2d Lt Aaron Bonner

Phase Retrieval Module

1. Prompt

Original - Laser speckle data was collected via the experimental setup shown in the diagram below [not shown]. The lens diameter is in inch and the wavelength of the light is 532 nm. The pixel pitch of the camera is 5.5 micrometers. You will need to interpolate the data found in :Laser Speckle Data. Once interpolated you can use the Gerchberg-Saxton phase retrieval algorithm to recover the phase of the shower glass. Compute a histogram of the phase to determine the distribution of the phase imparted by the glass. does this make sense given what you know about random phasor sums? Explain.

Modified – Collect laser speckle data using a stable laser that has a beam spreader at its output to ensure an even distribution of intensity instead of a focused point at the aperture plane. Once collected, process the data and use the the Gerchberg-Saxton phase retrieval algorithm to recover the phase of the phase screen at the aperture. Compute a histogram of the phase to determine the distribution of the phase imparted by the glass. does this make sense given what you know about random phasor sums? Explain.

2. Data Collection

Data was collected using a HeNe laser with a wavelength of $632.8nm$. Directly at the output of the laser was a $75mm$ len. The purpose of this lens was to act as a beam spreader to even out the intensity of the laser distribution that exited the laser itself. The light from the lens then reached the aperture of the detection box, where it passed through a phase screen and then propagated a short distance to the detector. The diameter of the aperture was approximately $6mm$ and the propagation distance between the aperture and sensor was approximately $9cm$. The distance between the laser-lens and the aperture of the detection box was lengthened until the circular beam was evenly spread over the entire aperture. The purpose of this adjustment was to ensure that the light entering the aperture had approximately the same intensity and phase, so it could be considered a plane wave impacting the phase screen. Then, an image of the detector plane was captured using a Raspberry Pi HD camera module, which features a $1.55um$ pixel size and resolution of 4056 by 3040 pixels. The image was cropped to 2000 by 2000 pixels and converted to greyscale during image capture. During the entire data capture process, a set of five images were captured in short succession, but the following procedures were done on a single image for simplicity. The image used, named $image\_001.jpg$, is seen in Figure 1.

