MODULE 1: ENGINEER COSTS

Project 1 ? Project 2 → Political, Economic, Environmental.

Maximize our benefits, minimize costs.

Costs

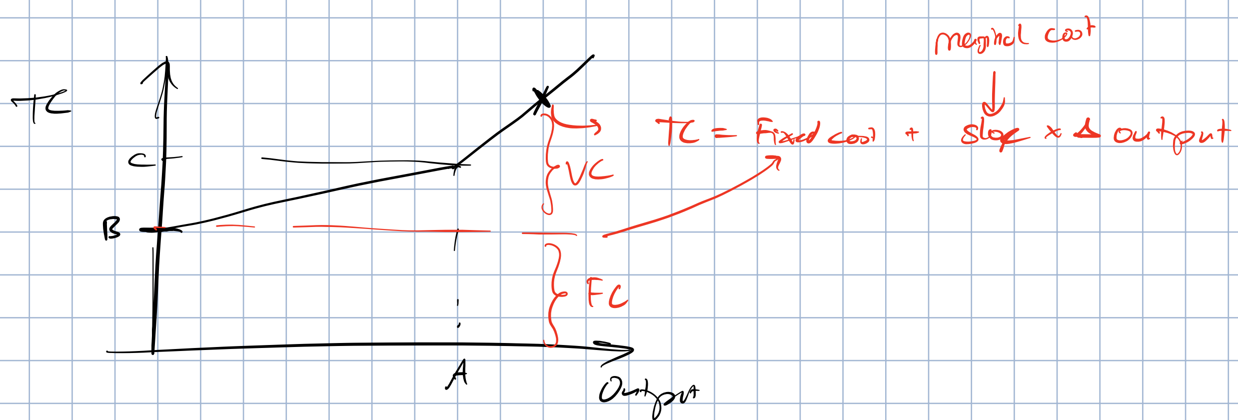
Total cost → Fixed costs $\not\propto$ output

Total cost → Variable Costs $\propto$ output

Marginal cost: cost of producing 1 more unit of output

Average cost: $\dfrac{\text{total cost}}{\text{\# of output units}}$

Analyze in a graph



Break-even: Revenue - costs = 0

Types of costs:

- Sunk cost: cannot recover
- Opportunity cost
- Recurring costs vs. non-recurring.
- Incremental costs: alt. diff.
- Cash cost
- Book costs
- Life cycle costs.

Cost Estimate

4 ways:

1. Per-unit estimate: find cost/unit $\Rightarrow$ multiply # of units.
2. Segment estimate: MECE analysis.
3. Cost index: historical data $\rightarrow$ cost in the future.

$$C_f = C_0 \left(\frac{Index_f}{Index_0} \right)$$

$C_0 \Rightarrow$ cost in past
$C_f \Rightarrow$ cost in future.

4. Power sizing: economies of scale.

Same time point

$$\frac{\text{Cost of equip A}}{\text{Cost of equip B}} = \left(\frac{\text{Size of equip A}}{\text{Size of equip B}} \right)^y$$

$$y = \begin{cases} 1: & \text{no economies of scale} \\ >1: & \text{diseconomies of scale} \\ <1: & \text{economies of scale.} \end{cases}$$

Module 2: Time Value Of Money

Interest & Interest Rates.

$$I = \underbrace{\text{money @ end paid}}_{\text{Future value}} - \underbrace{\text{money @ end that you got}}_{\text{Present value.}}$$

Connection between future + present value:

$$F = P(1+i)$$

↳ i is interest rate (% / time period)

Multiple periods of interest

Compound interest
Interest reinvested $\Rightarrow$ interests on interest.

$$F = P(1+i)^N$$

← Compounding periods

Periods should match.

Simple interest:
Constant amount of interest:

$$F = P + I = P + PiN$$

Compounding period different than given

Nominal

- Change to align w/ actual compounding period

Effective.

- Change to align w/ given period.

Nominal interest rate: interest rate, but actual comp. period diff.

$$i_{nominal} = i_{compound} \times m$$

$$i_{compound} = \frac{i_{nominal}}{m}$$

Problem tip: if compounding $\neq$ $i_{nominal}$ $\Rightarrow$ convert rate into right comp. time & change period!

Effective interest rate: assume actually compounding at given period $\Rightarrow$ ie changes!

$$i_e = \left(1 + \frac{i_n}{m}\right)^m - 1$$

# of comp. periods in given periods

Given interest rate.

