WI23_CSBR-NY_1_NC_INT2 HW7 (Q3 to Q7)

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TOTAL POINTS

28 / 37

QUESTION 1

1 Q3 **1 / 6**

- 0 pts All correct

✓ **- 1 pts** *8.2.2b incorrect*

- 0.5 pts 8.2.2 correct but lacks sufficient detail / explanation

or, did not show $$f(n) = \Omega(n^{3})$$

or, did not show $$f(n) = O(n^{3})$$

*Review ZyBooks Section 8.2

✓ **- 1 pts** *8.3.5a incorrect.*

correct:

The algorithm sorts the sequence so that all the numbers that are less than the input value p appear before all the numbers that are greater than or equal to p.

- 0.5 pts 8.3.5a incorrect. explained what the algorithm does step by step but not what it does in the grand scheme, i.e. what it accomplishes

Or, gave vague description of what happens without explicitly saying that numbers less than $$p$$ end up to the left of $$p$$ while numbers greater than $$p$$ end up to the right of $$p$$ (sorting data)

✓ **- 0 pts** *8.3.5b incorrect.*

correct:

The number of times i is incremented or j is decremented is exactly n-1, regardless of the values in the input sequence.

✓ **- 1 pts** *8.3.5c incorrect.*

correct:

Number of swaps does depend on input. Number of swaps is at minimum 0 and at maximum n/2.

- 0.5 pts 8.3.5 C: had only minimum number of times OR had only maximum number of times swapped.

MIN: 0

MAX: $$\frac{n}{2}$$

- 0.5 pts 8.3.5 C: explained what would maximize/minimize input but did not give a solution.

MIN: 0

MAX: $$\frac{n}{2}$$

✓ **- 1 pts** *8.3.5d incorrect.*

correct:

The number of times i is incremented or j is decremented is exactly n, so the time complexity of the algorithm is Ω(n).

- 0.5 pts 8.3.5 d: used Big O or Big Theta instead of Big Omega for lower bound

or, did not use any Big-O notation and just wrote

$$n$$

✓ **- 1 pts** *8.3.5e incorrect.*

correct:

O(n).

- 0.5 pts 8.3.5 e: used Big Omega or Big Theta instead of Big O for upper bound.

or, did not use any Big-O notation and just wrote $$n$$

- 6 pts Not submitted/handwritten/not tagged/completely incorrect

QUESTION 2

2 Q4 7 / 7

✓ **- 0 pts** *All correct*

- 1 pts Incorrect 5.1.2 b. Correct answer: 40^7 + 40^8 + 40^9 = 2.6886 x 10^14

- 1 pts Incorrect 5.1.2 c. Correct answer: 14(40^6 + 40^7 + 40^8) = 9.41 x 10^13

- 1 pts Incorrect 5.3.2 a. Correct answer: 3* 2^9 (1536). Three options for the first character (3), two options for every additional character (2^9)

- 1 pts Incorrect 5.3.3 b. Correct answer: 10 * 9 * 8 * 26^4 (329,022,720) (720* (26^4))

- 1 pts Incorrect 5.3.3 c. Correct answer: 10 * 9 * 8 * 26 * 25 * 24 * 23) (258336000)

- 1 pts Incorrect 5.2.3 a.

- Correct Answer: Let x ∈ B9. If the number of 1's in x is even, then f(x) = x0. If the number of 1's in x is odd, then f(x) = x1.

- Need to expand on the definition of a bijection and explain how the function is BOTH one-to-one and onto

- 1 pts Incorrect 5.2.3 b. Correct answer: |E10| = |B9| = 2^9 (512)

- 0.5 pts - Need to explain a bit more for 5.2.3 A

- or: Need to show how to go from a string in B9 to a string in E10 (Let x ∈ B9. If the number of 1's in x is even, then f(x) = x0. If the number of 1's in x is odd, then f(x) = x1.)