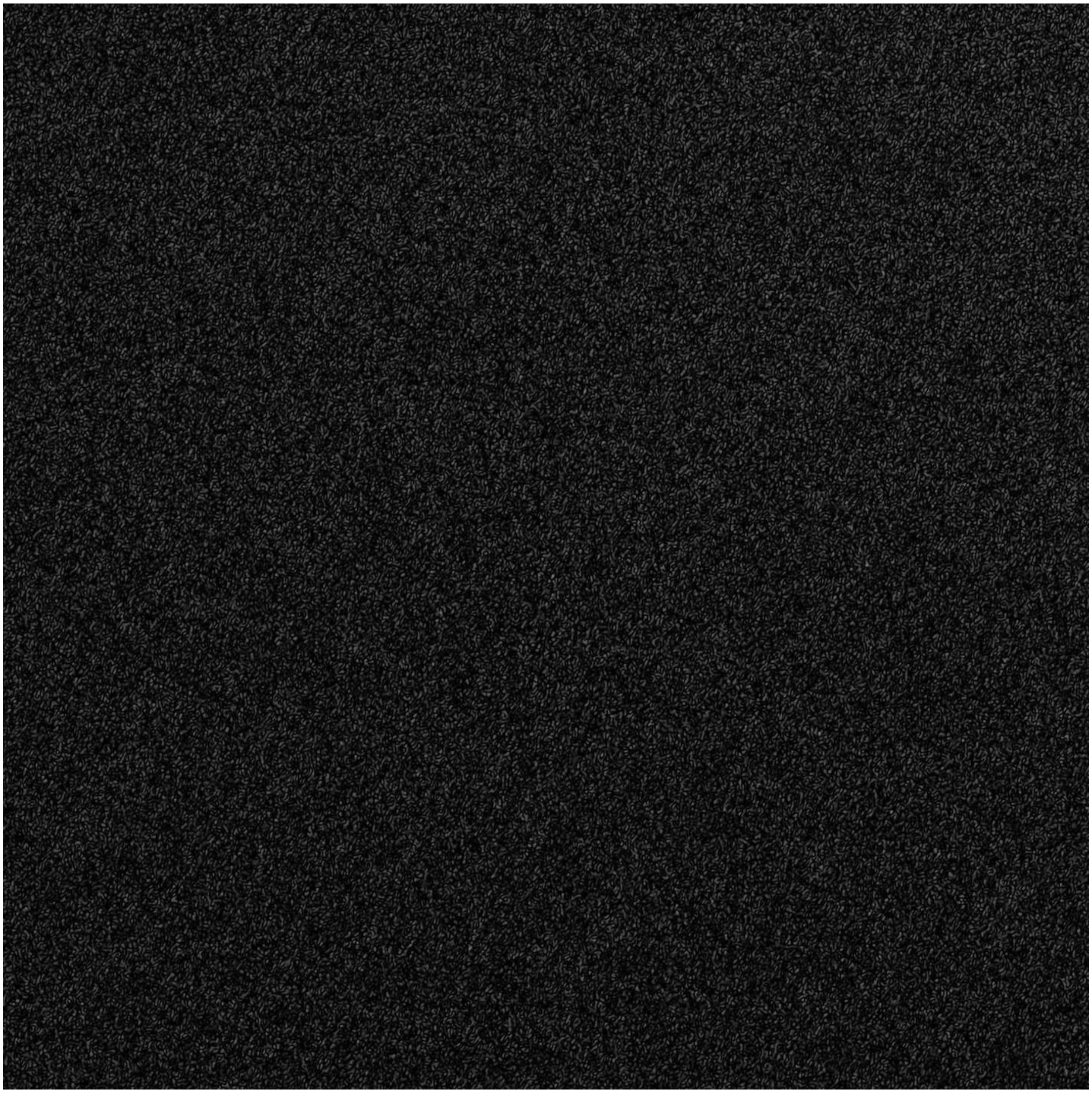

Figure 1. The image used in this paper.

3. Data Processing and Calculations

Following the initial image collection, several steps were taken to prepare the data for the requirements of Gerchberg-Saxton phase retrieval algorithm.

3.1 Intensity Normalization

The first step was to normalize the intensity of the data. After the image was loaded into memory, a dimension test was implemented to check if the data was still in RGB, and if so, it was converted to grayscale. Since our assumption during data collection was that the intensity was a plane wave at the aperture, by dividing each pixel by the sum of all pixels the data was normalized so that the total intensity summed to one. The amplitude of the normalized intensity was then calculated as the square root of the normalized intensity.

3.2 Aperture Mask

Next, the programmatic implementation of the aperture was implemented. In the experiment, the value of circular aperture was known to be approximately $6mm$. The first consideration for the conversion was that the sampling rate at the detector plane must exactly match the actual sampling rate that the camera had, $1.55um$. The derivation for the sampling rate in the aperture plane, Δx, started with the equation for the sampling rate of the detector plane, Δx_2, seen in Equation (3.1).

$$\Delta x_2 = \frac{\lambda z}{L} \tag{3.1}$$

In this equation, λ is the wavelength of the light being propagated, z is the distance between the aperture and the detector, and L is the size of the aperture plane. The value for L is the unknown, so it is broken down further using Equation (3.2).

$$L = \Delta x * N \tag{3.2}$$

In this equation, L is defined as the sampling rate Δx in the aperture plane multiplied by N, the number of values used for the x and y dimensions for the aperture plane. In this case the value of N was 2000 to match the original image dimensions. Rearranging Equation (3.1) to isolate Δx results in Equation (3.3).

$$\Delta x = \frac{\lambda z}{N \Delta x_2} \tag{3.3}$$

Inserting in the known values for variables on the RHS of the equation results in a Δx of $18.37um$. Then, a mesh grid of coordinates is created with X and Y values from $\left[-\frac{N}{2}, \frac{N}{2}\right]$ and separation of Δx. Then, a circle is drawn by programmatically checking the distance between a coordinate and the origin, or center, of the grid. Since the coordinates are a function of the real space Δx, if the distance is less than the radius of the aperture, $3mm$, then that point is inside the aperture. The output mask of this process is seen in Figure 2.

