Fundamentals of Statistical Inference

Measuring Uncertainty

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Outline

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- Descriptive vs. inferential statistics
- ▶ The normal distribution
- Comparing groups
- Statistical/practical significance

Resources

▶ Slides, data, and handouts available at:

bit.ly/umhb_dpt

Stigler (1986, p. 1):

[Modern statistics provides]...the logic and methodology for the measurement of uncertainty and for examination of the consequences of that uncertainty in the planning and interpretation of experimentation and observation.

- Experimentation and observation:
 - 1. Measurement of uncertainty
 - 2. Examination of the consequences of that uncertainty

- ► Two fundamental branches
 - 1. Descriptive statistics
 - Summarize data
 - ► Condense larger themes
 - 2. Inferential statistics
 - Infer meaning
 - Test predictions

Example

- Baystate Medical Center, Springfield, MA.
- Sample of 189 births in 1986
- Risk factors in low birth weight babies

| Age | Weight | Race | Smoker? | Birth Weight |
|-----|--------|-------|------------|--------------|
| 19 | 182 | Black | Non-Smoker | 5.56 |
| 33 | 155 | Other | Non-Smoker | 5.62 |
| 20 | 105 | White | Smoker | 5.64 |
| 21 | 108 | White | Smoker | 5.72 |
| 18 | 107 | White | Smoker | 5.73 |
| 21 | 124 | Other | Non-Smoker | 5.78 |

- ► How many babies were considered low birth weight (< 5.5 lbs.)?
- How many mothers smoked during pregnancy?
- How much did the average baby weigh?
 - By smoking status
 - By race

Question:

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How should we answer this question?

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[ON AVERAGE], do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

| Smoking Status | n | Min. | Max. | X | SD |
|----------------|-----|------|-------|------|------|
| Non-Smoker | 115 | 2.25 | 11.00 | 6.74 | 1.66 |
| Smoker | 74 | 1.56 | 9.34 | 6.11 | 1.46 |

Question:

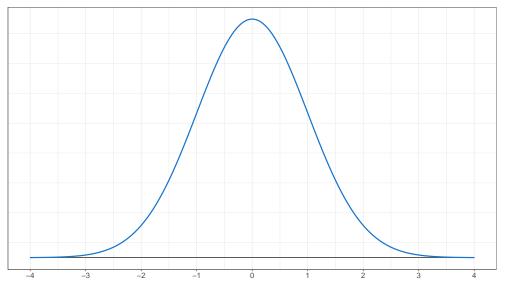
1. Based on our sample, what are we left to assume about the weights of babies *in the population* born to smoking and non-smoking mothers?

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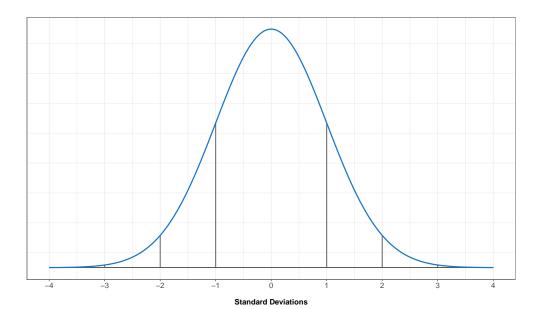
- 1. Based on our sample, what are we left to assume about the weights of babies *in the population* born to smoking and non-smoking mothers?
 - ▶ That the sample estimates represent the population parameters

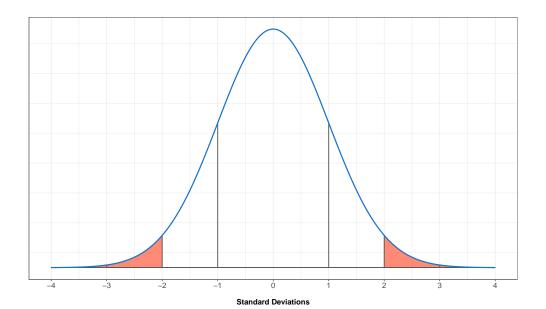
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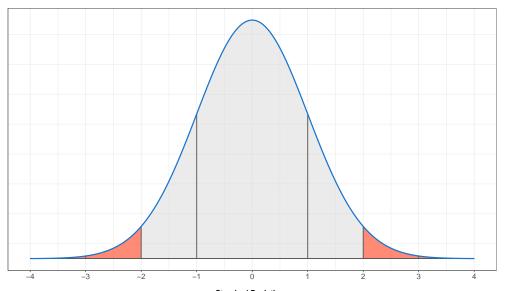
▶ In fact, we assume that the population distribution of baby weights is "normal"



