

Sample	Value
$(x_1, y_1)$	$(2, 4)$
$(x_2, y_2)$	$(3, 2)$
$(x_3, y_3)$	$(5, 3)$

$$\hat{y} = X \hat{\beta}$$

$\nwarrow$  predict      fit  $\swarrow$

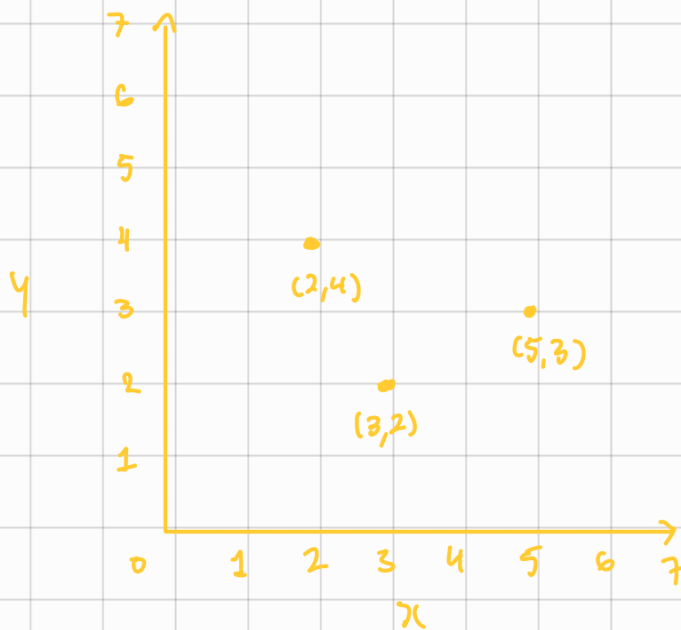
1)  $\hat{\beta} = ?$

RSS = ? ; i.e.  $\hat{y}_i = \hat{\beta}_0$

2)  $\hat{\beta}_0 = ?$

$\hat{\beta}_1 = ?$

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$



$$\hat{\beta} \rightarrow$$

$$X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} \quad (X^T \cdot X) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \end{bmatrix} \quad y = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \hat{\beta} = (X^T \cdot X)^{-1} \cdot (X^T \cdot y)$$

$$= \left( \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} \right)^{-1} \cdot \left( \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} \right)$$

$$\hat{\beta} = \begin{bmatrix} \frac{26}{7} \\ -\frac{3}{14} \end{bmatrix}$$

$$\hat{\beta}_0 = \frac{26}{7}, \quad \hat{\beta}_1 = \frac{-3}{14}$$

$$RSS = (y - \hat{y})^T \cdot (y - \hat{y})$$

$$\hat{y} = X \hat{\beta}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 26/7 \\ -3/14 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} 23/7 \\ 43/14 \\ 37/14 \end{bmatrix}$$

$$RSS = (y - \hat{y})^T \cdot (y - \hat{y})$$

$$= \left( \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 23/7 \\ 43/14 \\ 37/14 \end{bmatrix} \right)^T \cdot \left( \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 23/7 \\ 43/14 \\ 37/14 \end{bmatrix} \right)$$

$$= \begin{bmatrix} \frac{25}{14} \end{bmatrix}$$

$$RSS = [1.786]$$

$$2) \hat{\beta}_0 = \frac{26}{7}, \hat{\beta}_1 = \frac{-3}{14}$$

$$X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{y}_1 = \frac{26}{7} + \left( \frac{-3}{14} \cdot 2 \right)$$

$$\hat{y}_1 = 3.286$$