

Phys 153: Fundamentals of Physics II

Unit #4 – Old Quantum Theory

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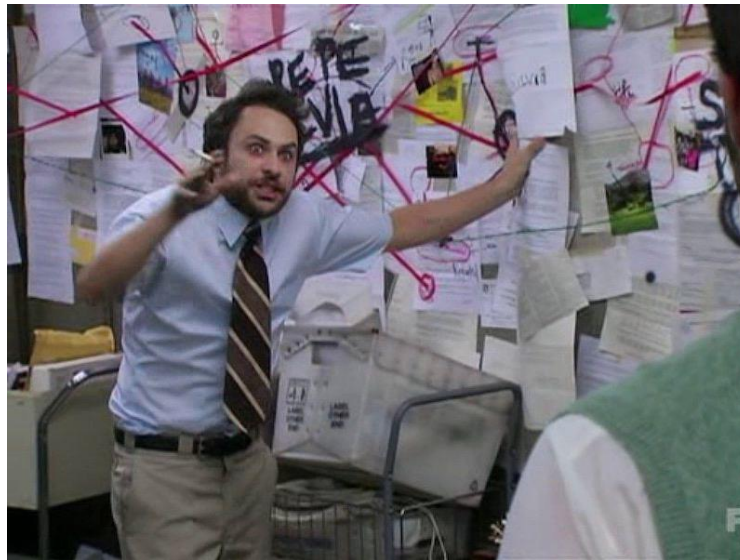
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Rethinking Light and Matter

Modern quantum theory was a culmination of intense research that took place during the period from ~1900 to 1924.

- Classical physics was unable to explain the behavior of light and matter in the microscopic domain.
- Arbitrary rules were invented to make predictions match observations.
- We're not just talking about esoteric concerns of academics—*the very stability of matter was on the line*. How embarrassing!



^average physicist from early 20th century trying to explain why atoms don't fall apart in < 1 ps

Why Do Hot Objects Glow?

One of the earliest failures of classical physics was the inability to explain why hot objects glow. When heated, a solid object emits radiation across a continuous distribution of wavelengths (a spectrum). As the temperature increases, the object becomes red, then yellow, then white.



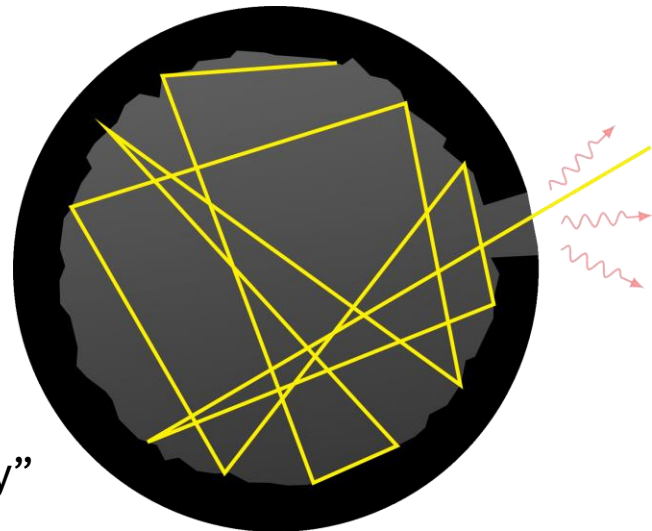
Ideal Blackbody

An ideal blackbody is an object that absorbs all incident light, reflecting none.

- An object in thermal equilibrium with its surroundings radiates as much energy as it absorbs.
- Hence, a blackbody is a perfect emitter as well as a perfect absorber of radiation.

The spectrum of a blackbody is universal, meaning it is independent of material composition.

- Provides insight into fundamental interactions between light and matter.



“cavity blackbody”

Blackbody Radiation

Empirical Findings:

1. Stefan's law: total power of emitted radiation increases with temperature.

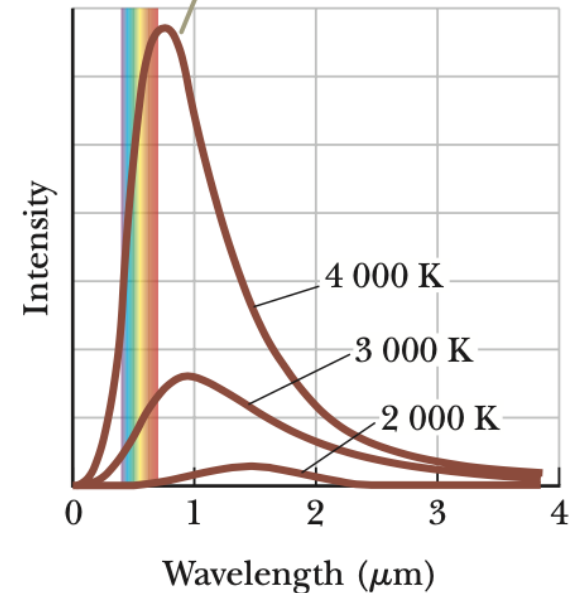
$$P = \sigma A \epsilon T^4$$

$$\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$$

2. Wien's law: the peak of the spectrum shifts to shorter wavelengths as temperature increases.

$$\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

The 4 000-K curve has a peak near the visible range. This curve represents an object that would glow with an appearance that is almost pure white.



Blackbody Spectrum

To describe the distribution of energy from a black body, we define $u_\nu(\nu, T)d\nu$ to be the energy density (energy per unit volume) of radiation in the frequency interval ν to $\nu + d\nu$.

The total energy density is

$$u = \int_0^\infty \underbrace{u_\nu(\nu, T)}_{\text{Energy density per unit frequency}} d\nu$$

Energy density per unit
frequency

Rayleigh-Jeans Law

Classical E&M and statistical mechanics can be applied to a cavity containing standing EM waves to **derive** the Rayleigh-Jeans law

$$u_\nu(\nu, T) = \frac{8\pi\nu^2}{c^3} k_B T$$

- $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant
- The Rayleigh-Jeans law works well for low frequencies, but it diverges at high frequencies
- Furthermore, it predicts infinite energy density, and hence infinite total energy!

“Ultraviolet catastrophe”

Planck's Math Trick

- Rayleigh-Jeans assumes energy exchange between radiation and matter is *continuous*.
- In 1900, Max Planck derived an accurate equation for blackbody spectrum by employing a “math trick”
 - Calculated the average energy by considering small but discrete energy exchange between radiation and the walls of the cavity.
 - Purely formal assumption, didn't believe in physical reality of discrete energy exchange at first.

