Casino

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The casino manager hired you as a consulting statistician. They would like to detect cheating players based on the number of wins. Cheating players will be approached by the security personnel and politely asked to leave the casino. Those players are then stored in a database. The image processing system of the casino will automatically detect those player if they should ever try to enter the casino again. Security personnel will prevent those players from entering the casino. The casino manager asked you to develop a decision rule with the following constraints and trade offs:

Let a player play 100 times (you can assume that plays are independent) Make a decision rule based on these 100 plays The decision rule should detect cheating players with a probability of 80% (or as close as possible to 80% but not below)

Develop your decision rule using hypothesis testing. Follow these steps:

a) What distribution can you use to model the number of wins in 100 plays? [10 points]

Each game has two possible outcomes win or lose, the winning probability for a normal player is of 1/37 and the wining probability for the cheater is of is of 1/20 so each game is a Bernoulli trial, as we are looking at 100 games and assuming that each game is independent the distribution is Binomial.

Distribution for normal player:

$$P_{player}(X = k) = \binom{100}{k} \frac{1^k}{37^k} \frac{36^{100-k}}{37^{100-k}}$$

$$X_{player} \sim Binom(100, \frac{1}{37})$$

Distribution for cheater player:

$$P_{cheater}(X = k) = \binom{100}{k} \frac{1^k}{20^k} \frac{19^{100-k}}{20^{100-k}}$$

$$X_{cheater} \sim Binom(100, \frac{1}{20})$$

b) What is the null hypothesis and its distribution (take fair players as the null)? [10 points]

The null hypothesis is that there are not cheaters so the distribution will be:

$$X_{cheater} \sim Binom(100, \frac{1}{20})$$

c) What is the alternative hypothesis and its distribution (take cheaters as the alternative)? [10 points]

The alternative hypothesis is that there are cheaters in the casino so the distribution will be:

$$X_{player} \sim Binom(100, \frac{1}{37})$$

d) What does the 80% correspond in the terminology of hypothesis testing? [10 points]

Type of error II is accepting the null Hypothesis when it is false, in this case would be accepting the player is fair when is not. The probability of type error II is defined by β in this case the 20% and finally the 80% corresponds to the power test, that's it probability of player is fair is rejected when is false $1 - \beta$.

