Phys 152: Fundamentals of Physics II

Unit #5 - Electric Current & Resistance

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Misconceptions Regarding Current

The following statements are either wrong or imprecise.

(i) Electric current is the flow of free electrons only.
(ii) Electricity flows at the speed of electrons.
(iii) Conductors allow electrons to flow without resistance.
(iv) Current gets "used up" in a circuit.
(v) Current always follows the path of least resistance.

Electric Current

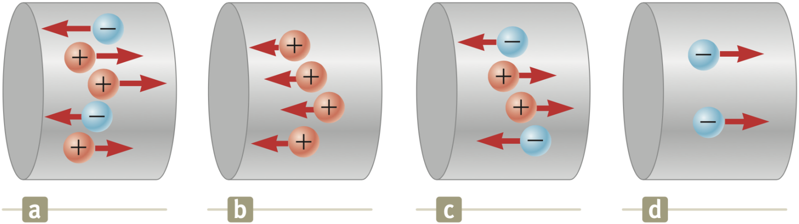
We begin by studying the "kinematics" of current and we will address its causes later. Electric current is defined as the rate of net charge transport through a surface:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad [I] = 1\ \mathrm{A} = 1\ \mathrm{C/s}$$

The SI unit of current is the **ampere** equal to one coulomb of charge passing through a surface in one second. When current is passing down a wire, it is conventional to assign to the current a direction that is the same as that of the flow of positive charge.

Conceptual Question 1

Consider positive and negative charges of equal magnitude moving horizontally through the four regions shown. Rank the current in these four regions from highest to lowest.



Electric Current in a Wire

A conducting wire has a **number density** n of mobile charge carriers, each with charge value q. The amount of charge that passes through a cross-sectional area A in a time Δt is contained within a cylinder of length Δx along the wire. The average current in the wire is

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA\Delta x}{\Delta t} = nqAv_d,$$

where v_d is the "drift velocity" or the average speed of charge carriers moving along the wire. It is not clear *why* the charge carriers would have a constant drift speed; for now, let's assume they do and we'll try to figure out why later.

Example 1: Drift Speed Estimation

The 12-gauge copper wire in a typical residential building has a cross-sectional area of about $3.3\times10^{-6}\ \mathrm{m}^2$. It carries a constant current of 10 A. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is roughly $8.9\ \mathrm{g/cm}^3$.

Current Density in a Wire

Notice that the current in a wire is proportional to the cross-sectional area A. Define the current density in a wire to be the average electric current divided by the cross-sectional area of the wire.

$$J = \frac{I}{A} = nqv_d \qquad [J] = \mathrm{A/m^2}$$

As a vector, the current density points in the direction of positive charge flow (or the opposite direction of negative charge flow).

Current Density and Steady Current

So far we've considered a model consisting of charge carriers moving along a wire. A more general kind of current involves charge carriers moving around in three-dimensional space.

The current flowing through *any surface* is the surface integral of the current density:

$$I = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}}$$

A *steady current* is when the current remains constant in time everywhere. The amount of charge passing through any surface does not change with time.

Charge Conservation

In the same way the charge density ρ tells you how much charge is contained in a little region of space around some point, the current density tells you how much current flows through a cross-sectional surface centered on some point.

Whether or not the current is steady, charge must be locally conserved. The net rate of charge transfer out of a closed surface equals the rate of change in the amount of charge enclosed:

$$\oint_S \vec{\mathbf{J}} \cdot d\,\vec{\mathbf{a}} = -\frac{d}{dt}\int_V \rho dV$$

The minus sign is due to the convention that positive flux corresponds to outward-pointing current density.

Electrical Conduction in Metals

When a constant potential difference, such as that provided by a battery, is applied across a metal at room temperature, it generates a steady current. The electric field inside the metal does not continuously accelerate the charges and lead to an infinite current.

In the 1820's, Georg Simon Ohm experimentally discovered a linear relationship between the applied potential and the resulting current in many metals near room temperature. He explained the lack of "infinite current" by introducing the concept of resistance, which is an intrinsic property of a material that opposes the flow of charges.

