Phys 152: Fundamentals of Physics II

Unit #1 - Electric Charges & Coulomb's Law

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The Universe is Electric

Electromagnetism is all around us. The structure and behavior of atoms and molecules are governed by electric forces. Our homes are powered by electric currents—lighting our rooms, running refrigerators, and charging our phones. Your brain sends signals through neurons using electrical impulses. Without electricity, you couldn't think, move, or even see, because light itself is electromagnetic.

We begin our study of electromagnetism by examining the nature of electric charge and the forces between them. We will introduce the concept of the electric field as a convenient way to describe interactions, and later in the course we will learn that the field is every bit as real as the particle that makes it!

Electric Charge

One fundamental property of electric charge is its existence in the two varieties that were historically named *positive* and *negative*. The observed fact is that all charged particles can be divided into two classes; all members of one class repel each other while attracting members of the other class.

Two positive charges or two negative charges repel each other.
A positive charge and a negative charge attract each other.

Electric charge is a scalar quantity that can be measured in terms of the force between charges a certain distance apart. Two important aspects of charge are its *conservation* and *quantization*.

Conceptual Question 1

Three objects are brought close to each other, two at a time. When objects A and B are brought together, they repel. When objects B and C are brought together, they also repel. Which of the following are true? Select all that apply.

(A) Objects A and C possess charges of the same sign.
(B) Objects A and C possess charges of opposite sign.
(C) All three objects possess charges of the same sign.
(D) One object is neutral.
(E) Additional experiments must be performed to determine the signs of the charges.

The Structure of Matter

The structure of atoms can be described in terms of three particles: the negatively charged electron, the positively charged proton, and the uncharged neutron. The number of protons or electrons in a neutral atom of an element is called the atomic number of the element. If one or more electrons are removed from an atom, what remains is called a positive ion. A negative ion is an atom that has gained one or more electrons. This gain or loss of electrons is called ionization.

In most cases, negatively charged (and highly mobile) electrons are added or removed, and a positively (or negatively) charged object is one that has lost (or gained) electrons. When we speak of the charge of an object, we always mean its net charge.

Conservation of Charge

The total electric charge in an isolated system is a constant and equals the algebraic sum of the positive and negative charges present.

$$Q_{\mathrm{net}} = q_1 + q_2 + \cdots$$

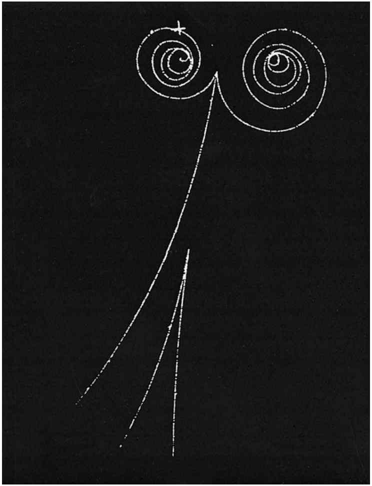
In any charging process, charge is not created or destroyed; it is merely transferred from one object to another, usually in the form of mobile electrons which carry negative charge.

When a glass rod is rubbed with silk, the silk acquires a negative charge and the glass rod acquires a positive charge. In fact, Benjamin Franklin defined a positive charge as the charge acquired by a glass rod when rubbed with silk, and this convention has persisted.

Pair-Creation Events

Conservation of charge is thought to be a *universal conservation law*. No experimental evidence for any violation of this principle has ever been observed. Even in high-energy interactions in which particles (with mass) are created and destroyed, the total charge of any closed system is exactly constant.

For every particle in nature, there can exist an *antiparticle*. The antiparticle carries charge of the opposite sign. A vacuum exposed to gamma rays might become the scene of a "pair-creation" event in which a high-energy photon vanishes with the creation of an electron and a *positron*. Two electrically charged particles can be created, but the net change in total charge is zero.



Quantization of Charge

Every observable amount of electric charge is an integer multiple of a basic unit called the fundamental charge which is equal to the magnitude of charge of the electron or proton.

$$Q_{\text{net}} = \pm Ne$$

where N is an integer and e is the fundamental charge. It is remarkable that the proton and the electron have the same magnitude of charge to within 1 part in 10^{20}. Perhaps a proton can decay into a positron and uncharged particles, but this has never been observed.

