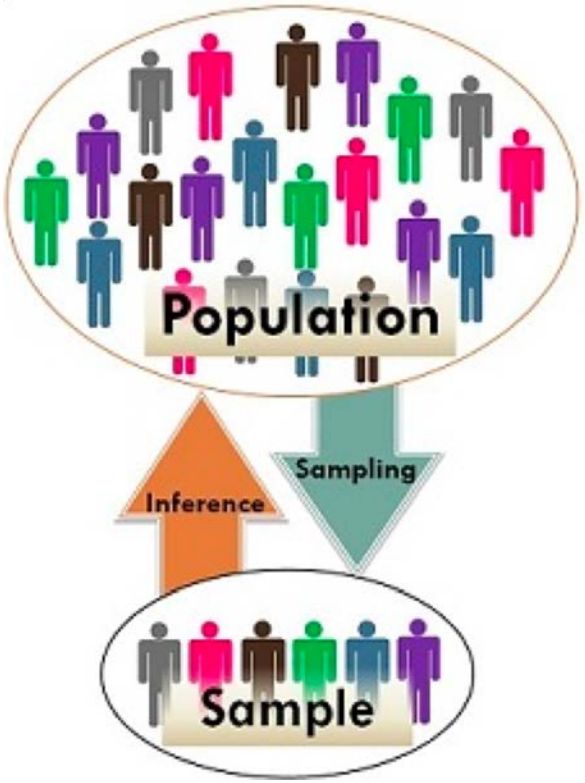
Descriptive Statistical Measures

Learning objectives

- Explain the difference between Populations and Samples
- Understand and be able to distinguish and apply the different measures:
  - Measures of Location (Mean, Median, Mode),
  - Measures of Dispersion (Range, Variance, Standard Deviation, Chebyshev's Theorem, Coefficient of Variation)
  - Measures of Shape (Skewness, Kurtosis)
  - Measures of Association (Covariance and Correlation)
- Be able to identify outliers

Populations and Samples



- Population - all items of interest for a particular decision or investigation
  - *all* married drivers over 25 years old
  - *all* subscribers to Netflix
- Sample - a subset of the population
  - a list of married drivers over 25 years old who bought a new car in the past year
  - a list of individuals who rented a comedy from Netflix in the past year
- Purpose of sampling is to obtain sufficient information to draw a valid inference about a population

Measures of Location – Mean

For a population of size *N:*

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

For a sample of *n* observations*:*

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Mean is also commonly known as average

Measures of Location – Mean

Example: Computing Mean Cost per Order (Using *Purchase Orders* data)

- Using formula:

Mean = $2,471,760/94
= $26,295.32

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

	A	B	C	D	E	F	G	H	I	J
1	**Purchase Orders**									
2										
3	**Supplier**	**Order No**	**Item No.**	**Item Description**	**Item Cost**	**Quantity**	**Cost per order**	**A/P Terms (Months**	**Order Date**	**Arrival Date**
4	Spacetime Technologies	A0111	6489	O-Ring	$ 3.00	900	$ 2,700.00	25	10/10/11	10/18/11
5	Steelpin Inc.	A0115	5319	Shielded Cable/ft.	$ 1.10	17,500	$ 19,250.00	30	08/20/11	08/31/11
6	Steelpin Inc.	A0123	4312	Bolt-nut package	$ 3.75	4,250	$ 15,937.50	30	08/25/11	09/01/11
7	Steelpin Inc.	A0204	5319	Shielded Cable/ft.	$ 1.10	16,500	$ 18,150.00	30	09/15/11	10/05/11
8	Steelpin Inc.	A0205	5677	Side Panel	$195.00	120	$ 23,400.00	30	11/02/11	11/13/11
9	Steelpin Inc.	A0207	4312	Bolt-nut package	$ 3.75	4,200	$ 15,750.00	30	09/01/11	09/10/11
10	Alum Sheeting	A0223	4224	Bolt-nut package	$ 3.95	4,500	$ 17,775.00	30	10/15/11	10/20/11
11	Alum Sheeting	A0433	5417	Control Panel	$255.00	500	$ 127,500.00	30	10/20/11	10/27/11
12	Alum Sheeting	A0443	1243	Airframe fasteners	$ 4.25	10,000	$ 42,500.00	30	08/08/11	08/14/11
13	Alum Sheeting	A0446	5417	Control Panel	$255.00	406	$ 103,530.00	30	09/01/11	09/10/11
14	Spacetime Technologies	A0533	9752	Gasket	$ 4.05	1,500	$ 6,075.00	25	09/20/11	09/25/11
15	Spacetime Technologies	A0555	6489	O-Ring	$ 3.00	1,100	$ 3,300.00	25	10/05/11	10/10/11

Measures of Location – Median

~ middle value of the data when arranged from least to greatest

Example: Finding the Median Cost per Order (*Purchase Orders* data)

Sort the data in column B.
Since $n = 94$,
Median = $15,656.25
(average of 47th & 48th observations)

	A	B	C	D
1	Rank	Cost per order		
2	1	$68.75		
3	2	$82.50		
4	3	$375.00		
5	4	$467.50		
6	5	$525.00		
44	43	$13,650.00		
45	44	$14,910.00		
46	45	$14,910.00		
47	46	$15,087.50		
48	47	$15,562.50		$15,562.50
49	48	$15,750.00		$15,750.00
50	49	$15,937.50	Average	$15,656.25
51	50	$16,276.75		

NOTE: If n is odd number, then median is the value of the middle observation

Using R to compute mean and median

```
mean(x, trim = 0, na.rm = FALSE, ...)
```

Arguments

`x`	An R object. Currently there are methods for numeric/logical vectors and date, date-time and time interval objects. Complex vectors are allowed for `trim = 0`, only.
`trim`	the fraction (0 to 0.5) of observations to be trimmed from each end of `x` before the mean is computed. Values of trim outside that range are taken as the nearest endpoint.
`na.rm`	a logical value indicating whether `NA` values should be stripped before the computation proceeds.

```
median(x, na.rm = FALSE, ...)
```

Arguments

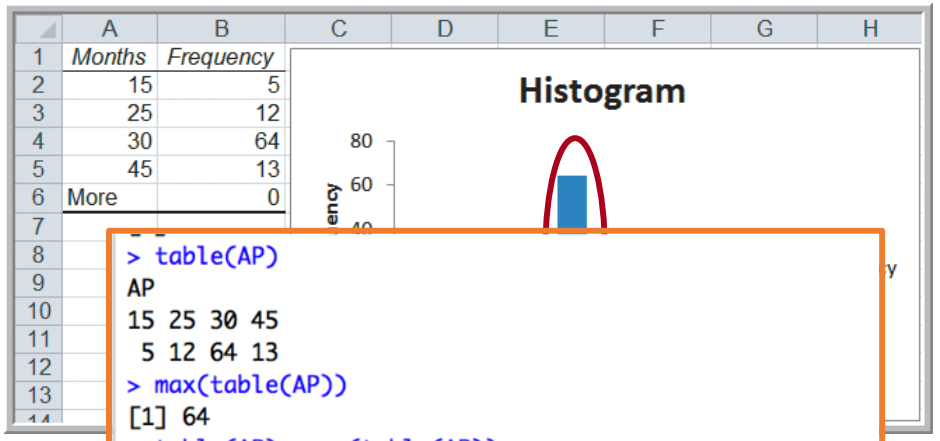
`x`	an object for which a method has been defined, or a numeric vector containing the values whose median is to be computed.
`na.rm`	a logical value indicating whether `NA` values should be stripped before the computation proceeds.

Measures of Location – Mode

~ observation that occurs most often or, for grouped data, the group with the greatest frequency.

Eg for observation data: Finding the Mode of A/P terms
(*Purchase Orders* data)

- Mode of A/P terms:
  = 30 months




```
> Purchase_Orders <- read_excel("~/Dropbox/Teaching/BT
+     sheet = "Data", col_types = c("text",
+         "text", "text", "text", "numeric",
+         "numeric", "numeric", "numeric",
+         "date", "date"), skip = 2)
> View(Purchase_Orders)
> AP<-Purchase_Orders$`A/P Terms (Months)`
> names(table(AP))[table(AP)==max(table(AP))]
[1] "30"
```

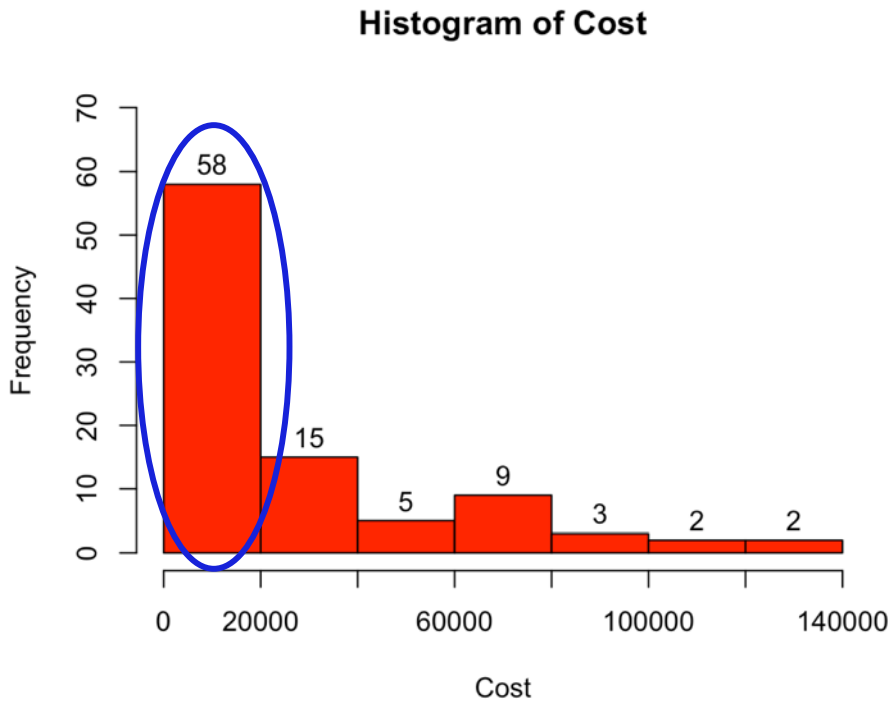
```
> table(AP)
AP
15 25 30 45
 5 12 64 13
> max(table(AP))
[1] 64
> table(AP)==max(table(AP))
AP
   15    25    30    45
FALSE FALSE  TRUE FALSE
> table(AP)[table(AP)==max(table(AP))]
30
64
> names(table(AP)[table(AP)==max(table(AP))])
[1] "30"
```

Measures of Location – Mode

~ observation that occurs most often or, for grouped data, the group with the greatest frequency.