Resistance

Define resistance as the ratio of the potential difference across a conductor to the current in the conductor.

$$R \equiv \frac{\Delta V}{I}, \qquad [R] = \Omega = \mathrm{V}/\mathrm{A}$$

The SI unit of resistance is the **ohm**. One volt per ampere is defined to be one ohm. You may recognize this as "Ohm's law," but we will treat this as the definition of resistance.

Think of the potential difference as the "cause" of the current. For a given potential difference, one material with a large resistance will develop a smaller current than one with a smaller resistance.

Drude Model of Electrical Conduction

We will study a model of electrical conduction in metals that was first proposed by Paul Drude (1863–1906) in 1900.

1. Conductor is a lattice of ions containing free electrons that we treat as point particles.
2. Electrons exhibit thermal motion like an ideal gas. They move in random directions and only change directions after collisions with the lattice or with other electrons.
3. A uniform electric field imparts a drift velocity with a magnitude smaller than the average speed of thermal motion.
4. Excess energy of the electrons provided by the field is transferred to the atoms in the conductor during collisions.

Kinetic Theory of Free Electrons

Let Δt be the time between collisions for a single electron. The electron's final momentum will be $\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i - e\vec{\mathbf{E}}\Delta t$. Taking the average over all electrons in the conductor,

$$\langle \vec{\mathbf{p}}_f \rangle = \langle \vec{\mathbf{p}}_i \rangle - e\vec{\mathbf{E}} \langle \Delta t \rangle = -e\vec{\mathbf{E}}\tau$$

where $\langle \vec{\mathbf{p}}_i \rangle = 0$ since the thermal motion is random, and $\tau = \langle \Delta t \rangle$ is the **mean time between collisions**. In terms of the drift velocity,

$$\langle \vec{\mathbf{p}}_f \rangle = m\langle \vec{\mathbf{v}}_f \rangle = -e\vec{\mathbf{E}}\tau \quad \rightarrow \quad \vec{\mathbf{v}}_d \equiv \langle \vec{\mathbf{v}}_f \rangle = -\frac{e\tau}{m}\vec{\mathbf{E}}$$

Conductivity and Ohm's Law

According to the Drude model, the drift speed is proportional to the driving force of an electric field. The current density in a metallic wire with an internal electric field along the wire is

$$J = nev_d = \frac{ne^2\tau}{m}E = \sigma E,$$

where σ is a parameter called the conductivity that depends only on material properties and temperature.

Ohm's law is an empirical law that states the current density in a conductor is directly proportional to an applied electric field.

Resistivity

Resistivity is the inverse of the conductivity, and according to the Drude model,

$$\sigma = \frac{ne^2\tau}{m}, \qquad \rho \equiv \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

Although Drude's model is consistent with Ohm's law, correctly predicting that current density is proportional to the applied electric field,it does not correctly predict the values of resistivity or the behavior of the resistivity with temperature.

A quantum mechanical model is needed to fully explain electron conduction in a metallic wire.

Example 2: Testing Drude's Model

The metal sodium contains 2.5×10^{28} atoms per cubic meter, and each atom provides one conduction electron.

(a) Determine the time τ between collisions if the conductivity at room temperature is measured in experiments to be $\sigma = 2.1 \times 10^{7}\ \Omega \cdot \mathrm{m}$.

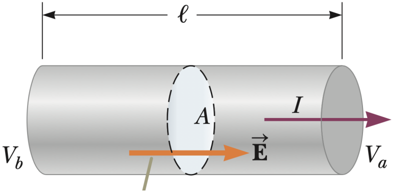
(b) How far should an electron move in that time if according to kinetic theory it would have a speed of about 10^{5} m/s at room temperature?

(c) Compare the result of part (b) with the observation that ions in a sodium metal have an average spacing of $3.8 \times 10^{-10}\ \mathrm{m}$.

