Physics 438A – Lecture #6
Bound States and Scattering

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1 Energy Diagrams

In the previous lecture, we introduced the wave function and solved for the dynamics of a free particle–one for which the potential energy was a constant value at all positions. We will now consider potential energy functions that vary with position as in Figure 1.1.

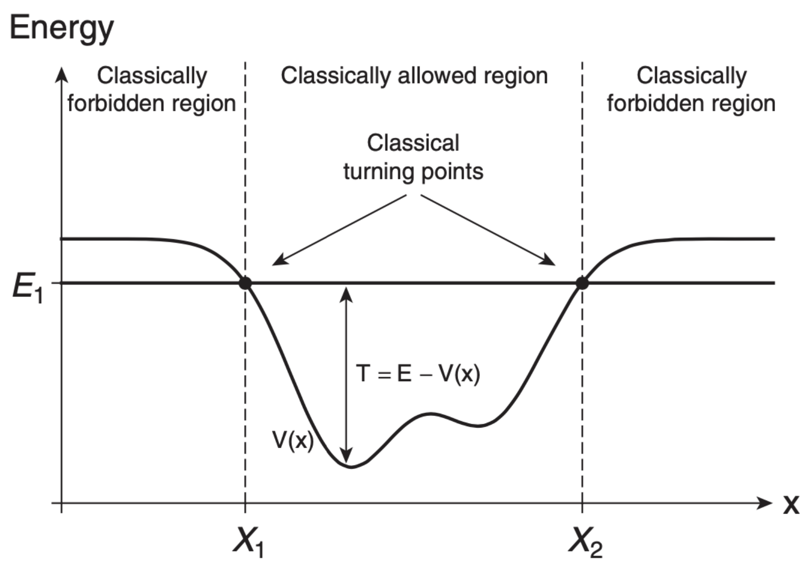

Figure 1.1: A generic potential energy well (McIntyre, 2022).

For a classical particle, the kinetic energy cannot be negative, so a classical particle with the energy E_1 chosen in Figure 1.1 has its motion constrained to the region between x_1 and x_2. These extreme points of the classical motion are called **classical turning points** and the region within the turning points is called the **classically allowed region**, while the regions beyond are called **classically forbidden regions**. Particles that have their motion constrained by the potential well are said to be in **bound states**. Particles with energies above the top of the potential well do not have their motion constrained and so are in **unbound states**. Note that the extent of the classically forbidden and allowed regions depends on the specific value of the energy, E_1, for a particular bound state.

2 Infinite Square Well

Consider a "super ball" bouncing between two perfectly elastic walls. We call this system a **particle in a box**. We observe three important characteristics of this classical system: (1) the ball flies freely between the walls, (2) the ball is reflected perfectly at each bounce, and (3) the ball remains in the box no matter how large its energy. These three observations are consistent with (1) zero force on the ball when it is between the walls, (2) infinite force on the ball at the walls, and (3) infinite potential energy outside the box.

The potential energy function for the classical super ball is a piecewise function

$$V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{otherwise} \end{cases} \tag{2.1}$$

and we posit that the quantum mechanical potential energy operator is simply $V(\hat{x})$ where x is "promoted" to an operator. We're interested in the definite-energy states of this system, and for a particle with finite energy E, $\psi(x) = 0$ outside the box, where $\psi(x)$ is the time-independent portion of the definite-energy state wave function. Inside the box,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

which is the same differential equation we encountered for the free particle. Define

$$k^2 = 2mE/\hbar^2 \qquad \text{(wave vector squared)}$$

and with foresight, let's write the general solution in terms of real functions

$$\psi(x) = A\sin(kx) + B\cos(kx) \tag{2.2}$$

Since $\psi(x)$ must vanish at the two boundaries, we have $B = 0$ and

$$\sin(ka) = 0 \tag{2.3}$$

This is a transcendental equation that places limitations on the allowed values of the wave vector k. The wave vectors that satisfy this equation are

$$k_n = \frac{n\pi}{a}, \quad \text{where } n = 1, 2, \ldots \tag{2.4}$$

where the negative n values can be absorbed into the constant A since $\sin(-\theta) = -\sin(\theta)$

and $n = 0$ is excluded because $\psi(x) = 0$ everywhere is not physical (normalizable). Only discrete wave vectors are allowed, so this is called a **quantization condition**. The index n is the **quantum number**, which we use to label the quantized states and energies. The wave vector quantization condition leads directly to energy quantization:

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad \text{where } n = 1, 2, \ldots \tag{2.5}$$

Any time the energy of this system is measured, the only possible values are given by Eq. (2.5), and the wave function will collapse to the energy eigenstate corresponding to whatever value of energy is actually obtained in the measurement.

The normalized energy eigenstates, labeled by the quantum number n, are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad \text{where } n = 1, 2, \ldots \tag{2.6}$$

Each solution looks just like the standing waves on a string of length a. We call the solution with lowest energy, ψ_1, the ground state. All solutions with higher energy are excited states. All together, the functions $\psi_n(x)$ constitute a countably infinite set of eigenfunctions.

Recall our discussion of quantum dynamics in Lecture #3. Even though we assumed the system was finite-dimensional, the same analysis applies to discrete systems which may be infinite dimensional. Due to the quantization of energy eigenvalues, the Hamiltonian for a particle in a 1D box admits a spectral decomposition

$$H = \sum_{n=1}^{\infty} E_n |E_n\rangle\langle E_n|$$

where the upper bound now goes to infinity, E_n are the energy eigenvalues, and $|E_n\rangle$ are the energy eigenstates in Dirac notation. Given an initial state vector

$$|\psi(0)\rangle = \sum_n c_n |E_n\rangle,$$

the most general time-dependent solution to the Schrödinger equation is

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

In some sense, the energy eigenstates form the "preferred basis" in which to expand a general quantum state vector, with the time evolution determined by phase factors dependent on the energy of each component state.

We can now translate everything into the position basis. The energy eigenfunctions are

$$\psi_n(x) = \langle x|E_n\rangle$$

and the initial wave function is given by

$$\psi(x,0) = \langle x|\psi(0)\rangle = \sum_n c_n \psi_n(x)$$

We can solve for the coefficients c_n using orthonormality of the energy eigenstates and the completeness relation for the position eigenstates:

$$c_n = \langle E_n|\psi(0)\rangle = \langle E_n| \int_{-\infty}^{\infty} dx |x\rangle\langle x|\psi(0)\rangle = \int_{-\infty}^{\infty} dx\ \psi_n^*(x)\psi(x,0)$$

Physically, the coefficients are related to the probabilities for measuring the system to have one of the energy eigenvalues. For instance, the probability of measuring E_n is

$$\mathcal{P}(E_n) = |c_n|^2$$

and the expectation value of the Hamiltonian can be calculated as

$$\langle H\rangle = \sum_n |c_n|^2 E_n$$

Once we have the wave function expansion coefficients in the energy basis, we can predict the future time evolution of the system. Then we can calculate any physical quantities we need to, such as probabilities and expectation values.

Example 2.1: Infinite Square Well

A particle in an infinite square well has the initial wave function

$$\psi(x,0) = Ax(a-x), \qquad (0 \le x \le a)$$

for some constant A. Outside the well, of course, $\psi = 0$. Find $\psi(x,t)$.

