Phys 152: Fundamentals of Physics II

Unit #6 - Magnetism & Magnetic Force

Aaron Wirthwein

Department of Physics and Astronomy
University of Southern California

Magnetism

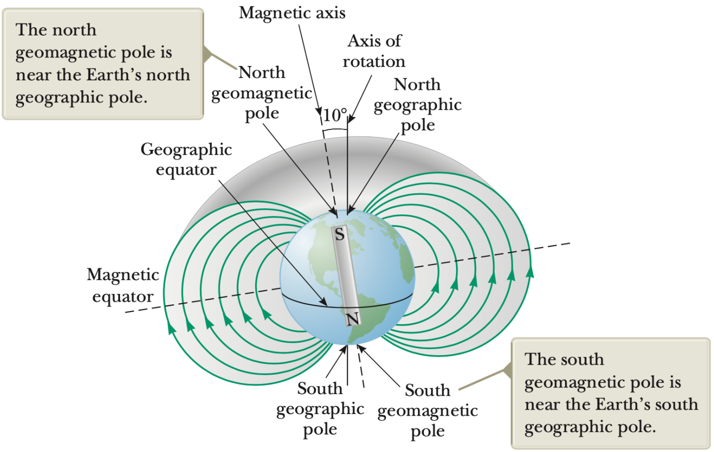
Around 500 BC, Greeks discovered magnetite, an iron ore with the unique property of attracting and repelling similar materials. Every magnet has two inseparable poles—one north and one south. Like poles repel, while unlike poles attract. Since magnetic forces act over a distance, we infer the existence of a magnetic field.

Ferromagnets, such as iron oxide, are materials that exhibit natural magnetism. No matter how much a ferromagnet is divided, it will always have both north and south poles. Even a single iron atom has both north and south magnetic poles!

Earth's Magnetic Field

The Chinese were among the first to make practical use of magnetism. By the 2nd century BC, they developed the first compasses using lodestones to align with Earth's magnetic field. In 1600, English scientist William Gilbert published *De Magnete*, a seminal work on magnetism. He proposed that Earth itself behaves like a giant magnet, with north and south poles.

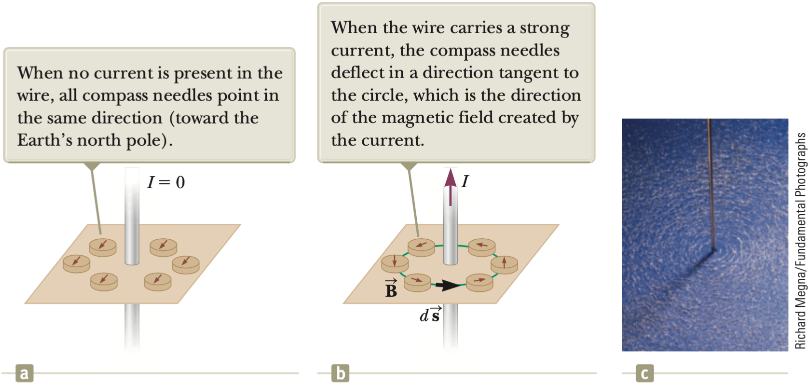
Gilbert defined the magnetic field direction to be the direction in which the north pole of a compass needle points. Earth's geographic north pole is actually a south magnetic pole (not exactly; the "magnetic axis" does not align with the "spin axis").


The north geomagnetic pole is near the Earth's north geographic pole.
Magnetic axis
Axis of rotation
North geomagnetic pole
North geographic pole
10°
Geographic equator
S
N
Magnetic equator
South geographic pole
South geomagnetic pole
The south geomagnetic pole is near the Earth's south geographic pole.

Oersted's Discovery

Until the winter of 1819-1820, electricity and magnetism were believed to be completely unrelated phenomena. Danish physicist and chemist Hans Christian Oersted suspected there might be a hidden connection. During a lecture at the University of Copenhagen, Oersted made a groundbreaking discovery: when a compass needle is placed near a current-carrying wire, it may deflect, revealing for the first time the link between electricity and magnetism.

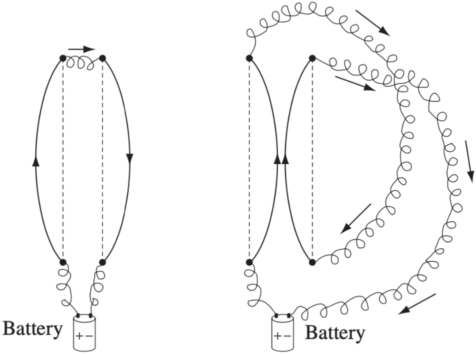
Interestingly, no deflection occurs if the current flows perpendicular to the needle. But when the current wire runs parallel to the needle, it rotates until it aligns perpendicular to the wire.


