

Starter Burr Set



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Figure 1. An assembled six-piece burr puzzle.

The object of a *six-piece burr puzzle* is to assemble six puzzle pieces into the interlocking shape shown Figure 1. While all six-piece burrs have this same basic assembled shape, they vary widely in difficulty and solution technique, depending on which selection of six pieces is involved.

The Starter Burr Set is a collection of 15 such pieces, out of which dozens of six-piece burrs can be formed, providing many hours of entertainment in a compact package. The 15 pieces have been intentionally selected so that several puzzles of theoretical or historical interest may be constructed, ranging in difficulty from easy to moderately difficult. An optional extension of 9 additional pieces further expands the range of puzzles to include some very unique and challenging designs.

The Starter Burr Set is intended for 3D printing, and 3D-printable models are distributed free of charge as part of the Printable Puzzle Project at puzzlehub.org. There is much more to discover: even the expanded set of 24 pieces barely scratches the surface of possibilities in this puzzle genre, and there are many options for exploring further, some of which are discussed below.

A First Burr Puzzle

Each piece in the Burr Starter Set is stamped on one end with its **Känel number**, a unique identifying number that distinguishes that piece. (Several Känel numbers are

visible in the picture in Figure 1.) If you're eager to dive right in to the puzzles, here's a simple example to get you started. In addition to being relatively easy, it's also believed to be the oldest known six-piece burr puzzle, dating to at least 1733! Find the pieces with the following Känel numbers and assemble them into the shape shown in Figure 1:

1 52 256 928 1024 1024

Overview of Burr Puzzles

The six-piece burr puzzle is one of the oldest and best-studied mechanical puzzle forms. Surviving examples date to at least the 18th century, and a systematic burr set (similar to the Starter Burr Set) appeared as early as 1926, in a *Popular Science* article by the puzzle expert Arthur Smith. Most examples published before around 1970 are **solid, notchable burrs**: “solid” because the interior space of the final assembly is completely filled (there are no internal voids); and “notchable” because each piece can be made from a single length of wood by carving out a series of straight notches.

In the 1970s, the pioneering work of Stewart Coffin—perhaps the greatest designer of interlocking puzzles of all time—hinted at the vastly greater possibilities available if one removes these constraints. An exhaustive computer analysis was undertaken in the 1980s and 1990s by Bill Cutler, a collaborator of Coffin's. Cutler determined that among the solid, notchable burrs, there are exactly 314 physically constructible assemblies—that is, 314 ways of putting together some combination of six notchable pieces into the solid shape of Figure 1. But if one considers *all* notchable assemblies, solid or not, then there are more than 26 million; and if one includes general pieces (not necessarily notchable), there are over 71 *billion*!¹

Now most of these 71 billion “puzzles” are of little interest: some are entirely trivial; others are simply unremarkable. To help bring order to this vast landscape, puzzle theorists have developed various ways of classifying burr puzzles by difficulty.

The most basic measure of a burr puzzle's difficulty is its **level**. This is the number of distinct movements necessary to remove the first piece from the assembled shape. Higher-level puzzles tend to be harder, since it is progressively more difficult to visualize how the pieces fit together.²

¹You might see the number 35 billion quoted elsewhere. It depends on how one treats mirror-image assemblies: if mirror images are counted as distinct, then there are around 71 billion total assemblies; if mirror images are treated as identical, then the number is around 35 billion.

²*Solid* burrs are always level 1, because if a piece is able to move at all, then it must be unobstructed and hence slide straight out. Six-piece burr assemblies with level greater than 5 are rare, and Cutler's analysis showed that level 12 is the absolute limit for six-piece burrs.

Another measure of difficulty is the number of **false assemblies** that a puzzle has. We've talked a lot about *assemblies* rather than *puzzles* or *sets of pieces*, and there's a subtle difference: some sets of pieces have more than one constructible assembly; or put another way, some puzzles have more than one solution. In fact, among the 314 solid, notchable assemblies found by Cutler, there are only 221 distinct sets of pieces, each having anywhere from 1 to as many as 6 solutions.

