

MAT033 Pre-Algebra Worksheet 2 Solutions

Part a: Fractions I

1. What fraction of an hour are each of the following? Express your answers in lowest terms, and as mixed numbers when appropriate.

(a) 30 minutes

There are 60 minutes in an hour, so 30 minutes is

$$\frac{30}{60} = \frac{30}{60} \div \frac{30}{30} = \frac{1}{2}$$

(b) 150 minutes

$$\begin{aligned} \frac{150}{60} &= \frac{150}{60} \div \frac{10}{10} \\ &= \frac{15}{6} \\ &= \frac{15}{6} \div \frac{3}{3} \\ &= \frac{5}{2} = 2\frac{1}{2} \end{aligned}$$

(c) 1 hour and 51 minutes

$$\begin{aligned} &1\frac{51}{60} \\ &= 1\frac{51}{60} \div \frac{3}{3} \\ &= 1\frac{17}{20} \end{aligned}$$

2. What fraction of a dollar are each of the coins: the dollar coin, the quarter, the dime, the nickel and the cent?

A dollar is worth 100 cents.

(a) a dollar coin is worth $1/1 = \$1$

(b) a quarter is worth 25 cents. This is

$$\begin{aligned} & \frac{25}{100} \\ &= \frac{5}{20} \\ &= \frac{1}{4} \end{aligned}$$

A quarter is worth 1/4 of a dollar (which is why, of course, it's called a quarter).

(c) a dime is worth 10 cents This is

$$\begin{aligned} & \frac{10}{100} \\ &= \frac{1}{10} \end{aligned}$$

A dime is worth 1/10 of a dollar.

(d) a nickel is worth 5 cents. This is

$$\begin{aligned} & \frac{5}{100} \\ &= \frac{1}{20} \end{aligned}$$

A nickel is worth 1/20 of a dollara

(e) One cent is worth 1/100 of a dollar.

3. Convert the following mixed numbers to fractions in lowest terms.

(a) $1\frac{2}{3}$

Using the formula in Section 2.3, $1\frac{2}{3}$

$$= \frac{(1)(3) + 2}{3} = \frac{5}{3}$$

(b) $5\frac{5}{8}$

$$= \frac{(5)(8) + 5}{8} = \frac{40 + 5}{8} = \frac{45}{8}$$

(c) $-4\frac{4}{6}$

$$= -\frac{(4)(6) + 4}{6} = -\frac{28}{6} = -\frac{14}{3}$$

4. Put the following fractions in lowest terms.

(a) $\frac{16}{18}$

$$= \frac{8}{9}$$

(divide numerator and denominator by 2)

(b) $\frac{2}{3}$

This fraction is already in its lowest terms.

(c) $\frac{51}{99}$

$$= \frac{17}{33}$$

(divide numerator and denominator by 3).

(d) $\frac{11}{121}$

$$= \frac{1}{11}$$

(divide numerator and denominator by 11).

5. Fred's rent is \$600 per month, and his paycheck is \$1500 per month. What fraction of his paycheck goes to rent? Express your answer as a fraction in lowest terms.

As a fraction, Fred's rent is $\frac{600}{1500}$

$$\begin{aligned} &= \frac{600}{1500} \div \frac{100}{100} \\ &= \frac{6}{15} \\ &= \frac{2}{5} \end{aligned}$$

6. You can compare two fractions and find which has the greater value if they are both expressed over the same denominator. In the following pairs of fractions, identify which one is greater:

(a) $\frac{3}{5}$ and $\frac{4}{5}$

$$\frac{3}{5} < \frac{4}{5}$$

(b) $-\frac{3}{5}$ and $\frac{4}{5}$

A negative number is always less than a positive one, so $-\frac{3}{5} < \frac{4}{5}$

(c) $-\frac{3}{5}$ and $-\frac{4}{5}$

Here, both numbers are negative. $-\frac{4}{5}$ is further to the left on the number line so it is the smaller number. $-\frac{3}{5} > -\frac{4}{5}$

7. Arnold eats $\frac{3}{7}$ of a cake and Bob eats $\frac{2}{5}$ of a cake.

(a) Who ate more cake?

Which is larger, $\frac{3}{7}$ or $\frac{2}{5}$? First express these fractions using a common denominator

$$\frac{3}{7} \times \frac{5}{5} = \frac{15}{35} \text{ and } \frac{2}{5} \times \frac{7}{7} = \frac{14}{35}$$

$$\frac{15}{35} > \frac{14}{35}$$

so $\frac{3}{7} > \frac{2}{5}$

(b) Find an amount of cake that is in between the amounts that Arnold and Bob ate.

$$\frac{14}{35} \times \frac{2}{2} = \frac{28}{70}$$

$$\frac{15}{35} \times \frac{2}{2} = \frac{30}{70}$$

The rational number $\frac{29}{70}$ is between $\frac{3}{7}$ and $\frac{2}{5}$.