Resistance of a Wire

According to Ohm's law, the resistance of a wire with length ℓ and cross-sectional area A is

$$R = \frac{\Delta V}{I} = \frac{E\ell}{I} = \frac{J\ell}{\sigma I} = \frac{1}{\sigma}\frac{\ell}{A} = \rho\frac{\ell}{A}.$$



Conceptual Question 2

A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, does the resistance of the wire

(a) increase

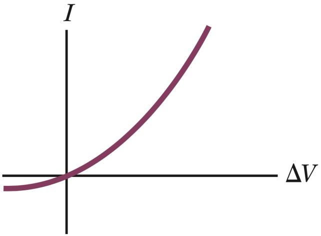
(b) decrease

(c) stay the same

Conceptual Question 3

In the figure, as the applied voltage increases, does the resistance of the diode

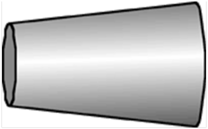
(a) increase
(b) decrease
(c) stay the same



Conceptual Question 4

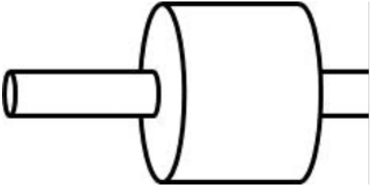
An ohmic conductor is carrying a current. The cross-sectional area of the wire changes from one end of the wire to the other. Which of the following quantities vary along the wire?

(a) resistivity
(b) current
(c) current density
(d) electric field



Group Activity

Consider a conducting wire in which the diameter changes as shown in the figure below. The diameter in the portion to the left is D and the diameter in the central portion is $2D$. The left and central portions have the same length L. Current I flows in from the left, and the resistivity of the wire is constant throughout.



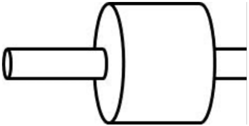
Problem 1. At which of the following points is the electric potential the greatest?

(a) The potential is constant because the wire is a conductor.

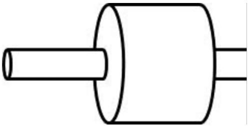
(b) At the left end.

(c) In the center.

(d) At the right end.



Problem 2. The electric field in the left portion is E_0. The electric field in the central portion is:



(a) $2E_0$

(b) $4E_0$

(c) E_0

(d) $E_0/2$

(e) $E_0/4$

Example 3: Resistance

Compare the current and resistance in 10.0-m lengths of 22-gauge copper wire ($1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$) and Nichrome wire ($1.0 \times 10^{-6}\ \Omega \cdot \mathrm{m}$). Both wires have a potential difference of 20.0 V across their lengths. 22-gauge wire has a diameter of 0.64 mm.

Is the magnitude of the electric field the same or different in the two wires?

Summary: Electrical Conduction

Ohm's law states that $\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$, where σ is a constant. From the definition of resistance $R \equiv \Delta V / I$, we find that for Ohmic matter, σ is constant, so R is constant. Drude's model is an attempt to understand why R is constant for many metals, but it ultimately fails to predict the value of the resistance and its dependence on temperature. We dive deeper into this problem in PHYS 153.

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}, \qquad \rho = \frac{1}{\sigma}, \qquad R = \frac{\Delta V}{I}$$

Electrical Power

The flow of current in a resistor involves the dissipation of energy. Suppose a steady current I, in amps, passes through a resistor of R ohms. In a time Δt, a charge of $I\Delta t$ coulombs is transferred through a potential difference of ΔV volts, where $\Delta V = IR$. Hence the work done in time Δt is $(I\Delta t)\Delta V = I^2 R\Delta t$. Electrical power is the rate of work done on moving charges:

$$P = \frac{W}{\Delta t} = I\Delta V = I^2 R \quad \text{(power delivered to resistor)}$$

$$[P] = \mathrm{J/s} = \mathrm{W}$$

Electromotive Force

In a DC circuit, energy is transferred from a source to a load. A DC source is characterized by its electromotive force, the highest possible potential difference it can provide. The origin of the electromotive force is some mechanism that transports charge carriers in a direction opposite to that in which the electric field is trying to move them.

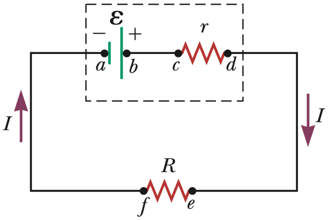
In ordinary batteries, chemical reactions result in charge carriers moving through a region where the electric field opposes their motion. That is, a positive charge carrier may move to a place of higher electric potential if by so doing it can engage in a chemical reaction that will yield more energy than it costs to climb the "electrical hill."

