Finance 5330 - Financial Econometrics

Prices, Returns and Price Discovery

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Asset Returns

One-Period Simple Return

- Hold asset from period $t-1$ to t
- Simple Gross Return:

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

or

$$P_t = P_{t-1}(1 + R_t)$$

Multiperiod Simple Return

- Holding the asset k period between dates $t-k$ and t gives a k-period gross return:

$$\begin{aligned}
1+R_t[k] &= \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}}\frac{P_{t-1}}{P_{t-2}}\cdots\frac{P_{t-k+1}}{P_{t-k}} \\
&= (1+R_t)(1+R_{t-1})\cdots(1+R_{t-k+1}) \\
&= \prod_{j=0}^{k-1}(1+R_{t-j})
\end{aligned}$$

- ***NB:*** the k-period simple gross return is the product of the k one-period simple gross returns. This is called a compound return.
- The k-period simple net return is: $R_t[k] = \frac{P_t - P_{t-k}}{P_{t-k}}$

Annualized Returns

- To facilitate comparison we usually standardize to a given duration (annual, monthly, quarterly, etc)
- The annual return is

$$\text{Annualized}\{R_t[k]\} = \left[\prod_{j=0}^{k-1}(1 + R_{t-j})\right]^{1/k} - 1$$

- ***NB:*** this is a geometric mean of the k one-period simple gross returns
- It can also be computed as

$$\text{Annualized}\{R_t[k]\} = \exp\left[\frac{1}{k}\sum_{j=0}^{k-1}\ln\left(1 + R_{t-j}\right)\right] - 1$$

Continuous Compounding

For continuous compounding we can use the following formula:

$$A = C \exp(r \times n)$$

where

- $A =$ net asset value
- $C =$ initial capital
- $r =$ interest rate per annum
- $n =$ number of years

With this in mind, we can also define the ***present value*** relation:

$$C = A \exp(-r \times n)$$

Continuously Compounded Return

- The natural log of the simple gross return

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = \ln P_t - \ln P_{t-1}$$

- Advantages
  - Multiperiod returns

$$\begin{aligned} r_t[k] &= \ln(1 + R_t[k]) \\ &= \ln[(1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1} \end{aligned}$$

Continously Compounded Return (Continued)

- Symmetric up/down moves
- Set up some imaginary prices

```
p1 = 100.0
p2 = 105.0
p3 = 100.0
```

Continously Compounded Return (Continued)

- The simple net return

```
R2 = p2/p1 - 1.0
print(R2)
```

```
## 0.050000000000000044
```

```
R3 = p3/p2 - 1.0
print(R3)
```

```
## -0.04761904761904767
```

Continously Compounded Return (Continued)

- Continously compounded prices

```
import numpy as np
r2 = np.log(p2) - np.log(p1)
print(r2)
```

```
## 0.04879016416943127
```

```
r3 = np.log(p3) - np.log(p2)
print(r3)
```

```
## -0.04879016416943127
```

Portfolio Returns

$$R_{p,t} = \sum_{i=1}^{N} \omega_i R_{i,t}$$

where

- $p =$ index representing the portfolio
- $N =$ number of assets in the portfolio
- $R_{i,t} =$ period t simple net return on aset i
- $\omega_i =$ weight on asset i

NB: continously compounded portfolio returns do not have this convenient property, but for simple returns $R_{i,t}$ small in magnitude, we have

$$r_{p,t} \approx \sum_{i=1}^{N} \omega_i r_{i,t}$$

Accounting for Dividend Payments

- We often have to account for dividends

$$R_t = \frac{(P_t + D_t)}{P_{t-1}} - 1$$

$$r_t = \ln(P_t + D_t) - \ln P_{t-1}$$

Excess Return

$$Z_t = R_t - R_{0,t}$$

where $R_{0,t}$ is a reference asset (e.g. the risk-free rate)

$$z_t = r_t - r_{0,t}$$

NB: Excess return can be thought of as the payoff to an arbitrage strategy that goes long the asset and short the reference rate with no initial investment.

Summary of Relationships

- $r_t = \ln(1 + R_t) \implies R_t = \exp(r_t) - 1$
- If R_t and r_t are in percentages then
  - $r_t = 100 \ln\left(1 + \frac{R_t}{100}\right)$
  - $R_t = 100\left(e^{r_t/100} - 1\right)$
- Temporal aggregation produces
  - $1 + R_t[k] = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})$
  - $r_t[k] = r_t + r_{t-1} + \cdots + r_{t-k+1}$
- Present and future values:
  - $A = C \exp(r \times n)$
  - $C = A \exp(-r \times n)$

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