Sports Analytics

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Contents

About

This book serves as the course textbook for the following courses at Colorado State University:

- STAT 351 (Sports Statistics and Analytics 1)
- STAT 451 (Sports Statistics and Analytics 2

CSU students contributed to the creation of this book. Many thanks to the following student collaborators:

- Levi Kipp
- Ellie Martinez
- Isaac Moorman

Current Tasks

Updated: "2022-06-06"

Team Tasks and Tips

1. Find datasets from various sports to use as examples for EDA and later chapters
2. Show how to get basic sumamry statistics from these datasets using dplyr, tidy
3. Describe and calculate useful team and individual (descriptive statistics). Example: Baseball: calculate AVG, OBP, OPS, WOBA
4. (High quality) Visualizations using ggplot
5. Look for relevant "sports" R packages
6. Include examples from CSU and Colorado sports teams when possible
7. Sports to be included: Baseball/Softball, Football, Basketball, Soccer, Hockey, Volleyball
8. Sports to be potentially included: Lacrosse, Cricket, Handball,

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Aaron:

Sports:

Chapters: Currently working to add content to chapters 1-4

Ellie:

Sports: Soccer, Volleyball

Chapters: EDA, Probability

Levi:

Sports: Basketball, Hockey

Chapters: EDA, Probability

Isaac:

Sports: Baseball, Football, Tennis

Chapters: EDA, Scraping

Chapter 1

Exploratory Data Analysis

1.1 Getting Started With R

1.1.1 Installing R

For this class, you will be using R Studio to complete statistical analyses on your computer.

To begin using R Studio, you will need to install "R" first and then install "R Studio" on your computer.

Step 1: Download R
(a) Visit https://www.r-project.org/
(b) Click **CRAN** under **Download** (c) Select any of the mirrors
(d) Click the appropriate link for your type of system (Mac, Windows, Linux)
(e) Download R on this next page.
(For Windows, this will say **install R for the first time**. For Mac, this will be under **Latest release** and will be something like **R-4.1.0.pkg** – the numbers may differ depending on the most recent version)
(f) Install R on your computer

Step 2: Download R Studio
(a) Visit https://www.rstudio.com/products/rstudio/download/#download
(b) Click to download
(c) Install R Studio on your computer

Step 3: Verify R Studio is working
(a) Open R Studio
(b) Let's enter a small dataset and calculate the average to make sure everything is working correctly.
(c) In the console, type in the following dataset of Sammy Sosa's season home run totals from 1998–2002:

```
sosa.HR <- c(66, 63, 50, 64, 49)
```

(d) In the console, calculate the average season home run total for Sammy Sosa between 1998–2002:

```
mean(sosa.HR)
```

```
## [1] 58.4
```

(e) Did you find Slammin' Sammy's average home run total from 1998–2002 was 58.4? If so, you should be set up correctly!

1.1.2   Some R Basics

For the following examples, let's consider Peyton Manning's career with the Denver Broncos. In his four seasons with the Broncos, Manning's passing yard totals were: 4659, 5477, 4727, 2249. Let's enter this data into R. To enter a vector of data, use the **c()** function.

```
peyton <- c(4659, 5477, 4727, 2249)
```

To look at the data you just put in the variable *peyton*, type *peyton* into the console and press enter.

```
peyton
```

```
## [1] 4659 5477 4727 2249
```

Some basic function for calculating summary statistics include **summary**, **mean()**, **median()**, **var()**, and **sd()**.

```
summary(peyton)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    2249    4056    4693    4278    4914    5477
```

```
mean(peyton)
```

```
## [1] 4278
```

```
sd(peyton)
```

```
## [1] 1402.522
```

R allows you to install additional packages (collections of functions) that aren't offered in the base version of R. To install a package, use **install.packages()** and to load a package, use **library()**.

One package that we will use frequently is **tidyverse**. This package includes several other packages and functions such as **ggplot** (plotting function), **dplyr** (data manipulation package), and **stringr** (string manipulation package).

```
install.packages("tidyverse")
library("tidyverse")
```

You will also need to know how to load datasets from files. For this class, we will typically provide data files is .csv format.

Here is how to load a file:

```
# load readr package and load example dataset
library(readr)
NFL_2021_Team_Passing <- read_csv("data/NFL_2021_Team_Passing.csv")

# we can look at the header (first few entries) using 'head()'
head(NFL_2021_Team_Passing)
```

```
## # A tibble: 6 x 25
##      Rk Tm              G   Cmp   Att `Cmp%`   Yds    TD `TD%`   Int `Int%`   Lng
##   <dbl> <chr>       <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl> <dbl>
## 1     1 Tampa Bay~     17   492   731   67.3  5229    43   5.9    12    1.6    62
## 2     2 Los Angel~     17   443   674   65.7  4800    38   5.6    15    2.2    72
## 3     3 Dallas Co~     17   444   647   68.6  4800    40   6.2    11    1.7    73
## 4     4 Kansas Ci~     17   448   675   66.4  4791    37   5.5    13    1.9    75
## 5     5 Los Angel~     17   406   607   66.9  4642    41   6.8    18    3      79
## 6     6 Las Vegas~     17   429   628   68.3  4567    23   3.7    14    2.2    61
## # ... with 13 more variables: `Y/A` <dbl>, `AY/A` <dbl>, `Y/C` <dbl>,
## #   `Y/G` <dbl>, Rate <dbl>, Sk <dbl>, SKYds <dbl>, `Sk%` <dbl>, `NY/A` <dbl>,
## #   `ANY/A` <dbl>, `4QC` <dbl>, GWD <dbl>, EXP <dbl>
```

1.2 Descriptive Statistics

1.2.1 Definitions

Definition 1.1. A ***population*** is a well-defined complete collection of objects.

Definition 1.2. A ***sample*** is a subset of the population.

Example 1.1. Suppose we are interested in studying Peyton's Manning's season passing yards totals. How could you define the population and what is one possible sample?

Definition 1.3. ***Quantitative data*** is numeric data or numbers. It can be broken into two further categories: discrete and continuous data.

Definition 1.4. ***Discrete data*** is quantitative data with a finite or countably infinite number of values.

Definition 1.5. ***Continuous data*** is quantitative data with an uncountably infinite number of values or data taken from an interval.

Example 1.2. What are possible discrete and continuous data associated with Peyton Manning?

Definition 1.6. ***Qualitative data*** refers to names, categories, or descriptions. It can also be broken down into two further categories, nominal data and ordinal data.

Definition 1.7. ***Nominal data*** is qualitative data with no natural ordering.

Definition 1.8. ***Ordinal data*** is qualitative data with a natural ordering.

Example 1.3. What are possible nominal and ordinal data associated with Peyton Manning?

1.2.2   Descriptive Statistics

While we will learn about some descriptive statistics that are unique to specific sports, there are some descriptive statistics that are frequently used in many applications.

1.2.2.1   Descriptive Statistics for Quantitative Data

There are different descriptive statistics depending on the type of data you are analyzing. We will begin by looking at descriptive statistics for quantitative data.

To begin, let $x_1, x_2, \dots, x_n$ represent a numerical dataset with a sample of size n, where x_i is the ith value in the dataset.

Definition 1.9. The ***sum*** of the data values is given by: $\sum_{i=1}^{n} x_i = x_1 + x_2 + \dots + x_n$

Definition 1.10. The ***sample mean*** (or sample average), $\bar{x}$, of the numerical dataset is given by $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

Definition 1.11. The ***population mean*** (or population average), μ, is the mean value for the entire population.

The mean can be thought of as a measure of center or more generally, a measure of location.

Example 1.4. Recall that Peyton Manning's season passing yards total while with the Broncos were: 4659, 5477, 4727, 2249. Calculate the sample mean of these values.

```
# Calculate the sample of Peyton Manning's passing yards season totals with
# Colts
peyton.broncos <- c(4659, 5477, 4727, 2249)
mean(peyton.broncos)
```

[1] 4278

In sports statistics, we often have to choose between using a descriptive statistic that summarizes a quantity versus a descriptive statistic that summarizes a rate. For instance, in basketball, we can compare two players based on how many points they score in a game (total quantity) or we can compare two players based on how many points per minute played (rate statistic). Many applications in sports analytics focus more on rate statistics rather than quantity statistics. Why?

We can measure the spread or variability of a dataset using *variance* and *standard devatiation*.

Definition 1.12. The ***sample variance***, s^2, of the numerical dataset is a measure of spread and is given by $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

Definition 1.13. The ***sample standard deviation***, s, of the numerical dataset is a measure of spread and is given by $s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$

Definition 1.14. The ***population variance***, σ^2, is the variance for an entire population.

Definition 1.15. The ***population standard deviation***, σ, is the standard deviation for an entire population.

We often prefer to work with standard deviations as a measure of spread as opposed to variance because standard deviations are given in our original units.

```
# Calculate the variance and standard deviation of Peyton Manning's passing
# yards season totals with Broncos
var(peyton.broncos)   # units: yards^2
```

[1] 1967068

```
sd(peyton.broncos)   # units: yards
```

[1] 1402.522

Definition 1.16. The **sample median**, $\tilde{x}$, of a numerical dataset is the middle value when the data are ordered from smallest to largest. In other words, let $x_1, x_2, \ldots, x_n$ be the (unordered) dataset and let $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ be the same dataset but ordered from smallest to largest. If n is odd, then $\tilde{x} = x_{(n+1)/2}$ and if n is even, then $\tilde{x} = \frac{1}{2} \cdot \left[x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n+1}{2}\right)} \right]$.

Example 1.5. Calculate the sample median of Peyton Manning's season passing yards total while with the Colts (3739, 4135, 4413, 4131, 4200, 4267, 4557, 3747, 4397, 4040, 4002, 4500, 4700).

Like sample mean, sample median is a measure of center. It gives you an idea of where the "middle" of your dataset is.

We can calculate sample mean and sample median in R as follows:

```
# Calculate the median of Peyton Manning's passing yards season totals with
# Broncos and Colts
peyton.colts <- c(3739, 4135, 4413, 4131, 4200, 4267, 4557, 3747, 4397, 4040,
4002,
    4500, 4700)
median(peyton.broncos)
```

```
## [1] 4693
```

```
median(peyton.colts)
```

```
## [1] 4200
```

Definition 1.17. A *percentile* is a measure of relative standing. The pth percentile is the number where at least p% of the data values are less than or equal to this number.

Definition 1.18. A *quantile* is a measure of relative standing and are the cut points for breaking a distribution of values into equal sized bins.

