Principal Component Analysis

Consider a trivial matrix X of dimension 5 x 4 consisting of the following five (5) samples:

Sample	Input Vector
x_1	(0, 4, 2, 2)
x_2	(2, 4, 2, 0)
x_3	(2, 3, 3, 0)
x_4	(1, 4, 2, 1)
x_5	(-1, 4, 2, 0)

The eigen decomposition of XTX, assuming X has been centered, returned the following V and D matrices:

$$V = \begin{bmatrix} -0.22 & 0 & -0.9 & 0.37 \\ -0.67 & -0.71 & 0.2 & 0.09 \\ 0.67 & -0.71 & -0.2 & -0.09 \\ 0.22 & 0 & 0.32 & 0.92 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7.85 & 0 \\ 0 & 0 & 0 & 2.75 \end{bmatrix}$$

Step 1: Calculate the mean of each column in X.

x1	0	4	2	2
x2	2	4	2	0
x3	2	3	3	0
x4	1	4	2	1
x5	-1	4	2	0
Mean	0.8	3.8	2.2	0.6

Step 2: Subtract mean from each cell in the column.

x1	-0.8	0.2	-0.2	1.4
x2	1.2	0.2	-0.2	-0.6
x3	1.2	-0.8	0.8	-0.6
x4	0.8	0.2	-0.2	0.4
x5	-1.8	0.2	-0.2	-0.6

Step 3: Find the 2 largest values in D:

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7.85 & 0 \\ 0 & 0 & 0 & 2.75 \end{bmatrix}$$

Largest values: 7.85, 2.75

Step 4: Grab the corresponding columns from V.

-0.9	0.37
0.2	0.09
-0.2	-0.09
0.32	0.92

Step 5: Transpose the columns from V:

-0.9	0.2	-0.2	0.32
0.37	0.09	-0.09	0.92

Step 6: Grab x1 and x3.

x1
-0.8
0.2
-0.2
1.4

x3
1.2
-0.8
0.8
-0.6

Step 7: Matrix Multiply Step 5 with each of x1 and x3

Z1	1.248
	1.028
Z3	-1.592
	-0.252