In addition to these *solutions*, a set of pieces may have one or more *false assemblies*. A false assembly is an arrangement of pieces that is theoretically possible, but is not physically achievable: the assembly could be formed if one could “teleport” the pieces into their respective locations; but there is no way to actually maneuver them into position in physical space without them obstructing one another. Puzzles with lots of false assemblies tend to be harder, because it is less obvious what the final assembled configuration should be.

In the puzzle tables that follow, each puzzle will be given with its level and number of assemblies. For example, puzzle #5 has 8 total assemblies, of which 6 are solutions (and the other 2 are false).

Starter Set: Solid, Notchable Burrs

#	Piece Selection						Level	Assemblies
1	1	52	256	928	1024	1024	1	1/1
According to Rob Stegmann, this one “[m]ay be the earliest known burr, depicted in a 1733 book by Pablo Minguet y Irol . . . [it] appeared as the Small Devil's Hoof in a 1785 catalogue.”								
2	1	256	824	928	975	1024	1	2/2
A popular design that has been made many times; Jürg von Känel called it The Well-Known One . Its two solutions are mirror images.								
3	1	188	256	960	975	1024	1	2/2
This design appeared in Louis Hoffmann's 1893 <i>Puzzles Old and New</i> .								
4	1	188	824	975	1024	1024	1	2/2
The Yamato Block Puzzle , of Japanese origin, believed to date to the 1920s.								

5	52	792	824	911	975	1024	1	6/8
This design has the largest number of distinct assemblies of any solid, notchable six-piece burr. Try to find them all!								

6	52	615	792	960	975	992	1	1/1
The celebrated Burr No. 305 , so named because it was number 305 in Cutler's original list of the 314 solid, notchable assemblies. It is traditionally regarded as one of the two most difficult solid, notchable puzzles (the other, Burr No. 306, can be made with the extended piece set; see puzzle #41 on page 6).								

Here are all the remaining solid, notchable burrs that can be constructed with the basic set. Some of them have "mirror images" that can also be constructed; these have been omitted from the table, so that each puzzle on this list is distinct.								
7	1	52	256	960	992	1024	1	3/3
8	1	52	960	975	1024	1024	1	3/3
9	1	103	256	928	1024	1024	1	1/1
10	1	103	256	960	992	1024	1	3/3
11	1	103	960	975	1024	1024	1	3/3
12	1	188	256	911	1024	1024	1	1/1
13	1	256	792	960	975	1024	1	2/2
14	1	256	824	960	975	992	1	2/2
15	1	256	911	960	975	1024	1	1/1
16	1	615	928	975	1024	1024	1	1/1
17	1	615	960	975	992	1024	1	2/2
18	1	824	911	975	1024	1024	1	1/1
19	52	103	824	928	975	1024	1	2/2
20	52	103	824	960	975	992	1	2/2
21	52	256	824	911	928	975	1	2/2
22	52	615	792	928	975	1024	1	1/1

Starter Set: Some Higher-Level Burrs

The Burr Starter Set can also be used to construct numerous assemblies of level 2 through 5. Here's a sampling of some interesting ones, though it's certainly not an exhaustive list.

23	615	792	824	911	992	1024	2	2/2
A relatively straightforward Level 2 puzzle with no false assemblies.								

24	103	188	615	824	1024	1024	2	1/6
25	103	256	911	928	975	1024	2	1/9
26	103	188	256	911	992	1024	3	1/7
27	103	188	911	960	992	1024	3	1/18
With unique solutions and multiple false assemblies, these Level 2 and 3 puzzles step up the difficulty substantially.								

28	103	188	256	928	975	1024	5	1/7
29	188	256	615	928	975	1024	5	2/16
These are the highest-level assemblies constructible with the Starter Set. (One can also construct the mirror image of #28 by using piece 824 in place of 975.) One of the two solutions to puzzle #29 derived from #28, but the other is distinct.								

30	103	188	615	960	992	1024	2	8/9
Unusually, <i>all but one</i> of this burr's assemblies are constructible. An interesting puzzle is to find the unique false assembly.								