Solution: First we need to determine a suitable value for A that ensures ψ is a normalized wave function. Normalizing the wave function, we find

$$1 = \int_0^a dx\ |\psi(x,0)|^2 = |A|^2 \int_0^a dx\ x^2(a-x)^2 = |A|^2 \frac{a^5}{30} \to A = \sqrt{\frac{30}{a^5}}$$

The nth coefficient in the expansion is

$$c_n = \sqrt{\frac{2}{a}} \int_0^a dx \ \sin\left(\frac{n\pi x}{a}\right) \sqrt{\frac{30}{a^5}} x(a-x)$$

$$= \frac{2\sqrt{15}}{a^3}\left[a\int_0^a dx\ x\sin\left(\frac{n\pi x}{a}\right) - \int_0^a dx\ x^2 \sin\left(\frac{n\pi x}{a}\right)\right]$$

$$= \begin{cases} 0, & n \text{ even} \\ 8\sqrt{15}/(n\pi)^3, & n \text{ odd} \end{cases}$$

Thus,

$$\psi(x,t) = \sqrt{\frac{30}{a^5}}\left(\frac{2}{\pi}\right)^3 \sum_{n=1,3,5,\dots} \frac{1}{n^3}\sin\left(\frac{n\pi x}{a}\right)\exp\left(-\frac{in^2\pi^2\hbar t}{2ma^2}\right) \tag{2.7}$$

As an interesting side-quest, let's consider the sum of the absolute squares of the coefficients. Since the initial state is normalized, it must be the case that

$$\sum_{n=1,3,5,\dots} |c_n|^2 = 1$$

Hence,

$$\frac{960}{\pi^6}\left(\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \cdots\right) = 1$$

and indeed, one can show from purely a mathematical argument that

$$\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \cdots = \frac{\pi^6}{960}$$

The expectation value of the Hamiltonian is

$$\langle H \rangle = \sum_n E_n |c_n|^2 = \sum_n \frac{n^2\pi^2\hbar^2}{2ma^2}\left(\frac{8\sqrt{15}}{n^3\pi^3}\right)^2 = \frac{480\hbar^2}{\pi^4 ma^2}\sum_{n=1,3,5,\dots}\frac{1}{n^4} = \frac{5\hbar^2}{ma^2}$$

since

$$\sum_{n=1,3,5,\dots}\frac{1}{n^4} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}$$

The value of $\langle H \rangle$ is very close to E_1, as one might expect given the similarity of the initial state to that of the ground state.

3 Finite Square Well

Consider the finite square well potential energy function given by

$$V(x) = \begin{cases} 0, & x < -a, \\ -V_0, & -a < x < a, \\ 0, & x > a \end{cases} \tag{3.1}$$

For now we will look for bound state solutions; we'll save the unbound states for the next section. In this case, bound state solutions correspond to a particle with energy $-V_0 < E < 0$, the maximum of the potential energy. Since there is a discontinuity in $V(x)$ at $x = \pm a$, we split the wave function into three regions and enforce the following boundary conditions:

1) The wave function $\psi(x)$ is everywhere continuous.

2) The first derivative $\frac{d}{dx}\psi(x)$ is continuous except where $V = \infty$.

The energy eigenvalue equation for the region $x < -a$ is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \qquad (E < 0) \tag{3.2}$$

and the general solution to this differential equation can be written as

$$\psi(x) = Ae^{-qx} + Be^{qx}, \quad \text{where } q = \frac{\sqrt{2m|E|}}{\hbar} > 0 \tag{3.3}$$

which corresponds to a superposition of exponential growth and decay. The first term grows without bound as $x \to -\infty$, so we must choose $A = 0$ to have a physical wave function.

In the region $-a < x < a$, the energy eigenvalue equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi \qquad (E < 0)$$

or

$$\frac{d^2\psi}{dx^2} = -k^2\psi, \quad \text{where } k = \frac{\sqrt{2m(V_0 - |E|)}}{\hbar} > 0$$

The general solution is oscillatory, and it can be written in a form that makes it easier to implement the boundary conditions:

$$\psi(x) = C\sin(kx) + D\cos(kx) \tag{3.4}$$

The last region, $x > a$, is identical to the first region where $x < -a$, expect we want the wave function to vanish when $x \to \infty$. The energy eigenstate must be constructed by connecting solutions in the three regions. We write the general solution as

$$\psi(x) = \begin{cases} Be^{qx}, & x < -a \\ C\sin(kx) + D\cos(kx), & -a < x < a \\ Ge^{-qx}, & x > a \end{cases} \tag{3.5}$$

