

Sample Hashing and Binary Tree Questions

These problems focus on hashing and binary trees because I already gave out a bunch of sample problems on counting and permutations/combinations.

1. Prove by induction that the number of nodes in a binary tree of height h is at most $2^{h+1} - 1$.
2. Prove by induction that the number of edges in a binary tree with $n \geq 1$ nodes is $n - 1$. Is this true if nodes in the tree can have any number of children?
3. Prove that the number of nodes in a non-empty *full* binary tree is odd.
4. Insert 7, 14, 21, 28 into a hash table of size 7 using linear probing with $h(k) = k \bmod 7$. Show the contents of the table.
5. Insert 7, 14, 21, 28 into a hash table of size 7 using quadratic probing with $h(k) = k \bmod 7$. Show the contents of the table.

What is wrong with the probe sequence $h(k) \bmod N, h(k)+1^2 \bmod N, h(k)+2^2 \bmod N, \dots, h(k)+(N-1)^2 \bmod N$? and why would some authors suggest $h(k) \bmod N, h(k)+1^2 \bmod N, h(k)-1^2 \bmod N, h(k)+2^2 \bmod N, h(k)-2^2 \bmod N, \dots, h(k)+(\frac{N-1}{2})^2 \bmod N, h(k)-(\frac{N-1}{2})^2 \bmod N$?

6. How many sequences of 23 birthdays (each birthday is an integer from 1 to 365) have at least two identical birthdays?

How many sequences of 23 birthdays are there?

What is the probability that 23 birthdays (each chosen at random) contains at least two identical birthdays?