Phys 152: Fundamentals of Physics II

Unit #10 - Electromagnetic Waves

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James Clerk Maxwell

James Clerk Maxwell was a Scottish physicist responsible for the classical theory of electromagnetism. Born the year Faraday discovered changing magnetic fields create electric fields, it is fitting Maxwell established that *changing electric fields create magnetic fields.*

At the time, the only known source of a magnetic field was a steady current. In the case of a changing current, Maxwell demonstrated that Ampère's law conflicts with conservation of charge. By introducing a correction term to Ampère's law, Maxwell was able to create a consistent set of equations describing all electromagnetic phenomena and even predict electromagnetic waves in the process!

Conservation of Charge

Recall that the conservation of charge implies that any current flowing into or out of a region of space must be equal to the rate of change of charge in that region. Mathematically, we can express that as a relationship between the flux of the current density and the volume integral of the charge density:

$$\oint_{\mathcal{S}} \vec{\mathbf{J}} \cdot d\,\vec{\mathbf{a}} = -\frac{d}{dt}\int_{\mathcal{V}} \rho dV$$

The flux of charge through a closed surface $\mathcal{S}$ must equal the rate of change of charge in the region $\mathcal{V}$ bounded by $\mathcal{S}$.

Conservation of Charge

The minus sign reflects the convention that positive flux corresponds to the outward direction through a closed surface. For instance, if 2 protons enter $\mathcal{S}$ per second, then the flux of charge is $-2e$ per second and the charge in the region $\mathcal{V}$ *increases* by $2e$ per second. Don't let the symbols confuse you!

If you fill up a bucket with water at a rate of 2 kg per second, then the mass of the bucket must be increasing by 2 kg every second. If it isn't, then water must be spilling over the side or through a hole somewhere. The water that goes into the bucket is negative flux, the water that spills over the side is zero flux, and the water leaving the bucket through a hole is positive flux.

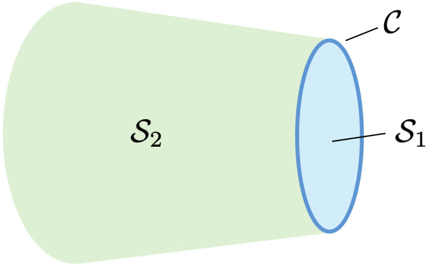
The problem with Ampère's law

The diagram below shows a circular closed curve C bounding two open surfaces $\mathcal{S}_1$ and $\mathcal{S}_2$. As the circumference of C shrinks to zero, the area of S_1 shrinks but that of $\mathcal{S}_2$ remains finite and becomes a *closed surface* (think of tying up a garbage bag for instance).

By Ampère's law, as we shrink the circumference to zero,

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 \oint_{\mathcal{S}_2} \vec{\mathbf{J}} \cdot d\,\vec{\mathbf{a}} = 0$$

Ampère's law implies there must never by any net current through a closed surface, but this contradicts the observation that there can be net current through a closed surface!


$\mathcal{C}$
$\mathcal{S}_2$
$\mathcal{S}_1$

Fixing Ampère's law

Physical laws can be non-intuitive but never contradictory or inconsistent with observation. Maxwell fixed Ampère's law with charge conservation and Gauss's law. Starting with conservation of charge,

$$\oint_{\mathcal{S}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} + \frac{d}{dt}\int_{\mathcal{V}} \rho dV = 0$$

$$\oint_{\mathcal{S}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} + \varepsilon_0 \frac{d}{dt}\oint_{\mathcal{S}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = 0$$

This relationship suggests a modification to Ampère's law that may apply to situations with non-steady current (next slide).

Ampère-Maxwell Law

In the case of a non-steady current, the Ampére-Maxwell law relates a magnetic field to moving charges and changing electric flux:

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 \int \vec{\mathbf{J}} \cdot d\,\vec{\mathbf{a}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{a}}$$

A magnetic field is present whenever there are moving charges and a changing electric flux.

The second term (divided by μ_0) is sometimes called *displacement current* but it does not correspond to any moving charges. It is only present when there is a changing electric flux through some surface.

Conceptual Question 1

In an RC circuit, the capacitor begins to discharge. During the discharge, in the region of space between the plates of the capacitor, there is/are

(a) conduction current but no displacement current

(b) displacement current but no conduction current

(c) both conduction and displacement current

(d) no current of any type

Conceptual Question 2

In the same region of space, there is/are

(a) an electric field but no magnetic field
(b) a magnetic field but no electric field
(c) both electric and magnetic fields
(d) no fields of any type

Example 1

A parallel-plate capacitor consisting of circular conducting plates each with radius R is connected to a DC circuit. Determine the magnetic field between the plates when the current in the circuit is I. The distance between the plates is much smaller than the radius.

