

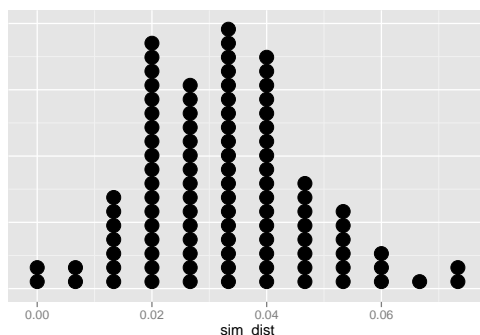
## Sample MT2

1. Diamond clarity is a quality of diamonds relating to the existence and visual appearance of internal characteristics of a diamond called inclusions, and surface defects called blemishes. Below is a table of the currently used clarity grading scales for diamonds.<sup>1</sup>

Category	Flawless	Internally Flawless	Very Very Slightly Included		Very Slightly Included		Slightly Included		Included		
Grade	FL	IF	VVS <sub>1</sub>	VVS <sub>2</sub>	VS <sub>1</sub>	VS <sub>2</sub>	SI <sub>1</sub>	SI <sub>2</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>

Flawless diamonds are very rare, and so are internally flawless diamonds. In a random sample of 150 diamonds, 5 diamonds are internally flawless (IF).

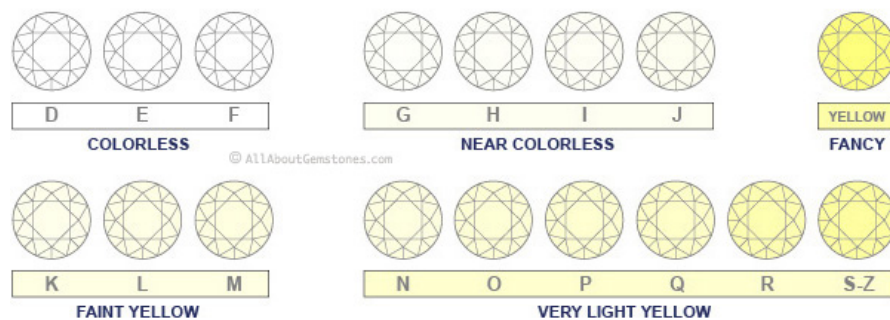
- (a) What methods (theoretical, simulation, or both) can we use to construct a confidence interval for the proportion of diamonds that are internally flawless. Explain your reasoning.
- (b) Regardless of your answer to part (a), we will construct a confidence interval using a simulation – bootstrapping. Explain, in your own words, how we can construct a bootstrapping distribution of 100 bootstrap statistics.
- (c) Below is a bootstrap distribution of 100 bootstrap statistics. What does each dot on this distribution represent?



<sup>1</sup>[http://en.wikipedia.org/wiki/Diamond\\_clarity](http://en.wikipedia.org/wiki/Diamond_clarity)

- (d) The mean of the above distribution is 0.033 and the standard deviation is 0.015. Calculate the 90% bootstrap confidence interval using the percentile method as well as the standard error method.
- (e) Interpret the above confidence interval in context of the question.

2. Another criterion that determines the quality of the diamond (and hence its price) is the color. Below is a table of the currently used clarity grading scales for diamonds.<sup>2</sup>



The table below shows the distribution of clarity and color in the same random sample of 150 diamonds. Clarity is divided into two levels: **VVS+** which represented very very slightly included or higher, and **VS-** very slightly included or lower.

	colorless	near colorless
VVS+	15	15
VS-	64	56

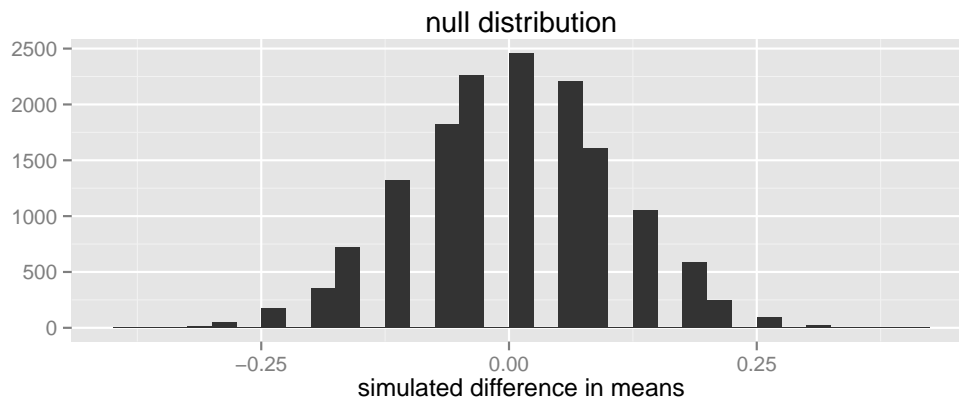
- What percent of VVS+ diamonds are colorless?
- What percent of VS- diamonds are colorless?
- Write the hypotheses (in words and using notation) for testing if the proportion of colorless diamonds is different between the VVS+ and VS- diamonds.
- What is the parameter of interest? (in words and using notation)
- What is the point estimate? (in words and using notation) Calculate this value.