Eg for grouped data: Finding the Mode of Cost per order (*Purchase Orders* data)

- **Mode is the group between $0 and $20,000**



Measures of Location: Application

Problem: Quoting Computer Repair Times

Data set (Computer Repair Times) includes 250 repair times for customers.

- What repair time would be reasonable to quote to a new customer?

```
> df8<-Computer_Repair_Times
# Use R functions for Mean & Median
> mean(df8$`Repair Time (Days)`)
[1] 14.912
> median(df8$`Repair Time (Days)`)
[1] 14
```

```
# Compute Mode
> x<-df8$`Repair Time (Days)`
> names(table(x))[table(x)==max(table(x))]
[1] "12" "15"
```

```
# Use Table function to obtain frequencies for each value of X
> table(x)
x
 5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 25 26 27 29 31 34 36 37 38 39 40
 1  2  5 12 14 19 19 23 22 20 23 18 16  9  9  8  5  4  4  3  2  2  2  1  1  2  1  1  1  1
```

Mean repair time is about 15 days
Median repair time: 2 weeks
Mode is 12 and 15 days

	Sample	Repair Time (Days)
1	1	18
2	2	15
3	3	17
4	4	9
5	5	37
6	6	15

	Sample	Repair Time (Days)
241	241	21
242	242	7
243	243	12
244	244	16
245	245	23
246	246	18
247	247	31
248	248	6
249	249	17
250	250	13

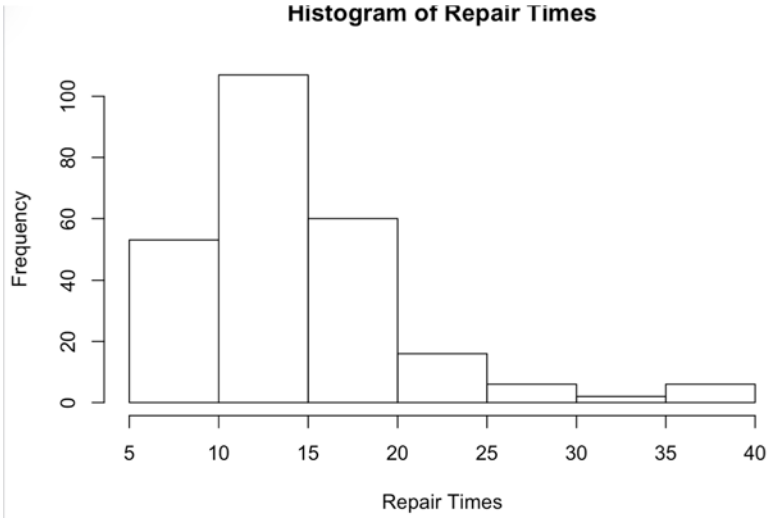
Measures of Location: Application

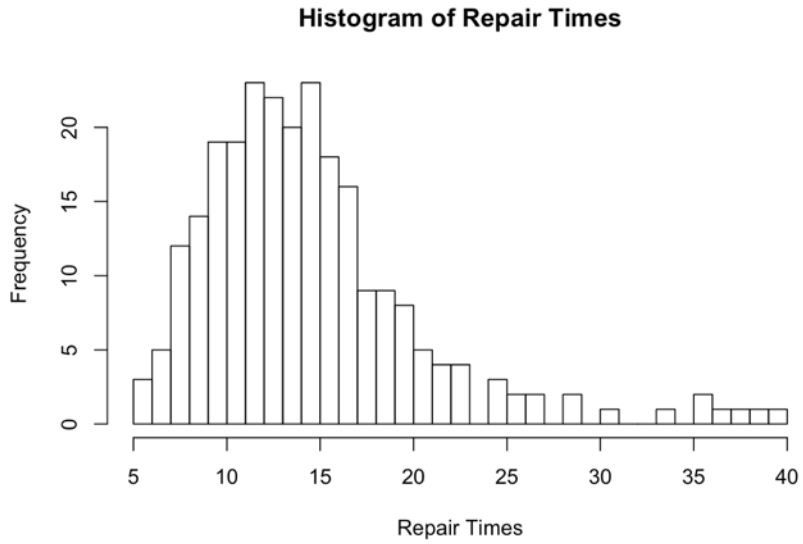
Problem: Quoting Computer Repair Times

Data set (Computer Repair Times) includes 250 repair times for customers.

- **What repair time would be reasonable to quote to a new customer?**
- **Mean repair time is about 15 days**
- **Median repair time: 2 weeks**
- **Mode is 12 and 15 days**

```
hist(x, main="Histogram of Repair Times",xlab="Repair Times")
hist(x, breaks=36, xlim=range(5,40), main="Histogram of Repair Times",xlab="Repair Times")
```





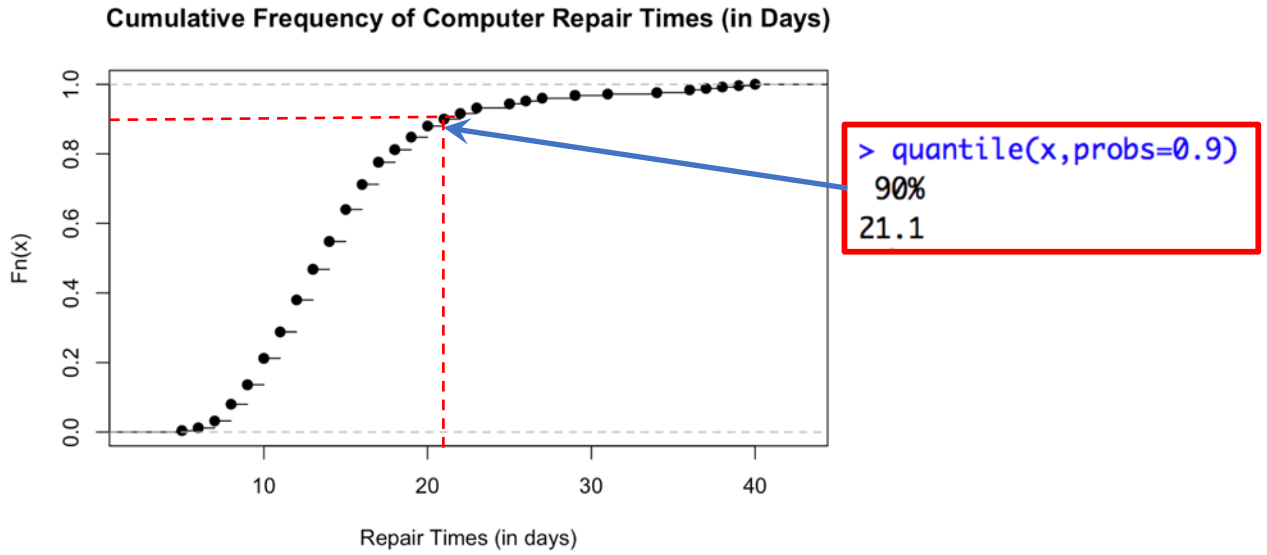
Measures of Location: Application

Problem: Quoting Computer Repair Times

Data set (Computer Repair Times) includes 250 repair times for customers.

- **What repair time would be reasonable to quote to a new customer?**
- Mean repair time is about 15 days
- Median repair time: 2 weeks
- Mode is 12 and 15 days

```
plot(ecdf(x),main ="Cumulative Frequency of Computer Repair Times (in Days)", xlab="Repair Times (in days)")
```



Measures of Dispersion

- Dispersion refers to the degree of variation (numerical spread or compactness) in the data
  - Range is the difference between the maximum and minimum data values
  - Interquartile range difference between the first and third quartiles (Q3 – Q1) (uses middle 50% data)
  - Variance is an average of squared deviations from mean (uses all data values) **var ( )**
  - Standard Deviation is the square root of the variance **sd ( )**

x is the computer repair times variable

install.packages (psych)
library (psych)

```
> summary(x)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   5.00   11.00   14.00   14.91   17.00   40.00
> describe(x)
   vars   n  mean   sd median trimmed  mad min max range skew kurtosis   se
X1    1 250 14.91 5.96     14   14.12 4.45   5  40    35 1.67     3.92 0.38
```

Range=?
IQR =?

