Chapter2

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1 Chapter 2: Linear Time Series Analysis and Its Applications

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1.1 Introduction

These notes are based on Chapter 2 of the book *Analysis of Financial Time Series 3rd Ed* by Ruey Tsay.

Understanding the simple time series models introduced here will go a long way to better appreciate the more sophisticated financial econometric models of later chapters.

Treating an asset return (e.g. log return r_t of a stock) as a collection of random variables over time., we have a time series $\{r_t\}$. The Linear time series models of this chapter are a natural first attemp at modeling such dynamic behavior.

The theories of linear time series discussed include:

- stationarity
- dynamic dependence
- autocorrelation function
- modeling
- forecasting

The econometric models introduced include:

- (a) simple autoregressive (AR) models
- (b) simple moving-average (MA) models
- (c) mixed autoregressive moving-average (ARMA) models
- (d) unit-root nonstationarity
- (e) regression models with times series errors
- (f) fractionally differenced models for long-range dependence

1.2 Section 2.1 Stationarity

The foundation of time series analysis is stationarity. A time series $\{r_t\}$ is said to be *strictly stationary* if the joint distribution of $(r_{t_1+t}, \ldots, r_{t_k+t})$ for all t, where k is an arbitrary positive integer and $(t_1, \ldots, t_k)$ is a collection of k positive integers.

Strict stationarity requires that the joint distribution of $(r_{t_1+t}, \ldots, r_{t_k+t})$ is invariant under time shift. This is a very strong requirement that is challenging to verify empirically. For this reason, we often employ a simpler form of stationarity.

A time series is $\{r_t\}$ *weakly stationary* if both the mean of r_t and the covariance between r_t and r_{t-l} are time invariant, where l is an arbitrary integer.

More specifically, $\{r_t\}$ is weakly stationary if:

- (a) $E(r_t) = \mu$, which is constant
- (b) $Cov(r_t, r_{t-l}) = \gamma_l$, which only depends on l

In practice, suppose that we have observed T data points $\{r_t | 1, \ldots, T\}$. Weak stationarity implies that a time plot of the data would show that the T values fluctuate with constant variation around a fixed level. In application, weak stationarity enables one to make inference concerning future observations (e.g. prediction).

Implicitly, in the condition of weak stationarity, we assume that the first two moments of r_t are finite. From the definitions, if r_t is strictly stationary and its first two moments are finite, then r_t is also weakly stationary. The converse is not true in general.

If the time series r_t is normally distributed, then weak stationarity is equivalent to strict stationarity.

We will be mainly concerned with weakly stationary time series.

The covariance $\gamma_l = Cov(r_t, r_{t-1})$ is called the lag-l autocovariance of r_t. It has two important properties:

- (a) $\gamma_0 = Var(r_t)$
- (b) $\gamma_{-l} = \gamma_l$

The second property holds because $Cov(r_t, r_{t-(-l)}) = Cov(r_{t-(-l)}, r_t) = Cov(r_{t+l}, r_t) = Cov(r_{t_1}, r_{t_1-l})$, where $t_1 = t + l$.

In the finance literature, is common to assume that an asset return series is weakly stationary. We can check this empirically given a sufficient number of historical returns observations. In particular, we can divide the historical returns into subsamples and check the consistency of the results obtained across subsamples.

1.3 Section 2.2 Correlation and Autocorrelation Function

Recall that the correlation between two random variables X and Y can be defined as:

$$\rho_{x,y} = \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}} = \frac{E[(X - \mu_x)(Y - \mu_y)]}{\sqrt{E[(X - \mu_x)^2]E[(Y - \mu_y)^2]}}$$

This coefficient measures the strength between X and Y, and can be shown that $-1 \leq \rho_{x,y} \leq +1$, and that $\rho_{x,y} = \rho y, x$. The two random variables are uncorrelated if $\rho_{x,y} = 0$. In addition, if both X and Y are normally distributed random variables then the condition that $\rho_{x,y} = 0$ also indicates that they are independent.

When the sample $\{(x_t, y_t)\}_{t=1}^{T}$ then the population parameter can be estimated by its sample counterpart:

$$\hat{\rho}_{x,y} = \frac{\sum_{t=1}^{T}(x_t - \tilde{x})(y_t - \tilde{y})}{\sqrt{\sum_{t=1}^{T}(x_t - \tilde{x})^2)\sum_{t=1}^{T}(y_t - \tilde{y})^2}}$$

where $\tilde{x} = \frac{1}{T}\sum_{t=1}^{T} x_t$ and $\tilde{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$ are the sample mean of X and Y, respectively.

Simulating Correlated Data We can simulate correlated data with the following algorithm:

1. Draw $z_1 \sim N(0,1)$
2. Draw $z_2 \sim N(0,1)$
3. Set $\epsilon_1 = z_1$
4. Set $\epsilon_2 = \rho z_1 + \sqrt{1-\rho^2} z_2$, where ρ is value of the correlation coefficient desired.

We can do this in Python as follows:

```
In [4]: import numpy as np

        M = 10000
        z1 = np.random.normal(size=M)
        z2 = np.random.normal(size=M)
        rho = 0.5
        e1 = z1
        e2 = rho * z1 + np.sqrt(1 - rho**2) * z2

        np.corrcoef(e1,e2)

Out[4]: array([[1.        , 0.50372534],
               [0.50372534, 1.        ]])
```

1.4 Section 2.3 White Noise and Linear Time Series

1.5 Section 2.4 Simple AR Models

Properties of AR Models

1.5.1 Identifying AR Models in Practice

1.5.2 Goodness of Fit

1.5.3 Forecasting

1.6 Section 2.5 Simple MA Models

1.6.1 Properties of MA Models

1.6.2 Identifying MA Order

1.6.3 Estimation

1.6.4 Forecasting Using MA Models

1.7 Section 2.6 Simple ARMA Models

1.7.1 Properties of ARMA(1,1) Models

1.7.2 General ARMA Models

1.7.3 Identifying ARMA Models

1.7.4 Forecasting Using an ARMA Model

1.7.5 Three Model Representations for an ARMA Model

1.8 Section 2.7 Unit-Root Nonstationarity

1.8.1 Random Walk

1.8.2 Random Walk with Drift

1.8.3 Trend-Stationary Time Series

1.8.4 General Unit-Root Nonstationary Models

1.8.5 Unit-Root Test

1.9 Section 2.8 Seasonal Models

1.9.1 Seasonal Differencing

1.9.2 Multiplicative Seasonal Models

1.10 Section 2.9 Regression Models with Time Series Errors

1.11 Section 2.10 Consistent Covariance Matrix Estimation

1.12 Section 2.11 Long-Memory Models

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