# FOUNDATIONS OF STATISTICAL DECISION MAKING

Relationships and Prediction

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PHTH 7147: Critical Inquiry I

# OUTLINE

# Outline

- Correlation
- Predicting outcomes (Regression)

# Recap

- Statistical variables
- Multiple group comparisons (ANOVA)

# Resources

- Slides, data, and handouts available at:

`bit.ly/umhb_dpt`

# Data

- Today's example data are from the 2002-2004 National Education Longitudinal Study (NELS)
- Nationally representative, longitudinal study of U.S. high school students
- Surveys of students, their parents, math and English teachers, and school administrators
- Student assessments in math (10th & 12th grades) and English (10th grade)

# Data

- Variables:
    1. `grades`: GPA of student in 2002
    2. `pared`: Highest education of parent (in years)
    3. `hwork`: Amount of time spent doing homeworkd during the week (in hours)

# Data

- Let's look at the NELS data

| | grades | pared | hwork |
|---|---|---|---|
| 1 | 78 | 13 | 2 |
| 2 | 79 | 14 | 6 |
| 3 | 79 | 13 | 1 |
| 4 | 89 | 13 | 5 |
| 5 | 82 | 16 | 3 |
| 6 | 77 | 13 | 4 |
| ... | ... | ... | ... |
| 100 | 74 | 12 | 4 |

# Data

Variable correlations

| | Grades | Parent Education | Homework |
|---|---|---|---|
| Grades | 1.00 | — | — |
| Parent Education | 0.29 (0.08) | 1.00 | — |
| Homework | 0.33 (0.11) | 0.28 (0.08) | 1.00 |

# CORRELATION

# Correlation

- Statistical technique used to determine the degree to which two variables are related
- Two numerical variables: Pearson's $r$
- The degree of relationship between two variables can vary from -1.0 to 1.0
- This is sometimes referred to as magnitude
- The closer the relationship is to -1.0 or 1.0, the stronger the magnitude or degree of relation between two variables

# Correlation

- Correlation coefficients describe two characteristics:
  1. The degree to which two variables are related
  2. The direction, or type of effect one variable has on the other (i.e., positive or negative)

# Correlation

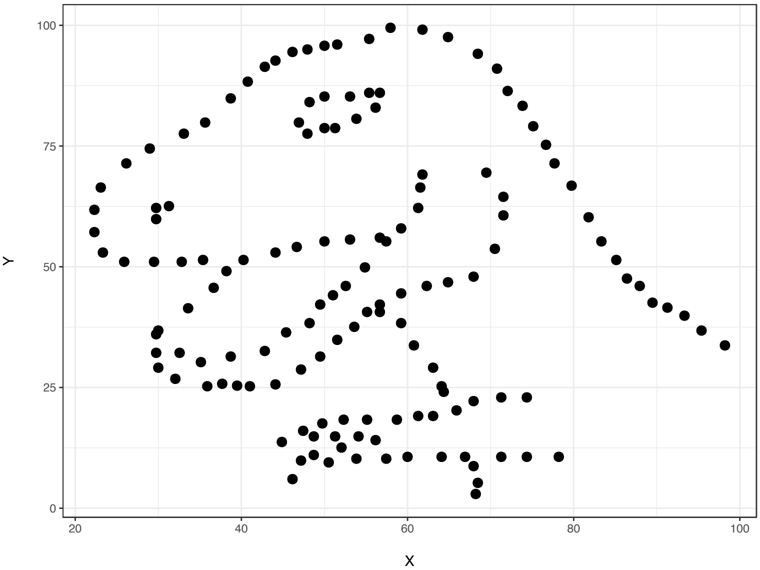
- Two types of correlation:
  1. Positive Correlation:
     - Higher scores on one variable associated with higher scores on a second variable
  2. Negative Correlation:
     - Higher scores on one variable associated with lower scores on a second variable

# Correlation

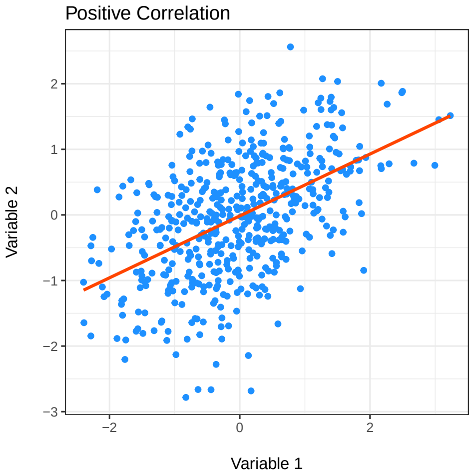
- It's always recommended that you visualize your correlational data first
- The results may yield more information than the $r$ alone
- For example, imagine we have the following data with $r = -0.064$

