

Problem Set 6
CSSS 505/ SOC 512
Due: February 17, 2021

1. Let Y be a uniform random variable on the interval $[2,10]$.
 - (a) Compute the expected value ($E[Y]$).
 - (b) Compute the variance ($Var[Y]$).
2. A couple has 4 pets, let X denote the number of cats. Assume cats and dogs are assigned to families with the probability of receiving a cat being 0.3.
 - (a) Write down the probability distribution of X . Hint: start by writing down the sample space and count the number of ways each event could occur. Slide 10 from Lecture 6 should be helpful.
 - (b) Compute the expected value ($E[X]$).
 - (c) Compute the variance ($Var[X]$).
3. Toss a coin 4 times, let X denote the number of heads.

- (a) Write down the probability distribution of X . Hint: start by writing down the sample space and count the number of ways each event could occur. Slide 10 from Lecture 6 should be helpful.
- (b) Compute the expected value ($E[X]$).
- (c) Compute the variance ($Var[X]$).
4. John pays \$40 per year for towing insurance. He thinks the probability that he will need to have his car towed is 10% and the probability that he will need to have it towed more than once is zero. Without insurance the cost of towing is \$100, but the cost is zero if insured. Let X =John's expenses next year for towing and/or insurance.
- (a) If he buys insurance, what is the value of X ?
- (b) If he doesn't buy insurance, what two values can X take?
- (c) Find $E[X]$ for both (a) and (b). Should he buy the insurance?
5. Let X represent the number of jobs held during the past year for students at a school, and suppose X has the following probability distribution:

X -Value	0	1	2	3	4
$P(X = x_i)$	0.15	0.28	0.36	0.10	0.11

- (a) What is the probability that a randomly selected student has fewer than two jobs?

(b) Find $P(X > 0)$.

(c) Find $P(X > 2)$.

6. Suppose that $E[X] = 10$ and $Var[X] = 5$. Compute the following:

(a) $E[2X + 3]$

(b) $E[-X]$

(c) $Var\left[\frac{X}{n}\right]$

(d) $Var[0.1X]$

(e) $Var[10X + 3]$

7. Are the following valid probability distributions? If so, compute $E[X]$ and $Var[X]$:

Table 1: Part (a)

X -Value	0	1	2	3	4
$P(X = x_i)$	0.05	0.08	0.36	0.10	0.10

Table 2: Part (b)

X -Value	0	1	10
$P(X = x_i)$	0.77	0.20	0.03