# CALGARY JUNIOR HIGH
# TEAM MATH ATTACK - PART A

**Saturday, December 9th, 2017**

**Sponsored by the University of Calgary Department of Mathematics and Statistics**

**Time: 30 minutes**

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.
Students are not allowed to leave the room during testing time.

**Instructions**

1. Do not open the contest paper until you are told to do so.
2. You may use rulers, compasses and paper for rough work, but all problems can be solved without additional aid.
3. Write your team name, member names, and school at the top of the response sheet. Print clearly. You will submit the response sheet for marking, but you may keep the problem booklet.
4. This is mostly a short answer test. A blank space to place your answer follows each question on the response sheet. To receive full marks, you must simply write your numerical answer in the blank space.
5. Scoring: Each correct answer in Part 1 is worth 1 point. Each correct answer in Part 2 is worth 2 points. Each correct answer in Part 3 is worth 3 points. There is no penalty for an incorrect answer.
6. Tiebreakers: In the event that two teams have the same number of total points, the team that correctly answered more questions wins. In the event two teams have the same number of points and correctly answered the same number of questions, the team that misses the earlier question loses the tiebreaker.
7. If there are words in italics at the end of the question, it is the source that the problem is drawn from. It does not change the question in any way.
8. When your supervisor tells you to begin, you will have 30 minutes of working time.

**Good luck!**

**CJHTMA 2017-18**
**PART A PROBLEMS**

## Part 1 - Easy

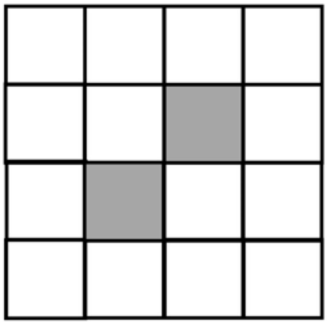
1. Let $2ab - 5a + 6b - 17 = 21$. If both $a$ and $b$ are positive integers, what is the value of $\frac{(a+1)}{b}$?

2. Four people compete in a rock paper scissors tournament: Alex, Bill, Charlise, and Damian. Alex plays Bill, and Charlise plays Damian with the respective winners facing off to determine the champion. However, Alex has a 75% chance of winning any match she plays, and matches without her have a 50-50 probability. What is the chance of Charlise becoming the champion?

3. How many sets of three positive distinct one-digit numbers exist such that they sum to 15?

4. Let O be the center of a circle. Assuming points A, B and C are on the circle, let point A pass through O to point C, creating a diameter. Point B creates a chord with point C, and angle ACB = 40°. What is angle BOC, in degrees?

5. In a 4x4 grid, two opposite center cells are shaded. How many rectangles and squares with sides parallel to the grid exist in the grid and do not contain either one of the shaded cells?



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**PART A PROBLEMS**

## Part 2 - Medium

1. In a school, 64 students take art classes and 32 students take drama classes. If they are allowed to take take both art and drama classes, but are required to take at least one of them, how many possible values are there for the number of students in the school?

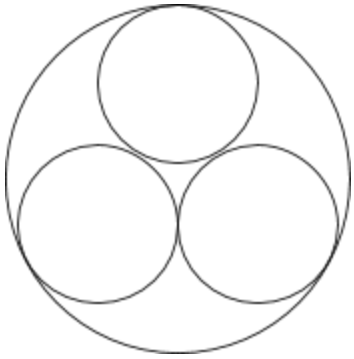
2. Given the following magic square, where the sum of each row and each column is the same, find w + a + y + z.

| | | |
|---|---|---|
| 12 | 18 | y |
| w | a | 13 |
| 16 | 11 | z |

3. Two circles of radius 4 are centered at (4 , 0) and (0 , 4). What is the area of the intersection of these two circles?

4. What is the probability that an integer in the set {1, 2, 3, … 100} is divisible by 6 but not 8?

5. Three circles (of radius 3) are externally tangent to each other and internally tangent to a larger circle. What is the radius of the larger circle?



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**Part 3 - Hard**

1. Determine all rational numbers $r$ for which all the solutions of the equation are integers.
$rx^2 + (r+1)x + (r-1) = 0 \in \mathbb{I}$

2. For how many real values of $a$ is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$ ? *(2005 AMC 10B #8)*

3. Two real numbers (a and b), between 0 and 1, are selected. What is the probability that $\sqrt{a^2 + b^2} \geq \sqrt{2}/2$ ?

4. $\sqrt{10 - x^2} = \frac{5}{x}$. Find all possible values of $x$.

5. Evaluate $i + 2i^2 + 3i^3 + 4i^4 + ... + 64i^{64}$.