Maxwell Equations

The Maxwell equations completely describe the electric and magnetic fields produced by moving and stationary charges.

$$\oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{a}} = \frac{Q_{\mathrm{enc}}}{\varepsilon_0} \qquad \oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{a}} = 0 \qquad \oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{\mathrm{thr}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Source-Free Maxwell Equations

Even in a region of space absent of moving or stationary charges, there can still be electric and magnetic fields.

$$\oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{a}} = 0 \qquad \oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{a}} = 0 \qquad \oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

A changing electric field supports a magnetic field and a changing magnetic field supports an electric field.

Maxwell Predicts Electromagnetic Waves

Using the source-free equations, Maxwell predicted the existence of electromagnetic waves that consist of oscillating electric and magnetic fields capable of traveling through empty space with a fixed speed denoted by the letter c and showed that

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299,792,458 \text{ m/s}$$

This figure was in good agreement with measurements of the speed of light (by Fizeau in 1849, for instance). Today the figure is exact in the SI system of units.

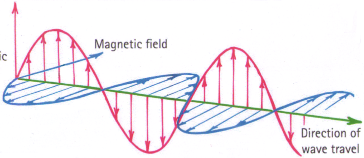
Hertz's Experiments

Heinrich Hertz experimentally verified Maxwell's predictions about electromagnetic waves in 1887-1888 by generating them using a *spark-gap transmitter* and detecting them with a *wire loop resonator*. He demonstrated that these waves exhibited properties like *reflection, refraction, interference, diffraction, and polarization*, confirming their similarity to light waves. By measuring their wavelength and speed, he showed they traveled at the speed of light, precisely as Maxwell predicted. Hertz provided the first direct evidence of electromagnetic waves before tragically passing away at the age of 36. Today we honor him with the unit of frequency, $1\ \mathrm{Hz} = 1\ \mathrm{s}^{-1}$.

Click here to watch a video on Hertz's experiments.

Conceptual Question 3

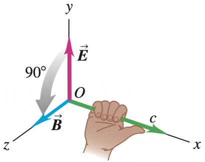
An electromagnetic wave consists of oscillating electric and magnetic fields that mutually induce each other. The existence of electromagnetic waves depends on



A. a critical speed of propagation.
B. equal-energy balanced fields.
C. the conservation of energy.
D. All of these.
E. Actually, none of these.

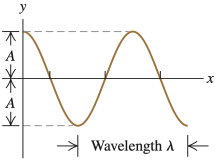
Properties of Electromagnetic Waves

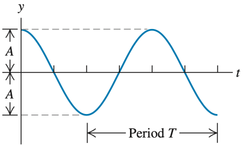
1. EM waves travel in vacuum with a definite and unchanging speed c. They require no medium unlike mechanical waves.
2. EM waves are **transverse**; both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and to the direction of propagation given by the direction of the vector product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.
3. There is a definite ratio between the magnitudes: $E = cB$



Sinusoidal Waves

A sinusoidal wave is characterized by its **wavelength**, **frequency**, and **amplitude**. Imagine forcing one end of a string to execute simple harmonic motion with frequency $f = 1/T$ (1 over the period). The disturbance that travels away from your hand is a sine wave, and the distance between any two points that are in phase with each other (like two crests or troughs of the wave) is the wavelength.





Electromagnetic Plane Waves

The simplest electromagnetic waves are (linearly polarized) sinusoidal plane waves. The electric field oscillates along a single direction, and at any instant, the fields are uniform over any plane perpendicular to the direction of propagation. The EM waves produced by an oscillating point charge or an antenna are not plane waves. Sufficiently far from any sources of EM waves, the plane wave approximation is valid. Since a traveling waves covers a distance of one wavelength in a time of one period, the speed of propagation satisfies

$$c = \frac{\lambda}{T} = \lambda f$$

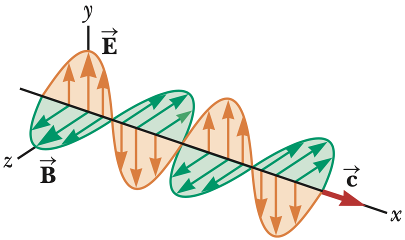
Fields of an EM Plane Wave

Consider an EM wave propagating in the $+x$ direction with an electric field that is polarized along the y direction. The fields can be mathematically described by the following set of equations:

$$\vec{\mathbf{E}} = E_{\max} \cos(kx - \omega t)\hat{\mathbf{j}}$$
$$\vec{\mathbf{B}} = B_{\max} \cos(kx - \omega t)\hat{\mathbf{k}}$$

where $k = 2\pi/\lambda$ is the **wave number** and $\omega = 2\pi/T = 2\pi f$ is the angular frequency of the wave. Notice that $\omega/k = c$.