Boundary at the left wall

The boundary conditions at the left wall where $x = -a$ give

$$\begin{aligned} \psi(-a): &\quad Be^{-qa} = -C\sin(ka) + D\cos(ka) \\ \psi'(-a): &\quad -qBe^{-qa} = kC\cos(ka) + kD\sin(ka) \end{aligned} \tag{3.6}$$

Boundary at the right wall

The boundary conditions at the right wall where $x = a$ give

$$\begin{aligned} \psi(a): &\quad Ge^{-qa} = C\sin(ka) + D\cos(ka) \\ \psi'(a): &\quad qGe^{-qa} = kC\cos(ka) - kD\sin(ka) \end{aligned} \tag{3.7}$$

Divide derivative equations by k, then add and subtract the equations to get

$$\begin{aligned} 2D\cos(ka) &= (B+G)e^{-qa} \\ 2C\sin(ka) &= (-B+G)e^{-qa} \\ 2C\cos(ka) &= \frac{q}{k}(B-G)e^{-qa} \\ 2D\sin(ka) &= \frac{q}{k}(B+G)e^{-qa} \end{aligned}$$

Eliminate B and G from the equations to find

$$\begin{aligned} 2C\left[\sin(ka) + \frac{k}{q}\cos(ka)\right] &= 0 \\ 2D\left[\cos(ka) - \frac{k}{q}\sin(ka)\right] &= 0 \end{aligned}$$

So either $C = 0$ or $D = 0$ (both cannot be zero, otherwise the wave function would be zero). This means we have either cosine or sine solutions; the energy eigenstates have definite parity (even or odd) just like the infinite square well. For the even cosine solutions ($C = 0$),

$$k \tan(ka) = q \tag{3.8}$$

and for the odd sine solutions ($D = 0$),

$$-k \cot(ka) = q \tag{3.9}$$

Equations (3.8) and (3.9) constitute quantization conditions; the solutions for the allowed energies come in discrete values which must be solved for numerically. This will easier if we work with dimensionless variables. To find a natural set of units, let's write the eigenvalue equation inside the well as

$$\frac{\hbar^2}{2ma^2}\left(a^2\frac{d^2\psi}{dx^2}\right) = -(V_0 - |E|)\psi$$

We define units such that

$$[\text{energy}] = \frac{\hbar^2}{2ma^2}, \qquad [\text{length}] = a$$

which is equivalent to setting $\hbar = m = a = 1$. In dimensionless units,

$$k^2 + q^2 = V_0, \quad \text{or} \quad q = \sqrt{V_0 - k^2}$$

The quantization conditions can be written in terms of dimensionless variables as

$$\begin{aligned} k \tan k &= \sqrt{V_0 - k^2} \\ -k \cot k &= \sqrt{V_0 - k^2} \end{aligned} \tag{3.10}$$

which are transcendental equations which must be solved numerically/graphically. Once the value of k is found, we can then find the dimensionless energy E using

$$E = -q^2 = k^2 - V_0,$$

and then simply multiply by $\hbar^2/2ma^2$ to recover ordinary units. For $V_0 = 36$, we find four points of intersection between the left- and right-hand sides of Eq. (3.10), as shown in

