WI23_CSBR-NY_1_NC_INT2 HW2 (Q5 to Q9)

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TOTAL POINTS

18.5 / 20

QUESTION 1

1 Q5 **4 / 8**

- 0 pts Correct

1.12.2b

- 1 pts 1.12.2 B

$$\neg q$$ H2

$$ \neg q \vee \neg r$$ Addition 1

$$\neg (q \land r)$$ De Morgan's Law

$$p \rightarrow (q \land r)$$ H1

$$\neg p $$ Modus Tollens 3, 4

or

$$p \rightarrow (q \land r)$$ H1

$$\neg p \vee (q \land r)$$

Conditional

$$ (\neg p \vee q) \land (\neg p \vee r)$$

Distributive

$$(\neg p \vee q)$$ Simplification

$$p \rightarrow q$$

Conditional

$$\neg q$$ H2

$$\neg p $$ Modus Tollens 5,6

✓ **- 0.5 pts** *Missing/Incorrect logic statements*

- 0.1 pts Line 2 rule incorrect

- 0.1 pts Missing step between line 2 and 3

- 0.1 pts Line 3 mistake - should be or

- 0.1 pts Line 2 mistake should be and between q and r

- 0.1 pts Incorrect question

1.12.2e

- 1 pts 1.12.2 E

$$p \vee q$$ H1

$$ \neg p \vee r$$ H2

$$q \vee r$$ Resolution 1, 2

$$\neg q$$ H3

$$r$$ Disjunctive Syllogism 3, 4

- 0.5 pts Missing/Incorrect logic statements

- 0.1 pts Line 5 incorrect

- 0.1 pts Line 3 incorrect

1.12.3c

- 1 pts 1.12.3 C

$$p \vee q$$ H1

$$ \neg (\neg p) \vee q$$ Double Negation

$$\neg p \rightarrow q$$ Conditional Identity 2

$$\neg p$$ H2

$$q$$ Modus Ponens 3,4

- 0.1 pts Careful with parentheses in line 2

✓ **- 0.5 pts** *Missing/Incorrect logic statements*

- 0.1 pts Line 4 incorrect

- 0.1 pts Should be -p ^ q for lines 4 - 7

1.12.5C

✓ **- 1 pts** *1.12.5 C*

$$(C \land H) \rightarrow J$$

$$\neg J$$

$$\therefore \neg C$$

explanation:

The argument is not valid. If c = T, and j = h = F, then the hypotheses are both true, but the conclusion ¬c is false.

- **0.5 pts** Missing/Incorrect explanation
- **0.5 pts** Incorrect argument form
- **0.1 pts** Missing predicate definitions

1.12.5D

✓ **- 1 pts** *1.12.5 D*

$$(C \land H) \rightarrow J$$

$$\neg J$$

$$H$$

$$\therefore \neg C$$

$$\neg J $$ H2

$$(C \land H) \rightarrow J$$ H1

$$\neg (C \land H)$$ Modus Tollens 1,2

$$\neg C \vee \neg H$$ De Morgan's Law

$$\neg H \vee \neg C$$ Commutative

$$H$$ H3

$$\neg (\neg H)$$ Double Negation

$$\neg C$$ Disjunctive Syllogism

- **0.5 pts** Missing/Incorrect proof
- **0.1 pts** Missing double negation
- **0.5 pts** Missing/Incorrect argument form
- **0.5 pts** Missing/Incorrect logic statements
- **0.1 pts** Missing predicate definitions

1.13.3b

- **1 pts** 1.13.3 B

P Q

a F T

b F F

or

P Q

a F F

b F T

Conclusion would be false since there does not exist an x such that P(x) = T. Both hypotheses would be true, e.g., P(a) v Q(a) = F v T = T and ~Q(b) = T.

- **0.5 pts** Incorrect explanation
- **0.1 pts** Incorrect problem

1.13.5d

✓ **- 1 pts** *1.13.5 D*

M(x): x missed class

D(x): x received detention

$$\forall x (M(x) \rightarrow D(x))$$

Penelope is a particular student

$$\neg M(Penelope)$$

$$\therefore \neg D(Penelope)$$

explanation: The argument is not valid. Consider a class that consists only of one student, which is Penelope. If D(Penelope) = T and M(Penelope) = F, then the hypotheses are all true and the conclusion is false. In other words, Penelope got a detention but did not miss class.

