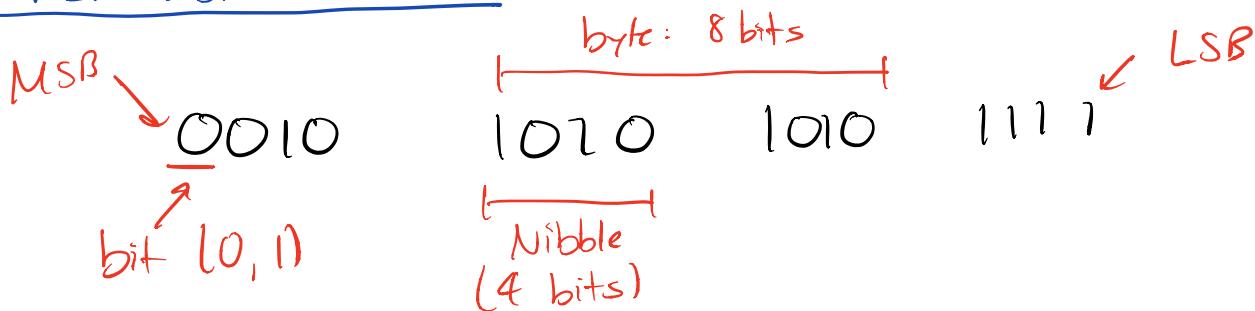


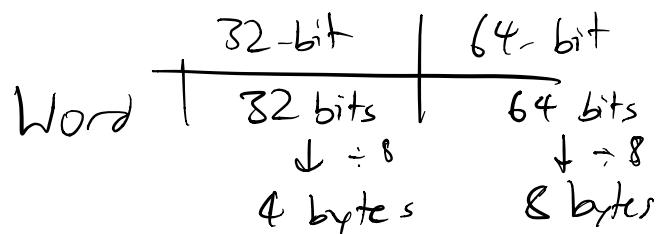
# CS 241

## DATA REPRESENTATIONS



- Word: dependent processor

o 32-bit / 64-bit  $\Rightarrow$



Binary number

Unsigned      Signed      Characters

1. Unsigned integer:  $n \geq 0$

a) UI  $\leftrightarrow$  decimal:

$\hookrightarrow$  UI  $\rightarrow$  decimal:

$$b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0 = (2^0 \times b_0) + (2^1 \times b_1) + (2^2 \times b_2) + \dots + (b_7 \times 2^7)$$

$\hookrightarrow$  Decimal  $\rightarrow$  UI

i) Group into largest powers of 2

$$\begin{aligned} 68 &= 64 + 4 \\ &= 2^6 + 2^2 \end{aligned}$$

$$1000100 = 68$$

$$\begin{array}{ccccccc} & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ \nearrow 2^6 & & \nearrow 2^5 & \nearrow 2^4 & \nearrow 2^3 & \nearrow 2^2 & \nearrow 2^1 & \nearrow 2^0 \end{array}$$

ii) Repeated division : Use remainder to construct digit

Num	$\frac{Q}{2}$	R
68	34	0
34	17	0
17	8	1
8	4	0
4	2	0
2	1	0
1	0	1

$\boxed{Q=0}$

$$68 = 1000100$$

b) Arithmetic: overflow

$$\begin{array}{r}
 \begin{smallmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{smallmatrix} \\
 + \begin{smallmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix} \\
 \hline
 \begin{smallmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{smallmatrix}
 \end{array}$$

2. Signed integers: neg. + pos.

Negative?

Use this!

MSB as indicator

$\hookrightarrow 0 \rightarrow \oplus$

$\hookrightarrow 1 \rightarrow \ominus$

$10 \dots 0$

$00 \dots 0$

2's complement

Neg: flip all bits + 1

-8  $\Rightarrow$  ① Positive integer rep.

1000

② Flip

0111

③ Add 1:

$$1000 \Rightarrow -8$$

- o 2's complement trick: flip trick:

Take the right-most 1 bit  $\Rightarrow$  flip everything to left

$$\text{Ex: } -8 \Rightarrow 0001000 \quad \left. \begin{array}{c} \\ \downarrow \text{Flip} \end{array} \right\} -68: 1000100$$

$$1 \dots 111000 \quad \left. \begin{array}{c} \\ \downarrow \text{Flip} \end{array} \right\} \dots 0111100 \Rightarrow -68$$

Proof:

$$\uparrow 8 \text{ bit} \Rightarrow 0 - (2^8 - 1)$$

$$2^{8n} \approx 0$$

$$\downarrow \text{Make } 2^8 \approx 0 \Rightarrow \text{system mod } 2^8 \pmod{256}$$

$$2^8 - 1 = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

Represent -68:

$$2^8 - 1 = 2^7 + \cancel{2^6} + 2^5 + 2^4 + 2^3 + \cancel{2^2} + 2^1 + 2^0$$

$$2^8 - (2^6 + 2^2) = (2^7 + 2^5 + 2^4 + 2^3 + 2^1 + 2^0) \boxed{+1}$$

$$\left. \begin{array}{c} \overbrace{- (2^6 + 2^2)} \\ = -68 \end{array} \right\} \text{Every other bit is flipped}$$

Hexadecimal: base 16

$$0 \longrightarrow 16$$

$$\left. \begin{array}{l} 0-9: 0-9 \\ 10-11: A-F \end{array} \right\} AF8_{16} = 8 \times 16^0 + F \times 16^1 + A \times 16^2 \\ = 8 \times 16^0 + 15 \times 16^1 + 10 \times 16^2 \\ = \dots$$

Binary  $\rightarrow$  hexadecimal:

1) Separate binary into nibbles:

$$\underline{0} \underline{1} \underline{0} \underline{0} \underline{1} \underline{0} \underline{1} \underline{0} \Rightarrow 0101 \ 0010 \ 1011$$

2) Convert each nibble  $\rightarrow$  hexadecimal

$$0101 \ 0010 \ 1011 \Rightarrow \boxed{\text{S2B}_{16}}$$

3. Characters:

ASCII / UTF / ...  $\Rightarrow$  encodings

- o ASCII table (man ascii)

### Operations

bitwise AND: AND of bit pair (&)

$$1001 \ \& \ 1100 = \begin{array}{r} (100\ 1) \\ \& (110\ 0) \\ \hline 1000 \end{array}$$

bitwise OR: OR of bit pair (|)

$$\begin{array}{r} (100\ 1) \\ | \quad (110\ 0) \\ \hline 1101 \end{array}$$

bitwise NOT ( $\sim$ )

bitwise XOR ( $\wedge$ ):  $a \wedge b$

Shifts:

o Left-shift ( $<<$ ):  $x << n = x \times \underline{2^n}$

$$1001 << 3 = \underline{1001} \underline{000}$$

o Right-bit ( $>>$ ):  $x >> n = x \div 2^n$  (remainders are zero)

$$\underline{1001} >> 3 = 000\dots01\underline{001} >> 3$$

$$2^0 = 000 \dots 0001$$

- Ex://

unsigned char c = 1;  $\Rightarrow$  00001  
 digit

unsigned char d = 25  $\Rightarrow$  11001

c <<= 3;  $\Rightarrow$  001000

c |= 1;  $\Rightarrow$  1001

c = c & d

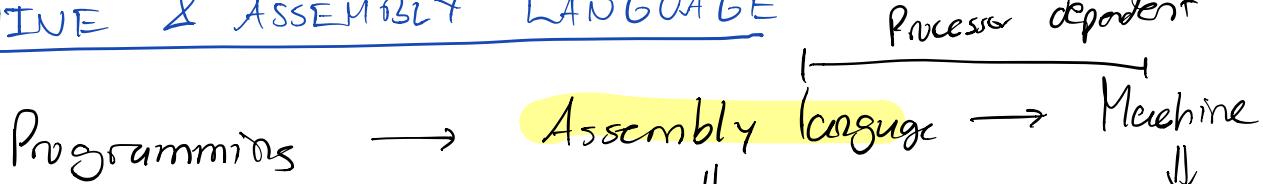
printf ('%d', c);

integer: 9

$$\begin{array}{r} 001000 \\ - - - - 1 \\ \hline 1001 \end{array}$$

$$\begin{array}{r} 01001 \\ & \& 11001 \\ \hline 01001 \\ | \\ 9 \end{array}$$

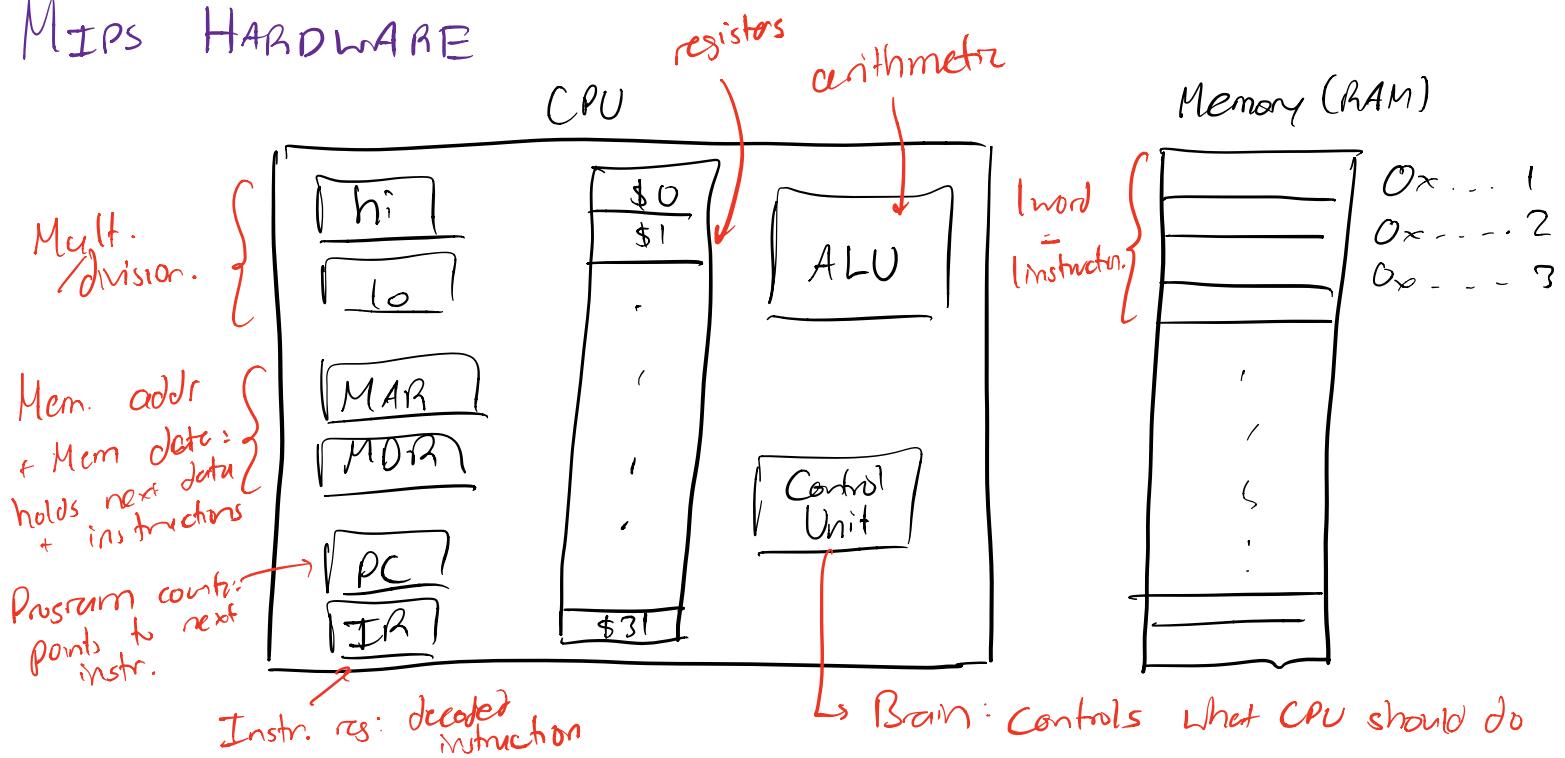
## MACHINE & ASSEMBLY LANGUAGE



Somewhat readable  
code s.t. processor  
will understand

Sequence of  
0's or 1's

## MIPS HARDWARE



- Special registers:
  - \$0: holds 0
  - \$29: frame ptr.
  - \$30: stack ptr.
  - \$31: return register
- Processor doesn't know difference between data and instruction:
  - Different data representations for a sequence of bits, processor makes no assumptions.
  - Know the representation before hand.
- Fetch - increment - execute cycle:
  1. Load into start of memory, set up PC to start of memory
  2. Fetch: IR fetches word from RAM based on PC
    - ↳ PC, PC+1, PC+2, PC+3
  3. Increment: PC += 4 (already at next instruction)
  4. Execute: take IR instructions → CU will decide what to do

## MIPS LANGUAGE

- In general: every MIPS command has an encoding into bits.
- Ex: / add \$s, \$t, \$d : 000000 tttttt dddddd ssssss 000000
  - 1000000
  - ↳ 5 bits: Range of s bit is 0-31
- Addition & subtraction:
  - add \$s, \$t, \$d  $\Rightarrow$  \$s = \$t + \$d
  - sub ll  $\Rightarrow$  \$s = \$t - \$d
- Jump register: move the PC to the address in \$s

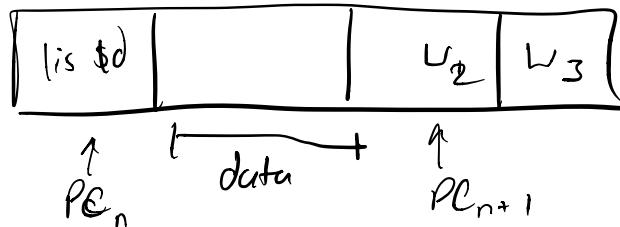
jr \$s

- jr \$31: returning from the program

- Load immediate + skip instruction: loading immediates/constants into a register.

lis \$d

· word 0/a/memory addr.