<sup>2</sup>[http://www.allaboutgemstones.com/diamonds\\_4cs\\_color.html](http://www.allaboutgemstones.com/diamonds_4cs_color.html)

- (f) Describe, in your own words, how you would conduct a randomization test using index cards.

- (g) Below is the histogram of a randomization distribution resulting from such randomization test using 15,000 simulations. Which of the following best describes the p-value? Circle one.

very small (less than 0.001)      small (bet. 0.001 - 0.05)      large (bet. 0.05 - 0.50)      very large (more than 0.50)



- (h) What is the conclusion of the hypothesis test?
- (i) Would you expect a confidence interval for the difference between the proportions of colorless VVS+ and VS- diamonds to include 0? Explain your reasoning. (Do not calculate a confidence interval, use the result of your hypothesis to answer this question.)

3. On March 23, 2012 SurveyUSA conducted a poll in Florida on the shooting of Trayvon Martin. The table below shows the distribution of political ideology of respondents and the degree to which they think the victim's race was a factor in this shooting.

	conservative	moderate	liberal	total
not a factor	64	43	17	124
small factor	54	79	13	146
major factor	80	179	99	358
not sure	40	44	26	110
total	238	345	155	738

- (a) What are the cases in this survey, and how many cases are there?
- (b) What are the variables in this study? Identify each variable as categorical or numerical.
- (c) Name one inference method that is appropriate for examining the relationship between the variables in this study? Be specific.
- (d) Write the hypotheses for testing for a relationship between these two variables. You can avoid notation and simply write the hypotheses in words.
- (e) If the variables in the study are not related, how many liberal respondents would we expect to have responded “not a factor”?

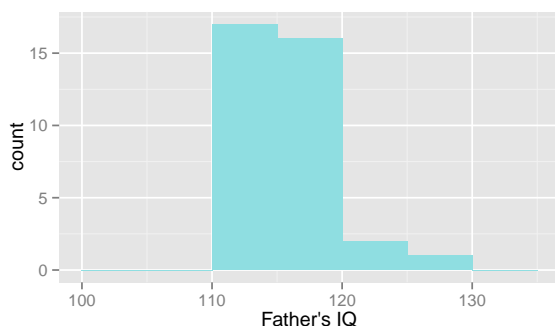
(f) The test statistic is calculated as 55.55. What is the p-value? Make sure to show all your work.

(g) What is the conclusion of the hypothesis test at the 5% significance level? Interpret your conclusion in the context of this question.

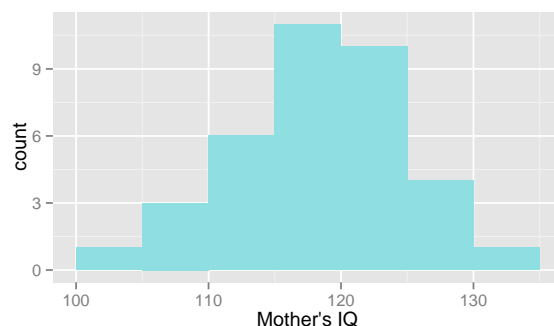
4. The 2010 General Social Survey asked the question; “Do you think the use of marijuana should be made legal, or not?” 48% of the 1,259 respondents said it should be made legal.
- (a) Is the number “48%” a sample statistic or a population parameter? Explain.
- (b) Construct a 95% confidence interval for the proportion of Americans who think marijuana should be made legal.
- (c) Interpret this confidence interval in the context of this question.
- (d) A critic points out that this 95% confidence interval is only accurate if the statistic follows a normal distribution, or if the normal model is a good approximation. Is this true for these data? Explain.
- (e) A news piece on this study’s findings states; “Majority of Americans think marijuana should be legalized.” Based on your confidence interval, is this news piece’s statement justified?