Definition 1.19. A *quartile* is a measure of relative standing and are the cut points for breaking a distribution of values into four equal parts.

```
# Calculate the 10th and 90th percentile of Peyton Manning's passing yards
# season totals with Colts
quantile(peyton.colts, 0.1)
```

```
##   10%
## 3798
```

```
quantile(peyton.colts, 0.9)
```

```
##    90%
## 4545.6
```

```
quantile(peyton.colts, c(0.1, 0.9))
```

```
##    10%    90%
## 3798.0 4545.6
```

Special percentiles:
1. 25th percentile = 1st quartile = Q_1

2. 50th percentile = 2nd quartile = $Q_2 = \tilde{x}$
3. 75th percentile = 3rd quartile = Q_3

Definition 1.20. *Range* is a measure of spread, measures the full width of a dataset, and is given by: $Range = Max - Min$.

Definition 1.21. *Interquartile range* is a measure of spread, measures the width of the middle 50% of a dataset, and is given by: $IQR = Q_3 - Q_1$.

Definition 1.22. A *five number summary* describes the center, spread, and edges of a dataset and is given by: $(Min, Q_1, Q_2, Q_3, max)$.

```
summary(peyton.colts)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    3739    4040    4200    4218    4413    4700
```

```
quantile(peyton.colts, c(0, 0.25, 0.5, 0.75, 1))
```

```
##   0%   25%   50%   75% 100%
## 3739 4040 4200 4413 4700
```

1.2.2.2 Descriptive Statistics for Qualitative Data

In sports statistics, we also encounter qualitative (categorical) data which is names or labels which has its own descriptive statistics.

To begin, let $x_1, x_2, ..., x_n$ represent a categorical dataset with a sample of size n, where x_i is the ith value in the dataset.

Definition 1.23. The *proportion* of sampled data that fall into a category is given by: $p = \frac{\# \text{ in category}}{\# \text{ total}}$

''Proportion" and "Probability" are often used interchangeably. Both have a minimum value of 0 and a maximum value of 1.

Definition 1.24. The *percentage* of sampled data that fall into a category is given by: $P\% = 100 \cdot p = 100 \cdot \frac{\# \text{ in category}}{\# \text{ total}}$

Percentages in this context can have a minimum value of 0% and a maximum value of 100%.

Example 1.6. In 2014, Peyton Manning started as quarterback for the Denver Broncos. The result of the Broncos' 16-game season was:
Win, Win, Loss, Win, Win, Win, Win, Loss, Win, Loss, Win, Win, Win, Win, Loss, Win

Calculate the proportion and percentage of Broncos' winning games in 2014.

```r
broncos2014 <- c("Win", "Win", "Loss", "Win", "Win", "Win", "Win", "Loss", "Win",
    "Loss", "Win", "Win", "Win", "Win", "Loss", "Win")
broncos.prop <- sum(broncos2014 == "Win")/length(broncos2014)
broncos.prop
```

```
## [1] 0.75
```

```r
broncos.perc <- 100 * broncos.prop
broncos.perc
```

```
## [1] 75
```

We can also build a frequency table that summarizes the categories and their occurrences using **table()** in R. Note that **table()** works for quantitative and qualitative data.

```r
table(broncos2014)
```

```
## broncos2014
## Loss  Win
##    4   12
```

1.3   Visualizations

Conveying information visually is also an important part in providing a description of a dataset.

R provides some basic plotting functions such as **plot**, **hist**, and **barplot**. These plotting functions are simple and not always very clean looking.

In this class, we will use analogous plotting functions in **ggplot2** that are much improved plotting functions.

If you have already installed the **tidyverse** package, it should have also installed the **ggplot2** package.

```r
# You have likely already installed the tidyverse package but if not, use the
# following command without the '#' install.packages('tidyverse')

# Load the tidyverse package (which includes ggplot2)
library(tidyverse)
```

```
## -- Attaching packages ------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.6     v dplyr   1.0.9
## v tibble  3.1.7     v stringr 1.4.0
## v tidyr   1.2.0     v forcats 0.5.1
## v purrr   0.3.4

## -- Conflicts ---------------------------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

Let's load the file "NFL_2021_Team_Passing.csv" which contains NFL Team Passing Statistics, 2021

```
library(readr)
NFL_2021_Team_Passing <- read_csv("data/NFL_2021_Team_Passing.csv")
```

Histograms are one of the most common and basic ways to visualize a dataset's distribution of values. To make a histogram, you will use **ggplot** and **geom_histogram**.

Example 1.7. Create a histogram of the NFL Team Passing Yards in 2021.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds)) + geom_histogram()
```
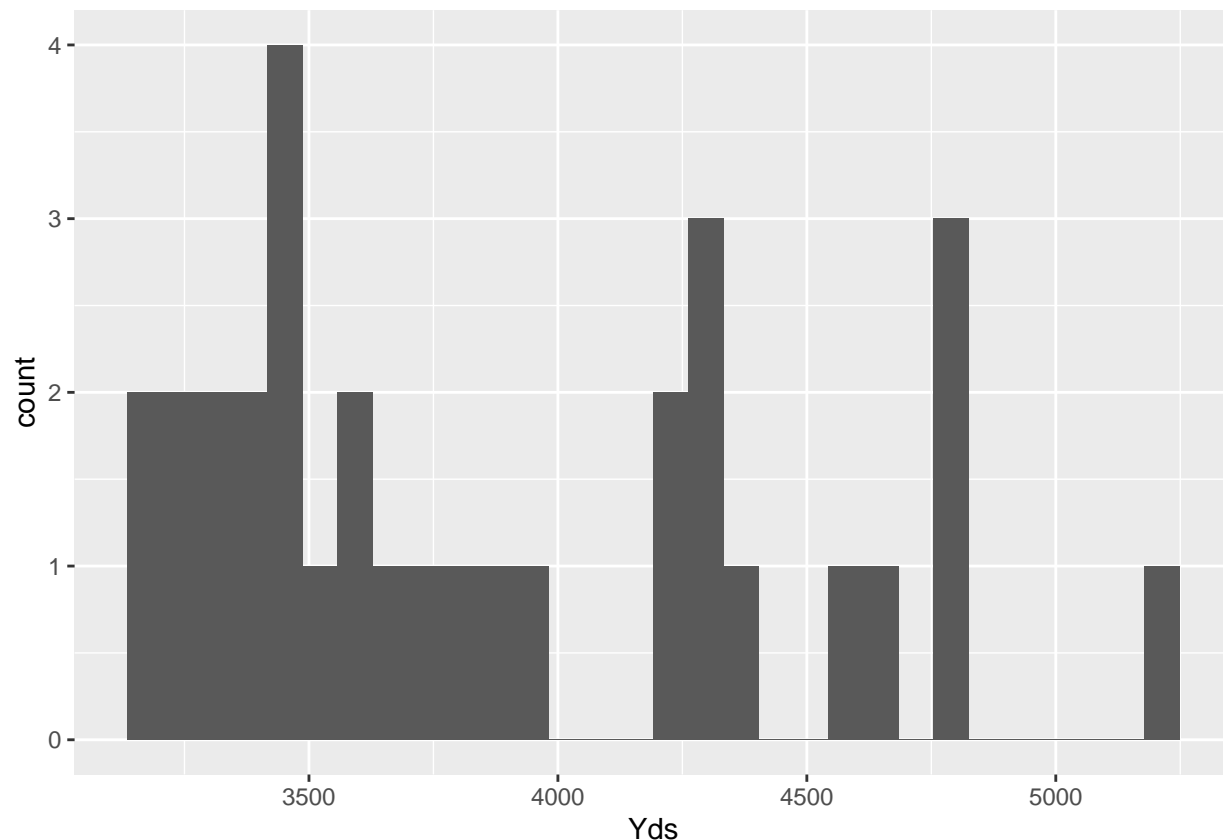

Notice how **%>%** is used to **pipe** the dataset into ggplot. This is using the pipe function from the **dplyr** package.

By default, **geom_histogram** uses 30 bins but this is customizable. Let's make the bins have a width of 200.

All good visualizations have good labels. Let's improve the axis labels and give the figure a title.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds)) + geom_histogram(binwidth = 200) + labs(x = "Team
    Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021")
```

NFL Team Passing Yards, 2021
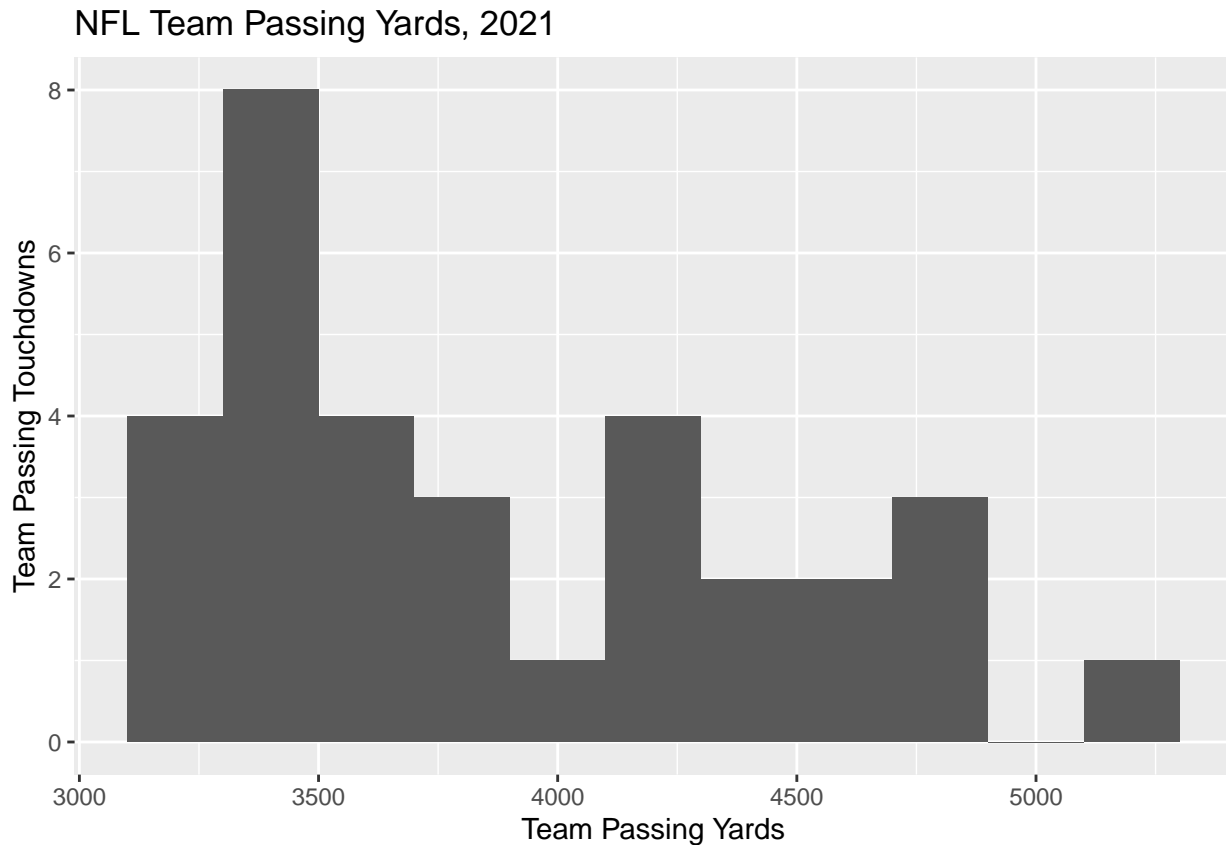
We also have numerous options to change the appearance of plots when using **ggplot**. Let's change the bins color to *blue* and change the bin borders to *white*.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds)) + geom_histogram(color = "white", fill = "blue",
    binwidth = 200) +
    labs(x = "Team Passing Yards", y = "Team Passing Touchdowns", title = "NFL
    Team Passing Yards, 2021")
```

