Unit 2: Probability and distributions

1. Probability and conditional probability

Sta 101 - Spring 2016

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_s16*

1. Housekeeping

2. Readiness assessment

3. Main ideas
   1. Disjoint and independent do not mean the same thing
   2. Application of the addition rule depends on disjointness of events
   3. Bayes' theorem works for all types of events

4. Summary

- Piazza – enroll and use regularly (post questions, answer others, read answers)
- Get started on PS 2

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- 15 minutes individual – turn your clicker over when you're done
- 10 minutes team – put your team name on the front of the scratch off sheet + Lab time + note if anyone from your team is missing

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1. Disjoint and independent do not mean the same thing

- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A\ and\ B) = 0$

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  - A voter cannot register as a Democrat and a Republican at the same time
  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A\ and\ B) = 0$
- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)
  - If A and B are independent:
    - $P(A \mid B) = P(A)$
    - $P(A\ and\ B) = P(A) \times P(B)$

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- *General addition rule:* P(A or B) = P(A) + P(B) - P(A and B)
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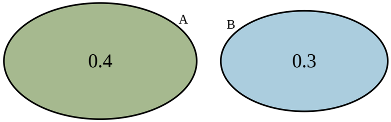
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0 = 0.7



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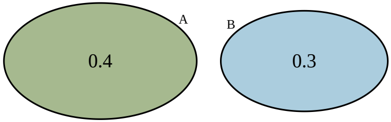
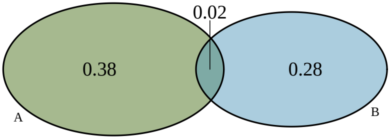


non-disjoint events:
P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0.02 = 0.68



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  = *0* × P(B) = 0

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= *P(A)* $\times$ P(B)

Application exercise: 2.1 Probability and conditional probability

See the course website for instructions.

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