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Estadística Aplicada

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Actividad 6

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① $Y_t = \varepsilon_t$ $\quad \varepsilon \sim iid\, N(0, \sigma_\varepsilon^2)$ $\quad E(\varepsilon_j \varepsilon_i) = 0 \quad i \neq j$

a) Encontrar media

$E(Y_t) = E(\varepsilon_t) = 0$

b) muestre que $var(Y_t) = \sigma_\varepsilon^2$ y que $\sigma_\varepsilon^2 = E(\varepsilon_t^2)$

$Var(Y_t) = E(Y_t - \mu)^2$
$= E(\varepsilon_t - 0)^2$
$= E(\varepsilon_t)^2 = \sigma_\varepsilon^2$

$* \; E(\varepsilon_j \varepsilon_i) = 0 \quad i \neq j$
$E(\varepsilon_t - 0)^2 = E(\varepsilon_t^2) = \sigma_\varepsilon^2$

c) Calcule covarianza

$Cov(Y_t, Y_{t+k}) = E[(Y_t - \mu_t)(Y_{t+k} - \mu_{t+k})]$
$= E[(\varepsilon_t - 0)(\varepsilon_{t+k} - 0)]$
$= E[\varepsilon_t \varepsilon_{t+k}] = 0$

$* \; E(\varepsilon_i \varepsilon_j) = 0 \quad i \neq j$

d) Señale si es estacionario y por qué

Media: 0
Varianza: σ_ε^2
Covarianza: 0

Sí es estacionario, tiene media, varianza y covarianza constante

② $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$

a) ¿Qué condición garantiza la convergencia de la sumatoria infinita del proceso?

El modelo AR(2) es una suma geométrica, al $|\phi| < 1$ significa que los valores se harán más pequeños, siendo eventualmente 0. Por esta razón la sumatoria infinita converge

b) Encuentre su media

$E(Y_t) = c + \phi_1 E(Y_{t-1}) + E(Y_{t-2}) + E(\varepsilon_t)$

$\mu = c + \phi_1 \mu + \phi_2 \mu + 0$

$\mu(1 - \phi_1 - \phi_2) = c$ $\qquad \phi_1 + \phi_2 \neq 1$

$\mu = \dfrac{c}{(1 - \phi_1 - \phi_2)}$

c) Encuentre su varianza

$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$

$Y_t = \mu(1 - \phi_1 - \phi_2) + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$

$\tilde{Y}_t = \phi_1 \tilde{Y}_{t-1} + \phi_2 \tilde{Y}_{t-2} + \varepsilon_t$

$\Rightarrow$

$\gamma_0 = Var(Y_t) = E(\tilde{Y}_t^2)$

$= E[(\phi_1 \tilde{Y}_{t-1} + \phi_2 \tilde{Y}_{t-2} + \varepsilon_t)^2]$

$= E[\phi_1^2 \tilde{Y}_{t-1}^2 + \phi_2^2 \tilde{Y}_{t-2}^2 + \varepsilon_t^2 + 2\phi_1\phi_2 \tilde{Y}_{t-1}\tilde{Y}_{t-2} + 2\phi_1 \tilde{Y}_{t-1}\varepsilon_t + 2\phi_2 \tilde{Y}_{t-2}\varepsilon_t]$

$= \phi_1^2 \gamma_0 + \phi_2^2 \gamma_0 + \sigma_\varepsilon^2 + 2\phi_1\phi_2 \gamma_1$

$\rightarrow$

$\gamma_0 = E[(Y_t - \mu)^2] = E(\tilde{Y}_t^2)$

$\gamma_1 = E[(Y_t - \mu)(Y_{t-1} - \mu)] = E[\tilde{Y}_t \tilde{Y}_{t-1}]$

$\Rightarrow$

$= E[(\phi_1 \tilde{Y}_{t-1} + \phi_2 \tilde{Y}_{t-2} + \varepsilon_t)(\tilde{Y}_{t-1})]$

$= E[\phi_1 \tilde{Y}_{t-1}^2 + \phi_2 \tilde{Y}_{t-1}\tilde{Y}_{t-2} + \varepsilon_t \tilde{Y}_{t-1}]$

$\gamma_1 = \phi_1 \gamma_0 + \phi_2 \gamma_1$

$\gamma_1 - \phi_2 \gamma_1 = \phi_1 \gamma_0$

$\gamma_1 = \dfrac{\phi_1 \gamma_0}{1 - \phi_2}$

Regresando a la ecuación original

$\gamma_0 = \phi_1^2 \gamma_0 + \phi_2^2 \gamma_0 + \sigma_\varepsilon^2 + 2\phi_1\phi_2 \left(\dfrac{\phi_1 \gamma_0}{1 - \phi_2}\right)$

$$\gamma_0 = \frac{\sigma_\varepsilon^2}{1-\phi_1^2-\phi_2^2-2\phi_1\phi_2\left(\frac{\phi_1}{1-\phi_2}\right)}$$

$$\gamma_0 = \frac{(1-\phi_2)\,\sigma_\varepsilon^2}{(1+\phi_2)\left[(1-\phi_2)^2-\phi_1^2\right]}$$

d) Encuentra la covarianza con el primer rezago $Cov(Y_t, Y_{t-1})$

$$\gamma_1 = Cov(Y_t, Y_{t-1}) = E[(Y_t-\mu)(Y_{t-1}-\mu)] = E[\tilde{Y}_t\tilde{Y}_{t-1}]$$