When no current is present in the wire, all compass needles point in the same direction (toward the Earth's north pole).
I = 0
When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current.
I
B
ds
Richard Megna/Fundamental Photographs
a
b
c

Ampère's Discovery

Soon after Oersted's discovery, André Marie Ampère found that two parallel wires carrying electric current experience a force between them: they attract each other if the currents flow in the same direction and repel each other if they flow in opposite directions.

In summary, current-carrying wires produce magnetic fields and experience magnetic forces. While a compass needle may not look anything like a DC circuit, we now know, as Ampère was the first to suspect, that magnetized iron is full of perpetually moving charges-electric currents on an atomic scale. *Magnetic fields are produced by and act upon moving charges.*



(a) Currents in opposite directions repel.

(b) Currents in same directions attract.

Magnetic Field

While an electric force will act on any charge regardless of its velocity, the magnetic force on a charged particle depends on its velocity. The magnetic field is the vector $\vec{\mathbf{B}}$ such that the force on a particle with charge q and velocity $\vec{\mathbf{v}}$ is given by

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \qquad [B] = \frac{\mathrm{N}}{\mathrm{C} \cdot \mathrm{m/s}} = \frac{\mathrm{N}}{\mathrm{A} \cdot \mathrm{m}} = \mathrm{tesla}\ (\mathrm{T})$$

Another common unit is the gauss where $10^4\ \mathrm{G} = 1\ \mathrm{T}$. The magnetic field of the Earth is about 0.5 G. MRI machines operate with very large magnetic fields of about 1.5-3.0 T.

Cross Product

Given two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ with smallest angle ϕ between them, a third vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is called the cross product if

$$|\vec{\mathbf{C}}| = C = AB\sin\phi$$

and the direction of $\vec{\mathbf{C}}$ is such that if you place the fingers of your right hand along $\vec{\mathbf{A}}$ and curl them in the direction of $\vec{\mathbf{B}}$, then $\vec{\mathbf{C}}$ points along the direction of your outstretched thumb.

Notice that the cross product of two parallel vectors is zero!

Properties of Cross Product

(i) Distributive: $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{A}} \times \vec{\mathbf{C}}$

(ii) Anti-commutative: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -(\vec{\mathbf{B}} \times \vec{\mathbf{A}})$

(iii) Compatible with scalar product: $(a\overrightarrow{\mathbf{A}}) \times \overrightarrow{\mathbf{B}} = a(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}})$

(iv) No cancellation: $\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{C}}$ does not imply $\overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{C}}$

(v) Product rule for differentiation (using prime notation):

$$(\overrightarrow{\mathbf{A}}(t) \times \overrightarrow{\mathbf{B}}(t))' = \overrightarrow{\mathbf{A}}'(t) \times \overrightarrow{\mathbf{B}}(t) + \overrightarrow{\mathbf{A}}(t) \times \overrightarrow{\mathbf{B}}'(t)$$

Cross Product with Components

The base vectors are useful in deriving a general rule for evaluating the cross product of two vectors in terms of their components.

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}, \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_y B_z - A_z B_y)\hat{\mathbf{i}} + (A_z B_x - A_x B_z)\hat{\mathbf{j}} + (A_x B_y - A_y B_x)\hat{\mathbf{k}}$$

We can also write this as the determinant of a 3×3 matrix:

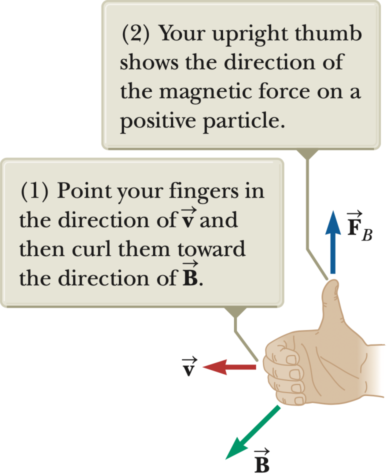
$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Magnetic Force

While the magnetic field is defined in terms of the force it produces on a moving charge, let's suppose we already know the magnetic field in a region of space. Let θ be the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. The magnitude of the magnetic force on a charged particle is

$$F_B = qvB\sin\theta$$

We find the direction of the magnetic force on a charged particle by using the right-hand rule for the cross product of $\vec{\mathbf{v}}$ with $\vec{\mathbf{B}}$.


