Phys 152: Fundamentals of Physics II

Unit #4 - Capacitance & Dielectrics

Aaron Wirthwein

Department of Physics and Astronomy
University of Southern California

Capacitors

A capacitor is a device made of two conductors that is used to store electrical charge and electrical energy. Capacitors are used to tune the frequency of radio receivers, as energy storage devices for cardiac defibrillators, and as accelerometers in smartphones.

In a parallel plate capacitor, a battery provides a constant potential difference between two conducting plates. Charge Q builds up on one plate, and $-Q$ builds up on the other. When we refer to "the charge on the capacitor," we mean the charge on the positive plate. The space between the plates can be empty or filled with insulating material called a dielectric.

Definition of Capacitance

The quantity of charge Q on a capacitor is linearly proportional to the potential difference ΔV between the conductors. The proportionality constant depends on the shape and separation of the conductors. Define capacitance as the ratio of the charge on the capacitor to the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V} \qquad [C] = \text{farad (f)} = \frac{\mathrm{C}}{\mathrm{V}}$$

$$\Delta V = V_+ - V_- = -\int_{(-)}^{(+)} \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{r}}$$

Parallel Plate Capacitor

Model a parallel plate capacitor as two infinite planes carrying uniform surface charge densities $+\sigma$ and $-\sigma$. In reality, the plates are separated by a distance d and have area $A \gg d^2$.

The electric field strength between the plates is $E = |\sigma|/\varepsilon_0$, the potential difference is $\Delta V = Ed$, and the capacitance is

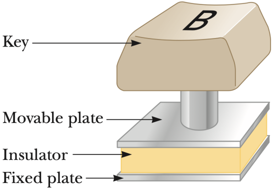
$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \varepsilon_0 \frac{Q}{|\sigma| d} = \varepsilon_0 \frac{A}{d} \qquad [\varepsilon_0] = \frac{\mathrm{f}}{\mathrm{m}}$$

This equation only works for <u>parallel plate capacitors</u>!

Conceptual Question 1

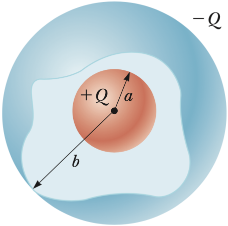
Many computer keyboard buttons are constructed of capacitors as shown in the figure. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, what happens to the capacitance?

(a) It increases
(b) It decreases
(c) It remains the same



Example 1: Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius b concentric with a smaller conducting sphere of radius a. Find the capacitance of this device. **What if** $b \to \infty$?



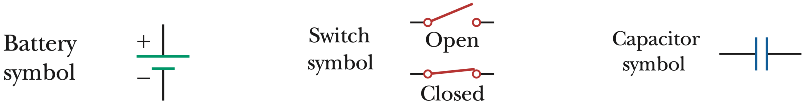
Grounding a Conductor

In the previous example, we found the "self-capacitance" of a sphere. The Earth is a very large spherical conductor whose self-capacitance is extraordinarily large compared to any laboratory conductor. Since the charge of the Earth is finite, if $C_{\text{Earth}} \to \infty$ then $V_{\text{Earth}} \to 0$.

Finite amounts of charge may be added to or taken away from the Earth without significantly changing its potential from zero. When we ground a conductor, we fix its potential to zero by connecting it to Earth using a fine conducting wire. A ground can serve as a reference point for voltage measurements in an electric circuit.

Introduction to Electric Circuits

An electric circuit is an interconnected network of electrical components (that technically forms a closed loop for current flow). In studying electric circuits, we use a simplified pictorial representation called a circuit diagram. Such a diagram uses circuit symbols to represent various circuit elements. The circuit symbols are connected by straight lines that represent the wires between the circuit elements.



Combinations of Capacitors

Manufactured capacitors may not be the ones you actually need in a particular application. You can obtain the capacitance you need by combining capacitors in one of two (simple) ways:

(i) In a series combination, capacitors are connected end-to-end, so the same charge accumulates on each capacitor.

(ii) In a parallel combination, capacitors are connected side-by-side, so each capacitor experiences the same voltage.