Ex:// 12% interest rate. Compounding semiannually. What is \$2,000 in 5 years

① Nominal strategy:

1. Find $i_{compounding}$:

$$i_c = \frac{i_n}{m} = \frac{0.12}{2} = 0.06$$

2. Future value:

$$F = 2{,}000\,(1 + 0.06)^{10}$$
$$= \$3581$$

② Effective strategy:

1. i_e:

$$i_e = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 12.36\,\%\ /\text{year}$$

2. F:

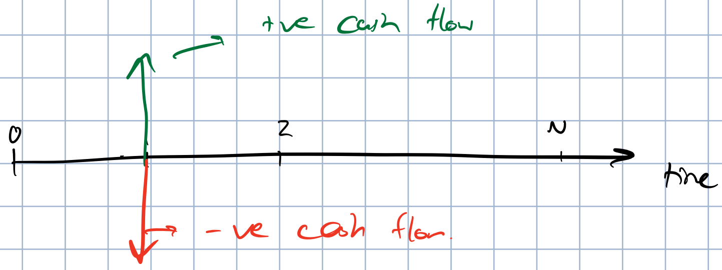
$$F = 2{,}000\,(1 + 0.1236)^5$$
$$= \$3581$$

Continuous compounding:

$$i_e = \lim_{m \to \infty} \left(1 + \frac{i_n}{m}\right)^m - 1$$ (really small periods)

$= e^{ir} - 1 \Rightarrow$ Maximum i_e

Cash Flow Diagram



All cash flows are considered at end of period!

Net cash flow: +ve cash − -ve cash

Cash Flow Analysis:

Non-recurring cash flow:

Future value given present: $F = P\boxed{(1+i)^N}$
$= P\underline{(F/P, i, N)}$

Present value given future: $P = \dfrac{F}{(1+i)^N} = F(P/F, i, N)$ ← Present worth factor.

Ex: $2,000 invested, 12%/yr compounded monthly. Future value? (20 yrs.)

Strategy 1: Nominal

$12\%/yr \Rightarrow 1\%/month$

$\therefore F = 2{,}000(1+0.01)^{240}$

Strategy 2: Effective:

$i_e = \left(1 + \dfrac{0.12}{12}\right)^{12} - 1$

$F = 2{,}000(1+i_e)^{20}$

Constant recurring cash flows:

⇒ Same value across all cash flows.

Annuities

Find future value given annuity + # of recurrences:

↳ Uniform series compound amount factor:

$$F = \frac{A(1+i)^N - A}{i} = A\left(\frac{(1+i)^N - 1}{i}\right)$$
$$= A(F/A, i, N)$$

Find annuity given future value + # of recurrence:

↳ Sinking fund factor

$$A = F \cdot \frac{i}{(1+i)^N - 1} = F(A/F, i, N)$$

Find annuity given present value + # of recurrence rate:

↳ Capital recovery factor:

$$A = P \cdot \frac{i(1+i)^N}{(1+i)^N - 1} = P(A/P, i, N)$$

Find present value given annuity

↳ Series present worth factor:

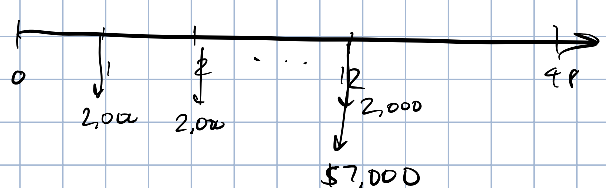
$$P = A \cdot \frac{(1+i)^N - 1}{i(1+i)^N} = A \cdot (P/A, i, n)$$

Ex:// \$80,000 in 48 months. Paid \$2,000/month + \$7,000 in 1^{st} year.
What is the effective & nominal interest rate:

① Figure out what info you have.

P.V. = \$80,000   1-time cash flow: \$7,000
A: \$2,000/month i : ?

② Cash flow:



③ Break up cash flow into parts.