When we deal with macroscopic charged objects, we will usually ignore the quantization of charge, but it is worth mentioning!

Conductors and Insulators

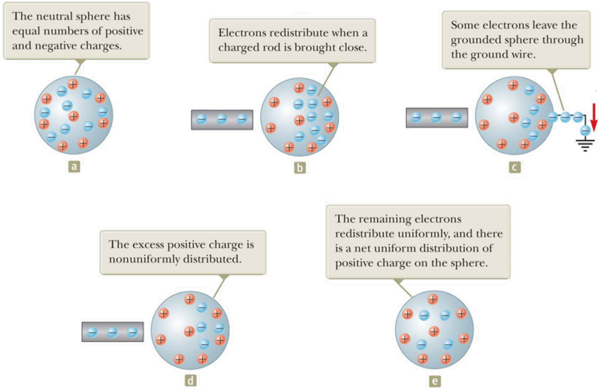
The motion of charges (e.g. electrons, ions, charged molecules) is called electrical conduction. Electrical conductors have free electrons that are not bound to particular atoms. Electrons in insulators are bound to atoms and cannot move freely through the material.

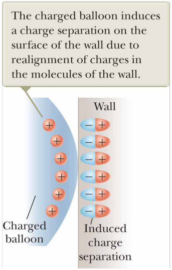
Most metals are good conductors, while most nonmetals are insulators. Within a solid metal such as copper, one or more outer electrons in each atom become detached and move freely throughout the material. Some materials called *semiconductors* are intermediate in their properties between good conductors and good insulators.

Charging by Induction

Charging by induction involves bringing a charged object close to a neutral conductor. This causes electrons in the conductor to move, creating regions of positive and negative charge. When the charged object is removed, the neutral conductor retains a temporary charge called an induced charge that is opposite to the charged object.

Induction requires no contact with the object inducing the charge. A process similar to induction in conductors takes place in insulators. Electrical forces cause a slight shift of charge within the molecules of the neutral insulator, an effect called polarization.


The neutral sphere has equal numbers of positive and negative charges.
a
Electrons redistribute when a charged rod is brought close.
b
Some electrons leave the grounded sphere through the ground wire.
c
The excess positive charge is nonuniformly distributed.
d
The remaining electrons redistribute uniformly, and there is a net uniform distribution of positive charge on the sphere.
e


The charged balloon induces a charge separation on the surface of the wall due to realignment of charges in the molecules of the wall.
Wall
Charged balloon
Induced charge separation

Conceptual Question 2

Three objects are brought close to one another, two at a time. When objects A and B are brought together, they attract. When objects B and C are brought together, they repel. Which of the following are necessarily true? Selet all that apply.

(A) Objects A and C possess charges of the same sign.

(B) Objects A and C possess charges of opposite sign.

(C) All three objects possess charges of the same sign.

(D) One object is neutral.

(E) Additional experiments must be performed to determine information about the charges on the objects.

Conceptual Question 3

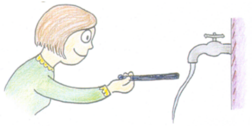
A thin stream of water bends toward a positively charged rod as the electrons in the stream are attracted toward the rod. When a negatively charged rod is placed near the stream, it will

____

(A) bend in the same direction.

(B) bend in the opposite direction.

(C) not bend at all.



Coulomb's Law (Qualitative)

Charles Coulomb measured the magnitudes of the electric forces between charged objects using a torsion balance, which he invented in 1784 and was later used by Cavendish to to study the much weaker gravitational interaction.

We use the term point charge to refer to a charged particle of "zero size." From experimental observations, we find that the magnitude of the electric force between two point charges is given by Coulomb's law: two stationary charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

Review of Vectors

A vector is a directed line segment with magnitude and direction. Represented with an arrow and/or boldface : $\vec{\mathbf{A}}$ is a vector.

A vector is part of an *equivalence class* of all directed line segments with the same magnitude and direction. Any two vectors with the same magnitude and same direction are equal.

A scalar is a single number that can be used to scale or modify a vector. Multiplication of a vector by a scalar has the effect of *scaling the magnitude* of the vector by the absolute value of the scalar.