NFL Team Passing Yards, 2021
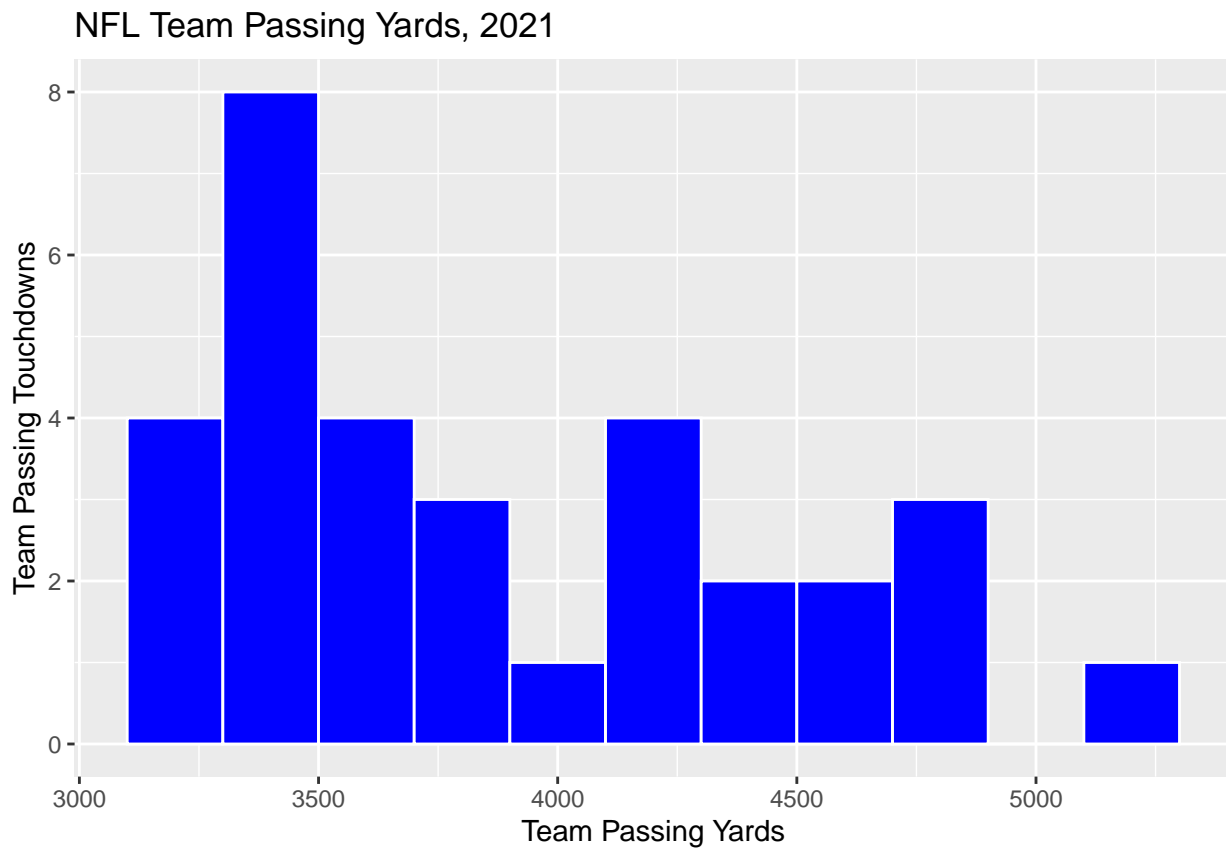
We can also create bar plots using ggplot using the **geom_bar** function.

Example 1.8. Create a bar plot with teams on the horizontal axis and passing touchdowns on the vertical axis.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Tm, y = Yds)) + geom_bar(stat = "identity")
```
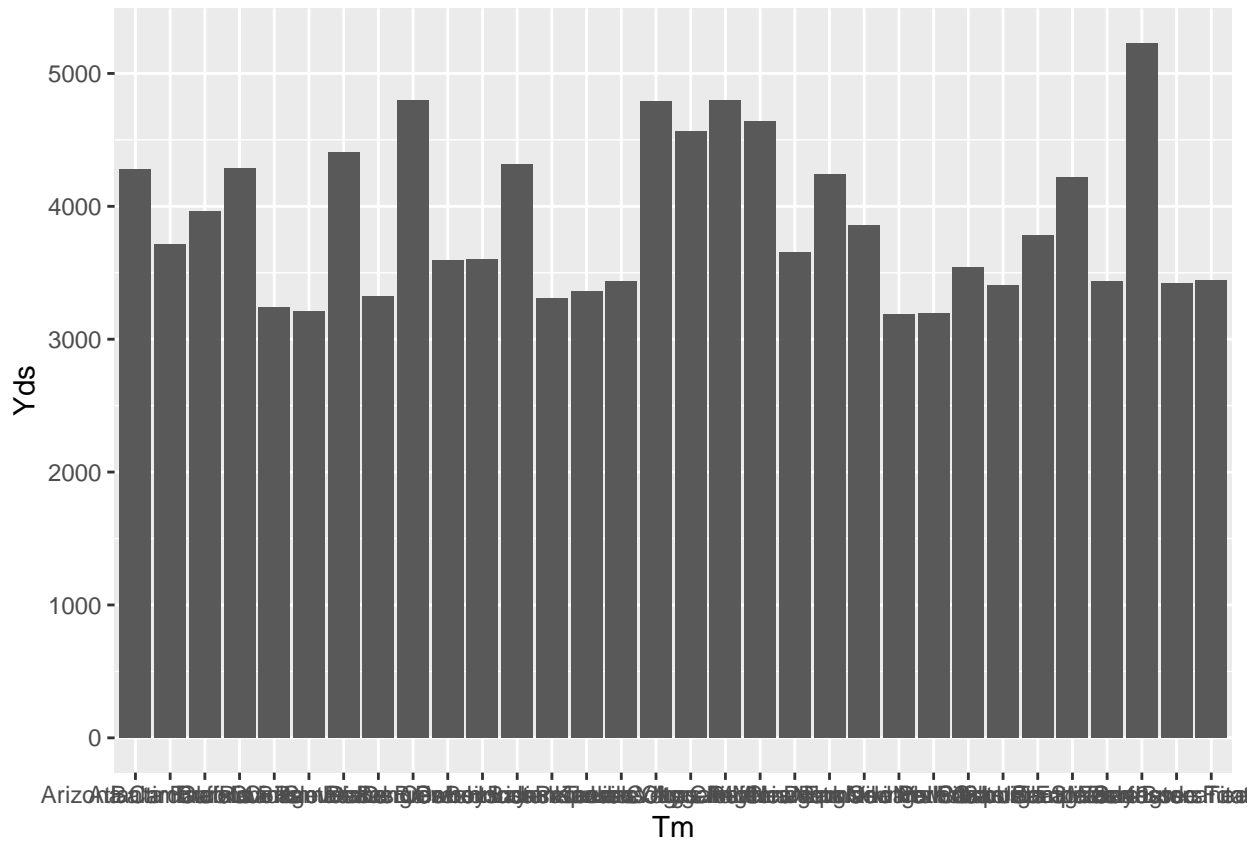
The team labels are a complete mess. Let's fix this and make some adjustments to the axis labels and figure title.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Tm, y = Yds)) + geom_bar(stat = "identity") + labs(x = "Team
    Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021") +
    theme(axis.text.x = element_text(angle = 90,
    vjust = 0.5, hjust = 1))
```