- **0.5 pts** Missing/Incorrect argument form
- **0.5 pts** Missing/Incorrect explanation
- **0.1 pts** Missing predicate definitions
- **0.1 pts** Incorrect predicate - P(x)

1.13.5e

- 1 pts 1.13.5 E

M(x): x missed class

D(x): x received detention

A(x): x got an A

$$\forall x (M(x) \vee D(x)) \rightarrow \neg A(x)$$

Penelope is a particular student

$$A(Penelope)$$

$$\therefore \neg D(Penelope)$$

$$\forall x (M(x) \vee D(x)) \rightarrow \neg A(x)$$ H1

Penelope is a student in the class H2

$$M(Penelope) \vee D(Penelope) \rightarrow \neg A(Penelope) $$ Universal Instatiation 1,2

$$A(Penelope)$$ H3

$$\neg (\neg A(Penelope))$$ Double Negation

$$\neg (M(Penelope) \vee D(Penelope))$$ Modus Tollens 3,5

$$\neg M(Penelope) \land \neg D(Penelope)$$ De Morgan's Law 6

$$\neg D(Penelope) \land \neg M(Penelope)$$ Commutative Law 7

$$\neg D(Penelope)$$ Simplification 8

- 0.5 pts Missing/Incorrect logic statements

- 0.5 pts Missing/Incorrect argument form

- 0.1 pts Missing predicate definitions

- 0.1 pts First line of argument form incorrect

- 0.1 pts Didn't instantiate p in Line 3

- 1 pts difficult to read or skipping rules or misnamed/misapplied rules

- 8 pts Entirely incorrect or missing

- 0.5 pts Missing logic statements

- 0.5 pts 1.13.5E Missing logic statements

- 8 pts Not tagged

- 0.1 pts Incorrect logic rule applied for 1.12.3C - line 5

- 0.5 pts Missing

QUESTION 2

2 Q6 **1.5 / 2**

- 0 pts Correct

- 1 pts 2.4.1 D Incorrect

- 0.5 pts 2.4.1 D Incomplete proof

- 1 pts 2.4.3 B Incorrect

✓ **- 0.5 pts** *2.4.3 B Incomplete proof*

- 1 pts 2.4.3 B Incorrect, plugged in numbers (not a proof)

- 2 pts Not submitted or not tagged

1 need more explanation between 4th and 5th

QUESTION 3

3 Q7 **2 / 4**

- 0 pts Correct: clear proofs, thoroughly explained

- 4 pts not tagged properly

- 1 pts 2.5.1 (d) is incorrect or missing. Proof could be something like this:

Proof.
We will assume that n is an even integer and show that $n^{2} - 2n + 7$ is an odd integer.

If n is an even integer, then $n=2k$ for some integer k. Plugging in the expression $2k$ for n in $n^{2} - 2n + 7$ gives $4k^{2} -4k+7 = 4k^{2} - 4k + 6 + 1 = 2(2k^{2} - 2k + 3) + 1$

Since k is an integer, $(2k^{2} - 2k + 3)$ is also an integer - call it m. Thus we have $2m + 1$. From the substitutions we can see that $n^{2} - 2n + 7$ is equal to two times an integer plus 1, and therefore, $n^{2} - 2n + 7$ is an odd integer. ■

✓ **- 0.5 pts** *2.5.1 (d) proof correct overall but has mistakes or is incomplete*

assigned usually due to incomplete factoring. should've factored to $2(2k^2-2k+3)+1$ i.e. 2(int)+1

- 1 pts 2.5.4A is incorrect or missing.
Proof could be something like this:
Proof.
We assume for real numbers x and y that $x>y$ and we prove that $x^3 + xy^2 > x^{2}y + y^{3}$.

The square of any real number is $\geq$ 0. Therefore $x^{2} \geq 0$ and $y^{2} \geq 0$. Since $x > y$, at least one of x and y is not zero, which means that at least one of x^{2} and y^{2} is not zero. Since x^{2} and y^{2} are both non-negative, at least one of x^{2} and y^{2} is positive. The sum of two non-negative numbers, at least one of which is positive, must be a positive number. Therefore, $x^{2} + y^{2} > 0$. Multiplying both sides of $x-y > 0$ by $x^{2} + y^{2}$ gives

$x-y > 0$
$(x^{2}+y^{2})(x-y) > (x^{2} + y^{2})0$
$x^{3} + xy^{2} - x^{2}y - y^{3} > 0$
$x^{3} + xy^{2} > x^{2}y + y^{3}$
■

✓ **- 0.5 pts** *2.5.4A did not prove that $(x^{2}+y^{2})$ is strictly larger than 0, and so it can be divided/multiplied to both sides of the inequality without changing the direction of the inequality.*

- 0.5 pts 2.5.4A proof correct overall but has mistakes or is incomplete

- 1 pts 2.5.4B is incorrect or missing. Proof could be something like this:
Proof.
We assume that for real numbers x and y that it is not the case that $x > 10$ or $y > 10$ and prove that $x+y \leq 20$.
Since it is not true that $x > 10$ or $y > 10$, by DeMorgan's Law, the inequalities $x > 10$ and $y > 10$ are both False. Therefore, $x \leq 10$ and $y \leq 10$. Adding $x \leq 10$ and $y \leq 10$ gives that:

$x + y \leq 10 + 10 = 20$. Therefore $x + y \leq 20$.■

- 0.5 pts 2.5.4B Proof correct overall but has mistakes or is incomplete.