Numerous additional Level 2 burrs can be constructed with the Starter Set. Here are some selected puzzles with unique solutions.								
31	103	188	615	975	1024	1024	2	1/5
32	103	256	615	792	1024	1024	2	1/6
33	103	256	615	824	928	1024	2	1/7

34	103	256	615	824	960	992	2	1/7
35	103	256	615	911	1024	1024	2	1/5
36	103	256	615	928	975	1024	2	1/5
37	103	256	615	960	975	992	2	1/5
38	103	615	792	824	1024	1024	2	1/9
39	103	615	824	928	960	1024	2	1/6
40	103	615	911	975	1024	1024	2	1/5

Expanded Set

The Starter Burr Set Expansion adds an additional nine pieces, greatly expanding the possibilities, particularly for higher-level burrs.

41	52	615	871	960	975	992	1	1/1
Burr No. 306 , named for its position in Cutler's 314; traditionally one of the two hardest solid, notchable puzzles (see also Burr No. 305, puzzle #6 on page 4).								

42	256	824	911	928	943	1024	4	1/3
43	256	792	911	943	960	1024	4	1/4
44	256	824	911	943	960	1024	4	1/4
45	871	911	943	960	1007	1024	4	1/8

A series of notchable Level 4 puzzles discovered by Bruno Curfs, in order of increasing difficulty (according to Curfs).

46	359	871	928	943	1007	1024	3	1/37
Bruno Curfs's Monster . According to Stegmann, this is perhaps the most difficult burr made with notchable pieces. 36 false assemblies.								

47	103	760	960	992	996	1024	5	1/24
Bill's Baffling Burr , found by Cutler. This one might be even more difficult, but it requires the non-notchable piece 760.								

48	359	615	943	960	992	1024	2	4/156
Numerous burrs can be found with over 150 total assemblies. This one is unusual in that only four of its assemblies are constructible.								

49	369	871	956	960	992	1000	6	1/5
50	824	928	992	996	1000	1024	6	1/20
51	369	871	928	956	1000	1024	8	1/5
These are the only burrs above Level 5 that are constructible with the expanded set. The Level 8 burr was discovered by Bill Cutler and published as Computer's Choice 4-Hole . Six-piece burrs at this level are rare and often call for exotic pieces—in this case, the rarely encountered piece 369.								

52	103	615	928	956	1024	1024	4	1/51
53	359	615	928	956	992	1024	4	1/80
54	615	928	996	1000	1024	1024	4	1/33
Some burrs have a unique solution and many false assemblies!								

Numerous additional solid, notchable burrs can be constructed with the expansion.								
55	1	52	928	1007	1024	1024	1	1/1
56	1	52	960	992	1007	1024	1	2/2
57	1	103	928	1007	1024	1024	1	1/1
58	1	103	960	992	1007	1024	1	2/2
59	1	188	256	928	1007	1024	1	1/1
60	1	188	256	960	992	1007	1	1/1
61	1	188	824	992	1007	1024	1	1/1
62	1	188	871	928	1024	1024	1	1/1
63	1	188	871	960	992	1024	1	3/3
64	1	188	911	1007	1024	1024	1	1/1
65	1	188	960	975	1007	1024	1	1/1

66	1	256	792	928	1007	1024	1	1/1
67	1	256	792	960	992	1007	1	1/1
68	1	256	824	928	992	1007	1	2/2
69	1	359	824	928	1024	1024	1	1/1
70	1	615	928	992	1007	1024	1	2/2
71	1	792	824	992	1007	1024	1	2/2
72	1	824	911	992	1007	1024	1	1/1
73	1	824	928	975	1007	1024	1	1/1
74	1	824	960	975	992	1007	1	1/1
75	1	871	943	975	1024	1024	1	1/1
76	1	871	943	992	1007	1024	1	1/1
77	1	871	960	975	1007	1024	1	2/2
78	52	103	824	928	992	1007	1	1/2
79	52	103	871	928	960	992	1	1/3
80	52	188	824	928	975	1007	1	1/1
81	52	359	824	928	943	992	1	1/2
82	52	615	871	928	975	1024	1	1/1

Finally, here's a batch of selected Level 5 puzzles with unique solutions.								
83	103	760	928	996	1024	1024	5	1/12
84	188	760	928	996	1007	1024	5	1/6
85	188	760	960	992	996	1007	5	1/6
86	188	824	960	992	996	1007	5	1/5
87	256	760	824	871	928	960	5	1/6
88	359	928	956	996	1000	1024	5	1/9
89	359	956	960	992	996	1000	5	1/9
90	369	824	956	960	975	992	5	1/8