Part b: Adding & subtracting fractions

1. Calculate the following (and express your answers in their lowest terms):

(a) $\frac{4}{15} + \frac{2}{15}$

Both denominators are the same - 15 - so you can just add the numerators:

$$\frac{4}{15} + \frac{2}{15} = \frac{4+2}{15} = \frac{6}{15} = \frac{2}{5}$$

Here, the fraction 6/15 was reduced to its lowest terms by dividing the numerator and the denominator by the highest common factor, 3.

(b) $\frac{1}{17} + \frac{7}{17} + \frac{9}{17}$

All three denominators are the same - 17 - so you can just add the numerators:

$$\frac{1}{17} + \frac{7}{17} + \frac{9}{17} = \frac{1+7+9}{17} = \frac{17}{17} = 1$$

Where you were able to divide the numerator and the denominator by the HCF, 17.

(c) $\frac{1}{41} + \frac{5}{41} + \frac{16}{41} - \frac{14}{41}$

$$\frac{1}{41} + \frac{5}{41} + \frac{16}{41} - \frac{14}{41} = \frac{1+5+16-14}{41} = \frac{22-14}{41} = \frac{8}{41}$$

$$(d) \frac{1}{99} + \frac{5}{99} + \frac{7}{99} + \frac{2}{99}$$

$$\frac{1}{99} + \frac{5}{99} + \frac{7}{99} + \frac{2}{99} = \frac{1+5+7+2}{99} = \frac{15}{99} = \frac{5}{33}$$

2. Calculate the following (and express your answers in their lowest terms):

$$(a) \frac{4}{15} + \frac{3}{20}$$

The lowest common multiple of 15 and 20 is 60 ($15 = 3 \times 5$, $20 = 4 \times 5$, so the LCM = $3 \times 4 \times 5 = 60$. This is the *lowest common denominator*. Make both of your fractions have 60 as the denominator:

$$\frac{4}{15} = \frac{16}{60},$$

$$\text{while } \frac{3}{20} = \frac{9}{60}.$$

Now you can add them:

$$\frac{16}{60} + \frac{9}{60} = \frac{16+9}{60} = \frac{25}{60} = \frac{5}{12},$$

where you reduced the fraction by dividing the numerator and the denominator by the highest common factor, 5.

$$(b) \frac{3}{5} - \frac{7}{9} + \frac{4}{15}$$

The LCD is 45. So

$$\frac{3}{5} - \frac{7}{9} + \frac{4}{15} = \frac{27}{45} - \frac{35}{45} + \frac{12}{45} = \frac{27-35+12}{45} = \frac{4}{45}$$

$$(c) \frac{2}{3} - \frac{1}{6}$$

$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$(d) 1 - \frac{7}{8}$$

$$1 - \frac{7}{8} = \frac{8}{8} - \frac{7}{8} = \frac{1}{8}$$

3. Calculate the following (and express your answers in their lowest terms):

$$(a) \ 1\frac{1}{2} + \frac{3}{4}$$

$$1\frac{1}{2} + \frac{3}{4} = \frac{3}{2} + \frac{3}{4} = \frac{6}{4} + \frac{3}{4} = \frac{6+3}{4} = \frac{9}{4} = 2\frac{1}{4}$$

$$(b) \ 4\frac{2}{3} - 3\frac{1}{2}$$

$$4\frac{2}{3} - 3\frac{1}{2} = \frac{14}{3} - \frac{7}{2} = \frac{28}{6} - \frac{21}{6} = \frac{28-21}{6} = \frac{7}{6} = 1\frac{1}{6}$$

$$(c) \ 9\frac{16}{17} - 2$$

$$9\frac{16}{17} - 2 = 9 - 2 + \frac{16}{17} = 7\frac{16}{17}$$

$$(d) \ 4\frac{3}{4} + 8\frac{2}{3}$$

$$4\frac{3}{4} + 8\frac{2}{3} = 4 + 8 + \frac{3}{4} + \frac{2}{3} = 12 + \frac{9}{12} + \frac{8}{12} = 12 + \frac{17}{12} = 12 + 1\frac{5}{12} = 13\frac{5}{12}$$

$$(e) \ 5\frac{2}{3} + 3\frac{1}{2} - 4\frac{1}{4}$$

$$5\frac{2}{3} + 3\frac{1}{2} - 4\frac{1}{4} = 4 + \frac{2}{3} + \frac{1}{2} - \frac{1}{4} = 4 + \frac{8}{12} + \frac{6}{12} - \frac{3}{12} = 4 + \frac{11}{12} = 4\frac{11}{12}$$