NFL Team Passing Yards, 2021
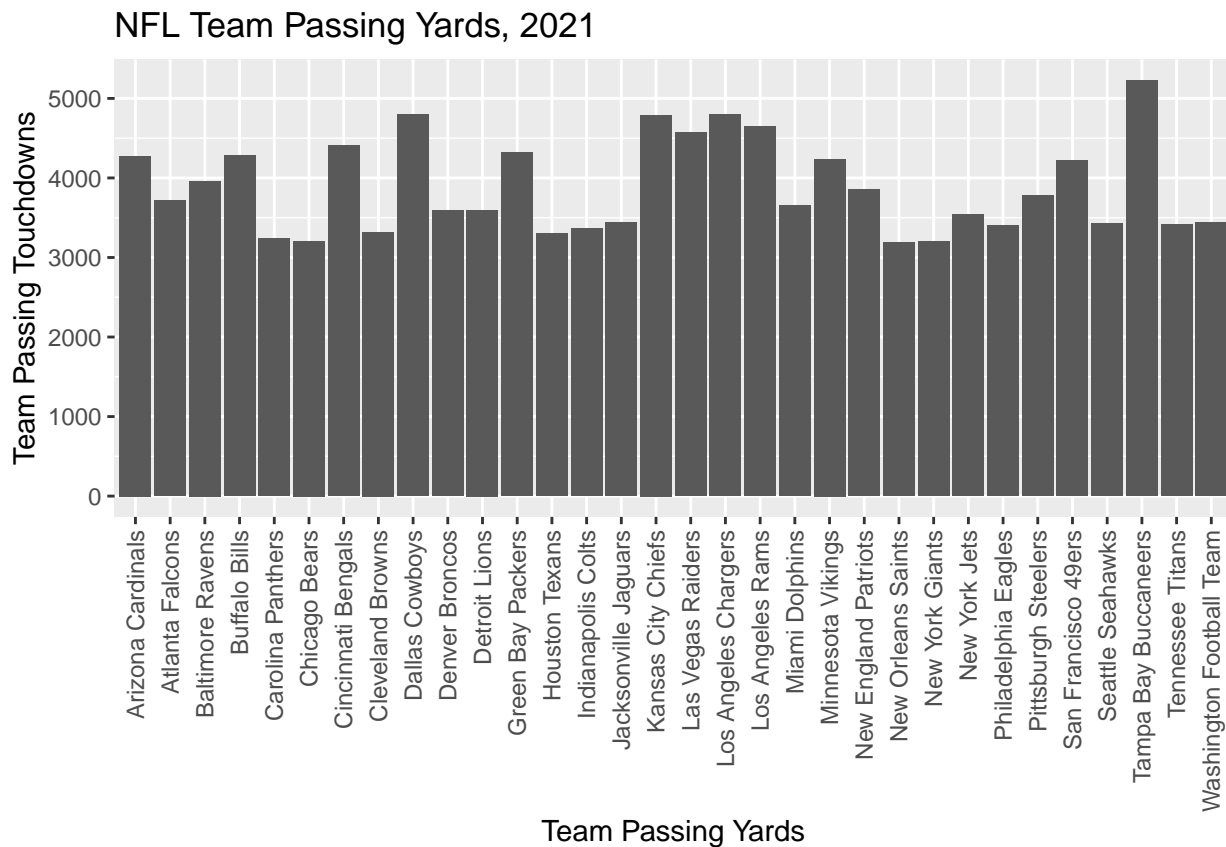
We can flip this graph if we like as well. Note that when we flip the graph, our labels get in reverse ordering, so this can be fixed using **fct_rev()** which is part of the **forcats** package.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = fct_rev(Tm), y = Yds)) + geom_bar(stat = "identity") + labs(x
    = "Team Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021") +
    coord_flip()
```
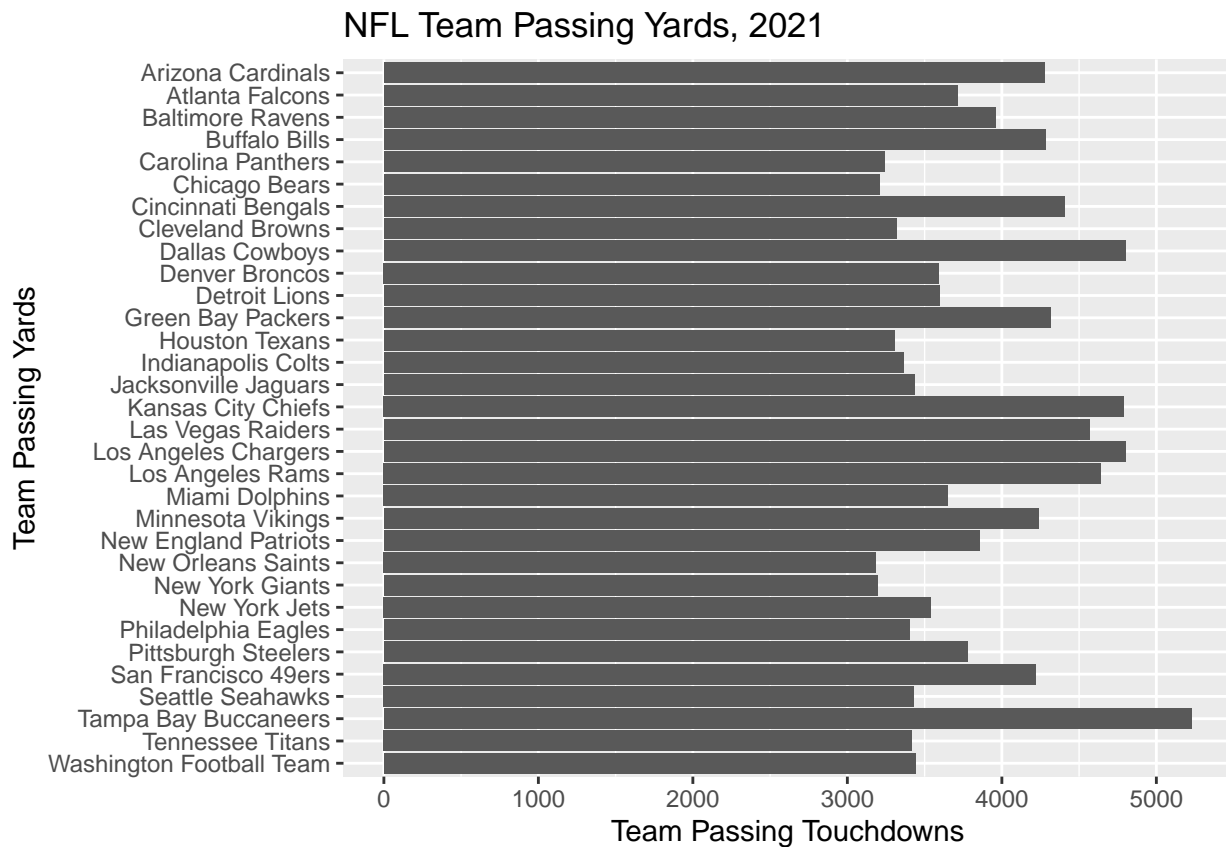
Another common and useful visualization is a scatterplot which shows the relationship between two numeric variable. In ggplot, you use **geom_point()**.

Example 1.9. Create a scatterplot of Team Passing Yards and Team Passing Touchdowns from the NFL 2021 dataset.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds, y = TD, label = Tm)) + geom_point() + labs(x = "Team
    Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021")
```

NFL Team Passing Yards, 2021
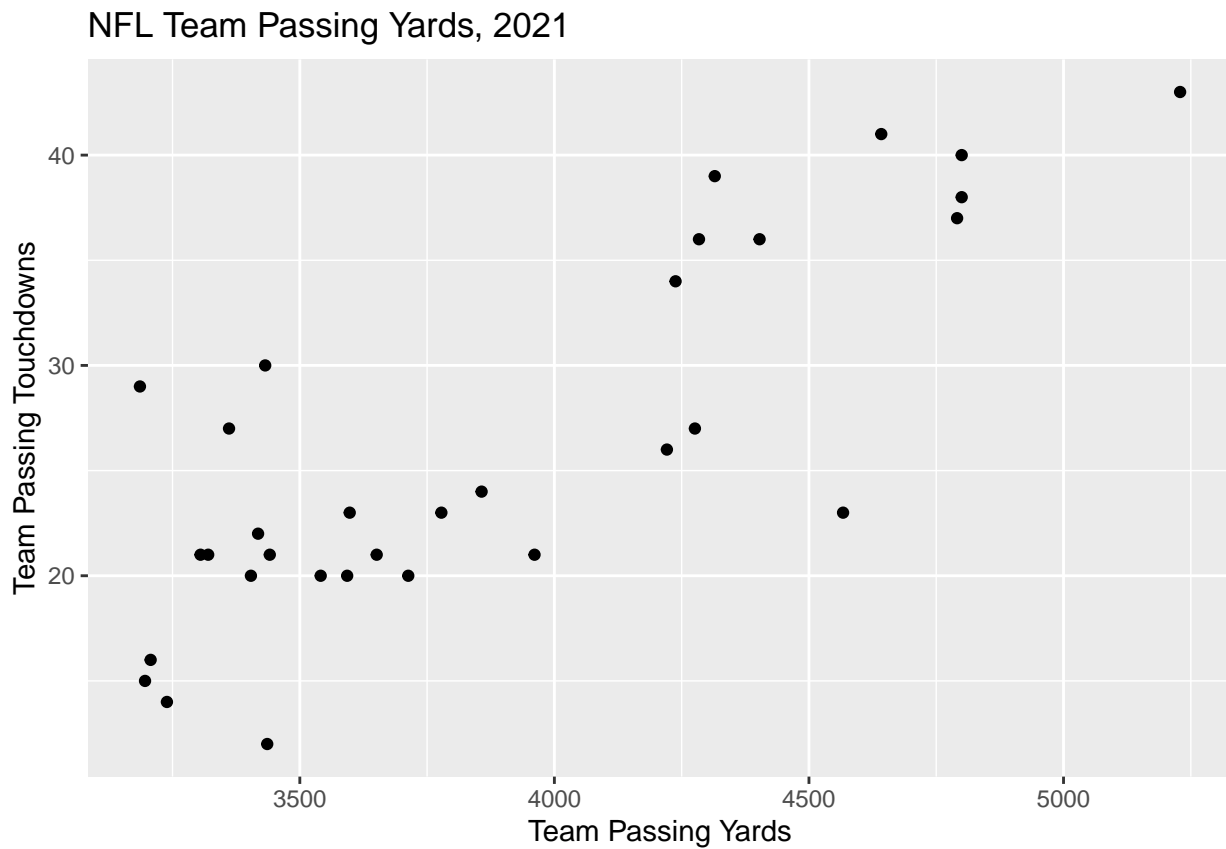
We may want to include team labels on this plot, however, it can get messy very quickly with a lot of points.

```
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds, y = TD, label = Tm)) + geom_point() + labs(x = "Team
    Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021") +
    geom_text()
```

NFL Team Passing Yards, 2021
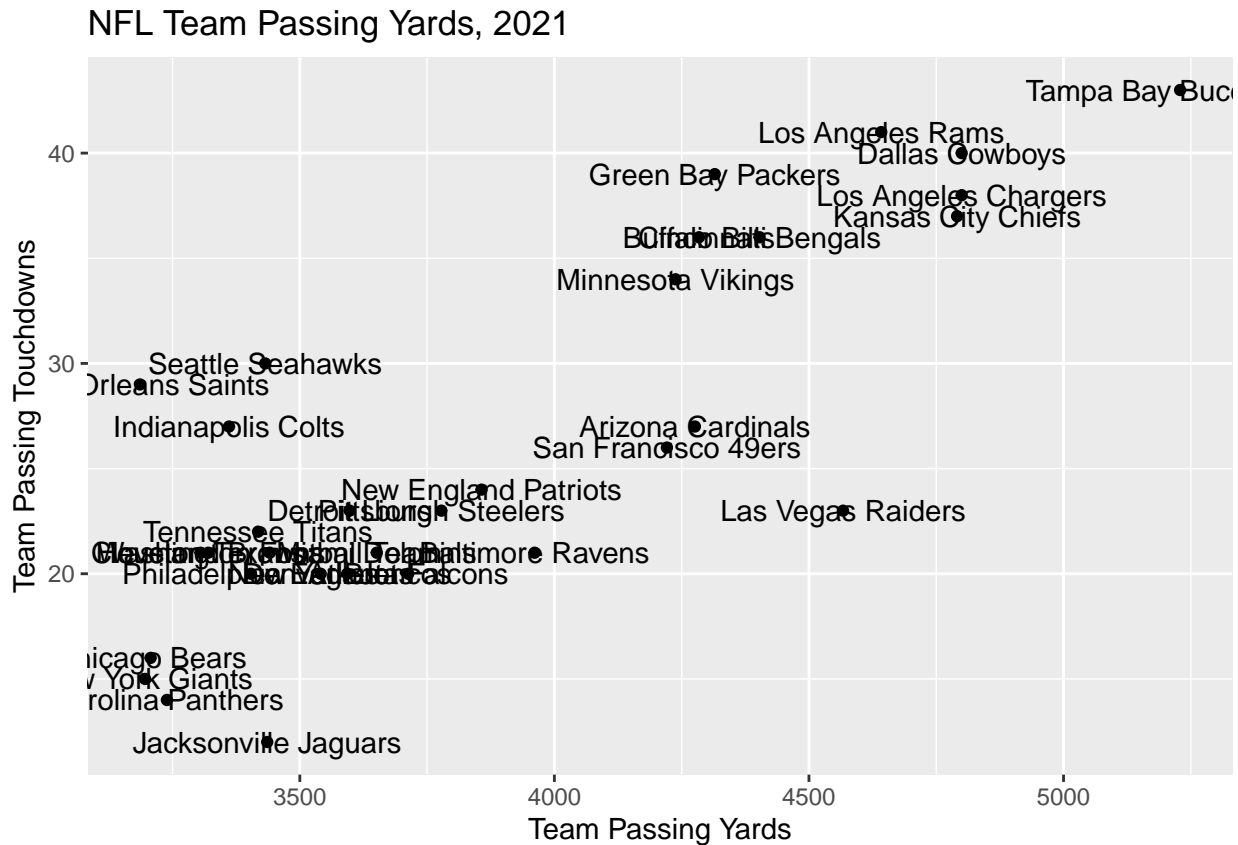
Many sports leagues have around 30 teams, so a clean scatterplot with labels can be tricky to make. Here are some options below.

```
# install ggrepel package
library(ggrepel)
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds, y = TD, label = Tm)) + geom_point() + labs(x = "Team
    Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021") +
    geom_text_repel()
```

```
## Warning: ggrepel: 9 unlabeled data points (too many overlaps). Consider
## increasing max.overlaps
```