$\Rightarrow$

$$= E[(\phi_1\tilde{Y}_{t-1}+\phi_2\tilde{Y}_{t-2}+\varepsilon_t)(\tilde{Y}_{t-1})]$$

$$= E[\phi_1\tilde{Y}_{t-1}^2+\phi_2\tilde{Y}_{t-1}\tilde{Y}_{t-2}+\varepsilon_t\tilde{Y}_{t-1}]$$

$$\gamma_1 = \phi_1\gamma_0+\phi_2\gamma_1$$

$$\gamma_1 = \frac{\phi_1\gamma_0}{1-\phi_2}$$

e) Encuentre la covarianza con $Cov(Y_t, Y_{t+1})$

$$\gamma_1 = Cov(Y_t, Y_{t-1}) = E[(Y_t-\mu)(Y_{t+1}-\mu)] = E[\tilde{Y}_t\tilde{Y}_{t+1}]$$

$\Rightarrow$

$$= E[(\phi_1\tilde{Y}_{t+1}+\phi_2\tilde{Y}_{t+2}+\varepsilon_t)(\tilde{Y}_{t+1})]$$

$$= E[\phi_1\tilde{Y}_{t+1}\tilde{Y}_{t+1}+\phi_2\tilde{Y}_{t+1}\tilde{Y}_{t+2}+\varepsilon_t\tilde{Y}_{t+1}]$$

$$= \phi_1\gamma_0+\phi_2\gamma_1+0$$

$\Rightarrow$

$$\gamma_1 = \frac{\phi_1\gamma_0}{1-\phi_2}$$

F) Calcula ρ_1

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\left(\frac{\phi_1}{1-\phi_2}\right)\gamma_0}{\gamma_0} = \frac{\phi_1}{1-\phi_2}$$

③ $Y_t = \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t$

Media

a) $E[Y_t] = E[\theta_1 \varepsilon_{t-1}] + E[\theta_2 \varepsilon_{t-2}] + E[\varepsilon_t] + E[\mu]$

$= 0 + \mu = \underline{\mu}$

b) Varianza

$\gamma_0 = Var(Y_t) = E[(Y_t - \mu)^2]$

$= E[(\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t + \mu - \mu)^2] = E[(\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t)^2]$

$= E(\theta_1^2 \varepsilon_{t-1}^2 + 2\theta_1 \varepsilon_{t-1} \theta_2 \varepsilon_{t-2} + 2\theta_1 \varepsilon_{t-1} \varepsilon_t + \theta_2^2 \varepsilon_{t-2}^2 + 2\theta_2 \varepsilon_{t-2} \varepsilon_t + \varepsilon_t^2)$

$= \theta_1^2 \sigma_\varepsilon^2 + \theta_2^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2$

$= \underline{(1 + \theta_1^2 + \theta_2^2) \sigma_\varepsilon^2}$

c) Covarianza rezago k $Cov(Y_t, Y_{t-k})$

$\gamma_k = Cov(Y_t, Y_{t-k}) = E[(Y_t - \mu)(Y_{t-k} - \mu)]$

$= E[(\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t)(\theta_1 \varepsilon_{t-k-1} + \theta_2 \varepsilon_{t-k-2} + \varepsilon_{t-k})]$

$= E[\theta_1^2 \varepsilon_{t-1} \varepsilon_{t-k-1} + \theta_1 \theta_2 \varepsilon_{t-2} \varepsilon_{t-k-1} + \theta_1 \varepsilon_t \varepsilon_{t-k-1} + \theta_1 \theta_2 \varepsilon_{t-1} \varepsilon_{t-k-2} + \theta_2^2 \varepsilon_{t-2} \varepsilon_{t-k-2} + \theta_2 \varepsilon_t \varepsilon_{t-k-2}$

$+ \theta_1 \varepsilon_{t-1} \varepsilon_{t-k} + \theta_2 \varepsilon_{t-2} \varepsilon_{t-k} + \varepsilon_t \varepsilon_{t-k}]$

$= \underline{0}$

d) Encuentra ρ_k

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{0}{(1+\theta_1^2+\theta_2^2)\,\sigma_\varepsilon^2} = 0$$

e) Grafica la función de autocorrelación, $\rho_1, \ldots, \rho_{10}$

$$\rho_1 = \frac{\theta_1 + \theta_1\theta_2}{1+\theta_1^2+\theta_2^2} \qquad \rho_2 = \frac{\theta_2}{1+\theta_1^2+\theta_2^2}$$