Planck's Distribution

Planck found the energy density per unit frequency of the radiation emitted from a blackbody is

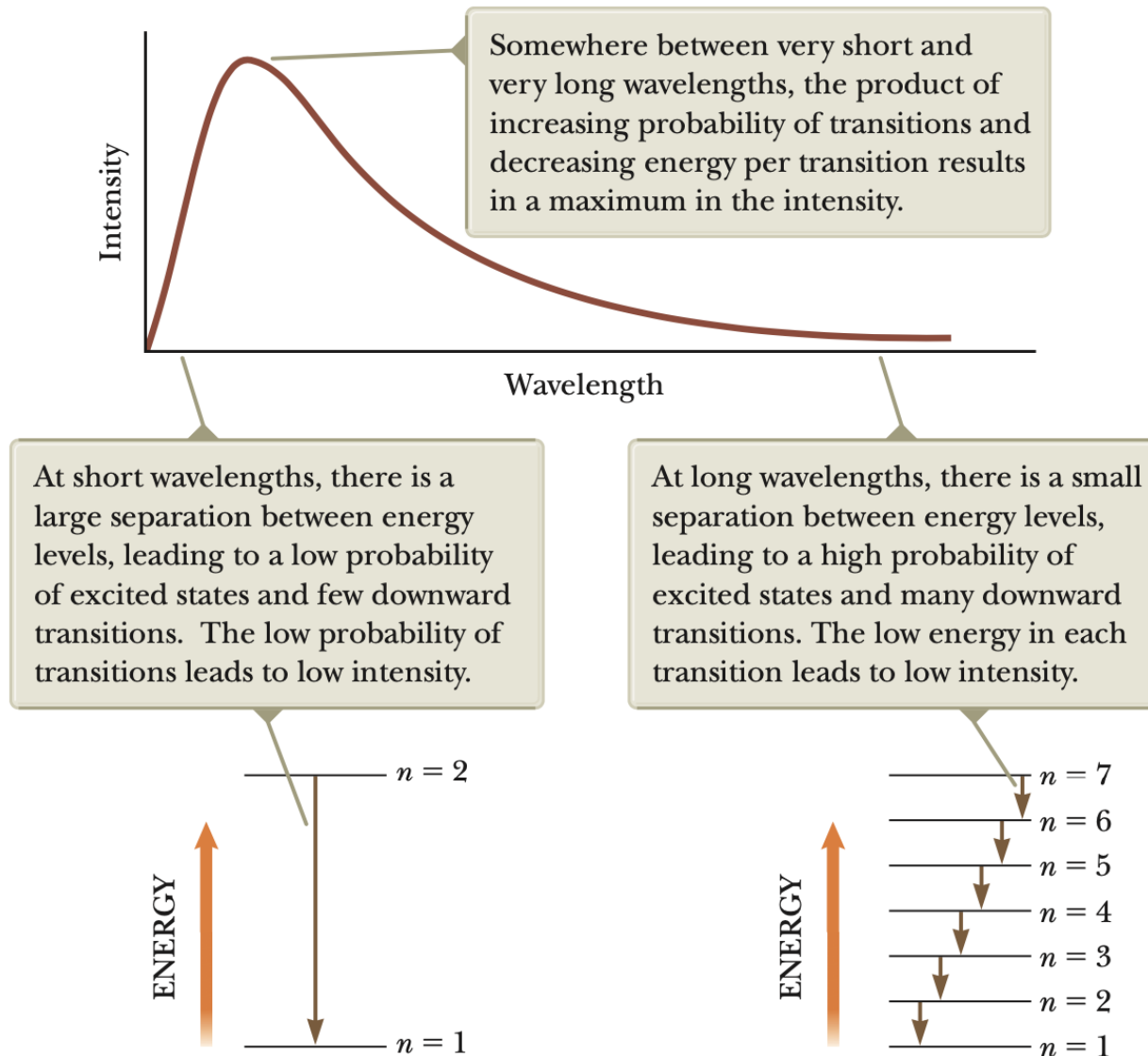
$$u_\nu(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

- Fits the data *exactly* as long as

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

- Predicts finite total energy, matches Rayleigh-Jeans in the low frequency limit, and leads to the empirical findings of Stefan and Wien.

Planck's Distribution

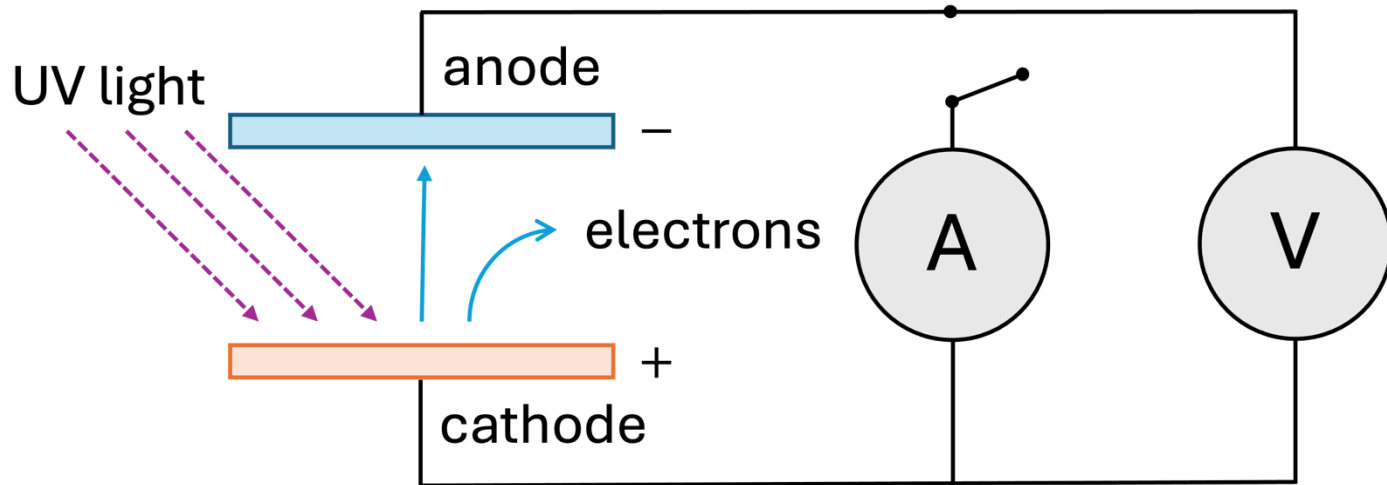


Example 1: Blackbody Radiation

Show that the maximum of Planck's distribution occurs for a wavelength $\lambda_{\max} = b/T$ where T is the absolute temperature (in kelvin) and b is a constant that needs to be estimated. The result is known as Wien's displacement law.

Photoelectric Effect

First discovered by Heinrich Hertz in 1887, the photoelectric effect describes the ability of light to dislodge electrons from a metallic surface.



Experimental design similar
to that of Lenard (1902)

Photoelectric Effect

List of experimental findings:

1. At high frequencies and intensities, rate of ejected electrons is proportional to intensity of light.
2. Even at low intensities, electrons are ejected essentially instantly with delays less than 10 ns.
3. If the intensity is held constant, the rate of ejected electrons decreases with increasing frequency in a similar way for all metals.