- Ex:// lis \$3

· word 7

lis \$7

· word 3

add \$2, \$3, \$7

jr \$31

- Multiply & divide:

- Multiplication:

Signed: mult \$s, \$d  $\Rightarrow$  \$s  $\times$  \$d

Unsigned: multu \$s, \$d  $\nearrow$

▷ Storage: hi holds MSW, lo holds LSU (ment for overflow)

- Division:

Signed: div \$s, \$d  $\Rightarrow$  \$s / \$d

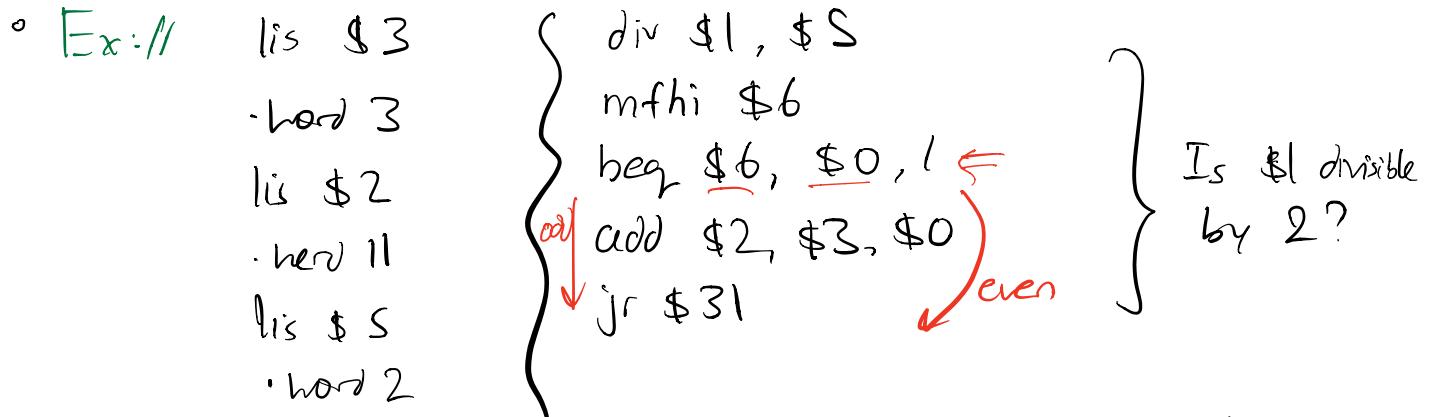
Unsigned: divu \$s, \$d  $\nearrow$

▷ Storage: quotient is in lo, remainder is in hi

- Moving from hi/lo: mflo / mfhi \$d (move lo/hi  $\rightarrow$  \$d)

- Branching:

- Branch if equal:  $\text{beq } \$s, \$t, i$
  - Branch if not equal:  $\text{bne } \$s, \$t, i$
- } skip  $i$  instructions if  
 $\$s == \$t$  or  $\$s != \$t$   
 $\uparrow$   
 $\text{beq}$   
 $\uparrow$   
 $\text{bneq}$



Essentially moves PC i words away based on branching

Tips:

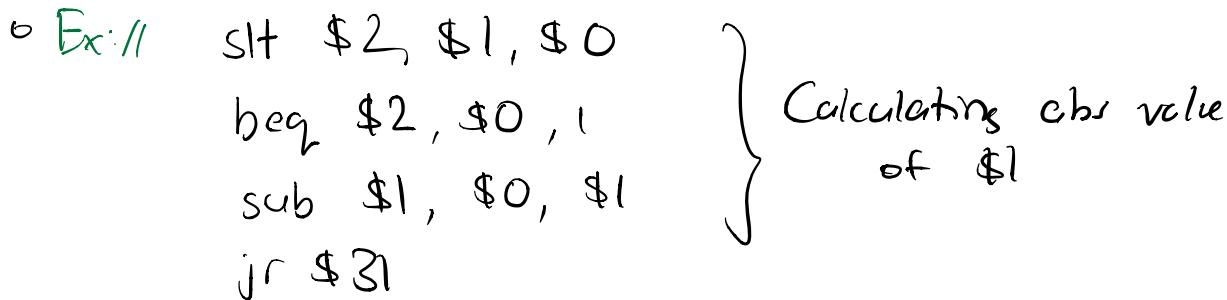
- Opposite can is addressed right after branch
- Makes sure comparison is correct

- Set less than:

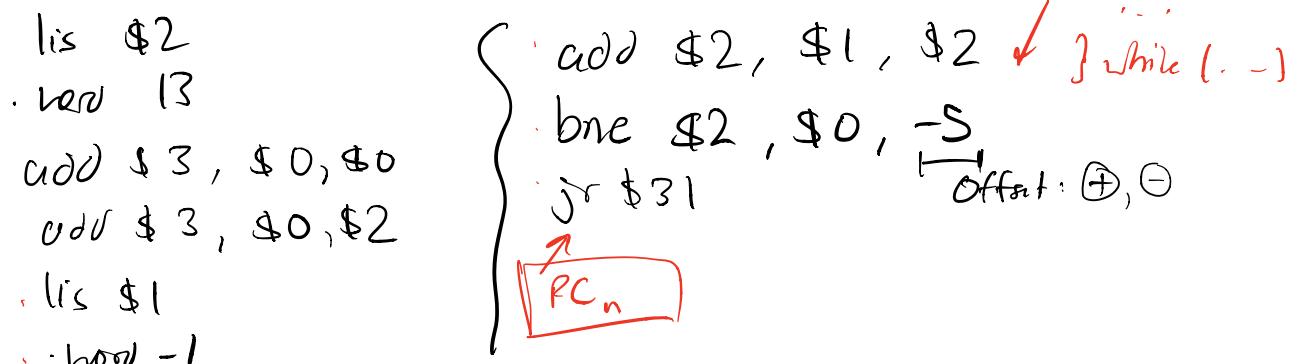
slt / sltu \$d, \$s, \$t

$\$d = 1$  if  $\$s < \$t$ , otherwise  $\$d = 0$

Use in conjunction w/ branching.



- Conditional loops: use bnez, bne to loop



Use labels instead: labels holds memory address

lis \$2  
 . word \$13  
 lis \$1  
 . word -1  
 add \$3, \$0, \$0

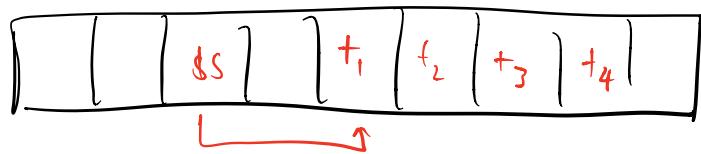
loop:  
 add \$3, \$2, \$3  
 add \$2, \$2, \$1  
 bne \$2, \$0, loop  
 jr \$31

⇒ Formula: offset =  $\frac{\text{label} - \text{PC}}{4}$

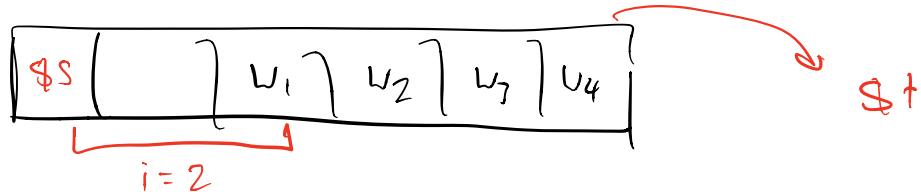
## - Storing and loading words

- Storing: sw \$t, i(\$s) ⇒ store the value in \$t and put it in  $\text{MEM}[\$s + i]$

◦ i: # of bytes! Not words



- Loading: lw \$t, i(\$s) ⇒ loading in word from  $\text{MEM}[\$s + i]$  -  $\text{MEM}[\$s + i + 3]$  and put it into \$t



- Aligned access: memory address should be multiple of 4

⇒  $(\$s + i) \% 4 = 0$

- Ex:// \$1 contains address to base of arr. \$2 contains # of elements. Refine a[3]

Loading 4<sup>th</sup> word ⇒ skip 3 words ⇒  $3 \times 4 = 12$  bytes

lis \$5  
 . word 3  
 lis \$4  
 . word 4

mult \$5, \$4  
 mflo \$5  
 add \$1, \$5, \$1  
 lw \$3, 0(\$1)

already added offset!  
 Tip to do when programming  
 no specifying necessary

- I/O:

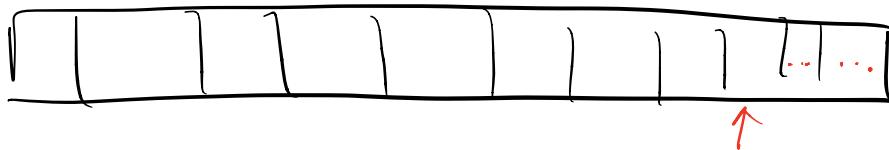
- Input: load from  $0xFFFF\ 0004 \Rightarrow$  read 1 character (LSB)?
- Output: store from  $0xFFFF\ 000C$

- Procedures: functions

- Issues: registers have global scope, transfer control, recursion arguments / return values

- Addressing register usage:

- Trick: store registers in RAM  $\rightarrow$  procedure  $\rightarrow$  restore register
- Use stack pointer ( $\$30$ ): points to memory of last allocated memory.



- ① stored in  $\$30$
- ② stored in  $\$30 - 1$
- ③ stored in  $\$30 - 2$

- Flow:

- Pushing to mem. stack*
  - 1. Store register in RAM using neg. offsets from  $\$30$
  - 2. Update  $\$30$  s.t. it is pointing to last word
  - 3. Perform operations
  - 4. Restore registers +  $\$30 \rightarrow$  Popping from stack

- Ex://



Pushing to memory

$sw \$1, -4 (\$30)$

lis \$4

word 4

$sub \$30, \$30, \$4$

\$30

Pop from Memory?

lis \$4

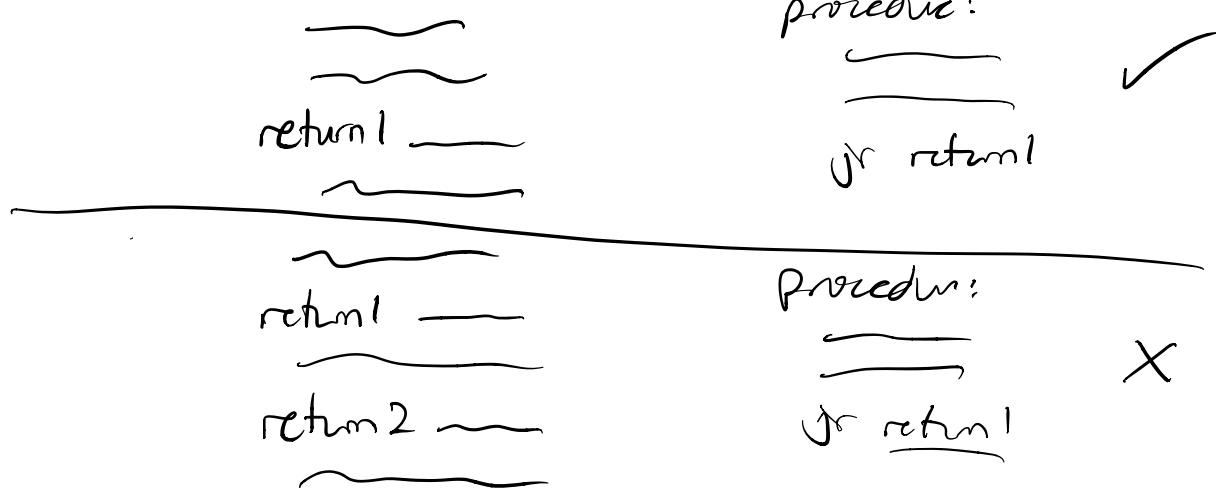
word 4

$add \$30, \$30, \$4$

$lw \$1, -4 (\$30)$

- Calling & returning from registers.

- Problem w/ labels:



↳ Calling procedure from 2 places

does not work w/ labels

- Jump register: same issue
- Use jalr instead

▷ Ex: // lis \$21  
· word foo  
jalr \$21



foo: - - ,

jr \$31

1. Updates \$31 to PC  
(pointing to next instr.)  
2. PC = \$21 (to execute procedure)

▷ Store \$31 in RAM to prevent overwite.

*pushing \$31 to mem.*

sw \$31, -4(\$30)  
lis \$31  
word 4  
sub \$30, \$30, \$31

*pop from mem.*

lis \$31  
word 4  
add \$30, \$30, \$31  
lw \$31, -4(\$30)

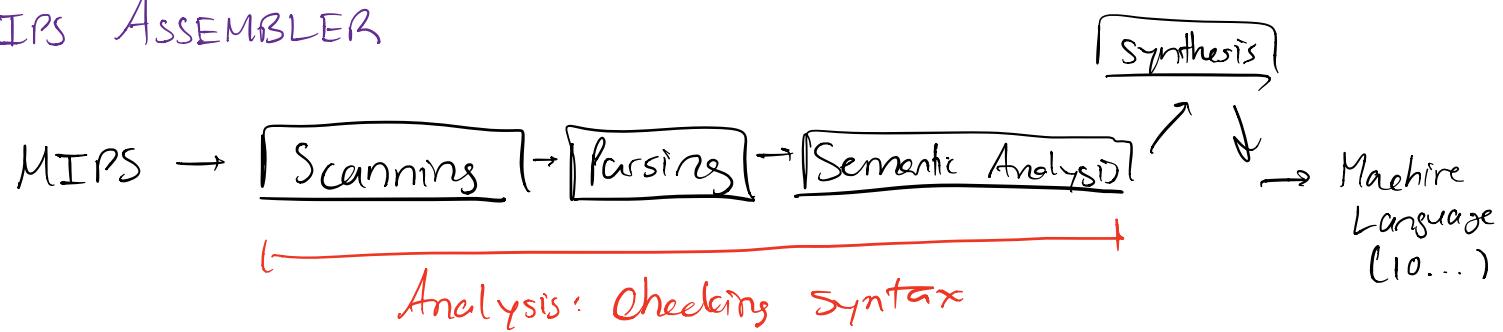
*call procedure*

lis \$21  
word foo  
jalr \$21

- Recursion: continually update stack pointer + push in memory
- Could potentially run out of stack space

- Pass in parameters: either use registers + persist, use stack just like method before for storing res.

## MIPS ASSEMBLER



### - Analysis:

- Scanning: characters → tokens
- Parsing: syntax checking
- Semantic analysis: unique labels, semantic

### - Synthesis: labels, instruction encoding, generating output

- Label: associate labels w/ addresses

#### D) Symbol table:

Label	Address	
:	:	
:	:	
:	:	
:	:	

⇒ Semantic analysis  
Can check unique labels  
easily.

#### D) Problem: labels used before definition

↳ 2 passes: first → scanning, parsing, symbol table  
Second pass → semantic analysis, synthesis

#### D) Ex://

0x00	main: lis \$2
0x04	.word beyond
0x08	lis \$1
0x0C	.word 2
0x10	add \$3, \$0, \$0
0x10	top: begin:

label	addr.
main	0x00
top	0x10
begin	0x10

## ◦ Instruction encoding:

D Ex:// Generate encoding for add \$3, \$2, \$4

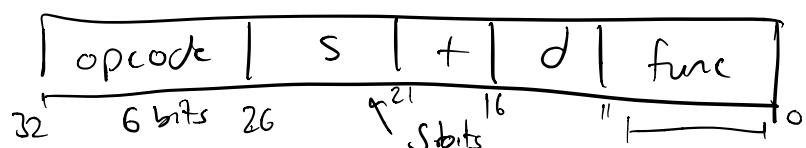
i) Encoding formally:

add \$d,\$s,\$t: 000000 ssss tttt ddjjjj  
00000 100000

## 2) Assemble mapping

<u>Value:</u>	<u>Decimal</u>	<u>Binary</u>
Op:	0	- - -
Result	2	1
{ S	4	.
+	3	J
)	32	J
0		
Fun		

3) Encoding → correct stop



Op code: shift left by 26 bits

g : shift left by 21

+ : shift left by 16

$\text{J} = \text{shift left by } 11$

fine: don't shift

4) Combine : bitwise or

$$\begin{array}{r}
 1100\ 00. \quad \rightarrow 0 \\
 - 0111 \quad \quad \quad \\
 \hline
 1100\ 0111 \quad \quad \quad
 \end{array}$$

5) Offsets: 32 bit constants:

- Masking: bitwise AND w/ 0...01...1

$$\begin{array}{r} 1011 \\ & \underline{\quad 0111} \\ & \boxed{0011} \end{array} \left. \begin{array}{l} \text{Put 1's in places where we} \\ \text{miniz the number, 0's where} \\ \text{we ignore.} \end{array} \right\}$$