5. The following histogram shows the distribution of IQ scores of fathers and mothers of a random sample of 36 students who were identified as “gifted” soon after they turned four. Relevant summary statistics for both distributions are also provided.<sup>3</sup>

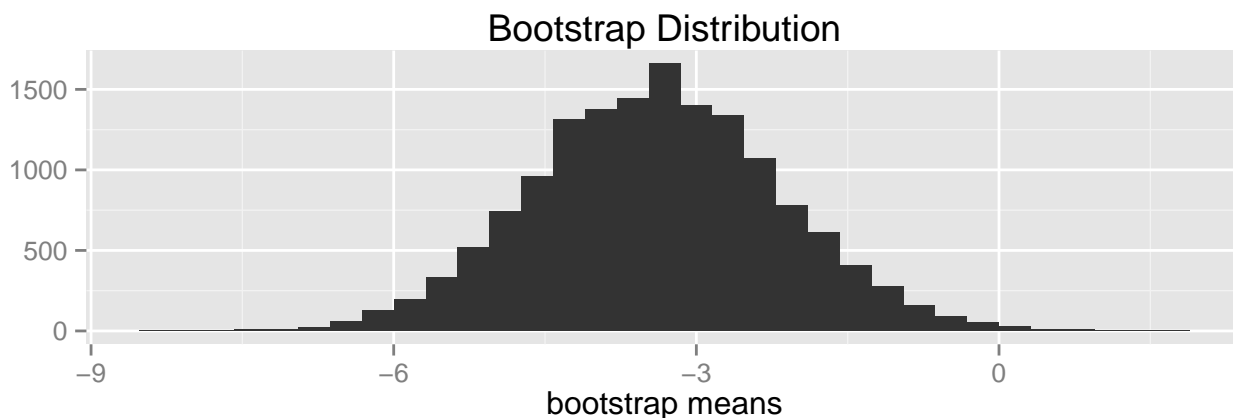


Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
110.0	112.0	115.0	114.8	116.2	126.0



Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
101.0	113.8	118.0	118.2	122.2	131.0

- (a) Given below is a dot plot of the bootstrap distribution of means of 15,000 bootstrap samples taken from the original sample of differences between the IQ scores of father and mother of a child (father's IQ score - mother's IQ score).



Also provided are the cutoffs for certain percentiles of this bootstrap distribution.

1%	2.5%	5%	10%	20%	80%	90%	95%	97.5%	99%
-6.17	-5.75	-5.39	-4.94	-4.42	-2.39	-1.83	-1.36	-1.00	-0.47

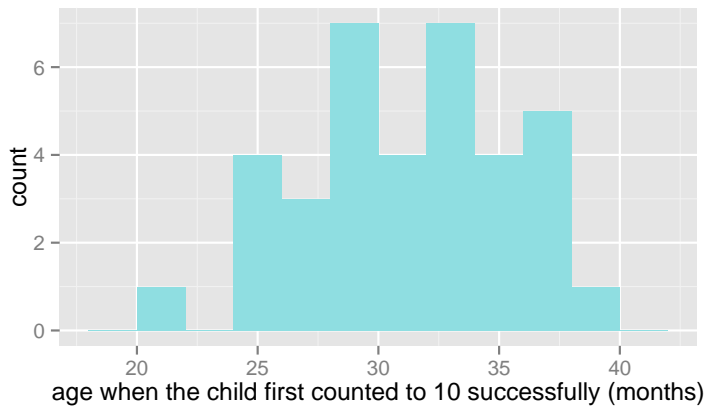
Based on this distribution, estimate a 95% confidence interval for the true average difference between the IQ scores of fathers and mothers of gifted children.

- (b) Interpret this interval in context of this question.

<sup>3</sup>Graybill, F.A. & Iyer, H.K., (1994) Regression Analysis: Concepts and Applications, Duxbury, p. 511-6.

- (c) You overhear two researchers talking about this study and one of them says; “This confidence interval shows that mothers of gifted children on average have significantly higher IQ scores than their fathers.” Is this statement justified? Explain your reasoning.
- (d) A hypothesis test for testing for the difference between the mean IQ scores of fathers and mothers of gifted children yields a p-value of 0.0099. Interpret the meaning of this probability in context of this research question.