Terminal Voltage

Real batteries have some internal resistance, so when you connect them to a circuit, there will be a slight reduction in the electromotive force (EMF). The terminal voltage of a battery is the voltage reading between its terminals when connected to a circuit.

$$\Delta V = \mathcal{E} - Ir$$

$$I = \frac{\Delta V}{R}$$



Kirchoff's Rules

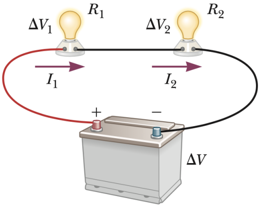
Any conceivable DC circuit, no matter how complicated, must satisfy the following conditions known as Kirchoff's rules:

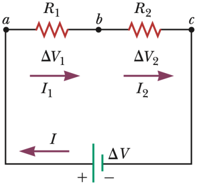
1. At a juction of the circuit, a point where three or more connecting wires meet, the algebraic sum of the currents into the node must be zero. (This is the charge conservation in circuit language.)
2. The sum of the potential differences taken in order around a loop of the network, a path beginning and ending at the same node, is zero. (This is because $\oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = 0$ for any closed path involving an electrostatic field.)

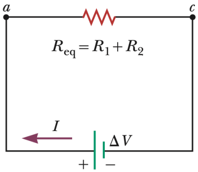
Resistors in Series

For resistors in series, the current through each resistor is the same due to charge conservation and the total potential divides between the resistors. We can define the equivalent resistance such that

$$\Delta V = I R_{\text{eq}} \qquad \text{if} \qquad R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$$



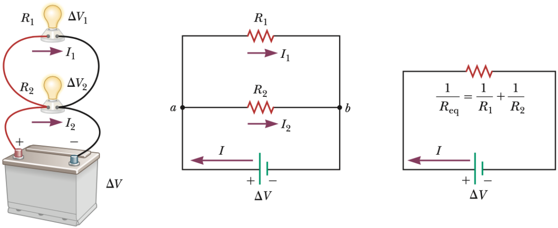




Resistors in Parallel

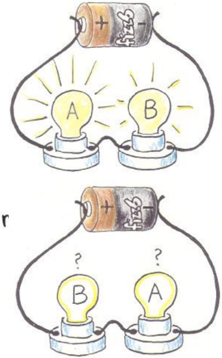
For resistors in parallel, the potential differences across each resistor are the same and the total current divides between the resistors.

$$\Delta V = I R_{\mathrm{eq}} \qquad \text{if} \qquad \frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$



Conceptual Question 5

When the series circuit shown to the right is connected, Bulb A is brighter than Bulb B. If the positions of the bulbs were reversed,



a) Bulb A would again be brighter

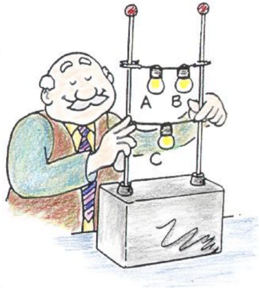
b) Bulb B would be brighter.

c) either of the above could occur.

Conceptual Question 6

Three identical lamps of resistance 12 ohms are connected to a 12-V battery as shown.

1. What is the current in each lamp?
2. What is the voltage across each lamp?
3. What is the power dissipated in each lamp?
4. How does the power dissipated in lamp A change when lamp C is unscrewed?



Example 4

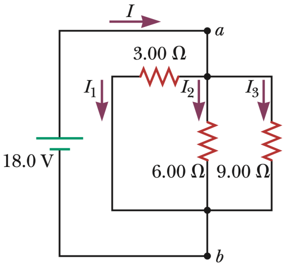
Three resistors are connected as shown in the figure. A potential difference of 18.0 V is maintained between points a and b.

(a) Discuss why all of the current I **does not** go through the 3-Ω resistor only.

(b) Calculate the equivalent resistance of the circuit.

(c) Find the current in each resistor.

(d) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.



Tips for Multi-Loop Circuits

(i) Apply Kirchhoff's rules to obtain as many independent equations as you have unknowns.

(ii) Draw the circuit diagram and assign labels and symbols to all known and unknown quantities.

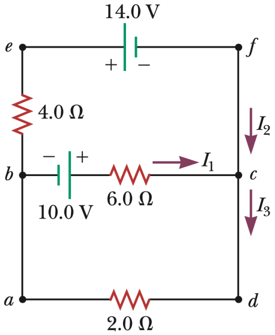
(iii) Assign directions to the currents. The direction must be the same along any single wire of the circuit.