✓ **- 1 pts** *2.5.5C is incorrect or missing. Proof could be something like this:*

Proof.

Let x be a non-zero real number and assume that $$\frac{1}{x}$$ is not irrational. We will prove that x must be rational. If x is a non-zero real number, then $$\frac{1}{x}$$ is a well-defined real number. Every real number is rational or irrational. Since $$\frac{1}{x}$$ is not irrational, then it must be rational. Therefore there are integers $$a$$ and $$b$$ such that $$\frac{1}{x} = \frac{a}{b}$$ and $$b \neq 0$$.

Since x is non-zero, we can divide both sides of the inequality $$1 \neq 0$$ by x to get that $$\frac{1}{x} \neq 0$$. Therefore the numerator $$a$$ in $$\frac{a}{b} = \frac{1}{x}$$ is also not equal to 0.

We have that $$x = \frac{1}{1/x} = \frac{1}{a/b} = \frac{b}{a}$$. Since $$x = \frac{b}{a}$$, where $$a$$ and $$b$$ are integers and $$a \neq 0$$, x is a rational number. ■

- 0.25 pts 2.5.5C Proof is correct overall but has minor mistakes

- 0.5 pts 2.5.5C Proof is mostly incomplete

- 1 pts - proofs are correct overall but work is difficult to read or messy

OR

- proofs could be clearer, such as by writing the contrapositive out clearly in one line

OR

- there are minor mistakes that do not change the validity of the proof

OR

- proofs do not show enough work

- 4 pts Missing OR very incorrect OR incomplete

2 need to prove this

QUESTION 4

4 Q8 **4 / 4**

✓ **- 0 pts** *Correct; good job!*

- 1 pts Incorrect or insufficiently explained math / properties / proof strategy for 2.6.6 (c)

- 0.5 pts 2.6.6 (c) proof is a little unclear; could use a little further explanation to accompany math

- 1 pts Incorrect or insufficiently explained math / properties / definitions for 2.6.6 (d)

- 0.5 pts 2.6.6 (d) proof is a little unclear; could use a little further explanation to accompany math

- 0.25 pts didn't state that example of smaller integer was an integer (2.6.6(d)) (i.e. should state i - 1 is also an integer since i is an integer), need this to prove full claim

- 2 pts 2.6.6(c) completely incorrect or missing

- 2 pts 2.6.6(d) completely incorrect or missing

- 4 pts Not complete or not submitted

- 4 pts Not Tagged

- 100 pts Do not copy someone else's work!

QUESTION 5

5 Q9 **2 / 2**

✓ **- 0 pts** *Correct*

- 1 pts Failed to prove one of two use cases, or error in proof logic

- 0.5 pts Mostly correct, needs a bit more to be robust

- **- 0.5 pts** Other minor error
- **- 2 pts** Late assignment
- **- 2 pts** Proof entirely incorrect or missing

QUESTION 6

6 Extra credit for typing **5 / 0**

✓ **+ 5 pts** *Entirely typed*

+ 0 pts Not entirely typed

QUESTION 7

7 Tagging / Starting questions on new pages / Formatting equations **0 / 0**

✓ **- 0 pts** *Correct*

- 10 pts Untagged / Did not start questions on new pages / Too many unformatted equations

Question #5a

1.12.2.b)

-q	Hypothesis
p -> (q and r)	Hypothesis
p -> q	Simplification
-p	Modus Tollens

1.12.2.e)

-q	Hypothesis
p or q	Hypothesis
p	Disjunctive Syllogism
-p or r	Hypothesis
r	Disjunctive Syllogism

1.12.3.c)

-p	Hypothesis
-p or q	Addition
P -> q	Conditional Identities
p or q	Hypothesis
p	Idempotent Laws
P P -> q q	Modus Ponens

1.12.5.c)

J(x) -> (C(x) and H(x))

<u>-J(x)</u>

-C(x)

Proof: Argument Valid

-J(x)	Hypothesis
-J(x) -> (-C(x) and –H(x)	Hypothesis
-J(x) -> -C(x)	Simplification
-C(x)	Modus Tollens

1.12.5.d)

J(x) -> (C(x) and H(x))

-J(x)

H(x)

-C(x)

Proof: Argument Invalid: Premise 1 states that if I get a job then I will get a car and a house while premise two states that I will not get a job but premise 3 states that I will buy a house. All 3 premises cannot be True at the same time therefore the argument is invalid.