The figure below represents a plane wave but is somewhat misleading. Electric and magnetic field vectors fill entire planes perpendicular to the x axis.



Click here for a better visualization of plane waves.

Why the minus sign?

I get this question a lot, so here's an easy way to understand it. Pretend for a moment that your friend can travel alongside the EM wave with speed c. According to your friend, the wave should be stationary. If your friend passes you at $t = 0$, any point in your frame is related to any point in their frame by $x = x' + ct$. Hence,

$$E = E_{\max} \cos\left(kx - \omega t\right) = E_{\max} \cos\left(k(x' + ct) - \omega t\right)$$

$$E = E_{\max} \cos\left(kx'\right)$$

This is purely mathematical; there is no "valid reference frame" where EM waves appear stationary–they don't require a medium!

Conceptual Question 4

An electromagnetic wave propagates in the negative y direction. The electric field at a point in space is momentarily oriented in the positive x direction. In which direction is the magnetic field at that point momentarily oriented?

(a) the negative x direction

(b) the positive y direction

(c) the positive z direction

(d) the negative z direction

Example 2

A plane electromagnetic sinusoidal wave propagates in the $+x$ direction. Suppose the wavelength is 50.0 m and the electric field vibrates in the xy plane with an amplitude of 22.0 V/m. Calculate

(a) the frequency of the wave and

(b) the magnetic field when the electric field has its maximum value in the negative y direction.

(c) Write an expression for the magnetic field $\vec{\mathbf{B}}(x, t)$

EM Waves Carry Energy

EM waves carry energy in their electric and magnetic fields. These fields can exert forces and do work on charges (i.e. make them move). The energy is present whether it is absorbed or not. The total energy density is the sum of the electric and magnetic field energy densities:

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

Since $E = cB$ and $c^2 = 1/\varepsilon_0\mu_0$, the energy density is

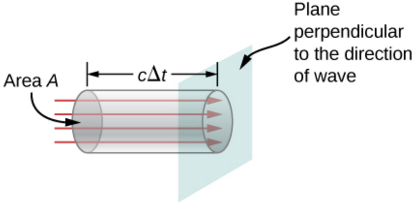
$$u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} = \frac{EB}{\mu_0} \cdot \frac{1}{c}$$

Power Density

The **power density** of an EM wave is the energy per unit time *per unit area* passing through a plane perpendicular to the wave. During a time interval Δt, a tube with length $c\Delta t$ and cross-sectional area A is the volume that passes through a window with area A. Hence,

$$S = \frac{u(Ac\Delta t)}{A\Delta t} = uc = \frac{EB}{\mu_0}$$

$$[S] = \mathrm{W/m^2}$$



Poynting Vector

The power density delivered to a surface clearly depends on the orientation of the surface and direction of wave propagation. We define the **Poynting vector** as the vector that *points* in the direction of the wave propagation and has magnitude equal to the power density:

$$\vec{\mathbf{S}} \equiv \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

The power delivered to a surface is

$$P = \int \vec{\mathbf{S}} \cdot d\,\vec{\mathbf{a}}$$

Wave Intensity

Typical frequencies of EM waves are very high, so most measuring devices are only sensitive to averages. The **wave intensity** is defined as the time-average magnitude of the Poynting vector:

$$I \equiv S_{\text{avg}}$$

In the case of a sinusoidal plane wave,

$$I = \frac{1}{2} c \varepsilon_0 E_{\max}^2 = \frac{c B_{\max}^2}{2\mu_0} = \frac{E_{\max} B_{\max}}{2\mu_0}$$

Conceptual Question 5

A point source of electromagnetic waves is isotropic–it emits radiation equally in all directions. If a detector at a distance d measures intensity I, what will be the intensity of the radiation when the detector moves to a distance $3d$ from the point source?