Standard Deviations







Standard Deviations

- More useful than descriptives
- Allow for making predictions or generalizations
- Key to hypothesis testing

Question:

Do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

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Do babies born to mothers who smoked during pregnancy weigh [STATISTICALLY SIGNIFICANTLY] less than those born to mothers who did not?

How should we answer this question?

- What do we mean by statistical significance?
- Observed differences which exceed "normality."

- We usually consider differences beyond ± 2 *SDs* from *M* to be "statistically significant"
- **NOTE**: Statistical significance \neq practical significance

Question:

▶ Do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

Hypotheses:

- ▶ H_0 : There is no mean difference in the birth weight of babies born to mothers who did and did not smoke during pregnancy $(\mu_N \mu_S = 0)$
- ▶ H_1 : There is some difference in the birth weight of babies born to mothers who did and did not smoke during pregnancy $(\mu_N \mu_S \neq 0)$

- ▶ Let's test our hypothesis using an independent-samples *t*-test
 - IV: Mothers' smoking status (smoker, non-smoker)
 - DV: Baby birth weight

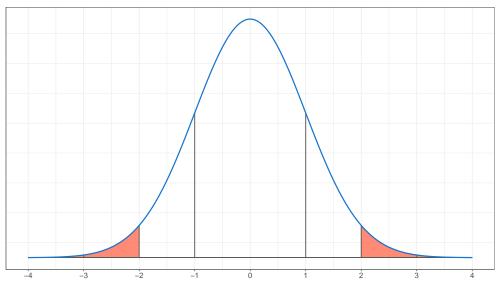
$$t = rac{\overline{X}_{non-smokers} - \overline{X}_{smokers}}{\sqrt{rac{s_{non-smokers}^2}{N_{non-smokers}} + rac{s_{smokers}^2}{N_{smokers}}}}$$

Table 1: Results of independent-samples *t*-test

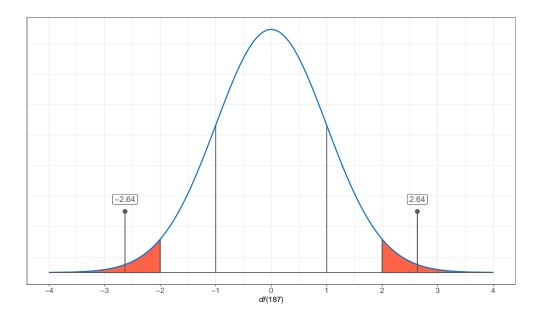
| $\overline{ n M SD n M}$ | SD | t(187) | - | 2 |
|---|------|--------------|-------|----------|
| | JD | $\iota(101)$ | ρ | ω |
| Baby birth weight 115 6.74 1.66 47 6.11 1 | 1.46 | 2.63 | 0.009 | 0.008 |

Note: M = Mean; SD = Standard deviation

- Assuming the null hypothesis, in reality, is true, the probability of obtaining a mean difference in birth weight ≥ 0.62 lbs. is $0.009 \ (0.90\%)$
- Birth weights appear to differ statistically significantly



Standard Deviations



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- ▶ But, is the difference of M = 0.62 lbs. meaningful?
- ▶ A meaningful difference implies practicality or usefulness in the real world
- Effect size (ω^2) : Proportion of variance explained in the model
- ► Smoking status explains 0.009 (0.90%) of the variance in baby birth weight
- ▶ Thus, 100% 0.991% = 99.10% of the variance in baby birth weight is left unexplained

Recap

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- Descriptive statistics allow us to summarize data from a sample
- Inferential statistics allow us to predict and generalize about a population
- Hypothesis testing allows us to construct a sense of meaning about the world

Next Time

- Making decisions using hypothesis testing and prediction
 - Statistical variables
 - Multiple group comparisons (ANOVA)
 - Predicting outcomes (Regression)