Unit 8: Final Review

1. Bayesian vs. frequentist inference

Sta 101 - Spring 2016

Duke University, Department of Statistical Science

Dr. Çetinkaya-Rundel

Slides posted at *http://bit.ly/sta101_s16*

1. Housekeeping

2. Bayesian vs. Frequentist Inference
    1. Frequentist inference
    2. Bayesian inference
    3. Comparison

Announcements

- Project questions?

Outline

1. Housekeeping

2. Bayesian vs. Frequentist Inference
    1. Frequentist inference
    2. Bayesian inference
    3. Comparison

- We have a population of M&Ms. The percentage of yellow M&Ms is either 10% or 20%.
- You have been hired as a statistical consultant to decide whether the true percentage of yellow M&Ms is 10%. You are being asked to make a decision, and there are associated payoff/losses that you should consider.

	True state of the population	
Decision	% yellow = 10%	% yellow = 20%
% yellow = 10%	*Your boss gives you a bonus, and I bring you candy on Monday*	*You lose your job, and no candy for you*
%yellow = 20%	*You lose your job, and no candy for you*	*Your boss gives you a bonus, and I bring you candy on Monday*

- I will show you a random sample from the population, but you pay $200 for each M&M, and you must buy in $1000 increments.
- That is, you may buy 5, 10, 15, or 20 M&Ms.

Outline

1. Housekeeping

2. Bayesian vs. Frequentist Inference
    1. Frequentist inference
    2. Bayesian inference
    3. Comparison

- Hypotheses:
  - H_0: 10% yellow M&Ms
  - H_A: more than 10% yellow M&Ms
- Your test statistic is the number of yellow M&Ms you observe in the sample.
- The p-value will be the probability of observing this many or more yellow M&Ms given the null hypothesis is true.

Application exercise: Set up -- data

How many M&Ms would you buy? Decide as a team and vote.

(a) 5 (b) 10 (c) 15 (d) 20

Application exercise: Set up -- significance level

Then, discuss at what significance level you will reject the null hypothesis.

Now we will take a sequence of M&Ms, and you record the number of yellows in the first n draws.

RGYBO *BBGOY* *YRBRR* *GORBY*

Application exercise: Frequentist inference

- Number of yellows in the first n draws = _____ = k
- Calculate the p-value using the Binomial distribution:
  p-value = P(k or more yellows | n, %yellow is 10%) = _____
- Do you reject the null hypothesis? _____

See next slide for hints...

Remember $Binomial(n, k) = \binom{n}{k} p^k (1-p)^{(n-k)}$

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Say you picked $n = 5$, and hence $k = 1$.

$$
\begin{aligned}
& P(1 \text{ } or \text{ } more \text{ } yellows \mid n = 5, \ \%yellow \text{ } is \text{ } 10\%) \\
= \quad & P(K=1) + P(K=2) + P(K=3) + P(K=4) + P(K=5)
\end{aligned}
$$

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= \ & P(K=1) + P(K=2) + P(K=3) + P(K=4) + P(K=5) \\
= \ & \left[\binom{5}{1} 0.1^1 \times 0.9^4\right] + \left[\binom{5}{2} 0.1^2 \times 0.9^3\right] + \cdots + \left[\binom{5}{5} 0.1^5 \times 0.9
\end{aligned}
$$

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\\
= \ & 0.32805 + 0.0729 + 0.0081 + 0.00045 + 0.00001
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\end{aligned}
$$

$$
\begin{aligned}
= \ & 0.32805 + 0.0729 + 0.0081 + 0.00045 + 0.00001 \\
\approx \ & 0.41
\end{aligned}
$$

Alternatively:

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```
# P(K = 1, n = 5, p = 0.1)
dbinom(1, 5, 0.1)
```

```
0.32805
```

... and

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```
# P(K = 1, n = 5, p = 0.1)
dbinom(1, 5, 0.1)
```

```
0.32805
```

... and

```
# P(K >= 1, n = 5, p = 0.1)
sum(dbinom(1:5, 5, 0.1))
```

```
0.40951
```

Now we will start over. Start with 1:1 odds that the percentage of yellows is 10%:20%.

- H_1: 10% yellow M&Ms $\rightarrow$ P(10% yellow) = 0.5
- H_2: 20% yellow M&Ms $\rightarrow$ P(20% yellow) = 0.5

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- H_2: 20% yellow M&Ms $\rightarrow$ P(20% yellow) = 0.5

Application exercise: Bayesian inference

Using the same data and Bayes' theorem to calculate the probability the percentage of yellow is 10% and 20% given the observed data.

See next slide for hints...

P(10% yellow | data):

$$P(10\%yellow|data) \quad = \quad \frac{P(data|10\%yellow) \times P(10\%yellow)}{P(data)}$$

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$$= \frac{P(data|10\%yellow) \times P(10\%yellow)}{P(data|10\%yellow) \times P(10\%yellow) + P(data|20\%yellow) \times P(20\%y}$$

P(10% yellow | data):

$$
\begin{aligned}
P(10\%yellow|data) &= \frac{P(data|10\%yellow) \times P(10\%yellow)}{P(data)} \\
&= \frac{P(data|10\%yellow) \times P(10\%yellow)}{P(data|10\%yellow) \times P(10\%yellow) + P(data|20\%yellow) \times P(20\%y} \\
&= \frac{0.33 \times 0.5}{0.33 \times 0.5 + 0.41 \times 0.5}
\end{aligned}
$$

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$$
\begin{aligned}
P(10\%yellow|data) &= \frac{P(data|10\%yellow) \times P(10\%yellow)}{P(data)} \\
&= \frac{P(data|10\%yellow) \times P(10\%yellow)}{P(data|10\%yellow) \times P(10\%yellow) + P(data|20\%yellow) \times P(20\%y} \\
&= \frac{0.33 \times 0.5}{0.33 \times 0.5 + 0.41 \times 0.5} \\
&= 0.44
\end{aligned}
$$

Application exercise: Bayesian vs. Frequentist inference

Regardless of the choices you made earlier about n, fill out the table below for all possible choices of n and the resulting k.

	Frequentist: p-value		Bayesian: Posterior	
Number of yellow M&Ms in first	P(K $\geq$ k | 10% yellow)	Decision	P(10% yellow | n,k)	P(20% yellow | n,k)
$n = 5 : k = 1$	0.41	Fail to reject H_0	0.44	
$n = 10 : k = 2$				
$n = 15 : k = 3$				
$n = 20 : k = 4$				

- We know that the true % yellow in these data is 20%:

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- However the Frequentist approach (using p-values) would not allow us to reject the null hypothesis of 10% yellow.
- On the other hand, the Bayesian approach yields a higher posterior probability for 20% yellow.