MAT033 Pre-Algebra
HW 1: Solutions

Part a: Integers

(1) Order from smallest to largest:

1. $-5, 2$ **A:** $-5 < 2$
2. $6, 10, -200, 0$ **A:** $-200 < 0 < 6 < 10$

(2) Draw number lines to represent the following, and find the answers:

1. $0 + -5$ **A:** -5
2. -2×-4 **A:** 8

(3) Solve:

$x = -3 + -5$ **A:** $x = -8$ $y = 5 + -3$ **A:** $y = 2$

$n = -3 + 2 - 6$; $2 - 9$ **A:** $n = -7$

$q = -3 \times 5$ **A:** $q = -15$ $r = 3 \times 5$ **A:** $r = 15$

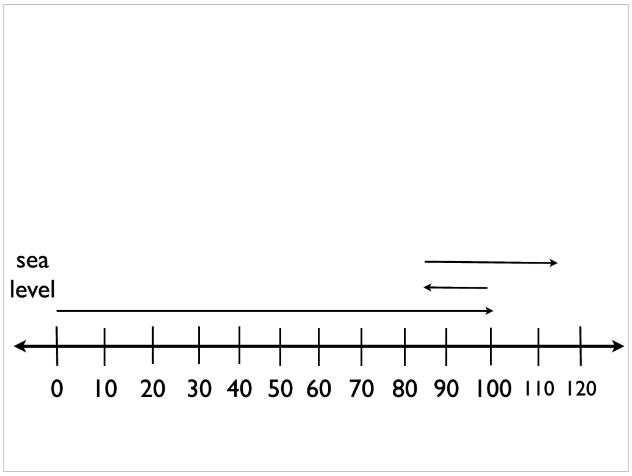
$x = 3 + 2 \times 4$; **MULTIPLY FIRST** $x = 3 + 8$ **A:** $x = 11$

$y = (3 + 2) \times 4$; y $= 5 \times 4$ **A:** $y = 20$

(4) Solve:

A plane carrying a sky-diver takes off from sea-level. It flies 100 m up, then dives 12 m, then climbs 25 m.

- Draw a number line to describe the airplane flight. *(see below)*



- How high above sea level is the airplane?

$$height = 100 - 12 + 25 = 125 - 12 = 113m \tag{1}$$

- If the sky-diver falls 40 m and then opens a parachute, how high above sea-level is he?

$$height = 113 - 40 = 73m \tag{2}$$

- If the sky-diver sinks 25 m into the ocean, how far is the distance between the sky-diver and the airplane?

$$distance = 113m \text{ (height of the airplane)} + 25 \text{ (depth into the ocean)} = 138m \tag{3}$$

Part b: Factors

1. Find all of the prime numbers between 70 and 100.

   Answer:

   First, reject the obvious non-prime numbers. None of the even numbers can be a prime because they can be divided by two. None of the numbers whose last digit is 5 is prime because it divides by 5. Every third odd number (check them of course) can be divided by 3 (see Worksheet 2). So: for the odd numbers between 70 and 100:

75, 85, 95 are divisible by 5

75, 81, 87, 93, 99 are divisible by 3

This leaves 71, 73, 77, 79, 83, 89, 91, 97. Every 7th odd number divides by 7 - these numbers are 77 and 91 ($77 = 7 \times 11$ and $91 = 7 \times 13$.) The next prime number is 11, but $97 \div 11 = 8$ plus remainder 9, i.e. the quotient is smaller than 11. So there are no more prime number factors. The prime numbers between 70 and 100 are **71, 73, 79, 83, 89, 97**.

2. Find the Greatest Common Factors of the following pairs of numbers, first using Method 1, and then using the Euclidean Algorithm: (a) (16,48) (b) (42,63) (c) (21,16) (d) (52,39)

   Answer:

   (a)
   $16 = 2\times2\times2\times2$
   $48 = 2\times2\times2\times2\times3$

   so the Greatest Common Factor (GCF) is $2\times2\times2\times2 = 16$

   (b)
   $42 = 2\times3\times7$
   $63 = 3\times3\times7$

   so the GCF is $3\times7 = 21$

   (c)
   $21=3\times7$
   $16=2\times2\times2\times2$

   the GCF is 1, i.e. there isn't one

   (d)
   $52 = 2\times2\times13$
   $39=3\times13$

CGF = 13

Now use Euclid's algorithm:

(a)
16, 48: $48 \div 16 = 3$, remainder 0
so GFC = 16

(b)
42, 63: $63 \div 42 = 1$, remainder 21
$42 \div 21 = 2$, remainder 0
So GCF = 21