- or: Needed to show one-to-one AND onto but only explained one (or minimally explained both):

-- For x, y ∈ B9, if x ≠ y, then f(x) ≠ f(y) because two different binary strings can not be made equal by adding a bit to the end of the strings. Therefore f is one-to-one.

-- Let y be a string in E10. Let x be the string consisting of the first 9 bits of y. If the last bit of y is 0, then the number of 1's in x must be even and f(x) = x0 = y. If the last bit of y is 1, then the number of 1's in x must be odd and f(x) = x1 = y. Therefore, f is onto.

- 0.5 pts 5.2.3 B: stated that $$|E_{10}| = |B_{9}|$$ but did not state the actual numerical value (which is $$2^{9}$$ or 512)

- 7 pts Not submitted/not tagged/handwritten/all incorrect

QUESTION 3

3 Q5 12 / 16

- 0 pts Correct

- 1 pts 5.4.2 A There are two choices for the first three digits (824 or 825). Each of the remaining 4 digits can be any one of the ten digits, so there are 10^4 ways to pick the last four digits. The total number of 7-digit phone numbers that start with 824 or 825 is **2 x 10^4 = 20,000**

- 1 pts 5.4.2 B There are two choices for the first three digits (824 or 825). In selecting the remaining 4 digits, 4 digits are selected from a set of 10 with no repetitions, so there are P(10, 4) choices. The total number of 7-digit phone numbers that start with 824 or 825 and have no repeated digits among the last four digits is **2 x P(10, 4) = 2 x 10 x 9 x 8 x 7 =10,080**

- 1 pts 5.5.3 A Two choices for each of the 10 bits: **2^10 = 1024** binary strings of length 10.

- 1 pts 5.5.3 B The first three bits of the string are determined. Each of the remaining 7 bits can be 0 or 1. Therefore there are **2^7 = 128** binary strings of length 10 that start with 001.

- 1 pts 5.5.3 C There are 2^7 10-bit strings that begin with 001. There are 2^8 10-bit strings that begin with 10. A string can not begin with 001 and 10, so the sum rule applies. Therefore the number of 10-bit strings that begin with 001 or 10 is **2^7 + 2^8 = 384**.
Alternate notation: 3*2^7

✓ **- 1 pts** *5.5.3 D Each of the first 8 bits can be either 0 or 1. Once the the first 8 bits are determined, the last two bits must match the first two bits, so there are no remaining choices for the string. Thus, the number of strings in which the first two bits are the same as the last two bits is 2^6*2^2= **2^8 = 256***

- 1 pts 5.5.3 E There are (10, choose 6) ways to select where the six 0's will be placed among the ten possible locations. Once the 0's are placed, the four 1's go in the unfilled locations. Therefore the number of 10-bit strings with exactly six 0's is **(10 choose 6)=210**. Alternate notation: 10 choose 4 = 210

- 1 pts 5.5.3 F The six 0's can be placed in any of the 10 locations, except the first location. There are (9, Choose 6) ways to select the six locations for the 0's among the last nine locations in the string. Once the 0's are placed, the four 1's go in the unfilled locations. Therefore the number of 10-bit strings with exactly six 0's and a 1 in the first location is **(9 choose 6) = 84**. Alternate notation: 9 choose 3 = 84

- 1 pts 5.5.3 G There are (5, Choose 1) ways to select the location for the 1 among the first five locations. There are (5, Choose 3) ways to select the location for the three 1's among the last five locations. Once the 1's are placed, the 0's go in all the unfilled locations. Since the location for the 1 in the first half and the locations for the 1's in the second half must both be determined to specify the string, the product rule applies. Therefore the number of 10-bit strings with one 1 in the first half and three 1's in the second half is **(5 choose 1) * (5 choose 3) = 5*(5 choose 3) = 50**

- 1 pts 5.5.5 A

There are (30 Choose 10) ways to select a subset of 10 boys from a set of 30 boys. There are (35 Choose 10) ways to select a subset of 10 girls from a set of 35 girls. Since the choir director must select the girls and the boys for the chorus, the product rule applies, and there are a total of **(30 Choose 10)*(35 Choose 10) $$\approx 5.5 \mathrm{e}15$$** ways to make the selection.