Question #5b

1.13.3.b) Show that the given argument is invalid by giving values for the predicates P and Q over the domain {a, b}.

	P	Q
a	F	T
b	F	F

1.13.5.d)

Ax (M(x) -> D(x)

S(Penelope)

-M(Penelope)

-D(Penelope)

Proof:

S(Penelope)	Hypothesis (True)
-M(Penelope)	Hypothesis
Ax (M(x) -> D(x)	Hypothesis
M(Penelope) -> D(Penelope)	Universal Instantiation
-D(Penelope)	Modus Tollens

1.13.5.e)

Ax (M(x) v D(x)) -> -A(x)

S(Penelope)

<u>A(Penelope)</u>

-D(Penelope)

S(Penelope)	Hypothesis (True)
A(Penelope)	Hypothesis
Ax (M(x) v D(x)) - > -A(x)	Hypothesis
(M(Penelope) v D(Penelope)) - > -A(Penelope)	Universal Instantiation
-(M(Penlope v D(Penelope))	Modus Tollens
-M(Penelope) and –D(Penelope)	De Morgan's Law
-D(Penelope)	Simplification

Question #6

2.4.1.b)

1st - Let x and y be two odd integers, which can be expressed as x = 2k + 1 and y = 2j + 1 for some integers k and j.

2nd – Then, the product of x and y can be expressed as

$$(2k + 1) * (2j + 1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$$

3rd – Since 2kj + k + j is an integer, the product of x and y is an odd integer, which can be expressed as 2(2kj + k + j) + 1.

4th – Thus, the product of two odd integers is an odd integer.

2.4.3.b)

1st - Let x be a real number such that x <= 3. Then, we have to show that 12 - 7x + x^2 >= 0.

2nd – We substitute x = 3; 12 – 7(3) + 3^2 = 12 – 21 + 9 = 0

3rd – therefore, 0 >= 0 which proves the inequality to be true

4th – since x <= 3 that means all values of x will be less than or equal to 3

1 Q5 **4 / 8**

- **- 0 pts** Correct

1.12.2b

- **- 1 pts** 1.12.2 B

$$\neg q$$ H2

$$ \neg q \vee \neg r$$ Addition 1

$$\neg (q \land r)$$ De Morgan's Law

$$p \rightarrow (q \land r)$$ H1

$$\neg p $$ Modus Tollens 3, 4

or

$$p \rightarrow (q \land r)$$ H1

$$\neg p \vee (q \land r)$$ Conditional

$$ (\neg p \vee q) \land (\neg p \vee r)$$ Distributive

$$(\neg p \vee q)$$ Simplification

$$p \rightarrow q$$ Conditional

$$\neg q$$ H2

$$\neg p $$ Modus Tollens 5,6

✓ **- 0.5 pts** *Missing/Incorrect logic statements*

- **- 0.1 pts** Line 2 rule incorrect
- **- 0.1 pts** Missing step between line 2 and 3
- **- 0.1 pts** Line 3 mistake - should be or
- **- 0.1 pts** Line 2 mistake should be and between q and r
- **- 0.1 pts** Incorrect question

1.12.2e

- **- 1 pts** 1.12.2 E

$$p \vee q$$ H1

$$ \neg p \vee r$$ H2

$$q \vee r$$ Resolution 1, 2

$$\neg q$$ H3

$$r$$ Disjunctive Syllogism 3, 4

- **- 0.5 pts** Missing/Incorrect logic statements
- **- 0.1 pts** Line 5 incorrect
- **- 0.1 pts** Line 3 incorrect

1.12.3c

- 1 pts 1.12.3 C

$$p \vee q$$ H1

$$ \neg (\neg p) \vee q$$ Double Negation

$$\neg p \rightarrow q$$ Conditional Identity 2

$$\neg p$$ H2

$$q$$ Modus Ponens 3,4

- 0.1 pts Careful with parentheses in line 2

✓ **- 0.5 pts** *Missing/Incorrect logic statements*

- 0.1 pts Line 4 incorrect

- 0.1 pts Should be -p ^ q for lines 4 - 7

1.12.5C

✓ **- 1 pts** *1.12.5 C*

$$(C \land H) \rightarrow J$$

$$\neg J$$

$$\therefore \neg C$$

explanation:

The argument is not valid. If c = T, and j = h = F, then the hypotheses are both true, but the conclusion ¬c is false.