(2) Your upright thumb shows the direction of the magnetic force on a positive particle.
(1) Point your fingers in the direction of $\vec{\mathbf{v}}$ and then curl them toward the direction of $\vec{\mathbf{B}}$.
$\vec{\mathbf{F}}_B$
$\vec{\mathbf{v}}$
$\vec{\mathbf{B}}$

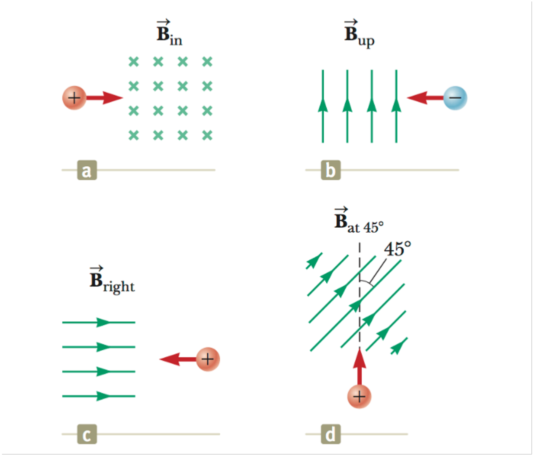
Relative Motion

Notice there is no magnetic force on stationary charges. When charges are stationary, their electric fields do not affect magnets. However, when charges move, they produce magnetic fields that exert forces on other magnets and moving charges. When there is relative motion, a connection between electric and magnetic forces emerges.

It turns out that the magnetic force is a consequence of Coulomb's law, charge invariance, and the principle of relativity! With just these three ideas, we could have predicted the magnetic force, but as always, hindsight is 20/20.

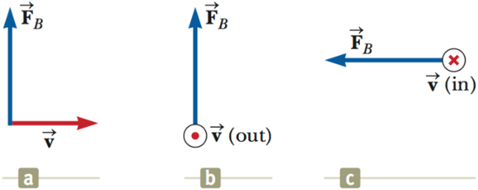
Conceptual Exercise 1

Determine the initial deflection experienced by the moving charges.



Conceptual Exercise 2

Find the direction of the magnetic field acting on a positively charged particle moving in the various situations shown in the figure. Assume $\vec{\mathbf{v}} \perp \vec{\mathbf{B}}$.



Example 1: Magnetic Force Calculation

An **alpha particle** is a positively charged particle consisting of two protons and two neutrons, bound together into a structure identical to a helium-4 nucleus (^{4}He). It is one of the common types of particles emitted during *radioactive decay*, specifically *alpha decay*.

An alpha particle with charge $q = 3.2 \times 10^{-19}$ C moves with velocity $\vec{\mathbf{v}} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times 10^4$ m/s through a region of space with a uniform magnetic field $\vec{\mathbf{B}} = 1.5\hat{\mathbf{k}}$ T.

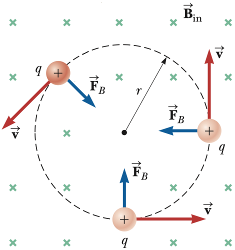
What is the magnetic force on the alpha particle?

Cyclotron Motion

Consider the special case of a charged particle moving in a uniform magnetic field with its initial velocity vector perpendicular to the field. The particle will move in a circle because the magnetic force is always perpendicular to both the velocity and the magnetic field.

$$\sum F_r = qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m} \quad \text{(cyclotron frequency)}$$



Example 2: Cyclotron Motion

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm.

(a) What is the magnitude of the magnetic field?

(b) What is the angular speed of the electrons?

Nonuniform Magnetic Fields

The motion of a charged particle in a nonuniform magnetic field can be complex. For example, in a magnetic field that is strong at the ends and weak in the middle, the particles can oscillate between two positions. This configuration is known as a magnetic bottle because charged particles can be trapped within it.

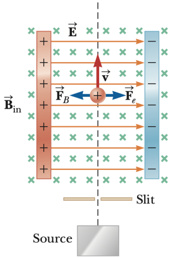
The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions. The particles, trapped by the Earth's nonuniform magnetic field, spiral around the field lines. Near the poles, the particles sometimes collide with atoms in the atmosphere, causing the atoms to emit visible light. Such collisions are responsible for the aurora borealis (northern lights).

Velocity Selector

Many experiments involving charged particles require that all particles move with the same velocity. An appropriate combination of electric and magnetic fields can serve as a velocity selector. Only particles having desired speed v pass undeflected through mutually perpendicular electric and magnetic fields:

$$\sum \vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$$

$$qE = qvB \rightarrow v = \frac{E}{B}$$



Example 3: Velocity Selector

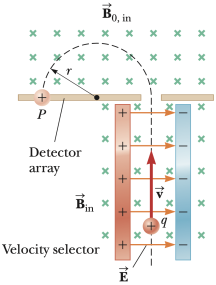
In a velocity selector, the magnitudes of the electric and magnetic fields are chosen so that $qE = qvB$, the charged particle experiences zero net force and moves in a straight vertical line through the region of the fields. A certain velocity selector consists of electric and magnetic fields described by the expressions $\vec{\mathbf{E}} = E_0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B_0\hat{\mathbf{j}}$ with $B_0 = 15.0\ \mathrm{mT}$. Find the value of E_0 such that a 750-eV electron moving in the negative x direction is undeflected.

