WI23_CSBR-NY_1_NC_INT2 HW5 (Q3 to Q5)

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TOTAL POINTS

24 / 30

QUESTION 1

1 Q3 6.5 / 7

✓ **- 0 pts** *Correct for parts not otherwise noted*

- 7 pts Incorrect/Missing

- 1 pts A: 4.1.3 B

Answer: Not a function. f is not well-defined for x=2. I.e. 1/0 is undefined and not a real number, therefore not mapping to the target

Note on interval notation with infinity: use \[0, inf) more info here https://web.nmsu.edu/~kberver/Unit1/Unit19.html

- 1 pts B: 4.1.3 C

Answer: f is a function. f is well defined. the range is the set of all non-negative reals

✓ **- 0.5 pts** *B: 4.1.3 C correct but missing range/incorrect range. The range is the set of all non-negative reals.*

- 1 pts B: 4.1.5 B

Answer: {4, 9, 16, 25}

- 1 pts B: 4.1.5 D

Answer: {0, 1, 2, 3, 4, 5}

- 1 pts B: 4.1.5 H

Answer: { (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }
i.e. flip the order of the elements of each cartesian product

- 1 pts B: 4.1.5 I

Answer: { (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4) }

- 1 pts B: 4.1.5 L

Answer: { ∅, {2}, {3}, {2,3} }

Note: {∅} != ∅

QUESTION 2

2 Q4 8 / 11

- 0 pts Correct

- 1 pts 4.2.2 C Not onto. For example, there is no integer x, such that x3 = 2. One-to-one.

- 1 pts 4.2.2 G One-to-one, but not onto. There is no pair (x, y) such that f(x, y) = (1, 1).

- 1 pts 4.2.2.K Neither one-to-one, nor onto. There is no (x, y) such that f(x, y) = 1. f(2, 2) = f(1, 4).

- 1 pts 4.2.4 B. Neither one-to-one nor onto. The function f is not one-to-one because, for example,

f(000) = f(100) = 100. The function f is not onto because there is no input x such that f(x) starts with a 0. For example, there is no x such that f(x) = 000.

- 1 pts 4.2.4 C. One-to-one and onto.

- 1 pts 4.2.4 D. One-to-one, but not onto. The output string always has the property that the first bit is the same as the last bit, so there is no x, for example, such that f(x) = 1000.

- 1 pts 4.2.4 G. The function is not one-to-one. For example f({1,2}) = f({2}) = {2}. The function is not onto because the output set never contains 1 as an element. So, for example, there is no X ⊆ A such that f(X) = {1}.

✓ **- 1 pts** *II A. Function does fulfill the conditions of being one-to-one, but not onto*

Example:

$$x^2 + 1, x \ge 0$$

$$x^2 + 2, x < 0$$

- 1 pts II B. Function does fulfill the conditions of being onto but not one-to-one

Example f(x) = |x| + 1

✓ **- 1 pts** *II C. Function does fulfill the conditions of being one-to-one and onto*

Example:

$$2x , x > 0$$

$$-2x + 1 , x \le 0$$

✓ **- 1 pts** *II D. Function does not fulfill the conditions of being neither one-to-one nor onto*

Example:

$$f(x) = x^2 + 1$$

- 11 pts No submission

+ 1 pts Adjustment for the same mistake

- 11 pts Incorrectly Tagged or not Typed

1 if x=-1 f(x)=-1; not in target

QUESTION 3

3 Q5 **9 / 12**

- 0 pts Correct

- 1 pts 4.3.2C is incorrect. Answer is

f^-1 (x) = (x - 3)/2

note: f^-1 (x) = (y - 3)/2 or similar is wrong. y is unbound in this case so its incorrect

✓ **- 1 pts** *4.3.2D is incorrect. Answer is*

Not well-defined. The function f is not one-to-one.

- 1 pts 4.3.2G is incorrect. Answer is

f^-1 = f. For x ∈ {0, 1}^3, f(x) = y if and only if f(y) = x.