- Generate output: ASCII & standard output

## REGULAR LANGUAGES

### Formal Languages

- Alphabet ( $\Sigma$ ): non-empty finite set of symbols
  - Ex://  $\Sigma = \{a, \dots, z\}$ ,  $\Sigma = \{0, 1\}$
- Word: finite sequence of symbols chosen from  $\Sigma$ 
  - $\Sigma^*$ : set of all words from alphabet  $\Sigma$
  - $|w|$ : length of word
  - $\epsilon$ : empty word  $\Rightarrow \forall \Sigma, \epsilon \in \Sigma^* \wedge |\epsilon| = 0$
- Language: set of strings w/ some constraint
  - Ex:// 1.  $L = \emptyset, L = \{\}$   $\Rightarrow$  empty language
  - 2.  $L = \{\epsilon\} \Rightarrow$  language made of empty word
  - 3.  $L = \{ab^n a : n \in \mathbb{N}\} \Rightarrow$  strings from  $\Sigma = \{a, b\}$   
s.t. a followed by 0 or more b followed by an a
- Recognition algorithm: determines if word fits in language

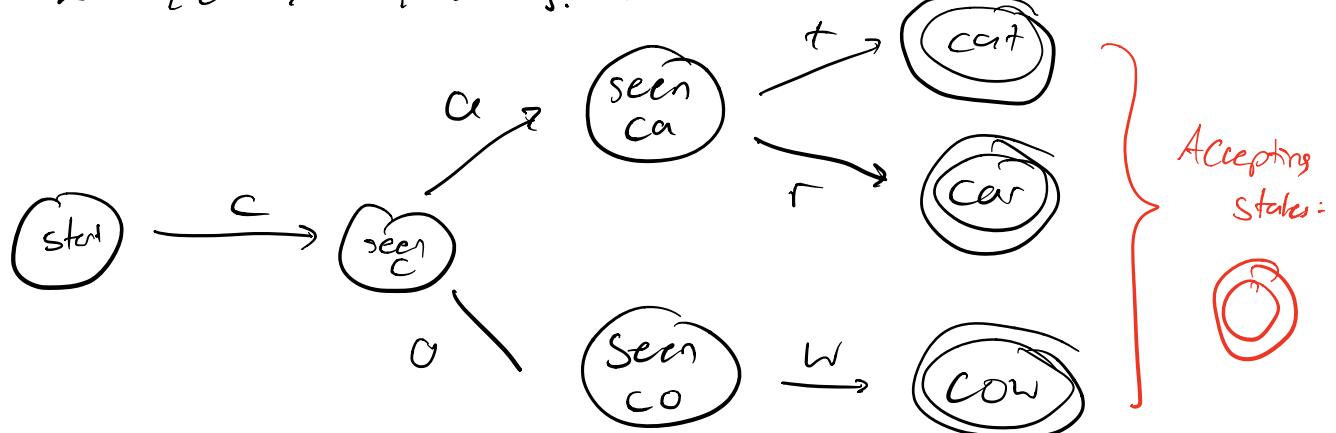
### Finite Languages

- Finite # of words  $\in L$
- Recognition algorithm:
  1. List all words + check if our word belongs (inefficient)
  2. state diagrams
- State diagram:
  - Rules:
    1. States: unique order of letters seen so far
      - start state, accepting states (legit word seen)

2. Transitions: tell us how to go from 1 state to another based on letter seen

▷ If no transitions defined for letter  $\Rightarrow$  error state

- Ex://  $L = \{\text{cat}, \text{cow}, \text{car}\}$ . Determine if  $w \in L$



## Regular Languages

- Definition of regular:

1.  $\{\}$  or  $\{\epsilon\}$  are regular

2.  $\{a\}$  for some  $a \in \Sigma$

3. Language built using union, concat. or Kleene star of 2 regular languages

- Building regular languages:

• Union:  $L_1 \cup L_2 = \{x : x \in L_1 \text{ or } x \in L_2\}$

▷ Ex://  $L_1 = \{\text{car}, \text{cat}\}$ ,  $L_2 = \{\text{boat}, \text{bar}\}$

$$L_1 \cup L_2 = \{\text{car}, \text{cat}, \text{boat}, \text{bar}\}$$

• Concatenation:  $L_1 \cdot L_2 = \{xy : x \in L_1, y \in L_2\}$

▷ Ex://  $L_1 = \{\text{car}, \text{cat}\}$ ,  $L_2 = \{\text{boat}, \text{bar}\}$

$$L_1 \cdot L_2 = \{\text{carboat}, \text{carbar}, \text{catboat}, \text{catbar}\}$$

• Kleene star:  $L^* \Rightarrow 0/\text{more concatenating strings from } L$

▷ Ex://  $a^* = \emptyset, a, aa, aaa\dots$

▷ Ex. //  $L = \{a, b\}$ .  $L^* = \{\epsilon, a, b, ab, abb, aab, \dots\}$

▷ Best way to think about this: regex \*

◦ Order of operations: \*, ., ∪

◦ Tips:

1. If we ever need  $a^n \Rightarrow a^*$

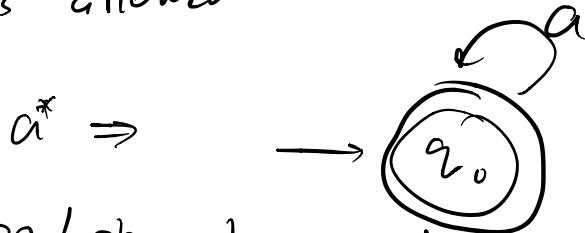
2. Even/odd: consider  $2n+1$

Odd #: of a:  $b^* a b^* (ab^* a b^*)^* \Rightarrow 2^{n+1}$  letters

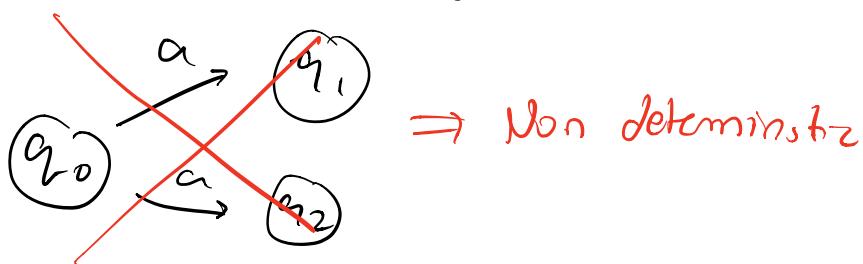
- Deterministic finite automata: state diagram for reg. L

◦ Rules:

▷ Self loops allowed:



▷ 1 transition / character  $\Rightarrow$  deterministic



◦ Formal definition: DFA is a 5 tuple:  $(\Sigma, Q, q_0, A, \delta)$

◦  $\Sigma$ : alphabet

◦  $Q$ : set of states (finite, non-empty)

◦  $q_0$ : starting state

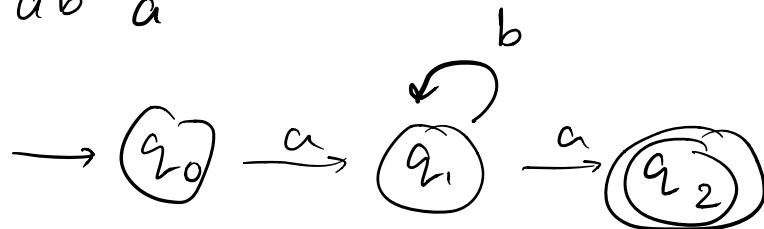
◦  $A$ : set of accepting states ( $A \subseteq Q$ )

◦  $\delta$ :  $\delta(Q, w \in \Sigma) \rightarrow Q_2$  (way to transition state  $h_{\text{state}}$ )

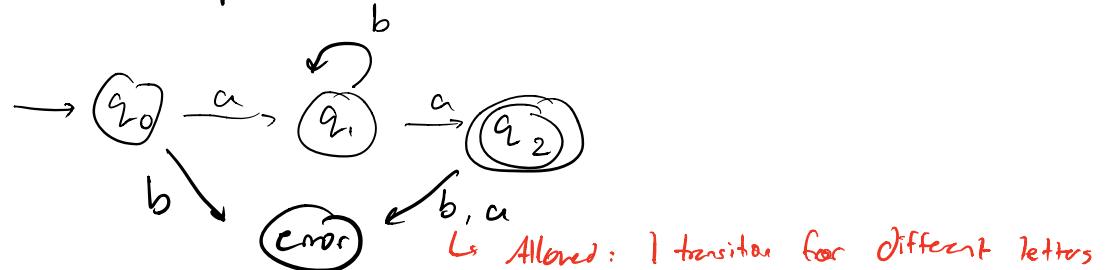
◦ Recognition algorithm:  $s^*(q_0, w) \in A$

▷ Take word character by character and use transitions to determine whether word belongs in DFA

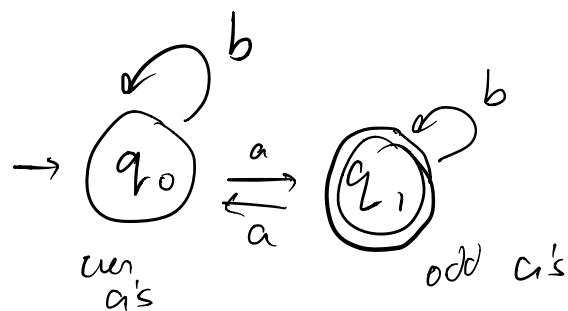
Ex://  $ab^*a$



Ex://  $ab^*a$  w/ explicit error state ( $\Sigma = \{a, b\}$ )

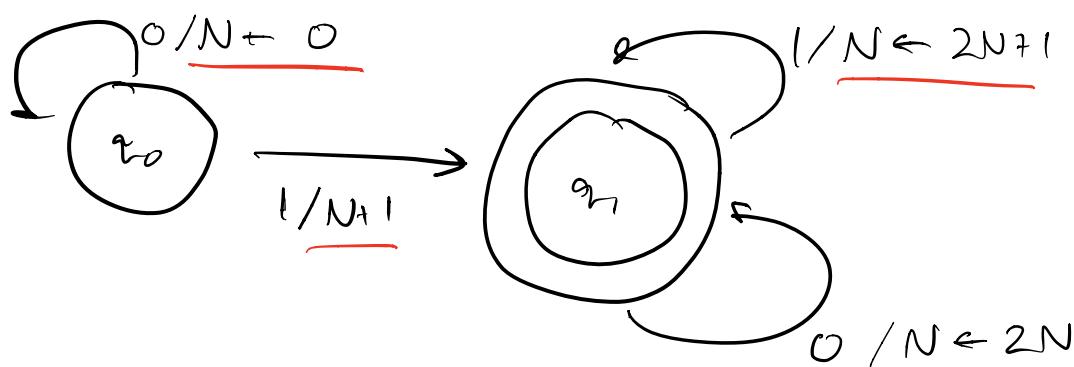


Ex://  $L = b^*a(b^*ab^*a)^*$



$L$  is regular  $\Leftrightarrow$  DFA for language

Finite transducers: action / transition

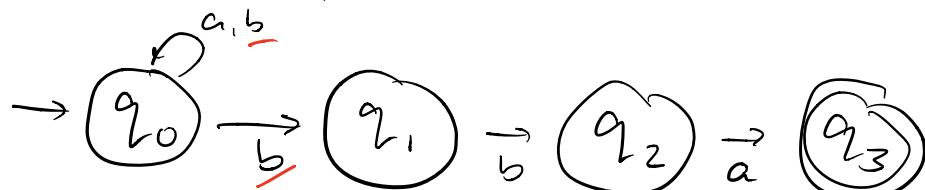


Used in scanning stage of compiler

- Non-deterministic finite automata:

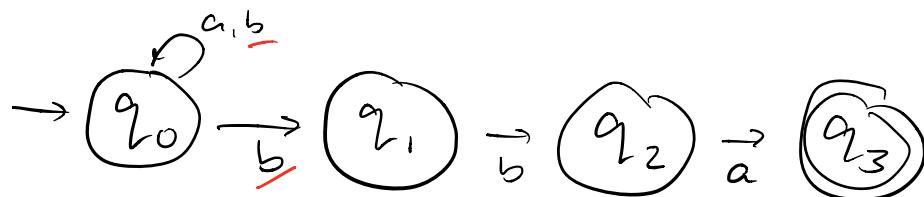
Non-deterministic: multiple transitions for a character in a state.

Ex://



- Formal definition: NFA is a \$ tuple  $(\Sigma, Q, q_0, A, \delta)$ 
  - Diff:  $\delta$ : stack, character  $\rightarrow$  set of states
- Recognition algorithm:
  1. Move to initial state
  2. Get set of states based on  $\delta$
  3. For each state in set of states, apply transition
  4. If end in set of states w/ accepting state, accept

◦ Ex:// Determine if abbbba will be accepted



	Seen	Remaining	Set of States
1.	$\epsilon$	abbbba	$\{q_0\}$
2.	a	bbbba	$\{q_0\}$
3.	ab	bbba	$\{q_0, q_1\}$
4.	abb	ba	$\{q_0, q_1, q_2\}$
5.	abbb	a	$\{q_0, q_1, q_2\}$
6.	abbbba	$\epsilon$	$\{q_0, q_3\}$ ✓

◦ L is regular  $\Leftrightarrow$  NFA represented

- NFA  $\rightarrow$  DFA:

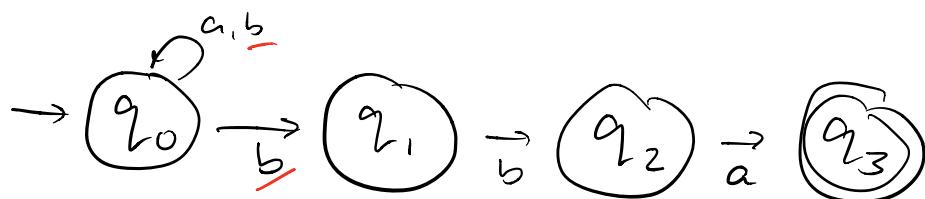
- Basic idea: Create a state for each unique set of NFA states
- Algo:

1. Start w/  $S_0 = \{q_0\}$

2. Note down resulting states for each character

3. Note down new set of states and make them accepting states if they contain an NFA accept state.

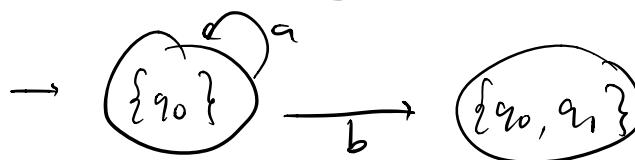
o Ex://



1.



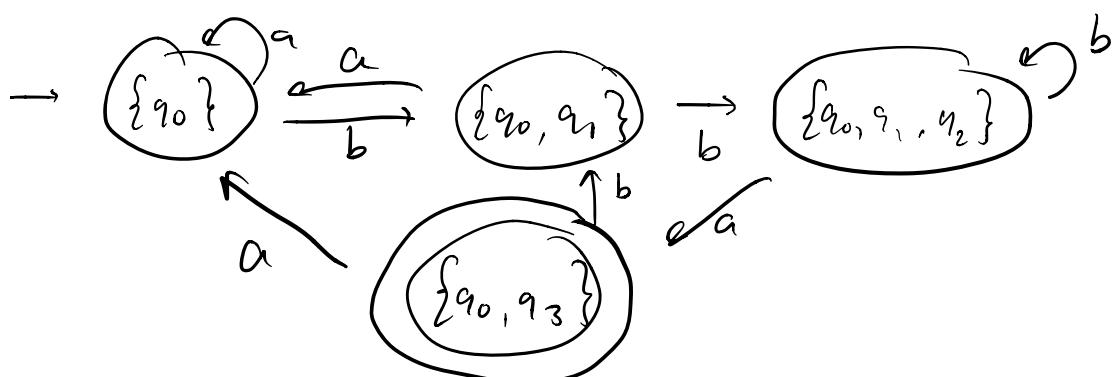
2.