✓ **- 1 pts** *5.5.8 C There are **(26 Choose 5) = 65780** ways to select a subset of 5 cards from the set of 26 hearts and diamonds in the deck. Alternate notation: 2 ((13 choose 1)(13 choose 4) + (13 choose*

2)(13 choose 3) + (13 choose 5))

- 1 pts 5.5.8 D There are 13 ways to select a rank. Once a rank has been chosen, all four of the cards with that rank will be in the hand. Then there are 48 ways to select the remaining card that does not have the same rank as the four already chosen. The total number of ways to select a five-card hand with four cards of the same rank is **13 x 48 = 624**

- 1 pts 5.5.8 E There are 13 ways to select the rank for the two cards with the same rank. The three cards with the same rank must have a different rank than the rank chosen for the pair, so there are 12 ways to select the rank for the three cards with the same rank. Once the rank has been chosen for the two cards that have the same rank, there are (4 choose 2) ways to select two cards from the four cards with that rank. Once the rank has been chosen for the three cards that have the same rank, there are (4 choose 3) ways to select three cards from the four cards with that rank. The choices are put together by the product rule, so that the number of ways to select a hand that is full house is:

$$(13)(12){4 \choose 2}{4 \choose 3}$$ = 3744

- 1 pts 5.5.8 F There are 13 different possible ranks. The number of ways to select 5 distinct ranks from 13 possible ranks is (13 choose 5). For each rank chosen, there are four possible cards with that rank that can be selected. Therefore once the ranks have been determined, there are 4^5 ways to select the cards in the hand. The total number of ways to select a five-card hand in which no two cards have the same rank is

$$4^{5} {13 \choose 5}$$ = 1,317,888
Alternate notation: **(52*48*44*40*36)/5!**
Why is it divided by (5!)? Because of the k-to-1 rule.

- 1 pts 5.6.6 A (44 choose 5) * (56 choose 5). The committee must have 5 Demonstrators and 5 Repudiators. There are (44 choose 5) ways to select the 5 committee members from the 44 Demonstrators. There are (56 choose 5) ways to select the 5 committee members from the 44 Repudiators. Therefore there are **$${44 \choose 5}{56 \choose 5} \approx 4.148\mathrm{e}12$$** ways to select the entire committee.

- 1 pts 5.6.6 B **P(44,2) x P(56,2) = 1892 x 3080 = 5,827,360**
Alternate notations: 44 * 43 * 56 * 55 = C(44,1)*C(43,1)*C(56,1)*C(55,1)

There are 44 ways to select the speaker from the Demonstrators. Once that person is chosen, there are 43 ways to select the vice speaker from the remaining Demonstrators. Therefore, there are P(44, 2) = 44 · 43 ways to select the speaker and vice speaker from the Demonstrators. Similarly, there are P(56, 2) ways to select the speaker and vice speaker from the Repudiators. The choices are combined using the product rule because a speaker and vice speaker are selected from both parties. Therefore there are a total of P(44,2) · P(56,2) ways to select the speaker and vice speaker from the two parties.

- 16 pts Entirely incorrect/no submission/not tagged/handwritten

- 1 pts division notation instead of combination notation

✓ **- 2 pts** *insufficient work shown*

- 0.25 pts incorrect notation

QUESTION 4

4 Q6 **4 / 4**

✓ **- 0 pts** *Correct; good job*

- 1 pts 5.7.2a is incorrect or missing. Should be (52 choose 5) - (39 choose 5) or 2,023,203 or equivalent.

- 1 pts 5.7.2b is incorrect or missing. Should be (52 choose 5) - (4^5)*(13 choose 5) or 1,281,072 or equivalent.

- 1 pts 5.8.4a is incorrect or missing. Should be 5^{20} or 95,367,431,640,625 or equivalent.

