Pythagorean Record

Baseball statistican (sabermetrican) Bill James proposed **Pythagorean expectation** to estimate the percentage (or number) of games that a team is expected to win based on the number of runs they score and the number of runs they allow.

Definition 6.6. The (basic) ***Pythagorean expectation*** for a team's win percentage and win total are given by the following:

$$\text{Pythagorean Win Pct} = PythWin\% = 100 * \frac{RS^2}{RS^2 + RA^2}$$

$$\text{Pythagorean Win Total} = PythWin = N \cdot \frac{RS^2}{RS^2 + RA^2}$$

where for a given team, N is the number of games, RS is the number of runs scored, and RA is the number of runs allowed.

Note that RS and RA can be based on season totals or per game averages.

Example 6.18. In 2022, the Colorado Rockies baseball team scored an average of 4.3 runs per game and allowed an average of 5.4 runs per game. The Rockies' record in 2022 was 69-94 (0.420 win pct). Calculate the Pythagorean win percentage and win total. Did the Rockies underperform or overperform based on these results?

$PythWin\% = 100 * \frac{RS^2}{RS^2+RA^2} = \frac{4.3^2}{4.3^2+5.4^2} = 38.8\%$

$PythWin\% = N \cdot \frac{RS^2}{RS^2+RA^2} = 162 * \frac{4.3^2}{4.3^2+5.4^2} = 63$

```
( pythwinpct = 100*(4.3^2)/(4.3^2+5.4^2) )
```

```
## [1] 38.80378
```

```
( pythwins = 162*(4.3^2)/(4.3^2+5.4^2) )
```

```
## [1] 62.86212
```

It turns out that we can find a more optimal estimate of Pythagorean wins by using an exponent different from 2.

For example, Baseball Reference (https://www.sports-reference.com/blog/baseball-reference-faqs/) uses an exponent of 1.83.

Definition 6.7. The (general) ***Pythagorean expectation*** for a team's win percentage and win total are given by the following:

$$\text{Pythagorean Win Pct} = PythWin\% = 100 * \frac{RS^n}{RS^2 + RA^n}$$

$$\text{Pythagorean Win Total} = PythWin = N \cdot \frac{RS^n}{RS^n + RA^n}$$

where for a given team, N is the number of games, RS is the number of runs scored, and RA is the number of runs allowed. n is optimized for predictive accuracy over a large dataset.

It will be helpful to have a function to find the optimal Pythagorean exponent. Such a function called `pyth_opt` is given below that minimizes mean squared error. Use `plot_flag=1` to generate a plot.

