CHAPTER 49

OPTIMAL MONEY MANAGEMENT

The Kelley Growth Criterion

Suppose we believe we have an almost sure bet on Colts –12. We believe the Colts have a 90% chance of covering the spread. This would probably never happen, but humor us and assume that such a bet really exists! What fraction of our capital should we allocate to this bet? If we bet many times all our money on bets with a 90% chance of winning, eventually we will be wiped out when we first lose a bet. Therefore, no matter how good the odds, we must be conservative in determining the optimal fraction of our capital to bet.

Edward Kelley (1956) determined the optimal fraction of our capital to bet on a gamble. The story of Kelley's work is wonderfully told in William Poundstone's *Fortune's Formula* (2005). Kelley assumes our goal is to maximize the expected **long run** percentage growth of our portfolio, measured on a per gamble basis. We will soon see, for example, if we can pick 60% winners against the spread, on each bet we should bet 14.55% of our bankroll, and in the long run our capital will grow by an average of 1.8% per bet. Kelley's solution to determining the optimal bet fraction is as follows. Assume we start with \$1. Simply choose the fraction to invest

which maximizes the expected value of the natural logarithm of your bankroll after the bet. The file Kelley.xlsx contains an Excel Solver model to solve for the optimal bet fraction given the following parameters:

- WINMULT = The profit we make per \$1 bet on a winning bet.
- LOSEMULT = Our loss per \$1 bet on a losing bet.
- PROBWIN = Probability we win bet.
- PROBLOSE = Probability we lose bet.

For a typical football point spread bet, WINMULT = 1 and LOSEMULT = 1.1. For a Super Bowl money line bet on the Colts –240, WINMULT = 100/240 = .417 and LOSEMULT = 1. For a Super Bowl money line bet on the Bears +220, WINMULT = 220/100 = 2.2 and LOSEMULT = 1. Kelley tells us to maximize the expected value of the logarithm of our final asset position Given a probability p of winning the bet we should choose f = our fraction of capital to bet to maximize:

$$p \cdot \ln(1+\mathrm{WINMULT}*f)+(1-p)*\ln(1-\mathrm{LOSEMULT}*f)\ (1). \quad (1)$$

We find the optimal value for f by setting the derivative of the above expression to 0. This derivative is:

$$\frac{p*\mathrm{WINMULT}}{(1+\mathrm{WINMULT}*f)}-\frac{(1-p)*\mathrm{LOSEMULT}}{(1-\mathrm{LOSEMULT}*f)}.$$

and will be equal to 0 if:

$$f=\frac{p\mathrm{WINMULT}-(1-p)\mathrm{LOSEMULT}}{\mathrm{WINMULT}*\mathrm{LOSEMULT}}.$$

The numerator of the equation for f is our expected profit on a gamble per dollar bet (often called the "edge"). Our equation shows that the optimal bet fraction is a linear function of the probability of winning a bet, a really elegant result!

	M	N	P
14	Prob Win	Fraction	Expected growth per gamble
15	0.54	0.0309	0.053%
16	0.55	0.0500	0.138%
17	0.56	0.0691	0.264%
18	0.57	0.0882	0.430%
19	0.58	0.1073	0.639%
20	0.59	0.1264	0.889%
21	0.60	0.1455	1.181%
22	0.61	0.1645	1.516%
23	0.62	0.1836	1.896%
24	0.63	0.2027	2.320%
25	0.64	0.2218	2.790%
26	0.65	0.2409	3.307%
27	0.66	0.2600	3.873%
28	0.67	0.2791	4.488%
29	0.68	0.2982	5.154%
30	0.69	0.3173	5.873%
31	0.70	0.3364	6.647%
32	0.71	0.3555	7.478%
33	0.72	0.3745	8.368%
34	0.73	0.3936	9.319%
35	0.74	0.4127	10.335%
36	0.75	0.4318	11.418%
37	0.76	0.4509	12.572%
38	0.77	0.4700	13.800%
39	0.78	0.4891	15.106%
40	0.79	0.5082	16.496%
41	0.80	0.5273	17.973%
42	0.81	0.5464	19.544%
43	0.82	0.5655	21.214%
44	0.83	0.5845	22.991%
45	0.84	0.6036	24.883%
46	0.85	0.6227	26.898%
47	0.86	0.6418	29.047%
48	0.87	0.6609	31.342%
49	0.88	0.6800	33.795%
50	0.89	0.6991	36.424%
51	0.90	0.7182	39.246%

FIGURE 49.1 Kelley Growth Strategy and Average Growth Rate of Bankroll as Function of Win Probability.