- 1 pts 5.8.4b is incorrect or missing. Should be (20!)/($(4!)^5$) or C(20,4)*C(16,4)*C(12,4)*C(8,4)*C(4,4) or 305,540,235,000 or equivalent.

- 4 pts Entirely incorrect/no submission/not tagged/handwritten

QUESTION 5

5 Q7 **4 / 4**

✓ **- 0 pts** *Correct; good job*

- 1 pts Part a is incorrect. Should be 0.

- 1 pts Part b is incorrect. Should be P(5,5) or 120 or equivalent.

- 1 pts Part c is incorrect. Should be P(6, 5) or 720 or equivalent.

- 1 pts Part d is incorrect. Should be P(7,5) or 2520 or equivalent.

- 4 pts Entirely incorrect/no submission/not tagged/handwritten

- 0.1 pts 5! = 120 not 125

Question #3:

Section A: 8.2.2

b) C = 3, N_0 = 2. We will prove that for any n >= 2, f(n) <= 3 * g(n)

$$f(n) = n^3 + 3n^2 <= n^3 + 3n^2 + 4$$

For n>= 2, n^2 <= n^3, so

$$f(n) = n^3 + 3n^2 + 4 <= n^3 + 3n^3$$

Finally, $n^3 + 3n^3 = 3n^3$ = 3 * g(n). Putting the inequalities together, we get that for any n >= 2,

$$f(n) = n^3 + 3n^2 <= n^3 + 3n^2 + 4 <= n^3 + 3n^3$$

And therefore, f(n) <= 3 * g(n) which means that f = O(g) or f = O(n^3). Since the degree of f is 3, therefore f = O(n^3) implies that f = Theta(n^3)

Section B: 8.3.5

a) This algorithm swaps the position of a_i and a_j , There are a few factors that go into how many times the swap is executed. Let's assume that the input was (-5, 5) with i = 1, p = 0 and j = 2 (len of the sequence). The initial while loop would commence since I < j, the first interior while loop would commence since I < j and a_i = -5 < p, I would now be equal to 2, the remainder of the while loops would not commence since I = 2 and j = 2, hence the return statement would be –5,5.

b) the total times that each of the incrementing or decrementing statements would be executed is half the length of sequence len n because you have two different incrementors/decrements converging towards each other. The total times though, is dependent on both the len of n and the actual input of numbers within the sequence since the loops also include an and which is contingent on a_i < p and a_j > p. At a minimum, the lines would be executed 0 times if the input was only 1 number because I = j from the inception. At a maximum, you can have a long list of ascending inputs such as (-50, -45, ….. , 0, ….. , 45, 50), with a p = 0 this would cause the swap to occur n / 2 times since the number p is the middle number in the input.

c) At a minimum, the lines would be executed 0 times if the input was only 1 number because I = j from the inception. At a maximum, you can have a long list of ascending inputs such as (-50, -45, ….. , 0, ….. , 45, 50), with a p = 0 this would cause the swap to occur n / 2 times since the number p is the middle number in the input.

d)

e)

1 Q3 **1 / 6**

- 0 pts All correct

✓ **- 1 pts** *8.2.2b incorrect*

- 0.5 pts 8.2.2 correct but lacks sufficient detail / explanation

or, did not show $$f(n) = \Omega(n^{3})$$

or, did not show $$f(n) = O(n^{3})$$

*Review ZyBooks Section 8.2

✓ **- 1 pts** *8.3.5a incorrect.*

correct:

The algorithm sorts the sequence so that all the numbers that are less than the input value p appear before all the numbers that are greater than or equal to p.

- 0.5 pts 8.3.5a incorrect. explained what the algorithm does step by step but not what it does in the grand scheme, i.e. what it accomplishes

Or, gave vague description of what happens without explicitly saying that numbers less than $$p$$ end up to the left of $$p$$ while numbers greater than $$p$$ end up to the right of $$p$$ (sorting data)

✓ **- 0 pts** *8.3.5b incorrect.*

correct:

The number of times i is incremented or j is decremented is exactly n-1, regardless of the values in the input sequence.