P = annuity + first-yr

$$80{,}000 = A(P/A, i, N) + F(P/F, i, N)$$

$$80{,}000 = \left(\frac{(1+i_s)^{48}-1}{(1+i_s)^{48}}\right) + 7{,}000\left(\frac{1}{(1+i_s)^{12}}\right)$$

④ Solve for Unknown:

Methods:

i) Algebra

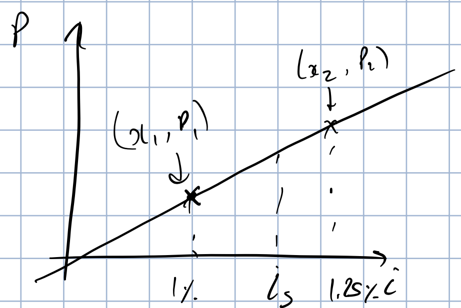
ii) Newton's method: BST

1. Make function = 0

$$P(x) = \left(\frac{(1+i_s)^{48}-1}{(1+i_s)^{48}}\right) + 7{,}000\left(\frac{1}{(1+i_s)^{12}}\right) - 80{,}000$$

2. Plug in i_s s.t. close to 0.

iii) Linear approximation:



$$\frac{P - P_1}{x - x_1} = \frac{P_2 - P_1}{x_2 - x_1}$$

$$x = x_1 + (x_2 - x_1)\frac{P - P_1}{P_2 - P_1}$$

$\vdots$

$= 1.126\% \Rightarrow$ Monthly rate.

Nominal:

$$i_N = 1.126 \times 12$$

$$i_e = \left(1 + i_N\right)^{12} - 1$$

Non-constant recurring cash flow:

Non-constant

Arithmetic series.
Constant amount of change

Geometric series
Constant % of change

Arithmetic gradient series:

G: constant amount of change.

Find present val. given i, G, $N \Rightarrow$ $P = G\left[\frac{(i+1)^N - iN - 1}{i^2(1+i)^N}\right]$

Find constant annuities equivalent to gradient series:

$$A = G(A/G, i, N) = G\left(\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right)$$

Only converts gradient, not entire flow!

Geometric gradient series.

g: rate of growth

Growth-adj rate: $i^\circ = \frac{1+i}{1+g} - 1$

Present val sum growth rate:

$$(P/A, g, i, N) = \frac{(P/A, i^\circ, N)}{1+g}$$

Problem Solving Strat.

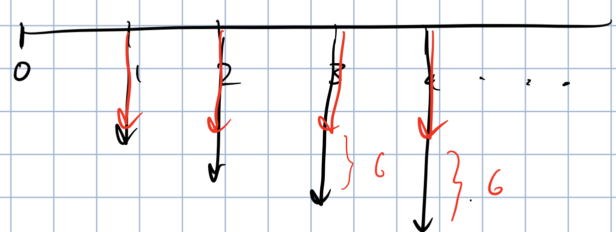
1. Cash flow diagram
2. Try to combine cash flows of same interest + compounding periods
3. Split up cash flows + analyze separately

   OR

   Change compounding period to match + change interest rate.

Ex:// P.W. of bills for Prius accumulating in 4 yrs. Repairs every 6 months w/ cost increasing \$50/6 months starting at \$500. $i = 12\%$ / year. P.W.?

① Cash flow diagram



② Separate out parts:

1. \$500 base repair
2. \$50 arithmetic series.

③ Combine:

$$A^T = A' + G\,(A/G, i, N)$$

i is wrong!

↳ Monthly comp. rate:

$$i_e = (1 + 0.01)^6 - 1$$
$$= 6.152\%$$

$$\therefore\ A^T = A' + G\,(A/G, i, N)$$
$$= \$659.39$$

④ Find calc:

$$P = A\,(P/A, i, N)$$

Net Present Worth Analysis (NPW)

Steps:

1. Determine interest rate, i
2. Determine lifetime service of proj., N
3. Determine inflow & outflow / period.
4. Determine net cash flow: $CF_k = \text{Inflow}_k - \text{Outflow}_k$
5. Discount to year 0: $CF_k^{(0)} = CF_k\,(P/F, i, k)$
6. Add all discounted cash flows:

$$NPW = \sum_{k=0}^{N} CF_k^{(0)}$$

Depreciation

Defn: Value of asset ↓ as time → ∞

Depreciation

Economic

Physical / monetary loss

Accounting:

Book / tax purposes.

