Finance 5330 - Financial Econometrics

Introduction to Time Series Analysis

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The Tradition Use of Time Series Models Was for Forecasting

If we know

$$y_{t+1} = a_0 + a_1 y_t + \varepsilon_{t+1}$$

then

$$E_t(y_{t+2}) = a_0 + a_1 y_t$$

and since

$$\begin{aligned} y_{t+2} &= a_0 + a_1 y_{t+1} + \varepsilon_{t+2} \\ E_t(y_{t+2}) &= a_0 + a_1 E_t(y_t + 1) \\ &= a_0 + a_1(a_0 + a_1 y_t) \\ &= a_0 + a_1 a_0 + (a_1)^2 y_t \end{aligned}$$

Capturing Dynamic Relationships

- With the advent of modern dynamic economic models, the newer uses of time series models involve
  - Capturing dynamic economic relationships
  - Hypothesis testing
- Developing "stylized facts"
  - In a sense, this reverses the so-called scientific method in that modeling goes from developing models that follow from the data

The Random Walk Hypothesis

$$y_{t+1} = y_t + \varepsilon_t$$

or

$$\Delta y_{t+1} = \varepsilon_t$$

where $y_t =$ the price of a share of stock on day t , and $\varepsilon_t =$ a random disturbance term that has an expected value of zero.
Now consider the more general stochastic difference equation

$$\Delta y_{t+1} = a_0 + a_1 y_t + \varepsilon_{t+1}$$

The random walk hypothesis requires the testable restriction:

$$a_0 = a_1 = 0$$

The Unbiased Forward Rate (UFR) Hypothesis

Given the UFR hypothesis, the following forward/spot exchange rate relationship is:

$$s_{t+1} = f_t + \varepsilon_{t+1}$$

where ε_{t+1} has a mean value of zero from the perspective of time period t.
Consider the regression

$$s_{t+1} = a_0 + a_1 f_t + \varepsilon_{t+1}$$

The hypothesis requires $a_0 = 0, a_1 = 1$, and that the regression residuals $\varepsilon t + 1$ have mean value of zero from the perspective of time period t.
The spot and forward markets are said to be in *long-run equilibrium* when $\varepsilon_{t+1} = 0$.
Whenever, s_{t+1} turns out to differ from f_t, some sort of equilibrium adjustment must occur to restore the equilibrium in the subsequent period. Consider the adjustment process

$$s_{t+2} = s_{t+1} - a[s_{t+1} - f_t] + \varepsilon_{s,t+2} \quad a > 0$$
$$f_{t+1} = f_t + b[s_{t+1} - f_t] + \varepsilon_{f,t+1} \quad b > 0$$

where $\varepsilon_{s,t+2}$ and $\varepsilon_{f,t+1}$ both have an expected value of zero.

Trend-Cycle Relationships

- We can think of a time series as being composed of:

$$y_t = \text{trend} + \text{"cycle"} + \text{noise}$$

- Trend: permanent
- Cycle: predictable (albeit temporary)
  - (Deviations from trend)
- Noise: unpredictable

Consider the Function $y_{t^*} = f(t^*)$

$$\begin{aligned}\Delta y_{t^*+h} &\equiv f(t^*+h) - f(t^*)\\ &\equiv y_{t^*+h} - y_{t^*}\end{aligned}$$

We can then form the **first differences:**

$$\begin{aligned}\Delta y_t &= f(t) - f(t-1) \equiv y_t - y_{t-1}\\ \Delta y_{t+1} &= f(t+1) - f(t) \equiv y_{t+1} - y_t\\ \Delta y_{t+2} &= f(t+2) - f(t+1) \equiv y_{t+2} - y_{t+1}\end{aligned}$$

More generally, for the forcing process x_t a n-th order linear process is

$$y_t = a_0 + \sum_{i=1}^{n} a_i y_{t-i} + x_t$$

What is a Solution?

A **solution** to a difference equation expresses the value of y_t as a function of the elements of the x_t sequence and t (and possibly some given values of the y sequence called **initial conditions**).

$$y_t = a_0 + \sum_{i=1}^{n} a_i y_{t-i} + x_t$$

The key property of a solution is that it satisfies the difference equation for all permissible values of t and x_t.

Solution by Iteration

Consider the first-order equation

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$$

Given the value of y_0, it follows that y_1 will be given by

$$y_1 = a_0 + a_1 y_0 + \varepsilon_1$$

In the same way, y_2 must be

$$\begin{aligned} y_2 &= a_0 + a_1 y_1 + \varepsilon_2 \\ &= a_0 + a_1[a_0 + a_1 y_0 + \varepsilon_1] + \varepsilon_2 \\ &= a_0 + a_0 a_1 + (a_1)^2 y_0 + a_1 \varepsilon_1 + \varepsilon_2 \end{aligned}$$

Continuing the process in order to find y_3, we obtain

$$\begin{aligned} y_3 &= a_0 + a_1 y_2 + \varepsilon_3 \\ &= a_0[1 + a_1 + (a_1)^2] + (a_1)^3 y_0 + a_1^2 \varepsilon_1 + a_1 \varepsilon_2 + \varepsilon_3 \end{aligned}$$

From

$$y_3 = a_0[1 + a_1 + (a_1)^2] + (a_1)^3 y_0 + a_1^2 \varepsilon_1 + a_1 \varepsilon_2 + \varepsilon_3$$

you can verify that for $t > 0$, repeated iteration yields

$$y_t = a_0 \sum_{i=0}^{t-1} a_i^i + a_1^i y_0 + \sum_{i=0}^{t-1} a_i^i \varepsilon_{t-i}$$

If $|a_1| < 1$, in the limit

$$y_t = a_0/(1 - a_1) + \sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i}$$

Backwards Iteration

Iteration from y_t back to y_0 yields exactly the formula given by (above).
Since $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$, it follows that

$$\begin{aligned} y_t &= a_0 + a_1[a_0 + a_1 y_{t-2} + \varepsilon_{t-1}] + \varepsilon_t \\ &= a_0(1 + a_1) + a_1\varepsilon_{t-1} + \varepsilon_t + a_1^2[a_0 + a_1 y_{t-3} + \varepsilon_{t-2}] \end{aligned}$$

If $|a_1| < 1$, in the limit

$$y_t = a_0/(1 - a_1) + \sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i}$$

Lag Operators

The lag operator L is defined to be:

$$L^i y_t = y_{t-i}$$

Thus, L^i preceding y_t simply means to lag y_t by i periods.
The lag of a constant is a constant: $Lc = c$.
The distributive law holds for lag operators. We can set:

$$(L^i + L^j)y_t = L^i y_t + L^j y_t = y_{t-i} + y_{t-j}$$

Lag Operators (Continued)

- Lag operators provide a concise notation for writing difference equations. Using lag operators, the p-th order equation

$$y_t = a_0 + a_1 y_{t-1} + \ldots + a_p y_{t-p} + \varepsilon_t$$

can be written as:

$$(1 - a_1 L - a2L^2 - \ldots - a_p L^p) y_t = \varepsilon_t$$

or more compactly as:

$$A(L) y_t = \varepsilon_t$$

As a second example,

$$\begin{aligned} y_t &= a_0 + a_1 y_{t-1} + \ldots + a_p y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \ldots + \beta_q \varepsilon_{t-q} \\ &= A(L) y_t + B(L) \varepsilon_t \end{aligned}$$

where: $A(L)$ and $B(L)$ are polynomials of orders p and q respectively.