Photoelectric Effect

Experimental findings continued:

4. If the frequency is below a material-dependent cutoff frequency, no electrons are ejected, no matter how high the intensity is.
5. If the frequency is held constant, the maximum kinetic energy of electrons K_{\max} is independent of the light's intensity.
6. Above the cutoff frequency, K_{\max} is directly proportional to the frequency with the same slope for all metals.

Wave Model Falls Flat

According to the classical wave model of radiation,

1. At high intensities, the rate at which electrons are ejected will be proportional to the intensity of the light.
2. At low intensities, there will be a significant delay between the illumination of the plate and the emission of electrons.
3. The rate of electron ejection depends in a metal-specific way on the frequency of the light (due to resonance).
4. The maximum kinetic energy of the ejected electrons will increase with increasing intensity (and may also depend on the light's frequency due to resonance).

In summary, the wave model is unable to account for many of the experimentally verified features of the photoelectric effect!

Photon Model of Light

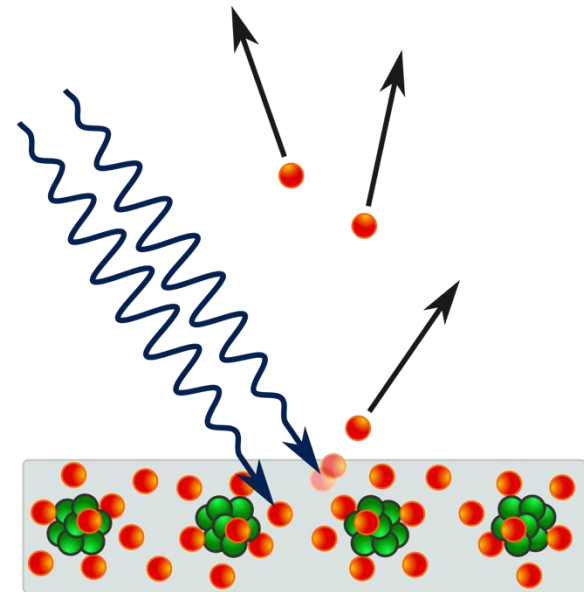
The **photon model of light** and its application to the photoelectric effect was first proposed by Albert Einstein in 1905 and later confirmed by Roger Millikan in 1917.

- Each photon of incident light carries energy hf where f is the frequency of the associated light wave.
- Each electron is ejected from the metal as a result of collision (and subsequent absorption of) a single photon of light.

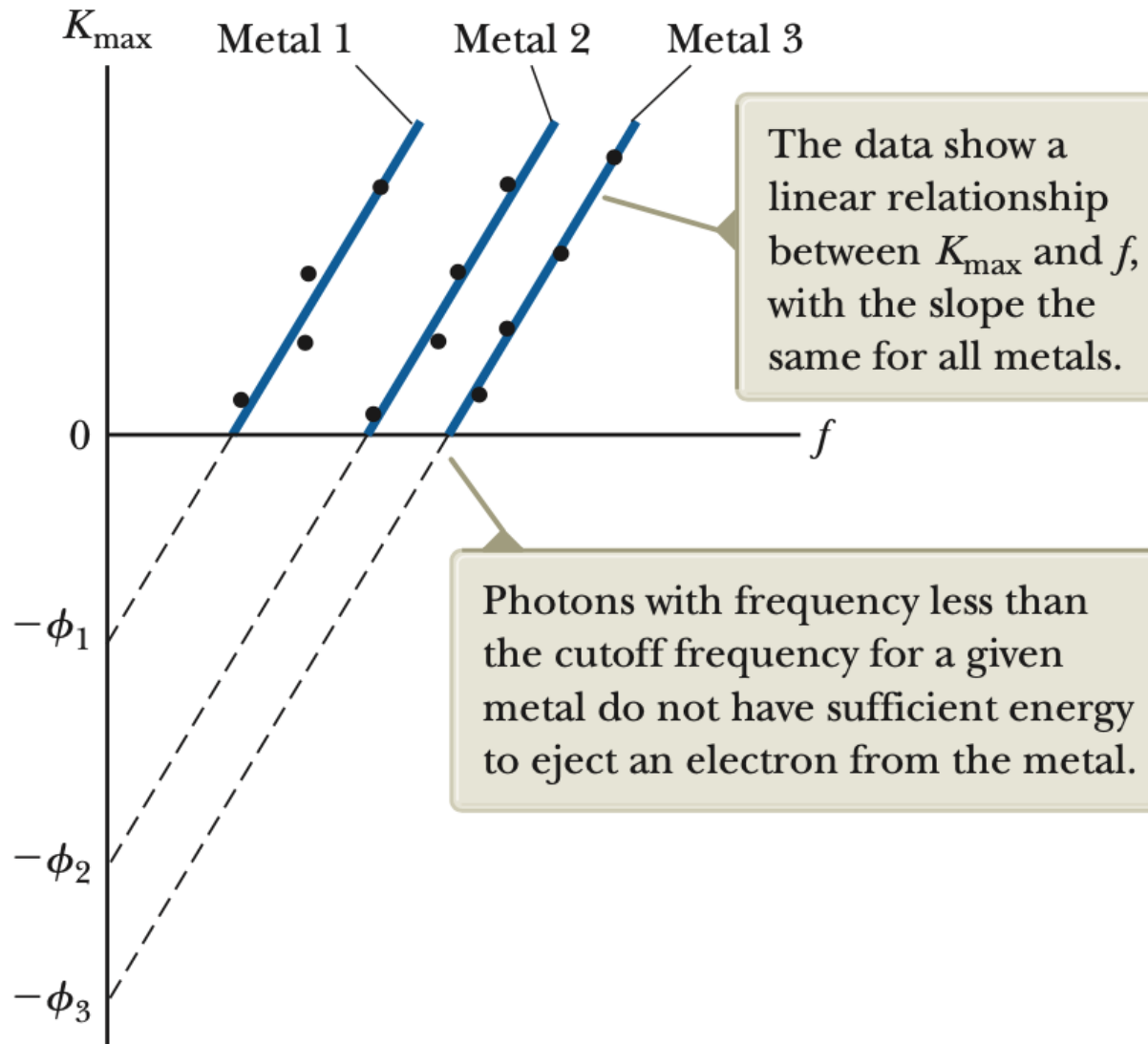
Maximum kinetic energy of electrons:

$$K_{\max} = hf - \phi$$

- ϕ is the **work function** of the metal – the minimum energy required to free an electron (on the order of electron-volts).



Photoelectric Effect



Example 2: Photoelectric Effect

When ultraviolet light of wavelength 250 nm falls on an aluminum surface, the measured maximum kinetic energy of ejected electrons is 0.88 eV, where 1 eV (electron-volt) is the energy required to accelerate a single electron from rest through a potential difference of 1 volt. When light of wavelength 210 nm falls on the same plate, the maximum kinetic energy is 1.82 eV. Check that the value of hc implied by these data is 1240 eV · nm and find the value of the work function for aluminum.

Recall that for electromagnetic radiation, $c = \lambda f$.