✓ **- 1 pts** *8.3.5c incorrect.*

correct:

Number of swaps does depend on input. Number of swaps is at minimum 0 and at maximum n/2.

- 0.5 pts 8.3.5 C: had only minimum number of times OR had only maximum number of times swapped.

MIN: 0

MAX: $$\frac{n}{2}$$

- 0.5 pts 8.3.5 C: explained what would maximize/minimize input but did not give a solution.

MIN: 0

MAX: $$\frac{n}{2}$$

✓ **- 1 pts** *8.3.5d incorrect.*

correct:

The number of times i is incremented or j is decremented is exactly n, so the time complexity of the algorithm is Ω(n).

- 0.5 pts 8.3.5 d: used Big O or Big Theta instead of Big Omega for lower bound

or, did not use any Big-O notation and just wrote $$n$$

✓ **- 1 pts** *8.3.5e incorrect.*

correct:

O(n).

- 0.5 pts 8.3.5 e: used Big Omega or Big Theta instead of Big O for upper bound.

or, did not use any Big-O notation and just wrote $$n$$

- 6 pts Not submitted/handwritten/not tagged/completely incorrect

Question #4:

Section A: 5.1.2

b) $40^7 + 40^8 + 40^9$ = 163,840,000,000 + 6,553,600,000,000 + 262,144,000,000,000 = 268,861,440,000,000 total combinations

c) $(14 * 40^6) + (14 * 40^7) + (14 * 40^8)$ = 57,344,000,000 + 2,293,760,000,000 + 91,750,400,000,000 = 94,101,504,000,000 total combinations

Section B: 5.3.2

a) $3 * 2^9$ = 1,536

Section C: 5.3.3

b) 10 * 26 * 26 * 26 * 26 * 9 * 8 = 329,022,720 -> non-repeating digits

c) 10 * 26 * 25 * 24 * 23 * 9 * 8 = 258,336,000 -> non-repeating digits or letters

Section D: 5.2.3

a) Since B^9 is an odd number, we know that B if there are an odd number of 1's then there will be an even number of 0's, also we know that if there is an odd number of 0's then there will be an even number of 1's. Therefore, we know that B^9 will have either an odd or even number of 1's. Our function can determine if there is an odd number of 1's and concatenate a 1 if so, it can also determine if we have an even number of 1's and concatenate a 0, keeping the 1's always even.

Assuming each B^9 is unique, using our function described above, we would add a new bit in order to create a unique new string of 10 bits. For example, f(110010110) = 1100101101 (E_{10}), since each element in the domain maps to a unique element in the codomain, that means this function is one-to-one.

We can also perform an inverse function f^{-1} which removes the last digit from E_{10} . Since E_{10} is a set where B^9 has a 1 or 0 bit concatenated to it, we can say that removing the last bit from E_{10} produces a unique value of B^9 for example, f^{-1} (1100101101) = 110010110 (B^9), which means E_{10} can be mapped to one unique B^9 meaning the function is also onto.

Because the designed function is both one-to-one and onto, it is then also a bijection.

b) Because there is a bijection between B^9 and E_{10} , we know that $|E^{10}| = |B^9|$, since $B^9 = 2^9$, and we can independently choose the last bit then we also know that $|E^{10}| = 2^9$

2 Q4 **7 / 7**

✓ **- 0 pts** *All correct*

- 1 pts Incorrect 5.1.2 b. Correct answer: 40^7 + 40^8 + 40^9 = 2.6886 x 10^14

- 1 pts Incorrect 5.1.2 c. Correct answer: 14(40^6 + 40^7 + 40^8) = 9.41 x 10^13

- 1 pts Incorrect 5.3.2 a. Correct answer: 3* 2^9 (1536). Three options for the first character (3), two options for every additional character (2^9)

- 1 pts Incorrect 5.3.3 b. Correct answer: 10 * 9 * 8 * 26^4 (329,022,720) (720* (26^4))

- 1 pts Incorrect 5.3.3 c. Correct answer: 10 * 9 * 8 * 26 * 25 * 24 * 23) (258336000)

- 1 pts Incorrect 5.2.3 a.

