Unit 7: Multiple linear regression

3. Transformations & case study

Sta 101 - Spring 2016

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_s16*

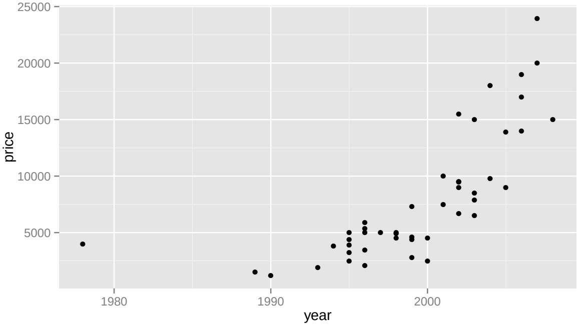
1. Housekeeping

2. Transformations

3. Case study

- Poster session on Thursday – see email for details, come to The Edge Workshop Room during your regular lab time
- I won't hold OH on Thursday (during the poster sessions)

The scatterplot below shows the relationship between year and price of a random sample of 43 pickup trucks. Describe the relationship between these two variables.



From: *http://faculty.chicagobooth.edu/robert.gramacy/teaching.html*

Let's remove trucks older than 20 years, and only focus on trucks made in 1992 or later.

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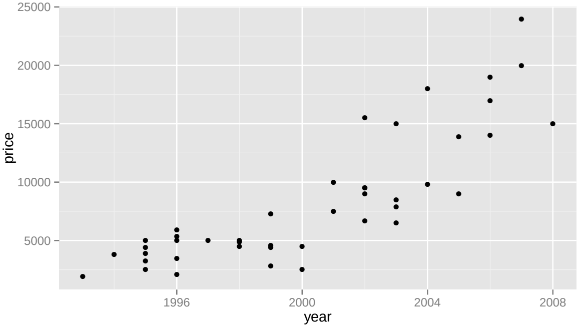
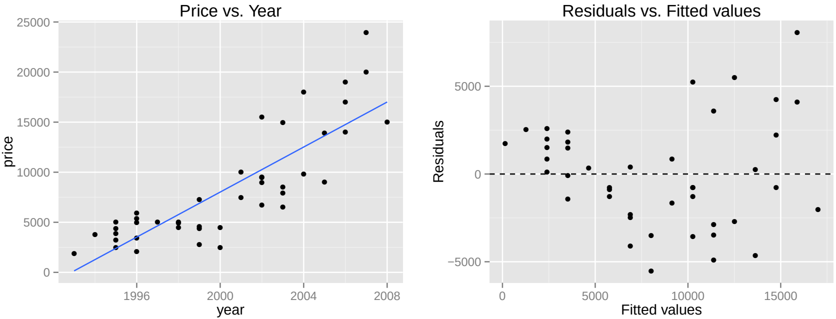
Now what can you say about the relationship?



Model: $\widehat{price} = b_0 + b_1 \ year$



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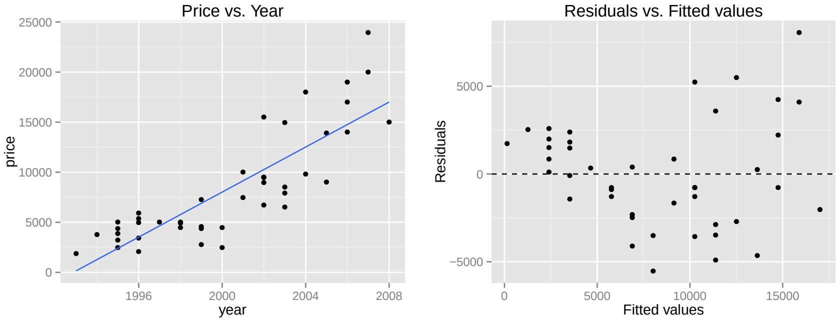
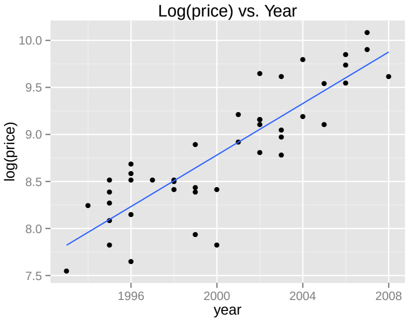
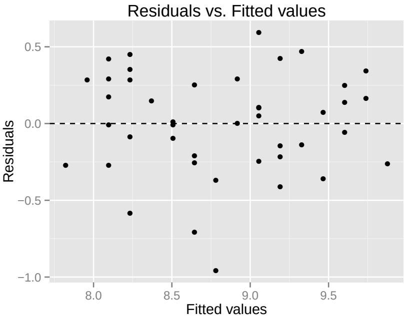


The linear model doesn't appear to be a good fit since the residuals have non-constant variance.