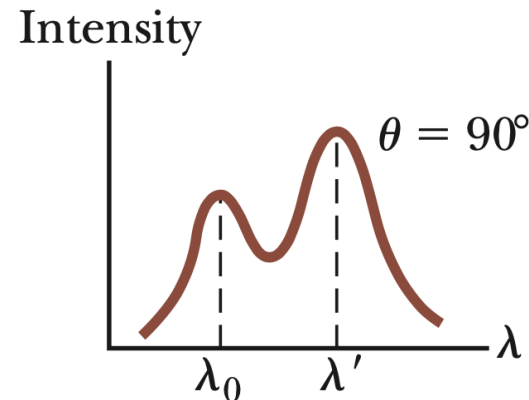
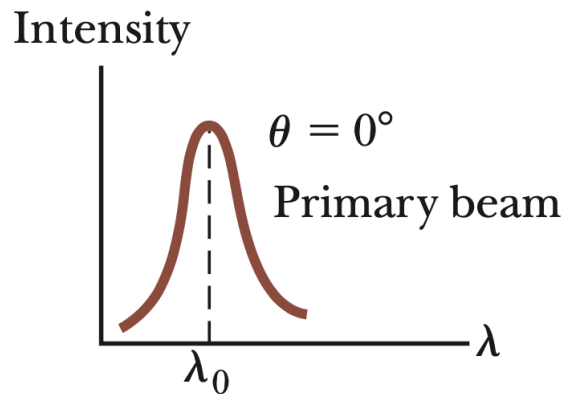
Example 3: Photon Model

A 100-W incandescent light bulb uses 100 W of electrical power but only radiates about 10 W of actual visible light. Estimate the number of photons that strike your face per second when you stand 2 m away from the light bulb.

Compton Effect

In 1923, Compton provided the most conclusive confirmation of the particle aspect of radiation.

- Scattered X-rays off free electrons (frequency too high for absorption)
- Found that wavelength of scattered radiation is larger than wavelength of incident radiation, in a way that depends on the angle of the scattered radiation



Wavelength shift is proportional to scattering angle

Wave-Particle Duality of Light

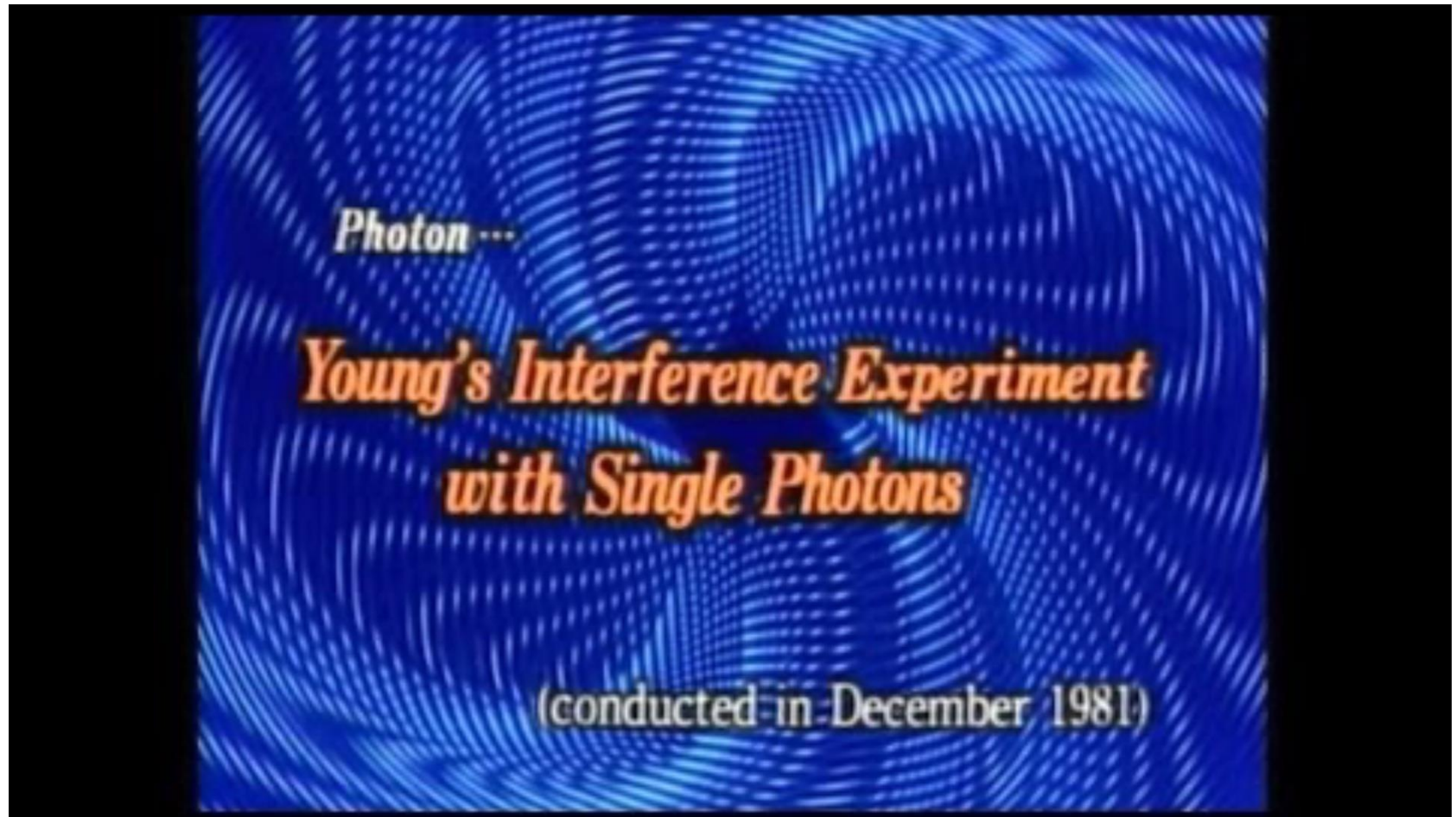
Within the framework of classical physics,

- Wave model: Young's double slit experiment, diffraction
- Photon model: photoelectric effect, Compton scattering