- Correct Answer: Let x ∈ B9. If the number of 1's in x is even, then f(x) = x0. If the number of 1's in x is odd, then f(x) = x1.
- Need to expand on the definition of a bijection and explain how the function is BOTH one-to-one and onto

- 1 pts Incorrect 5.2.3 b. Correct answer: |E10| = |B9| = 2^9 (512)

- 0.5 pts - Need to explain a bit more for 5.2.3 A

- or: Need to show how to go from a string in B9 to a string in E10 (Let x ∈ B9. If the number of 1's in x is even, then f(x) = x0. If the number of 1's in x is odd, then f(x) = x1.)
- or: Needed to show one-to-one AND onto but only explained one (or minimally explained both):

-- For x, y ∈ B9, if x ≠ y, then f(x) ≠ f(y) because two different binary strings can not be made equal by adding a bit to the end of the strings. Therefore f is one-to-one.

-- Let y be a string in E10. Let x be the string consisting of the first 9 bits of y. If the last bit of y is 0, then the number of 1's in x must be even and f(x) = x0 = y. If the last bit of y is 1, then the number of 1's in x must be odd and f(x) = x1 = y. Therefore, f is onto.

- 0.5 pts 5.2.3 B: stated that $$|E_{10}| = |B_{9}|$$ but did not state the actual numerical value (which is $$2^{9}$$ or 512)

- 7 pts Not submitted/not tagged/handwritten/all incorrect

Question #5:

Section A: 5.4.2

a) 2 * 10 * 10 * 10 * 10 = 20,000

b) 2 * (10 * 9 * 8 * 7) = 2 * P(10,4) = 10,080

Section B: 5.5.3

a) 2^{10}= 1,024

b) 2^7 = 128

c) $2^7 + 2^8$ = 384

d) 2^6 = 64

e) C(10,6) = 10! / 6!(4!) = 210

f) C(9,6) = 9! / 6!(3!) = 84

g) C(5,1) * C(5,3) = (5! / 1!(4!)) * (5! / 3!(2!)) = 5 * 10 = 50

Section C: 5.5.5

a) C(30,10) * C(35,10) = (30! / 10!(20!)) * (35! / 10!(25!)) = 30,045,015 * 183,579,396 = 5,515,645,706,510,940

Section D: 5.5.8

c) C(13,5) * 2 = (13! / 5!(8!)) * 2 = 1,287 * 2 = 2,574

d) C(13,1) * C(48,1) = 13 * 48 = 624

e) C(13,1) * C(4,3) * C(12,1) * C(4,2) = 13 * 4 * 12 * 6 = 3744

f) C(13,5) * 4^5 = 1,287 * 1024 = 1,317,888

Section E: 5.6.6

a) C(44,5) * C(56,5) = (44! / 5! * 39!) * (56! / 5! * 51!) = 1,086,008 * 3,819,816 = 4,148,350,734,528

b) P(44,2) * P(56,2) = (44! / 42!) * (56! / 54!) = 1,892 * 3,080 = 5,827,360

3 Q5 **12 / 16**

- 0 pts Correct

- 1 pts 5.4.2 A There are two choices for the first three digits (824 or 825). Each of the remaining 4 digits can be any one of the ten digits, so there are 10^4 ways to pick the last four digits. The total number of 7-digit phone numbers that start with 824 or 825 is **2 x 10^4 = 20,000**

- 1 pts 5.4.2 B There are two choices for the first three digits (824 or 825). In selecting the remaining 4 digits, 4 digits are selected from a set of 10 with no repetitions, so there are P(10, 4) choices. The total number of 7-digit phone numbers that start with 824 or 825 and have no repeated digits among the last four digits is **2 x P(10, 4) = 2 x 10 x 9 x 8 x 7 =10,080**

- 1 pts 5.5.3 A Two choices for each of the 10 bits: **2^10 = 1024** binary strings of length 10.

