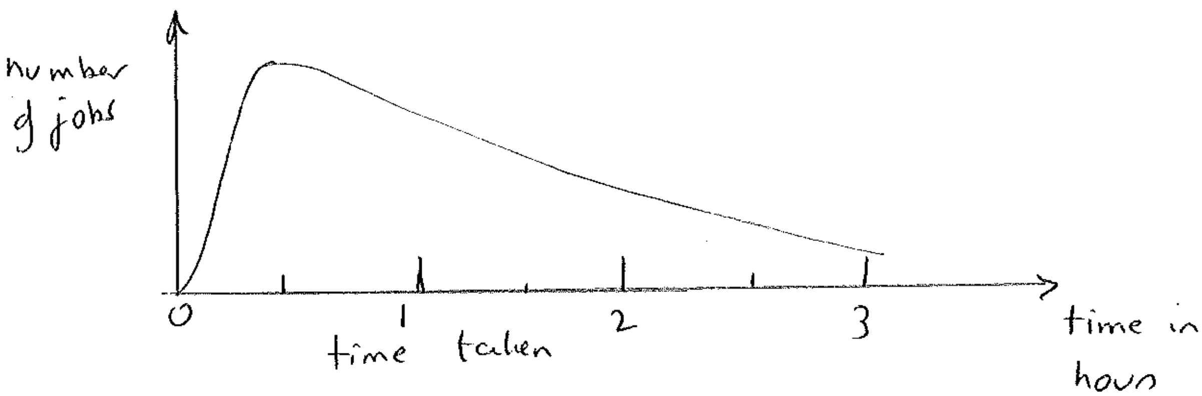
MAT 125 WORKSHEET 10 SOLUTIONS

(1) The distribution of service times described in the question probably looks something like this:



i.e. nothing like a normal distribution. However the central limit theorem says that the sampling distribution of the mean time is approximately normal for a large enough set of samples (> 30 is a good rule of thumb) so we can use the normal distribution to do this problem.

The distribution has mean μ = 1 hour
dispersion σ = 1 hour

According to the central limit theorem the sampling distribution of the mean maintenance time for 70 units is approximately normal with mean 1 hour and standard deviation:

$$SD = \frac{1 \text{ hour}}{\sqrt{70}} = 0.12$$

The z-score of a mean time of 1.1 hours is then

$$z = \frac{1.1 - 1}{0.12} = 0.83$$

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Then, looking up the z-score table,

$$P(\text{mean} > 1.1) = P(z > 0.83)$$

$$= 0.5 - 0.297 = 0.203$$

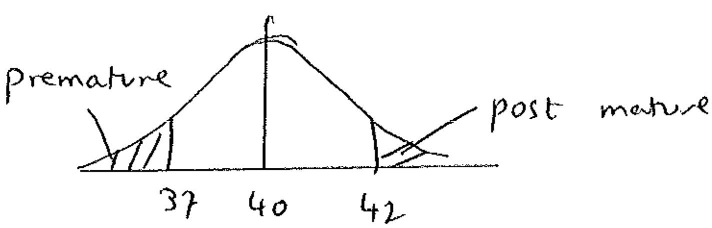
So there is a probability of $\boxed{0.203, \ \sim 20\%,}$ that the technicians will not complete the work in the allotted time.

2. Length of pregnancy normally distributed, with $\mu = 40$ weeks, $\sigma = 2.5$ weeks

Premature birth: born 37 weeks or less into the pregnancy

Postmature birth: born 42 weeks or later into the pregnancy

(a) Probability that the next infant will not be premature:



$$P(\text{time} > 37 \text{ weeks})$$

$$z = \frac{37-40}{2.5} = -1.2$$

Look up the z-table: $P(z < -1.2) = 0.115$

$$\therefore P(z \geq -1.2) = 1 - 0.115 = 0.885$$

The probability that the next baby born is <u>not</u> premature is $0.885 = \boxed{88.5\%}$

(b) Probability that the next baby born is not postmature:

post-mature = 42 weeks or more

~~[illegible]~~ z for 42 weeks $= \frac{42-40}{2.5} = 0.8$

So probability that a baby is post-mature (from z-score table) is 0.212, and probability that a baby is <u>not</u> post-mature is $1 - 0.212 = 0.788 = \boxed{78.8\%}$

