


# CS Bridge Module 12 Recursions Part 1

## 1. Recursions - Part 1


### 1.1 CS Bridge: Recursions



CS Bridge: Recursions

Module 12 Part 1  
Itay Tal

### 1.2 Mathematical Induction overview



#### Mathematical Induction Overview

Mathematical induction is a technique to prove universal statements.

**Conclusion:** "P(n) is true for any natural number n."

There are 2 steps in such proofs:


- Base Case:** We prove that P(1) is true.
- Inductive Step:** We show that for any  $n \geq 2$ ,  $P(n-1) \rightarrow P(n)$ .

(That is we assume that P(n-1) is true, and under this assumption, we show that P(n) is true)


Notes:

## 1.3 Mathematical Induction Example

### Mathematical Induction Example



Claim  
For any natural number  $n$ :  $2 \cdot n^2 + 5 \cdot n - 6 \geq 0$ .




## 1.4 Walkthrough



Notes:

## 1.5 Strong Induction Overview

### Strong Induction Overview



Strong induction is a variation of the previous technique, also used to prove universal statements:

"P(n) is true for any natural number n"

$P(1) \rightarrow P(2)$

There are 2 steps in such proofs:

- I. Base Case: We prove that P(1) is true
- II. Inductive Step: We show that for any  $n \geq 2$ ,  $[P(k) \text{ is true for all } k < n] \rightarrow P(n)$

(That is we assume that for any  $k < n$ , P(k) is true, and under this assumption we show that P(n) is true)

P(1)

P(2)


P(3)

P(4)

...

## 1.6 Strong Induction Example Part 1

### Strong Induction Example



Claim

Every natural number  $n$  can be written in the form:

$n = 2^i \cdot j$ , where  $i$  is a non-negative integer and  $j$  is odd.

Examples:

$40 = 2^3 \cdot 5$

$6 = 2^1 \cdot 3$

$7 = 2^0 \cdot 7$

NYU TANDON ONLINE

## 1.7 Strong Induction Example Part 2

### Strong Induction Example

We have 2 cases:

Claim: Every natural number  $n$  can be written in the form:

$n = 2^i \cdot j$  where  $i$  is a non-negative integer and  $j$  is odd

In this case, if we take  $i=0$  and  $j=n$ , we get  $n = 2^0 \cdot n$

$i$  and  $j$  are as requested by strong induction:

I) Base Case: for  $n=1$

If we take  $i=0$  and  $j=1$

II) Inductive Step: We assume that for non-negative  $i$  and odd  $j$  we have  $k = 2^i \cdot j$

In this case since  $n = k + 1$ , by the inductive hypothesis for  $k = \frac{n}{2}$

II) Inductive Step: We assume that for non-negative  $i$  and odd  $j$  we have  $k = 2^i \cdot j$

Where  $i$  is a non-negative integer and  $j$  is odd

Now we can show that  $n$  can be written as  $n = 2^i \cdot j$

and  $i$  and  $j$  as described

If we take  $i=i+1$  and  $j=j$  we get  $n$  written in the requested form

NYU TANDON ONLINE

Notes:

## 1.8 Knowledge Check

(Sequence Drag-and-Drop, 10 points, 4 attempts permitted)

Knowledge Check

NYU TANDON  
ONLINE

Order the steps used to prove a statement via induction:

1. Base Case: Prove that  $P(n)$  holds for  $n=0$ , or  $n=1$
2. Induction Hypothesis: Since the base case holds, assume that  $P(k)$  also holds
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

Correct Order
Base Case: Prove that $P(n)$ holds for $n=0$ , or $n=1$
Induction Hypothesis: Since the base case holds, assume that $P(k)$ also holds
Inductive Step: Show that $P(k)$ implies $P(k+1)$

**Feedback when correct:**

That's right! You selected the correct response.

**Feedback when incorrect:**

You did not select the correct response.

## Correct (Slide Layer)

Knowledge Check

NYU TANDON  
ONLINE

Order the steps used to prove a statement via induction:

Correct

That's right! You selected the correct response.

Continue

1. Base Case: Show that  $P(1)$  holds
2. Induction Hypothesis: Assume that  $P(k)$  holds
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

## Incorrect (Slide Layer)

Knowledge Check

NYU TANDON  
ONLINE

Order the steps used to prove a statement via induction:

Incorrect

You did not select the correct response.

Continue

1. Base Case: Show that  $P(1)$  holds
2. Induction Hypothesis: Assume that  $P(k)$  holds
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

## Try Again (Slide Layer)

Knowledge Check

NYU TANDON  
ONLINE

Order the steps used to prove a statement via induction:

Incorrect

That is incorrect. Please try again.