Truck prices - log transform of the response variable

Model: $\widehat{log(price)} = b_0 + b_1 \; year$





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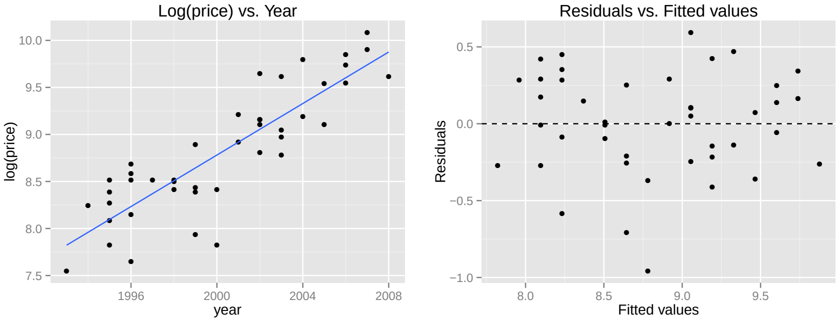

We applied a log transformation to the response variable. The relationship now seems linear, and the residuals no longer have non-constant variance.

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(Intercept)	-265.073	25.042	-10.585	0.000
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- which is not very useful...

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- We can these identities to "undo" the log transformation

The slope coefficient for the log transformed model is 0.137, meaning the log price difference between cars that are one year apart is predicted to be 0.14 log dollars.

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For each additional year the car is newer (for each year decrease in car's age) we would expect the price of the car to increase on average *by a factor of 1.15*.

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- Another useful transformation is the square root: $\sqrt{y}$, especially useful when the response variable is counts.

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- Another useful transformation is the square root: $\sqrt{y}$, especially useful when the response variable is counts.
- These transformations may also be useful when the relationship is non-linear, but in those cases a polynomial regression may also be needed.

1. Housekeeping

2. Transformations

3. Case study

1. `income`: Yearly income (wages and salaries)
2. `employment`: Employment status, not in labor force, unemployed, or employed
3. `hrs_work`: Weekly hours worked
4. `race`: Race, White, Black, Asian, or other
5. `age`: Age
6. `gender`: gender, male or female
7. `citizens`: Whether respondent is a US citizen or not
8. `time_to_work`: Travel time to work
9. `lang`: Language spoken at home, English or other
10. `married`: Whether respondent is married or not
11. `edu`: Education level, hs or lower, college, or grad
12. `disability`: Whether respondent is disabled or not
13. `birth_qrtr`: Quarter in which respondent is born, jan thru mar, apr thru jun, jul thru sep, or oct thru dec

```
acs_emp <- acs %>%
  filter(employment == "employed", income > 0)
```

```
acs_emp %>%
  select(employment) %>%
  table()
```

```r
acs_emp %>%
  select(employment) %>%
  table()
```

```
not in labor force         unemployed           employed
                 0                  0                787
```

```
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  select(employment) %>%
  table()
```

```
not in labor force          unemployed            employed
                 0                   0                 787
```

```
acs_emp <- droplevels(acs_emp) # overwrite acs_emp

acs_emp %>%
  select(employment) %>%
  table()
```

```
employed
     787
```

Suppose we only want to consider the following explanatory variables: `hrs_work`, `race`, `age`, `gender`, `citizen`.

```
m_full = lm(income ~ hrs_work + race + age + gender
    + citizen, data = acs_emp)
```

