Finance 5330 - Financial Econometrics

Prices, Returns and Price Discovery

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Introduction to Financial Data: Price and Returns

Price Disovery in Financial Markets

Asset Returns

One-Period Simple Return

- Hold asset from period $t-1$ to t
- Simple Gross Return:

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

or

$$P_t = P_{t-1}(1 + R_t)$$

Multiperiod Simple Return

- Holding the asset k period between dates $t-k$ and t gives a k-period gross return:

$$\begin{aligned}1+R_t[k] &= \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}}\frac{P_{t-1}}{P_{t-2}}\cdots\frac{P_{t-k+1}}{P_{t-k}}\\ &= (1+R_t)(1+R_{t-1})\cdots(1+R_{t-k+1})\\ &= \prod_{j=0}^{k-1}(1+R_{t-j})\end{aligned}$$

- ***NB:*** the k-period simple gross return is the product of the k one-period simple gross returns. This is called a compound return.
- The k-period simple net return is: $R_t[k] = \frac{P_t - P_{t-k}}{P_{t-k}}$

Annualized Returns

- To facilitate comparison we usually standardize to a given duration (annual, monthly, quarterly, etc)
- The annual return is

$$\text{Annualized}\{R_t[k]\} = \left[\prod_{j=0}^{k-1}(1 + R_{t-j})\right]^{1/k} - 1$$

- ***NB:*** this is a geometric mean of the k one-period simple gross returns
- It can also be computed as

$$\text{Annualized}\{R_t[k]\} = \exp\left[\frac{1}{k}\sum_{j=0}^{k-1}\ln\left(1 + R_{t-j}\right)\right] - 1$$

Continuous Compounding

For continuous compounding we can use the following formula:

$$A = C \exp\left(r \times n\right)$$

where

- $A =$ net asset value
- $C =$ initial capital
- $r =$ interest rate per annum
- $n =$ number of years

With this in mind, we can also define the ***present value*** relation:

$$C = A \exp\left(-r \times n\right)$$

Continuously Compounded Return

- The natural log of the simple gross return

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = \ln P_t - \ln P_{t-1}$$

- Advantages
    - Multiperiod returns

$$\begin{aligned} r_t[k] &= \ln(1 + R_t[k]) \\ &= \ln[(1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1} \end{aligned}$$

Continously Compounded Return (Continued)

- Symmetric up/down moves
- Set up some imaginary prices

```
p1 = 100.0
p2 = 105.0
p3 = 100.0
```

Continously Compounded Return (Continued)

- The simple net return

```
R2 = p2/p1 - 1.0
print(R2)
```

```
## 0.050000000000000044
```

```
R3 = p3/p2 - 1.0
print(R3)
```

```
## -0.04761904761904767
```

Continously Compounded Return (Continued)

- Continously compounded prices

```
import numpy as np
r2 = np.log(p2) - np.log(p1)
print(r2)
```

```
## 0.04879016416943127
```

```
r3 = np.log(p3) - np.log(p2)
print(r3)
```

```
## -0.04879016416943127
```

Portfolio Returns

$$R_{p,t} = \sum_{i=1}^{N} \omega_i R_{i,t}$$

where

- $p =$ index representing the portfolio
- $N =$ number of assets in the portfolio
- $R_{i,t} =$ period t simple net return on aset i
- $\omega_i =$ weight on asset i

NB: continously compounded portfolio returns do not have this convenient property, but for simple returns $R_{i,t}$ small in magnitude, we have

$$r_{p,t} \approx \sum_{i=1}^{N} \omega_i r_{i,t}$$

Accounting for Dividend Payments

- We often have to account for dividends

$$R_t = \frac{(P_t + D_t)}{P_{t-1}} - 1$$

$$r_t = \ln\left(P_t + D_t\right) - \ln P_{t-1}$$

Excess Return

$$Z_t = R_t - R_{0,t}$$

where $R_{0,t}$ is a reference asset (e.g. the risk-free rate)

$$z_t = r_t - r_{0,t}$$

NB: Excess return can be thought of as the payoff to an arbitrage strategy that goes long the asset and short the reference rate with no initial investment.

Summary of Relationships

- $r_t = \ln(1 + R_t) \implies R_t = \exp(r_t) - 1$
- If R_t and r_t are in percentages then
  - $r_t = 100 \ln\left(1 + \frac{R_t}{100}\right)$
  - $R_t = 100\left(e^{r_t/100} - 1\right)$
- Temporal aggregation produces
  - $1 + R_t[k] = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})$
  - $r_t[k] = r_t + r_{t-1} + \cdots + r_{t-k+1}$
- Present and future values:
  - $A = C \exp(r \times n)$
  - $C = A \exp(-r \times n)$

The Role of Prices

Customer: How much are these?
Merchant: A buck fifty.
Customer: I'll take some.
Merchant: They're a buck fifty-one.
Customer: Um, you said a buck fifty.
Merchant: That was before I knew you wanted some.
Customer: You can't do that.
Merchant: It's my shop.
Customer: But I need to buy a hundred!
Merchant: A hundred? Then it's a buck fifty-two.
Customer: You're ripping me off.
Merchant: Supply and demand, pal. You want 'em or not?

The Price Mechanism

The ***Price Mechansim:*** Used with reference to the free market system and the way in which prices act as automatic signals which coordinate the actions of individual decision making units. By means of this role, the price system provides a mechanism whereby changes in demand and supply conditions can affect the *allocative efficiency* of resources.

Price Disovery

Price Discovery is the process whereby information is incorporated into prices through the trading process via the price mechanism. Buyers seek the lowest possible prices whilst sellers seek the highest possible prices. Both are constrained by supply and demand conditions prevalent in the market. Traders seek to gain advantage from their information and thereby cause prices to become informative through their trading.

See the following defition at the CFTC.