Try Again

1. Base Case: Show that  $P(1)$  holds
2. Induction Hypothesis: Assume that  $P(k)$  holds
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

## 1.9 Knowledge Check

(Sequence Drag-and-Drop, 10 points, 4 attempts permitted)

Knowledge Check

NYU TANDON  
ONLINE

Order the steps used to prove a statement via strong induction:

1. Base Case: Prove that  $P(n)$  holds for  $n=0$ , or  $n=1$
2. Induction Hypothesis: Since the base case holds, assume that  $P(k)$  also holds for all values  $n$  such that  $n$  is less than or equal to  $k$
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

Correct Order
Base Case: Prove that $P(n)$ holds for $n=0$ , or $n=1$
Induction Hypothesis: Since the base case holds, assume that $P(k)$ also holds for all values $n$ such that $n$ is less than or equal to $k$
Inductive Step: Show that $P(k)$ implies $P(k+1)$

### Feedback when correct:

That's right! You selected the correct response.

### Feedback when incorrect:

You did not select the correct response.

## Correct (Slide Layer)

### Knowledge Check

Order the steps used to prove a statement via strong induction:

Correct

That's right! You selected the correct response.

Continue

1. Base Case
2. Induction Hypothesis: Assume that  $P(k)$  is true for all  $k$  such that  $n$  is less than or equal to  $k$
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

## Incorrect (Slide Layer)

### Knowledge Check

Order the steps used to prove a statement via strong induction:

Incorrect

You did not select the correct response.

Continue

1. Base Case
2. Induction Hypothesis: Assume that  $P(k)$  is true for all  $k$  such that  $n$  is less than or equal to  $k$
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

## Try Again (Slide Layer)

### Knowledge Check

Order the steps used to prove a statement via strong induction:

Incorrect

That is incorrect. Please try again.

Try Again

1. Base Case
2. Induction Hypothesis: Assume that  $P(k)$  is true for all  $k$  such that  $n$  is less than or equal to  $k$
3. Inductive Step: Show that  $P(k)$  implies  $P(k+1)$

## 1.10 Knowledge Check

(Matching Drag-and-Drop, 10 points, 2 attempts permitted)

Knowledge Check

NYU TANDON  
ONLINE

Match the descriptions for induction and strong induction

Induction

Strong Induction

If  $T(k)$  is true, then  $T(k+1)$  is also true

If  $T(i)$  is true for all values of  $i$  less than or equal to  $k$ , then  $T(k+1)$  is true

Correct	Choice
Induction	If $T(k)$ is true, then $T(k+1)$ is also true
Strong Induction	If $T(i)$ is true for all values of $i$ less than or equal to $k$ , then $T(k+1)$ is true

### Feedback when correct:

That's right! Regular (weak) induction only considers one case to imply the final case. Strong induction considers every iteration to imply the final case.

### Feedback when incorrect:

You did not select the correct response.



### Correct (Slide Layer)

Knowledge Check

NYU TANDON  
ONLINE

Match the descriptions for induction and strong induction

Induction

Strong Induction

Correct

That's right! Regular (weak) induction only considers one case to imply the final case. Strong induction considers every iteration to imply the final case.

Continue

is also true

if  $T(k)$  is true, then  $T(k+1)$  is true

### Incorrect (Slide Layer)

Knowledge Check

NYU TANDON  
ONLINE

Match the descriptions for induction and strong induction

Induction

Strong Induction

Incorrect

You did not select the correct response.

Continue

is also true

if  $T(k)$  is true, then  $T(k+1)$  is true

### Try Again (Slide Layer)

Knowledge Check

NYU TANDON  
ONLINE

Match the descriptions for induction and strong induction

Induction

Strong Induction

Incorrect

That is incorrect. Please try again.

Try Again


is also true

if  $T(k)$  is true, then  $T(k+1)$  is true

## 1.11 Results Slide

(Results Slide, 0 points, 1 attempt permitted)

Results



Your Score:

%Results.ScorePercent%% (%Results.ScorePoints% points)

Passing Score:

%Results.PassPercent%% (%Results.PassPoints% points)

Result:

Retry Quiz

Review Quiz

Results for
1.8 Knowledge Check
1.9 Knowledge Check
1.10 Knowledge Check


Result slide properties

Passing80%

Score

## Success (Slide Layer)

Results



Your Score: %Results.ScorePercent%% (%Results.ScorePoints% points)

Passing Score: %Results.PassPercent%% (%Results.PassPoints% points)

---

Result:


✓ Congratulations, you passed.

Retry Quiz

Review Quiz

## Failure (Slide Layer)

Results



Your Score: %Results.ScorePercent%% (%Results.ScorePoints% points)

Passing Score: %Results.PassPercent%% (%Results.PassPoints% points)

---


Result:

✗ You did not pass.

Retry Quiz

Review Quiz

## 1.12 End of Module



NYU TANDON  
ONLINE

End of Module

Exit