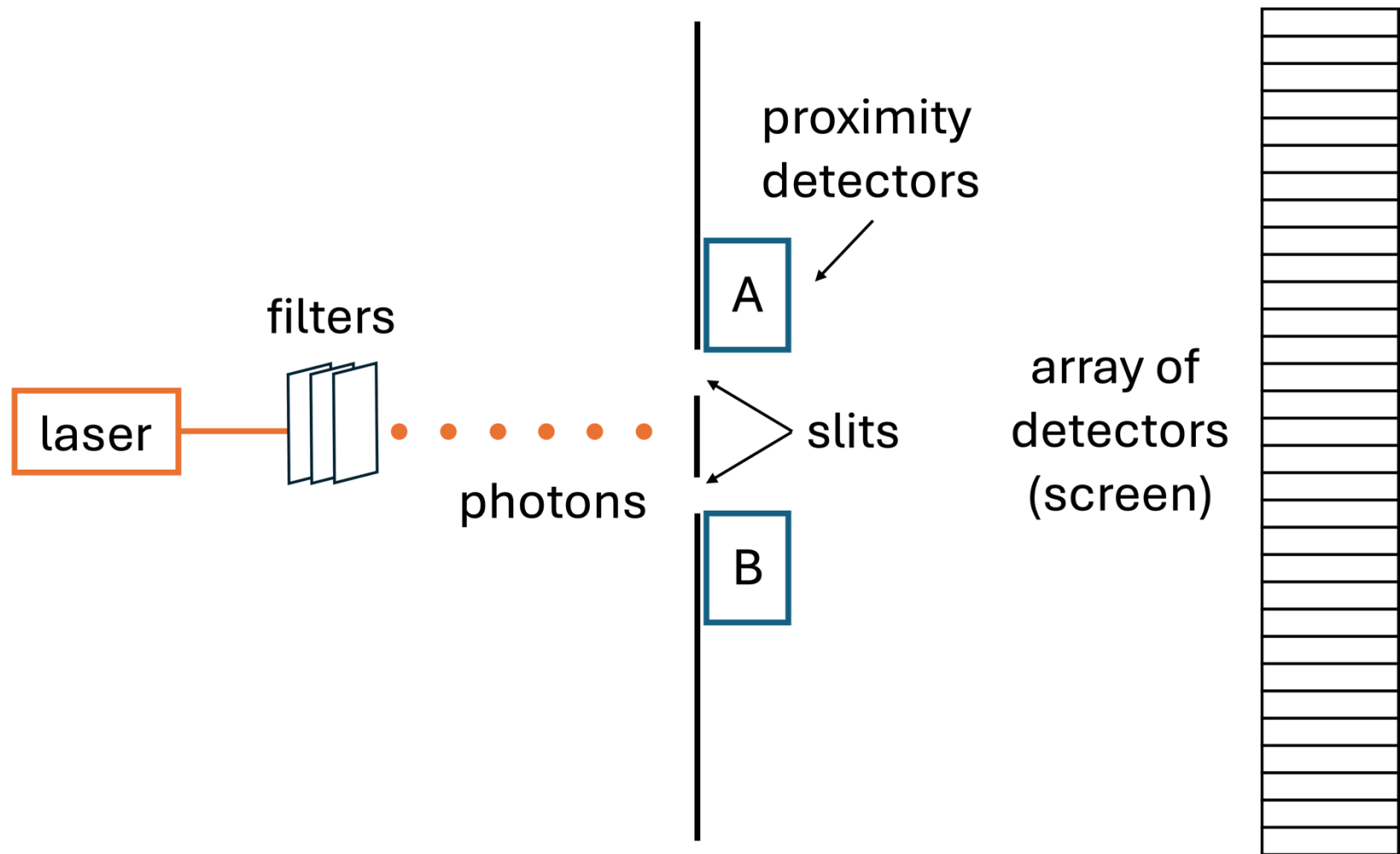
Maybe classical physics is alright; light just behaves like a wave sometimes and other times like a particle...

- Even in situations where we are almost certainly dealing with individual photons, light may still exhibit wave-like behavior, namely interference!
- In 1909, G.I. Taylor used a low intensity light source to demonstrate that even when individual photons pass through one by one through a double-slit, an interference pattern builds up over time.

Single-Photon Interference



Single-Photon Interference



Example 4: Single-Photon Source

Assume that a laser emits about 10^{15} photons per second. If the distance between the filters and the detector array is about 3 m, then it will take a photon about 10^{-8} s to cover this distance. If we arrange things so that an average of one photon every 10^{-5} s (i.e., an average of 10^5 photons per second) gets through the last filter, then there will be only a 1/1000 chance that a photon is in flight at any given time. So, we want to reduce the 10^{15} photons per second emitted by the laser to roughly 10^5 photons per second emerging from the last filter.

If each filter transmits only 5% of incident light, how many filters are needed to create a single-photon source?

Wave Model of Matter

In 1923, Louis de Broglie suggested that wave-particle duality was universal, and that material particles should also have a literal wave nature.

- Hypothesized that any material object with nonzero rest mass has energy and momentum given by

$$E = \hbar\omega \qquad p = |\vec{\mathbf{p}}| = \hbar k$$

$$\hbar = \frac{h}{2\pi} \qquad \omega = 2\pi f \qquad k = \frac{2\pi}{\lambda}$$

reduced Planck's constant

The *de Broglie relations* connect the energy and momentum of a particle with the frequency and wavelength of the corresponding “matter wave.”

Note: this is NOT how we think of “wave functions” today!

Example 5: de Broglie Wavelength

Calculate the de Broglie wavelength for

(a) a proton of kinetic energy 70 MeV and

(b) a 100 g bullet moving at 900 m/s.

Recall the relation for relativistic momentum:

$$pc = \sqrt{K(K + 2mc^2)}$$

Example 6: Electron Gun

The de Broglie relations imply that electrons with the same momentum will have the same wavelength.

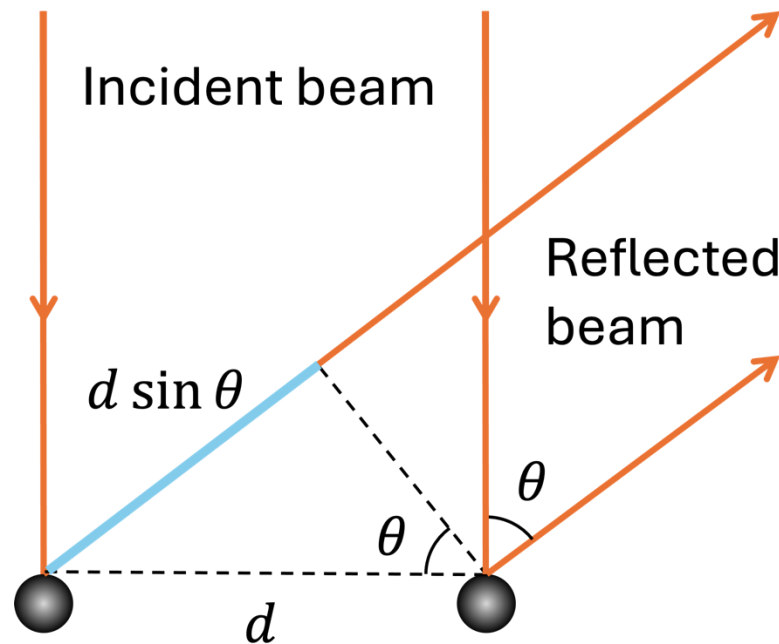
An **electron gun** is a device that accelerates electrons from rest through a fixed potential difference, thereby creating a monochromatic beam of electrons.

- (a) Find the wavelength of electrons accelerated by a potential difference of 100 V.
- (b) Why can we use the non-relativistic relation $p = \sqrt{2mK}$ in part (a)?



Example 7: Davisson-Germer Exp.