- 0.5 pts Missing/Incorrect explanation

- 0.5 pts Incorrect argument form

- 0.1 pts Missing predicate definitions

1.12.5D

✓ **- 1 pts** *1.12.5 D*

$$(C \land H) \rightarrow J$$

$$\neg J$$

$$H$$

$$\therefore \neg C$$

$$\neg J $$ H2

$$(C \land H) \rightarrow J$$ H1

$$\neg (C \land H)$$ Modus Tollens 1,2

$$\neg C \vee \neg H$$ De Morgan's Law

$$\neg H \vee \neg C$$ Commutative

$$H$$ H3

$$\neg (\neg H)$$ Double Negation

$$\neg C$$ Disjunctive Syllogism

- 0.5 pts Missing/Incorrect proof

- 0.1 pts Missing double negation

- 0.5 pts Missing/Incorrect argument form

- 0.5 pts Missing/Incorrect logic statements

- 0.1 pts Missing predicate definitions

1.13.3b

- 1 pts 1.13.3 B

P Q

a F T

b F F

or

P Q

a F F

b F T

Conclusion would be false since there does not exist an x such that P(x) = T. Both hypotheses would be true, e.g., P(a) v Q(a) = F v T = T and ~Q(b) = T.

- 0.5 pts Incorrect explanation

- 0.1 pts Incorrect problem

1.13.5d

✓ **- 1 pts** *1.13.5 D*

M(x): x missed class

D(x): x received detention

$$\forall x (M(x) \rightarrow D(x))$$

Penelope is a particular student

$$\neg M(Penelope)$$

$$\therefore \neg D(Penelope)$$

explanation: The argument is not valid. Consider a class that consists only of one student, which is Penelope. If D(Penelope) = T and M(Penelope) = F, then the hypotheses are all true and the conclusion is false. In other words, Penelope got a detention but did not miss class.

- 0.5 pts Missing/Incorrect argument form

- 0.5 pts Missing/Incorrect explanation

- 0.1 pts Missing predicate definitions

- 0.1 pts Incorrect predicate - P(x)

1.13.5e

- 1 pts 1.13.5 E

M(x): x missed class

D(x): x received detention

A(x): x got an A

$$\forall x (M(x) \vee D(x)) \rightarrow \neg A(x)$$

Penelope is a particular student

$$A(Penelope)$$

$$\therefore \neg D(Penelope)$$

$$\forall x (M(x) \vee D(x)) \rightarrow \neg A(x)$$ H1

Penelope is a student in the class H2

$$M(Penelope) \vee D(Penelope) \rightarrow \neg A(Penelope) $$ Universal Instatiation 1,2

$$A(Penelope)$$ H3

$$\neg (\neg A(Penelope))$$ Double Negation

$$\neg (M(Penelope) \vee D(Penelope))$$ Modus Tollens 3,5

$$\neg M(Penelope) \land \neg D(Penelope)$$ De Morgan's Law 6

$$\neg D(Penelope) \land \neg M(Penelope)$$ Commutative Law 7

$$\neg D(Penelope)$$ Simplification 8

- 0.5 pts

Missing/Incorrect logic statements

- 0.5 pts Missing/Incorrect argument form

- 0.1 pts Missing predicate definitions

- 0.1 pts First line of argument form incorrect

- 0.1 pts Didn't instantiate p in Line 3

- 1 pts difficult to read or skipping rules or misnamed/misapplied rules

- 8 pts Entirely incorrect or missing

- 0.5 pts Missing logic statements

- 0.5 pts 1.13.5E

Missing logic statements

- 8 pts Not tagged

- 0.1 pts Incorrect logic rule applied for 1.12.3C - line 5

- 0.5 pts Missing

1.13.5.e)

Ax (M(x) v D(x)) -> -A(x)

S(Penelope)

A(Penelope)

-D(Penelope)

S(Penelope)	Hypothesis (True)
A(Penelope)	Hypothesis
Ax (M(x) v D(x)) - > -A(x)	Hypothesis
(M(Penelope) v D(Penelope)) - > -A(Penelope)	Universal Instantiation
-(M(Penlope v D(Penelope))	Modus Tollens
-M(Penelope) and –D(Penelope)	De Morgan's Law
-D(Penelope)	Simplification

Question #6

2.4.1.b)

1st - Let x and y be two odd integers, which can be expressed as x = 2k + 1 and y = 2j + 1 for some integers k and j.

2nd – Then, the product of x and y can be expressed as

$$(2k + 1) * (2j + 1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$$

3rd – Since 2kj + k + j is an integer, the product of x and y is an odd integer, which can be expressed as 2(2kj + k + j) + 1.

4th – Thus, the product of two odd integers is an odd integer.

