WI23_CSBR-NY_1_NC_INT2 HW8 (Q7 to Q10)

Aaron Bengochea

TOTAL POINTS

23 / 26

QUESTION 1

1 Q7 **7 / 7**

✓ **- 0 pts** *Correct*

QUESTION 2

2 Q8 **8 / 8**

✓ **- 0 pts** *Correct*

QUESTION 3

3 Q9 **5 / 6**

✓ **- 1 pts** *6.6.4b is incorrect. Answer:*

E\[Y] = 0·(1/8) + 1·(3/8) + 4·(3/8) + 9·(1/8) = 24/8 = 3

QUESTION 4

4 Q10 **3 / 5**

✓ **- 1 pts** *6.8.1.d completely incorrect or incomplete:*

expected number of batches with defects: 50·(0.01) = 0.5. There are two circuit boards in each batch, so the expected number of boards with defects is (0.5)·2 = 1. The expectation is the same as the case in which the boards are made separately.

There are at least two circuit boards with defects unless none of the batches have defects. The probability that at least two circuit boards have defects is $$1 - (0.99)^{50} ≈ 0.395$$. It is much more likely (probability 0.395 compared to probability 0.264) that there are at least two boards with defects in the situation in which boards are made in batches.

✓ **- 1 pts** *6.8.3.b incorrect. The correct conclusion is reached if there are fewer than four heads. The probability that there are 0, 1, 2, or 3 heads is approximately 0.65*

The probability that the incorrect conclusion is reached is approximately 0.35.

$$(0.7)^{10} + {10 \choose 1}(0.7)^{9}(0.3) + {10 \choose 2}(0.7)^{8}(0.3)^2 + {10 \choose 3}(0.7)^{7}(0.3)^3 \approx 0.65$$

Answer: probability of incorrect conclusion reached is : 1- 0.65 = 0.35

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Question #7

Section A: 6.1.5

b) $C(13,1) * C(4,3) * C(12,2) * C(4,1) * C(4,1) = 13 * 4 * 66 * 4 * 4 = 54{,}912$
$C(52,5) = 2{,}598{,}960$
Three of a Kind Probability = 54,912 / 2,598,960 = 0.021128

c) $C(4,1) * C(13,5) = 4 * 1{,}287 = 5{,}148$
$C(52,5) = 2{,}598{,}960$
5 Same Suits Probability = 5,148 / 2,598,960 = 0.00198

d) $C(13,1) * C(4,2) * C(12,3) * C(4,1) * C(4,1) * C(4,1) = 13 * 6 * 220 * 4 * 4 * 4 = 1{,}098{,}240$
$C(52,5) = 2{,}598{,}960$
Single Pair Probability = 1,098,240 / 2,598,960 = 0.42256

Section B: 6.2.4

a) Solution by compliment
$C(39,5) = 575{,}757$
$C(52,5) = 2{,}598{,}960$
No Clubs Probability = 575,757 / 2,598,960 = .22153
At Least One Club = 1 - 0.22153 = 0.778466

b) Solution by compliment
$C(13,5) * C(4,1) * C(4,1) * C(4,1) * C(4,1) * C(4,1) = 1{,}287 * 4 * 4 * 4 * 4 * 4 = 1{,}317{,}888$
$C(52,5) = 2{,}598{,}960$
No Two Cards have the same rank = 1,317,888 / 2,598,960 = 0.50708
At Least One Pair = 1 - 0.50708 = 0.492917

c)
Exactly One Club = Exactly One Spade = $C(13,1) * C(39,4) = 13 * 82{,}251 = 1{,}069{,}263$
One Club and One Spade = $C(13,1) * C(13,1) * C(26,3) = 13 * 13 * 2{,}600 = 439{,}400$
$C(52,5) = 2{,}598{,}960$
Exactly One Club or Exactly One Spade = ((2 * 1,069,263) - 439,400) / 2,598,960
Exactly One Club or Exactly One Spade = 1,699,126 / 2,598,960 = 0.65377

d) Solution by compliment
$C(26,5) = 65{,}780$
$C(52,5) = 2{,}598{,}960$
Hand with no Spades and no Clubs = 65,780 / 2,598,960 = 0.025310
Hand with at least One Spade or at least One Club = 1 - 0.025310 = 0.974689

1 Q7 **7 / 7**

✓ **- 0 pts** *Correct*

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Question #8

Section A: 6.3.2

a) $|A| = 6!$
$P(A) = 6! / 7! = 1 / 7$ or 0.14285

$|B| = 7! / 2$
$P(B) = 7! / 2 * 7! = 1 / 2$ or 0.50

$|C| = 5 * 4! = 5!$
$P(C) = 5! / 7! = 1 / 7 * 6 = 1 / 42$ or 0.023809

b) $|A \cap C| = 2 * 3! = 12$
$P(A|C) = |A \cap C| / |C| = 12 / 5! = 1 / 5 * 2 * 1 = 1 / 10$ or 0.1000

c) $|B \cap C| = 5! / 2$
$P(B|C) = |B \cap C| / |C| = 5! / 2 * 5! = 1 / 2$ or 0.5000

d) $|A \cap B| = 3 * 5!$
$P(A|B) = |A \cap B|/|B| = 3 * 5! / 7! / 2 = 2 * 3 * 5! / 7! = 2 * 3 / 7 * 6 = 6 / 42 = 1 / 7$ or 0.1428

e) $P(A|C) = 1 / 10$ while $P(A) = 1 / 7$ therefore they are not independent
$P(B|C) = 1 / 2$ while $P(B) = 1 / 2$ therefore they are independent
$P(A|B) = 1 / 7$ while $P(A) = 1 / 7$ therefore they are independent