Unit Vectors

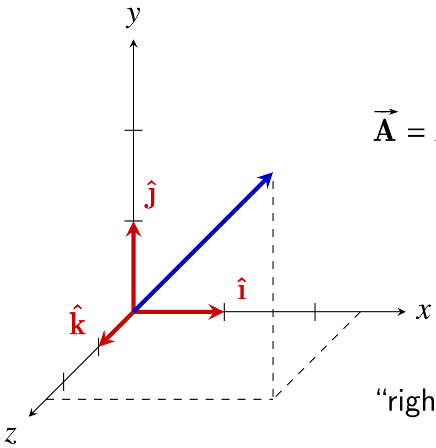
The magnitude of a vector is its length denoted by $|\vec{\mathbf{A}}| = A$; it is a scalar quantity greater than or equal to zero.

A unit vector has a magnitude of one. If $\vec{\mathbf{A}}$ is a vector, then the unit vector pointing in the same direction as $\vec{\mathbf{A}}$ is

$$\hat{\mathbf{A}} = \frac{\vec{\mathbf{A}}}{A} \quad \text{or} \quad \vec{\mathbf{A}} = A\hat{\mathbf{A}}.$$

The carat symbol is used to denote unit vectors.

Basis Vectors



$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

"right-handed coordinates"

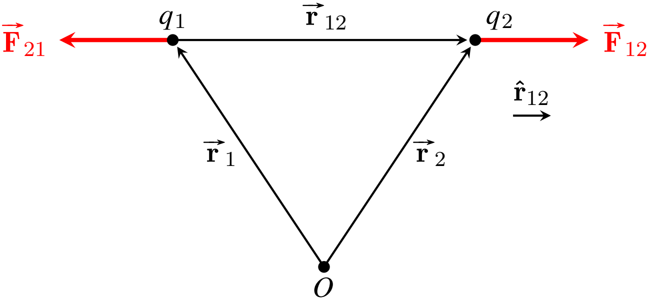
Coulomb's Law (Quantitative)

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12} \qquad \text{(force of 1 on 2)}$$

Here q_1 and q_2 are the values of the point charges, $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from charge 1 to charge 2.

The unit vector $\hat{\mathbf{r}}_{12}$ shows that the force is parallel to the line joining the charges. By Newton's third law, $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$.

The constant k_e is the Coulomb constant with a value that depends on the choice of units. We will stick to SI units in this class.


$\vec{\mathbf{F}}_{21}$
q_1
$\vec{\mathbf{r}}_{12}$
q_2
$\vec{\mathbf{F}}_{12}$
$\hat{\mathbf{r}}_{12}$
$\vec{\mathbf{r}}_1$
$\vec{\mathbf{r}}_2$
O

Units of Charge

The fundamental charge e is considered one of seven SI defining constants. The SI unit of charge is the coulomb (C) defined by taking the fixed numerical value of the fundamental charge to be

$$e = 1.602\ 176\ 634 \times 10^{-19}\ \mathrm{C}$$

There are about 6.24×10^{18} electrons in $-1\ \mathrm{C}$ of charge! The Coulomb constant is calculated from experiments to be

$$k_e = 8.987\ 551\ 792 \times 10^{9}\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$$

Two like charges, each of 1 coulomb, repel one another with a force of about 8.99×10^{9} newtons when they are 1 meter apart.

Permittivity of Free Space

The Coulomb constant is also written in the form

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

where ε_0 is the permittivity of free space and has the value

$$\varepsilon_0 = 8.854\ 187\ 819 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

The reason for this will become clear later, but for now there is no conceptual advantage to using ε_0.

Example 1: Hydrogen atom

The electron and proton of a hydrogen atom are separated (on average) by approximately 5.3×10^{11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Conceptual Exercise 4

Two oppositely charged particles, an alpha particle with 2 positive charges and a less-massive electron with a single negative charge are attracted to each other.

1. Compared to the force that the alpha particle exerts on the electron, the electron exerts a force on the alpha particle that is
   (A) greater. (B) the same. (C) less.
2. The particle with the greatest acceleration is the
   (A) alpha particle. (B) electron. (C) same for each.
3. As the particles get closer, each experiences an **increase** in
   (A) force. (B) speed. (C) acceleration. (D) all of the above.
   (E) none of the above.

Principle of Superposition

There are two essential features of Coulomb's law: (i) inverse-square dependence and (ii) the implication that electric charge is additive in its effects. When more than two charges are present, the force between any pair of them is given by Coulomb's law. The resultant electrostatic force on any single point charge is the vector sum of forces exerted by other charges.