3.



4.



o My algorithm:

```

for ( $a \in \Sigma$ ) {
    Set = {}
    for ( $q_i \in S$ ) {
        Set.add( $\delta(q_i, a)$ )
    }
}
  
```

if set already exists :

transition to existing set

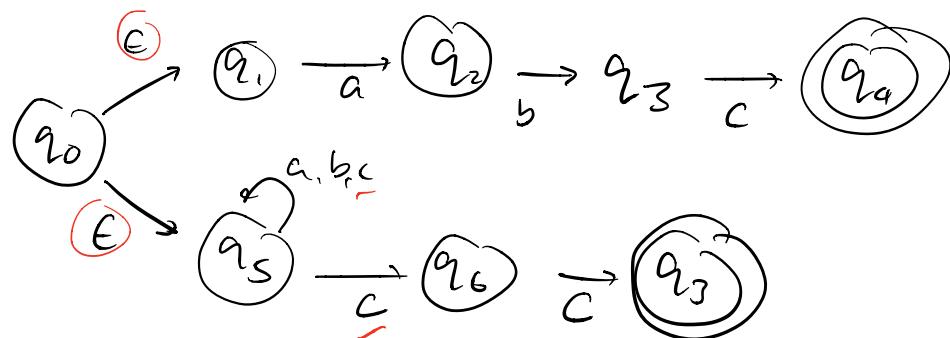
else:  
draw a new state

}

-  $\epsilon$  - Non-deterministic finite automata:

- $\epsilon$  transition: transition without character needed
- Usage: concatenate NFA

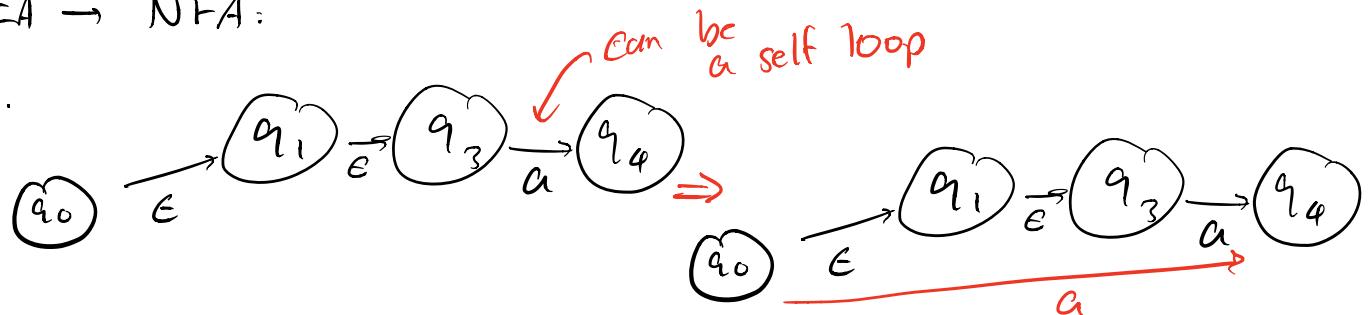
◦ Ex://



- Definition change:  $\epsilon$  cannot be part of alphabet
- Epsilon closure:  $E(S)$  = set of states reachable from  $S$  using 0/more epsilon transitions.
- Recognition algorithm: look at epsilon closure for  $\delta$

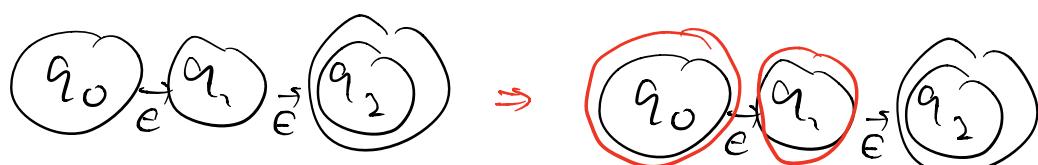
-  $\epsilon$  NFA  $\rightarrow$  NFA:

1.



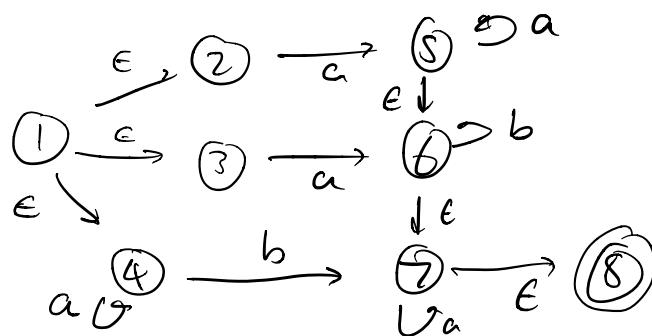
Given sequence of  $\epsilon$  transitions ending w/ non  $\epsilon$  transition,  
write a transformation from src to dest on last non  $\epsilon$  trans.

2.

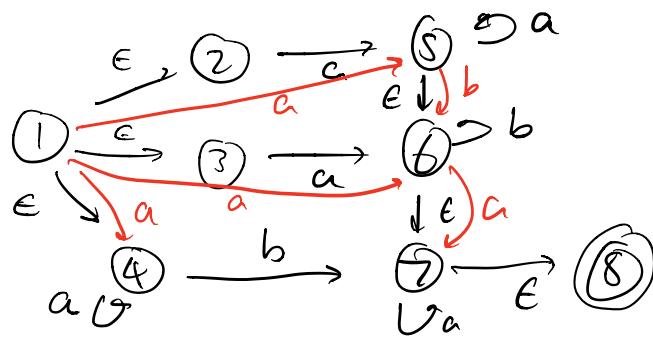


3. Remove  $\epsilon$  transitions + unreachable states.

◦ Ex:// Convert the following to NFA:

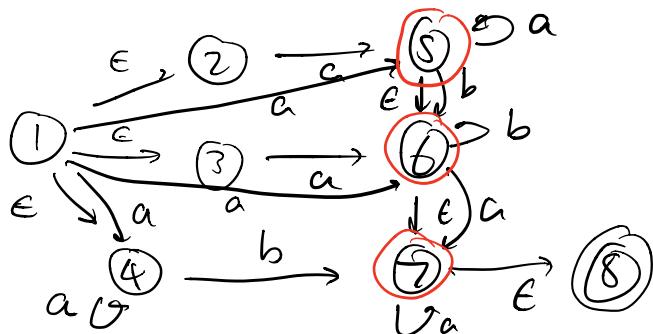


1.



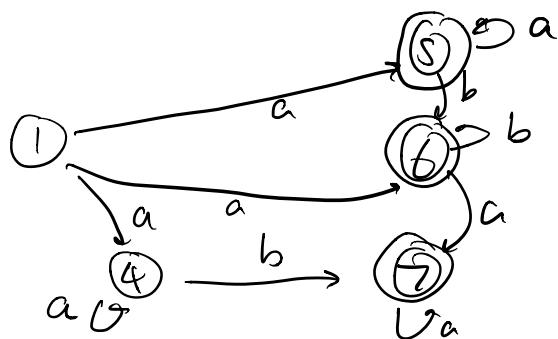
Edge creation

2.



Accepting state propagation

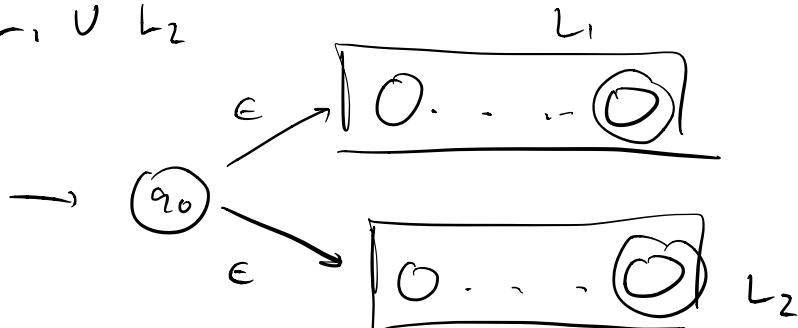
3.



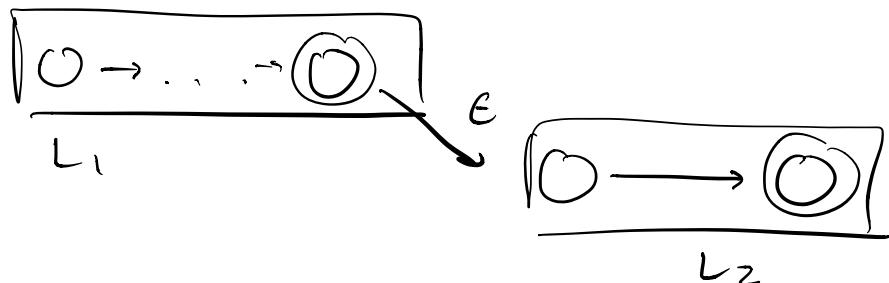
Cleanup

- Kleene's Theorem: showing regular languages  $\rightarrow \epsilon\text{-NFAs}$

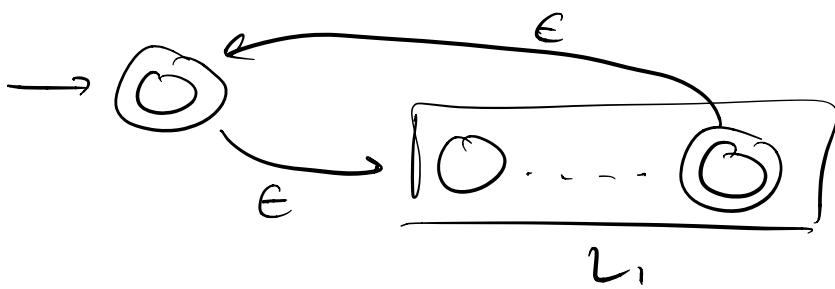
- Union:  $L_1 \cup L_2$



- Concatenation:  $L_1 L_2$

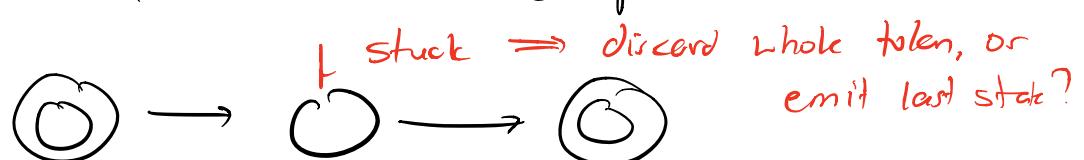


- Kleene Star:  $L^*$



## Scanning

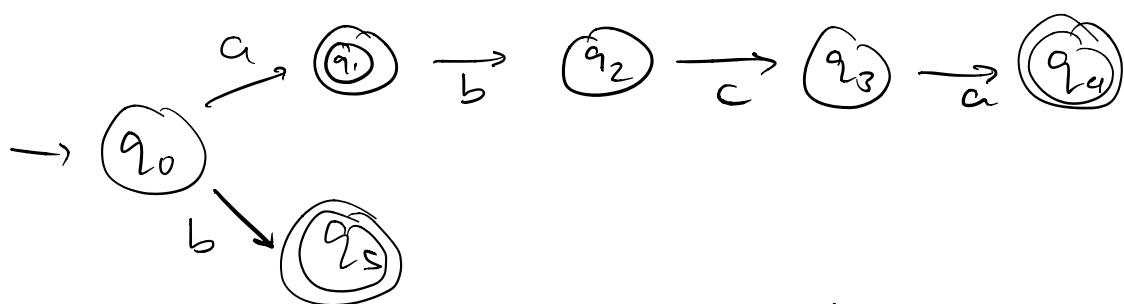
- Use DFA to determine if word belongs to a language
  - Issue: no clarity on when to output token



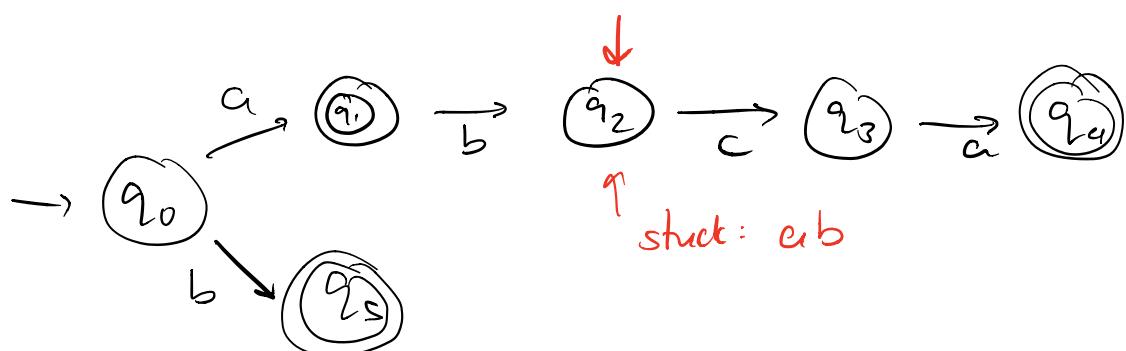
- Maximal match algorithm: greedy algorithm.

$$\Sigma = \{a, b, c\}, \quad L = \{a^*, b, abc^*c\}, \quad s = ababca$$

① NFA:

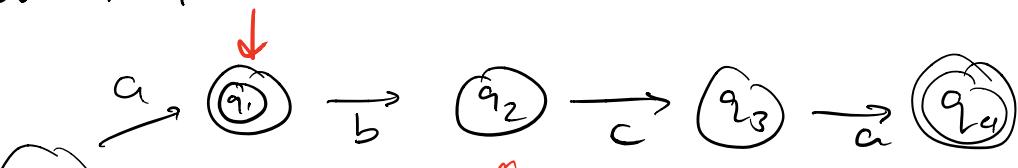


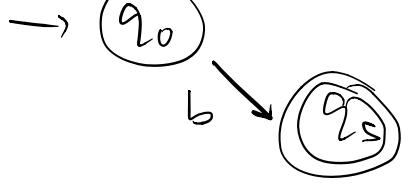
② Go as far as possible until stuck (no transitions left (end), or transitions don't match characters)



$$s = abca$$

③ Back to last emitted state + emit token

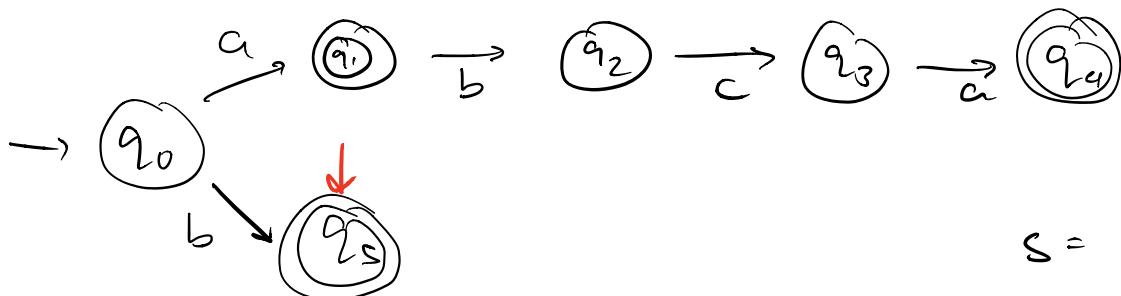




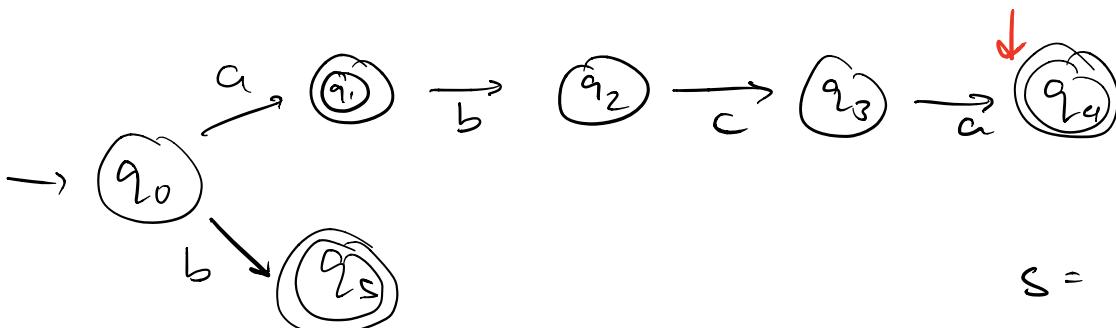
↑ stuck: c  
↓  
Remove character  
Like backtracking

Output = 'a'  
 $s = babcba$

- ④ Repeat until exhausted string



$s = abcba$   
Output = 'a', 'b'



$s = abcba$   
Output = 'a', 'b'  
'abcba'

- Greedy:

- Ex://



$s = "aacaac" \Rightarrow$  Expected token: 'aa', 'aa'

Maximal match = 'aaca' → reject!