Measures of Dispersion – Variance

~ average of squared deviations from mean

Computing the Variance

- For a population: $$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

- For a sample: $$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}$$

- Note the difference in denominators

If sample data is also population data, then n=N, to compute population variance:
[(N-1)/N]*var(X)

In R, sample variance is var(X)

Measures of Dispersion – Standard Deviation

~ square root of the variance

(popular measure of risk)

Computing the Standard Deviation

- For a population:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}}$$

Sqrt(((N-1)/N)*var(X))

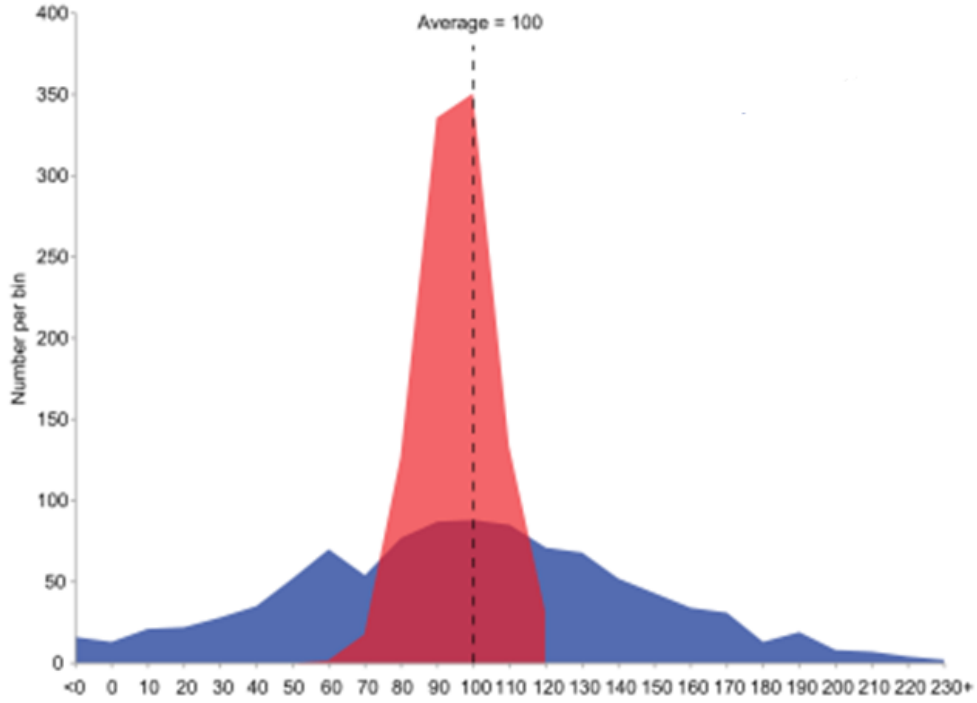
- For a sample:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$$

sd (X)

Measures of Dispersion – Standard Deviation

- Which has a higher standard deviation?



Source: Wikipedia (Standard Deviation)

Measures of Dispersion - Application

Mean & Standard Deviation of Closing Stock Prices

Intel (INTC):
Mean = $18.81
Stdev. = $0.50

General Electric (GE):
Mean = $16.19
Stdev. = $0.35

Whose risk may be higher?

	A	B	C	D	E	F
1	Closing Stock Prices					
2						
3	Date	IBM	INTC	CSCO	GE	DJ Industrials Index
4	9/3/2010	$127.58	$18.43	$21.04	$15.39	10447.93
5	9/7/2010	$125.95	$18.12	$20.58	$15.44	10340.69
6	9/8/2010	$126.08	$17.90	$20.64	$15.70	10387.01
7	9/9/2010	$126.36	$18.00	$20.61	$15.91	10415.24
8	9/10/2010	$127.99	$17.97	$20.62	$15.98	10462.77
9	9/13/2010	$129.61	$18.56	$21.26	$16.25	10544.13
10	9/14/2010	$128.85	$18.74	$21.45	$16.16	10526.49
11	9/15/2010	$129.43	$18.72	$21.59	$16.34	10572.73
12	9/16/2010	$129.67	$18.97	$21.93	$16.23	10594.83
13	9/17/2010	$130.19	$18.81	$21.86	$16.29	10607.85
14	9/20/2010	$131.79	$18.93	$21.75	$16.55	10753.62
15	9/21/2010	$131.98	$19.14	$21.64	$16.52	10761.03
16	9/22/2010	$132.57	$19.01	$21.67	$16.50	10739.31
17	9/23/2010	$131.67	$18.98	$21.53	$16.14	10662.42
18	9/24/2010	$134.11	$19.42	$22.09	$16.66	10860.26
19	9/27/2010	$134.65	$19.24	$22.11	$16.43	10812.04
20	9/28/2010	$134.89	$19.51	$21.86	$16.44	10858.14
21	9/29/2010	$135.48	$19.24	$21.87	$16.36	10835.28
22	9/30/2010	$134.14	$19.20	$21.90	$16.25	10788.05
23	10/1/2010	$135.64	$19.32	$21.91	$16.36	10829.68

Measures of Dispersion

Chebyshev's Theorem

- For any data set, the proportion of values that lie within k ($k > 1$) standard deviations of the mean is *at least* $1 - 1/k^2$

Substituting values of k, we get:

- For k = 2: at least ¾ or 75% of the data lie within two standard deviations of the mean
- For k = 3: at least 8/9 or 89% of the data lie within three standard deviations of the mean

Why is this useful?

- Able to use mean and standard deviation to find percentage of total observations that fall within a given interval about the mean

Example: For Cost per order data in Purchase orders database

Purchase Orders

Supplier	Order No.	Item No.	Item Description	Item Cost	Quantity	Cost per order	A/P Terms (Months)	Order Date	Arrival Date
Alum Sheeting	Aug11002	1243	Airframe fasteners	$ 4.25	10,000	$ 42,500.00	30	08/08/11	08/14/11
Alum Sheeting	Sep11002	5417	Control Panel	$ 255.00	406	$ 103,530.00	30	09/01/11	09/10/11
Alum Sheeting	Sep11008	1243	Airframe fasteners	$ 4.25	9,000	$ 38,250.00	30	09/05/11	09/12/11
Alum Sheeting	Oct11016	1243	Airframe fasteners	$ 4.25	10,500	$ 44,625.00	30	10/10/11	10/17/11
Alum Sheeting	Oct11022	4224	Bolt-nut package	$ 3.95	4,500	$ 17,775.00	30	10/15/11	10/20/11
Alum Sheeting	Oct11026	5417	Control Panel	$ 255.00	500	$ 127,500.00	30	10/20/11	10/27/11
Alum Sheeting	Oct11028	5634	Side Panel	$ 185.00	150	$ 27,750.00	30	10/25/11	11/03/11
Alum Sheeting	Oct11036	5634	Side Panel	$ 185.00	140	$ 25,900.00	30	10/29/11	11/04/11
Durrable Products	Aug11008	7258	Pressure Gauge	$ 90.00	100	$ 9,000.00	45	08/25/11	08/28/11
Durrable Products	Sep11009	7258	Pressure Gauge	$ 90.00	120	$ 10,800.00	45	09/05/11	09/09/11
Durrable Products	Sep11027	1369	Airframe fasteners	$ 4.20	15,000	$ 63,000.00	45	09/25/11	09/30/11
Durrable Products	Sep11031	1369	Airframe fasteners	$ 4.20	14,000	$ 58,800.00	45	09/27/11	10/03/11
Durrable Products	Sep11034	1369	Airframe fasteners	$ 4.20	10,000	$ 42,000.00	45	09/29/11	10/04/11
Durrable Products	Oct11002	9399	Gasket	$ 3.65	1,250	$ 4,562.50	45	10/01/11	10/06/11
Durrable Products	Oct11007	9399	Gasket	$ 3.65	1,450	$ 5,292.50	45	10/03/11	10/08/11
Durrable Products	Oct11009	9399	Gasket	$ 3.65	1,985	$ 7,245.25	45	10/05/11	10/11/11

Example: For Cost per order data in Purchase orders database

- Applying two std dev interval (i.e. k=2)

```
> mean(Cost)
[1] 26295.32
> sd(Cost)
[1] 29842.83
> mean(Cost)-2*(sd(Cost))
[1] -33390.34
> mean(Cost)+2*(sd(Cost))
[1] 85980.98
> (length(Cost[Cost>-33390.34 & Cost<85980.98]))/94
[1] 0.9468085
```

94.68% falls within 2 sd of the mean.

- **k= 2; 75% of according to Chebyshev's theorem**

- Applying three std dev interval (i.e. k=3)

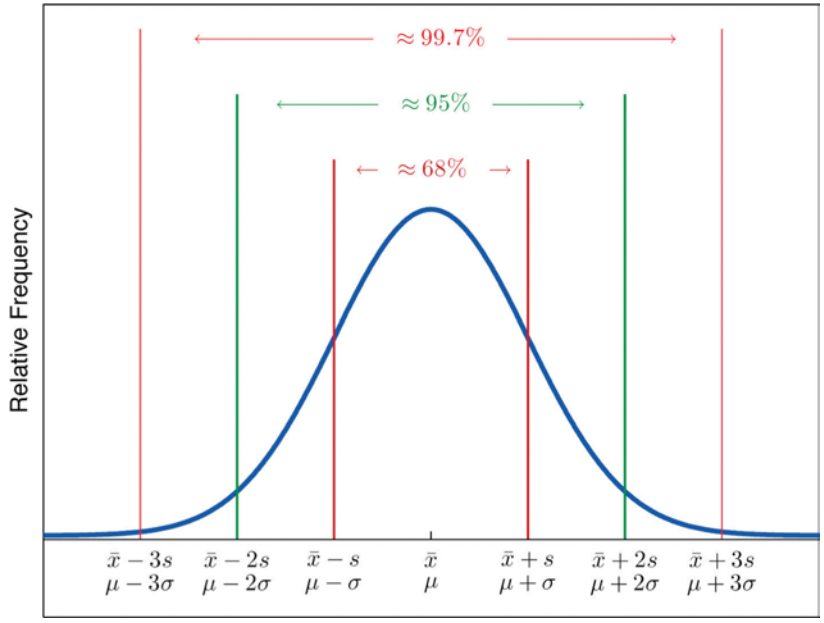
```
> mean(Cost)
[1] 26295.32
> sd(Cost)
[1] 29842.83
> mean(Cost)-3*(sd(Cost))
[1] -63233.17
> mean(Cost)+3*(sd(Cost))
[1] 115823.8
> (length(Cost[Cost>-63233.17 & Cost<115823.8]))/94
[1] 0.9787234
```