(c) Probability that infant is either premature or postmature

$P =$ probability (premature) + probability (post mature)

(this is an OR problem):

$P(\text{age} < 37 \text{ weeks}) + P(\text{age} > 42 \text{ weeks})$

$= P(z < -1.2) + P(z > 0.8)$

$= 0.115 + 0.212 = 0.327$

Probability that an infant is either pre- or post-mature is $\boxed{0.327 = 32.7\%}$

(d) Probability that <u>mean</u> gestational age of next 49 infants born is in the "normal" range, ie.

between 37 and 42 weeks:

$$SD \text{ for } 49 \text{ infants} = \frac{2.5}{\sqrt{49}} = \frac{2.5}{7} = 0.36$$

$$z\text{-score, } 37 \text{ weeks} = \frac{37-40}{0.36} = -8.3$$

$$z\text{-score, } 42 \text{ weeks} = \frac{42-40}{0.36} = 5.6$$

So $P(-8.3 < z < 5.6) \doteq 1.0$

(3.) Let's focus on the fair coin. But first:

(a), (b) the answers are yes and yes, obviously: the law of large numbers says that the fraction of heads will tend to 0.5 and 0.75 respectively and that you can therefore know, if you toss the coin enough times, which one you've chosen. The question is, how many times do you have to toss the coin?

For the fair coin, we have two probabilities: r = expected rate of heads = theoretical probability = 0.5. p = observed fraction of heads = empirical probability. We're asked that the observed ~~number~~ fraction of heads comes within 0.025 of the expected number. Call the difference between the empirical and theoretical probability the error, E:

$$E = |p - r| \leq 0.025$$

What is the <u>expected</u> error rate? We calculate the standard deviation as follows. Tails turn up half the time, heads turn up half the time, so, remembering the binomial theorem,

$$SD = \sqrt{\frac{r(1-r)}{n}} = \sqrt{\frac{0.5 \times 0.5}{n}} = \frac{0.5}{\sqrt{n}} = \frac{1}{2\sqrt{n}}$$

We want the result to be within 0.025 of the expected number, so look up $A = 0.5 - 0.025 = 0.475$ in the z-score table: this gives ~~$\mathcal{B}$~~ $z = 1.96$. To make the calculations easier, let's just take $z \approx 2$.

Then $$z = \frac{E}{SD} = \frac{0.025}{1/2\sqrt{n}}.$$

$$= \cancel{2 \times 0.025} \quad 2\sqrt{n} \times 0.025$$

$$\cancel{2} = \cancel{2} \times 0.025\sqrt{n}$$

$$n = \frac{1}{(0.025)^2} = 1600$$

So we need at least 1600 tosses.

The standard deviation of the biased coin is a bit different, $\sqrt{\frac{0.75 \times 0.25}{n}}$ is $\frac{1}{2.3\sqrt{n}}$.

We'll have a proper discussion of the expected difference in the discussion of analysis of variance in MAT126.

4. See problem 3. The expected SD $= \frac{0.5 \times 0.5}{\sqrt{n}} = \frac{1}{2n} = \cancel{0}$ $= \frac{1}{20}$. The expected number $= 100 \times \frac{1}{20} = 5$

(a) $z = \frac{15}{5} = 3$ $A = 0.499$ so $P = 0.5 - 0.499$ $= 0.001 = \boxed{0.1\%}$

(b) Sampling mean $= 0.5 \times 10{,}000 = 5{,}000$

$\sigma = \frac{5}{\sqrt{100}} = \frac{5}{10} = 0.5$ ~~so~~ SD of Sampling mean

~~Number of coins = 0.5×10,000~~

$\frac{5050}{10{,}000} = 0.505$

$z = \frac{\cancel{0.5 - 0.505}\; 0.505 - 0.5}{0.5} = \frac{+0.005}{0.5} = +0.01$

According to the table $A = 0.004$. So

$P = 0.5 - 0.004 = 0.496 = \boxed{49.6\%}$