- $O(n^2)$  b/c of backtracking, optimization possible

- Real life:

int y = 10;

int \*z = &y

int x = y /\*z

reject:

/\* = start of comment

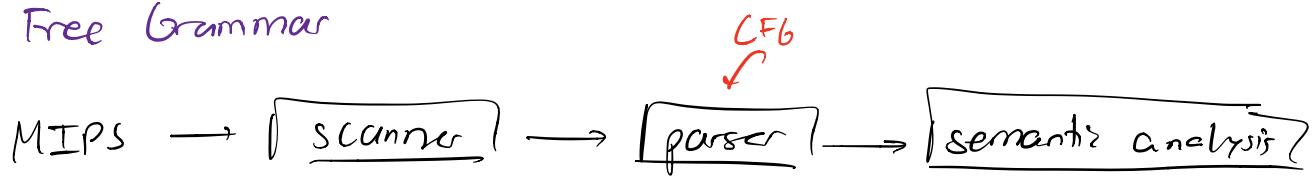
- Simplified maximal match:

- Don't back track ⇒ if stuck at some unaccepting state ⇒ reject
- $O(n) \Leftarrow$  no backtracking

## CONTEXT - FREE GRAMMARS

- DFAs are quite restrictive: Nested statements, infinite states

### Context Free Grammar



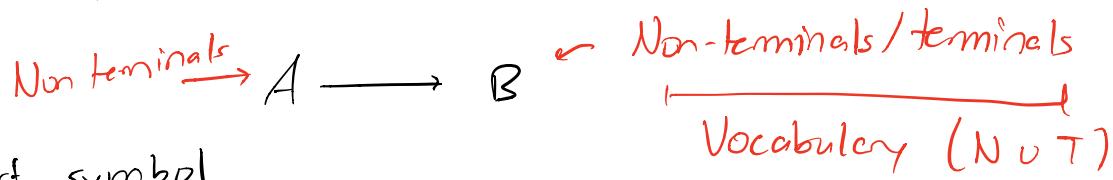
- Formal defn:  $(N, T, P, S)$

- $N$ : finite, set of non-terminals symbols  $\Rightarrow$  can be expanded

- $T$ : alphabet (finite, set of terminal symbols)

↳ Cannot be expanded

- $P$ : set of productions/rules



- $S$ : start symbol

- CFG converting non-terminals (variables) into another expression via production rules

- Ex://  $N: \{\text{expr}\}$

- $T: \{\text{ID}, \text{OP}, \text{LPAREN}, \text{RPAREN}\}$

- $P: \{\text{expr} \rightarrow \text{ID}, \text{expr} \rightarrow \text{expr OP expr}, \text{expr} \rightarrow \text{LPAREN/RPAREN}\}$

- $S: \{\text{expr}\}$

- Rewrite start symbol into another expression via production rules + continue this until reach goal

- Ex:// ID OP LPAREN ID OP ID RPAREN in CFG?

$$\begin{aligned}
 \text{expr} &\Rightarrow \text{expr OP} \boxed{\text{expr}} \\
 &\Rightarrow \boxed{\text{expr}} \text{ OP } \boxed{\text{LPAREN}} \boxed{\text{expr}} \boxed{\text{RPAREN}} \\
 &\Rightarrow \text{ID } \text{OP } \text{LPAREN } \boxed{\text{expr OP expr}} \text{ RPAREN} \\
 &\Rightarrow \text{ID OP LPAREN ID OP ID RPAREN}
 \end{aligned}$$

- Context-free language: all words that can be derived w/ CFG
  - Language is context-free  $\Leftrightarrow \exists$  CFG G such that  $L = L(G)$
  - Regular language is also a context-free language
  - Ex://  $T = \{a, b\}$ . Find a CFG G that represents  $\{a^n b^n : n \in \mathbb{N}\}$ .  
Find a derivation for  $aabb$

① Define parts of CFG

$$N: \{S\} \quad S: \{S\}$$

$$T: \{a, b\}$$

② Define our productions

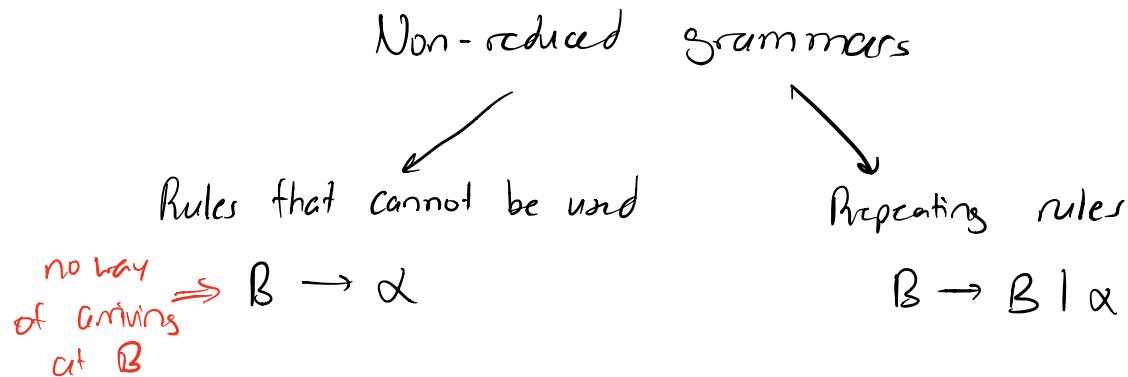
$$\begin{array}{l} S \rightarrow a S b \\ S \rightarrow \epsilon \end{array} \quad \begin{array}{l} \text{Look @ patterns of our language} \\ + \text{make it recursive} \end{array}$$

Derivation:

$$\boxed{\text{To combine: } S \rightarrow \epsilon \mid a S b}$$

$$S \Rightarrow a S b \Rightarrow a a S b b \Rightarrow a a a S b b b \Rightarrow a a a b b b$$

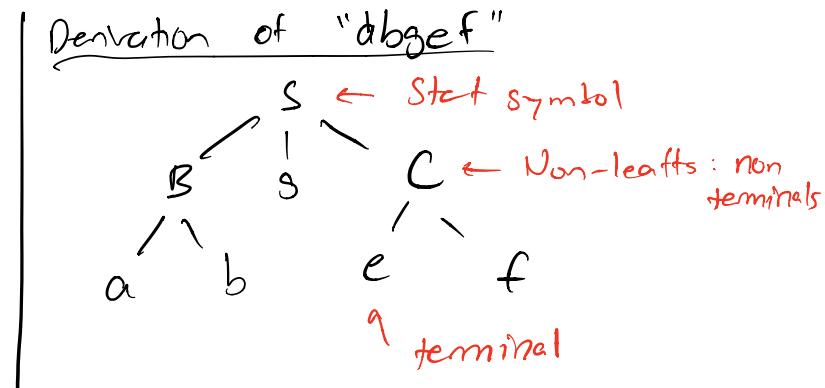
## Reduced Grammars



## Parse Trees

- Representation of derivation of a particular string

- Ex://  $N = \{S, B, C\}$
- $T: \{a, b, \dots, h\}$
- $P: \begin{aligned} S &\rightarrow B \cup C \\ B &\rightarrow ab \mid cd \\ C &\rightarrow h \mid ef \end{aligned}$



## - Properties:

1. A derivation uniquely defines a parse tree  
↳ 2 diff. derivations  $\Rightarrow$  2 diff. parse trees
2. Multiple derivations for string  $\Rightarrow$  multiple parse trees for string

## - Order of derivation matters:

1. Leftmost derivation: expand left-most non terminal
2. Rightmost derivation: || right most ||

## Ambiguous Grammars

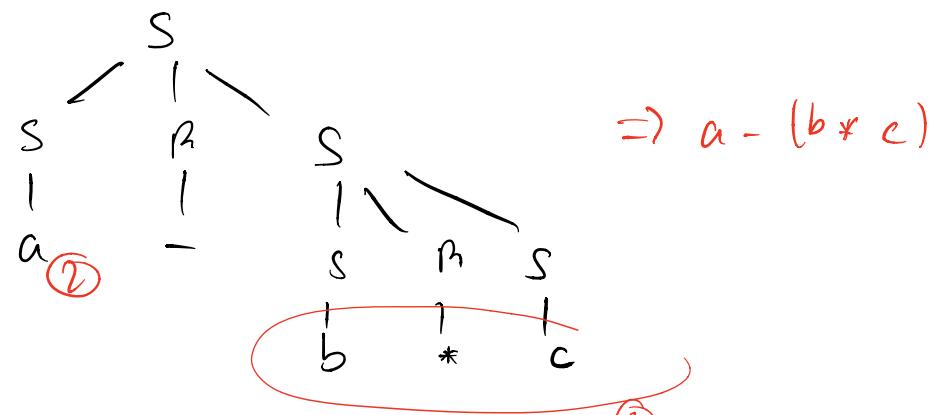
### - Ambiguous grammars: > 1 leftmost/rightmost derivations

Ex: //  $S \rightarrow a \mid b \mid c \mid SRS$

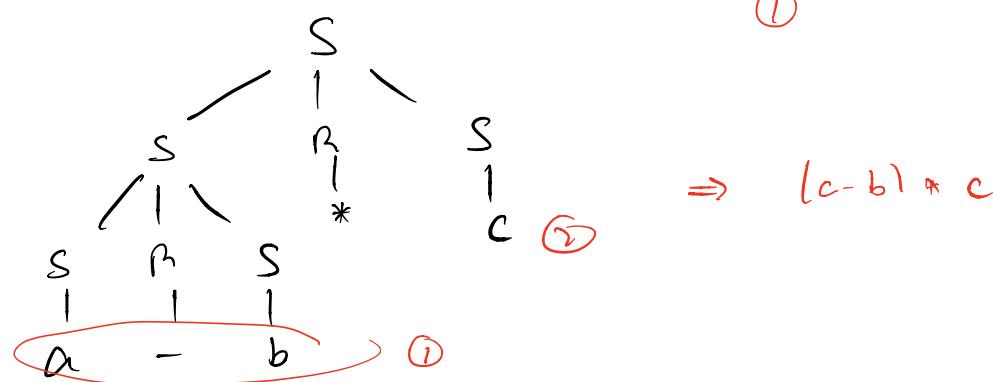
$R \rightarrow + \mid - \mid * \mid /$

$a - b * c$  (leftmost derivation)

1)



2)



• Constructing meanings out of parse trees that are ambiguous is hard

## - Ways to solve:

1. Use parentheses to force precedence
2. Make left/right associative based on location of non-terminal on RHS of production rule.

- 2<sup>nd</sup> method: remember, DFS traversal

o Ex: //

$a - b \times c$

Right associative

$S \rightarrow L R S \mid L$

$L \rightarrow a \mid b \mid c$

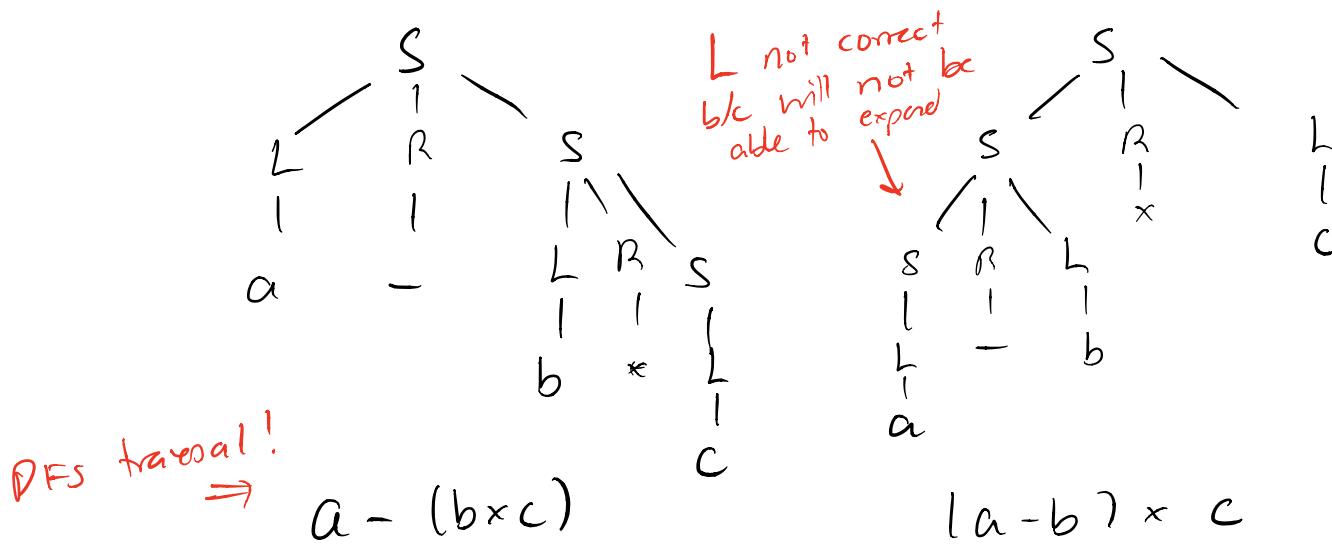
$R \rightarrow + \mid - \mid * \mid /$

Left associative

$S \rightarrow S M 2 \mid L$

$L \rightarrow a \mid b \mid c$

$M \rightarrow + \mid - \mid * \mid /$



o Reason why it works:

1. Split recursive non-terminal s.t. it only lead to other non-terminals OR only terminals
2. Order: expand non-terminal in the desired position  
(left assoc.  $\rightarrow S$  in left most, right assoc.  $\rightarrow S_{\text{right}}$ )

- No guarantee that a non-ambiguous grammar exists for CFL
- No algorithm to determine if grammar is ambiguous
- No way of knowing that 2 CFGs represent same language

- Parsing: find derivations  $\Rightarrow$  define parsing tree
- Method of derivations: pushdown automata
  - Not practical for converting different pushdown automata

## TOP DOWN PARSING

Start symbol  $\rightarrow$  leftmost rules  $\rightarrow$  show string in lang.