NFL Team Passing Yards, 2021


```
NFL_2021_Team_Passing$Abbr <- c("TB", "LAC", "DAL", "KC", "LAR", "LV", "CIN",
"GB",
    "BUF", "AZ", "MN", "SF", "BAL", "NE", "PIT", "ATL", "MIA", "DET", "DEN",
    "NYJ",
    "WAS", "JAC", "SEA", "TEN", "PHI", "IND", "CLE", "HOU", "CAR", "CHI", "NYG",
    "NO")
NFL_2021_Team_Passing %>%
    ggplot(aes(x = Yds, y = TD, label = Abbr)) + geom_point() + labs(x = "Team
    Passing Yards",
    y = "Team Passing Touchdowns", title = "NFL Team Passing Yards, 2021") +
    geom_text_repel(box.padding = 0.3)
```
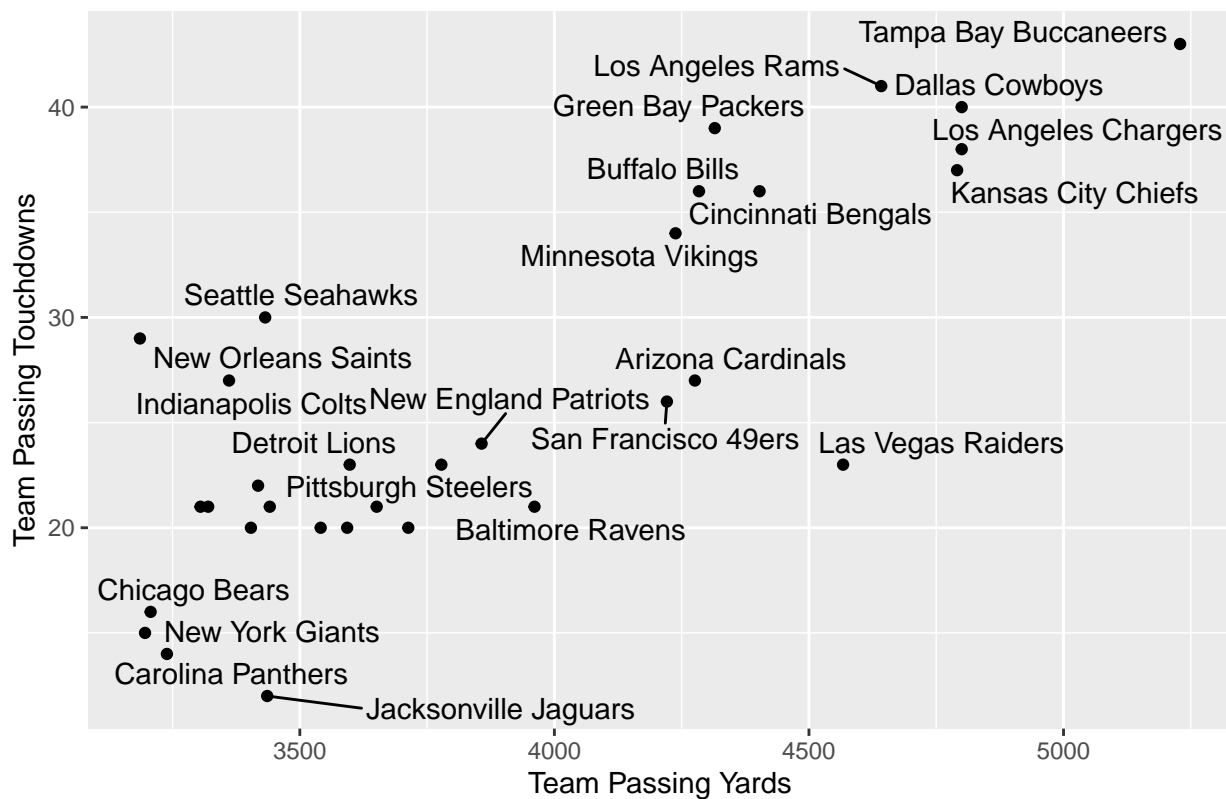
NFL Team Passing Yards, 2021
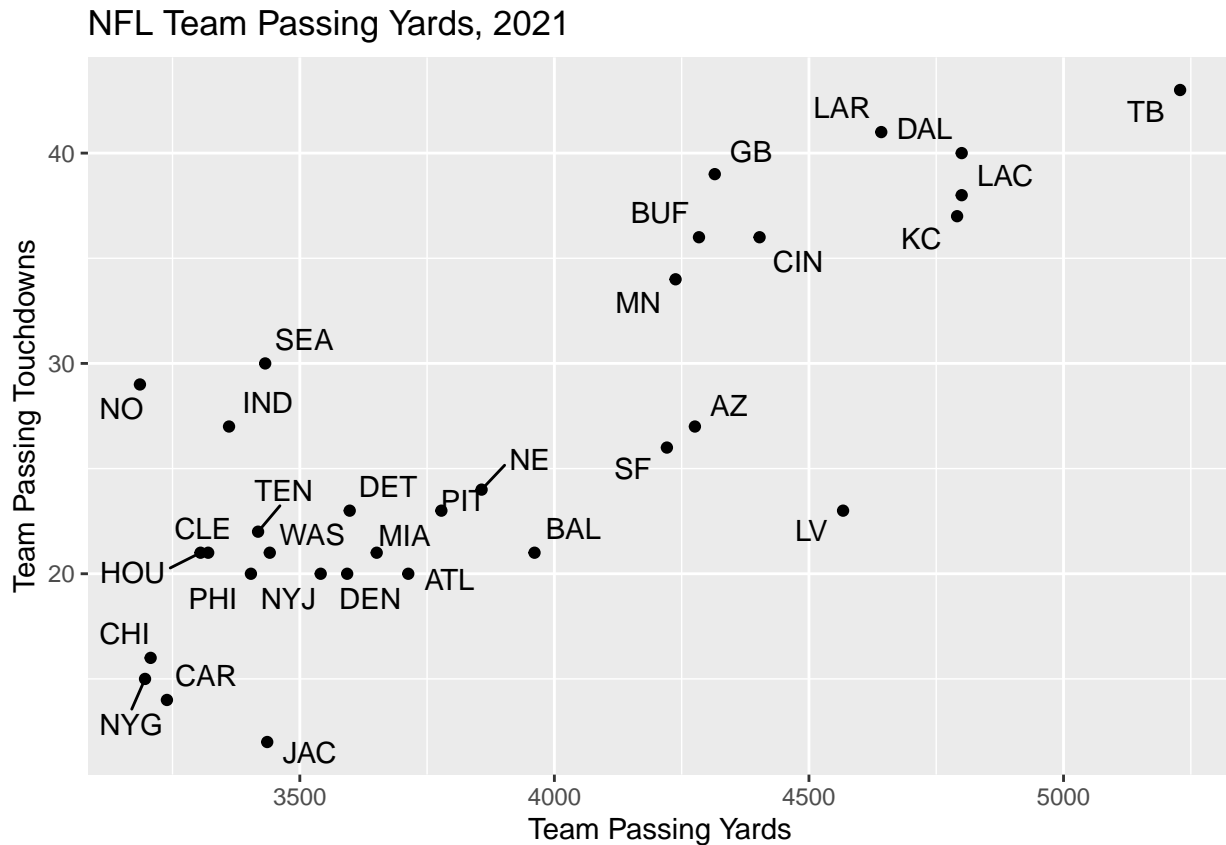
1.4 Baseball

1.5 Football

1.6 Basketball

1.7 Soccer

1.8 Volleyball

1.9 Hockey

For this example, we'll use a set of NHL data from moneypuck.com. First, let's load the data into
R and open the data frame.

```
nhl_2022_data <-
read_csv("https://moneypuck.com/moneypuck/playerData/seasonSummary/2021/regular/teams.csv")

head(nhl_2022_data)

## # A tibble: 6 x 107
```

name	situation	games_played	xGoalsPercentage	corsiPercentage
WPG	other	82	0.49	0.50
WPG	all	82	0.49	0.50
WPG	5on5	82	0.49	0.49
WPG	4on5	82	0.16	0.14
WPG	5on4	82	0.86	0.86
CBJ	other	82	0.52	0.49
CBJ	all	82	0.45	0.48
CBJ	5on5	82	0.45	0.48

```
##    team...1 season name  team...4 position    situation games_played
##    <chr>      <dbl> <chr> <chr>    <chr>        <chr>            <dbl>
## 1 WPG          2021 WPG   WPG      Team Level other              82
## 2 WPG          2021 WPG   WPG      Team Level all                82
## 3 WPG          2021 WPG   WPG      Team Level 5on5               82
## 4 WPG          2021 WPG   WPG      Team Level 4on5               82
## 5 WPG          2021 WPG   WPG      Team Level 5on4               82
## 6 CBJ          2021 CBJ   CBJ      Team Level other              82
## # ... with 100 more variables: xGoalsPercentage <dbl>, corsiPercentage <dbl>,
## #   fenwickPercentage <dbl>, iceTime <dbl>, xOnGoalFor <dbl>, xGoalsFor <dbl>,
## #   xReboundsFor <dbl>, xFreezeFor <dbl>, xPlayStoppedFor <dbl>,
## #   xPlayContinuedInZoneFor <dbl>, xPlayContinuedOutsideZoneFor <dbl>,
## #   flurryAdjustedxGoalsFor <dbl>, scoreVenueAdjustedxGoalsFor <dbl>,
## #   flurryScoreVenueAdjustedxGoalsFor <dbl>, shotsOnGoalFor <dbl>,
## #   missedShotsFor <dbl>, blockedShotAttemptsFor <dbl>, ...
```

We can create nice looking tables using the "kableExtra'' package. Let's look at the first eight rows and a samll selection of columns of the data frame and format the table output using a kable table.

```
library("kableExtra")
```

```
##
## Attaching package: 'kableExtra'

## The following object is masked from 'package:dplyr':
##
##     group_rows
```

```
nhl_2022_data[1:8, c(3, 6:9)] %>%
    kbl() %>%
    kable_styling()
```

This dataset includes a *lot* of covariates. It also splits these data by different game situations: even-strength (5 on 5), power play (5 on 4), etc. Let's subset the data to include all game situations.