Equivalent capacitance refers to the single capacitance value that could replace a combination of capacitors in a circuit. In what follows, assume that all capacitors are initially uncharged.

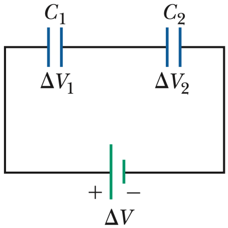
Series Combination

Two capacitors in series have the same charge and the total potential difference is the sum of potential differences across each capacitor:

$$Q = Q_1 = Q_2, \quad \Delta V = \Delta V_1 + \Delta V_2$$

$$\frac{1}{C_{\mathrm{eq}}} = \frac{\Delta V}{Q} = \frac{\Delta V_1}{Q_1} + \frac{\Delta V_1}{Q_2}$$

$$\boxed{\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}}$$



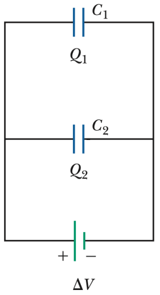
Parallel Combination

Two capacitors in parallel have the same potential difference and the total charge is the sum of the charges on the individual capacitors:

$$Q = Q_1 + Q_2, \quad \Delta V = \Delta V_1 = \Delta V_2$$

$$C_{\text{eq}} = \frac{Q}{\Delta V} = \frac{Q_1}{\Delta V_1} + \frac{Q_2}{\Delta V_2}$$

$$\boxed{C_{\text{eq}} = C_1 + C_2}$$



Conceptual Question 2

Two capacitors are identical. They can be connected in series or in parallel. If you want the smallest equivalent capacitance for the combination, how should you connect them?

(a) In series

(b) In parallel

(c) Either way because both combinations have the same capacitance.

Summary: Combinations of Capacitors

(i) When two or more capacitors are connected in series,

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots ,$$

and C_{eq} is less than any individual capacitance.

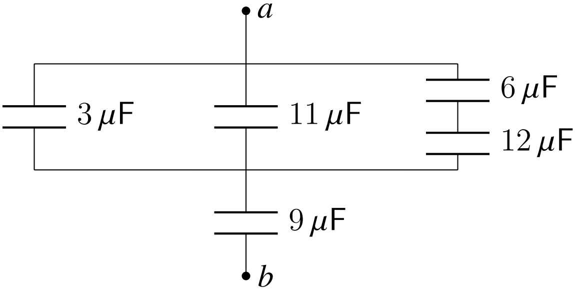
(ii) When two or more capacitors are connected in parallel,

$$C_{\mathrm{eq}} = C_1 + C_2 + C_3 + \cdots ,$$

and C_{eq} is greater than any individual capacitance.

Example 2: A Capacitor Network

In the capacitor network below, $\Delta V_{\mathrm{ab}} = 20.0\ \mathrm{V}$. Find the equivalent capacitance between a and b and the charge on each capacitor.



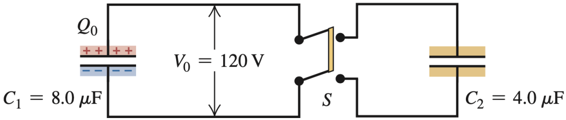
Energy Stored in a Capacitor

Start with two neutral conducting plates. The amount of work required to move a small amount of charge dq from one plate to the other depends on ΔV. At first, $\Delta V = 0$, but once charge begins to transfer, $\Delta V = q/C$, and the work required to move dq becomes $dW = \Delta V dq$. The stored potential energy of the capacitor is equal to the total work done in the charging process:

$$U_E = \int_0^Q \Delta V dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} Q(\Delta V) = \frac{1}{2} C(\Delta V)^2$$

Example 3: Transferring energy between capacitors

We connect a capacitor $C_1 = 8.0\ \mu\mathrm{F}$ to a power supply, charge it to a potential difference $V_0 = 120\ \mathrm{V}$, and disconnect the power supply. Switch S is open. (a) What is the charge Q_0 on C_1? (b) What is the energy stored in C_1? (c) Capacitor $C_2 = 4.0\ \mu\mathrm{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor? (d) What is the final energy of the system?