For example, if four charges are present, the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

Conceptual Question 5

Three point particles each have charge q and lie on a straight line. The separation distance a between charges A and B is the same as that between B and C. If $f = k_e q^2/a^2$, which equation accurately represents the magnitude of the electric force on charge C?

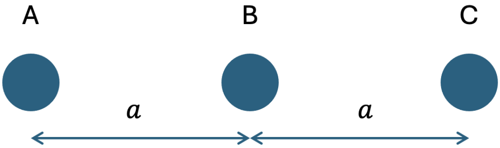
(A) f

(B) $2f$

(C) $3f/2$

(D) $5f/4$

(E) None of these

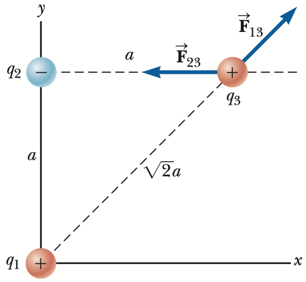


Example 2: Net force by vector addition

Consider three point charges located at the corners of a right triangle as shown in the figure, where $q_1 = q_3 = 5.00\ \mu\mathrm{C}$, $q_2 = -2.00\ \mu\mathrm{C}$, and $a = 0.100$ m.

Find the net force on q_3 where

$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$$



Example 3: Force equilibrium

Two charges $-Q_0$ and $-4Q_0$ are placed a distance ℓ apart. The charges do not move because there is a third charge being held nearby. What must be the magnitude and location of the third charge in order for the first two to be in equilibrium?

Concept of a Field

One hallmark of good science is the ability to criticize your own work. Newton was famously troubled by his law of gravity, despite its success. "That one body may act upon another at a distance through a vacuum without the mediation of anything else... is to me so great an absurdity."

In everyday life, we typically observe objects responding to their immediate surroundings (the technical term for this is locality). Coulomb's law is very similar to the law of gravity in several ways–most notably it appears to describe *instantaneous action at a distance* between point charges. The concept of an electric field was introduced by Michael Faraday to express the locality of electrical interactions. We say that electric charges create electric fields in the space surrounding them, and other charges interact with the electric field in their immediate surroundings. Electric charges both create and respond to electric fields!

Our task now is to figure out a mathematical description of the electric field that connects to the paradigm of forces introduced by Newton. Ultimately, we want a quantity that describes electrical interactions in a way that is independent of its effect on any particular object.

Force Vector Field

A vector field is a function that assigns a vector to every point in space. Consider a static configuration of source charges $q_1, q_2, \ldots,$ etc. We use a test charge q_0 to map out a vector field of forces that would be exerted on q_0 at all locations

$$\vec{\mathbf{F}}_0(\vec{\mathbf{r}}) = \sum_j \vec{\mathbf{F}}_{j0} = \sum_j k_e \frac{q_j q_0}{r_{j0}^2} \hat{\mathbf{r}}_{j0}$$

where $\vec{\mathbf{r}}_{j0} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_j$ is a vector pointing from source charge q_j to the test charge q_0.

Notice that the force on the test charge depends on the value of the test charge, so it cannot be characteristic of the space surrounding the source charges alone.

Electric Field

Define the electric field of a configuration of stationary source charges to be the force that they exert on a test charge divided by the value of the test charge:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) \equiv \frac{\vec{\mathbf{F}}_0(\vec{\mathbf{r}})}{q_0} = \sum_j k_e \frac{q_j}{r_{j0}^2} \hat{\mathbf{r}}_{j0} \qquad [E] = \mathrm{N/C}$$

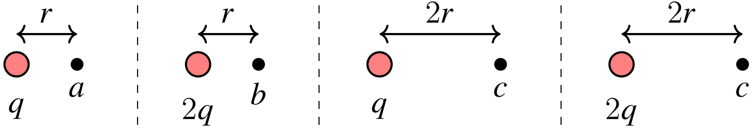
For a single source charge q located at the origin, we obtain

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = k_e \frac{q}{r^2} \hat{\mathbf{r}} \qquad \text{(electric field of a point charge)}$$

The electric field depends only on (i) the structure of the source charges and (ii) the position of the test charge only, not the actual value of the test charge.