Electrons with a kinetic energy of 54 eV are fired toward a crystal perpendicular to its face. A detector is mounted to receive electrons reflected at a variable angle θ from the direction of the incident beam. Given that the spacing between atomic rows in nickel is $d = 0.215$ nm, what is the smallest nonzero angle for constructive interference?



The Not-So-Atomic Atom

Atomos, the Greek word for atom, means indivisible.

- In the 1890's, radioactive materials were discovered, suggesting atoms have internal structure (they can fall apart).
- J.J. Thompson discovered β particles using cathode rays.
- Ernest Rutherford discovered α particles in the decay of uranium and other radioactive sources (like radon).

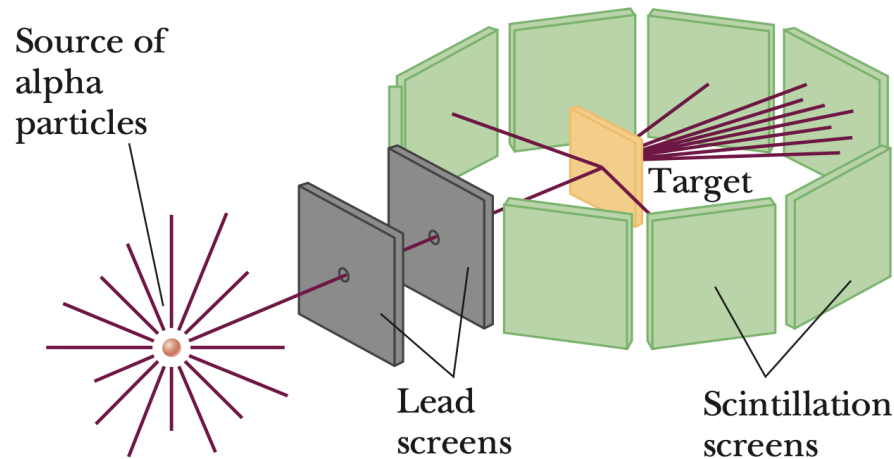


Uranium sample emitting particles into a cloud chamber.

Rutherford Scattering

In 1909, Rutherford began using α particles to probe atomic structure by directing them toward a gold foil.

- Most particles passed through with little-to-no deflection.
- On rare occasions (1 out of every 10^5 events) particles would scatter at a large angle, “as if you fired a 15-inch artillery shell at a piece of tissue paper and it came back and hit you.”

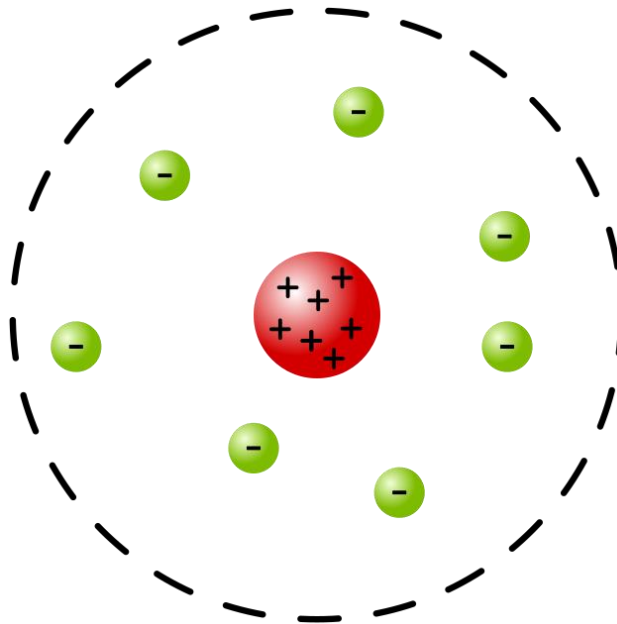


Rutherford's Planetary Model

Rutherford could only make sense of the scattering in the context of a “planetary model” where electrons orbit a very dense, positively charged nucleus.

Application of classical physics to this model predicts

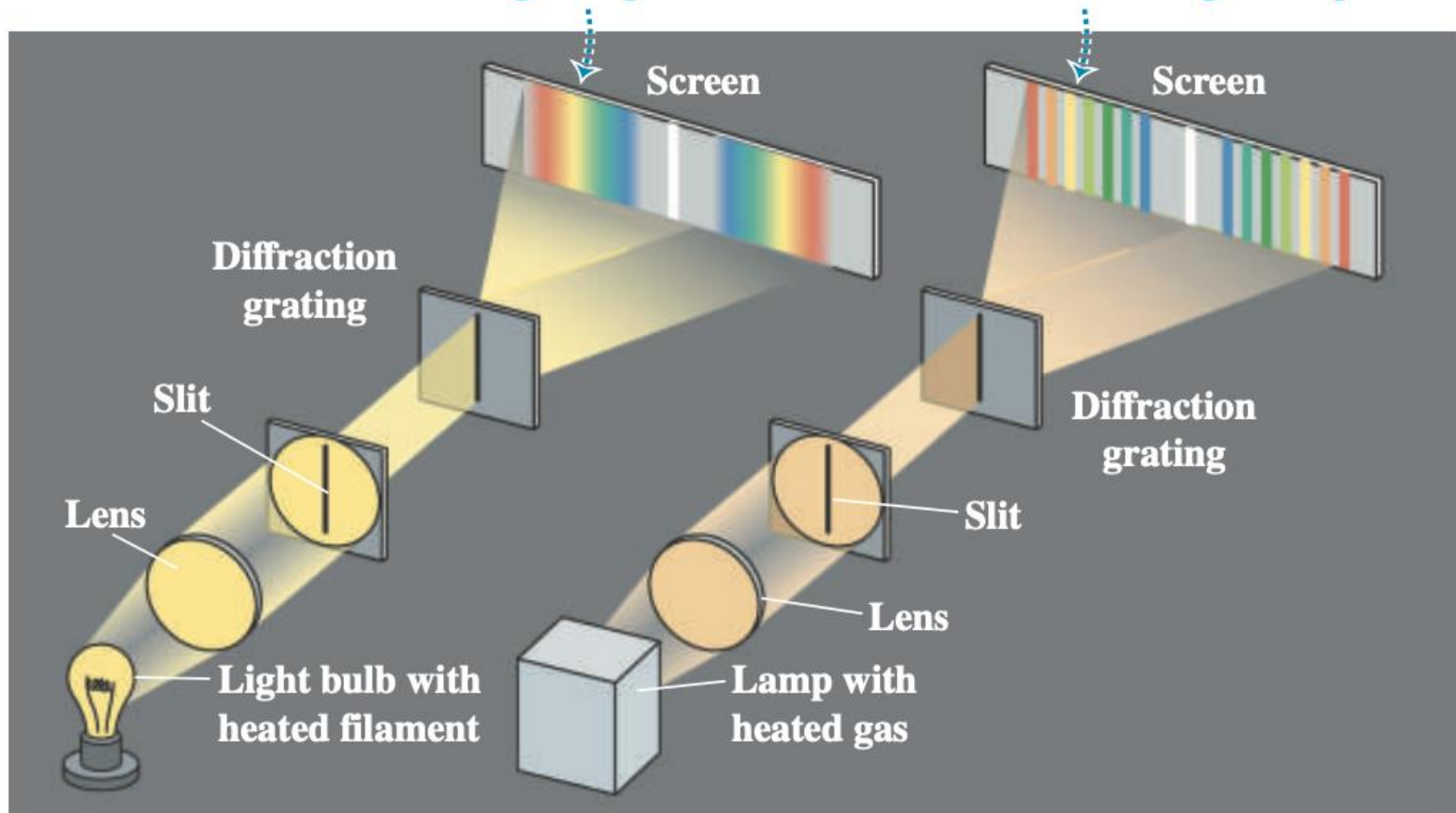
- i. Atoms should be unstable (with very short lifetimes).
- ii. Atoms should radiate energy over a continuous range of frequencies.



