Unit 6: Introduction to linear regression

2. Outliers and inference for regression

Sta 101 - Spring 2016

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_s16*

1. Housekeeping

2. Main ideas
    1. Predicted values also have uncertainty around them
    2. R^2 assesses model fit -- higher the better
    3. Inference for regression uses the t-distribution
    4. Conditions for regression
    5. Type of outlier determines how it should be handled

3. Summary

- PA 6 opens today, due Apr 10, Sun
- PS 6 due tonight
- RA 7 (last RA!) on Monday
- Project questions?
  - If you want to see sample posters from previous years, stop by office hours
  - Most important advice: Sketch out a meeting / working plan with your team **TODAY**

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- With any prediction we can (and should) also report a measure of uncertainty of the prediction.

A *prediction interval* for y for a given $x^\star$ is

$$\hat{y} \pm t^\star_{n-2} s \sqrt{1 + \frac{1}{n} + \frac{(x^\star - \bar{x})^2}{(n-1)s_x^2}}$$

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- Prediction level: If we repeat the study of obtaining a regression data set many times, each time forming a XX% prediction interval at $x^\star$, and wait to see what the future value of y is at $x^\star$, then roughly XX% of the prediction intervals will contain the corresponding actual value of y.

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predict(m_mur_pov, newdata, interval = "prediction", level = 0.95)
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       fit      lwr      upr
1 21.28663 9.418327 33.15493
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We are 95% confident that the annual murders per million for a county with 20% poverty rate is between 9.52 and 33.15.

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- For single predictor regression: R^2 is the square of the correlation coefficient, R.

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murder %>%
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      r_sq
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anova(m_mur_pov)
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Analysis of Variance Table

Response: annual_murders_per_mil
          Df  Sum Sq Mean Sq F value    Pr(>F)
perc_pov   1 1308.34 1308.34  43.064 3.638e-06 ***
Residuals 18  546.86   30.38
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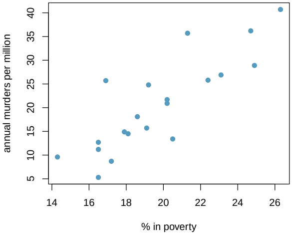
Clicker question

R^2 for the regression model for predicting annual murders per million based on percentage living in poverty is roughly 71%. Which of the following is the correct interpretation of this value?



(a) 71% of the variability in percentage living in poverty is explained by the model.

(b) 84% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

(c) 71% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

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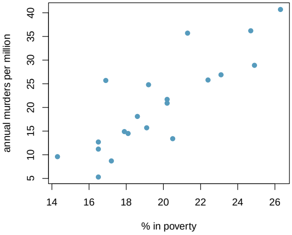