(iv) Solve the equations simultaneously for unknown quantities.

(v) If a current turns out to be negative, the magnitude will be correct, and the direction is opposite to what you assigned.

Example 5

Find the currents I_1, I_2 and I_3 in the circuit below.



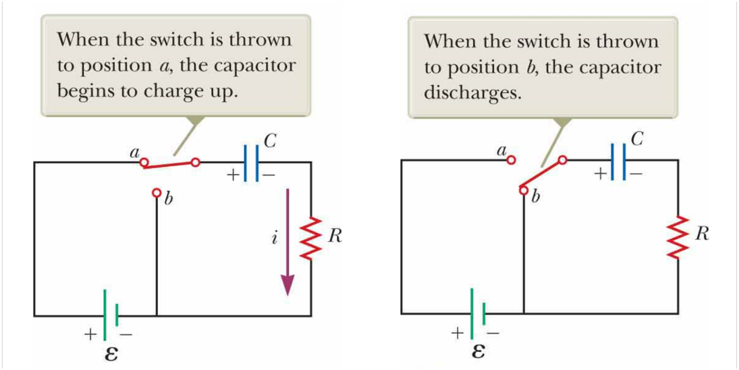
RC Circuits

In an RC circuit, the current either grows or vanishes with time in a way that depends on the resistance and capacitance present in the circuit. RC circuits can be used as timers for applications such as intermittent windshield wipers, pacemakers, and strobe lights. Later we will study the use of RC circuits with time-varying voltages.

With an uncharged capacitor in a DC circuit,

(i) At $t = 0$ (when the switch is closed) the capacitor acts like a piece of ideal wire very briefly, letting all the current through.

(ii) After a very long time, the capacitor doesn't let any current through, so the capacitor acts like an open switch.


When the switch is thrown to position a, the capacitor begins to charge up.
a
b
C
+
−
i
R
+
−
ε
When the switch is thrown to position b, the capacitor discharges.
a
b
C
+
−
R
+
−
ε

Charging an RC Circuit

During the charging phase, a battery is wired in series with a resistor and uncharged capacitor. By Kirchoff's loop rule,

$$\mathcal{E} - \frac{q}{C} - iR = 0 \qquad \longrightarrow \qquad RC\,\frac{dq}{dt} + q - C\mathcal{E} = 0$$

This equation can be solved to find the charge on the capacitor and the current in the circuit over time.

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) \quad \text{and} \quad i(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

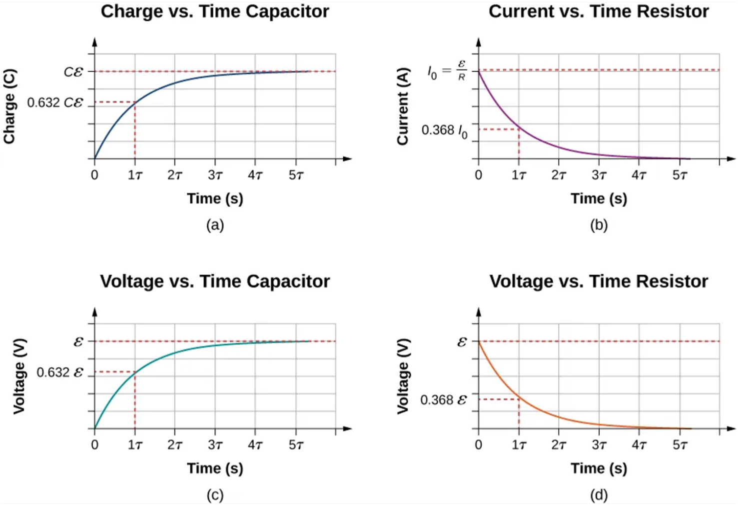
RC Time Constant

The quantity RC is called the time constant τ of the circuit:

$$\tau = RC$$

The time constant represents the time interval during which the current decreases to $1/e$ of its initial value, or equivalently, when the charge on the capacitor has reached 63.2% of its maximum.

While the charging process technically takes forever, by general agreement we consider the capacitor to be "fully charged" at $t = 5\tau$ when the charge has reached 99.3% of its theoretical maximum.