(c)
21, 16: $21 \div 16 = 1$, remainder 7
$16 \div 7 = 2$, remainder 2
$7 \div 2 = 3$, remainder 1
$2 \div 1 = 2$, so we can't go further

(d)
39,52: $52 \div 39 = 1$, remainder 13
$39 \div 13 = 3$, remainder 0
so the GCF = 13

3. Find the Least Common Multiple of the number pairs in the previous problem

   Answer

   Use the prime factorization from the previous problem:

   (a)
   16,48: $16 = 2\times2\times2\times2$; $48 = 2\times2\times2\times2\times3$
   LCM = $2\times2\times2\times2\times3 = 48$
   The lowest common multiple is 48

   (b)
   42,63; $42=2\times3\times7$; $63=3\times3\times7$
   LCM = $2\times3\times3\times7 = 126$

   (c)
   16,21: $16=2\times2\times2\times2$; $21=3\times7$

LCM = $2\times2\times2\times2\times3\times7=336$

(d)
39,52: $39=3\times13$; $52=2\times2\times13$
LCM = $2\times2\times3\times13 = 156$

4. A mother wishes to divide 6 chocolate bars evenly among 4 children. What is the smallest total number of pieces needed, into how many pieces must each bar be broken, and how many pieces does each child receive?

   Answer:

   Each child can have one whole bar of chocolate, with 2 left over ($6\div 4 = 1$, remainder 2). To share the 2 remaining bars equally among the four children, each must be broken into two equal halves. There are four halves, so each child gets one. Each child gets 1 whole bar plus one half bar, two pieces for each child, 8 pieces in all. If all the pieces have to be the same size, look at the least common multiple of 6 and 4: $6 = 2\times3$, $4=2\times2$. The LCM is $2\times2\times3 = 12$. She breaks each bar into two equal pieces, 12 pieces in all, and shares them among the 4 children, who get 3 pieces each.

5. Calculate the prime factors of the following numbers
   (a) 1620
   (b) 375

   Answer

   (a)
   1620 obviously divides by 10, leaving 162. Sum the integers to get $1+6+2 = 9$, which is divisible by 3. 162 is an even number, so must divide by 2. So far we have as prime factors 2,5 ($10 = 2\times 5$), 3, and 2. Divide 162 by 2 to get 81, and 81 by 3 to get 27. This is still divisible by 3. $27\div3 = 9$. $9\div3 = 3$. So the prime factors of 1620 are: $1620 = 2\times2\times3\times3\times3\times3\times5$.

   (b)
   $375 = 5\times75 = 5\times5\times15=5\times5\times5\times3$ (and you can check that 375 is divisible by 3 by adding the integers).

6. Which of the following numbers divides by 3 with no remainder: (a) 246,105 (b) 17 (c) -27 (d) 178,316,166 (e) 29,629,630

   Answer

   In each case, sum the integers.

(a)
$2+4+6+1+5 = 18$; $1+8 = 9$, which is divisible by 3

(b)
$1+7=8$, which is not divisible by 3

(c)
$2+7 = 9$, which is divisible by 3 (note we just need to check the absolute value)

(d)
$1+7+8+3+1+6+1+6+6 = 39$; $3+9 = 12$; $1+2=3$. So 178,316,166 is divisible by 3

(e) $2+9+6+2+9+6+3+0 = 37$; $3+7 = 10$, which is not divisible by 3.

So the numbers in (a),(c) and (d) are divisible by 3, and the others aren't.

7. A school PE coach is organizing sports teams for the school year. The coach wants to divide all of the children into complete teams. For example, if there are 14 students he could form three basketball teams, but the third team would be incomplete (it would only have 4 players, but for basketball it should have 5; the coach needs 10 or 15 students instead). If every student will play lacrosse, basketball, relay racing, and baseball, what is the smallest number of children the coach needs in order to divide all of them into complete teams for all of the sports? (There are 9 players on a baseball team, 4 on a relay racing team, 5 on a basketball team and 10 on a lacrosse team).

   Answer

   We want the lowest common multiple of 9, 4, 5 and 10. Do it in pairs. The LCM of 9 and 4 is 36 (with factors 2, 2, 3, and 3). The LCM of 36 and 5 is 180 (with factors 2,2,3,3,5). The factors of 10 are 2×5. So the LCM of these four numbers (9, 4, 5 and 10) is 180. If you have 180 kids at the school, each of them can be on a team to play all four sports.