- Augmented grammar:

- Start symbol has 1 production rule

$$CFG \quad G = \{N, T, P, S\} \Rightarrow G' = \{N', T', P', S'\}$$

$$N' = N \cup \{S'\}$$

$$T' = T \cup \{t, +\}$$

$$P' = P \cup \{S' \rightarrow t \ S - 1\}$$

- Ex // Non-augmented:

Augmented Grammar

$$\begin{array}{ccc} S \rightarrow P & \Rightarrow & S' \rightarrow t \ S + \\ S \rightarrow T & & S \rightarrow P \\ & & S \rightarrow T \end{array}$$

## Algorithm

- Vague description

1. Start w/  $S' \rightarrow t \ S +$

2. Loop through all derivations + determine if non-term or term

i) If terminal: match terminal character w/ character at m

a) If match: move to next string character

b) No match: parse error

ii) Non-terminal: determine if correct derivation rule + substitute.

3. Repeat until no more non-terminals / error

- Ex:// 1)  $S' \rightarrow t S t$       String:  $tabywx\downarrow t$
- 2)  $S \rightarrow A y B$
- 3)  $A \rightarrow ab \mid cd$
- 4)  $B \rightarrow z \mid wx$

1.  $\alpha_1 = t S t \Rightarrow$  Used rule 1

2.  $\alpha_2 = t A y B t \Rightarrow$  Used rule 2

3.  $\alpha_3 = t aby B t \Rightarrow$  Used rule 3 } Looked at which NT

4.  $\alpha_4 = t aby wx t \Rightarrow$  Used rule 4 } gives character needed

- How did we come up w/ appropriate rule?

- Lookahead to find next unmatched character

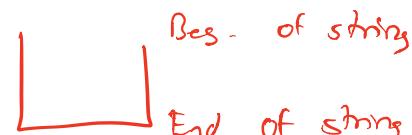
- Predict table: (unmatched char, NT) → derivation to apply

- Optimization: stack that has truncated part of string

- String to match = read string + stack

- Optimized algo:

- 1) Stack w/ t char



- 2) While TOS is a terminal, pop + match w/ string

- i) If match: move to next unmatched char

- ii) Else: error

- 3) If TOS is a non-terminal: Pop and query Predict[A][c]

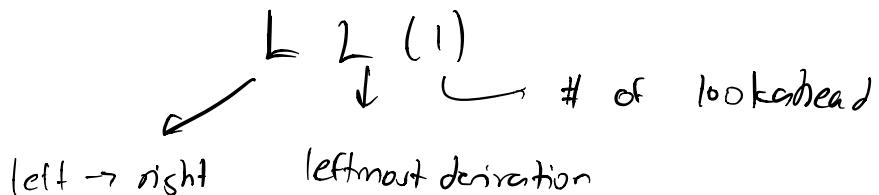
- i) If Predict[A][c] == rule: push symbols in reverse on stack

- ii) Else: error

- 4) Repeat until error / stack is empty

- LL(1):

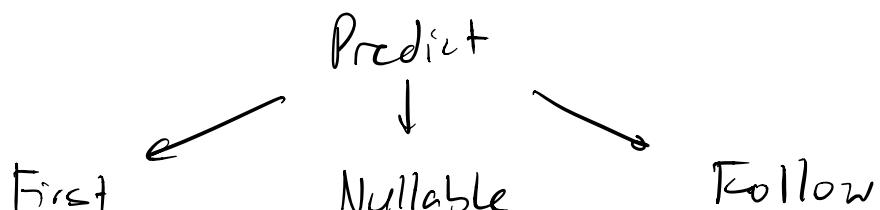
NT  
↓  
unmatch  
char



- $\text{LL}(1) \Leftrightarrow$  Each cell in Predict contains  $\leq 1$  rule

## Predict Table

- Format: Predict  $[A] [a]$   $\Rightarrow A \rightarrow B$  s.t.  $B \Rightarrow a\gamma$
- Other words: derive a rule st. first character is lookahead



- First:  $\text{First}(\beta) = \{a \in T' : \beta \Rightarrow a\gamma \text{ for some } \gamma \in V^*\}$ 
    - Set of first characters in  $\beta$
    - Ex://  $A \rightarrow ab \mid cd \mid zy \Rightarrow \text{First}(A) = \{a, c, d\}$
    - ∵ ∴ Predict  $[A] [a] = \{A \rightarrow B : a \in \text{First}(B)\}$
  - Dealing w/ nullable non-terminals
- |  |  |
|--|--|
| $\underline{\text{Nullable } (\beta)}$<br>true $\Leftrightarrow B \xrightarrow{*} \epsilon$<br>"can $B$ so to $\epsilon$ " | $\underline{\text{Follow } (A)}$<br>$\{b \in T' : S' \Rightarrow \alpha A b \beta \text{ for some } \alpha, \beta \in V^*\}$<br>"Set of terminals that occur right after $A$ " |
|--|--|
- ∴ ∴ Predict  $[A] [a] = \{A \rightarrow B : a \in \text{First}(B)\} \cup \{A \rightarrow B : \text{Nullable}(B) \wedge a \in \text{Follow}(A)\}$ 
    - Non-nullable rules:  $\xrightarrow{\quad}$  find rules st.  $a$  is first char
    - Nullable rule:  $\xrightarrow{\quad}$  find rule st.  $a$  follows nullled char

- Nullable construction:

- Observations

1. Nullable ( $\epsilon$ ): true
2. Nullable ( $\beta$ ): false if  $\beta$  has terminal
3. Nullable ( $A\beta$ ): Nullable ( $A$ )  $\wedge$  Nullable ( $\beta$ )

• Also:

1. Init Nullable( $A$ ) = false for all  $A \in N'$

2. Loop through rules:

i) If:  $(P \text{ is } A \rightarrow \epsilon) \vee (P \text{ is } A \rightarrow B_1 \dots B_n \text{ and } \bigwedge_{i=1}^n \text{Nullable}(B_i) = \text{true})$   
 $\text{Nullable}(A) = \text{true}$

3. Repeat until no changes

• Ex://

1)  $S' \rightarrow + S +$

2)  $S \rightarrow b S d$

3)  $S \rightarrow p S \alpha$

4)  $S \rightarrow C$

5)  $C \rightarrow IC$

6)  $C \rightarrow \epsilon$

Nullable

	Iter 0	Iter 1	Iter 2	Iter 3
$S'$	F	F	F	F
$S$	F	F	T	T
$C$	F	T	T	T

- First construction

• Also:

1. Init First( $A$ ) =  $\{\}$  for all  $A \in N'$

2. For each rule  $A \rightarrow B_1 \dots B_k$

3. For each  $i \in \{1, \dots, k\}$

4. If  $B_i \in T'$ :  $\text{First}(A) = \text{First}(A) \cup \{B_i\}$ . Break  $\Rightarrow B_i$  will be first terminal in rule b/c breaking to next rule

5. Else:  $\text{First}(A) = \text{First}(A) \cup \text{First}(B_i)$

i)  $\text{Nullable}(B_i) = \text{False} \Rightarrow \text{break}$ .

↳ If  $B_i$  is  $N'$  and is the first + not nullable,  $\text{First}(A) += \text{First}(B_i)$

6. Repeat until no change

• Generalized also to find First( $B$ ):

1. Result =  $\emptyset$

2. for  $i \in \{1, \dots, n\}$ :

3. If  $B_i \in T'$ : result = result  $\cup \{B_i\}$ . break

4. Else: result = result  $\cup \text{First}\{B_i\}$ .

5)  $\text{Nullable}(B_i) = \text{false} : \text{break}$

• Ex://

1)  $S' \rightarrow + S +$

2)  $S \rightarrow b S d$

3)  $S \rightarrow p S \alpha$

4)  $S \rightarrow C$

5)  $C \rightarrow IC$

6)  $C \rightarrow \epsilon$

	0	1	2	3
$S'$	$\{\}$	$\{+\}$	$\{+\}$	$=$
$S$	$\{\}$	$\{b, +\}$	$\{b, +\}$	$=$
$C$	$\{\}$	$\{I\}$	$\{I\}$	$=$

Fast way:

$S'$	$\vdash$
$S$	$b, p, \vdash$
$C$	$!$

1. Go through each rule + add first terminals
2. If rule has a non-terminal first, note for next pass
3. Add the terminal of non-terminal into First

- Follow construction

• Also:

1. Init  $\text{Follow}(A) = \{\}$  for all  $A \in N \Rightarrow$  Not including  $S'!!$

2. For each rule  $A \rightarrow B_1 \dots B_k$

3. For each  $i \in \{1 \dots k\}$

4. If  $B_i \in N$ :  $\text{Follow}(B_i) = \text{Follow}(B_i) \cup \text{First}(B_{i+1} \dots B_k)$

5. If  $\lambda_{m=i+1}^k$  Nullable ( $B_m$ ) = True or  $i=k$

6.  $\text{Follow}(B_i) = \text{Follow}(B_i) \cup \text{Follow}(A)$

7. Repeat until no change

↑ Set of terminals that follow  
 $B_i$   
 } If  $B_{i+1} \dots B_k \Rightarrow \epsilon$   
 or at end use  $\text{Follow}(A)$   
 $A \rightarrow B_1 \dots B_k$   
 $S \rightarrow A z$   
 $\text{Follow}(B_i) = z!!$

• Ex:// 1)  $S' \rightarrow \vdash S \vdash$

2)  $S \rightarrow b S \vdash$

3)  $S \rightarrow p S q$

4)  $S \rightarrow C$

5)  $C \rightarrow l C$

6)  $C \rightarrow \epsilon$

Follow

$S$	$\{\}$	$\{\vdash, \vdash, \vdash\}$	$=$
$C$	$\{\}$	$\{\vdash, \vdash, \vdash\}$	$=$

In Rule 4,  $C$  is last  $\Rightarrow \text{Follow}(C) = \text{Follow}(S)$

- Predict table construction.

• Also:

1. Init  $P[A][a] = \{\}$  for all  $A \in N', a \in T'$

2. For each rule  $A \rightarrow B$ :

3. For each  $a \in \text{First}(B)$ : add  $A \rightarrow B \wedge \text{Predict}[A][a]$

4. If Nullable ( $B$ ):

5. For each  $a \in \text{Follow}(B)$ , add  $A \rightarrow B \wedge \text{Predict}[A][a]$

• Ex:// 1)  $S' \rightarrow \vdash S \vdash$

2)  $S \rightarrow b S \vdash$

3)  $S \rightarrow p S q$

	Nullable	First	Follow
$S'$	F	$\{\vdash, \vdash\}$	$\{\}$
$S$	T	$\{b, p, \vdash\}$	$\{\vdash, \vdash, \vdash\}$

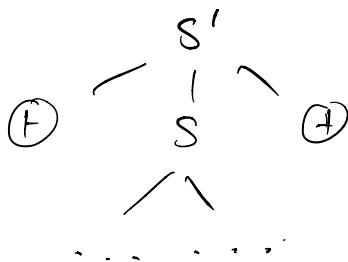
4)  $S \rightarrow C$ 5)  $C \rightarrow IC$ 6)  $C \rightarrow \epsilon$ 

C | T | {1} | {1, 0, 3}

Predict:

$s'$	T	+	b	d	p	$\epsilon_2$	1
S		4	2	4	3	4	4
C		6		6		6	S

Parse Tree



$\Rightarrow$  Record derivations during algorithm

### Limitations of LL(1) Grammar

- Ex://
  - 1)  $expr \rightarrow expr \ op \ expr$
  - 2)  $expr \rightarrow id$
  - 3)  $op \rightarrow +$
- Predict  $[expr][id]$ :
  - Rule 1:  $id \in \text{First}(expr)$
  - Rule 2:  $id \in \text{First}(expr)$

$\Rightarrow !LL(1)$
- Grammar is LL(1) if:
  1. No 2 distinct rules s.t. same LHS  $\rightarrow$  same first terminal

~~$A \rightarrow bS$~~   
 ~~$A \rightarrow cS$~~

  2. No nullable symbol A has same terminal in both first + follow sets.

~~$A \rightarrow aB \Rightarrow \text{First}(A) = \{a\}$~~   
 ~~$A \rightarrow a \qquad \qquad \qquad \text{Follow}(A) = \{a\}$~~   
 ~~$B \rightarrow AaC$~~

  3. Only 1 way to derive  $\epsilon$  from nullable
  4. Left recursive algorithms not LL(1)
- To convert left recursive  $\rightarrow$  right recursive
  1. Switch recursive statements s.t.  $N \rightarrow N$  is on Right+
    - o Common prefix problem
    - 1)  $S \rightarrow T + S$

2)  $S \rightarrow T$

$\Rightarrow$  Cannot decide on 1 or 2

3)  $T \rightarrow F * T$

b/c lookahead of 1 is not sufficient

4)  $T \rightarrow F$

S, 6, 7)  $F \rightarrow a \mid b \mid c$

## 2. Increase lookahead / factor

$A \rightarrow \alpha \beta_1 \mid \alpha \beta_2 \mid \dots \Rightarrow$

$A \rightarrow \alpha \beta$

$\beta \rightarrow \beta_1 \mid \beta_2 \mid \dots$

o Ex // Left recursive

1)  $S \rightarrow S + T$

① Right recursive

2)  $S \rightarrow T$

1)  $S \rightarrow T + S$

② Factor:

3)  $T \rightarrow T * F$

2)  $S \rightarrow T$

1)  $S \rightarrow T B$

4)  $T \rightarrow F$

3)  $T \rightarrow F * T$

2)  $B \rightarrow + S \mid \epsilon$

S, 6, 7)  $F \rightarrow a \mid b \mid c$

4)  $T \rightarrow F$

3)  $T \rightarrow F X$

4)  $X \rightarrow x T \mid \epsilon$

S, 6, 7)  $F \rightarrow a \mid b \mid c$

S, 6, 7)  $F \rightarrow a \mid b \mid c$

o Another way:

LEFT

$A \rightarrow A \alpha$

RIGHT

$A \rightarrow B \quad (B \text{ does not start w/ } A)$

$A \rightarrow B A'$

$A' \rightarrow \alpha A'^1 \epsilon$

o Ex // Left Recursion

1)  $S \rightarrow S + T$

Right recursive

2)  $S \rightarrow T$

1)  $S \rightarrow T S'$

3)  $T \rightarrow T * F$

2)  $S' \rightarrow + T S' \mid \epsilon$

4)  $T \rightarrow F$

3)  $T \rightarrow F T'$

S, 6, 7)  $F \rightarrow a \mid b \mid c$

4)  $T' \rightarrow x F T' \mid \epsilon$

S, 6, 7)  $F \rightarrow a \mid b \mid c$

## BOTTOM UP PARSING

Input string  $\xrightarrow{\text{derivations}}$  start symbol

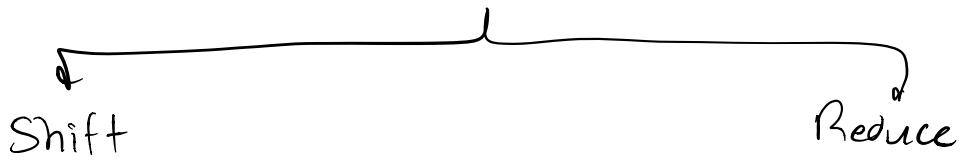
### Algorithm

- High level:

1. Read input symbols from left  $\rightarrow$  right

2. If RHS of rule seen  $\rightarrow$  replace L/ RHS of rule
3. Use a stack to keep track of converted input
4. Repeat until start symbol

### Operations



Consume symbol  $\rightarrow$  push to stack

Pop RHS from stack  $\rightarrow$  push LHS

- Ex:// 1.  $S' \rightarrow T S T$

2.  $S \rightarrow A y B$

String:  $T a b y w x T$

3.  $A \rightarrow a b | c d$

4.  $B \rightarrow z | w x$

Read	Unread	Stack	Actions
$E$	$T a b y w x T$		Shift $T$
$T$	$a b y w x T$	$T$	Shift $a$
$T a$	$b y w x T$	$T a$	Shift $b$
$T a b$	$y w x T$	$T a b$	Reduce $a b$
$T a b$	$y w x T$	$T A$	Shift $y$
$T a b y$	$w x T$	$T A y$	Shift $w$
$T a b y w$	$x T$	$T A y w$	Shift $x$
$T a b y w x$	$T$	$T A y w x$	Reduce $w x$
$T a b y w x$	$-$	$T A y B$	Reduce $A y B$
$T a b y w x$	$-$	$T S$ <small>rightmost</small>	Shift $-$
$T a b y w x T$	$E$	$T S - T$	Reduce $T S T$
$T a b y w x T$	$E$	$S'$	done

- Observations

1. Derivation at any iteration = Unread  $\rightarrow$  Stack

2. Acceptance : unread =  $\epsilon$   $\wedge$  stack = [start]

3. Reverse derivation

4. Right most derivation

- Items: prod. rule w/ bookmark indicating how far we have read

$$S \rightarrow \cdot a b C, S \rightarrow a b \cdot C, S \rightarrow a b C \cdot$$

◦ Non-reducible: not finished item. Reducible: finished item

- Problem: how to determine whether to shift or reduce.