⇓

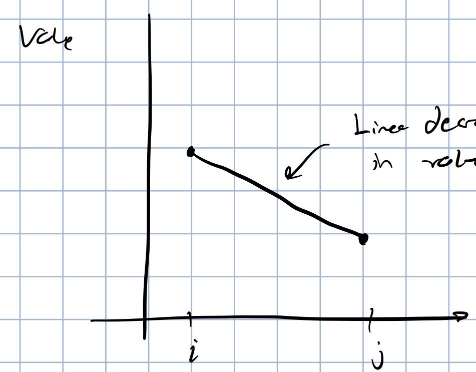
Spread out fixed cost in period

Cost basis: total cost of asset | Useful life: Service time | Salvage value: Value if asset sold at period k

How do you calculate depreciation cost:

Option A: Straight line method:

Assumption: constant rate of loss.



$$\text{Depreciation cost} = \frac{\text{Present value} - \text{Salvage value}}{N}$$

$$\text{Book value} = P - n\left(\frac{P-S}{N}\right) = \text{Book value @ period } n$$

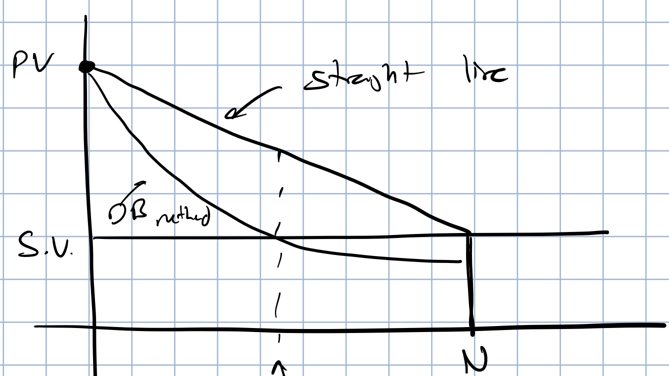
Option B: Declining balance method.

Assumption: loss in value is constant fraction.

Depreciation charge for period n = $dP(1-d)^{n-1}$ (d = rate of dep. P = Present value. n = period)

$\therefore$ $BV = P(1-d)^n$

Depreciates value faster:



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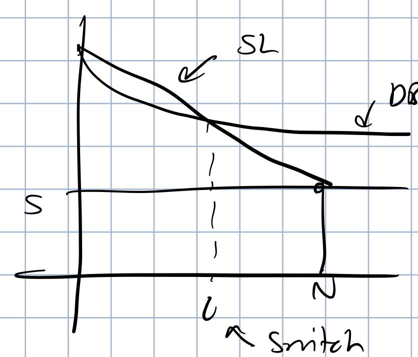


DB method depreciate quickly ⇒ at the point of salvage ⇒ switch to straight line method.

Ex:// Photocopier bought at \$10,000. Will in 5 yrs. of life, $S = \$3,277$

Find depreciation rate:

Assume that we are using declining balance method

① Depreciation rate via formula:

$$3,277 = 10,000(1-d)^5$$

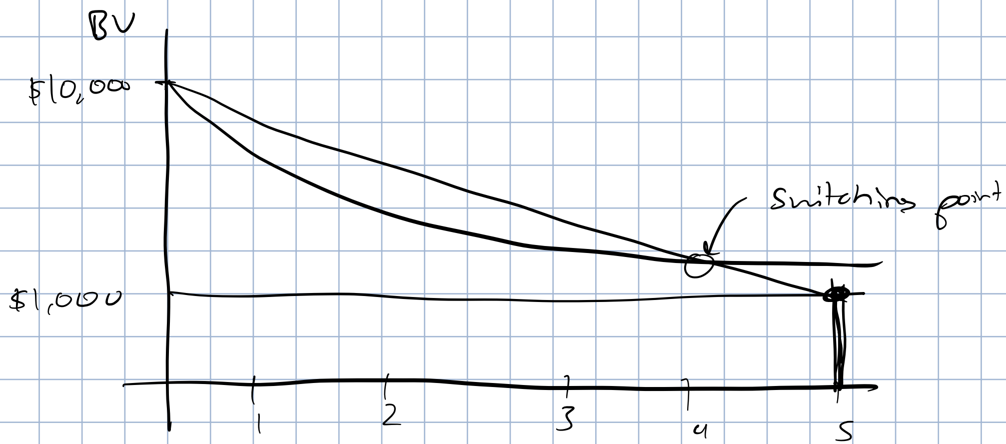
$$d = 0.2$$

② Annual depreciation formula:

$$BV_{dB_n} = 10,000\,(0.8)^n$$

Ex:// Prev. example but $S = \$1,000$ and $d = 0.2$. Find when to switch b/w declining balance → straight line:

① Graph:



② Equations:

$$BV_{SL} = P - n\left(\frac{P-S}{N}\right)$$

$$= 10,000 - 1,800n$$

$$BV_{DB} = 10,000\,(0.8)^n$$

③ Equality → solve for n

$$10,000 - 1,800n = 10,000\,(0.8)^n$$

⋮

$$\boxed{n = 2}$$

④ Depreciation chart:

	n	BV	D
DB	0	10,000	0
	1	8,000	2,000
	2	6,400	1,600
SL	3	4,600	1,800
	⋮	⋮	⋮

Module 3: Comparison Methods

Goal: compare different projects.