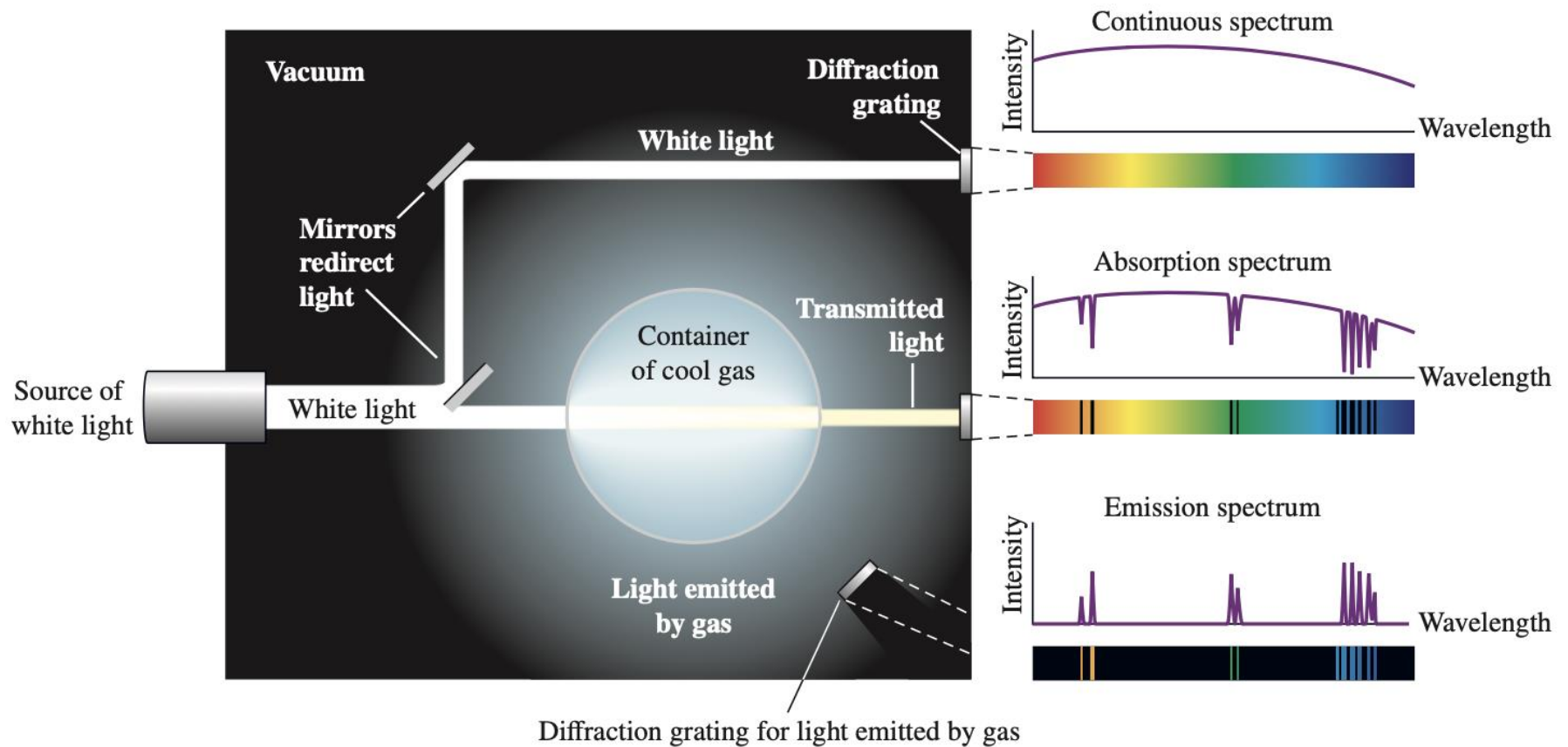
Continuous vs. Line Spectra

(a) Continuous spectrum: light of all wavelengths is present.

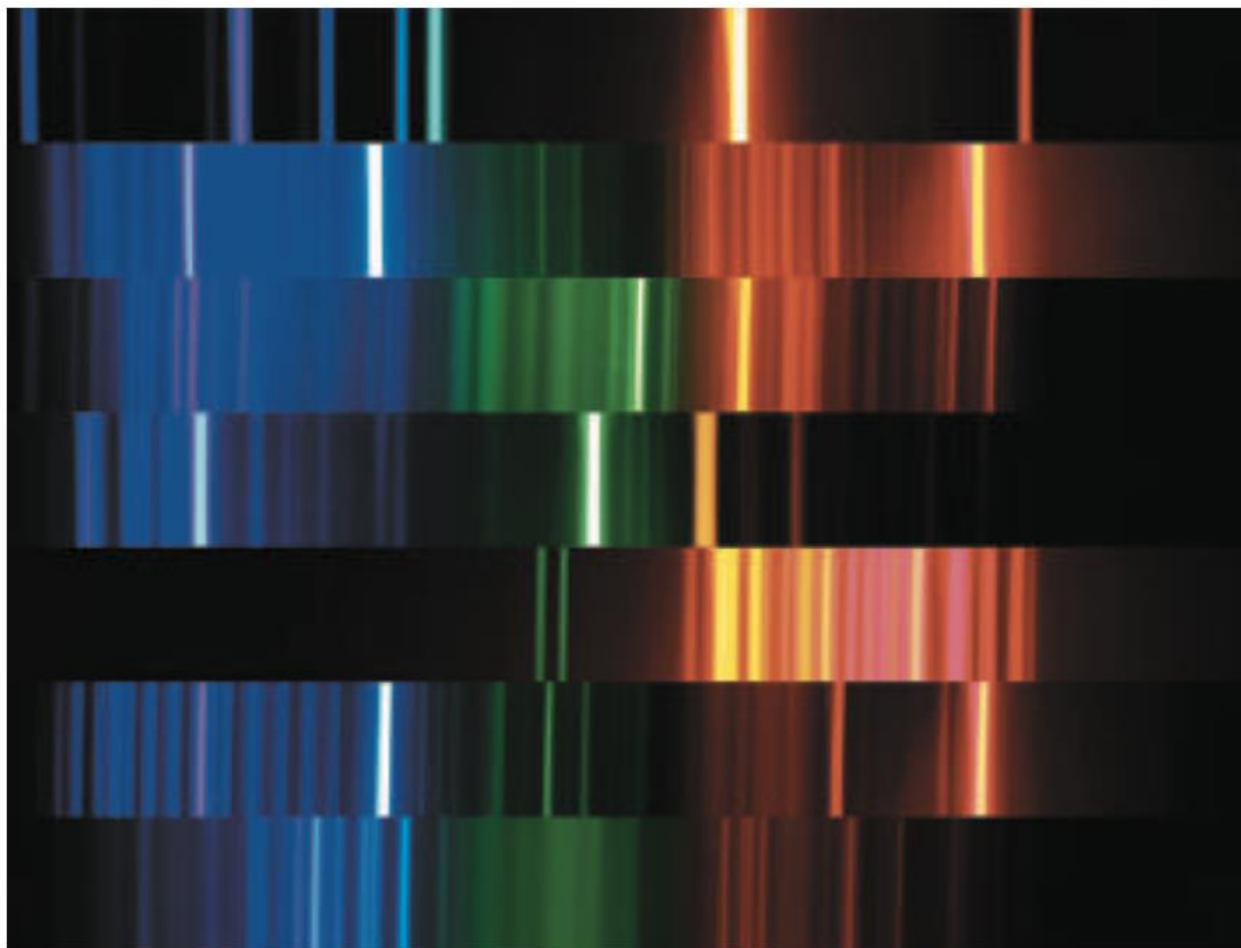
(b) Line spectrum: only certain discrete wavelengths are present.