◦ Soln: keep track of how far we are using items

$$\begin{array}{lll} E \downarrow & E \rightarrow \cdot E + T & S \rightarrow \cdot E - T \\ & E \rightarrow E \cdot + T & S \rightarrow E \cdot - T \\ + \downarrow & E \rightarrow E^+ \cdot T & S \rightarrow E^+ - T \\ & & \vdots \end{array}$$

◦ Use a DFA to keep track of items

1. Start state: fresh rule w/ start symbol

$$\rightarrow \boxed{S' \rightarrow \cdot E - I} q_i$$

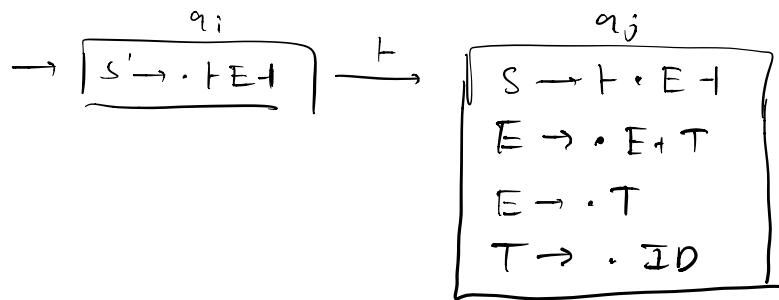
2. Find state w/ non-reducible item + transition on symbol following bookmark to state  $q_j$

$$\rightarrow \boxed{S' \rightarrow \cdot E - I} \xrightarrow{t} \boxed{q_j}$$

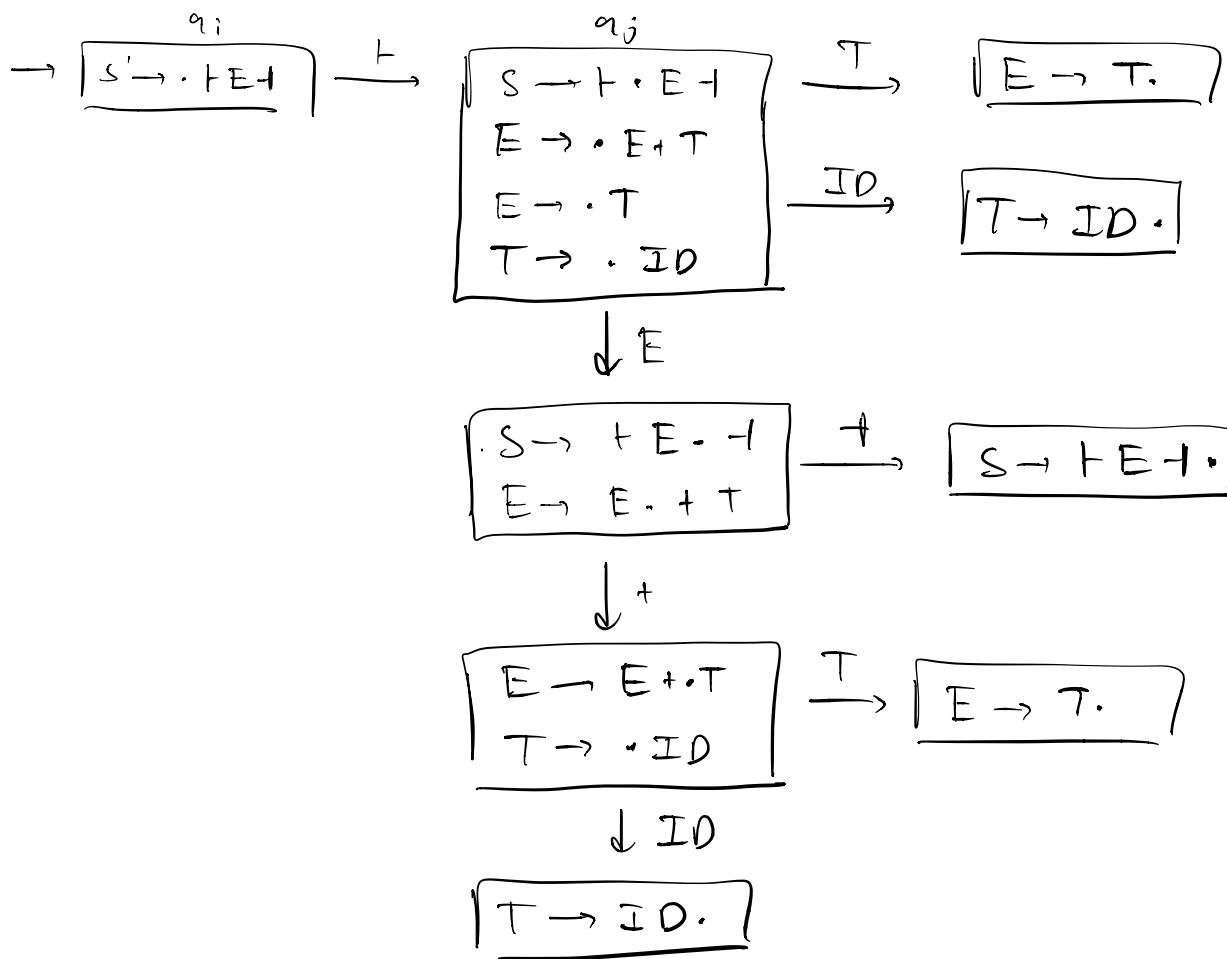
3. Put all items in  $q_i$  w/ bookmark followed by symbol into  $q_j$  w/ bookmark update

$$\rightarrow \boxed{S' \rightarrow \cdot E - I} \xrightarrow{t} \boxed{S \rightarrow \cdot E - I}$$

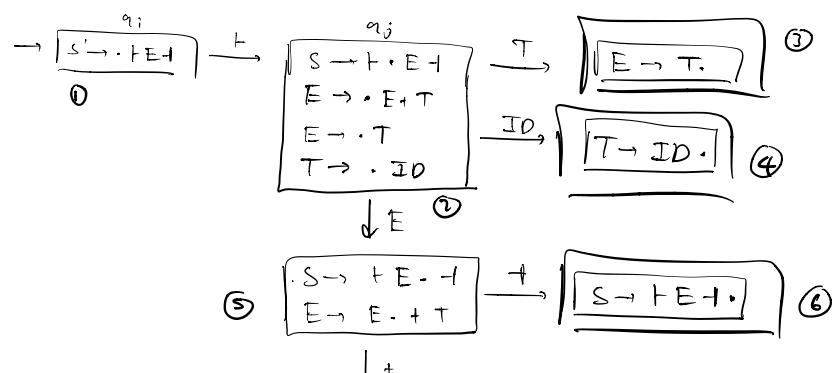
4. For all items in  $q_j$ : if bookmark followed by NT, add fresh item for non-terminal in  $q_j$ . Repeat

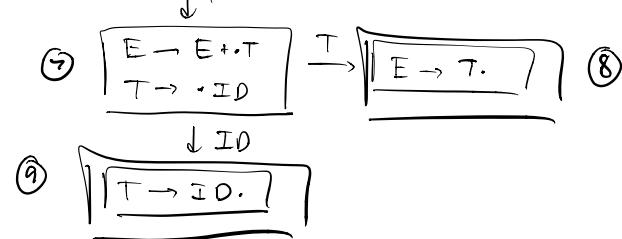


5. Repeat until no new states



6. Accepting states: states that contain scorable item



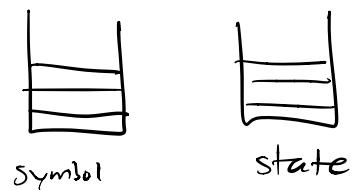


- Algorithm: run stack through DFA. If we land on reducible state  $\rightarrow$  reduce. Else: shift

- Problem: have to run through whole stack every time

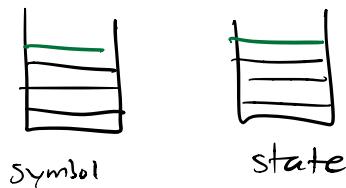
- Soln: stack stack (sequence of states till current state) + symbol stack (keeps track of derivations)

- Query the state stack



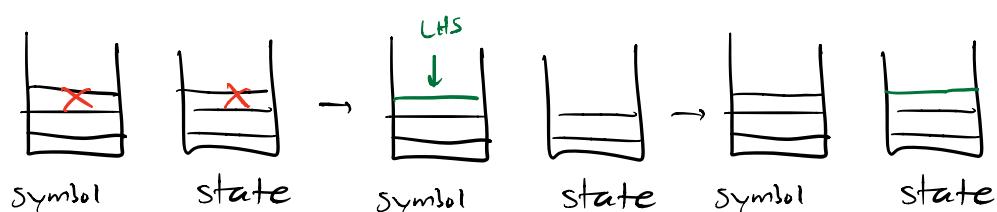
- State not reducible:

Shift next input symbol  
+ push new state



- State is reducible

- Pop n items from state  $\Rightarrow$  symbol ( $n = \#$  of RHS symbols)
- Push new symbol
- Push new state after symbol



- Ex://  $T ID + ID - 1$

Read	Unread	Symbol	State	Action
$\epsilon$	$T ID + ID - 1$		1	Shift on $T$
$T$	$ID + ID - 1$	$T$	12	Shift on $ID$
$+ ID$	$+ ID +$	$+ ID$	124	Reduce on 4
$+ ID$	$+ ID -$	$+ T$	123	Reduce on 5
$+ ID$	$+ ID - 1$	$+ E$	125	Shift on $+$
$:$	$:$	$:$	$:$	$:$

- LR(0) grammars: left  $\rightarrow$  right, right recursive, no lookahead

## Parsing Grammars

### Conflicts

#### Shift-reduce

State in DFA where

$A \rightarrow \alpha \cdot a\beta = \text{non-reducible}$

$B \rightarrow \gamma \cdot = \text{reducible}$

Cannot decide to reduce w/out lookahead

#### Reduce-reduce conflict

2 reducible rules in state

↓

Also doesn't know which rule to use in derivation

## Using lookaheads

- Methods of solving conflict:

1. SLR(1): use lookahead and check if in Follow set of LHS of reducible item

a. Note down Follow sets for all reducible items

b. Only reduce if lookahead is in Follow set

• Limitation: if lookahead in Follow set of multiple rules in state

▫ Soln: make Follow set local, i.e.  $\subseteq$  Follow that only applies to rule in state of question

2. LR(1): exponential states if large grammar is

3. LALR(1): use locally computed follow sets

- Changes in implementation:

1. DFA constructed accordingly

2. Determine if state is reducible using current state + lookahead

## CONTEXT-SENSITIVE ANALYSIS

Unique to your language! Following: WhP4

# CFG problems

Type checking

Unique variables

Usage before declaration Func calls

Parser

→ Concrete syntax tree

→ Tree Transformation

→ Abstract Syntax Tree

↳ Removing useless nodes

CSA

1. Unique variables / procedure names

Symbol Table

var name	type

1. For each procedure → create symbol table

2. Add variable-type pairs in table

Master symbol table

procedure name	symbol table
...	...

Ensures unique procedure names!

Note: identifiers within a function with same name as function treated as a variable

∴ int p(int p) {  
 }      return p(p);    ⇒ Would not compile

## Implementation:

1. Go through parse tree + check if producing procedure  
procedure → INT ID ...

main → INT MAIN ...

2. Check if procedure already defined (ID name in master table)

i) Reject if already defined

3. Create new row in <sup>master</sup> symbol table + keep track of proc.  
symbol table

4. Continue parse tree. Look for variable declarations.

dcl → type ID

5. Map in symbol table w/ type : ID lexeme

i) Catch if already in symbol table

2. Usage before declaration

Symbol table is finished before statements ← WLP4 spec.

1. Look for factor → ID or lvalue → ID

2. If ID lexeme n symbol table → ✓. Otherwise: catch.

3. Functions correctly called

"correctly called" ← correct # of param. & matching types

1. Function exists:

a) Look for factor → ID LPAREN RPAREN  
OR

factor → ID LPAREN arglist RPAREN

b) Does ID lexeme exist in master symbol?