Mass Spectrometer

A mass spectrometer separates ions according to their mass-to-charge ratio. In one version of this device, known as the *Bainbridge mass spectrometer*, a beam of ions first passes through a velocity selector and then enters a second uniform magnetic field that has the same direction as the magnetic field in the selector.

We can determine m/q by measuring the radius of curvature and knowing the field magnitudes.

$$\frac{m}{q} = \frac{rB_0}{v} = \frac{rB_0B}{E}$$



Magnetic Force on a Current Wire

When a current-carrying wire is placed in a magnetic field, there is a force on the wire consistent with the magnetic force law. A wire with mobile charge per unit length λ travels with speed v carries a current $I = \lambda v$ because a segment of length $v\Delta t$, carrying charge $\lambda v\Delta t$, passes a fixed point in the wire in a time interval Δt.

The force on an arbitrarily shaped current wire is given by

$$\vec{\mathbf{F}}_B = \int (\vec{\mathbf{v}} \times \vec{\mathbf{B}})dq = \int (\vec{\mathbf{v}} \times \vec{\mathbf{B}})\lambda ds = \int I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

since $\vec{\mathbf{v}}$ and $d\vec{\mathbf{s}}$ point in the same direction in the wire.

Magnetic Force on a Straight Wire

When a straight wire of length L carrying current I is placed in a uniform magnetic field $\vec{\mathbf{B}}$, it will experience a force

$$\vec{\mathbf{F}}_B = \int I d\vec{\mathbf{s}} \times \vec{\mathbf{B}} = I \left(\int d\vec{\mathbf{s}} \right) \times \vec{\mathbf{B}} = I \vec{\mathbf{L}} \times \vec{\mathbf{B}},$$

where $\vec{\mathbf{L}}$ is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment.

Conceptual Question 3

A wire carries current in the plane of this page toward the top of the page. The wire experiences a magnetic force toward the right edge of the page. Is the direction of the magnetic field causing this force

(a) in the plane of the page and toward the left edge,

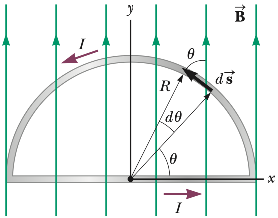
(b) in the plane of the page and toward the bottom edge,

(c) out of the page, or

(d) into the page?

Example 4: Force on a Semicircular Conductor

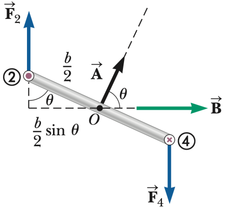
A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.



Torque on a Current Loop

At the heart of an electric motor is a coil of current-carrying wire in the presence of a magnetic field. A wire loop in a uniform magnetic field experiences zero net force and a nonzero net torque.

$$\begin{aligned} \tau &= F_2 \frac{b}{2}\sin\theta + F_4 \frac{b}{2}\sin\theta \\ &= (IaB)\frac{b}{2}\sin\theta + (IaB)\frac{b}{2}\sin\theta \\ &= IAB\sin\theta \end{aligned}$$



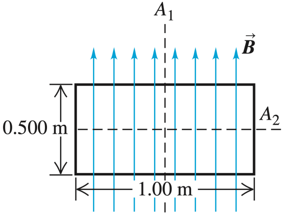
$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

Example 5

A uniform rectangular coil of total mass 212 g and dimensions 0.500 m $\times$ 1.00 m is oriented with its plane parallel to a uniform 3.00 T magnetic field. A current of 2.00 A is suddenly started in the coil.

(a) About which axis (A_1 or A_2) will the coil begin to rotate? Why?

(b) Find the initial angular acceleration of the coil just after the current is started.



Magnetic Dipoles

A closed loop of current constitutes a magnetic dipole with a magnetic dipole moment represented by the vector $\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}}$. In the presence of an external magnetic field, a magnetic dipole moment will experience a torque $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ that tends to align the dipole moment vector with the magnetic field lines.

Work must be done to bring the dipole moment out of alignment, and the work done is equal to the potential energy of the dipole-field system. The potential energy is given by

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

Note that, due to the scalar product, the potential energy is minimized when the dipole moment points in the same direction as the magnetic field, i.e. the two vectors are aligned with one another.