Charge vs. Time Capacitor
Charge (C)
$C\mathcal{E}$
$0.632\ C\mathcal{E}$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(a)
Current vs. Time Resistor
Current (A)
$I_0 = \frac{\mathcal{E}}{R}$
$0.368\ I_0$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(b)
Voltage vs. Time Capacitor
Voltage (V)
$\mathcal{E}$
$0.632\ \mathcal{E}$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(c)
Voltage vs. Time Resistor
Voltage (V)
$\mathcal{E}$
$0.368\ \mathcal{E}$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(d)

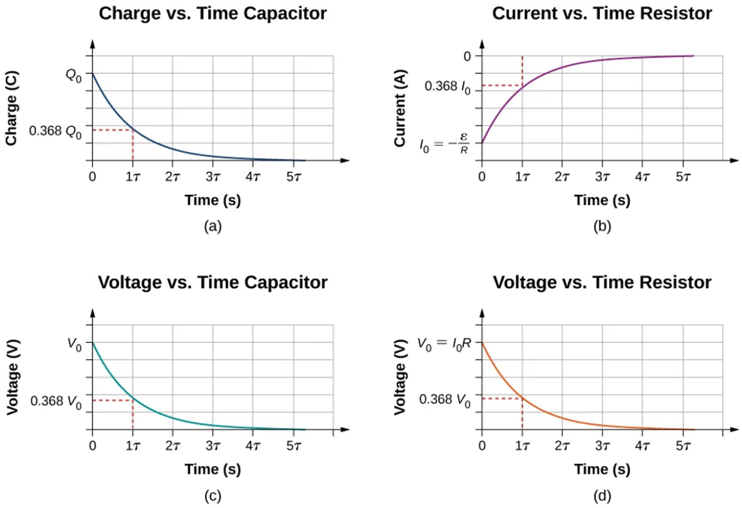
Discharging an RC Circuit

Start with a fully charged capacitor having charge Q_0 and let it discharge by placing it in series with a resistor only.

$$-\frac{q}{C} - iR = 0 \qquad \rightarrow \qquad RC\,\frac{dq}{dt} + q = 0$$

Let $q(0) = Q_0$ and solve for the charge and the current.

$$q(t) = Q_0 e^{-t/RC} \quad \text{and} \quad i(t) = -\frac{Q_0}{RC} e^{-t/RC}$$



Charge vs. Time Capacitor
Charge (C)
Q_0
$0.368\ Q_0$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(a)
Current vs. Time Resistor
Current (A)
0
$0.368\ I_0$
$I_0 = -\frac{\varepsilon}{R}$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(b)
Voltage vs. Time Capacitor
Voltage (V)
V_0
$0.368\ V_0$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(c)
Voltage vs. Time Resistor
Voltage (V)
$V_0 = I_0R$
$0.368\ V_0$
0
1τ
2τ
3τ
4τ
5τ
Time (s)
(d)

Example 6: Discharging an RC Circuit

Consider a capacitor with capacitance C that is being discharged through a resistor of resistance R.

(a) Show that all the electrical energy stored in a fully charged capacitor is released as thermal energy through the resistor.

(b) After how many time constants is the charge on the capacitor one-fourth its initial value?

(c) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value?

Relaxation Oscillators

A relaxation oscillator is an application of RC circuits used to produce low frequency periodic electrical signals by charging and discharging a capacitor. The capacitor is designed to discharge once it reaches a threshold voltage. Examples include blinking lights, windshield wipers, and electronic beepers.

The Peason-Anson oscillator (neon-lamp oscillator) is one of the simplest types of relaxation oscillators. It operates via the Pearson–Anson effect, discovered in 1922, which is the phenomenon of an oscillating voltage produced by a neon bulb connected across a capacitor, when a direct current is applied through a resistor.

Example 7: Neon-Lamp Oscillator

In the relaxation oscillator shown, the voltage source charges the capacitor until the voltage across the capacitor is 80 V. When this happens, the neon in the lamp breaks down and allows the capacitor to discharge through the lamp, producing a bright flash.

After the capacitor fully discharges through the neon lamp, it begins to charge again, and the process repeats. Assuming that the time it takes the capacitor to discharge is negligible, what is the time interval between flashes?