2.4.3.b)

1st - Let x be a real number such that x <= 3. Then, we have to show that 12 - 7x + x^2 >= 0.

2nd – We substitute x = 3; 12 – 7(3) + 3^2 = 12 – 21 + 9 = 0

3rd – therefore, 0 >= 0 which proves the inequality to be true

4th – since x <= 3 that means all values of x will be less than or equal to 3

5th – Hence, $12 - 7x + x^2 >= 0$ is true for all values of $x <= 3$

Question #7

2.5.1.d)

1st – Assume that n is even. We will prove that $n^2 + 2n + 7$ is odd.

2nd – Since n is even, we know that $n = 2k$ for some integer k.

3rd - $n^2 + 2n + 7 = (2k)^2 – 2(2k) + 7 = 4k^2 – 4k + 7$

4th – Since k is an integer then $4k^2 – 4k + 7$ is also an integer, the sum of an integer and 7 is always an odd number

5th – Therefore, $n^2 + 2n + 7$ is an odd number when n is even which shows contrapositive proof that $n^2 + 2n + 7$ is even when n is odd as truth

2.5.4.a)

1st – Assume $x > y$, we will show that $x^3 + xy^2 > x^2y + y^3$

2nd – Therefore, $x^2 > y^2$ and $x^3 > y^3$

3rd – Hence, $xy^2 > 0$ and $x^2y > 0$

4th – Since $x > y$, $x^3 + xy^2 > x^2y + y^3$ contrapositive statement is true

5th – Thus, the original statement is true by contrapositive

2.5.4.b)

1st – Assume $x <= 10$ and $y <=10$

2nd – Therefore, $x + y <= 10 + 10 = 20$

3rd – Since, $x + y <= 20$ while $x <= 10$ and $y <=10$, the contrapositive statement is true

4th – Thus, the statement is true by contrapositive

2.5.5.c)

Proof by Contrapositive

1st – Assume that 1/x is rational. We will show that x must be rational.

2nd – if 1/x is rational then $x = 1/y$ for some non-zero real number y.

3rd - Since multiplication of rational numbers is a rational number and since division is the inverse operation of multiplication, we can say that x is also rational.

4th – This proves that our contrapositive statement is true, thus the statement "For every non-zero real number x, if x is irrational, then 1/x is also irrational" is true

2 Q6 **1.5 / 2**

- 0 pts Correct

- 1 pts 2.4.1 D

Incorrect

- 0.5 pts 2.4.1 D

Incomplete proof

- 1 pts 2.4.3 B

Incorrect

✓ **- 0.5 pts** *2.4.3 B*

Incomplete proof

- 1 pts 2.4.3 B

Incorrect, plugged in numbers (not a proof)

- 2 pts Not submitted or not tagged

1 need more explanation between 4th and 5th

5th – Hence, $12 - 7x + x^2 >= 0$ is true for all values of $x <= 3$

Question #7

2.5.1.d)

1st – Assume that n is even. We will prove that $n^2 + 2n + 7$ is odd.

2nd – Since n is even, we know that n = 2k for some integer k.

3rd - $n^2 + 2n + 7 = (2k)^2 – 2(2k) + 7 = 4k^2 – 4k + 7$

4th – Since k is an integer then $4k^2 – 4k + 7$ is also an integer, the sum of an integer and 7 is always an odd number

5th – Therefore, $n^2 + 2n + 7$ is an odd number when n is even which shows contrapositive proof that $n^2 + 2n + 7$ is even when n is odd as truth

2.5.4.a)

1st – Assume x > y, we will show that $x^3 + xy^2 > x^2y + y^3$

2nd – Therefore, $x^2 > y^2$ and $x^3 > y^3$

3rd – Hence, $xy^2 > 0$ and $x^2y > 0$

4th – Since x > y, $x^3 + xy^2 > x^2y + y^3$ contrapositive statement is true

5th – Thus, the original statement is true by contrapositive

2.5.4.b)

1st – Assume x <= 10 and y <=10

2nd – Therefore, x + y <= 10 + 10 = 20

3rd – Since, x + y <= 20 while x <= 10 and y <=10, the contrapositive statement is true

4th – Thus, the statement is true by contrapositive

2.5.5.c)

Proof by Contrapositive

1st – Assume that 1/x is rational. We will show that x must be rational.

2nd – if 1/x is rational then x = 1/y for some non-zero real number y.

3rd - Since multiplication of rational numbers is a rational number and since division is the inverse operation of multiplication, we can say that x is also rational.