Conceptual Question 6

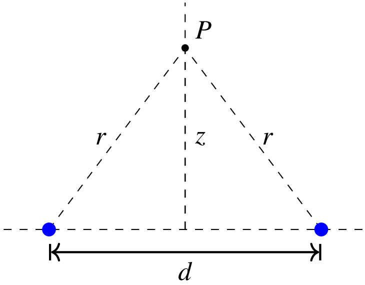
Rank in order, from largest to smallest, the electric field strengths E_a, E_b, E_c, and E_d at points a, b, c, and d. Don't worry about superposition–the point charges shown below are very far away from each other (hence the dashed lines).



Example 4: Electric Dipoles

(a) Find the electric field (magnitude and direction) a distance z above the midpoint between two equal charges $+q$ that are a distance d apart. Check that your result is consistent with what you'd expect when $z \gg d$.

(b) The same as part (a), only this time make the right-hand charge $-q$ instead of $+q$.



Electric Field Lines

An electric field line is a curve whose tangent, at any point, lies in the direction of the field at that point. Field lines always begin on positive charges and end on negative charges. Electric field lines are not paths of particles!

The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. The field lines are close together where the electric field is strong and far apart where the field is weak.

Field lines are smooth and continuous except at singularities like point charges and surface charges. Since the electric field is unique at every point in space, electric field lines can never cross. The field lines of different sources will add to produce a unique field vector at every point in space.

Charged Particle in an Electric Field

The electric force experienced by some charge q located by position vector $\vec{\mathbf{r}}$ in an electric field $\vec{\mathbf{E}}$ is

$$\vec{\mathbf{F}}_e(\vec{\mathbf{r}}) = q\vec{\mathbf{E}}(\vec{\mathbf{r}})$$

The electric field vector is all we need to know to predict the force that will act on any charge at that point. Consider a point charge with mass m in a region of space containing a uniform electric field. In the absence of other interactions,

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}} \quad \Rightarrow \quad \vec{\mathbf{a}} = \frac{q}{m}\vec{\mathbf{E}}$$

Even though electrons have a small charge and a small mass, the charge-to-mass ratio of the electron is about $1.76 \times 10^{11}\ \mathrm{C/kg}$ in SI units. For the proton, it's $9.58 \times 10^{7}\ \mathrm{C/kg}$.

Example 5: Uniform E-field

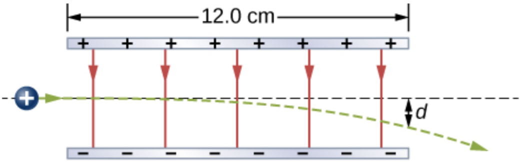
A uniform electric field $\vec{\mathbf{E}}$ is directed along the x axis between parallel plates of charge separated by a distance d. A positive point charge q of mass m is released from rest at a point A next to the positive plate and accelerates to a point B next to the negative plate.

(a) Find the speed of the particle at B by modeling it as a particle under constant acceleration.

(b) Find the speed of the particle at B using the work-energy theorem.

Example 6: Electric Projectile

A proton enters the uniform electric field produced by the two charged plates shown below. The magnitude of the electric field is 4.0×10^5 N/C, and the speed of the proton when it enters is 1.5×10^7 m/s. What distance d has the proton been deflected downward when it leaves the plates?



Uniform Charge Densities

We will learn how to calculate electric fields created by continuous charge distributions. These objects are characterized by charge density (the charge per unit length, area, or volume).

When the charge is spread evenly throughout the object, we say the object is uniformly charged. The charge density does not vary inside a uniformly charged object. We distinguish between linear charge density λ, surface charge density σ, and volume charge density ρ.

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{L}, \quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A}, \quad \rho = \frac{\text{charge}}{\text{volume}} = \frac{Q}{V}$$

Conceptual Question 7

A piece of plastic is uniformly charged with surface charge density σ. The plastic is then broken into a large piece with surface charge density σ_A and a smaller piece with surface charge density σ_B. Which of the following statements is true?

(a) $\sigma > \sigma_A > \sigma_B$

(b) $\sigma = \sigma_A = \sigma_B$

(c) $\sigma < \sigma_A < \sigma_B$

Follow up question: if the smaller piece has area a_B, what is its total charge in terms of σ and a_B?

Electric Field Models

The electric fields used in science and engineering are often caused by fairly complex distributions of charge. Sometimes these fields require exact calculations, but much of the time we can understand the essential physics by using simplified models of the electric field.