The following histogram shows the distribution of the ages (in months) at which a random sample of 36 children first counted to 10 successfully. These children were identified as “gifted” soon after they turned four.<sup>4</sup>



Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
21.00	28.00	31.00	30.69	34.25	39.00

6. Which of the following is the correct set of hypotheses for testing if the average age at which gifted children first count to 10 successfully is less than 32 months?
  - (a)  $H_0 : \mu = 32; H_A : \mu < 32$
  - (b)  $H_0 : \mu = 32; H_A : \bar{x} < 30.69$
  - (c)  $H_0 : p = 32; H_A : p < 32$
  - (d)  $H_0 : \bar{x} = 32; H_A : \bar{x} < 32$
7. Which of the following is an appropriate method for these data?
  - (a)  $\chi^2$  test of independence
  - (b)  $\chi^2$  test of goodness of fit
  - (c) ANOVA
  - (d) T-test
8. The test statistic is calculated as -1.82. Which of the below ranges contain the p-value?
  - (a) Less than 0.005
  - (b) Between 0.025 and 0.05
  - (c) Between 0.05 and 0.1
  - (d) Greater than 0.1
9. Which of the below is the best interpretation of the p-value for this hypothesis test?
  - (a) Probability that gifted children successfully count to 10 at the average age of 32 months.
  - (b) Probability that gifted children successfully count to 10 at the average age of less than 32 months.
  - (c) Probability of getting a random sample of 36 gifted children where the average age at which they count to 10 successfully is 30.69 or less, if in fact the true mean is 32 months
  - (d) Probability of getting a random sample of 36 gifted children where the average age at which they count to 10 successfully is 30.69 or less, if in fact the true mean is less than 32 months

<sup>4</sup>Graybill, F.A. & Iyer, H.K., (1994) Regression Analysis: Concepts and Applications, Duxbury, p. 511-6.

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The 2010 General Social Survey asked the question “After an average work day, about how many hours do you have to relax or pursue activities that you enjoy?” to a random sample of 1,155 Americans. A 95% confidence interval for the mean number of hours spent relaxing or pursuing activities they enjoy was

(1.38, 1.92)

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10. Which of the following is a valid interpretation of this interval?
- (a) 95% of all Americans spend between 1.38 to 1.92 hrs per day relaxing or pursuing activities they enjoy.
  - (b) If a new survey with the same sample size were to be taken, there is a 95% chance that the mean number of hours spent relaxing or pursuing activities enjoyed in the sample would be between 1.38 and 1.92.
  - (c) We are 95% confident that, were we to repeat this survey, the mean number of hours spent relaxing or pursuing activities they enjoy would be between 1.38 and 1.92.
  - (d) We are 95% confident that Americans spend an average of 1.38 to 1.92 hours per day relaxing or pursuing activities they enjoy.
11. If the researchers who conducted this survey wanted to report a confidence interval with a larger margin of error based on the same sample of 1,155 Americans, what would change?
- (a) the confidence level would go down
  - (b) the confidence level would go up
  - (c) the confidence level would stay the same
12. If a new survey were to be done with 2,500 Americans, which of the following would be true?
- (a) margin of error would be smaller
  - (b) margin of error would be larger
  - (c) margin of error would be about the same

13. Does Weight Watchers work? Researchers randomly divided 500 people into two equal-sized groups. One group spent 6 months on the Weight Watchers program. The other group received a pamphlet about controlling portion sizes. At the beginning of the study, the average difference in weights between the two groups was approximately 0. After the study, the average difference was about 8 pounds. The Weight Watchers group had the lower average weight. To test whether an average difference of 8 pounds could be due to chance, a statistician writes everyone's end-of-diet weight on an index card. He shuffles these cards together, and then deals them into two equal-sized groups. Which of the following best describes the expected result?
- (a) The average difference between the two stacks of cards will be about 8 pounds.
  - (b) The average difference between the two stacks of cards will be about 0 pounds.
  - (c) If Weight Watchers was effective, the average difference between the two stacks of cards will be more than 8 pounds.
14. Answer the following true / false questions. Each question is worth 1 point.
- (a) ( T / F ) With large sample sizes even small differences between the null value and the point estimate, also called the effect size, can be statistically significant.
  - (b) ( T / F ) If you found  $\chi^2 = 10$  and  $df=5$  you would fail to reject  $H_0$  at 5% significance level.
  - (c) ( T / F ) A cutoff of  $\alpha = 0.05$  is the ideal value for all hypothesis tests.
  - (d) ( T / F ) We should be concerned about the independence of observations in a sample if we sample more than 10% of the population without replacement.
  - (e) ( T / F ) If the p-value is sufficiently large you can reject  $H_A$ .
  - (f) ( T / F ) Power of a test and the probability of making a Type 1 error are complements.
  - (g) ( T / F ) The equivalent confidence level for a two-sided hypothesis test with  $\alpha = 0.05$  is 95%.