Unit 4: Inference for numerical data

3. Power

Sta 101 - Spring 2016

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_s16*

Outline

1. Housekeeping

2. Main ideas
    1. Not every statistically significant result is practically significant
    2. Hypothesis tests have error rates associated with them
    3. Type 1 error rate = significance level
    4. Calculating the power is a two step process
    5. Power goes up with effect size and sample size, and is inversely proportional with significance level and standard error
    6. A priori power calculations determine desired sample size

3. Summary

Announcements

- Watch ANOVA videos before next class
- Midterm course feedback:
  - 58% think pace is about right, nobody thinks it's too slow :)
  - 42% learn most from videos, 25% from problem sets, 15% from application exercises, 10% from reading the textbook
  - Clickers are a hit, TAs have been helpful, amount of work put into class doesn't vary much by previous background
  - Returning PS before midterm – huge ask from TAs, but I will solve any questions you ask about in office hours / midterm review / Piazza
  - Grading of PS – a randomly assigned TA grades it each time, for consistency throughout the semester

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Reminder: Not every statistically significant result is practically significant

- Real differences between the point estimate and null value are easier to detect with larger samples
- However, very large samples will result in statistical significance even for tiny differences between the sample mean and the null value (*effect size*), even when the difference is not practically significant
- This is especially important to research: if we conduct a study, we want to focus on finding meaningful results (we want observed differences to be real but also large enough to matter).
- The role of a statistician is not just in the analysis of data but also in planning and design of a study.

"To call in the statistician after the experiment is done may be no more than asking him to perform a post-mortem examination: he may be able to say what the experiment died of." – R.A. Fisher

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	H_A true		✓

- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true.

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- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true.
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true.
- We (almost) never know if H_0 or H_A is true, but we need to consider all possibilities.

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- This is why we prefer small values of α – increasing α increases the Type 1 error rate.

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	H_A true	*Type 2 Error, β*	

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- Type 2 error is failing to reject H_0 when you should have, and the probability of doing so is β (a little more complicated to calculate)

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- *Power* of a test is the probability of correctly rejecting H_0, and the probability of doing so is $1-\beta$
- In hypothesis testing, we want to keep α and β low, but there are inherent trade-offs.

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- The answer is not obvious, but
  - If the true population average is very close to the null hypothesis value, it will be difficult to detect a difference (and reject H_0).
  - If the true population average is very different from the null hypothesis value, it will be easier to detect a difference.

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  - If the true population average is very different from the null hypothesis value, it will be easier to detect a difference.
- Therefore, β must depend on the *effect size* (δ) in some way

To increase power / decrease β: increase n, increase δ, or increase α

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3. Summary

A medical research group is recruiting people to complete short surveys about their medical history. For example, one survey asks for information on a person's family history in regards to cancer. Another survey asks about what topics were discussed during the person's last visit to a hospital. So far, on average people complete an average of 4 surveys, with the standard deviation of 2.2 surveys. The research group wants to try a new interface that they think will encourage new enrollees to complete more surveys, where they will randomize a total of 300 enrollees to either get the new interface or the current interface (equally distributed between the two groups). What is the power of the test that can detect an increase of 0.5 surveys per enrollee for the new interface compared to the old interface? Assume that the new interface does not affect the standard deviation of completed surveys, and $\alpha = 0.05$.

The preceeding question can be rephrased as – How likely is it that we can reject a null hypothesis of $H_0 : \mu_{new} - \mu_{current} = 0$ if the new interface results in an increase of 0.5 surveys per enrollee, on average?

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Let's break this down intro two simpler problems:

1. Problem 1: Which values of $(\bar{x}_{new} - \bar{x}_{current})$ represent sufficient evidence to reject this H_0?
2. Problem 2: What is the probability that we would reject this H_0 if $\bar{x}_{new} - \bar{x}_{current}$ had come from a distribution with $\mu_{new} - \mu_{current} = 0.5$, i.e. what is the probability that we can obtain such an observed difference from this distribution?

Which values of ($\bar{x}_{new\ interface} - \bar{x}_{old\ interface}$) represent sufficient evidence to reject H_0?

$$H_0 : \mu_{new} - \mu_{current} = 0$$
$$H_A : \mu_{new} - \mu_{current} > 0$$
$$n_{new} = n_{current} = 150$$

Clicker question

What is the lowest t-score that will allow us to reject the null hypothesis in favor of the alternative?