Assumptions:

#1: Costs & benefits measured monetarily

#2: We can always determine future flows

#3: No inflation /deflation

#4: No taxes

#5: All investments are upfront cost

#6: Generated cash reinvested equal to discount rate.

Project Types

Independent:
Costs & benefits separate in each project
Ex:// Buy fridge and bed?

Mutually Exclusive
Can only impl. 1 project
Ex:// Fridge A or B?

Related, Not M.E.
Costs /benefits depend on other project.
⇓
Simplify by dividing M.E. feasible projects.
Ex:// Freezer, fridge or both?

MARR (minimum acceptable rate return): minimum return company expects from investment.

- MARR ≥ 0 ⇒ at least worth it

Analysis Techniques:

1. Present worth analysis:

   Do NPW analysis for each project

   Comp. { If proj. are M.E.: choose proj. w/ best NPW
   If proj. are indep.: choose proj. with +ve NPW }

2. Annual worth analysis:

Measures present worth in annuities

Formula: $AW_i = PW_i (A/P, i, N)$ ⇒ AW concl. will be the same as PW. analysis conclusion.

Annual equivalent cost = AW for M.E. projects.

Capital recovery cost: another measure of AW.

$$CR(i) = P(A/P, i, N) - S(A/F, i, N)$$
$$= (P-S)(A/P, i, N) + iS$$

Comparisons:

If indep: choose project where A.W. > 0

If M.E.: choose project w/ best A.W.

↳ If analysis is using AEC, choose proj. w/ smallest AEC

3. Future Worth Analysis: exact same as P.W. except calculate worth at period N.

$$FW(i) = \sum_{k=0}^{N} CF_k (F/P, i, N-k)$$

Note: may have diff. conclusion than P.W/A.W.

Q: What happens if projects have unequal lives

① Repeated lives method: Use LCM of service lives & compare, assuming cash flows equal.

② Study method: define minimum common service life $_N$ ⇒ find a salvage value of longer project at N

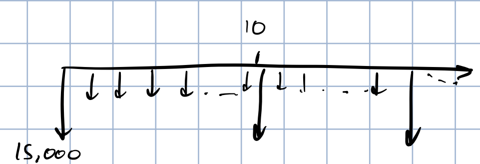
Ex://

	A1	A2
Initial	15,000	25,000
O&M	6,400	5,625
Service life	10	15

Repeated lives:

LCM (10, 15) : 30

A1:

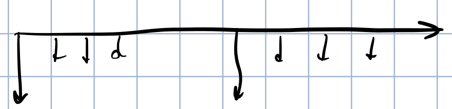


⇒ 3x repeat

$PW_1 = 15{,}000 + 15{,}000\,(P/F, i, 10) + 15{,}000\,(P/F, i, 20)$
$+ 6{,}400\,(P/A, i, 30)$

...

A2 :



$\Rightarrow$ 2x repeat

$PW_2 = 25{,}000 + 25{,}000\,(P/F, i, 15) + 5{,}625\,(P/A, i, 30)$

Study period:

A_2 has salvage value at yr. 10

$PW_1 = 15{,}000 + 6{,}400\,(P/A, i, 10)$

$PW_2 = 25{,}000 + 5{,}625\,(P/A, i, 10) + 5{,}000\,(P/F, i, 10)$

Note: you can calculate salvage value s.t. $PW_1 = PW_2$ and compare against posted salvage value. If calculated > posted $\Rightarrow$ other option. o.w. take current option.

Easier method of comparison: payback period:

$$\text{Pay back period} = \frac{\text{initial investment}}{\text{annual savings}}$$

If annual savings not constant $\Rightarrow$ manually subtract savings from previous year.

Con: discriminate long-term project, no discounting, no consideration about life.

