## **Principal Component Analysis**

Consider a trivial matrix X of dimension 5 x 4 consisting of the following five (5) samples:

| Sample                | Input Vector  |
|-----------------------|---------------|
| <i>x</i> <sub>1</sub> | (0, 4, 2, 2)  |
| X2                    | (2, 4, 2, 0)  |
| <i>x</i> <sub>3</sub> | (2, 3, 3, 0)  |
| <i>x</i> <sub>4</sub> | (1, 4, 2, 1)  |
| x5                    | (-1, 4, 2, 0) |

The eigen decomposition of XTX, assuming X has been centered, returned the following V and D matrices:

$$V = \begin{bmatrix} -0.22 & 0 & -0.9 & 0.37 \\ -0.67 & -0.71 & 0.2 & 0.09 \\ 0.67 & -0.71 & -0.2 & -0.09 \\ 0.22 & 0 & 0.32 & 0.92 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7.85 & 0 \\ 0 & 0 & 0 & 2.75 \end{bmatrix}$$

Step 1: Calculate the mean of each column in X.

| x1         | 0   | 4   | 2   | 2   |
|------------|-----|-----|-----|-----|
| x2         | 2   | 4   | 2   | 0   |
| <b>x</b> 3 | 2   | 3   | 3   | 0   |
| x4         | 1   | 4   | 2   | 1   |
| х5         | -1  | 4   | 2   | 0   |
| Mean       | 0.8 | 3.8 | 2.2 | 0.6 |

Step 2: Subtract mean from each cell in the column.

| x1 | -0.8 | 0.2  | -0.2 | 1.4  |
|----|------|------|------|------|
| x2 | 1.2  | 0.2  | -0.2 | -0.6 |
| х3 | 1.2  | -0.8 | 0.8  | -0.6 |
| х4 | 0.8  | 0.2  | -0.2 | 0.4  |
| x5 | -1.8 | 0.2  | -0.2 | -0.6 |

## **Step 3: Find the 2 largest values in D:**

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7.85 & 0 \\ 0 & 0 & 0 & 2.75 \end{bmatrix}$$

Largest values: 7.85, 2.75

Step 4: Grab the corresponding columns from V.

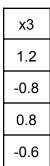
| -0.9 | 0.37  |
|------|-------|
| 0.2  | 0.09  |
| -0.2 | -0.09 |
| 0.32 | 0.92  |

**Step 5: Transpose the columns from V:** 

| -0.9 | 0.2  | -0.2  | 0.32 |
|------|------|-------|------|
| 0.37 | 0.09 | -0.09 | 0.92 |

Step 6: Grab x1 and x3.

| x1   |  |
|------|--|
| -0.8 |  |
| 0.2  |  |
| -0.2 |  |
| 1.4  |  |



Step 7: Matrix Multiply Step 5 with each of x1 and x3

| Z1 | 1.248  |
|----|--------|
|    | 1.028  |
| Z3 | -1.592 |
|    | -0.252 |