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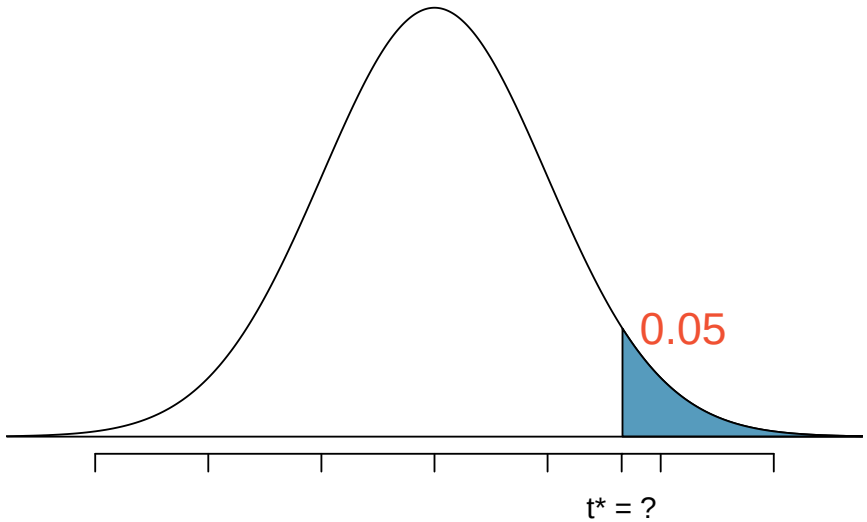
(a) 1.65
(b) 1.66
(c) 1.96
(d) 1.98
(e) 2.63



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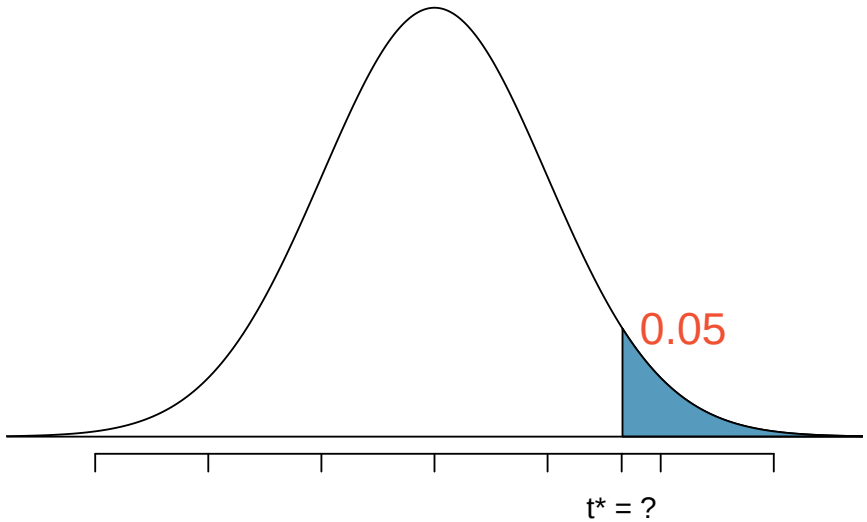

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(a) $\bar{x}_{new} - \bar{x}_{current} < -0.42$

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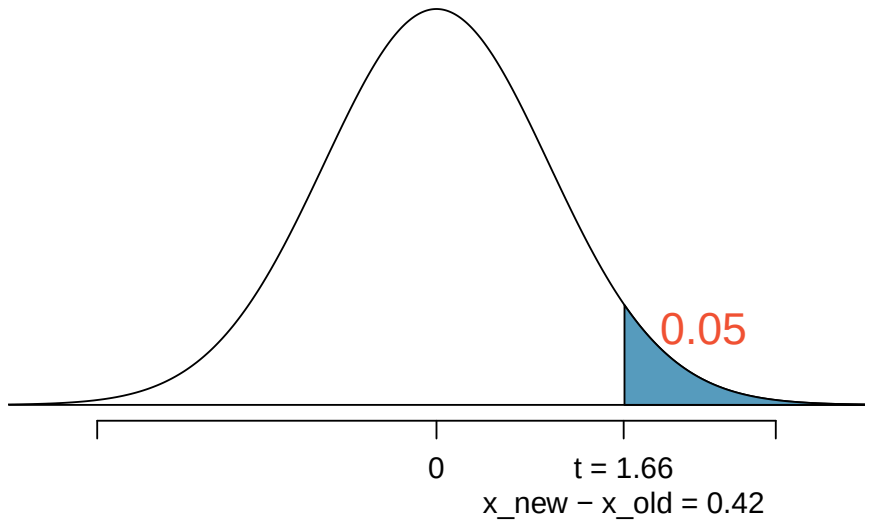

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What is the probability that we would reject this H_0 if $\bar{x}_{new} - \bar{x}_{current}$ had come from a distribution with $\mu_{new} - \mu_{current} = 0.5$, i.e. what is the probability that we can obtain such an observed difference from this distribution?