Further Explorations

The world of burr puzzles is staggeringly vast. Fortunately, there are several excellent references for exploring further. Rob Stegmann's *Puzzle Pages* contain a great deal of background material on six-piece burrs and a host of related puzzles:

- <http://robspuzzlepage.com/interlocking.htm>

Stegmann's pages were an essential reference in creating this pamphlet, and they're an easy recommendation for further reading. Most of the six-piece burrs mentioned by Stegmann cannot be made with the Starter Burr Set—but if you're not yet burned out on six-piece burrs, the much larger *Extensible Burr Set* contains essentially every notable standard burr piece (as well code for generating models of any less notable ones). It is 3D-printable and fully compatible with the Starter Burr Set:

- <https://www.thingiverse.com/thing:3370714>

With it you can construct any six-piece burr mentioned on Stegmann's site, including notable puzzles such as Coffin's *Interrupted Slide* and Marineau's *Piston Burr*.

Going beyond six-piece burrs, the possibilities are seemingly endless. There isn't space for more than a cursory mention of general interlocking puzzles here, but you can find some examples ready-made for 3D printing at the *Printable Puzzle Project*:

- <https://www.puzzlehub.org/ppp>

To really take the plunge, you can poke around Keiichiro Ishino's site *Puzzle Will Be Played*, which catalogues over 5,000 (!) distinct designs at the time of this writing.

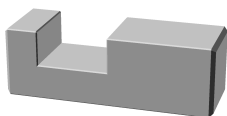
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Acknowledgement

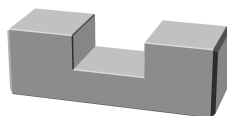
I wish to thank Rob Stegmann for providing a number of helpful suggestions and corrections to an earlier draft of this booklet.



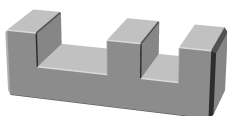
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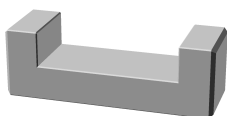
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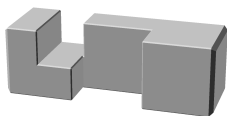
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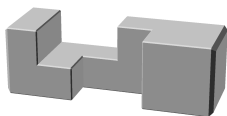
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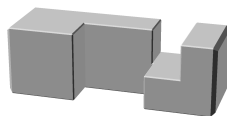
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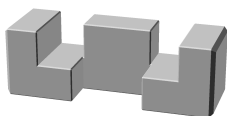
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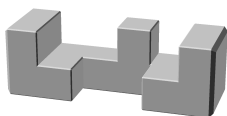
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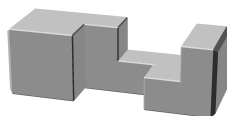
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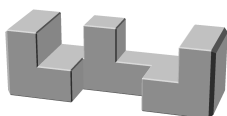
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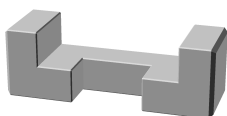
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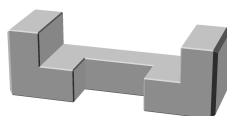
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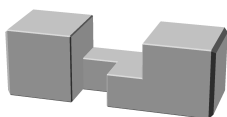
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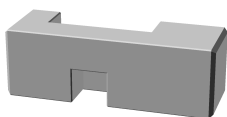
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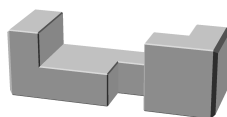
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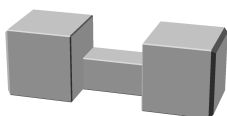
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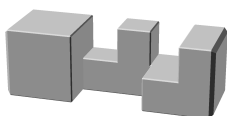
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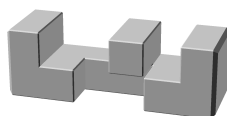
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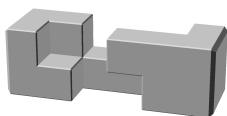
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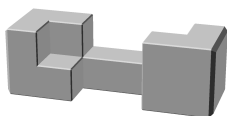
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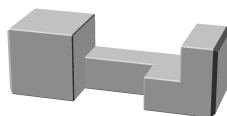
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