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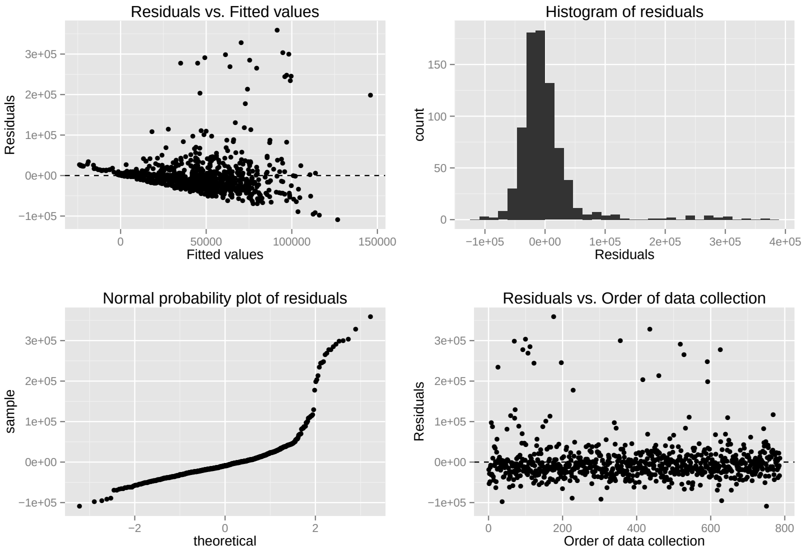
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17215.60	11399.81	-1.51	0.13
hrs_work	1251.31	153.14	8.17	0.00
raceblack	-13202.39	6373.05	-2.07	0.04
raceasian	32699.34	8903.66	3.67	0.00
raceother	-12032.88	7556.78	-1.59	0.11
age	760.99	129.71	5.87	0.00
genderfemale	-17246.91	3887.17	-4.44	0.00
citizenyes	-9537.20	8360.85	-1.14	0.25

What do you think?



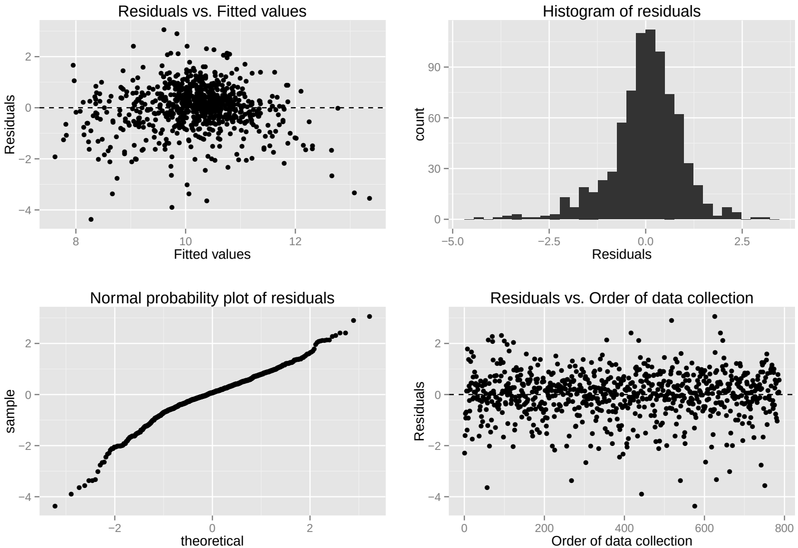
```
# residuals vs. fitted
qplot(data = m_full, y = .resid, x = .fitted, geom = "point") +
  geom_hline(yintercept = 0, linetype = "dashed") +
  xlab("Fitted values") +
  ylab("Residuals") +
  ggtitle("Residuals vs. Fitted values")

# histogram of residuals
qplot(data = m_full, x = .resid, geom = "histogram") +
  xlab("Residuals") +
  ggtitle("Histogram of residuals")

# normal prob plot of residuals
qplot(data = m_full, sample = .resid, stat = "qq") +
  ggtitle("Normal probability plot of residuals")

# order of residuals
qplot(data = m_full, y = .resid) +
  geom_hline(yintercept = 0, linetype = "dashed") +
  ylab("Residuals") +
  xlab("Order of data collection") +
  ggtitle("Residuals vs. Order of data collection")
```

```
m_full_log = lm(log(income) ~ hrs_work + race + age
    + gender + citizen, data = acs_emp)
```



Application exercise: 7.4 Interpreting models with a transformed response

See course website for more details