4th – This proves that our contrapositive statement is true, thus the statement “For every non-zero real number x, if x is irrational, then 1/x is also irrational” is true

3 Q7 **2 / 4**

- 0 pts Correct: clear proofs, thoroughly explained

- 4 pts not tagged properly

- 1 pts 2.5.1 (d) is incorrect or missing.

Proof could be something like this:

Proof.

We will assume that $$n$$ is an even integer and show that $$n^{2} - 2n + 7$$ is an odd integer.

If $$n$$ is an even integer, then $$n=2k$$ for some integer $$k$$. Plugging in the expression $$2k$$ for $$n$$ in $$n^{2} - 2n + 7$$ gives $$4k^{2} -4k+7 = 4k^{2} - 4k + 6 + 1 = 2(2k^{2} - 2k + 3) + 1$$

Since $$k$$ is an integer, $$(2k^{2} - 2k + 3)$$ is also an integer - call it $$m$$. Thus we have $$2m + 1$$. From the substitutions we can see that $$n^{2} - 2n + 7$$ is equal to two times an integer plus 1, and therefore, $$n^{2} - 2n + 7$$ is an odd integer. ■

✓ **- 0.5 pts** *2.5.1 (d) proof correct overall but has mistakes or is incomplete*

assigned usually due to incomplete factoring. should've factored to $$2(2k^2-2k+3)+1$$ i.e. 2(int)+1

- 1 pts 2.5.4A is incorrect or missing.

Proof could be something like this:

Proof.

We assume for real numbers $$x$$ and $$y$$ that $$x>y$$ and we prove that $$x^3 + xy^2 > x^{2}y + y^{3}$$.

The square of any real number is $$\geq$$ 0. Therefore $$x^{2} \geq 0$$ and $$y^{2} \geq 0$$. Since $$x > y$$, at least one of $$x$$ and $$y$$ is not zero, which means that at least one of $$x^{2}$$ and $$y^{2}$$ is not zero. Since $$x^{2}$$ and $$y^{2}$$ are both non-negative, at least one of $$x^{2}$$ and $$y^{2}$$ is positive. The sum of two non-negative numbers, at least one of which is positive, must be a positive number. Therefore, $$x^{2} + y^{2} > 0$$. Multiplying both sides of $$x-y > 0$$ by $$x^{2} + y^{2}$$ gives

$$x-y > 0$$

$$(x^{2}+y^{2})(x-y) > (x^{2} + y^{2})0$$

$$x^{3} + xy^{2} - x^{2}y - y^{3} > 0$$

$$x^{3} + xy^{2} > x^{2}y + y^{3}$$

■

✓ **- 0.5 pts** *2.5.4A did not prove that $$(x^{2}+y^{2})$$ is strictly larger than 0, and so it can be divided/multiplied to both sides of the inequality without changing the direction of the inequality.*

- 0.5 pts 2.5.4A proof correct overall but has mistakes or is incomplete

- 1 pts 2.5.4B is incorrect or missing. Proof could be something like this:

Proof.

We assume that for real numbers $$x$$ and $$y$$ that it is not the case that $$x > 10$$ or $$y > 10$$ and prove that $$x+y \leq 20$$.

Since it is not true that $$x > 10$$ or $$y > 10$$, by DeMorgan's Law, the inequalities $$x > 10$$ and $$y > 10$$ are both False. Therefore, $$x \leq 10$$ and $$y \leq 10$$. Adding $$x \leq 10$$ and $$y \leq 10$$ gives that:

$$x + y \leq 10 + 10 = 20$$. Therefore $$x + y \leq 20$$.■

- 0.5 pts 2.5.4B Proof correct overall but has mistakes or is incomplete.

✓ **- 1 pts** *2.5.5C is incorrect or missing. Proof could be something like this:*

Proof.

Let x be a non-zero real number and assume that $$\frac{1}{x}$$ is not irrational. We will prove that x must be rational. If x is a non-zero real number, then $$\frac{1}{x}$$ is a well-defined real number. Every real number is rational or irrational. Since $$\frac{1}{x}$$ is not irrational, then it must be rational. Therefore there are integers $$a$$ and $$b$$ such that $$\frac{1}{x} = \frac{a}{b}$$ and $$b \neq 0$$.

Since x is non-zero, we can divide both sides of the inequality $$1 \neq 0$$ by x to get that $$\frac{1}{x} \neq 0$$. Therefore the numerator $$a$$ in $$\frac{a}{b} = \frac{1}{x}$$ is also not equal to 0.