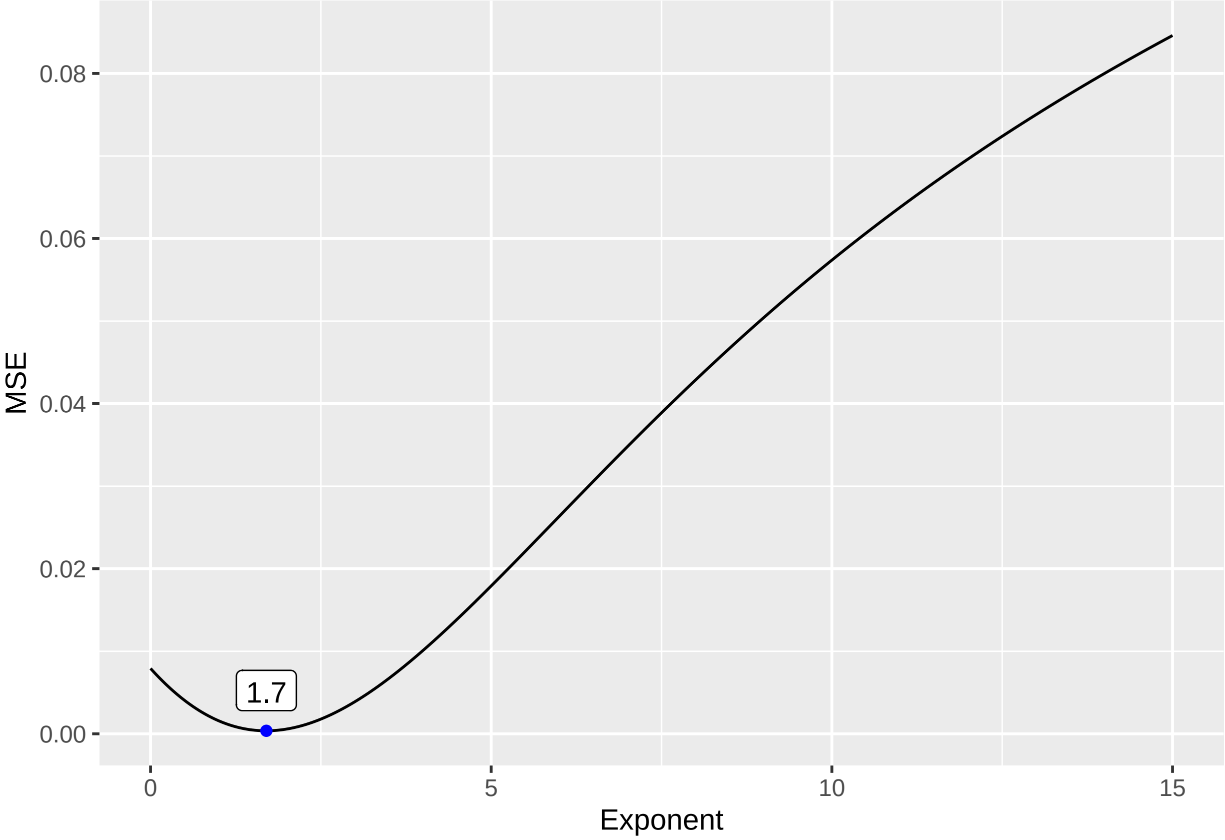
```r
pyth_opt = function(RS,RA,WinPct,digits,plot_flag){
  exps = seq(0,15,by=10^(-digits))
  n_exps = length(exps)
  MSE = rep(NA,n_exps)
  for(i in 1:n_exps){
    temp_exp = exps[i]
    PyWinPct = RS^temp_exp/(RS^temp_exp + RA^temp_exp)
    MSE[i] = mean((PyWinPct-WinPct)^2)
  }
  min_idx = which(MSE==min(MSE))
  pyth_opt = exps[min_idx]

  if(plot_flag){
    df = data.frame(Exponent=exps,MSE=MSE)
    df %>% ggplot(aes(x=Exponent,y=MSE)) +
      geom_line() +
      geom_point(data=df[min_idx,],color="blue")  +
      geom_label(data = df[min_idx,],
                 aes(x = Exponent, y = MSE, label = Exponent),vjust=-0.5)
  } else {
    return(pyth_opt)
  }

}
```

Example 6.19. Final season MLB standings and related statistics are given in `mlb_2022.csv`. Find the optimal value of n that minimizes mean squared error between actual wins and Pythagorean wins.

```
mlb_2022 = read_csv("data/mlb_2022.csv")
mlb_2022 %>% slice(1:10) %>% kable(booktabs=T,digits = 4)
```

Team	W	L	W-L%	R	RA	Rdiff	pythWL
Los Angeles Dodgers	111	51	0.685	5.2	3.2	2.1	116-46
Houston Astros	106	56	0.654	4.5	3.2	1.4	106-56
Atlanta Braves	101	61	0.623	4.9	3.8	1.1	100-62
New York Mets	101	61	0.623	4.8	3.7	1.0	99-63
New York Yankees	99	63	0.611	5.0	3.5	1.5	106-56
St. Louis Cardinals	93	69	0.574	4.8	3.9	0.8	95-67
Cleveland Guardians	92	70	0.568	4.3	3.9	0.4	88-74
Toronto Blue Jays	92	70	0.568	4.8	4.2	0.6	91-71
Seattle Mariners	90	72	0.556	4.3	3.8	0.4	89-73
San Diego Padres	89	73	0.549	4.4	4.1	0.3	86-76

```
pyth_opt(mlb_2022$R,mlb_2022$RA,mlb_2022$`W-L%`,2,1)
```



Example 6.20. For a more accurate estimate of the optimal Pythagorean exponent, use all MLB final standings data from 2000–2017. This is contained in `mlb_standings_long.csv`.