97.87% falls within 3 sd of the mean

- **k=3; 89% according to Chebyshev's theorem**

Measures of Dispersion

- Empirical Rules- For normally distributed data set, the proportion of values that lie within *k* (*k* > 1) standard deviations of the mean follows the empirical rules:
  - For k = 1: about 68% lie within one standard deviations of the mean
  - For k = 2: about 95% lie within two standard deviations of the mean
  - For k = 3: about 99.7% lie within three standard deviations of the mean



Source: Statistics Libretexts

Application of Empirical Rule - Process Capability Index

- Process capability index (C_p) is a measure of how well a manufacturing process can achieve specifications
- Using a sample of output, measure the dimension of interest, and compute the total variation using the third empirical rule.
- Compare results to specifications using:

$$Cp = \frac{\text{upper specification} - \text{lower specification}}{\text{total variation}}$$

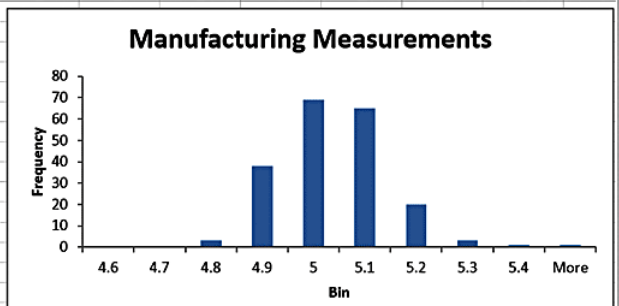
Eg: Using Empirical Rules to Measure the Capability of a Manufacturing Process

	A	B	C	D	E	F	G	H	I	J
1	**Manufacturing Measurements**								**Part dimension (cm)**	
2										
3	5.21	5.87	4.85	4.95	5.07	4.96	4.96	5.11	**Mean**	4.99
4	5.02	5.33	4.82	4.86	4.82	4.96	5.06	5.11	**Standard deviation**	0.117
5	4.90	5.11	5.02	5.13	5.03	4.94	4.86	5.08		
6	5.00	5.07	4.90	4.95	4.85	5.19	4.96	5.03	**Mean - 3*Stdev**	4.640
7	5.16	4.93	4.73	5.22	4.89	4.91	4.99	4.94	**Mean + 3*Stdev**	5.340
8	5.03	4.99	5.04	4.81	4.82	5.01	4.94	4.88	**Total variaton**	0.700
9	4.96	5.04	5.07	4.91	5.18	4.93	5.06	4.91		
10	5.04	5.14	4.81	4.95	5.02	5.05	4.95	4.86	**Lower Specification**	4.8
11	4.98	5.09	5.04	4.94	5.05	4.96	5.02	4.89	**Upper Specification**	5.2
12	5.07	5.06	5.03	4.81	4.88	4.92	5.01	4.91	**Specification range**	0.4
13	5.02	4.85	5.01	5.11	5.08	4.95	5.04	4.87		
14	5.08	4.93	5.14	4.81	4.98	5.08	5.01	4.93	**Cp**	0.57

Cp=0.4/0.7

Third Empirical rule

	A	B
1	*Bin*	*Frequency*
2	4.6	0
3	4.7	0
4	4.8	3
5	4.9	38
6	5	69
7	5.1	65
8	5.2	20
9	5.3	3
10	5.4	1
11	More	1

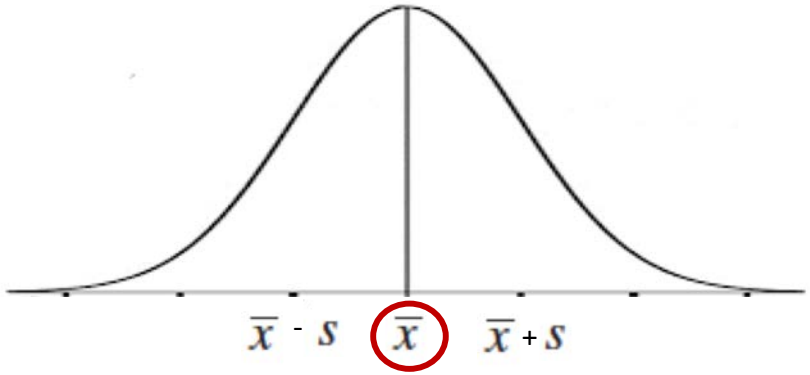


In practice:
AIM for Cp => 1.5

Standardized Values

- A standardized value, commonly called a *z*-score, provides a relative measure of the distance an observation is from the mean (independent of units of measurement)
- *z*-score for i^{th} observation in a data set is:

$$z_i = \frac{x_i - \bar{x}}{s}$$



Z = -1 (observation is 1 SD to the left of mean)

Z = 1 (observation is 1 SD to the right of mean)

EG: Computing z-Scores

- *Purchase Orders* Cost per order data

	A	B	C
1	Observation	Cost per order	z-score
2	x1	$2,700.00	-0.79
3	x2	$19,250.00	-0.24
4	x3	$15,937.50	-0.35
5	x4	$18,150.00	-0.27
6	x5	$23,400.00	-0.10
91	x90	$6,750.00	-0.65
92	x91	$16,625.00	-0.32
93	x92	$74,375.00	1.61
94	x93	$72,250.00	1.54
95	x94	$6,562.50	-0.66
96			
97	Mean	$26,295.32	
98	Standard Deviation	$29,842.83	

← df$zscore <- (df$cost -mean(df$cost))

Coefficient of Variation

- The coefficient of variation (CV) provides a relative measure of dispersion in data relative to the mean:

$$\text{CV} = \frac{\text{standard deviation}}{\text{mean}}$$

  - Sometimes expressed as a percentage (x100)
  - Provides a relative measure of risk to return
  - Useful when comparing variability of two or more data sets with different scales
  - Smaller CV → smaller risk
  - Reciprocal of CV → return to risk

Eg: Applying the Coefficient of Variation

- *Closing Stock Prices database*
- Which investment is most risky?
- Which investment would have least risk?

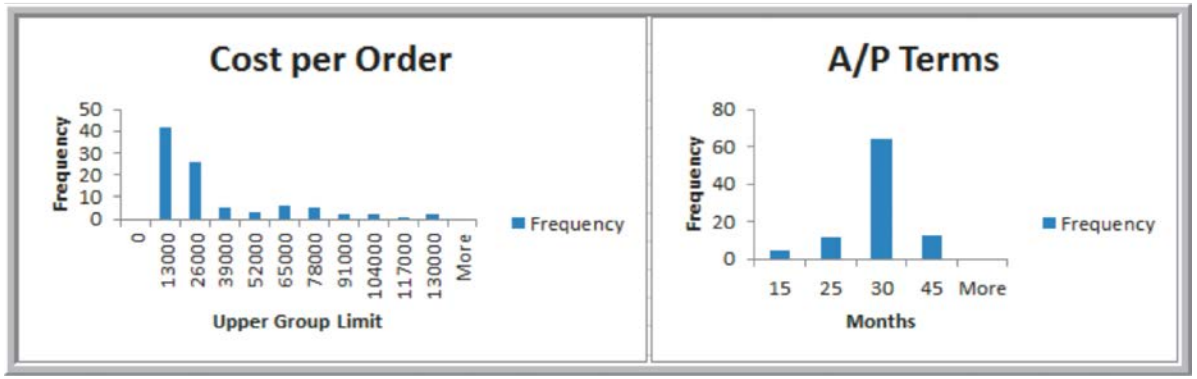
	A	B	C	D	E	F
1	**Closing Stock Prices**					
2						
3	**Date**	**IBM**	**INTC**	**CSCO**	**GE**	**DJ Industrials Index**
4	9/3/2010	$127.58	$18.43	$21.04	$15.39	10447.93
5	9/7/2010	$125.95	$18.12	$20.58	$15.44	10340.69
6	9/8/2010	$126.08	$17.90	$20.64	$15.70	10387.01
22	9/30/2010	$134.14	$19.20	$21.90	$16.25	10788.05
23	10/1/2010	$135.64	$19.32	$21.91	$16.36	10829.68
24	**Mean**	$130.93	$18.81	$21.50	$16.20	$10,639.98
25	**Standard Deviation**	$3.22	$0.50	$0.52	$0.35	$171.94
26	**Coefficient of Variation**	0.025	0.027	0.024	0.022	0.016

Intel (INTC) is slightly riskier than the other stocks.

Index fund has the least risk (lowest CV).

Measures of Shape: Skewness

- **Skewness** describes the lack of symmetry of data.
  - Distributions that tail off to the right are called positively skewed; those that tail off to the left are said to be negatively skewed.



Positively skewed

Symmetrical

Coefficient of Skewness

- Coefficient of Skewness (CS):

$$\mathrm{CS} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^3}{\sigma^3}$$

- CS is negative for left-skewed data.
- CS is positive for right-skewed data.
- $|CS| > 1$ suggests high degree of skewness.
- $0.5 \leq |CS| \leq 1$ suggests moderate skewness.
- $|CS| < 0.5$ suggests relative symmetry.