## 2. Parameters match:

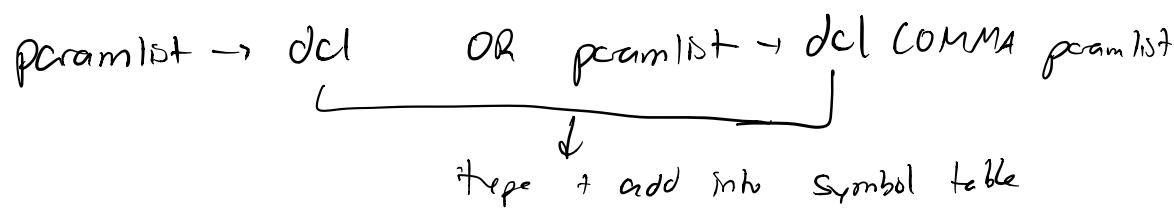
Master symbol table:

name	$\langle [\text{argslist types}], [\text{symbol table}] \rangle$
:	:
:	:
:	:
:	:

Ex:// int foo (int \* a, int b) { . . . }

foo	$\langle [\text{int*}, \text{int}], \{\dots\} \rangle$
-----	--

To figure out argslist types:



## 4. Types

Create typechecker for every single production

Ex:// typecheck (statement  $\rightarrow$  lvalue BECOMES expr SEMI) {  
    ltype := typecheck (lval)      } → Recursively find type  
    exprtype := typecheck (expr)      ↓  
    if ltype != exprtype: return error      → Check  
}

Not returning a type! Statement  
has no types

int mem () {  
 expr  
 int a = b + 10;  
}

typecheck (expr → expr PLUS term) }

ltype := typecheck (expr)

rtype := typecheck (term)

if ltype == int & rtype == int ⇒ return int

if ltype == int \* & rtype == int | ltype == int &

rtype == int \* ⇒ return int \*

Checks

}

Procedure calls

1. Check if function exists
2. Check function type
3. Check params + type match

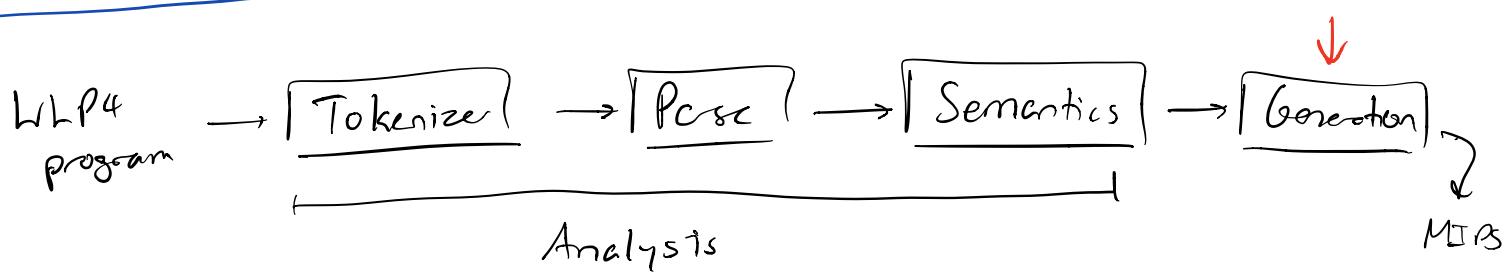
Booleans: welltyped if operand types are same

Control: welltyped if Boolean + child statements are welltyped

Other checks

- Variables initialised
  - Exhaust all possible returning
  - Dead code check
- } Do not happen in WLP4

## CODE GENERATION



Variables

## - Convention:

```
int rain (int a, int b) {  
    :  
    return _____ ← $3  
}
```

- Issue: mapping between variables + register

- Extend symbol tables

Name	Type	Location
a	int	\$1
b	int	\$2

- Issue: limited # of registers  $\rightarrow$  limited # of variables

- Keep track of stack offset for each variable

int main (int a , int b) {              lis \$4 } convention

```

return a;           ⇒   word 4
}                   {    SW $1, -4 ($30)
                        {    SW $2, -8 ($30)
                        {    sub $30, $30, $4
                        {    sub $30, $30, $4
    }   Store or stack
    }   update stack ptr

```

Name	Type	Offset
a	int	4
b	int	0

load into ~~ctrn~~  
register → lw \$3, 4(\$30)

restor

- Issue: \$30 (stack ptr) constantly moving

- o To use \$29 (frame pointer): rel. position of start for procedure
  - o Use offsets from \$29 rather than \$30

## Expressions

- Issue: expressions need temp. registers for storage

- o Use stack! Push everything stack  $\Rightarrow$  pop off sequentially to perform nested operations

- Ex:// int main (int a, int b) {
   
    return a-b;
   
}
   
 ⇒
   
 code (a)  $\Rightarrow$  load a into \$3
   
 push (\$3)
   
 code (b)  $\Rightarrow$  load b into \$3
   
 pop (\$5)  $\Rightarrow$  a is in \$5
   
 sub \$3, \$5, \$3
  
- Ex:// int main (int a, int b) {
   
    int c = 3;
   
    return a + (b - c);
   
}
   
 ⇒
   
 code (a)
   
 push (\$3)
   
 code (b)
   
 push (\$3)
   
 code (c)
   
 pop (\$5)
   
 sub \$3, \$5, \$3
   
 pop (\$5)
   
 add \$3, \$5, \$3

- Rule conversions:

1.  $S \rightarrow T$  statements  $\vdash$ :

$$\text{code}(S) = \text{code}(\text{statements})$$

2.  $expr \rightarrow term$ :

$$\text{code}(expr) = \text{code}(term)$$

3.  $factor \rightarrow LPAREN\ expr\ RPAREN$

$$\text{code}(factor) = \text{code}(expr)$$

4.  $expr_1 \rightarrow expr_2\ PLUS\ term$

$$\begin{aligned} \text{code}(expr_1) &= \text{code}(expr_2) + \text{push}(\$3) + \text{code}(term) + \\ &\quad \text{pop}(\$5) + \text{add}\ $3, \$3, \$5 \end{aligned}$$

5.  $statement \rightarrow lvalue\ BECOMES\ expr\ SEMI;\ lvalue \rightarrow ID$

$$\text{code}(statement) = \text{code}(expr) + \text{sw}\ $3, \text{offset}(\$29)$$

$\hookrightarrow$  ID of symbol table

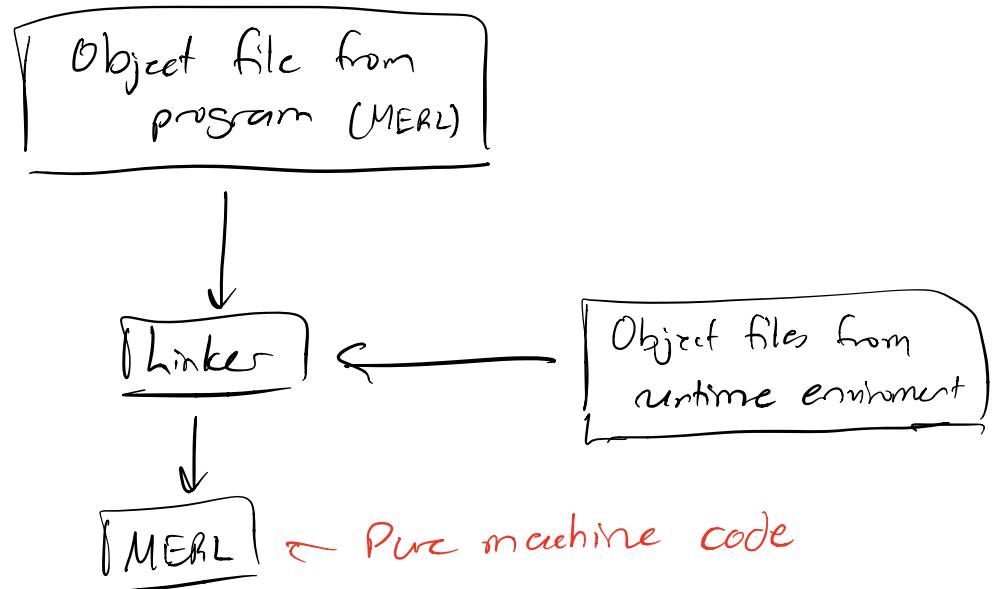
## Print Statements

2 options

Create print procedure  
+ call everywhere

Import print procedure from runtime environment.

- Runtime environment: exec. environment provided by OS that assists



- Code (println(expr)) = push \$1 + code(expr) + add \$1 \$3, \$0  
+ push (\$31) + lsr \$5 + word.print + jmr \$5  
+ pop (\$31) + pop (\$1)

## Comparisons

- test →  $\text{expr}_1 < \text{expr}_2$ : 1 if true, 0 if false

Code(test) = code(expr<sub>1</sub>) + push (\$3) + code(expr<sub>2</sub>) + pop (\$5)  
+ slt \$3, \$5, \$3

• Do the same for test →  $\text{expr}_1 > \text{expr}_2$

- test →  $\text{expr}_1 \neq \text{expr}_2$ :

$\text{code}(\text{to-1}) = \text{code}(\text{expr}_1) + \text{push } (\$3) + \text{code}(\text{expr}_2) + \text{pop } (\$5)$   
+ s1t \$6, \$3, \$5 + s1t \$7, \$5, \$3 + add \$3, \$6, \$7

o For equality: sub \$3, \$11, \$3 (\$11 = 1 \Rightarrow \text{flips})

-  $\geq \Rightarrow !(<)$ ,  $\leq \Rightarrow !(>)$

## Control flow statements

- Statement  $\rightarrow$  IF (test) {stmt1} ELSE {stmt2}

$\text{code}(\text{statement}) = \text{code}(\text{test}) + \text{beq } \$3, \$0, \text{else} + \text{code}(\text{stmt}_1)$   
+  $\text{beq } \$0, \$0, \text{endif} + \text{else:} + \text{code}(\text{stmt}_2)$   
+  $\text{endif:}$

o To have unique labels, use a counter + append to label

- Statement  $\rightarrow$  WHILE (test) {stmts}

$\text{code}(\text{statement}) = \text{loop:} + \text{code}(\text{test}) + \text{bca } \$3, \$0, \text{endwhile}$   
+  $\text{code}(\text{stmts}) + \text{beg } \$0, \$0, \text{loop} + \text{endwhile:}$

## Pointers

- NULL: misaligned address  $\Rightarrow$  add \$3, \$0, \$1 (0x01)

- Dereferences ptr:

$\text{factor}_1 \rightarrow \text{STAR factor}_2 \Rightarrow \text{code}(\text{factor}_1) = \text{code}(\text{factor 2})$  ↗  
+ lwr \$3, 0(\$3)

- Get address:

1) factor  $\rightarrow$  AMP lvalue (lvalue is ID)

$\Rightarrow \text{code}(\text{factor}) = \text{lis } \$3 + \text{.word [OffsetFromTable]} + \text{add } \$3, \underbrace{\$3, \$29}_{\text{Addr}}$

2) factor  $\rightarrow$  AMP lvalue (lvalue is STAR factor<sub>2</sub>)

$\text{code}(\text{factor 1}) = \text{code}(\text{factor 2})$

- Assignment via ptr dereference:

stmt → lvalue BECOMES expr SEMI (lvalue → STAR factor)

code(stmt) = code(expr) + push(\$?) + code(factor) + pop(\$\$)  
+ SW \$S, 0(\$?)

- Comparison: ptrs can be res ⇒ sltu

- Arithmetic:

1) expr<sub>1</sub> → expr<sub>2</sub> + term where type(item) == int + type(expr<sub>2</sub>) == int\*

code(expr<sub>1</sub>) = code(expr<sub>2</sub>) + push(\$3) + code(item) +  
mult \$3, \$4 + mfl \$3 + pop(\$5)  
+ add \$3, \$5, \$3

2) expr<sub>1</sub> → expr<sub>2</sub> + term but opposite types

Switch order ⇒ multiply expr × 4 ⇒ + to term

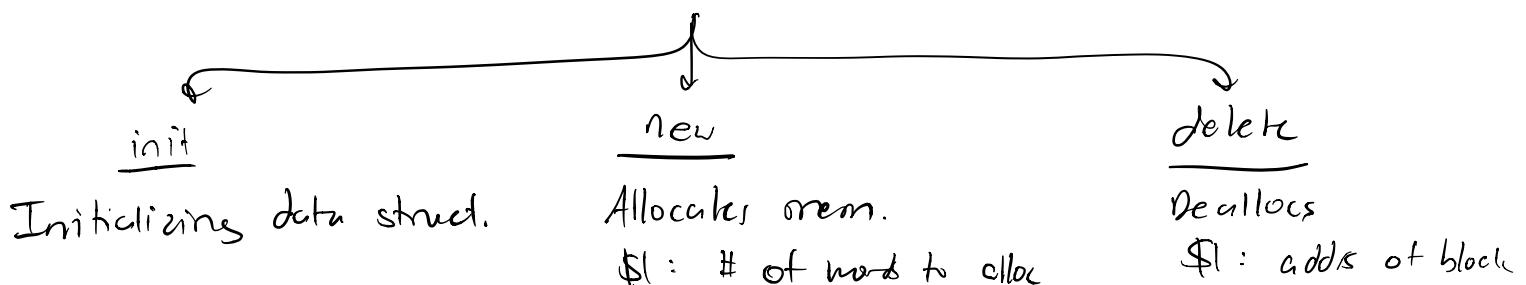
3) expr<sub>1</sub> → expr<sub>2</sub> - term where type(expr<sub>2</sub>) == int\*, type(item) = int  
expr<sub>2</sub> - (4 × item)

4) expr<sub>1</sub> → expr<sub>2</sub> - term but types switched

$$\frac{\text{expr}_2 - \text{term}}{4}$$

## Heap Allocation

MERL file: alloc.merl



$\$2$ : length of array  
(or 0)

$\$3$ : odds of block

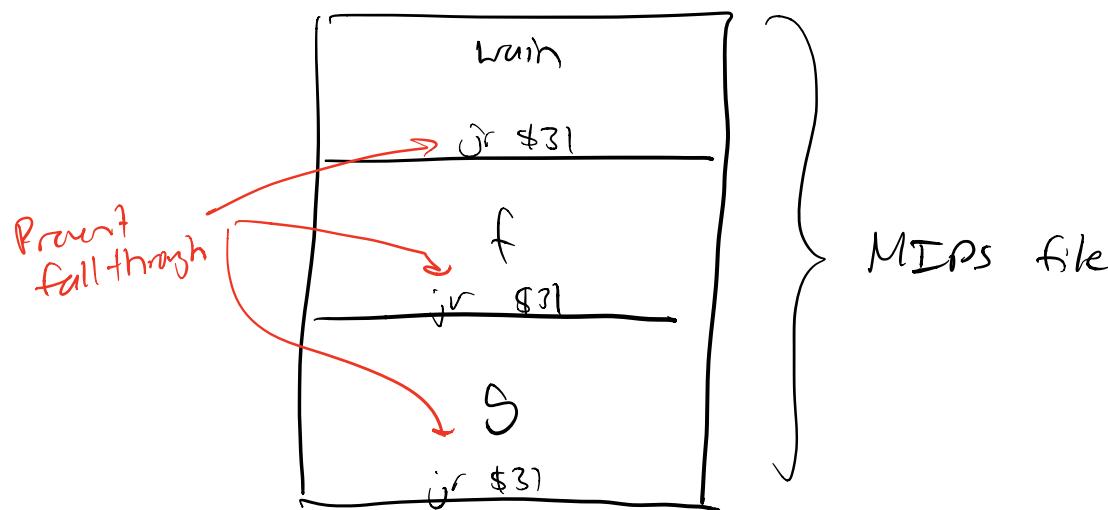
- new int [expr]

code(expr) + add \$1, \$3, \$0 + push (\$31) + lis \$5 + .word new  
+ jalr (\$5) + pop (\$31) + bne \$3, \$0, 1 + add \$3, \$1, \$0  
NULL

- delete [] expr

code(expr) + beq \$3, \$11, skipDelete + add \$1, \$3, \$0  
+ push (\$31) + lis \$5 + .word delete + jalr \$5 + pop (\$31) + skipDelete

## Other procedures



- Main procedure:

- 1) Call imports + initialize
- 2) Save \$1 + \$2 on stack
- 3) Code  $\Rightarrow$  rest stack  $\Rightarrow$  exit

- Saving + restoring registers: either caller or callee-saved

- o \$1 -> needs to be saved. \$30 needs to be restored
- o \$29:

Callee-saved: push(\$29) + add \$29, \$30, \$0

Caller-saved: save \$29 before proc. called

- Saving parameters: caller-saves

- factor → ID(expr1, ..., exprN)

code(factor) = push(\$29) + push(\$31) + code(expr1) + push(\$3)  
+ ... + code(exprN) + push(\$2) + li \$1  
+ word ID + jalr(\$5) + pop n times + pop(\$3) + pop(\$29)

- Procedure code:

procedure → INT ID(params) {dcls stmts RETURN expr};

code(procedure) = ID:

- + sub \$29, \$30, \$4
- + push registers to save
- + code(dcls)
- + code(expr)
- + pop saved reg.
- + add \$30, \$29, \$4
- + jr \$31

◦ Problem: params + local variable memory is non contiguous

1) Param i at  $4(n-i+1)$ , n is # of param

Local var i  $-4r - 4(i-1)$ , r is # of saved registers

2) Switch code order

3) Caller save

- Labels: F\_label