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- Use a T distribution for inference on the slope, with degrees of freedom $n-2$
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  - $T_{n-2} = \frac{b_1 - 0}{SE_{b_1}}$
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  - p-value = P(observing a slope at least as different from 0 as the one observed if in fact there is no relationship between x and y
- Confidence intervals for a slope:
  - $b_1 \pm T^{\star}_{n-2} SE_{b_1}$
  - In R:

```
confint(m_mur_pov, level = 0.95)
```

```
                 2.5 %     97.5 %
(Intercept) -46.265631 -13.536694
perc_pov      1.740003   3.378776
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- Constant variability of residuals (*homoscedasticity*) $\rightarrow$ no fan shape in the residuals plot
- Independence of residuals (and hence observations) $\rightarrow$ depends on data collection method, often violated for time-series data

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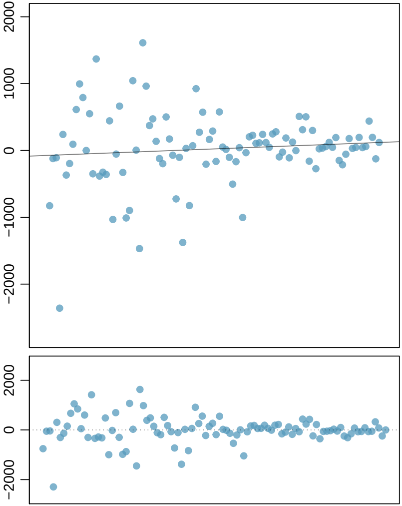
What condition is this linear model obviously and definitely violating?

(a) Linear relationship
(b) Non-normal residuals
(c) Constant variability
(d) Independence of observations



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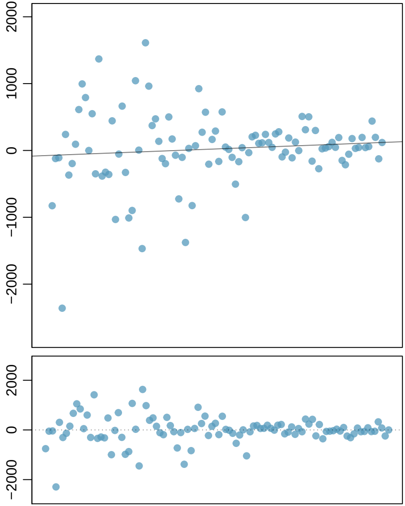

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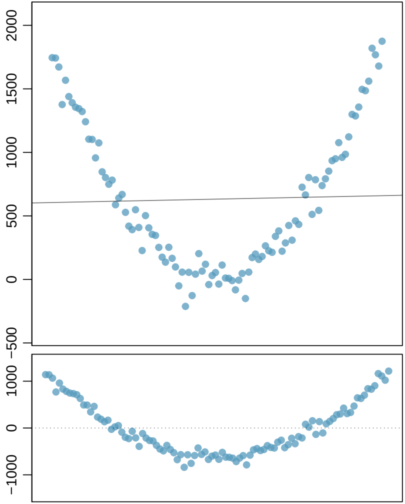

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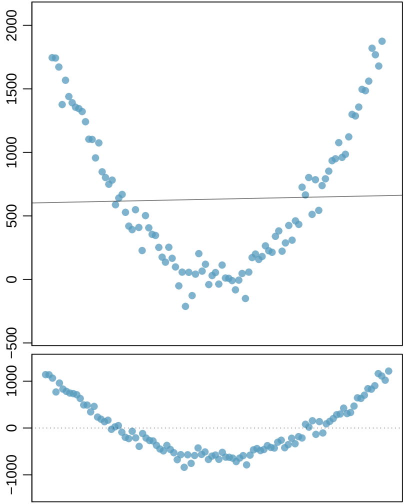
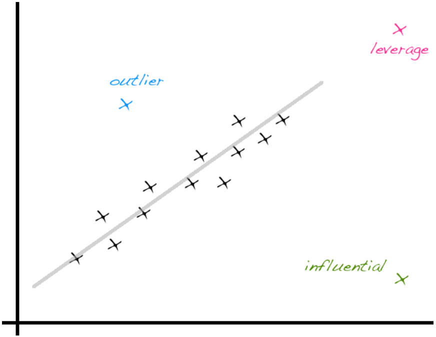


- *Leverage* point is away from the cloud of points horizontally, does not necessarily change the slope
- *Influential* point changes the slope (most likely also has high leverage) – run the regression with and without that point to determine



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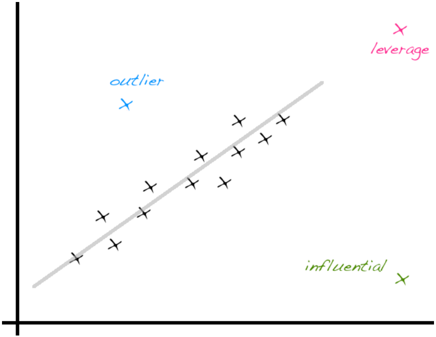

- *Outlier* is an unusual point without these special characteristics (this one likely affects the intercept only)
- If clusters (groups of points) are apparent in the data, it might be worthwhile to model the groups separately.

Application exercise: 6.2 Linear regression

See course website for details

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