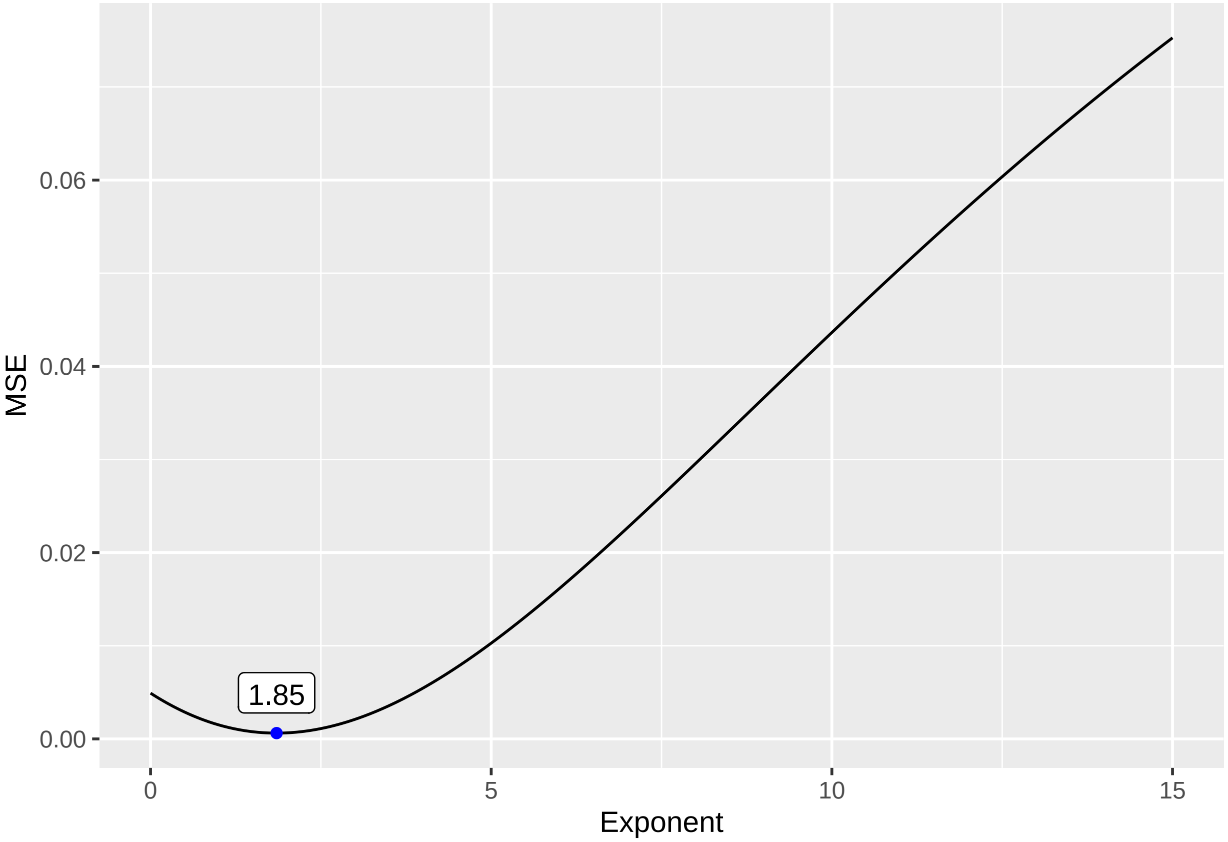
```
mlb_long = read_csv("data/mlb_standings_long.csv")
mlb_long %>% slice_head(n = 3) %>% kable(booktabs=T,digits = 4)
```

yearID	lgID	teamID	G	W	L	R	RA	RD	Wpct
2000	AL	ANA	162	82	80	864	869	-5	0.5062
2000	NL	ARI	162	85	77	792	754	38	0.5247
2000	NL	ATL	162	95	67	810	714	96	0.5864

```
mlb_long %>% slice_tail(n = 3) %>% kable(booktabs=T,digits = 4)
```

yearID	lgID	teamID	G	W	L	R	RA	RD	Wpct
2017	AL	TEX	162	78	84	799	816	-17	0.4815
2017	AL	TOR	162	76	86	693	784	-91	0.4691
2017	NL	WAS	162	97	65	819	672	147	0.5988