We have that $$x = \frac{1}{1/x} = \frac{1}{a/b} = \frac{b}{a}$$. Since $$x = \frac{b}{a}$$, where $$a$$ and $$b$$ are integers and $$a \neq 0$$, x is a rational number. ■

- 0.25 pts 2.5.5C Proof is correct overall but has minor mistakes

- 0.5 pts 2.5.5C Proof is mostly incomplete

- 1 pts - proofs are correct overall but work is difficult to read or messy

OR

- proofs could be clearer, such as by writing the contrapositive out clearly in one line

OR

- there are minor mistakes that do not change the validity of the proof

OR

- proofs do not show enough work

- 4 pts Missing OR very incorrect OR incomplete

2 need to prove this

Question #8

2.6.6.c)

1st – Assume there is an average of three real numbers a, b and c, which is less than one of the numbers a. We will prove that the average of three real numbers is less than one of the real numbers (a).

2nd – Then, $(a + b + c)/3 < a = a + b + c < 3a = b + c < 2a$

3rd – This is a contradiction because the sum of any three real numbers is always greater than or equal to the largest of the three numbers. Also, there is proof that a is two times that of b and c combined

4th – Therefore, our assumption that the average of a, b and c is less than on of the numbers is false

5th – Hence, since our contradiction was false, it means that the average of three real numbers is greater than or equal to at least one of the numbers is in fact true

2.6.6.d)

1st - Assume there is a smallest integer number r.

2nd – r is negative, r – 1 is also negative, since r is negative, $r > r - 1$

3rd – r – 1 is a smaller integer than r

4th – This contradicts the assumption that r is the smallest integer.

5th – Therefore, there is no smallest integer

Question #9

2.7.2.b)

Case 1 -

1st – x and y are both even, we can express them as $x = 2a$ and $y = 2b$ for some integer a and b

2nd – $x + y = 2a + 2b = 2(a + b)$, since a and b are integers, we have proven that the result is even.

Case 2 -

1st – x and y are both odd, we can express them as $x = 2a + 1$ and $y = 2b + 1$ for some integer a and b

2nd – $x + y = 2a + 1 + 2b + 1 = 2(a + b) + 2$, since a and b are integers, we have proven the result is even.

4 Q8 **4 / 4**

✓ **- 0 pts** *Correct; good job!*

- 1 pts Incorrect or insufficiently explained math / properties / proof strategy for 2.6.6 (c)

- 0.5 pts 2.6.6 (c) proof is a little unclear; could use a little further explanation to accompany math

- 1 pts Incorrect or insufficiently explained math / properties / definitions for 2.6.6 (d)

- 0.5 pts 2.6.6 (d) proof is a little unclear; could use a little further explanation to accompany math

- 0.25 pts didn't state that example of smaller integer was an integer (2.6.6(d)) (i.e. should state i - 1 is also an integer since i is an integer), need this to prove full claim

- 2 pts 2.6.6(c) completely incorrect or missing

- 2 pts 2.6.6(d) completely incorrect or missing

- 4 pts Not complete or not submitted

- 4 pts Not Tagged

- 100 pts Do not copy someone else's work!

Question #8

2.6.6.c)

1st – Assume there is an average of three real numbers a, b and c, which is less than one of the numbers a. We will prove that the average of three real numbers is less than one of the real numbers (a).

2nd – Then, $(a + b + c)/3 < a = a + b + c < 3a = b + c < 2a$

3rd – This is a contradiction because the sum of any three real numbers is always greater than or equal to the largest of the three numbers. Also, there is proof that a is two times that of b and c combined

4th – Therefore, our assumption that the average of a, b and c is less than on of the numbers is false

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Question #9

2.7.2.b)

Case 1 -

1st – x and y are both even, we can express them as $x = 2a$ and $y = 2b$ for some integer a and b

2nd – $x + y = 2a + 2b = 2(a + b)$, since a and b are integers, we have proven that the result is even.

Case 2 -

1st – x and y are both odd, we can express them as $x = 2a + 1$ and $y = 2b + 1$ for some integer a and b

2nd – $x + y = 2a + 1 + 2b + 1 = 2(a + b) + 2$, since a and b are integers, we have proven the result is even.

Finally, in either case, if x and y have the same parity, x + y will always be even

5 Q9 **2 / 2**

✓ **- 0 pts** *Correct*

- **- 1 pts** Failed to prove one of two use cases, or error in proof logic
- **- 0.5 pts** Mostly correct, needs a bit more to be robust
- **- 0.5 pts** Other minor error
- **- 2 pts** Late assignment
- **- 2 pts** Proof entirely incorrect or missing

6 Extra credit for typing **5 / 0**

✓ **+ 5 pts** *Entirely typed*

+ 0 pts Not entirely typed

7 Tagging / Starting questions on new pages / Formatting equations 0 / 0

✓ **- 0 pts** *Correct*

- 10 pts Untagged / Did not start questions on new pages / Too many unformatted equations