Emission vs. Absorption



Emission Lines are Fingerprints



Helium (He)

Hydrogen (H₂)

Krypton (Kr)

Mercury (Hg)

Neon (Ne)

Water vapor (H₂O)

Xenon

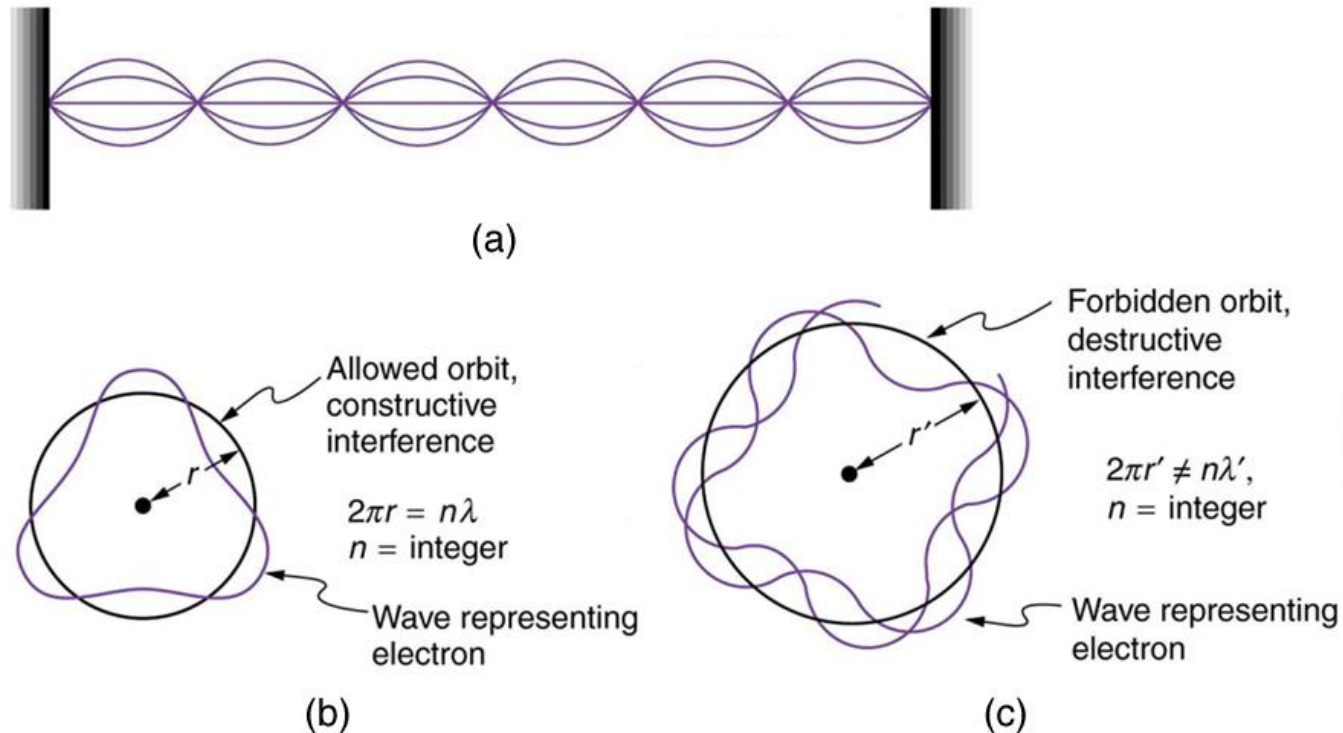
Bohr's Model of Hydrogen

In 1913, Niels Bohr proposed a model of hydrogen that gave an accurate account of the observed spectrum and a convincing argument for its stability.

- Only a discrete set of stable circular orbits are allowed, and each has definite energy.
- The angular momentum in each orbit is an integer multiple of \hbar .
- Emission or absorption of radiation occurs when an electron jumps from one orbit to another.
 - Energy of radiation must correspond exactly to the *energy difference* between orbits.

Circular Standing Waves

In the Bohr model, electrons in circular orbits are treated as *standing waves*, which means the electron's de Broglie wavelength must fit an integer number of times into the circumference of the orbit.



Example 8: Bohr's Model

Apply Bohr's "quantization rules" to Rutherford's planetary model of the hydrogen atom to find

- (a) the radius of the electron's circular motion for each stable orbit,
- (b) the energy of the atom for each stable orbit,
- (c) and the allowed wavelengths of radiation emitted by the atom.

Modern Quantum Mechanics

1. In the Spring of 1925, Werner Heisenberg developed **matrix mechanics** and ushers the first paradigm shift of quantum theory.
 - “The aim is to establish a formal connection between quantities which in principle are **observable**.”
 - Stop focusing on what the electron is “really doing” and instead look at the frequencies and intensities of light coming from the atom (for example).
2. In 1926, Erwin Schrödinger invented **wave mechanics** as an intuitive and visualizable approach.
 - Max Born points out that the wavefunction cannot describe a physical wave, but rather a “probability amplitude.”
 - The emphasis on **probability** was the second paradigm shift of quantum theory.
3. In 1928, Paul Dirac merged matrix and wave mechanics into a single, abstract framework—**quantum mechanics**.
 - A comprehensive set of postulates emerge that cover all known quantum phenomena!

Solvay Conference