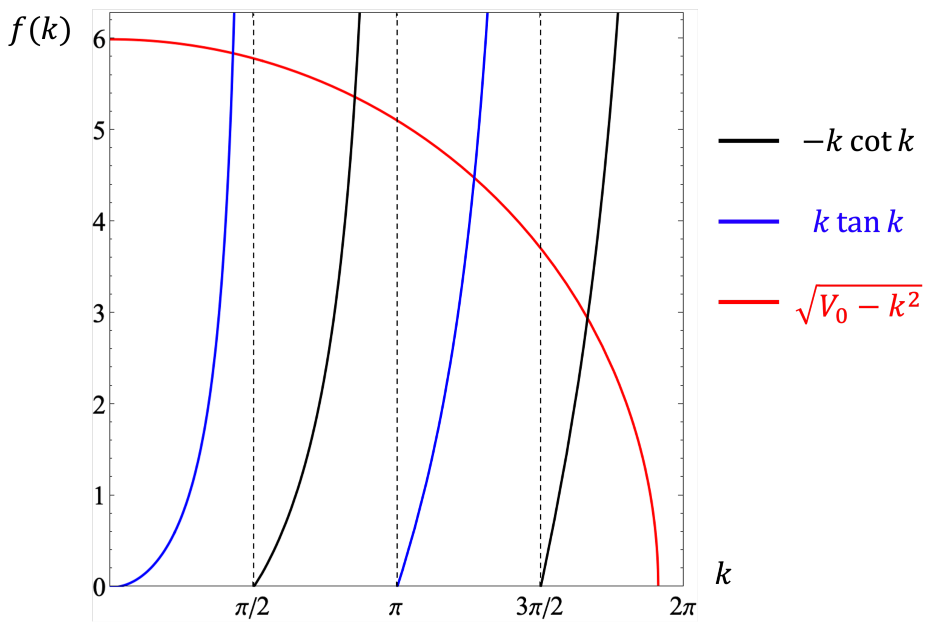

Figure 3.1: Graphical solution of the transcendental equations for the allowed energies of a finite square well for $V_0 = 36$ in dimensionless units.

Figure 3.1. The corresponding energy eigenvalues are, in ordinary units,

$$k_1 a = 1.34475: \; E_1 = -34.1916\frac{\hbar^2}{2ma^2}$$

$$k_2 a = 2.67878: \; E_2 = -28.8241\frac{\hbar^2}{2ma^2}$$

$$k_3 a = 3.98583: \; E_3 = -20.1132\frac{\hbar^2}{2ma^2}$$

$$k_4 a = 5.22596: \; E_4 = -8.68931\frac{\hbar^2}{2ma^2}$$

The limit of an infinitely deep well corresponds to the case that $V_0 \to \infty$. In other words, the radius of the quarter circle in Figure 3.1 tends to infinity and the allowed values of k become the asymptotes of $\tan k$ and $\cot k$. In this limit, the allowed values are given by