- 1 pts 5.5.3 B The first three bits of the string are determined. Each of the remaining 7 bits can be 0 or 1. Therefore there are **2^7 = 128** binary strings of length 10 that start with 001.

- 1 pts 5.5.3 C There are 2^7 10-bit strings that begin with 001. There are 2^8 10-bit strings that begin with 10. A string can not begin with 001 and 10, so the sum rule applies. Therefore the number of 10-bit strings that begin with 001 or 10 is **2^7 + 2^8 = 384**.
Alternate notation: 3*2^7

✓ **- 1 pts** *5.5.3 D Each of the first 8 bits can be either 0 or 1. Once the the first 8 bits are determined, the last two bits must match the first two bits, so there are no remaining choices for the string. Thus, the number of strings in which the first two bits are the same as the last two bits is 2^6*2^2= **2^8 = 256***

- 1 pts 5.5.3 E There are (10, choose 6) ways to select where the six 0's will be placed among the ten possible locations. Once the 0's are placed, the four 1's go in the unfilled locations. Therefore the number of 10-bit strings with exactly six 0's is **(10 choose 6)=210**. Alternate notation: 10 choose 4 = 210

- 1 pts 5.5.3 F The six 0's can be placed in any of the 10 locations, except the first location. There are (9, Choose 6) ways to select the six locations for the 0's among the last nine locations in the string. Once the 0's are placed, the four 1's go in the unfilled locations. Therefore the number of 10-bit strings with exactly six 0's and a 1 in the first location is **(9 choose 6) = 84**. Alternate notation: 9 choose 3 = 84

- 1 pts 5.5.3 G There are (5, Choose 1) ways to select the location for the 1 among the first five locations. There are (5, Choose 3) ways to select the location for the three 1's among the last five locations. Once the 1's are placed, the 0's go in all the unfilled locations. Since the location for the 1 in the first half and the locations for the 1's in the second half must both be determined to specify the string, the product rule applies. Therefore the number of 10-bit strings with one 1 in the first half and three 1's in the second half is **(5 choose 1) * (5 choose 3) = 5*(5 choose 3) = 50**

- 1 pts 5.5.5 A

There are (30 Choose 10) ways to select a subset of 10 boys from a set of 30 boys. There are (35 Choose 10) ways to select a subset of 10 girls from a set of 35 girls. Since the choir director must select the girls and the boys for the chorus, the product rule applies, and there are a total of
(30 Choose 10)*(35 Choose 10) $$\approx 5.5 \mathrm{e}15$$ ways to make the selection.

✓ **- 1 pts** *5.5.8 C There are **(26 Choose 5) = 65780** ways to select a subset of 5 cards from the set of 26 hearts and diamonds in the deck. Alternate notation: 2 ((13 choose 1)(13 choose 4) + (13 choose 2)(13 choose 3) + (13 choose 5))*

- 1 pts 5.5.8 D There are 13 ways to select a rank. Once a rank has been chosen, all four of the cards with that rank will be in the hand. Then there are 48 ways to select the remaining card that does not have the same rank as the four already chosen. The total number of ways to select a five-card hand with four cards of the same rank is **13 x 48 = 624**

- 1 pts 5.5.8 E There are 13 ways to select the rank for the two cards with the same rank. The three cards with the same rank must have a different rank than the rank chosen for the pair, so there are 12 ways to select the rank for the three cards with the same rank. Once the rank has been chosen for the two cards that have the same rank, there are (4 choose 2) ways to select two cards from the four cards with that rank. Once the rank has been chosen for the three cards that have the same rank, there are (4 choose 3) ways to select three cards from the four cards with that rank. The choices are put together by the product rule, so that the number of ways to select a hand that is full house is:

$$(13)(12){4 \choose 2}{4 \choose 3}$$ = 3744

- 1 pts 5.5.8 F There are 13 different possible ranks. The number of ways to select 5 distinct ranks from 13 possible ranks is (13 choose 5). For each rank chosen, there are four possible cards with that rank that can be selected. Therefore once the ranks have been determined, there are 4^5 ways to select the cards in the hand. The total number of ways to select a five-card hand in which no two cards have the same rank is

$$4^{5} {13 \choose 5}$$ = 1,317,888
Alternate notation: **(52*48*44*40*36)/5!**
Why is it divided by (5!)? Because of the k-to-1 rule.