Use the `nrow` command to check the number of columns in the new data frame. Check: Is it the same as the number of teams in the league for the 2021-2022 season?

```
nhl_data_all <- filter(nhl_2022_data, situation == "all")

nrow(nhl_data_all)
```

[1] 32

The dataset includes an Expected Goals statistic for each team in the `xGoalsFor` column. Let's plot this quantity against the team's actual number of goals scored; this is given by the `goalsFor` column.

(Remember to always have a good title and axis labels!)

```
ggplot(data = nhl_data_all, aes(x = xGoalsFor, y = goalsFor)) + labs(x =
"Expected Goals Scored",
    y = "Actual Goals Scored", title = "NHL Teams: Expected vs. Actual Goals,
    2021-22") +
    geom_point()
```
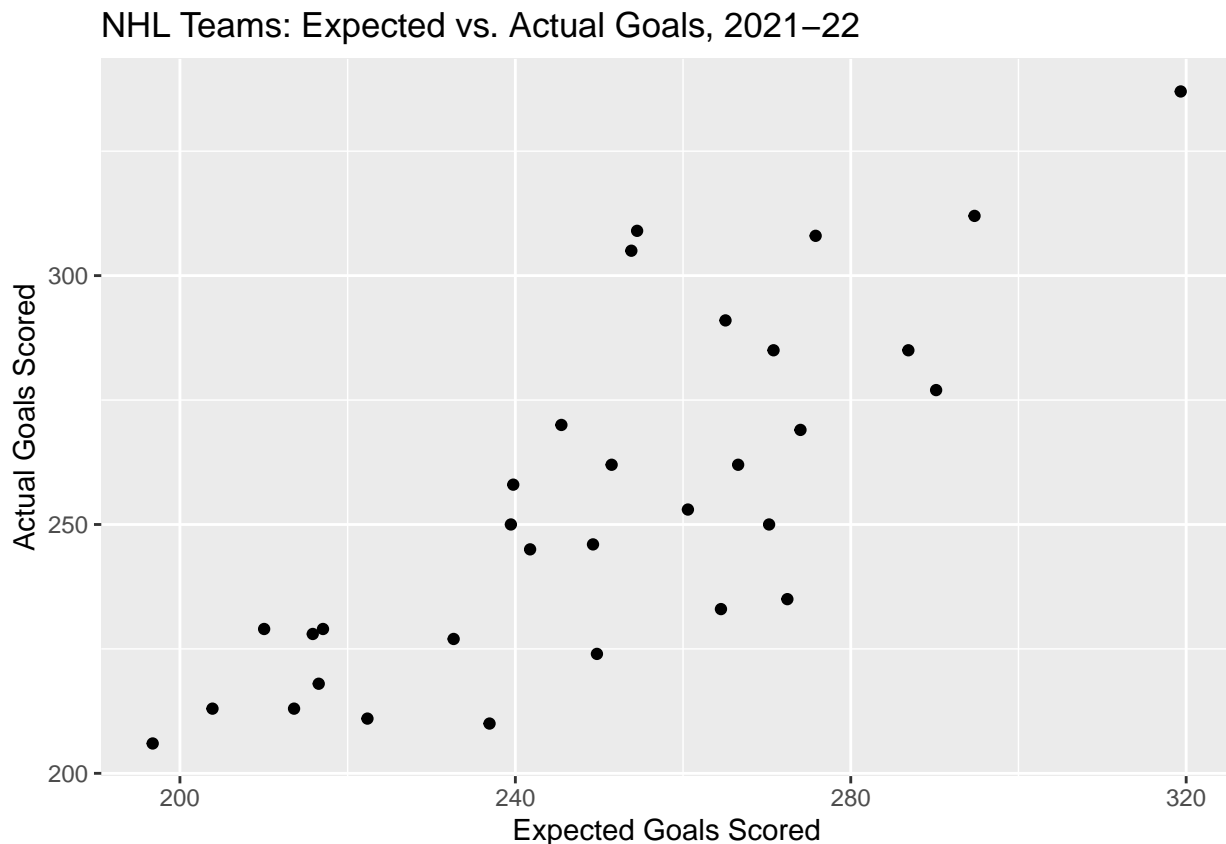

As expected, there is a general positive correlation between expected and actual goals ($r \approx 0.8$). However, there is some variability - for example, the Kings only scored 7 more actual goals than the Ducks, despite having 56.6 more expected goals.

Let's add a line to the graph using the `geom_abline` function corresponding to the line $y = x$, the line on which data points would fall if expected goals were equal to actual goals. We can also customize the line's color and type.

```
ggplot(data = nhl_data_all, aes(x = xGoalsFor, y = goalsFor)) + labs(x =
"Expected Goals Scored",
    y = "Actual Goals Scored", title = "NHL Teams: Expected vs. Actual Goals,
    2021-22") +
    geom_point() + geom_abline(intercept = 0, slope = 1, color = "red", linetype
    = "dashed")
```


NHL Teams: Expected vs. Actual Goals, 2021–22

Note: A slope of 0 and an intercept of 1 are actually the default parameters for the function.

Q: What does it mean for a team's data point to fall below this line? Above it?
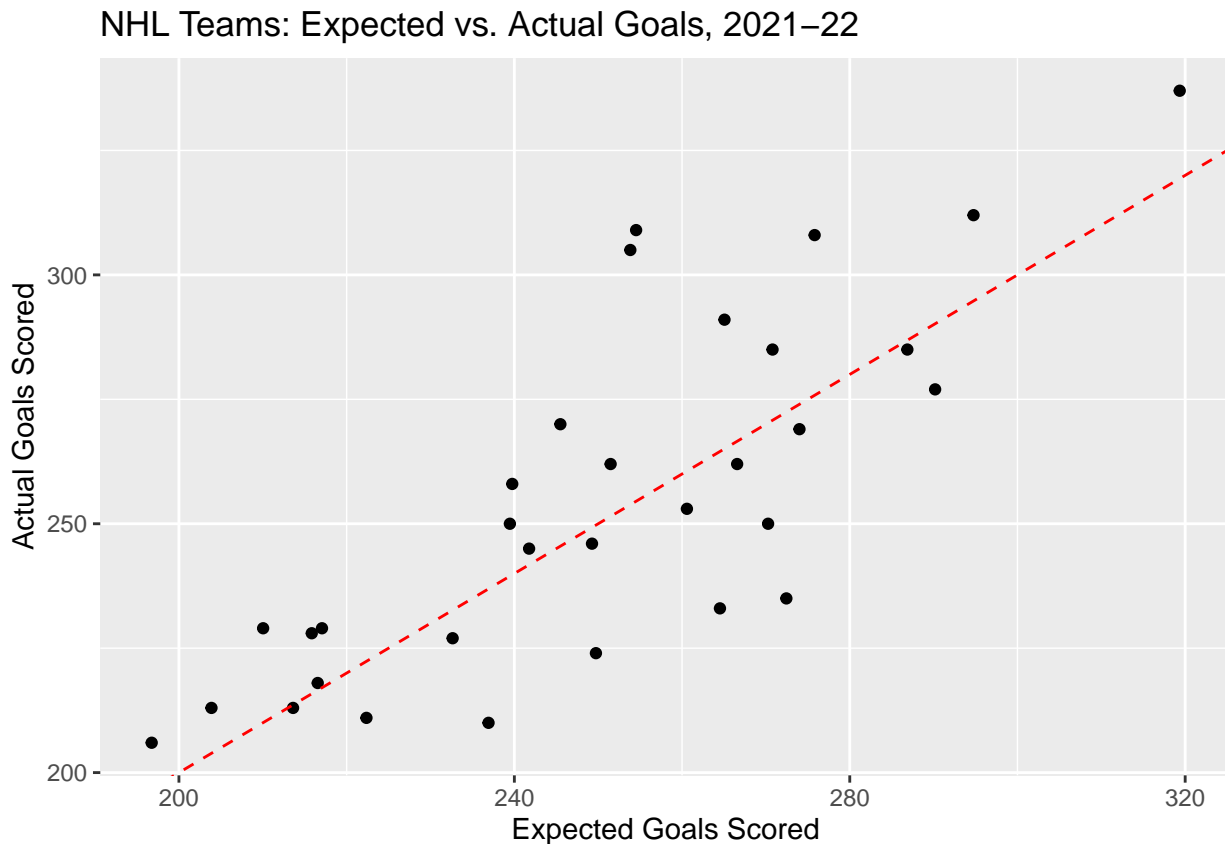
A: If the data point is below the line, it means the expected goals were greater than the actual goals; if the data point is above the line, it means the actual goals were greater than the expected goals.

Q: Do you think that a team's expected goals would be more likely to be closer to its actual goals for a ten-game stretch, an entire season, or five consecutive seasons? Why?

A: We would expect that as sample size increases, the result would become closer to expectation. So, actual goals would be most likely closer to expected goals over a span of five seasons.

Chapter 2

Probability

Chapter Preview

Simply put, probability is the study of randomness. In this chapter, we will define probability, learn rules of probability, and apply these rules to sports data.

2.1 Definitions

Definition 2.1. An ***experiment*** is any activity or process whose outcome is subject to uncertainty.

Definition 2.2. The ***sample space*** of an experiment, denoted by Ω or $\mathcal{S}$, is the set of all possible outcomes of that experiment.

Definition 2.3. An ***event*** is any collection (subset) of outcomes contained in the sample space, Ω.

Example 2.1.

Example 2.2.

2.2   Set Theory

For the following examples, suppose that we are interested in the batting outcomes of a plate appearance in softball.