$$k_n a = \frac{n\pi}{2} \Rightarrow k_n = \frac{n\pi}{2a}$$

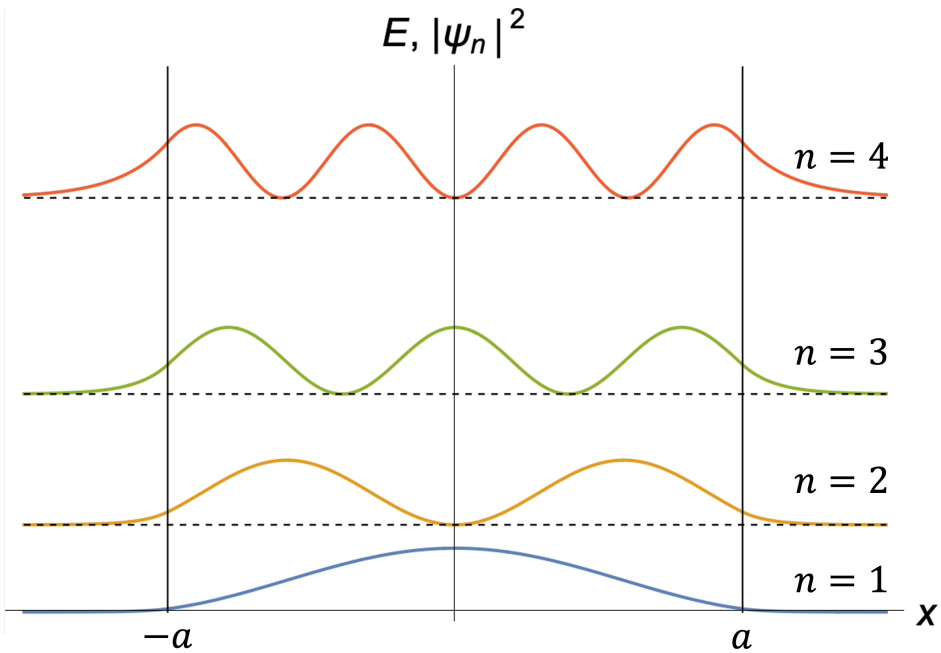

Figure 3.2: Square magnitude of the energy eigenstates for a finite square well with $V_0 = 36$ in dimensionless units.

which is exactly the same quantization condition for the infinite well. We can redefine the zero of potential energy by simply adding V_0 to the energy values, and we find

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$$

In other words, we recover the energy eigenvalues of the infinite energy well. On the other hand, when V_0 is very small, there is always at least one intersection that occurs for $ka < \pi/2$.

The energy eigenfunctions are constrained by the boundary conditions and normalization. In your homework (?), you'll discover that the even solutions are

$$\psi_{\text{even}}(x) = \sqrt{a + \frac{1}{q}} \begin{cases} e^{qa}\cos(ka)e^{-q|x|}, & |x| > a, \\ \cos(kx), & |x| < a \end{cases} \tag{3.11}$$

and the odd solutions are

$$\psi_{\text{odd}}(x) = \sqrt{a + \frac{1}{q}} \begin{cases} \text{sgn}(x)e^{qa}\sin(ka)e^{-q|x|}, & |x| > a, \\ \sin(kx), & |x| < a \end{cases} \tag{3.12}$$

where $\text{sgn}(x)$ is the "sign" function which is equivalent to $x/|x|$ when $x \neq 0$. The energy eigenfunctions are plotted in Figure 3.2 for the case $V_0 = 36$ in dimensionless units.

4 Unbound States and Scattering

Unbound states have an energy that is greater than the potential energy at infinity, in contrast to bound states, which have an energy that is less than the potential energy at infinity. Bound states must "fit" into the potential well, which leads to energy quantization, while unbound states lie above the well with sinusoidal wave functions that extend to infinity. Sinusoidal wave functions are not normalizable, but they are a useful way to analyze *scattering phenomena*, whereby unbound states are affected by the potential energy profile. In this context, we call them **scattering states**. Imagine we are working with wave packets that have very large spread in position such that sinusoidal functions serve as a good approximation.

Let's return to the finite square well, but this time the particle has energy $E > 0$, and thus occupies an unbound state. The energy eigenvalue equations are

$$\frac{d^2\psi}{dx^2} = -k_1^2\psi \qquad \text{for } |x| > a$$
$$\frac{d^2\psi}{dx^2} = -k_2^2\psi \qquad \text{for } |x| < a$$

where $k_1 = \sqrt{2mE}/\hbar > 0$ and $k_2 = \sqrt{2m(E+V_0)}/\hbar > 0$. Hence, in all regions we have sinusoidal solutions which must satisfy the boundary conditions at $x = \pm a$. Let

$$\psi(x) = \begin{cases} Ae^{ik_1x} + Be^{-ik_1x}, & x < -a \\ Ce^{ik_2x} + De^{-ik_2x}, & -a < x < a \\ Fe^{ik_1x} + Ge^{-ik_1x}, & x > a \end{cases} \tag{4.1}$$

When solving a scattering problem, we treat the energy E as an initial condition; we can prepare particles with energy E by accelerating them through a fixed potential energy difference. Let's imagine we prepare particles with energy E and send them as projectiles into the potential well from $x = -\infty$. The potential well represents an interaction with some other, much larger particle (a target). In this context, since e^{ikx} represents a traveling wave moving to the right and e^{-ikx} travels to the left, we can set $G = 0$; there is no probability to measure a projectile coming from $x = \infty$ due to the experimental design.

