WI23_CSBR-NY_1_NC_INT2 HW11 (Q5 & Q6)

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TOTAL POINTS

10 / 28

QUESTION 1

1 Q5 **5 / 10**

✓ **- 5 pts** *Part b is incorrect or missing. See comments. The idea is if k+1 is prime, it's a product of one prime: itself. If it's not prime, its factors are products of primes by the inductive hypothesis. So k+1 is the product of several numbers, each of which is the product of primes.*

QUESTION 2

2 Q6 **5 / 18**

✓ **- 1 pts** *7.4.1 F the inductive hypothesis is P(k)*

✓ **- 4 pts** *7.4.1 G*

✓ **- 4 pts** *7.4.3 C*

✓ **- 4 pts** *7.5.1 A Did not clearly prove that 4 evenly divides 3^(2k) -1*

Question 5:

a) Using mathematical Induction

Base Case: $n = 1$;
$n^3 + 2n = 1^3 + 2 * 1 = 3$

Therefore, for $n = 1$, 3 divides $n^3 + 2n$ is true

Inductive Step: We will show that for any positive int $k >= 1$, if 3 divides $k^3 + 2k$, then 3 also divides $(k + 1)^3 + 2(k + 1)$.

$(k + 1)^3 + 2(k + 1) = k^3 + 3k^2 + 3k + 1 + 2k + 2$

Next,

$k^3 + 3k^2 + 3k + 1 + 2k + 2 = (k^3 + 2k) + (3k^2 + 3k + 3)$

By induction, we know that 3 divides $k^3 + 2k$, 3 also divides $(3k^2 + 3k + 3)$.
Thus, we prove that 3 divides $(k + 1)^3 + 2(k + 1)$.

b)

1 Q5 **5 / 10**

✓ **- 5 pts** *Part b is incorrect or missing. See comments. The idea is if k+1 is prime, it's a product of one prime: itself. If it's not prime, its factors are products of primes by the inductive hypothesis. So k+1 is the product of several numbers, each of which is the product of primes.*

Question 6:

1) Exercise 7.4.1

a) P(3)
   (3(3 + 1)(2*3 + 1)) / 6 = 14
   14 = 1^2 + 2^2 + 3^2

b) Express P(k)
   (k(k + 1)(2k + 1)) / 6

c) Express P(k + 1)
   ((k + 1)(k + 2)(2k + 3)) / 6

d) P(1) must be proven to be true in the base case

e) For all positive ints k, P(k) implies P(k + 1)

f) P(k)

2 Q6 **5 / 18**

- ✓ **- 1 pts** *7.4.1 F the inductive hypothesis is P(k)*
- ✓ **- 4 pts** *7.4.1 G*
- ✓ **- 4 pts** *7.4.3 C*
- ✓ **- 4 pts** *7.5.1 A Did not clearly prove that 4 evenly divides 3^(2k) -1*