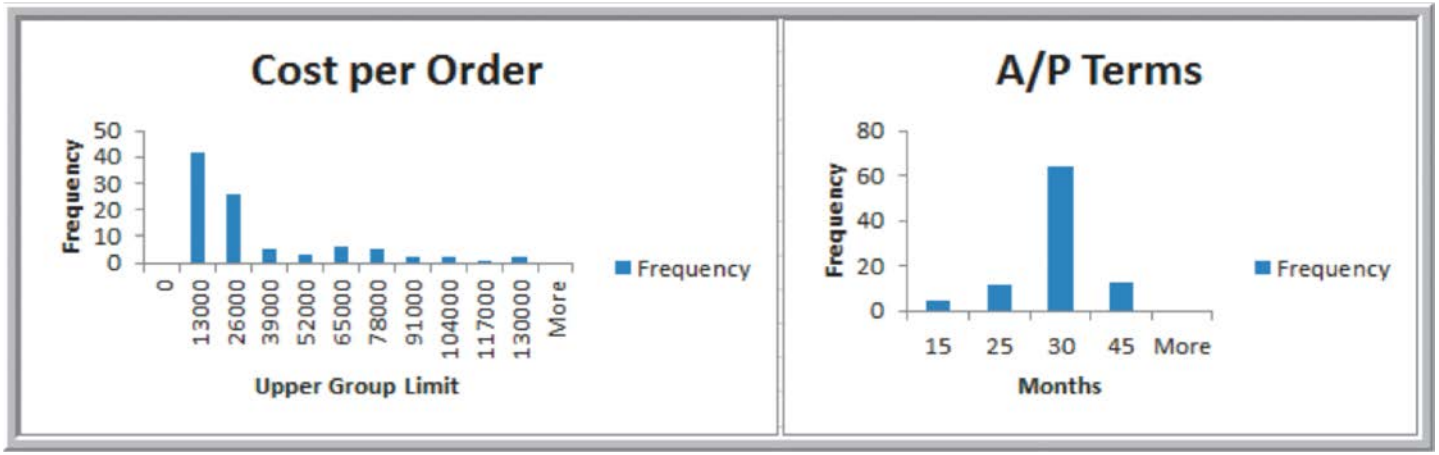
In R, can be obtained using the "skew" function in "psych" package.

Eg: Measuring Skewness

- Using *Purchase Orders* database

```
> skew(Cost)
[1] 1.611533
```

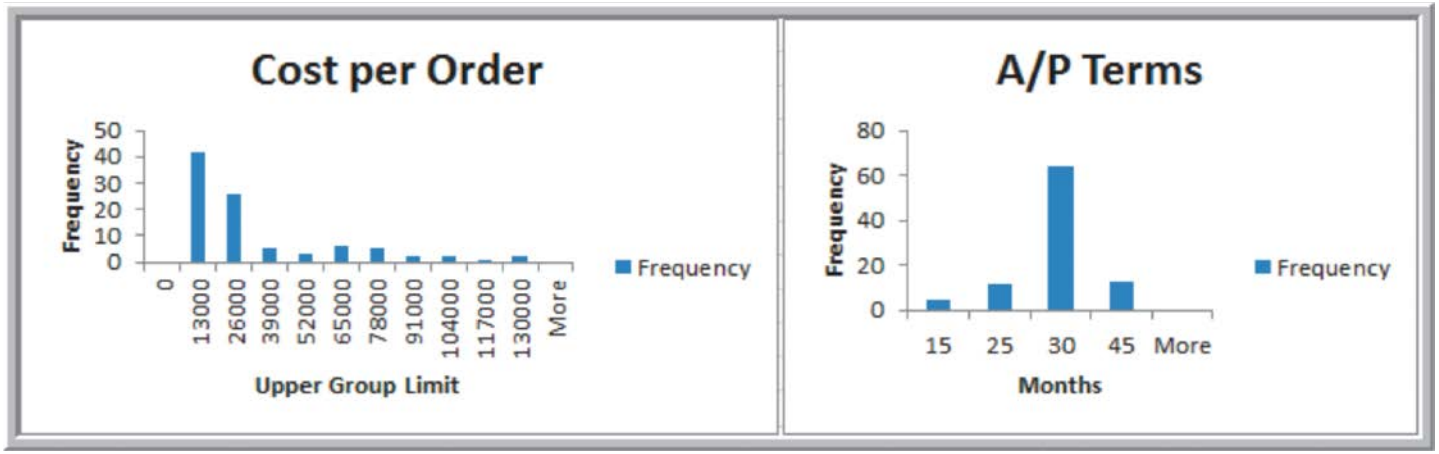
```
> skew(AP)
[1] 0.5802752
```



Eg: Measuring Skewness

- Using *Purchase Orders* database
- Cost per order data: CS = 1.61
- A/P terms data: CS = 0.58

Which has higher skewness?
Positive or Negative?

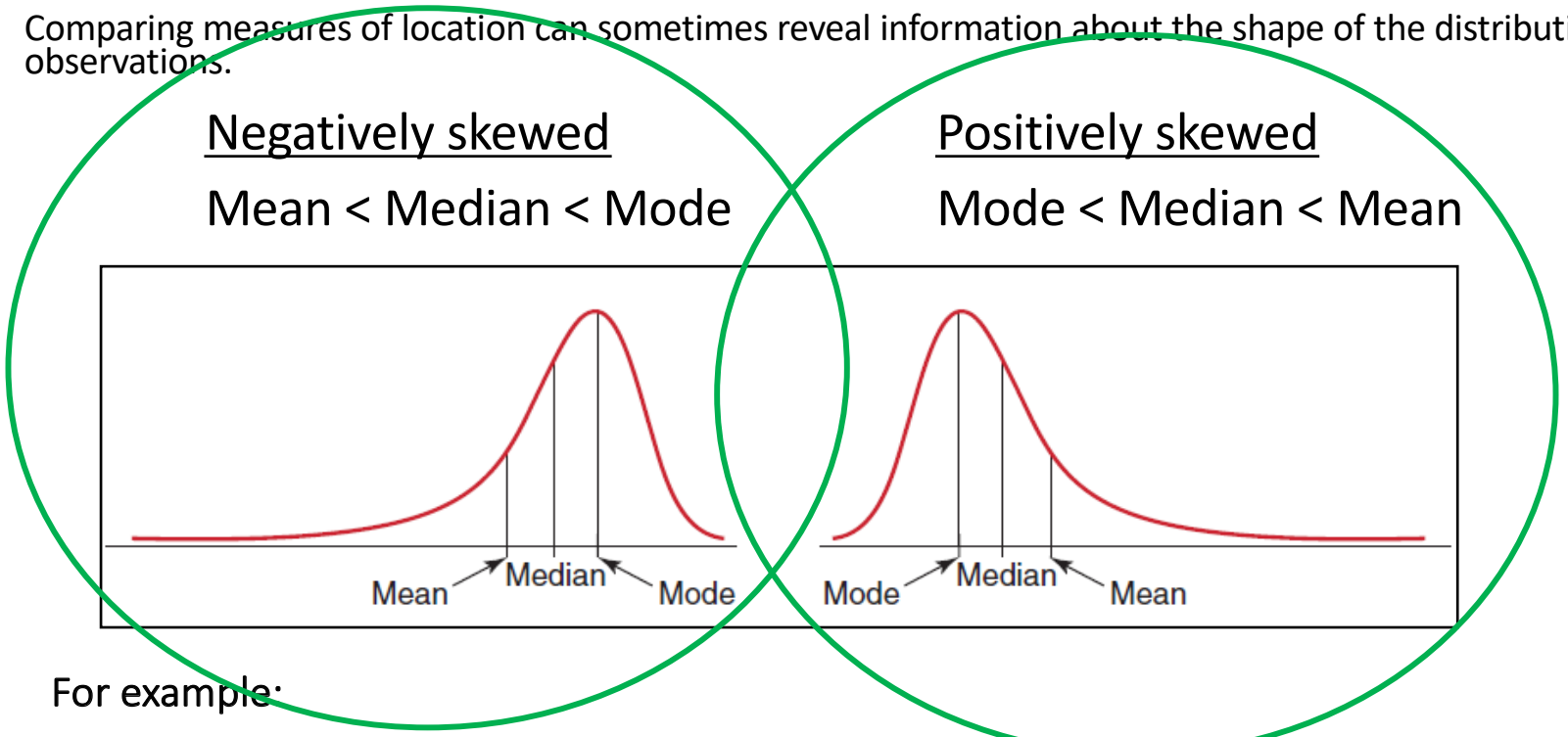


CS = 1.61
High positive skewness

CS = 0.58
Moderate positive skewness

Shape and Measures of Location

Comparing measures of location can sometimes reveal information about the shape of the distribution of observations.