Notice that we have five unknowns, A, B, C, D and F, but the boundary conditions give us only four constraints. We can eliminate A by effectively normalizing our solutions to the amplitude of the incoming wave. Letting $A = 1$, the constant B is the relative amplitude of *reflected waves* and F is the relative amplitude of *transmitted waves*. Altogether,

$$\psi(x) = \begin{cases} e^{ik_1x} + Be^{-ik_1x}, & x < -a \\ Ce^{ik_2x} + De^{-ik_2x}, & -a < x < a \\ Fe^{ik_1x}, & x > a \end{cases} \tag{4.2}$$

Boundary at the left wall

The boundary conditions at the left wall where $x = -a$ give

$$\begin{aligned} \psi(-a):\quad & e^{-ik_1a} + Be^{ik_1a} = Ce^{-ik_2a} + De^{ik_2a} \\ \psi'(-a):\quad & ik_1e^{-ik_1a} - ik_1Be^{ik_1a} = ik_2Ce^{-ik_2a} - ik_2De^{ik_2a} \end{aligned} \tag{4.3}$$

Boundary at the right wall

The boundary conditions at the right wall where $x = a$ give

$$\begin{aligned} \psi(a):\quad & Ce^{ik_2a} + De^{-ik_2a} = Fe^{ik_1a} \\ \psi'(a):\quad & ik_2Ce^{ik_2a} - ik_2De^{-ik_2a} = ik_1Fe^{ik_1a} \end{aligned} \tag{4.4}$$

After some tedious calculations, this system of equations can be solved for F and B to give

$$F = \frac{e^{-2ik_1a}}{\cos(2k_2a) - i\dfrac{k_1^2 + k_2^2}{2k_1k_2}\sin(2k_2a)}$$

$$B = -iF\frac{k_1^2 - k_2^2}{2k_1k_2}\sin(2k_2a)$$

The absolute square of F gives the relative probability T that an incident particle is transmitted through the potential well, which we call the **transmission coefficient**. The transmission coefficient for a finite square well is

$$T = \frac{1}{1 + \dfrac{(k_1^2 - k_2)^2}{4k_1^2k_2^2}\sin^2(2k_2a)} \tag{4.5}$$

Expressed in terms of the energy E and potential well depth V_0,

$$T = \frac{1}{1 + \dfrac{V_0^2}{4E(E + V_0)}\sin^2\left(\dfrac{2a}{\hbar}\sqrt{2m(E + V_0)}\right)} \tag{4.6}$$

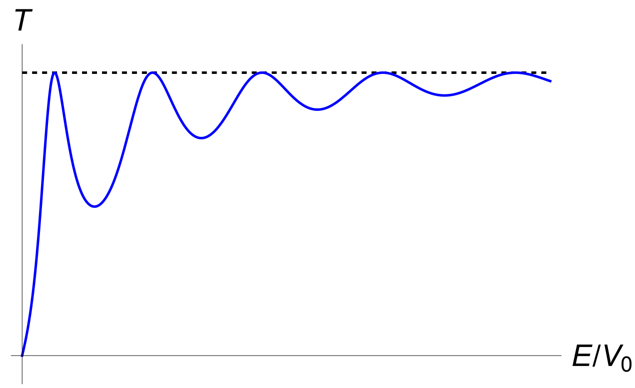

Figure 4.1: Transmission coefficient vs. energy for $V_0 = 12$ in dimensionless units.

The **reflection coefficient** $R = |B|^2$ is given by

$$R = \frac{1}{1 + \frac{(k_1^2 - k_2)^2}{4k_1^2 k_2^2}\sin^2(2k_2 a)} \tag{4.7}$$

In this finite square well problem, there is no absorption of particles by the well, so the reflection and transmission coefficients add up to unity:

$$R + T = 1$$

In contrast to quantum mechanical particles, classical particles do not reflect from potential wells. They merely speed up and then slow down as they traverse the well. The reflection of quantum mechanical particles is thus further evidence of the wave nature of particle motion.