$\Downarrow$

Discounted payback:
$$\frac{\text{initial investment}}{\sum_{k=0}^{N} \frac{\text{annual savings}_k}{(1+i)^k}}$$

Part 2

Rate of return: gain of investment / initial investment

Internal rate of return: interest rate i^* s.t. P.W. of outflows = P.W. of inflows.

$$① - \sum_{k=0}^{N} \frac{I_k}{(1+i^*)^k} = \sum_{k=0}^{N} \frac{O_k}{(1+i^*)^k}$$

Neglecting external factors.

IRR can be -ve $\Rightarrow$ losing money in project

How to calculate:

① Use formula in ①

② May need Newton's or algebraically

How is this related to project comp.:

If IRR $\geq$ MARR $\Rightarrow$ valid alternative.

Independent

Choose all projects where $i^* \geq$ MARR

Mutually Exclusive

Only choose highest i^* if equal lives.

Conclusion might not be same as NPW analysis.

Mutually exclusive projects & IRR:

1. Determine expensive & cheap options (look at abs value of initial investment)

2. Calculate incremental IRR:

$$\underbrace{(\text{expensive} - \text{cheap})}_{\text{initial cost}} + \underbrace{(\text{expensive} - \text{cheap})}_{\text{annuities}}(P/A, i^*, \ldots) = 0$$

3. Choose expensive if IRR > MARR. O.W., choose cheaper option

W/ more than 3 projects:

$\boxed{\text{Rank projects from cheap} \rightarrow \text{expensive via initial cost. Current best} = \text{cheapest}}$

$\downarrow$

$\boxed{\text{Challenger is next most expensive}}$

For incremental IRR ⇒ ≥ MARR

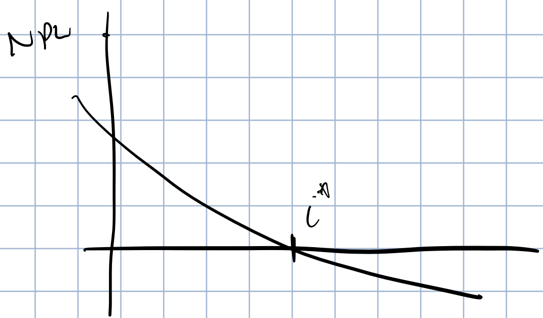
Yes → Current best = challenger

No → Discard challenger

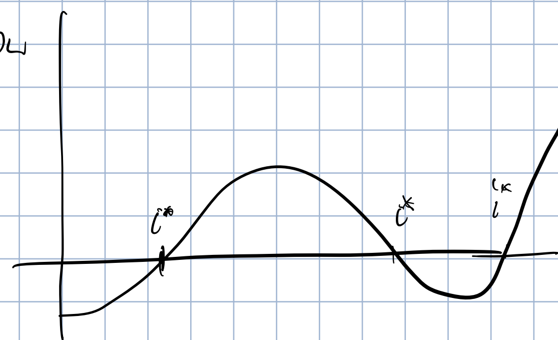
Part 3

Is possible to have multiple IRRs.

↳ # of possible solutions for i^* ≤ # of sign changes.



⇒



Classify projects based on # of sign changes:

1. Simple project: init CF < 0, 1 sign change ⇒ 1 unique IRR
2. Non-simple project: # of sign changes > 2 ⇒ Possible to have multiple IRR
3. Neither: start off @ +ve CF

Q: How do we select whether project is valid

Case A: Simple investment

↳ Calculate i^* ⇒ accept if $i^* \geq$ MARR

Case B: Non-simple investment

↳ Calculate i^* ⇒ only accept if i in interval like NPW > 0

Q: How to tell if multiple IRRs:

1. Descartes rule: # of i^* ≤ # of sign change
2. Nordstrom's Criterion: cumulative cash flow S_k. If $S_0 < 0$ & only changes signs once ⇒ unique +ve i^*

Q: Compare valid projects that have multiple IRR?

Use external rate of return: only 1 of them

External rate of return using project balance:

B_k = cumulative value of all cashflows up to year k compounded

$$B_3 = (1+i)^3 CF_0 + (1+i)^2 CF_1 + (1+i) CF_2 + CF_3$$

If $B_k \geq 0$: finds for k+1 period

else $B_k < 0$: investment

ERR is rate which should be applied at last period to make balance = 0.
All other periods compound @ MARR.

Ex:// CF following: MARR = 25%.

Year	CF
0	1000
1	-5,000
2	6,000

Find i_e^*

① Will this have multiple IRR?