For example:

- If distribution was perfectly symmetrical and unimodal, the mean, median, and mode would all be the same.
- If it were negatively skewed, mean < median < mode
- Positive skewness would suggest that mode < median < mean

Measures of Shape: Kurtosis

- Kurtosis refers to the peakedness (i.e., high, narrow) or flatness (i.e., short, flat-topped) of a histogram.
- Coefficient of kurtosis (CK) measures the degree of kurtosis of a population

$$\mathrm{CK} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^4}{\sigma^4}$$

- **CK < 3 indicates the data is somewhat flat with a wide degree of dispersion.**
- **CK > 3 indicates the data is somewhat peaked with less dispersion.**

In R, `kurtosi` function in the `psych` package can be used to compute CK (note however the cut off is 0 instead of 3)

```
> kurtosi(Cost)
[1] 1.803636
```

```
> kurtosi(AP)
[1] 1.277305
```

Kurtosis > 1, hence data is somewhat peaked with less dispersion

Descriptive Statistics for Grouped Data

- Population mean:

$$\mu = \frac{\sum_{i=1}^{N} f_i x_i}{N}$$

- Sample mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{n}$$

- Population variance:

$$\sigma^2 = \frac{\sum_{i=1}^{N} f_i (x_i - \mu)^2}{N}$$

- Sample variance:

$$s^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{n - 1}$$

Eg: Computing Statistical Measures from Frequency Distributions

- *Computer Repair Times*

	A	B	C	D	E	F
1	**Computer Repair Times**					
2						
3	**Days (x)**	**Frequency (f)**	**Frequency*Days**	**Days - Mean**	**(Days - mean)^2**	**Frequency*(Days - Mean)^2**
4	0	0	0	-14.912	222.368	0.000
5	1	0	0	-13.912	193.544	0.000
6	2	0	0	-12.912	166.720	0.000
7	3	0	0	-11.912	141.896	0.000
43	39	1	39	24.088	580.232	580.232
44	40	1	40	25.088	629.408	629.408
45	41	0	0	26.088	680.584	0.000
46	42	0	0	27.088	733.760	0.000
47	**Sum**	250	3728			8840.064
48						
49		**Mean**	14.912		**Variance**	35.50226506

```
> describe(x)
   vars   n  mean   sd median trimmed  mad min max range skew kurtosis   se
X1    1 250 14.91 5.96     14   14.12 4.45   5  40    35 1.67     3.92 0.38
```

$5.96^{2} = 35.50$

Eg: Computing Home Value by Type and Region

function in `psych` package

```
> describeBy(df11$`Market Value`,group=df11$`Type`)

 Descriptive statistics by group
group: A1
   vars  n     mean       sd median trimmed      mad   min    max
X1    1 12 94166.67 10261.52  90500   93450 10452.33 81500 114000
   range skew kurtosis      se
X1 32500 0.54    -1.11 2962.25
------------------------------------------------------------
group: A2
   vars  n  mean      sd median trimmed     mad   min    max range
X1    1 10 95000 8921.88  92300   93700 7042.35 87200 113200 26000
   skew kurtosis      se
X1 0.73    -0.91 2821.35
------------------------------------------------------------
group: B1
   vars n    mean       sd median trimmed     mad   min    max range
X1    1 8 88812.5 13569.23  85400 88812.5 4744.32 76600 120700 44100
   skew kurtosis      se
X1 1.49     0.91 4797.45
------------------------------------------------------------
group: B2
   vars n  mean      sd median trimmed     mad   min   max range
X1    1 5 89360 6209.91  91300   89360 3113.46 78800 94200 15400
    skew kurtosis      se
X1 -0.83    -1.22 2777.16
------------------------------------------------------------
group: C
   vars n     mean       sd median  trimmed     mad   min    max
X1    1 7 89942.86 12625.22  87600 89942.86 9340.38 79800 116100
   range skew kurtosis      se
X1 36300 1.12    -0.25 4771.88
```

	House Age	Square Feet	Market Value	Type	Region	Sub-Reg
1	33	1812	90000	A1	1	U
2	32	1914	104400	A2	1	X
3	32	1842	93300	B2	1	Z
4	33	1812	91000	A1	1	U
5	32	1836	101900	A1	2	U
6	33	2028	108500	A1	2	U
7	32	1732	87600	A2	2	U
8	33	1850	96000	A2	2	U
9	32	1791	89200	B2	3	U
10	33	1666	88400	A1	3	U
11	32	1852	100800	A1	3	U
12	32	1620	96700	A1	3	U

Descriptive Statistics for Categorical Data: The Proportion

- Proportion (*p)*, is the fraction of data that have a certain characteristic.
- Proportions are key descriptive statistics for categorical data, such as defects or errors in quality control applications or consumer preferences in market research.

Eg: Computing a Proportion

- Proportion of orders placed by Spacetime Technologies

	A	B	C	D	E	F	G	H	I	J
1	**Purchase Orders**									
2										
3	**Supplier**	**Order No.**	**Item No.**	**Item Description**	**Item Cost**	**Quantity**	**Cost per order**	**A/P Terms (Months)**	**Order Date**	**Arrival Date**
4	Spacetime Technologies	A0111	6489	O-Ring	$ 3.00	900	$ 2,700.00	25	10/10/11	10/18/11
5	Steelpin Inc.	A0115	5319	Shielded Cable/ft.	$ 1.10	17,500	$ 19,250.00	30	08/20/11	08/31/11
6	Steelpin Inc.	A0123	4312	Bolt-nut package	$ 3.75	4,250	$ 15,937.50	30	08/25/11	09/01/11
7	Steelpin Inc.	A0204	5319	Shielded Cable/ft.	$ 1.10	16,500	$ 18,150.00	30	09/15/11	10/05/11
8	Steelpin Inc.	A0205	5677	Side Panel	$ 195.00	120	$ 23,400.00	30	11/02/11	11/13/11
9	Steelpin Inc.	A0207	4312	Bolt-nut package	$ 3.75	4,200	$ 15,750.00	30	09/01/11	09/10/11
10	Alum Sheeting	A0223	4224	Bolt-nut package	$ 3.95	4,500	$ 17,775.00	30	10/15/11	10/20/11

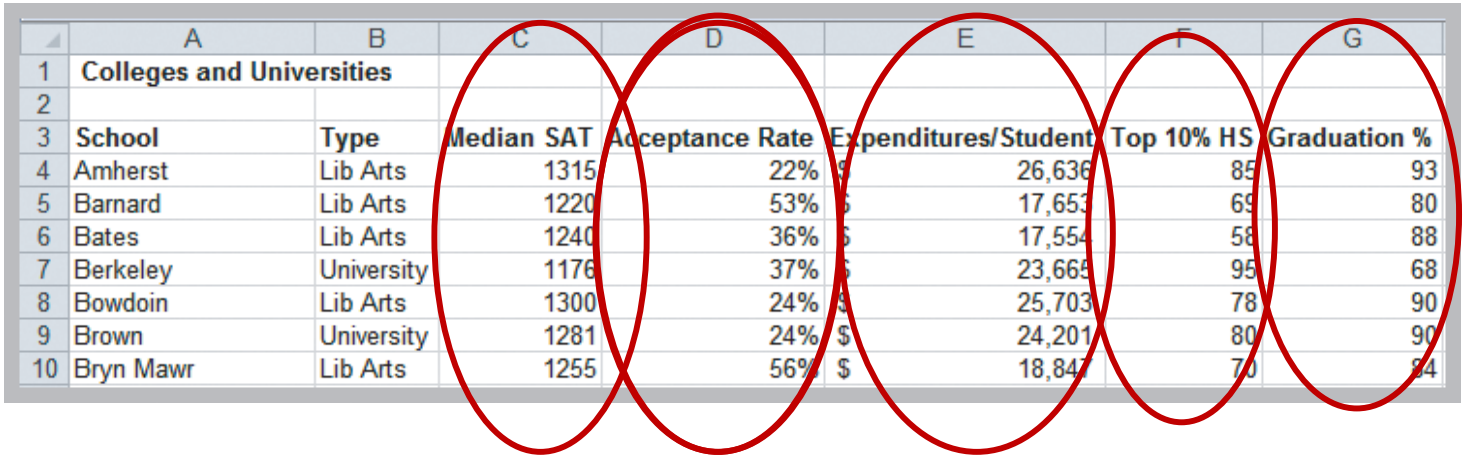
$$\text{Proportion} = \frac{\text{Number of Orders by SpaceTime Technologies}}{\text{Total number of Orders}}$$

```
> df9<-Purchase_Orders
> length(df9$Supplier[df9$Supplier=="Spacetime Technologies"])
[1] 12
> nrow(df9)
[1] 94
> length(df9$Supplier[df9$Supplier=="Spacetime Technologies"])/nrow(df9)
[1] 0.1276596
```

can also use filter function in dplyr

Measures of Association

- Data from 49 top liberal arts and research universities can be used to answer questions:
  - Is *Top 10% HS* related to *Graduation %*?
  - Is *Accept. Rate* related to *Expenditures/Student?*
  - Is *Median SAT* related to *Acceptance Rate?*



	A	B	C	D	E	F	G
1	**Colleges and Universities**						
2							
3	**School**	**Type**	**Median SAT**	**Acceptance Rate**	**Expenditures/Student**	**Top 10% HS**	**Graduation %**
4	Amherst	Lib Arts	1315	22%	$ 26,636	85	93
5	Barnard	Lib Arts	1220	53%	$ 17,653	69	80
6	Bates	Lib Arts	1240	36%	$ 17,554	58	88
7	Berkeley	University	1176	37%	$ 23,665	95	68
8	Bowdoin	Lib Arts	1300	24%	$ 25,703	78	90
9	Brown	University	1281	24%	$ 24,201	80	90
10	Bryn Mawr	Lib Arts	1255	56%	$ 18,847	70	84

Measures of Association - Covariance

- Covariance is a measure of the linear association between two variables, *X* and *Y*.
- For a population:

$$\operatorname{cov}(X, Y) = \frac{\sum_{i=1}^{N} (x_i - \mu_x)(y_i - \mu_y)}{N}$$

- For a sample:

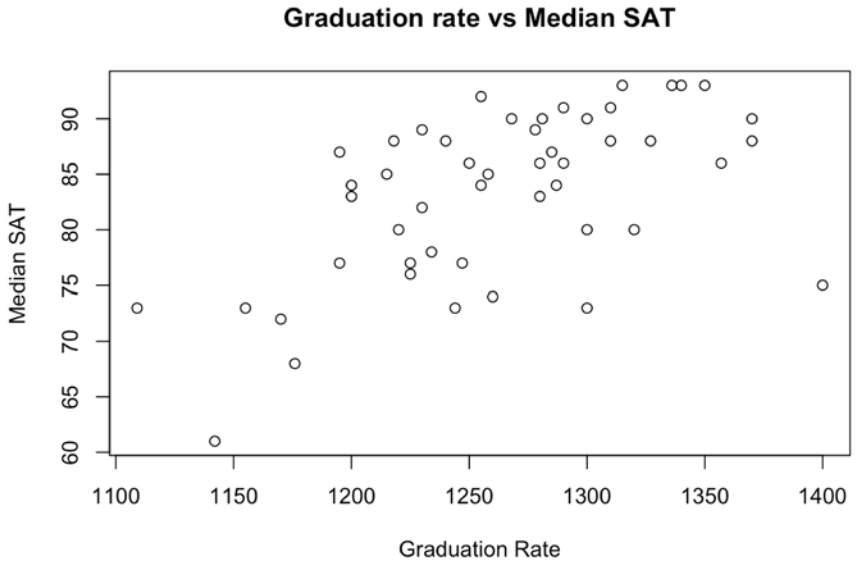
$$\operatorname{cov}(X, Y) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

Positive Covariance → direct relationship
Negative Covariance → inverse relationship
Magnitude → degree of association

Measures of Association - Covariance

Eg: Computing the Covariance

- Scatterplot of the *Colleges and Universities* data




```
> cov(df9$`Median SAT`,df9$`Graduation %`)
[1] 263.3703
```

Measures of Association - Correlation

- Correlation is a measure of the linear association between two variables, *X* and *Y* (not dependent on units of measurement)
- Correlation Coefficient formulas:

For a population:

$$\rho_{xy} = \frac{\operatorname{cov}(X, Y)}{\sigma_x \sigma_y}$$

For a sample:

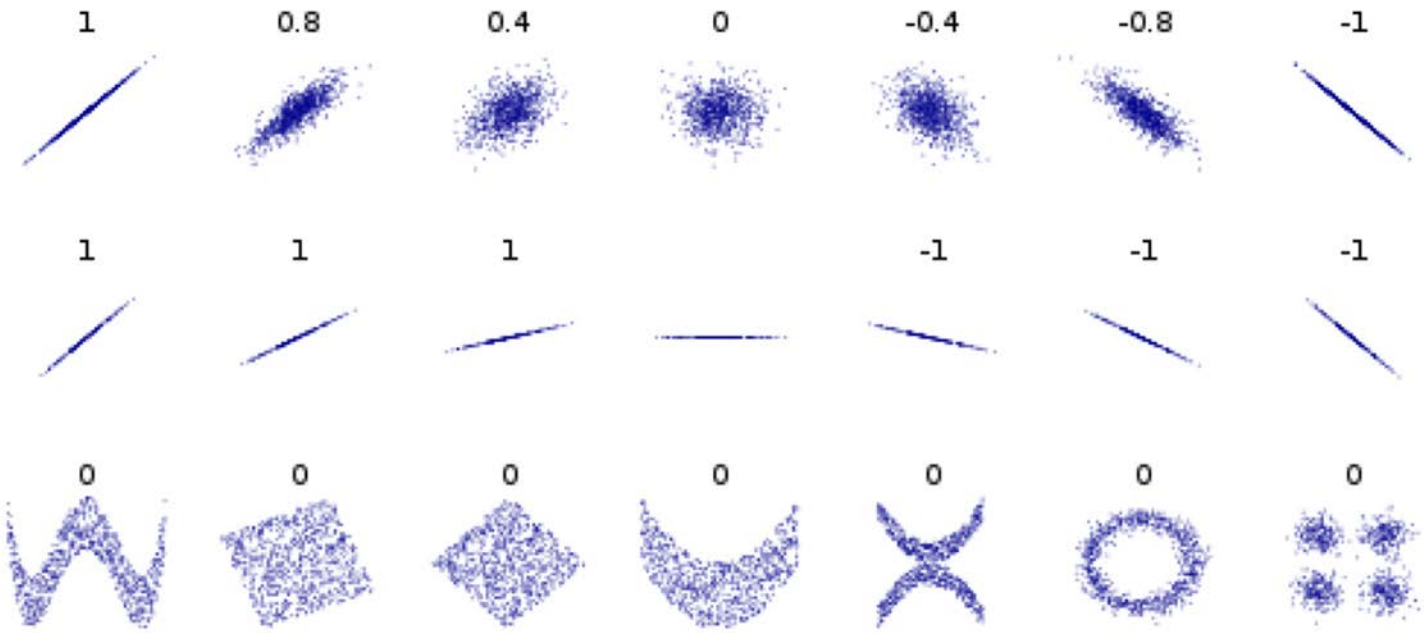
$$r_{xy} = \frac{\operatorname{cov}(X, Y)}{s_x s_y}$$

- Range: -1 (Strong negative) and 1 (Strong positive linear relationship)
- 0 indicates no linear relationship
- Also known as: Pearson product moment correlation or Pearson's correlation coefficient

```
> cor(df9$`Median SAT`,df9$`Graduation %`)
[1] 0.5641468
```

Measures of Association

- Correlation as a measure of LINEAR association



Source: Wikipedia (Correlation and dependence)

Computing Correlation of Multiple Variables

- Is *Top 10% HS* related to *Graduation %*?
- Is *Accept. Rate* related to *Expenditures/Student*?
- Is *Median SAT* related to *Acceptance Rate*?

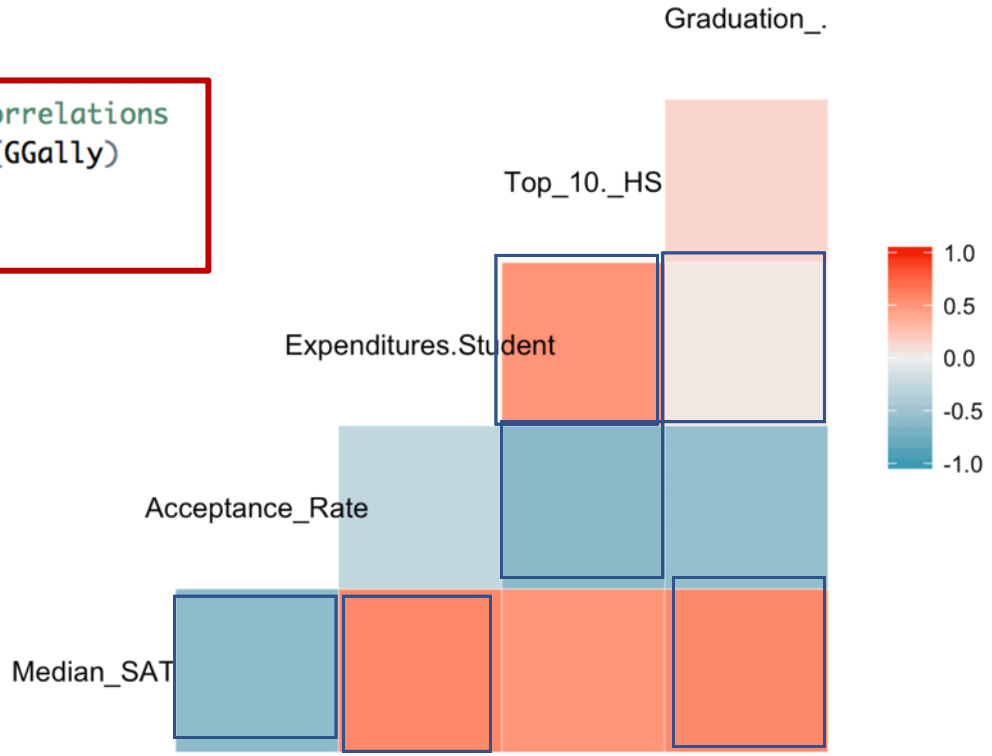
```
> library(psych)
> cts <- corr.test(df9[3:7])
> cts
Call:corr.test(x = df9[3:7])
Correlation matrix
                   Median SAT Acceptance Rate Expenditures/Student Top 10% HS Graduation %
Median SAT               1.00           -0.60                 0.57       0.50         0.56
Acceptance Rate         -0.60            1.00                -0.28      -0.61        -0.55
Expenditures/Student     0.57           -0.28                 1.00       0.51         0.04
Top 10% HS               0.50           -0.61                 0.51       1.00         0.14
Graduation %             0.56           -0.55                 0.04       0.14         1.00
Sample Size
[1] 49
Probability values (Entries above the diagonal are adjusted for multiple tests.)
                   Median SAT Acceptance Rate Expenditures/Student Top 10% HS Graduation %
Median SAT                  0           0.00                 0.00       0.00         0.00
Acceptance Rate             0           0.00                 0.14       0.00         0.00
Expenditures/Student        0           0.05                 0.00       0.00         0.77
Top 10% HS                  0           0.00                 0.00       0.00         0.68
Graduation %                0           0.00                 0.77       0.34         0.00

 To see confidence intervals of the correlations, print with the short=FALSE option
```

Plotting a correlation matrix

	Median SAT	Acceptance Rate	Expenditures/Student	Top 10% HS	Graduation %
Median SAT	1.00	-0.60	0.57	0.50	0.56
Acceptance Rate	-0.60	1.00	-0.28	-0.61	-0.55
Expenditures/Student	0.57	-0.28	1.00	0.51	0.04
Top 10% HS	0.50	-0.61	0.51	1.00	0.14
Graduation %	0.56	-0.55	0.04	0.14	1.00

```
#plot graph of correlations
install.packages(GGally)
library(GGally)
ggcorr(df9[3:7])
```