Energy Density of the Electric Field

We can consider the energy in a capacitor to be stored in the electric field. Consider a parallel plate capacitor.

$$U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\varepsilon_0 \frac{A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 (Ad)$$

$$u_E = \frac{U_E}{Ad} = \frac{1}{2}\varepsilon_0 E^2 \quad \text{(energy density of } \vec{\mathbf{E}}\text{)}$$

Electric Fields in Matter

The study of electric fields in matter is largely the study of electric dipoles. Electric fields can either rotate or stretch the atoms and molecules that make up an insulating material. In some cases the electric field polarizes the molecules (the atomic polarizability quantifies this effect). In other cases, the molecules are permanent dipoles, and the external field brings them into alignment.

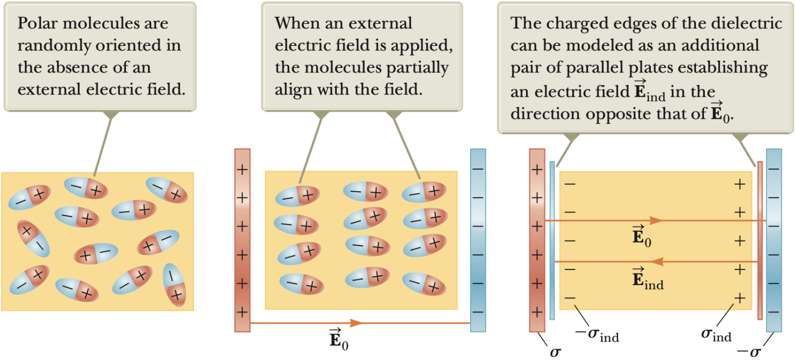
A dielectric is a nonconducting material such as rubber, glass, or waxed paper. Dielectrics can store and release electrical energy by polarizing in response to an electric field. *Filling a capacitor with a dielectric will always increase its capacitance and maximum voltage*.

Polar Molecules

Polar molecules have a permanent separation between positive and negative charges. Normally, polar molecules are randomly aligned within an insulating substance, but in the presence of an electric field, they can rotate and "line up." For weak electric fields, chemical forces hold the molecules in place.

Internal polarization generates an opposing electric field which partially cancels the incident field in the material. Experimentally, we find that the total field inside many insulating materials is directly proportional to the external field, but reduced by an overall constant.

Atomic Description of Dielectrics



Dielectric Constant

Define the dielectric constant as the ratio of the external electric field to the total electric field within a dielectric:

$$\kappa = \frac{E_{\text{ext}}}{E} \geq 1$$

For linear dielectrics, the dielectric constant is truly a constant that depends only on the material and its temperature.

Most dielectric constants range from 1 - 10 with strontium titanate having a dielectric constant of 233 (at $20^\circ\mathrm{C}$)!

Capacitor Connected to Battery

A parallel plate capacitor is hooked up to a battery that maintains a constant ΔV. The field between the plates will cause a dielectric to polarize such that a thin layer of opposing charge builds up on either side. For weak electric fields, the induced layer of charge on the dielectric is always less than the charge on the capacitor.

Since the capacitor is hooked up to a battery, the electric field must be the same as if there were no dielectric ($E = \Delta V/d = E_0$), so the total charge on and near the top plate in the dielectric-filled capacitor must be the same as it was in the empty capacitor (call it Q_0). The charge on the positive plate will increase such that $Q_{\mathrm{tot}} = Q_0$.

Capacitor Connected to Battery

Inserting a dielectric into the capacitor increases its charge. For a linear dielectric, let E' be the field due to charges on the capacitor plates which serves as the external field acting on the dielectric:

$$\kappa = \frac{E'}{E} = \frac{Q}{Q_0} \quad \rightarrow \quad Q = \kappa Q_0$$

ΔV is fixed by the battery, so the capacitance must increase

$$C = \frac{Q}{\Delta V} = \kappa \frac{Q_0}{\Delta V} = \kappa C_0.$$

where C_0 is the capacitance in the absence of a dielectric.