Let A be the event that the batter gets walked, let B be the event that the batter gets a hit, let C be the event that the batter strikes out, and let D be the event that the batter makes it to first base at the end of their at bat.

We will define a handful of set operations to help us when we begin calculating the probability of different events occurring.

Definition 2.4. The ***compliment*** of an event A, denoted by A^c or A', is the set of all outcomes in Ω that are not contained in A.

Example 2.3. Draw a Venn diagram illustrating A^c and describe the event.

Definition 2.5. The ***union*** of two events A and B, denoted by $A \cup B$ and read "A or B", is the event consisting of all outcomes that are either in A or B or in both.

Example 2.4. Draw a Venn diagram illustrating $A \cup D$ and describe the event.

Definition 2.6. The ***intersection*** of two events A and B, denoted by $A \cap B$ and read "A and B", is the event consisting of all outcomes that are in both A and B.

Example 2.5. Draw a Venn diagram illustrating $A \cap D$ and describe the event.

Definition 2.7. The ***difference*** of two events A and B, denoted by $A \,/\, B$ and read "difference of A and B", is the event consisting of all outcomes that are in A but not in B.

Example 2.6. Draw a Venn diagram illustrating $D \,/\, A$ and describe the event.

Definition 2.8. Two events A and B are said to be ***disjoint*** (or ***mutually exclusive***) if $A \cap B = \emptyset$

Example 2.7. Are the events A and B disjoint? How about A and D?

2.3 Probability Axioms and Properties

There are some basic assumptions of "axioms" which are the foundation of the theory of probability. Andrey Kolmogorov first described these axioms in 1933.

2.3.1 Axioms of Probability

1. $P(A) \geq 0$, for any event A

2. $P(\Omega) = 1$

3. If $A_1, A_2, A_3, ...$ is a collection of disjoint events, then:
   $P(\cup_{i=1}^{\infty} A_i) = P(A_1 \cup A_2 \cup ...) = \sum_{i=1}^{\infty} P(A_i)$

Note that all probabilities are between 0 and 1, that is, for any event A, $0 \leq P(A) \leq 1$.

We can convert to percentages by multiplying probabilities by 100, however, this is a set that is only done after all calculations have been completed.

2.3.2 Properties of Probability

- $P(\emptyset) = 0$

- $P(A^c) = 1 - P(A)$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- $P(A \cup B \cup C) =$
  $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

- $P([A \cup B]^c) = P(A^c \cap B^c)$

- $P([A \cap B]^c) = P(A^c \cup B^c)$

2.4   Laws of Probability

Definition 2.9. Let A and B be two events such that $P(B) > 0$. Then the ***conditional probability*** of A given B, written $P(A|B)$, is given by: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Example 2.8. In 2001, Barry Bonds broke the single season home run record with 73 home runs. In this season, he had 664 plate appearances, 156 hits, 177 walks and 9 hit by pitches. Given that Bonds reached base (via hit, walk, or HBP), what was the probability that he got a hit?

Theorem 2.1 (Multiplication Rule). *For any two events A and B, $P(A \cap B) = P(B|A) \cdot P(A)$.*

Definition 2.10. Events $A_1, A_2, \ldots, A_n$ are said to form a ***partition*** of a sample space Ω if both:
(i) $A_i \cap A_j = \emptyset$ $(i \neq j)$
(ii) $\cup_{i=1}^n A_i = \Omega$

Theorem 2.2 (Law of Total Probability). *Suppose events $A_1, A_2, \ldots, A_n$ form a partition of Ω, then:* $P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots P(B|A_n)P(A_n)$

Theorem 2.3 (Bayes Theorem: simple version). *Suppose events B and C form a partition of Ω, then:* $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|C)P(C)}$

Theorem 2.4 (Bayes Theorem). *Suppose events $B_1, B_2, \ldots, B_n$ form a partition of Ω, then:* $P(B_k|A) = \frac{P(B_k \cap A)}{P(A)} = \frac{P(A|B_k)P(B_k)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)}$

2.5   Combinatorics

Combinatorics is the mathematical study of counting, particularly with respect to permutations and combinations.

Definition 2.11. The ***factorial function*** *(n!)* is defined for all positive integers by: $n! = n \cdot (n - 1) \cdot \ldots 2 \cdot 1$

Note that $0! \equiv 1$ and $1! \equiv 1$.

Definition 2.12. An ordered subset is called a ***permutation***. The number of permutations of size k that can be formed from the n elements in a set is given by: $P_{n,k} = \frac{n!}{(n-k)!}$

Definition 2.13. An unordered subset is called a ***combination***. The number of combinations of size k that can be formed from the n elements in a set is given by: $C_{n,k} = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$

Theorem 2.5 (Product Rule for Ordered Pairs). *If the first element of an ordered pair can be selected in n_1 ways and for each of the these n_1 ways the second element of the pair can be selected in n_2 ways, then the number of pairs is $n_1 \cdot n_2$.*

Theorem 2.6 (Generalized Product Rule). *Suppose a set consists of k elements (k-tuples) and that there are n_1 possible choices for the first element, n_2 possible choices for the second element, $\ldots$, and n_k possible choices for the kth element, then there are $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ possible k-tuples.*

Rockies wins, X	0.000	1.000	2.000	3.000	4.000
Probability, p(X)	0.015	0.111	0.311	0.384	0.179

2.6 Odds and Gambling

2.7 Random Variables

Definition 2.14. Let Ω be the sample space of an experiment. A ***random variable*** is a rule that associates a number with each outcome in Ω. In other words, a random variable is a function whose domain is Ω and whose range is the set of real numbers.

Random variables are be broken down into subcategories:
1. ***Discrete random variables*** - random variables which have a sample space that is finite or countably infinite.
2. ***Continuous random variables*** - random variables which have a sample space that is uncountably infinite (such as an interval of real numbers)

Discrete and ***Continuous*** random variables use similar yet slightly different mathematical tools. Discrete random variables involve working with "sums" and continuous random variables involve working with "integrals".

Example 2.9.

Example 2.10.

Definition 2.15. A ***probability distribution*** is a function that gives probabilities of different possible outcomes for a given experiment.

The probability distribution for a discrete random variable, $p(x)$, is called a ***probability mass function (pmf)***.

The probability distribution for a continuous random variable, $f(x)$, is called a ***probability density function (pdf)***.

Example 2.11. Suppose the Colorado Rockies are playing a four game series against the Chicago Cubs and that the Rockies have a 65% chance of winning an individual game. Further, assume that the games are independent. The following PMF describes the outcomes (number of Rockies wins) and their probabilities.

What is the probability that the Rockies win zero games? What is the probability that the Rockies win at least two games? Why might the independence assumption be false?

We may be interested in describing the center or average value of our random variable. We can do this with the following definitions.

Definition 2.16. The *expected value* (or *population mean* or *average*) of a random variable X is given by:

(i) $E[X] = \mu = \sum_{x \in \Omega} x \cdot p(x)$ (for discrete random variables)

(ii) $E[X] = \mu = \int_{x \in \Omega} x \cdot f(x) dx$ (for continuous random variables)

For this class, evaluating integrals is not essential, so we will avoid using Calculus (integrals and derivatives) when possible.

Sometimes, it makes sense to calculate the expected value of a function of a random variable. This can be easily done with a slight modification to the previous definition. Let $h(X)$ be some function of a random variable X. The expected value of $h(X)$, $E[h(X)]$, is given by:

(i) $E[h(X)] = \sum_{x \in \Omega} h(x) \cdot p(x)$ (for discrete random variables)

(ii) $E[h(X)] = \int_{x \in \Omega} h(x) \cdot f(x) dx$ (for continuous random variables)

Example 2.12. For the Rockies/Cubs four game series example, calculate $E[X]$ and $E[X^2]$.

The spread or variability associated with a random variable can be calculated using expected values as well.

Definition 2.17. The *population variance* of a random variable X is given by:

(i) $Var(X) = \sum_{x \in \Omega} (x - \mu)^2 \cdot p(x)$ (for discrete random variables)

(ii) $Var(X) = \int_{x \in \Omega} (x - \mu)^2 \cdot f(x) dx$ (for continuous random variables)

There is also a shortcut formula for calculating variance:

Theorem 2.7. $Var(X) = E[X^2] - (E[X])^2$

Definition 2.18. The ***population standard deviation*** of a random variable X is given by:

$$SD(X) = \sigma = \sqrt{Var(X)} = \sqrt{E[X^2] - (E[X])^2}$$

2.8 Some examples

Over the course of a season, a hockey player scored a goal 30% of the time during a home game, and $P(player\ scores \mid away\ game) = .18$. Assume all games are either home or away.

Q: What is the probability the player scored a goal in any game if there were an equal number of home and away games?

A: $P(score) = P(score|home)P(home) + P(score|away)P(away) = .3(.5) + .18(.5) = .24$

Q: What is the probability the player scored a goal in any game if there were twice as many home games as away games?

A: $P(score) = P(score|home)P(home) + P(score|away)P(away) = .3(\frac{2}{3}) + .18(\frac{1}{3}) = .26$

Q: What is the probability the player scored a goal in any game if the ratio of home games to away games is 2:3?

A: $P(score) = P(score|home)P(home) + P(score|away)P(away) = .3(\frac{2}{5}) + .18(\frac{3}{5}) = .228$

2.8.1 Sets and Conditional Probability

100 sports fans in Colorado were polled and it was found that 64 had attended either a Denver Nuggets or Colorado Avalanche game at Ball Arena (formerly Pepsi Center). 34 people had seen only a Nuggets game, while 17 had seen both a Nuggets and an Avalanche game.

Q: How many people saw an Avalanche game but not a Nuggets game?

A: 64 - 34 - 17 = 13

Q: What is the probability that a randomly selected person in the poll had been to a Nuggets game?

A: (34 + 17) / 100 = .51

Q: What is the probability that a randomly selected person that had been to a game at Ball Arena had been to a Nuggets game?

A: (34 + 17) / 64 = .797

Q: What is the probability that a randomly selected person had been to a Nuggets game given they had been to an Avalanche game?

A: 17 / (17 + 13) = .567

2.8.2 Binomial Probability

Two baseball teams are playing a 4-game series. The home team has a .65 probability of winning each game, and the away team a .35 probability. Assume each game is independent.

I used baseball in this example because it's the sport that most often has 4-game series, but it could easily be replaced by another sport.

Find the following probabilities.

(a) The road team wins exactly 1 game.

$\binom{4}{1} .65^3 .35^1 = \binom{4}{3} .65^3 .35^1 \approx .384$

```
dbinom(1, 4, 0.35)
```

```
## [1] 0.384475
```

```
dbinom(3, 4, 0.65)
```

```
## [1] 0.384475
```

(b) The home team wins exactly 2 games.