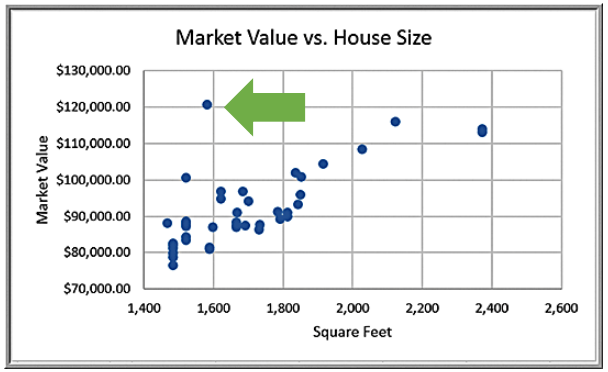
Outliers

- Mean and Range are sensitive to outliers
- No standard definition of what constitutes an outlier.
- How do we identify potential outliers?
- Some rules of thumbs:
  - *z*-scores > +3 or < -3 (< 0.3% for normal data)
  - Extreme outliers are > 3*IQR to the left of Q_1 or right of Q_3
  - Mild outliers are between (1.5to 3)*IQR to the left of Q_1 or right of Q_3

Eg: Investigating Outliers

- *Home Market Value* data

	A	B	C	D	E
1	Home Market Value				
2					
3	House Age	Square Feet	z-score	Market Value	z-score
4	33	1,812	0.5300	$90,000.00	-0.196
5	32	1,914	0.9931	$104,400.00	1.168
6	32	1,842	0.6662	$93,300.00	0.117
7	33	1,812	0.5300	$91,000.00	-0.101
41	27	1,484	-0.9592	$81,300.00	-1.020
42	27	1,520	-0.7957	$100,700.00	0.818
43	28	1,520	-0.7957	$87,200.00	-0.461
44	27	1,684	-0.0511	$96,700.00	0.439
45	27	1,581	-0.5188	$120,700.00	2.713
46	Mean	1,695		92,069	
47	Standard Deviation	220.257		10553.083	



- None of the *z*-scores exceed 3. However, while individual variables might not exhibit outliers, combinations of them might.
  - The last observation has a high market value ($120,700) but a relatively small house size (1,581 square feet) and may be an outlier.

What do you do with outliers?

- Leave them in the data if it is important
- Remove them if they are different from the rest
- Correct error in data entry

Statistical Thinking in Business DM

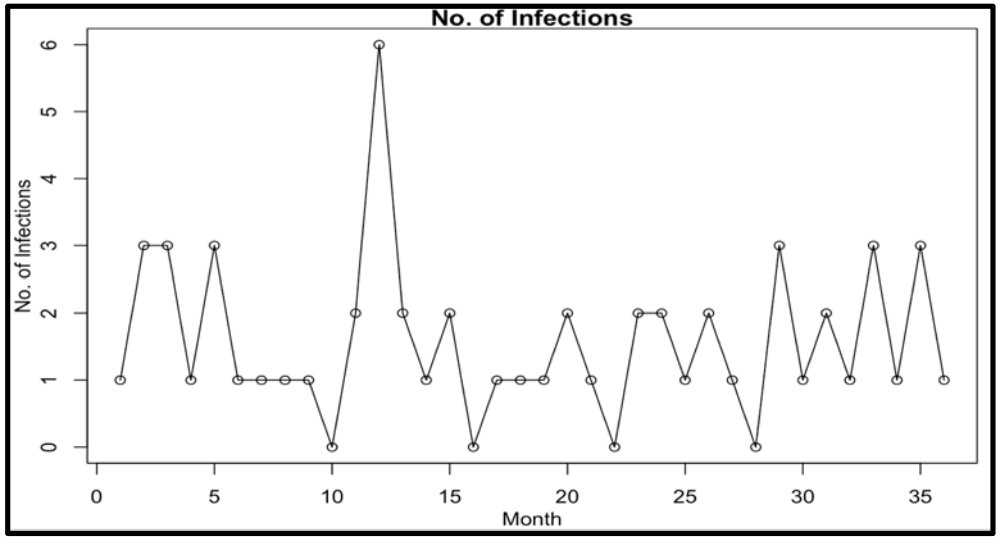
- Statistical Thinking is a philosophy of learning and action for improvement, based on principles that:
  - all work occurs in a system of interconnected processes
  - variation exists in all processes
  - better performance results from understanding and reducing variation
- Business Analytics provide managers with insights into facts and relationships that enables them to make better decisions.

Applying Statistical Thinking

- Excel file *Surgery Infections*
  - Is month 12 simply random variation or some explainable phenomenon?

	A	B
1	Surgery Infections	
2		
3	Month	Infections
4	1	1
5	2	3
6	3	3
7	4	1
8	5	3
9	6	1
10	7	1
11	8	1
12	9	1
13	10	0
14	11	2
15	12	6
16	13	2
17	14	1
18	15	2
19	16	0
20	17	1
21	18	1
22	19	1
23	20	2
24	21	1
25	22	0

```
> mean(df10$Infections)
[1] 1.583333
> sd(df10$Infections)
[1] 1.180194
```




```
> plot(df10$Month,df10$Infections, Main="No. of Infections",xlab="Month",ylab="No. of Infections")
> lines(df10$Month,df10$Infections)
```

Applying Statistical Thinking

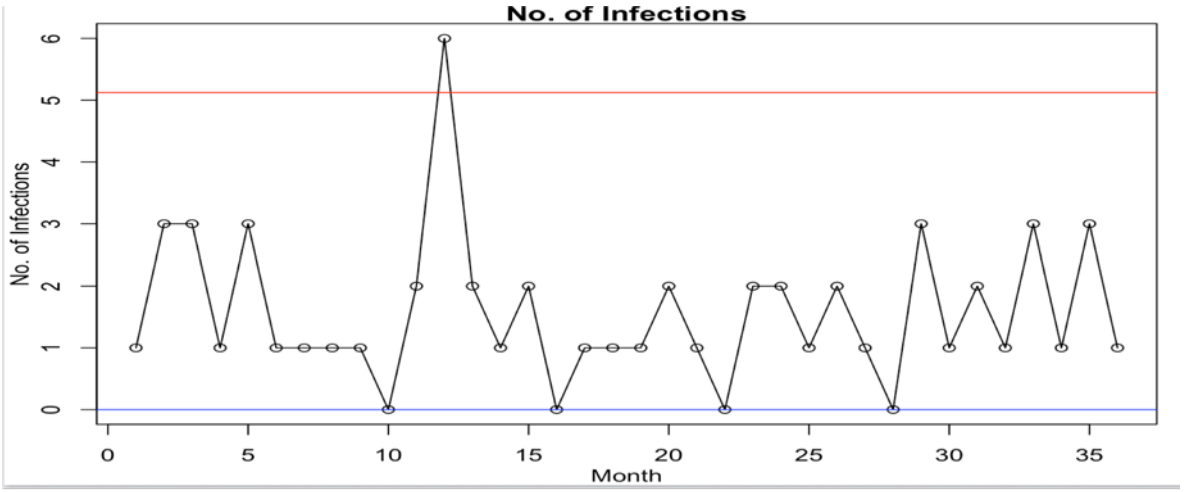
- Excel file *Surgery Infections*
  - Is month 12 simply random variation or some explainable phenomenon?

	A	B
1	Surgery Infections	
2		
3	Month	Infections
4	1	1
5	2	3
6	3	3
7	4	1
8	5	3
9	6	1
10	7	1
11	8	1
12	9	1
13	10	0
14	11	2
15	12	6
16	13	2
17	14	1
18	15	2
19	16	0
20	17	1
21	18	1
22	19	1
23	20	2
24	21	1
25	22	0

Applying the 3 std dev empirical rule!

```
> LL<- mean(df10$Infections)-3*(sd(df10$Infections))
> UL<- mean(df10$Infections)+3*(sd(df10$Infections))
> LL                > UL
[1] -1.957248   [1] 5.123914
```

```
abline(h=UL,col="red")
abline(h=0,col="blue") #infection rate cannot be less than 0
```



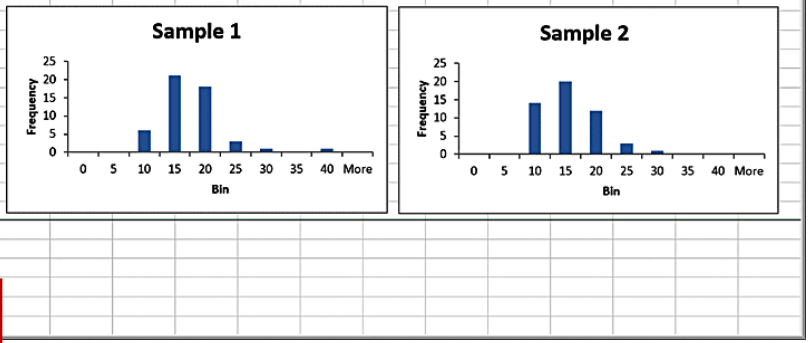
Variability in Samples

- Different samples from any population will vary
  - different means, standard deviations, and other statistical measures
  - differences in shapes of histograms
- Samples are extremely sensitive to the sample size – the number of observations included in the samples.

Eg: Variation in Sample Data

- Samples from *Computer Repair Times* data
- Population statistics: $\mu = 14.91$ days, $\sigma^2 = 35.5$ days^2
- Two samples of size 50:

Observation	Sample 1	Sample 2
1	12	12
2	10	14
3	10	27
4	15	9
5	21	9
6	12	18
7	20	8
8	19	13
9	12	14
10	16	13
47	15	16
48	9	7
49	18	16
50	29	20
Mean	15.78	13.62
Variance	23.15469	22.28122



- Two samples of size 25:

Observation	Sample 3	Sample 4
1	16	13
2	12	9
3	11	26
4	10	10
5	16	14
6	13	14
7	15	21
8	17	9
9	36	12
10	12	12
23	12	22
24	12	14
25	14	14
Mean	14.68	14.92
Variance	36.97667	42.74333