- 1 pts 5.6.6 A (44 choose 5) * (56 choose 5). The committee must have 5 Demonstrators and 5 Repudiators. There are (44 choose 5) ways to select the 5 committee members from the 44 Demonstrators. There are (56 choose 5) ways to select the 5 committee members from the 44 Repudiators. Therefore there are **$${44 \choose 5}{56 \choose 5} \approx 4.148\mathrm{e}12$$** ways to select the entire committee.

- **1 pts** 5.6.6 B **P(44,2) x P(56,2) = 1892 x 3080 = 5,827,360**

Alternate notations: 44 * 43 * 56 * 55 = C(44,1)*C(43,1)*C(56,1)*C(55,1)

There are 44 ways to select the speaker from the Demonstrators. Once that person is chosen, there are 43 ways to select the vice speaker from the remaining Demonstrators. Therefore, there are P(44, 2) = 44 · 43 ways to select the speaker and vice speaker from the Demonstrators. Similarly, there are P(56, 2) ways to select the speaker and vice speaker from the Repudiators. The choices are combined using the product rule because a speaker and vice speaker are selected from both parties. Therefore there are a total of P(44,2) · P(56,2) ways to select the speaker and vice speaker from the two parties.

- **16 pts** Entirely incorrect/no submission/not tagged/handwritten
- **1 pts** division notation instead of combination notation

✓ **- 2 pts** *insufficient work shown*

- **0.25 pts** incorrect notation

Question #6:

Section A: 5.7.2

a) C(52,5) - C(39,5) = 2,598,960 - 575,757 = 2,023,203

b) C(52,5) - C(13,5) * 4^5 = 2,598,960 - (1,287 * 1024) = 1,281,072

Section B: 5.8.4

a) 5^{20} = 95,367,431,640,625

b) 20! / $4!^5$ = 305,540,235,000

4 Q6 **4 / 4**

✓ **- 0 pts** *Correct; good job*

- 1 pts 5.7.2a is incorrect or missing. Should be (52 choose 5) - (39 choose 5) or 2,023,203 or equivalent.

- 1 pts 5.7.2b is incorrect or missing. Should be (52 choose 5) - (4^5)*(13 choose 5) or 1,281,072 or equivalent.

- 1 pts 5.8.4a is incorrect or missing. Should be 5^{20} or 95,367,431,640,625 or equivalent.

- 1 pts 5.8.4b is incorrect or missing. Should be (20!)/($(4!)^5$) or C(20,4)*C(16,4)*C(12,4)*C(8,4)*C(4,4) or 305,540,235,000 or equivalent.

- 4 pts Entirely incorrect/no submission/not tagged/handwritten

Question #7:

a) None because 5 is greater than 4.

b) 5! / (5 - 5)! = 5! / 0! = 120/1 = 120

c) 6! / (6 – 5)! = 6! / 1! = 720/1 = 720

d) 7! / (7 – 5)! = 7! / 2! = 5,040 / 2 = 2,520

5 Q7 **4 / 4**

✓ **- 0 pts** *Correct; good job*

- 1 pts Part a is incorrect. Should be 0.

- 1 pts Part b is incorrect. Should be P(5,5) or 120 or equivalent.

- 1 pts Part c is incorrect. Should be P(6, 5) or 720 or equivalent.

- 1 pts Part d is incorrect. Should be P(7,5) or 2520 or equivalent.

- 4 pts Entirely incorrect/no submission/not tagged/handwritten

- 0.1 pts 5! = 120 not 125