(a) The intensity remains the same because energy is conserved

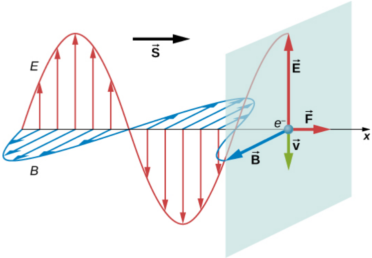
(b) The intensity decreases by a factor of three

(c) The intensity decreases by a factor of nine

(d) Not enough information

EM Waves Carry Momentum

Shake a string up and down all you want–you'll never exert a significant force in the same direction that transverse mechanical waves are propagating. An EM wave that is incident on an object *can exert a force in the direction of wave propagation*. This implies that EM waves carry momentum as well as energy!

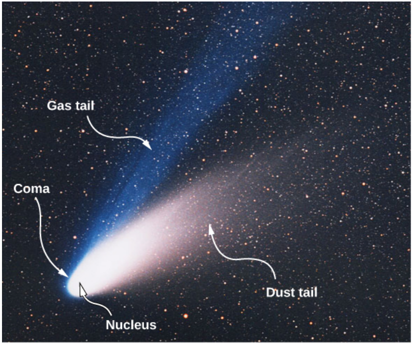


Radiation Pressure

The **radiation pressure** applied by an electromagnetic wave on a *perfectly absorbing surface* turns out to be equal to the energy density of the wave. If the material is *perfectly reflecting*, such as a metal surface, and if the incidence is along the normal to the surface, then the pressure exerted is twice as much because the momentum direction reverses upon reflection. We are generally interested in the **time-averaged radiation pressure** given by

$$\mathcal{P}_{\rm avg} = (1 + r)u_{\rm avg} = \frac{(1 + r)I}{c}$$

where r is the fraction of incident radiation that is reflected.


Gas tail
Coma
Dust tail
Nucleus

Conceptual Question 6

To maximize the radiation pressure on the sails of a spacecraft using solar sailing, the sheets should be

(a) very black to absorb as much sunlight as possible

(b) very shiny to reflect as much sunlight as possible

Example 3

At the upper surface of Earth's atmosphere, the time-averaged magnitude of the Poynting vector is $S_{\mathrm{avg}} = 1.362\,\mathrm{kW/m^2}$. This figure (the solar constant) measures the intensity of EM waves emitted by the Sun that are incident on the Earth!

(a) Assume the Sun's EM radiation is a plane wave and calculate the maximum values of E and B. Why can we treat the radiation as a plane wave?

(b) What is the total time-averaged power output of the Sun? The mean Sun-Earth distance is 1.50×10^{11} m.

(c) Assuming a reflectivity of 39%, what is the pressure on the sunlit hemisphere of the Earth?

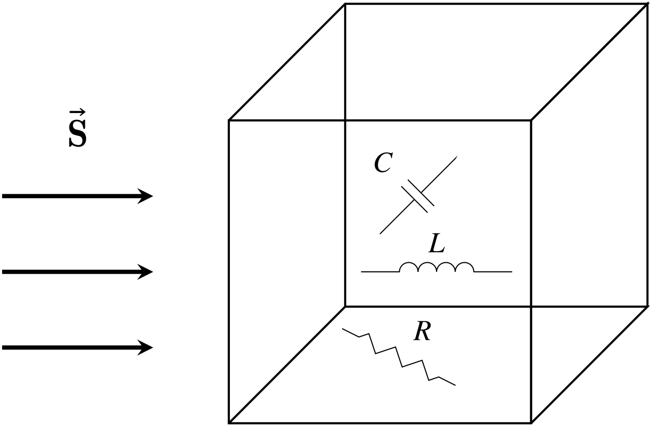
(d) Calculate the total force acting on the Earth as a result of radiation pressure.

Poynting's Theorem

The net electromagnetic power entering a region of space is equal to the rate of change of electromagnetic energy plus the rate of work done on moving charges by the electromagnetic fields:

$$-\oint \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}} = \frac{d}{dt}\int u dV + \int \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\, dV$$

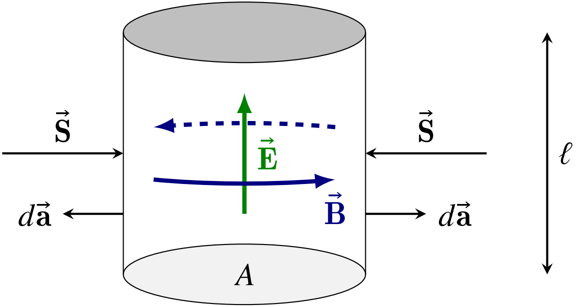
E.g. energy can be stored in capacitors and inductors, whereas in a resistor, work is done on moving charges. The second term involving the current density $\vec{\mathbf{J}}$ requires more explanation (see below); rather than derive Poynting's theorem, we'll look at a detailed example.



