Math 135: Intermediate Algebra Homework 7 Solutions

Factor

1. $\mathbf{2a^3 + 8a}$

$= 2a(a^2) + 2a(4)$

$= 2a(a^2 + 4)$

2. $\mathbf{32 - 40a + 24a^2}$

$= 8(4) - 8(5a) + 8(3a^2)$

$= 8(3a^2 - 5a + 4)$

3. $\mathbf{3x^4 + 7x^3 - 9x^2}$

$= x^2(3x^2) + x^2(7x) - x^2(9)$

$= x^2(3x^2 + 7x - 9)$

4. $\mathbf{12p^5 - 4p^4 - 24p}$

$= 4p(3p^4) - 4p(p^3) - 4p(6)$

$= 4p(3p^4 - p^3 - 6)$

5. $\mathbf{9x^2 - 28\,y^2}$

can't be factored

6. $\mathbf{6x^2y^2 + 18xy^3 - 36y^4}$

$= 6y^2(x^2) + 6y^2(3xy) - 6y^2(6y^2)$

$= 6y^2(x^2 + 3xy + 6y^2)$

7. $\mathbf{3x(x - 5) + 2(x - 5)}$

$= (3x + 2)(x - 5)$

8. $\mathbf{(2x - 3y)(3x) - (2x - 3y)(2y)}$

$= (3x - 2y)(2x - 3y)$

9. $\mathbf{x(x + y)^2 - y(x + y)^2}$

$= (x - y)(x + y)^2$

Factor by grouping

10. $\mathbf{x(y - 7) + 10(7 - y)}$

$= x(y - 7) - 10(y - 7)$

$= (x - 10)(y - 7)$

11. $\mathbf{4x(z - y) - (y - z)}$

$= 4x(z - y) + (z - y)$

$= (4x + 1)(z - y)$

12. $\mathbf{(2x - y) + 2x(y - 2x)}$

$= (2x - y) - 2x(2x - y)$

$= (1 - 2x)(2x - y)$

13. $\mathbf{ab + 5a + cb + 5c}$

$= a(b + 5) + c(b + 5)$

$= (a + c)(b + 5)$

14. $\mathbf{x^2 - 9x + 2x - 18}$

$= x(x - 9) + 2(x - 9)$

$= (x + 2)(x - 9)$

15. $\mathbf{9xy - 3y + 3x - 1}$

$= 3y(3x - 1) + (3x - 1)$

$= (3y + 1)(3x - 1)$

16. $\mathbf{10st - 6s - 25t + 15}$

$= 2s(5t - 3) - 5(5t - 3)$

$= (2s - 5)(5t - 3)$

17. The height above the ground (in feet) of a stone t sec after it is dropped from a bridge 720 ft. above the ground is given by the polynomial $\mathbf{720 - 16t^2}$.
a. Factor the polynomial:

$720 - 16t^2 = 16(45) - 16(t^2) = 16(45 - t^2)$

b. Use the factored form in part (a) to find the height of the stone 5 sec after it is dropped:

Let h be the height of the stone in feet:

$h = 16\ (45 - t^2)$ ft

where t is the time after the stone is dropped in seconds. So

$h = 16\ (45 - 5^2)$ ft

$= 16\ (45 - 25)$

$= 16\ (20)$

$= 320$ ft.

18. The area (in sq m) of an Olympic-size swimming pool is given by the expression $l^2 - 25l$, where l is the length of the pool.

a. Factor this expression

$l^2 - 25l = l(l - 25)$

b. The width of an Olympic-size swimming pool is 25 m. What is the length?

Area A = length $\times$ width

let w be the width (and l is the length)

$A = lw = l(l - 25)\ m^2$

$w = l - 25 = 25$ m

$l = 25 + 25 = 50$ m

19. After 2 yr, the total amount of money in an account that pays interest rate r (in decimal form), compounded annually, is given by $P + Pr + (P + Pr)r$. Factor to show that the given expression can be written as $P(1 + r)^2$.