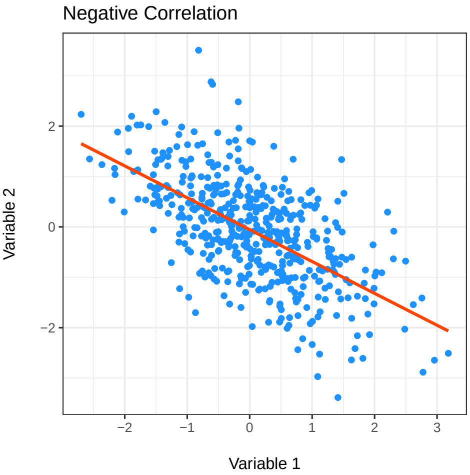
| | x | y |
|---|---|---|
| 1 | 55.38 | 97.18 |
| 2 | 51.54 | 96.03 |
| 3 | 46.15 | 94.49 |
| – | – | – |
| 142 | 44.10 | 92.69 |

# Correlation



# Correlation



# Correlation

- What determines a strong, medium, and small correlation?
  - Cohen (1988) suggested the following:
    - $r \leq$ 0.10 = small
    - $>$ 0.10 $r \leq$ 0.30 = medium
    - $r \geq$ 0.50 = large

# Correlation

- Once calculated, $r$ can be squared ($r^2$)
- This is called a coefficient of determination
- Proportion of variability in one variable that can be accounted for (or explained) by variability in the other variable
- The remaining proportion can be explained by factors other than your variables
  - **Ex.:** $r$ = 0.50 $\rightarrow r^2$ = 0.25

# Correlation

- We often examine correlations visually using a scatterplot
- Graphically depicts the relationship between 2 variables
- Typically, the predictor is on the X-axis and the outcome is on the Y-axis

# Correlation

| | **Quantitative X** | **Ordinal X** | **Nominal X** |
|---|---|---|---|
| **Quantitative Y** | Pearson's $r$ | — | — |
| **Ordinal Y** | Biserial $r_b$ | Spearman $\rho$ | — |
| **Nominal Y** | Point Biserial $r_{pb}$ | Rank Biserial $r_{rb}$ | Phi ($\phi$) |

Calkins (2005)

# PREDICTION AND REGRESSION

# Prediction and Regression

- Regression is a statistical procedure used to predict values of one variable from values of another variable
- It is a hypothetical model of the relationship between at least two variables
- The model used is a linear one
- Therefore, we describe the relationship using the equation of a straight line

# Prediction and Regression

- Imagine we suspect parents' education and time spent doing homework combine to predict students' grades

# Prediction and Regression

- Regression model equation:

$$Y = a + bX_1 + bX_2 + e$$

- $a$ = Intercept
  - Point where regression line crosses $Y$ axis
- $b$ = Slope of the line

# Prediction and Regression

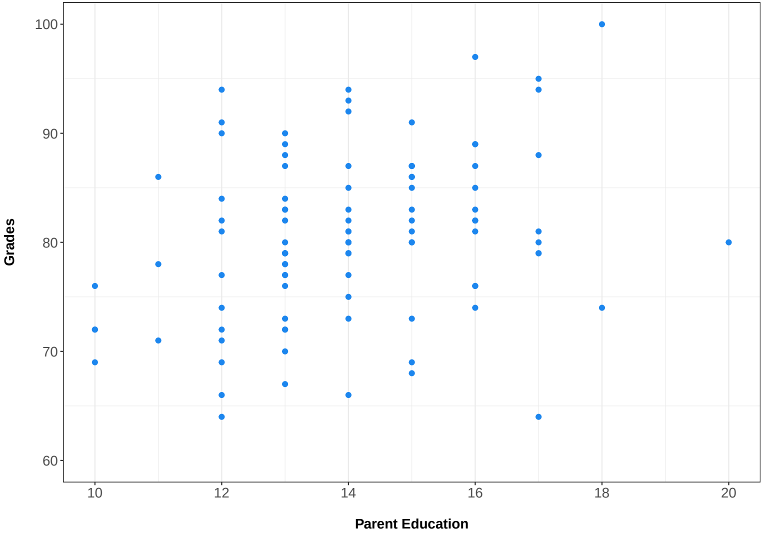
$$Y = a + bX_1 + bX_2 + e$$

- $Y$ = Criterion or dependent variable
    - Variable being measured and predicted
    - $Y$ = students' grades
- $X$ = Predictor or independent variable
    - Variable we use to predict the outcome
    - $X_1$ = parents' education
    - $X_2$ = homework

