# Problem Set 8: Maximum Likelihood Estimation & Course Review
# Due: March 10, 2021

1. You want to compute the HIV prevalence in Botswana. You conduct a nationwide survey of 1000 people, and 380 of them have HIV.

   (a) What is your estimate of the national HIV prevalence rate?

   (b) What is the population this estimate applies to?

   (c) What is a 95% confidence interval for your estimate?

2. Suppose you randomly sample 200 college students and test them for herpes. In your sample, 31 of them tested positive.

   (a) What is your estimate of the herpes rate?

   (b) What is the population that your estimated herpes rate applies to?

   (c) Compute a 95% confidence interval for your estimate.

# REVIEW

3. Let X denote the number of times that a student takes his qualifying exams. Suppose that if the student fails, he retakes the test until he passes. Students are permitted a maximum of three tries. A student has a 90% chance of passing on each try. Write down the sample space, the probability distribution of X, E(X), and Var(X).

4. Let $f(x) = 2x$.

   (a) Compte $F(x) = \int f(x)dx$. Check by differentiating.

   (b) Plot $f(x)$ and $F(x)$.

   (c) Compute the area under $f(x)$ on the interval [0.5,2] using the formula:
   Area $= F(b) - F(a)$

   (d) Show this area on both plots.

5. $f(x) = 4x^2 + log(x)$

   $f'(x) =$

   $f''(x) =$

   $f''(x) =$

6. $f(x) = \frac{sin(x)}{log(x)}$

   $f'(x) =$

7. $f(x) = e^{(6+x^3)^5}$

   $f'(x) =$

8. $f(x) = \frac{1}{x^4} + 12$

   $\int f(x)dx =$

9. $f(x) = \frac{1}{x} + e^x + cos(x)$

   $\int f(x)dx =$

10. $f(x) = (3 + 2x)^4$

    $\int f(x)dx =$

11. Find the critical point(s) of the function: $f(x) = 6 \cdot log(x) + \frac{12}{x}$. Are they maximums or minimums?

12. Compute the area under the curve:

$$\int_2^5 (2x+3)^2 dx$$

13. Suppose I flip a coin twice. What is the probability of getting two heads?

14. Suppose I roll two dice. What is the probability the sum of the dice equals 5?

15. Find the determinants. Do the inverses exist?

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 \\ 5 & 12 \end{pmatrix} \quad C = \begin{pmatrix} 8 & 7 \\ -9 & 3 \end{pmatrix}$$