$P + Pr + (P + Pr)r$

$= P(1 + r) + Pr(1 + r)$

$= (P + Pr)(1 + r)$

$= P(1 + r)(1 + r)$

$= P(1 + r)^2$

Fill in the missing factor

20. $\mathbf{x^2 - 4x + 3}$

$= (x - 1)(x - 3)$

$(3 = 3 \times 1)$

21. $\mathbf{x^2 + 12x + 35}$

$= (x + 5)(x + 7)$

$(35 = 7 \times 5)$

22. $\mathbf{x^2 + 2xy - 8y^2}$

$= (x - 2y)(x + 4y)$

$(-8y^2 = -2y \times 4y)$

Factor if possible

23. $\mathbf{x^2 + 7x + 12}$

$= (x + 3)(x + 4)$

(factors of 12:1,12,6,2,3,4)

$12 = 4\times3$

$7 = 4 + 3$

24. $\mathbf{n^2 - 12n + 35}$

$= (n - 5)(n - 7)$

(factors of 35:1,35,5,7)

$35 = 5\times7$

$12 = 5 + 7$

25. $\mathbf{t^2 + 2t - 48}$

$= (t + 8)(t - 6)$

(factors of 48:1,48,2,24,4,12,6,8)

$48 = 6\times8$

$2 = 8 - 6$

26. $\mathbf{x^2 - 3x - 54}$

$= (x - 9)(x + 6)$

(factors of 54:1,54,2,27,6,9)

27. $\mathbf{18 - 7y - y^2}$

$= -(y^2 + 7y - 18)$

$= -(y + 9)(y - 2)$

$= (y + 9)(2 - y)$

(factors of 18:1,18,2,9,3,6)

28. $\mathbf{x^2 + 12x + 36}$

$= (x + 6)(x + 6) = (x + 6)^2$

(factors of 36:1,36,2,18,4,9,3,12,6)

29. $\mathbf{x^2 - 5xy + 6y^2}$

$(x - 2y)(x - 3y)$

(factors of $6y^2$:1y,6y,2y,3y)

30. $\mathbf{a^2 - 2ab - 15b^2}$

$(a - 5b)(a + 3b)$

(factors of $15b^2$:1b,15b,3b,5b)

31. $\mathbf{3x^2 + 11x + 6}$

$= (3x + 2)(x + 3)$

factors:1,6,2,3

32. $\mathbf{2a^2 - 13a + 18}$

$= (2a - 9)(a - 2)$

factors:1,18,2,9,3,6

33. $\mathbf{4t^2 + 4t - 15}$

$= (2t - 3)(2t + 5)$

factors:1,15,3,5

34. $\mathbf{6n^2 - n - 12}$

$= (3n + 4)(2n - 3)$

factors:1,12,3,4,2,6

35. A city parks department increased the size of a rectangular ice-skating rink in its largest park by adding x ft to the length and width. The new rink has an area given by ($\mathbf{x^2 + 140x + 4000}$) sq. ft.

a. Factor the expression:
some factors: 1,4000,2,2000,4,1000,8,500,10,400,20,200,40,100,80,50
$x^2 + 140x + 4000 = (x + 40)(x + 100)$

b. What were the dimensions of the original ice-skating rink?:
Area = Length x Width = $(x + 40)(x + 100)$ sq. ft.
New length = $100 + x$ ft., so original length = 100 ft.
New width = $40 + x$ ft., so original width = 40 ft.

36. The height (in feet) above the ground of an object t sec after it is thrown downward from a height of 192 ft. with an initial velocity of 64 ft. per sec is given by the polynomial 192 $-$ 64t $-$ 16t^2. Factor this polynomial.

$-16(t^2 + 4t - 12)$
$= -16(t + 6)(t - 2) = 16(2 - t)(t + 6)$ ft.
factors: 1,12,2,6,3,4