The transmission coefficient equals one when either the energy of the projectiles overwhelms the potential energy well, or when the wavelength of the wave function matches a **resonance condition**:

$$2k_2 a = n\pi$$

$$2\left(\frac{2\pi}{\lambda_2}\right) a = n\pi \Rightarrow 2a = n\frac{\lambda_2}{2}$$

When this condition is met, the wave function will have nodes on either side of the well, so there it effectively "hops the gap" so to speak. In terms of the energy, the resonance condition is

$$\left(\frac{2a}{\hbar}\right)^2 2m(E + V_0) = n^2\pi^2$$

$$E = -V_0 + \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Thus, the energies of the transmission resonances (with respect to the bottom of the well) correspond to the bound-state energies of the infinite well. A similar effect is seen in atomic physics, where it is called the Ramsauer-Townsend effect.

5 Tunneling Through Barriers

If the energy of the particle is below the barrier height, then the barrier region is classically forbidden and a classical particle reflects perfectly from the barrier. In the quantum mechanical treatment there is a possibility that the particle can penetrate the barrier and come out on the other side! This is because the quantum mechanical wave function penetrates into the classically forbidden region. This phenomenon is called **quantum mechanical tunneling**. We can study the tunneling phenomenon by inverting the potential well into a potential barrier with $-V_0 \to +V_0$. The "incident particles" have energy E greater than zero but less than V_0. The energy eigenvalue equations in each region are

$$\frac{d^2\psi}{dx^2} = -k^2\psi \qquad \text{for } |x| > a$$
$$\frac{d^2\psi}{dx^2} = q^2\psi \qquad \text{for } |x| < a$$

where $k = \sqrt{2mE}/\hbar > 0$ and $q = \sqrt{2m(V_0 - E)}/\hbar > 0$. Using the same experimental design as in the last section, we can write the wave function as

$$\psi(x) = \begin{cases} e^{ikx} + Be^{-ikx}, & x < -a \\ Ce^{qx} + De^{-qx}, & -a < x < a \\ Fe^{ikx}, & x > a \end{cases} \tag{5.1}$$

After implementing the boundary conditions, we can solve for the **transmission coefficient** $T = |F|^2$, and we find

$$T = \frac{1}{1 + \dfrac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\dfrac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right)} \tag{5.2}$$

As for the reflection coefficient,

$$R = |B|^2 = 1 - T = \frac{1}{1 + \dfrac{4E(V_0 - E)}{V_0^2 \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right)}} \tag{5.3}$$

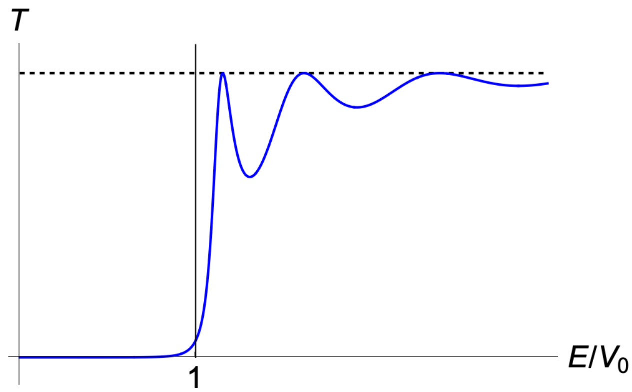

Figure 5.1: Transmission coefficient for scattering from a square barrier for $V_0 = 12$ in dimensionless units. Nonzero T for $E < V_0$ corresponds to quantum mechanical tunneling.

The transmission coefficient is plotted in Figure 5.1. In the tunneling case, the transmission is nearly zero except near the top of the barrier, where the tunneling probability increases exponentially. As the energy of the incident particle exceeds the barrier height, the transmission becomes large and exhibits the same resonances seen in the finite well problem. For large energy, the transmission goes to unity, which is to be expected because the potential barrier becomes insignificant.