Section B: 6.3.6

b) First 5 Heads = $(1 / 3) ^ 5 = 0.00411$
Last 5 Tails = $(2 / 3) ^ 5 = 0.13168$
Combined Probability = $0.00411 * 0.13168 = 0.00054$

c) First 1 Heads = $1 / 3 = 0.333$
Last 9 Tails = $(2 / 3) ^ 9 = 0.0260$
Combined Probability = $0.333 * 0.0260 = 0.00865$

Section C: 6.4.2

a) $P(F) = 1 / 2 = 0.5$
Biased Dice = $P(Y|F) = (.15 ^ 4) * (.25 ^ 2) = 0.000506 * 0.0625 = 0.0000316$
Fair Dice = $P(Y|F) = (1 / 6) ^ 6 = 0.0000214$

Bayes' Theorem → $P(F|Y) = P(Y|F) * P(F) / P(Y|F)* P(F) + P(Y|-F) * P(-F)$
Fair Dice Choice $P(F|Y) = 0.0000214 * 0.5 / 0.0000214 * 0.5 + 0.0000316 * 0.5$
Fair Dice Choice $P(F|Y) = 0.0000107 / 0.0000107 + 0.0000158 = 0.40377$

2 Q8 **8 / 8**

✓ **- 0 pts** *Correct*

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Question #9

Section A: 6.5.2

a) Range of A = {0,1,2,3,4}

b) D = C(5, x) * C(47, 4 - x) / C(52,4)
P(D = 0) = 1 * 178,365 / 270,725 = 0.658841
P(D = 1) = 5 * 16,215 / 270,725 = 0.299473
P(D = 2) = 10 * 1,081 / 270,725 = 0.039929
P(D = 3) = 10 * 47 / 270,725 = 0.001736
P(D = 4) = 5 * 1 / 270,725 = 0.000018

D = 0.658841 + 0.299473 + 0.039929 + 0.001736 + 0.000018 = 1

Section B: 6.6.1

a) Sample size = C(10,2) = 45
Number of ways to select 2 girls = C(7,2) = 21
Number of ways to select 1 girl and 1 boy = 7 * 3 = 21
Number of ways to select 0 girls = C(3,2) = 3
P(G = 2) = 21 / 45 = 7 / 15
P(G = 1) = 21 / 45 = 7 / 15
P(G = 0) = 3 / 45 = 1 / 15

E[G] = (2 * (7 / 15)) + (1 * (7 / 15)) + (0 * (1 / 15)) = 21 / 15 = 7 / 5 = 1.40

Section C: 6.6.4

a) E[X] = (1 * (1 / 6)) + (4 * (1 / 6)) + (9 * (1 / 6)) + (16 * (1 / 6)) + (25 * (1 / 6)) + (36 * (1 / 6))
E[X] = 0.1666 + 0.6666 + 1.5 + 2.6666 + 4.1666 + 6
E[X] = 15.1664

b) Y = 0 → {TTT}
Y = 1 → {TTH, THT, HTT}
Y = 4 → {HHT, HTH, THH}
Y = 9 → {HHH}
Total Outcomes = 2^3 = 8

E[Y] = (0 * (1 / 8)) + (1 * (3 / 8)) + (4 * (3 / 8)) + (9 * (1 / 8))
E[Y] = 0 + 0.375 + 1.5 + 3.375
E[Y] = 5.25

Section D: 6.7.4

a) E[C1] = (1 * (1 / 10)) + (0 * (1 / 10)); Total Children = 10
E[C1] = 10 * (1 / 10) = 1

3 Q9 **5 / 6**

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$E\[Y] = 0\cdot(1/8) + 1\cdot(3/8) + 4\cdot(3/8) + 9\cdot(1/8) = 24/8 = 3$

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Question #10

Section A: 6.8.1

a) Exactly 2 Defects = B(2, 100, .01)
B(2, 100, .01) = C(100,2) * (.01)^2 * (.99)^98 = 4,950 * 0.0001 * 0.37346 = 0.1848

b) B(1, 100, .01) = C(100,1) * (.01)^1 * (.99)^99 = 100 * 0.01 * 0.36972 = .36972
B(0, 100, .01) = C(100,0) * (.01)^0 * (.99)^100 = 1 * 1 * 0.36603 = 0.36603
At least 2 Defects = 1 - B(1, 100, .01) - B(0, 100, .01)
At least 2 Defects = 1 - .36972 - 0.36603 = 0.26425

c) N = 100, p = .01
Expected circuits with defects = 100 / 100 = 1

d)

Section B: 6.8.3

b)

4 Q10 **3 / 5**

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