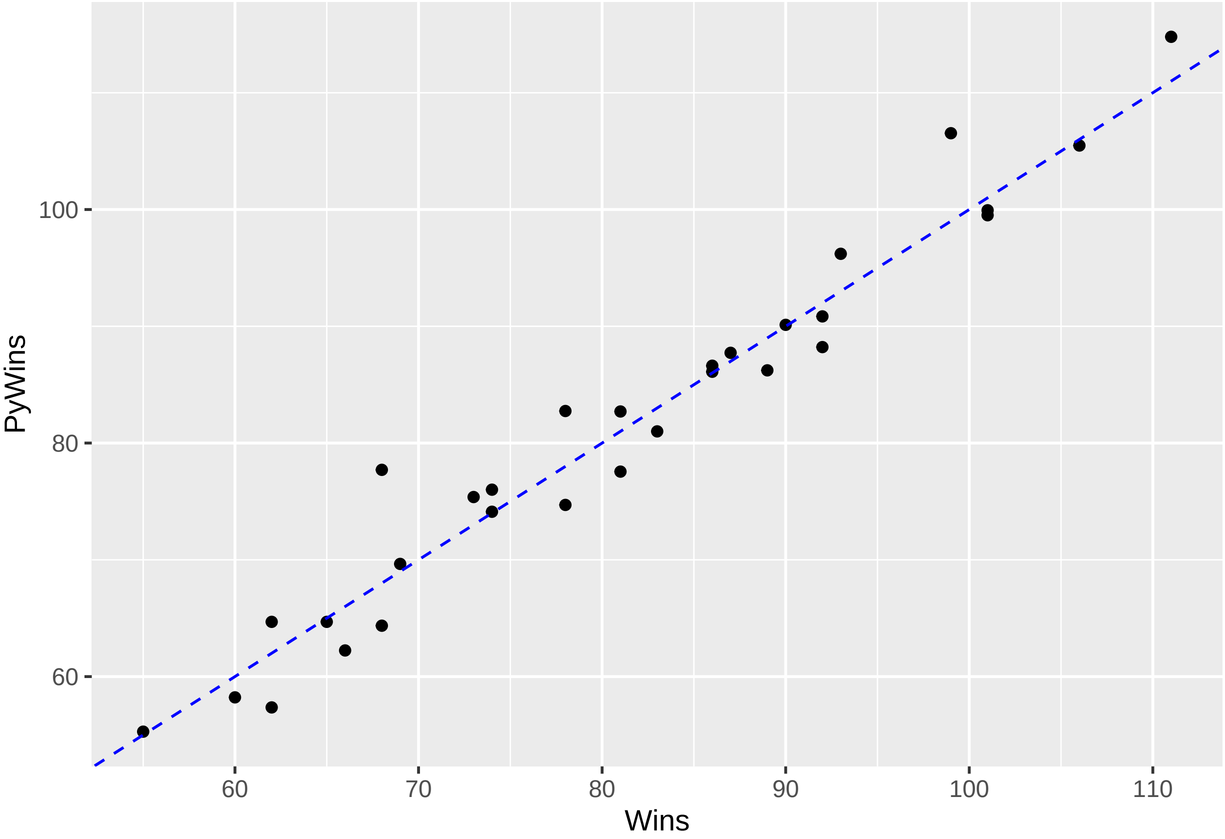
```
pyth_opt(mlb_long$R,mlb_long$RA,mlb_long$Wpct,2,1)
```



As previously mentioned, sabermetricans tend to use **PyExp = 1.83 for MLB**.

Example 6.21. Create a scatterplot to compare Team Wins and Team Pythagorean Wins in 2022 and calculate the correlation.

```
mlb_2022 = mlb_2022 %>% mutate(PyWins=162*R^1.83/(R^1.83+RA^1.83))
mlb_2022 %>% ggplot(aes(x=W,y=PyWins)) + geom_point() + labs(x="Wins") +
  geom_abline(intercept=0, slope=1, color="blue", linetype="dashed")
```



```
cor(mlb_2022$W,mlb_2022$PyWins)
```

```
## [1] 0.9764977
```

Example 6.22. The Rockies scored 4.31 runs per game and allowed 5.4 runs per game in 2022. Did the Rockies underperform or overperform based on their Pythagorean record?

```
mlb_2022 %>% filter(Team=="Colorado Rockies") %>%
  mutate(PyWins = 162*4.31^1.83/(4.31^1.83+5.39^1.83)) %>%
  kable(booktabs=T,digits = 4)
```

Team	W	L	W-L%	R	RA	Rdiff	pythWL	PyWins
Colorado Rockies	68	94	0.42	4.3	5.4	-1.1	65-97	64.6548