$\vec{S}$
C
L
R

Example 4

Consider a cylindrical conductor with resistivity ρ, length ℓ, and cross-sectional area A. Verify Poynting's theorem when there is a steady current I in the conductor resulting from a fixed potential difference across both ends.



Rate of Work Done on Moving Charges*

The infinitesimal work done by the electromagnetic field on a moving charge is the result of work done by the electric field only:

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = (q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}}dt = q\vec{\mathbf{v}} \cdot \vec{\mathbf{E}}\ dt$$

$$\Rightarrow \frac{dW}{dt} = q\,\vec{\mathbf{v}} \cdot \vec{\mathbf{E}}$$

For a continuous charge distribution moving with velocity field $\vec{\mathbf{v}}$

$$\frac{dW}{dt} = \int \rho\,\vec{\mathbf{v}} \cdot \vec{\mathbf{E}}\ dV = \int \vec{\mathbf{J}} \cdot \vec{\mathbf{E}}\ dV$$

Photons

In 1905, Einstein proposed the idea that light is made of particles called photons. Photons move at the speed of light and carry a *quantum of energy* that is proportional to the frequency f of the radiation they are associated with:

$$E_{\mathrm{photon}} = hf = \frac{hc}{\lambda}$$

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s} \quad \text{(Planck's constant)}$$

$$hc = 1240\ \mathrm{eV \cdot nm}$$

Photon Characteristics

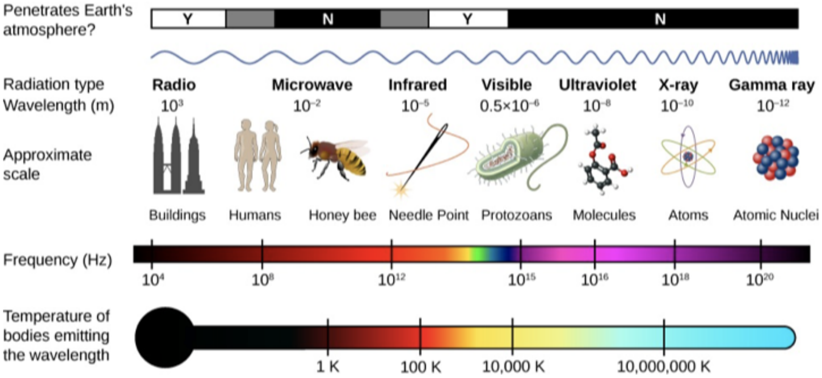
When photons interact with matter, they are either fully absorbed and transfer all of their energy, or no energy transfer occurs at all. Fractional absorption of EM radiation is therefore related to the number of photons that are absorbed. The energy of a photon strangely does not depend on the amplitude of the electric or magnetic field.

In 1923, Arthur Compton experimentally verified the prediction that **photons have momentum**:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Electromagnetic Spectrum

The **electromagnetic spectrum** is the range of all possible electromagnetic waves, classified by wavelength or frequency. It consists of radio waves (longest wavelength, from stars and artificial transmitters, used in communication), microwaves (produced by oscillating currents, used in radar and cooking), infrared (IR) radiation (emitted by warm objects, used in thermal imaging and remote controls), visible light (from the Sun and artificial sources, detected by the human eye), ultraviolet (UV) radiation (from the Sun and high-energy processes, can cause ionization and DNA damage), X-rays (decelerating high-energy electrons, used in medical imaging), and gamma rays (from nuclear reactions and cosmic events, highly penetrating and dangerous). Each region originates from different physical processes, from electron oscillations in antennas to nuclear transitions.



Penetrates Earth's atmosphere?
Y
N
Y
N
Radiation type
Wavelength (m)
Radio
Microwave
Infrared
Visible
Ultraviolet
X-ray
Gamma ray
10^{3}
10^{-2}
10^{-5}
0.5×10^{-6}
10^{-8}
10^{-10}
10^{-12}
Approximate scale
Buildings
Humans
Honey bee
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^{4}
10^{8}
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of bodies emitting the wavelength
1 K
100 K
10,000 K
10,000,000 K