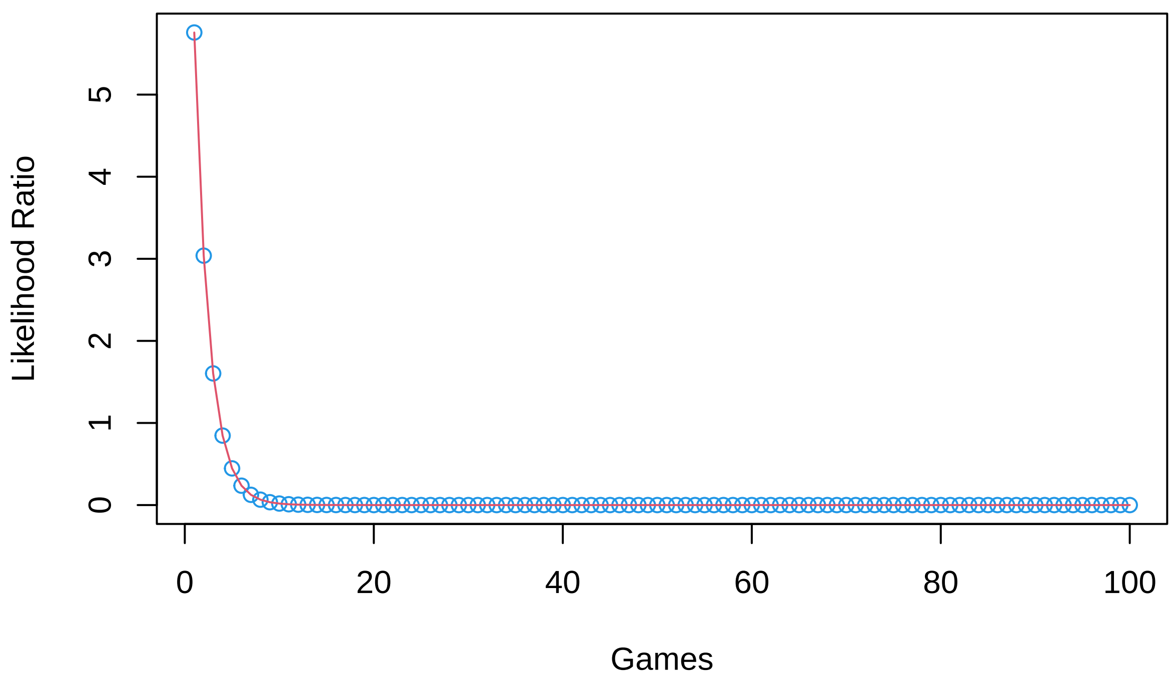
e) What is the decision rule that you propose? [20 points]

```
hypotesis <- function(n, prob){
  k <- seq(1,n)
  return(dbinom(k, games, prob))
}

likelihoodRatio <- function(H0, H1){

  likelihoodRatio <- P0/P1
  x <- seq(1, length(likelihoodRatio))

  plot(x, likelihoodRatio,
     type = "p",
     col = 4,
     ylab = "Likelihood Ratio",
     xlab = "Games")
  lines(x, likelihoodRatio,
     col = 2)

  return(likelihoodRatio)
}

powerTest <- function(c, likelihoodRatio, H1){

  beta <- 0
  x <-length(likelihoodRatio)

  for (i in  1:x) {
     if (likelihoodRatio[i] > c) {
       beta = beta + H1[i]
     }
  }

  return(1 - beta)
}

rule <- function(prob, maxC, likelihoodRatio, H1, alphaBeta){
  c <- seq(0, maxC, by=0.01)
  probability <- c()
  bigProb <- 0
  result <- c()

  for (i in 1:length(c)) {
    probability[i] <- powerTest(c[i], likelihoodRatio, H1)
```

```
  }

  for (i in 1:length(c)) {

      if(probability[i] > prob){
          bigProb <-i
          break
        }
  }

  result[1] <- probability[bigProb]
  result[2] <- c[bigProb]
  result[3] <- round(result[2])

  return(result)
}
```

We want a ratio of the probabilities of the two events in other words a likelihood ratio.

```
games <- 100
player <- 1/37
scammer <- 1/20

P0 = hypotesis(games, player)
P1 = hypotesis(games, scammer)

ratio <- likelihoodRatio(P0,P1)
```



Then I use the decision rule that I create using the power test and aplaying 80%

```r
rul <- rule(0.8, max(ratio), ratio, P1)

print(paste("Probability is ", rul[1]))
```

```
## [1] "Probability is  0.887657548035213"
```

```r
print(paste("Decision rule is X >= ", rul[3]))
```

```
## [1] "Decision rule is X >=  2"
```

f) What is the probability of calling a player a cheater who is in fact not cheating (and just got lucky) called in the terminology of hypothesis testing? [10 points]

Probability of error type I, that's it reject the null hypothesis when is true.

g) What is the probability of f) in your proposed decision rule? [10 points]

```r
alpha <- function(rule, ratio, H0){

  alpha <- 0

  for (i in 1:length(H0)) {

    if(ratio[i] < rule){

      alpha = alpha + H0[i]
    }
  }

  return(alpha)
}


print(paste("Probability of error type I is", alpha(rul[3], ratio, P0)))
```

```
## [1] "Probability of error type I is 0.509394401362411"
```

After you developed the system, the manager realizes that too many honest players are being flagged as cheaters.

h) What do you have to change in your decision rule to solve this problem? [10 points]

I will decrease the probability of detecting cheaters and i see that my error type 1 decreases.

```r
rulas <- rule(0.3, max(ratio), ratio, P1)

print(paste("Probability of error type I now is", alpha(rulas[2], ratio, P0)))
```

```
## [1] "Probability of error type I now is 0.0544169500121121"
```

i) How does this affect the probabilities of the two types of errors (use the terminology of hypothesis testing)? [10 points]

Now the casino wont detect as many cheaters as before but the casino will reduce the error type I so the players that are honest will not be flagged as cheaters.