# NELS Data

| pared | n | mean_grades | sd_grades | mean_hw | sd_hw |
|---|---|---|---|---|---|
| 10 | 4 | 72.25 | 2.87 | 3.75 | 1.26 |
| 11 | 3 | 78.33 | 7.51 | 3.33 | 1.15 |
| 12 | 13 | 78.08 | 9.80 | 4.46 | 1.81 |
| 13 | 23 | 79.22 | 6.07 | 4.78 | 2.19 |
| 14 | 19 | 81.37 | 6.85 | 5.32 | 1.57 |
| 15 | 15 | 81.67 | 6.82 | 5.53 | 2.45 |
| 16 | 12 | 83.42 | 6.54 | 5.75 | 2.30 |
| 17 | 8 | 82.50 | 9.96 | 5.62 | 2.39 |
| 18 | 2 | 87.00 | 18.38 | 6.00 | 1.41 |
| 20 | 1 | 80.00 | | 6.00 | |

# NELS Data



# NELS Regression

- Let's regress students' grades on parent education and time spent doing homework
- Notice the intercept term and coefficients for `pared` and `hwork`
- Interpretation can be tricky

# NELS Regression

```
##
##  LINEAR REGRESSION
##
##  Model Fit Measures
##  ───────────────────────────
##    Model    R        R²
##  ───────────────────────────
##        1    0.390    0.152
##  ───────────────────────────
##
##
##  MODEL SPECIFIC RESULTS
##
##  MODEL 1
##
##  Model Coefficients
##  ─────────────────────────────────────────────────────
##    Predictor    Estimate    SE       t        p
##  ─────────────────────────────────────────────────────
##    Intercept      63.227    5.240    12.07    < .001
##    pared           0.871    0.384     2.27     0.026
##    hwork           0.988    0.361     2.74     0.007
##  ─────────────────────────────────────────────────────
```

# NELS Regression

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## ─────────────────────────────────────────────────────────
```

### Interpretation

*For a student who spends 0 hours weekly doing homework and whose parent has 0 years of education, we would predict his/her GPA to be approximately 63.23.*

# NELS Regression

```
##
## MODEL 1
##
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##   Predictor    Estimate    SE       t        p
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##   Intercept      63.227    5.240    12.07    < .001
##   pared           0.871    0.384     2.27     0.026
##   hwork           0.988    0.361     2.74     0.007
## ─────────────────────────────────────────────────────
```

**Interpretation, contd.**

*For every 1 unit increase in parent education and time spent weekly doing homework, we would expect this students' GPA to increase by 0.871 and 0.988 points, respectively.*

# NELS Regression

What's wrong here?

# NELS Regression

- We need to mean center both `pared` ($M$ = 14.03, $SD$ = 1.93) and `hwork` ($M$ = 5.09, $SD$ = 2.06)
- This will allow more realistic interpretation

# NELS Regression

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##
##  MODEL 1
##
##  Model Coefficients
##  ─────────────────────────────────────────────────────────
##    Predictor       Estimate    SE       t         p
##  ─────────────────────────────────────────────────────────
##    Intercept          80.47    0.709    113.47    < .001
##    pared_center        1.68    0.742      2.27     0.026
##    hwork_center        2.03    0.742      2.74     0.007
##  ─────────────────────────────────────────────────────────
```

# NELS Regression

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##   hwork_center        2.03    0.742      2.74     0.007
## ──────────────────────────────────────────────────────────
```

### Interpretation

*For a student who spends M = 5.09 hours weekly doing homework and whose parent has M = 14.03 years of education, we would predict his/her GPA to be approximately 80.47.*

# NELS Regression

```
##
## MODEL 1
##
## Model Coefficients
## ───────────────────────────────────────────────────────
##    Predictor        Estimate    SE       t         p
## ───────────────────────────────────────────────────────
##    Intercept           80.47    0.709    113.47    < .001
##    pared_center         1.68    0.742      2.27     0.026
##    hwork_center         2.03    0.742      2.74     0.007
## ───────────────────────────────────────────────────────
```

**Interpretation, contd.**

*For every 1 unit change in parent education and time spent weekly doing homework, we would expect a students' GPA to change by 1.68 and 2.03 points, respectively.*

# NELS Regression

- Overall model interpretation
- In regression, we typically use $R^2$ as a meausre of effect size
- Proportion of variance explained by the model

```
##
##  Model Fit Measures
##  ───────────────────────────
##    Model    R        R²
##  ───────────────────────────
##        1    0.390    0.152
##  ───────────────────────────
```

# NELS Regression

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##
##  Model Fit Measures
##  ───────────────────────────
##    Model    R        R²
##  ───────────────────────────
##        1    0.390    0.152
##  ───────────────────────────
```

**Interpretation**

*Parents' education and the time spent doing homework combine to explain approximately 0.152 $\longrightarrow$ 15.20% of the variability in determining students' grades.*

# RECAP

# Recap

- Correlation and regression are used to predict outcomes using past data
- Interpretation can be tricky
- Causation cannot be assumed

# QUESTIONS?