Yes, b/c sign change.

② Guess an ERR

Initial guess: $i_e^* = MARR = 25\%$.

③ Simple investment:

Year 1: $\underbrace{1{,}000\,(F/P, 25\%, 1)}_{\text{Balance from prev. year, take FV to next year}} - \underbrace{5{,}000}_{\text{Current year cash flow}} = -3750$

⋮

Year k:

④ Last year, use i_e^* instead:

$$-3750\,(F/P, i_e, 1) + 6{,}000 = 0$$

⋮

$i_e = 60\%$.

Not super intuitive.

We like to calculate approximate ERR

i_{ea}^* is solution:

$$\sum_k CF_k^- (F/P, i_{ea}^*, N-k) = \sum_k CF_k^+ (F/P, MARR, N-k)$$

For FV of all disbursements discounts at i_{ea}^*

For FV of receipts at end, discounted @ MARR.

Ex: // Previous example:

$$-5{,}000\,(1+i_{ea}^*)^1 = 1{,}000\,(F/P, 25\%, 2) + 6{,}000$$

$$\Downarrow$$

$$i_{ea}^* = 51.25\%$$

Another method: modified internal rate of return

Uses 2 ERRs:

- i_{inv}: rate of investment
- i_{fin}: rate of borrowing

$\}$ Profit $\Rightarrow$ $i_{inv} > i_{fin}$

Formula:

$$\left(\sum_k CF_k^- (P/F, i_{fin}, k) \right) (F/P, MIRR, N) = \sum_k CF^+ (F/P, i_{inv}, N-k)$$

Part 4

Public projects require different analysis.

Public project →
- Sponsor's cost
- Social benefits: hard to estimate, monetary
- Disbenefits (social cost): undesirable consequences.

How to analyze:

① Benefits cost ratio:

$$BCR = \frac{PW\,(\text{social benefits} - \text{social costs})}{PW\,(\text{sponsor costs})}$$

$\}$ → FW & AW possible.

② Modified benefits cost ratio:

$$BCRM = \frac{PW\,(\text{social benefits} - \text{social costs} - \text{O\&M costs})}{PW\,(\text{capital cost})}$$

Leads to same conclusion as BCR, but also gives info on RoR on initial money.

Independent project:

BCR > 1 ⇒ accept

BCR < 1 ⇒ don't accept

BCR ~ 1 ⇒ look @ other factors

Mutually exclusive:

① Figure out PW / FW / AW for projects

② Order projects in order of increasing costs.

③ Put do-nothing option 1st

④ Calculate incremental BCR:

$$\Delta BCR = \frac{B_X - B_Y}{C_X - C_Y}$$

⑤ Analyze ΔBCR:

ΔBCR

If $C_X = C_Y$ ⇓ Choose alt. that has greater PW of benefit

$B_X = B_Y$ ⇓ Cheaper option

Choose X if BCR > 1, o.w. choose Y.

⑥ Repeat until 1 project remaining.

Ex: 2 systems replacing \$50,000/yr process over 5 yrs.

	System 1	System 2
Investment cost	10,000	15,000
Annual cost	45,000/yr	42,000/yr

① Calculate benefits & costs ⇒ order

	Cost (@ t = 0)	Benefit
Do nothing	0	0
System 1	10,000	5,000/yr
System 2	15,000	8,000/yr

② Incremental cost:

$$\Delta BCR_1 = \frac{5{,}000\,(P/A,\ 10\%,\ 5\text{ yrs})}{10{,}000} = 1.89 > 1 \quad \text{(System 1 better)}$$

$$\Delta BCR_2 = \frac{8{,}000(P/A, 10\%, 5yr) - 5{,}000(P/A, 10\%, 5)}{5{,}000}$$

$$= 2.27 > 1 \quad \text{(System 2 is better)}$$

MODULE 4: RISK & UNCERTAINTY

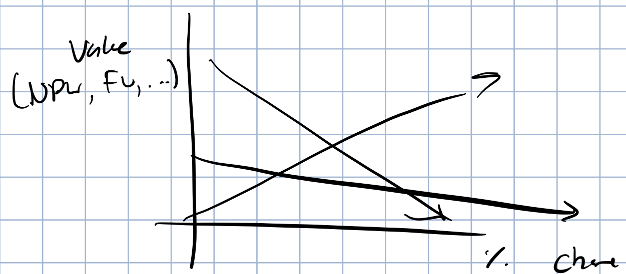
Q: How to deal w/ change?