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(d) 80%

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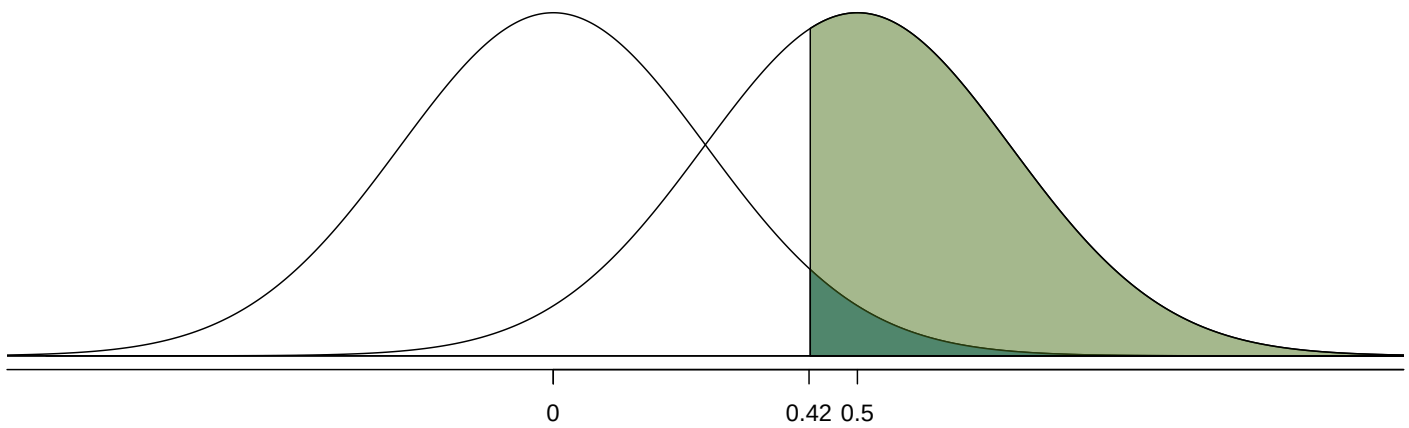

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What is β, the Type 2 error rate?

(a) 5%

(b) 38%

(c) 62%

(d) 80%

(e) 95%

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(c) 62%

(d) 80%

(e) 95%

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3. Increase α, which will make it more likely to reject H_0 (but note that this has the side effect of increasing the Type 1 error rate).
4. Consider a larger effect size. If the true mean of the population is in the alternative hypothesis but close to the null value, it will be harder to detect a difference.

- *Step 0:* Pick a meaningful effect size δ and a significance level α
- *Step 1:* Calculate the range of values for the point estimate beyond which you would reject H_0 at the chosen α level.
- *Step 2:* Calculate the probability of observing a value from preceding step if the sample was derived from a population where $\mu = \mu_{H_0} + \delta$

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How large a sample size would you need if you wanted 90% power to detect a 0.5 increase in average number of surveys taken at the 5% significance level?

$H_0 : \mu_{new} - \mu_{current} = 0$, $H_A : \mu_{new} - \mu_{current} > 0$

$n_{new} = n_{current} = ?$, $s_{new} = 2.2 = s_{current} = 2.2$

$\delta = 0.5$, $\alpha = 0.05$, power = 90%, $\beta = 0.10$

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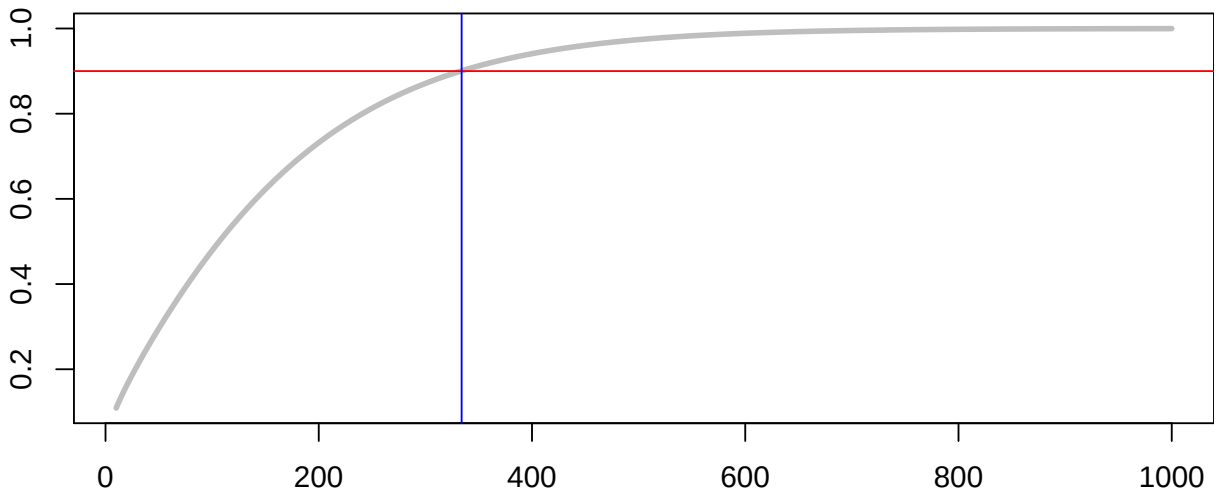

When $n > 334$, power is at least 90%.

```
s = 2.2
mu = 0
delta = 0.5

ns = 10:1000
power = rep(NA, length(ns))

for(i in 10:1000){
  n = i
  t_star = qt(0.95, df = n-1)
  se = sqrt((s^2 / n) + (s^2 / n))
  cutoff = t_star * se
  t_cutoff = (cutoff - (mu+delta)) / se
  power[i-9] = pt(t_cutoff, df = n-1, lower.tail = FALSE)
}

which_n = which.min(abs(power - 0.9))
power[which_n]
power[which_n + 1]
ns[which_n + 1]
```

Application exercise: 4.3

See course website for details.

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