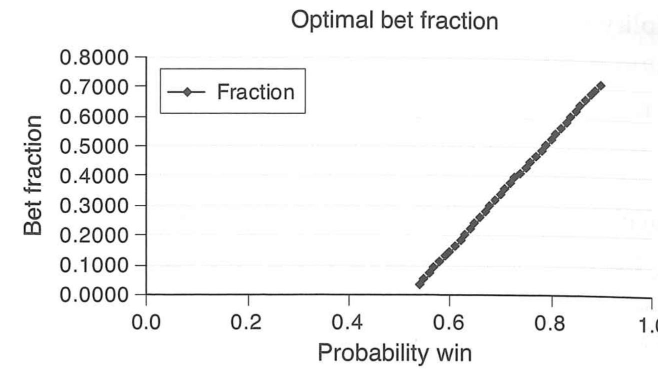

FIGURE 49.2 Optimal Bet Fraction as Function of Win Probability.

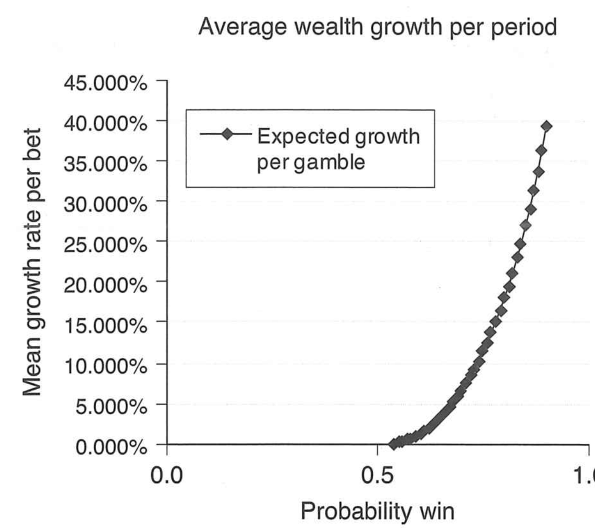

FIGURE 49.3 Average Wealth Growth per Period as a Function of Win Probability.

Kelley also showed that in the long run betting a fraction f of your bankroll each time leads to a long-term growth rate per gamble of $e^{\text{Expected LN final wealth}}$, where Expected Ln Final Wealth is given by equation (1).

Simplifying our expression for f we may rewrite our equation for f as:

$$f = \frac{p}{\text{LOSEMULT}} - \frac{q}{\text{WINMULT}}.$$

This equation makes it clear that an increase in the probability of a winning bet or an increase in WINMULT will increase our bet. Also, an increase in the probability of losing or an increase in LOSEMULT will decrease our bet.

As an example, let us compute our optimal bet fraction for an NFL point spread bet with a 60% chance of winning. We find that $f = \frac{.6(1) - .4(1.1)}{1(1.1)} = .145.$

Figure 49-1 summarizes the optimal bet fraction and expected percentage growth per gamble if we use the Kelley growth criterion.

Figures 49-2 and 49-3 summarize the dependence of the optimal bet fraction and expected long-term growth rate on our win probability (of the bet).

As stated earlier, the optimal bet fraction is a linear function of our (bet) win probability, but our average capital growth rate per gamble increases at a faster rate as our win probability increases.

	K	L
2	f	average growth rate
3	0.05	0.67%
4	0.10	1.06%
5	0.15	1.18%
6	0.20	1.01%
7	0.25	0.53%
8	0.30	–0.28%
9	0.35	–1.43%
10	0.40	–2.96%
11	0.45	–4.91%
12	0.50	–7.33%

FIGURE 49.4 Average Long-Term Growth Rate vs. Fraction Bet.

To show the importance of the optimal bet fraction, suppose that we can win 60% of our football point spread bets. Figure 49-4 shows how our long-term average growth rate per bet varies as a function of the fraction bet on each game.

Note that if we bet 30% or more of our money on each game, in the long run our capital will decline even though we win 60% of our bets!