✓ **- 1 pts** *4.3.2I is incorrect. Answer is*

f^-1 (x, y) = (x-5, y+2)

- 1 pts 4.4.8C is incorrect. Answer is

f o h(x) = 2x^2 + 5

- 1 pts 4.4.8D is incorrect. Answer is

h o f(x) = 4x^2 + 12x + 10

- 1 pts 4.4.2B is incorrect. Answer is

121

- 1 pts 4.4.2C is incorrect. Answer is

justification is incorrect/missing

16

✓ **- 1 pts** *4.4.2D is incorrect. Answer is*

h o f(x) = ceiling(x^2 /5)

- 1 pts 4.4.6C is incorrect. Answer is

111

- 1 pts 4.4.6D is incorrect. Answer is

{101, 111}

- 1 pts 4.4.6E is incorrect. Answer is

{001, 011, 101, 111}

- 12 pts No Submission

QUESTION 4

4 Extra credit Question **0.5 / 0**

- 0 pts Both incorrect/Did not attempt/Missing/Nothing Selected

+ 1 pts 4.4.4c correct. The answer should be something similar to "No. We will show that if f is not one-to-one, then g o f is not one-to-one. If f is not one-to-one, then there are x, x' ∈ X, such that x ≠ x' and f(x) = f(x'). Let y = f(x) = f(x'). g o f(x) = g(f(x)) = g(y). g o f(x') = g(f(x')) = g(y). Therefore g o f(x) = g o f(x') and therefore g o f is not one-to-one."

+ 1 pts 4.4.4d correct. The answer is yes and must also include a diagram or description of such a function.

✓ **+ 0.5 pts** *4.4.4C has correct answer ("no") but justification is incorrect/missing*

+ 0.5 pts 4.4.4D has correct answer ("yes") but

Question #3:

a) 4.1.3

b) Function is not well defined for x = 2

c) Function is well defined, f: R -> R, where $f(x) = |x| = \sqrt{x^2}$

b) 4.1.5

b) {4, 9, 16, 25}

d) {0, 1, 2, 3, 4, 5}

h) { (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }

i) { (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4) }

l) { {}, {2}, {3}, {2,3} }

1 Q3 6.5 / 7

✓ **- 0 pts** *Correct for parts not otherwise noted*

- 7 pts Incorrect/Missing

- 1 pts A: 4.1.3 B

Answer: Not a function. f is not well-defined for x=2. I.e. 1/0 is undefined and not a real number, therefore not mapping to the target

Note on interval notation with infinity: use \[0, inf)

more info here https://web.nmsu.edu/~kberver/Unit1/Unit19.html

- 1 pts B: 4.1.3 C

Answer: f is a function. f is well defined. the range is the set of all non-negative reals

✓ **- 0.5 pts** *B: 4.1.3 C correct but missing range/incorrect range. The range is the set of all non-negative reals.*

- 1 pts B: 4.1.5 B

Answer: {4, 9, 16, 25}

- 1 pts B: 4.1.5 D

Answer: {0, 1, 2, 3, 4, 5}

- 1 pts B: 4.1.5 H

Answer: { (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }

i.e. flip the order of the elements of each cartesian product

- 1 pts B: 4.1.5 I

Answer: { (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4) }

- 1 pts B: 4.1.5 L

Answer: { ∅, {2}, {3}, {2,3} }

Note: {∅} != ∅

Question #4 – Part 1:

a) 4.2.2

c) One-to-one but not onto, because there is no integer x, such that $x^3 = 2$

g) One-to-one but not onto, because there is no such (x,y) such that f(x,y) = (0, 1)

k) Neither One-to-one or onto

-Not onto because there is no such (x,y) such that f(x,y) = 1

-Not one-to-one because f(2,2) = f(1,4)

b) 4.2.4

b) Neither one-to-one nor onto

-Not onto because there is no such input x such that f(x) starts with a 0, no x such that f(x) = 000