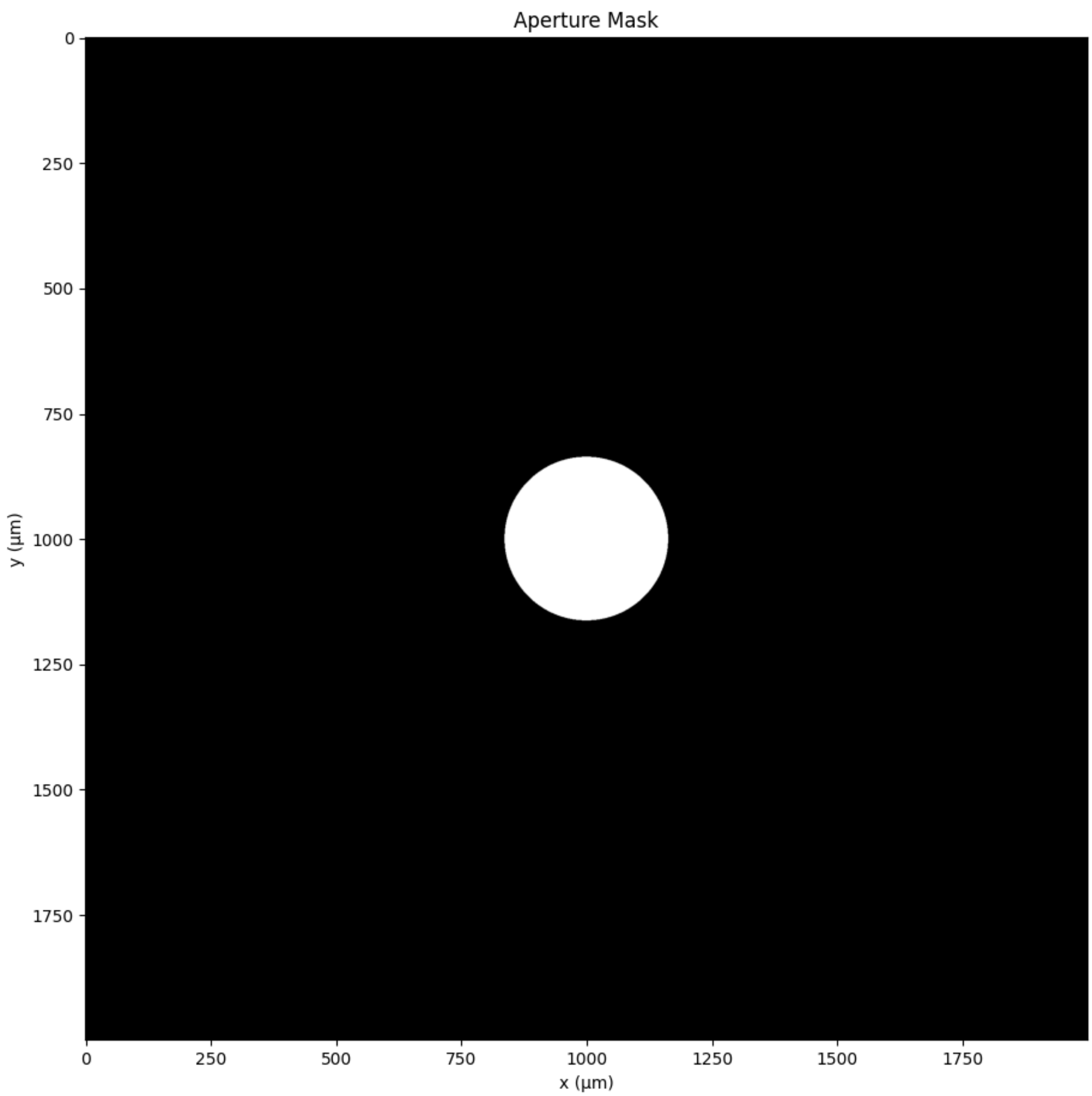

Figure 2. The aperture mask after calculating its sampling rate

To verify the validity of the calculations performed, the aperture size in pixel space was found to span from $X = 837$ to $X = 1163$, resulting in a diameter of 326 pixels. Multiplying this value by the sampling rate Δx calculated earlier results in diameter of $5.988mm$, indicating a successful aperture mask implementation.

3.3 Phase Screen Initial Guess

The phase screen was expected to follow a uniform distribution between 0 and 2π, so a 2000 by 2000 grid of random phases between 0 and 2π using $np.random()$. The initial guess was then calculated by performing the Inverse Fourier Transform of the amplitude calculated in Section 3.1 multiplied by the generated phase screen.