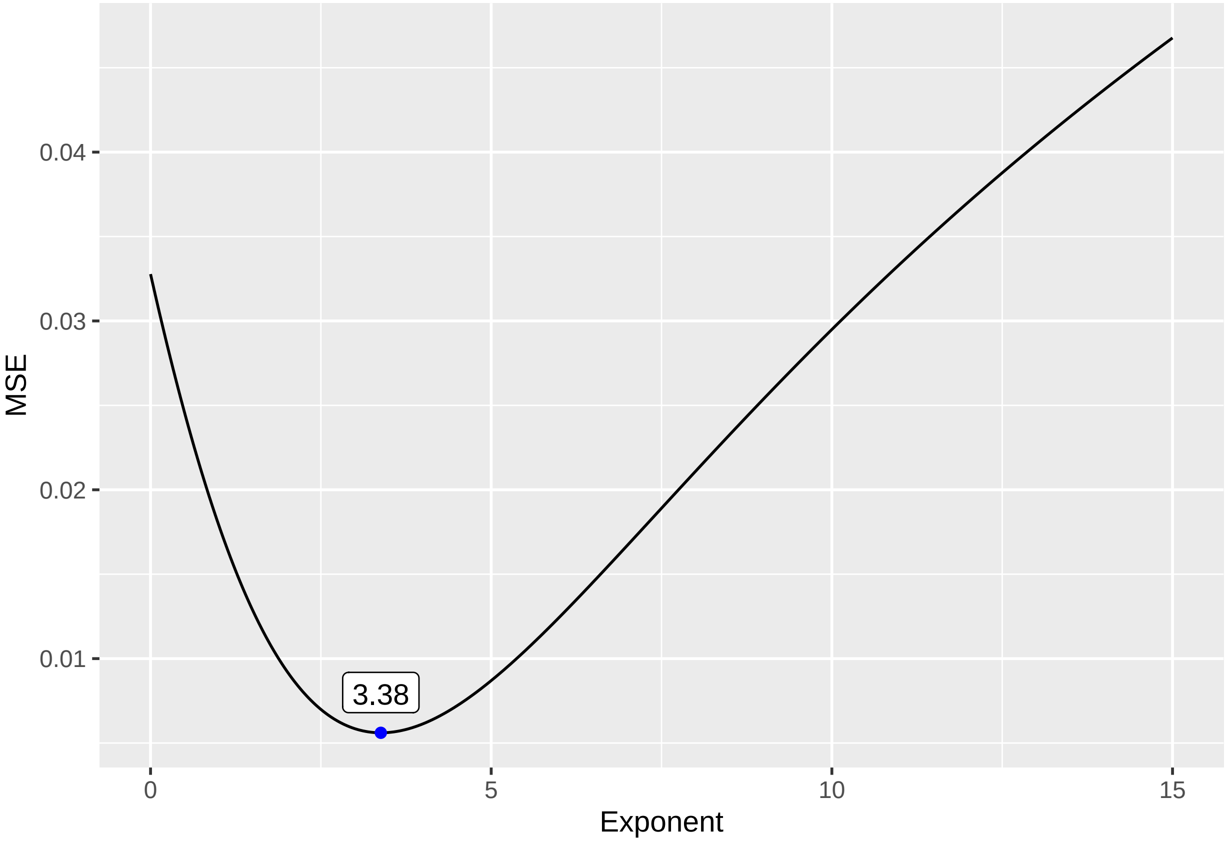
The Rockies won 68 games and were expected to win 65 games based on their Pythagorean record. The Rockies outperformed their Pythagorean record.

Example 6.23. Calculate the Pythagorean exponent for NFL using 2022 season totals. This data is contained in `nfl_2022.csv`.

```
nfl_2022 = read_csv("data/nfl_2022.csv")
nfl_2022 %>%
  slice_head(n=5) %>%
  kable(booktabs=T,digits = 4)
```

Team	W	L	T	W-L%	PF	PA
Buffalo Bills*	13	3	0	0.813	455	286
Miami Dolphins+	9	8	0	0.529	397	399
New England Patriots	8	9	0	0.471	364	347
New York Jets	7	10	0	0.412	296	316
Cincinnati Bengals*	12	4	0	0.750	418	322

```
pyth_opt(nfl_2022$PF,nfl_2022$PA,nfl_2022$`W-L%`,2,1)
```



For 2022, there is an optimal Pythagorean exponent of 3.38. Using a larger dataset of more seasons will give a better estimate.

Football Outsiders (https://www.footballoutsiders.com/stat-analysis/2017/presenting-adjusted-pythagorean-theorem) uses **PyExp = 2.37 for NFL**.

Similar analyses can be done for other sports as well. **PyExp = 13.91 is often used for NBA and PyExp = 2.15 for NHL.**