-Not one-to-one because f(000) = f(100) = 100

c) One-to-one and onto

d) One-to-one but not onto, there is no such input x such that f(x) ends with a 0, such that f(x) = 1000

g) Neither one-to-one nor onto

-Not onto because the subset {1} in the co-domain is not equal to any subset in the domain since f({1}) = {}

-Not one-to-one because subsets {2,3,4} and {1,2,3,4} both map to the same subset of {2,3,4}

Question #4 – Part 2:

a) One-to-one but not onto = f: Z -> Z^+, where f(x) = 2x + 1

b) Onto but not one-to-one = f: Z -> Z^+, where f(x) = |x| + 1

c) One-to-one and onto = f: Z -> Z^+, where f(x) = x +1

d) Neither one-to-one nor onto = f: Z -> Z^+, where f(x) = -2

2 Q4 **8 / 11**

- 0 pts Correct

- 1 pts 4.2.2 C Not onto. For example, there is no integer x, such that x3 = 2. One-to-one.

- 1 pts 4.2.2 G One-to-one, but not onto. There is no pair (x, y) such that f(x, y) = (1, 1).

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Example:

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$$f(x) = x^2 + 1$$

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+ 1 pts Adjustment for the same mistake

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1

if x=-1 f(x)=-1; not in target

Question #5:

A) 4.3.2

c) $f^{-1}(x) = (x - 3) / 2$

d)

g) $f^{-1} = f$, for $x \in \{0,1\}^3$, $f(x) = y$ if and only if $f(y) = x$

i)

B) 4.4.8

c)$f \circ h(x) = 2(x^2 + 1) + 3 = 2x^2 + 2 + 3 = 2x^2 + 5$

d)$h \circ f(x) = (2x + 3)^2 + 1 = (4x^2 + 6x + 6x + 9) + 1 = 4x^2 + 12x + 10$

C) 4.4.2

b) $(52/5)^2 = 11^2 = 121$

c) $2^{((4\text{^}2)/5)} = 2^{(16/5)} = 2^4 = 16$

d) $(x^2)/5$

D) 4.4.6

c) first f(010) = 110, then h(110) = 111

d) {101, 111}

e) {001, 011, 101, 111}

E) 4.4.4

c) No, because f and g are bijections of each other which implies they are both one-to-one and onto

d) No, because f and g are bijections of each other which implies they are both one-to-one and onto

3 Q5 **9 / 12**

- 0 pts Correct

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note: f^-1 (x) = (y - 3)/2 or similar is wrong. y is unbound in this case so its incorrect

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- **12 pts** No Submission

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c) $f^{-1}(x) = (x - 3) / 2$

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B) 4.4.8

c)$f \circ h(x) = 2(x^2 + 1) + 3 = 2x^2 + 2 + 3 = 2x^2 + 5$

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C) 4.4.2

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c) $2^{((4\text{^}2)/5)} = 2^{(16/5)} = 2^4 = 16$

d) $(x^2)/5$

D) 4.4.6

c) first f(010) = 110, then h(110) = 111

d) {101, 111}

e) {001, 011, 101, 111}

E) 4.4.4

c) No, because f and g are bijections of each other which implies they are both one-to-one and onto

d) No, because f and g are bijections of each other which implies they are both one-to-one and onto

4 Extra credit Question 0.5 / 0

- 0 pts Both incorrect/Did not attempt/Missing/Nothing Selected

+ 1 pts 4.4.4c correct. The answer should be something similar to "No. We will show that if f is not one-to-one, then g o f is not one-to-one. If f is not one-to-one, then there are x, x' ∈ X, such that x ≠ x' and f(x) = f(x'). Let y = f(x) = f(x'). g o f(x) = g(f(x)) = g(y). g o f(x') = g(f(x')) = g(y). Therefore g o f(x) = g o f(x') and therefore g o f is not one-to-one."

+ 1 pts 4.4.4d correct. The answer is yes and must also include a diagram or description of such a function.

✓ **+ 0.5 pts** *4.4.4C has correct answer ("no") but justification is incorrect/missing*

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