Method #1: Sensitivity Analysis:

① Parameterize 1 aspect of NPW/AW/FW ⇒ graph how that 1 param would affect value:

② Change parameter $\pm x\%$.

③ Draw a graph:



⇒ Do this / param.

Ex: // Project has following:

Capital investment = $-\$11{,}500$
Annual revenue = $\$5{,}000$
" expenses = $-\$2{,}000$

Salvage = $\$1{,}000$
$N = 6$ years
MARR = 10%.

Sensitivity analysis:

① Baseline NPW:

$$NPW = -\$11{,}500 + 3{,}000(P/A, 10\%, 6) + 1{,}000(P/F, 10\%, 6)$$

② Parametrize:

Vary each param by $p\%$.

$$NPW = -\left(1 + \frac{p}{100}\right)(11{,}500) \ldots \sim$$

$$NPW = -\$11{,}500 + \left(1 + \frac{p}{100}\right)5{,}000(\ldots) + \sim \_\_\_ -$$

⋮

Cons: 1. Can't extrapolate too far
2. Missing out on inter-param effects.

Method #2: Break-even analysis

Find param value ⇒ hit break-even / threshold

For comparison:

- Independent: choose any project > threshold
- M.E.: choose project w/ param. in threshold range.

Ex://

	A	B
Expected Life	7 yrs.	10 yrs.
First Cost	\$200,000	\$350,000
Maintenance	\$10k/yr 0.05/unit	\$20k + \$0.01/unit
Labor	\$1.25/unit	\$0.5/unit
Other Costs	\$6.5 k/yr + 0.95/unit	\$15.5 k/yr + \$0.55/unit
Salvage Value	\$ 5,000	\$20,000

Uncertain on:

- # of units
- Unitary 'other costs' for project B: \$0.45 → \$0.75/unit

Figure out which project.

MARR: 15%.

① Define parameters:

x = # of units, y = unitary other costs for B.

② PW/AW/FW per project

$$AC_A = 200{,}000\,(P/A, 15\%, 7) + 10k + 6.5k + (0.05 + 1.25 + 0.95)x$$

$$= 65{,}023.9 + 2.25x$$

$$AC_B = \ldots = 106{,}223 + (0.51 + y)x$$

③ Graph param range:

If graphing multiple params ⇒ keep everything but 1 constant s.t. same AW/PW/FW despite param.

Method #3: Risk-adjusted MARR

MARR ↑ ⇔ uncertain ↑

Have different MARR based on riskiness of project ⇒ given.

Con: uncertainty baked in MARR ⇒ hard to figure out actual uncertainty

Analysis methods stay same.

Method #3: Probability

Idea: Use expected value for PW/ Conb. ...

Ex:// Project has 60% chance of \$8,000 annual benefit, 30% it is \$5,000, also 2% chance of \$10,000. 6yr life is 2x more likely than 9yr life. First cost = \$25,000. MARR = 10%.

Find expected value of project.

① Make a matrix of possible combos:

Possibilities	Probability
\$5,000, 6yrs	p_1
\$5,000, 9yrs	p_2
\$8,000, 6yrs	p_3
\$8,000, 9yrs	p_4
\$10,000, 6yrs	p_5
\$10,000, 9yrs	p_6

p_i = P(1st outcome) · P(2nd outcome)

② Calculate PW/ AW ... per outcome.

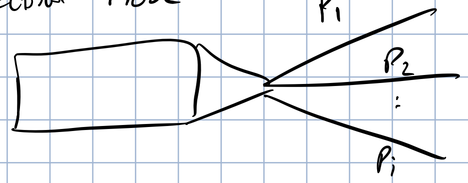
③ Expected value:

$$PW_1 \cdot p_1 + PW_2 \cdot p_2 + \ldots + PW_6 \, p_6 = \_\_\_\_$$

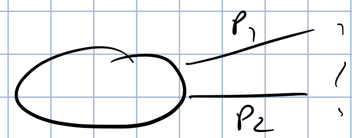
Note: do not take expectations of param ⇒ take E[PW]

Method #4: Decision Tree

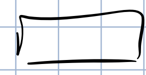
Decision node



Chance node



Outcome



MECE tree:

To find value:

1. Start from outcome nodes & work back
2. Come to chance node ⇒ expectation of monetary value.
3. Decision node ⇒ choose highest branch.