Four common electric field models can be used to understand a wide variety of electric phenomena.

(i) Point charge model (for small charged objects)

(ii) Infinite line charge model (for wires)

(iii) Infinite plane model (for electrodes)

(iv) Spherical charge model (far away from localized charges)

Infinite Line of Charge

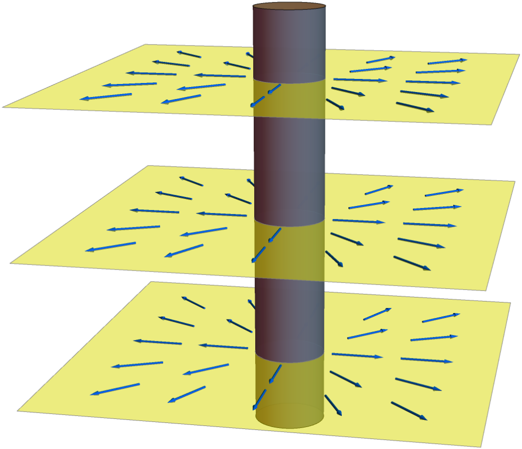
The electric field strength of an infinite line of charge with uniform linear charge density λ is

$$\vec{\mathbf{E}}_{\text{line}} = \left(\frac{|\lambda|}{2\pi\varepsilon_0 r} \,, \quad \begin{cases} \text{away from the line if } \lambda > 0 \\ \text{towards the line if } \lambda < 0 \end{cases} \right)$$

where r is the perpendicular distance from the line.

Unlike a point charge, for which the field decreases as $1/r^2$, the field of an infinitely long charged wire decreases more slowly–as only $1/r$. This approximates the field of a finite-length wire near its center.

Infinite Line of Charge



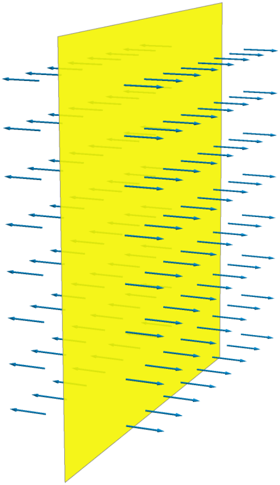
Infinite Plane of Charge

The electric field strength of an infinite plane of charge with uniform surface charge density σ is

$$\vec{\mathbf{E}}_{\text{plane}} = \left(\frac{|\sigma|}{2\varepsilon_0} \, , \quad \begin{cases} \text{away from the plane if } \sigma > 0 \\ \text{towards the plane if } \sigma < 0 \end{cases} \right)$$

Surprisingly, the field produced by an infinite plane is uniform above or below the plane! The strength of the field does not depend on distance. This approximates the field of a finite-sheet near its center.

Infinite Plane of Charge



Example 7: Parallel Plate Capacitor

Find the electric field everywhere resulting from two infinite planes with equal but opposite surface charge densities $\pm\sigma$.

What if there were two positively charged plates with equal charge densities?

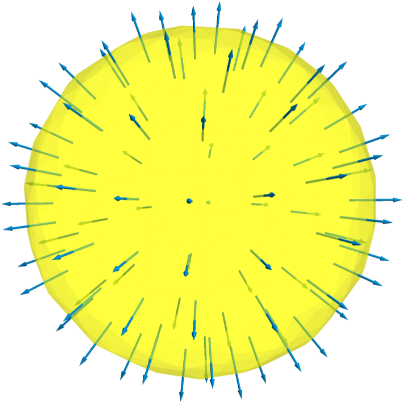
Spherical Charge

A sphere of charge Q and radius R could be a spherical shell with uniform σ over its surface or a solid sphere with uniform ρ throughout its volume. Either way, outside the sphere, the electric field is the same as that of a point charge Q located at the center of the sphere:

$$\vec{\mathbf{E}}_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad \text{for } r \geq R$$

This deceptively intuitive result is actually quite difficult to prove using Coulomb's law! We will learn a better way to do it.

Spherical Charge



Example 8: An orbiting proton

In a vacuum chamber, a proton orbits a 1.0-cm-diameter metal ball 1.0 mm above the surface with a period of 1.0 $\mu\mathrm{s}$. What is the charge on the ball?