so that I've been calculating these values, the formulas have evolved and they will continue to evolve. But the results are generally robust. The biggest change I made was the addition of offensive rebounds, which I did not include for years because they didn't definitely contribute a scoring possession. After adding them as a probabilistic contributor to scoring possessions, players who got a lot of offensive rebounds like Dennis Rodman received more credit and looked like better offensive players. Aside from that, improvements in the estimation of the difficulty of different actions have made only slight changes in the numerical results.

Endnotes

1. It turns out that this assumption implies that 75 to 80 percent of all passes that potentially could be assists reach the shooter successfully.
2. The more complicated form of the equation for q_5 is

$$q_5 = \sum_{i \neq n} \frac{\text{AST}_i}{\sum_{k \neq i} \text{FGM}_k}$$

where n represents the player for whom q_5 is being calculated. Basically, this sums the chances of each player assisting other players. It's an ugly formula, but it works pretty well. The simplified form in the text approximates this more complicated version fairly well.