$\binom{4}{2} .65^2 .35^2 \approx .311$

```
dbinom(2, 4, 0.65)
```

```
## [1] 0.3105375
```

```
dbinom(2, 4, 0.35)
```

```
## [1] 0.3105375
```

(c) The road team wins at least 2 games.

$\binom{4}{2} .65^2 .35^2 + \binom{4}{3} .65^1 .35^3 + .35^4 = 1 - [.65^4 + \binom{4}{1} .65^3 .35^1] \approx .437$

```
pbinom(1.9, 4, 0.35, lower.tail = F)
```

```
## [1] 0.4370187
```

```
pbinom(2, 4, 0.65, lower.tail = T)
```

```
## [1] 0.4370187
```

(d) The series ends in a sweep.

$.65^4 + .35^4 \approx .194$

```
dbinom(4, 4, 0.65) + dbinom(4, 4, 0.35)
```

```
## [1] 0.1935125
```

```
0.65^4 + 0.35^4
```

```
## [1] 0.1935125
```

2.8.3 Binomial Coefficient Symmetry

Playoff series for a certain sports league are played as a best-of-seven series, with one team hosting four games and the opposing team hosing three. An executive for the league wishes to know the number of ways the home and away games can be assigned. (One such combination is A-A-B-B-A-B-A, the format used by the NBA and NHL for their best-of-seven series.) What is the total number of combinations?

Answer: Since there are a fixed number of games (seven) and a fixed number of games that must be given to the lower-seeded team (four), there are $\binom{7}{4} = \frac{7!}{4! \cdot (7-4)!} = 35$ ways to create a home-away pattern for the seven-game series.

However, instead of thinking about the number of ways to assign the games to the team that gets four home games, what if we thought about the number of ways to assign games to the team that gets three home games?

That would be $\binom{7}{3}$. We can use the `choose` command in R to find this quantity.

```
choose(7, 3)
```

```
## [1] 35
```

It turns out that this binomial coefficient is also equal to 35.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$

$\binom{n}{n-k} = \frac{n!}{(n-k)! \cdot (n-(n-k))!} = \frac{n!}{(n-k)! \cdot k!} = \binom{n}{k}$

2.8.4 Binomials and Multinomials

Suppose we are curious about probabilities regarding the results of a soccer team's next five games.

Wait!!! A soccer game has three possible outcomes (win, lose, draw)! We can't use the binomial distribution, since it limits us to two possible outcomes!

It depends. If we are interested in the probability that a soccer team wins 2 of their next 5 games, we can use the binomial distribution. We can create the following partition of the sample space of outcomes: (Win) and (Win^C), where the second set includes both losing and drawing.

Then, the formula would be represented as:

$\binom{5}{2} P(Win)^2 P(Win^C)^{(5-2)}$

If we are interested in the probability of the team winning two of the next five games, drawing two, and losing one, we cannot use the binomial theorem. That involves three outcomes, and would be represented as a multinomial.

2.8.5 Geometric (First Success) RVs

Caution: Some references parameterize the Geometric distribution based on the number of failures before the first success, rather than the trial on which the first success occurs. This changes the PMF, mean, and variance, so be careful.

```
set.seed(2022)
geometric <- rgeom(100, 1/3)
head(geometric, 20)
```

```
## [1]  2  5  1  3 12  7  1  4  2  2  1  1  1  2  0  0  0  4  3  0
```

Some of the values were 0, which could not happen if R was considering the number of the trial on which the first success occurred. You can add 1 to the values given by R to arrive at the First Success distribution.

```
first_success <- geometric + 1
head(first_success, 20)
```

```
## [1]  3  6  2  4 13  8  2  5  3  3  2  2  2  3  1  1  1  5  4  1
```

```
mean(first_success)
```

```
## [1] 3.03
```

The mean of this sample of variables is 3.03, which is close to the expected mean of $\frac{1}{p} = 3$.

2.8.6 Geometric Distribution - Hockey

Suppose the number of shots needed by a hockey team in order to score their first goal, X, is modeled by a Geometric($\frac{1}{10}$) random variable.

Q: What is the probability that it takes more than 3 shots to score the first goal?

A: $P(X > 3) = P(X = 4) + P(X = 5) + P(X = 6) + ...$

This is an infinite series, so let's use the Law of Total Probability.

$P(X > 3) = 1 - P(X \le 3) = 1 - [P(X = 1) + P(X = 2) + P(X = 3)] = 1 - [(\frac{1}{10}) + (\frac{9}{10})^1(\frac{1}{10}) + (\frac{9}{10})^2(\frac{1}{10})] = .729$

Chapter 3

Monte Carlo Simulation

Chapter 4

Statistical Inference

4.1   One Sample and Two Sample t-tests and confidence intervals

Chapter 5

Correlation

Chapter 6

Linear Regression

Chapter 7

Data Scraping

Chapter 8

Principal Component Analysis

Chapter 9

Clustering

Chapter 10

Classification

Chapter 11

Decision Trees

11.1   Random Forests

11.2   Gradient Boosting

Chapter 12

Non-parametric Statistics

Chapter 13

Baseball

Chapter 14

Football

Chapter 15

Basketball

Chapter 16

Soccer

Chapter 17

Hockey

Chapter 18

Volleyball

18.1   Resources

Women's Volleyball D1 Statistics

Chapter 19

Other Sports

Chapter 20

Aaron's stuff

20.1 Notes for Chapter 2 (Probability)

Axioms of Probability:

1. $P(A) \geq 0$

2. $P(\Omega) = 1$

3. If $A_1, A_2, \ldots, A_n$ are disjoint events, then $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

Theorem 20.1 (Bayes theorem). *Let A and B be events in Ω such that $P(B) > 0$. Then we have the following:*

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

20.2 Suggested Readings

20.2.1 Moneyball

Moneyball, Chapter 2, How to Find a Ballplayer [Lewis, 2004]

Near the end of the chapter (page 40), Michael Lewis give a list of players the Oakland Athletics hoped to draft. How did these players turn out? Find the WAR for each of the players in their pre-free agency years and compare it against the Rockies draft picks in the same rounds from the same draft.

20.2.2 Future Value

Future Value, Chapter 7, How to Scout [Longenhagen and McDaniel, 2020]

If a player receives a running grade of 40, approximately what proportion of MLB players have a lower have a lower running grade?

For a given tool, about 95% of all player grades fall between what two bounds? (Consider the middle 95% of the distribution of grades.)

## 20.3   Notes for Chapter 4 (Simulation)

20.3.1   Baseball Simulation Example

```
library(tidyverse)
```

This is a baseball example for chapter 4.

```
set.seed(2022)
n.sims <- 1000
hits <- rep(0, n.sims)
avg <- 0.3
atbats.mean <- 450
atbats.sd <- 100
sim.atbats <- round(rnorm(n.sims, atbats.mean, atbats.sd))

for (i in 1:n.sims) {
    sim.hits <- rbinom(1, sim.atbats[i], avg)
    hits[i] = sim.hits
}
hits.df <- data.frame(Hits = hits)
hits.df %>%
    ggplot(aes(x = Hits)) + geom_histogram()
```
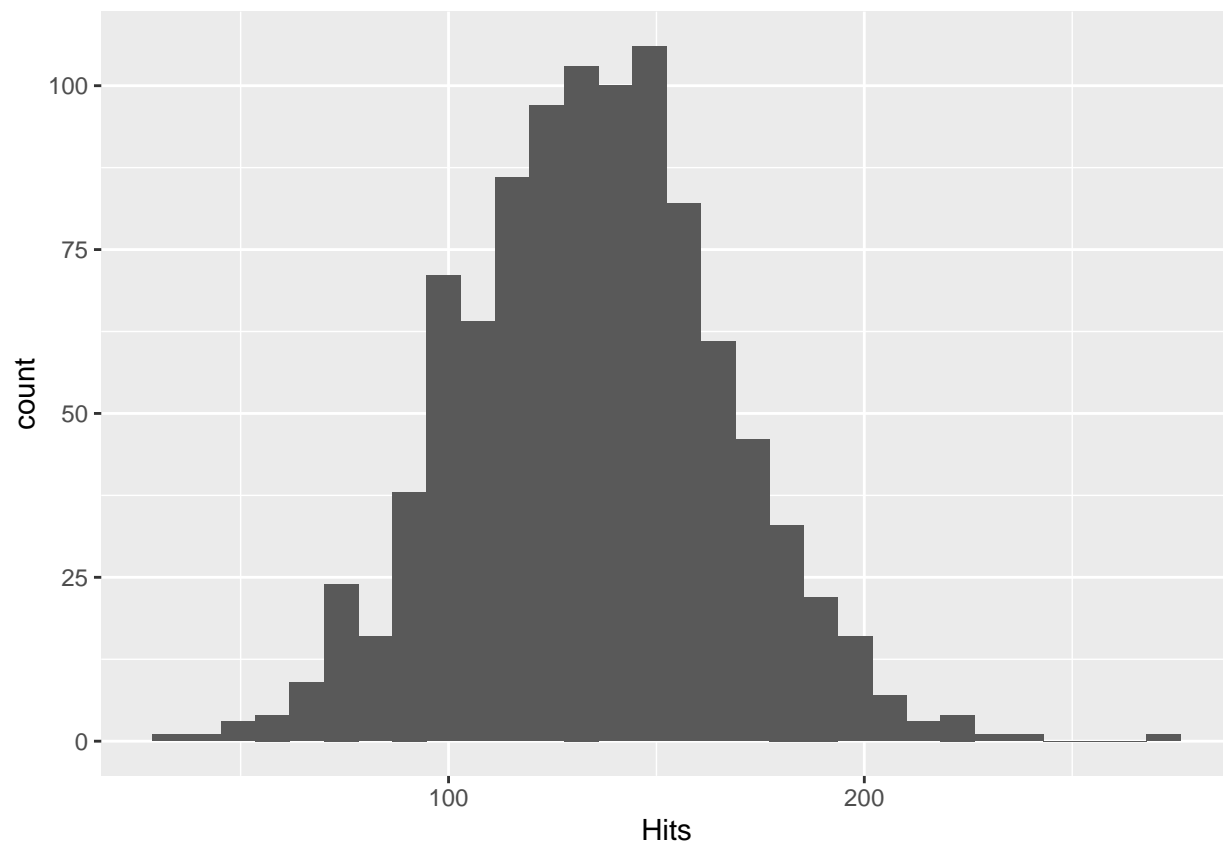
Bibliography

Michael Lewis. *Moneyball: The art of winning an unfair game.* WW Norton & Company, 2004.

Eric Longenhagen and Kiley McDaniel. *Future Value: The battle for baseball's soul and how teams will find the next superstar.* Triumph Books, 2020.