Charged Capacitor Disconnected from Battery

Suppose we begin with an isolated, fully-charged capacitor. The capacitor will maintain its charge, but the electric field and potential difference will both decrease when a dielectric is inserted:

$$E = \frac{E_0}{\kappa} \quad \rightarrow \quad \Delta V = \frac{\Delta V_0}{\kappa}$$

Q is fixed, so the capacitance must increase

$$C = \frac{Q}{\Delta V} = \kappa \frac{Q}{\Delta V_0} = \kappa C_0$$

Summary: Capacitors with Dielectrics

The results in the preceding slides turn out to be true for any capacitor. Whenever a dielectric is inserted into a capacitor, the capacitance always increases $C = \kappa C_0$.

(i) When a capacitor is connected to a battery,

$$\Delta V = \Delta V_0, \quad E = E_0, \quad Q = \kappa Q_0$$

(ii) When a charged capacitor is disconnected from a battery,

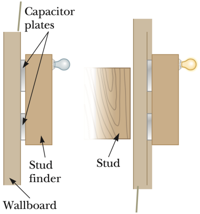
$$\Delta V = \frac{\Delta V_0}{\kappa}, \quad E = \frac{E_0}{\kappa}, \quad Q = Q_0$$

Conceptual Question 3

If you have ever tried to hang a picture or a mirror, you know it can be difficult to locate a wooden stud in which to anchor your nail or screw. A carpenter's stud finder is a capacitor with its plates arranged side by side instead of facing each other as shown in the figure. When the device is moved over a stud, the capacitance

(a) increases

(b) decreases



Dielectric Breakdown

When the magnitude of an external electric field becomes too large, the molecules of a dielectric start to ionize, and the materials becomes conducting. The dielectric strength of a substance is the critical value of the electrical field at which the molecules of an insulator become ionized.

The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation in a capacitor. The dielectric strength of air is $3\ \mathrm{MV/m}$. Filling a capacitor with most other dielectrics allows you to apply larger potential differences across the capacitor to store more charge and more energy.

Energy in a Dielectric

For a parallel plate capacitor, $C = \kappa C_0 = \kappa\varepsilon_0 A/d$.

$$U_E = \frac{1}{2}\kappa C_0(\Delta V)^2 = \frac{1}{2}\kappa\varepsilon_0 E^2(Ad)$$

$$u_E = \frac{U_E}{Ad} = \frac{1}{2}\kappa\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2,$$

where $\varepsilon = \kappa\varepsilon_0$ is the permittivity of the dielectric. When a capacitor is connected to a battery, a dielectric allows you to store more energy.

Conceptual Question 4

A capacitor with capacitance C is connected to a battery until charged, then disconnected from the battery. A dielectric having constant κ is inserted in the capacitor. What changes occur in the charge, potential and stored energy of the capacitor after the dielectric is inserted?

A. ΔV stays same, Q increases, U increases
B. ΔV stays same, Q decreases, U stays same
C. ΔV increases, Q decreases, U increases
D. ΔV decreases, Q stays same, U decreases
E. ΔV stays same, Q decreases, U stays same
F. None of the above

Example 4: A capacitor with a dielectric

A parallel-plate capacitor with area $A = 2,000\ \mathrm{cm}^2$ and plate separation $d = 1.00\ \mathrm{cm}$ is connected to a power supply and charged to a potential difference of $V_0 = 3.00\ \mathrm{kV}$, after which the power supply is disconnected. Next, a sheet of insulating plastic material is inserted between the plates, completely filling the space between them. We find that the potential difference decreases to $1.00\ \mathrm{kV}$ while the charge on each capacitor plate remains constant. Find

(a) the original capacitance C_0,

(b) the charge Q on the capacitor,

(c) capacitance C with dielectric,

(d) the dielectric constant κ,

(e) the permittivity ε,

(f) induced charge Q_i on each face of the dielectric,

(g) the original electric field E_0,

(h) and finally, the electric field E after the dielectric is inserted.

Example 5: A dielectric slab

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness fd is inserted between the plates, where $0 < f < 1$?