3.4 Gerchberg-Saxton Algorithm

Next the initial guess was looped through the Gerchberg-Saxton algorithm 100 times. This process is shown in Equation (3.4).

$$
\begin{aligned}
&\phi_{\text{initial}}(x,y) = \text{random}([0,2\pi]) \\
&A(x,y) = \text{IFT}\left(\text{Amplitude}_{\text{target}}(x,y)\cdot e^{j\phi_{\text{initial}}(x,y)}\right) \\
&\text{Repeat 100 times} \\
&\begin{bmatrix}
B(x,y) = \text{Amplitude}_{\text{source}}(x,y)\cdot e^{j\cdot\phi(A(x,y))} \\
C(u,v) = \text{FT}(B(x,y)) \\
D(u,v) = \text{Amplitude}_{\text{target}}(u,v)\cdot e^{j\cdot\phi(C(u,v))} \\
A(x,y) = \text{IFT}(D(u,v))
\end{bmatrix} \\
&\phi_{\text{retrieved}}(x,y) = \phi(A(x,y))
\end{aligned}
\tag{3.4}
$$

4. Results and Analysis

Following the Gerchberg-Saxton algorithm, the $\phi_{retrieved}$ grid was flattened from 2D to 1D. A histogram of the phase with 200 bins was generated as seen in Figure 3.

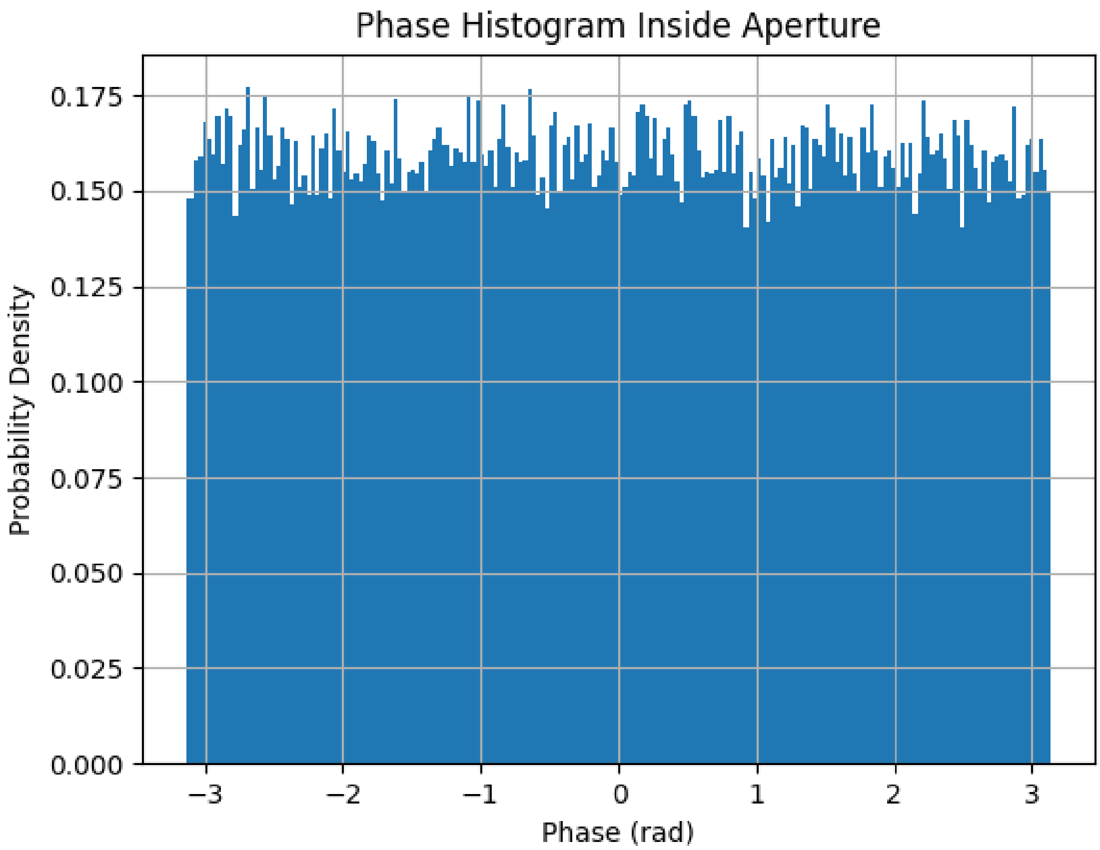

Figure 3. Histogram of the retrieved phases.

Examining this histogram, there is a clear indication that the phases calculated in the phase screen follow a uniform distribution. This implies that the phase screen adds a spatially random and uncorrelated phase shift to the starting plane wave as it propagates through the aperture. Then, considering each point in the aperture plane as a small random phasor sum, when propagating all the random phasor sums to the intensity plane, a developed speckle pattern is formed as seen in the starting image. This behavior follows the expectation of random phasor sum theory where uniformly distributed phase screens generate speckle patterns.