MAT033 Pre-Algebra Worksheet 1 Solutions

Part a: Integers

0.1 Solve:

Find the opposite:

-5 $\quad$ 5

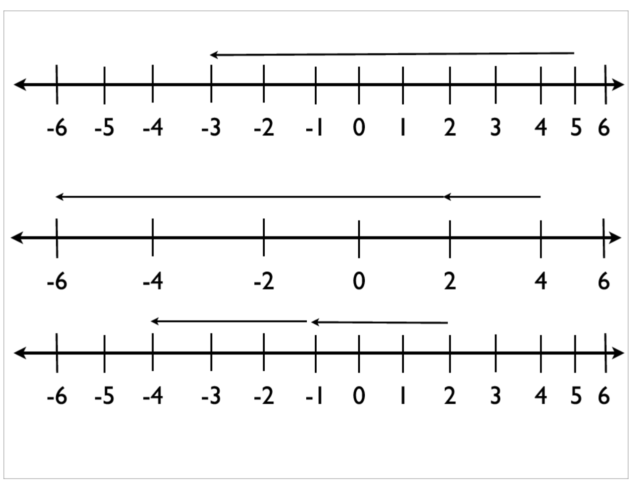
10 $\quad$ -10

Order from smallest to largest:

$-21, -20, 0$ $\quad$ $-21 < -20 < 0$

$-5, -3, 1, 2$ $\quad$ $-5 < -3 < 1 < 2$

Convert the following number lines into sums:



$5 + -8 = -3$

$4 - 2 - 8 = -6$

$2 - 3 - 3 = -4$

Solve:

$n = -5 + 3$ $\quad$ $n = -2$

$x = 10 + -4$ $\quad x = 6$

$n = 5 \times -2$ $\quad n = 3$

$y = 3 \times 0$ $\quad y = 0$

$q = -5 \times -3 \times 1$ $\quad q = 15$

$x = 37 \times -25 \times 0$ $\quad x = 0$

$x = -5 + -7$ $\quad x = -12$

$n + 2 = -5$ $\quad n = -7$

0.2 Word Problems

1. Christine has 20 dollars in the bank. She puts in (deposits) 10 dollars, withdraws 25 dollars, and then withdraws five dollars two more times.

   - Write an expression for the amount of money she has in the bank after all of these transactions.

     $m = 20 + 10 - 25 - (2 \times 5)$

   - How much money does she need to deposit or withdraw in order to have exactly zero dollars in the bank?
     $m = 20 + 10 - 25 - 10$

     $m = -5$

     She needs to put in or deposit 5 more dollars.

2. A man is searching for buried treasure. Every year he digs 10 meters further down.

   - After five years, how many meters down has the man dug?
     $h = 10 \times 5$ meters

     $h = 50$ meters

   - How about after twenty years?
     $h = 20 \times 5$ meters

$h = 100$ meters

- Every years the wind refills the hole the man is digging. It fills in two meters every year. After five years, how many meters has the wind filled in?

  $w = 2 \times 5$ meters

  $w = 10$ meters

- After five years, what is the net depth of the hole?

  The man has dug 50 meters, and the wind has filled in 10, so the hole is 40 meters deep after five years.

3. An apple tree grows upwards at a rate of five meters per year.

   - How tall is the tree after five years?

     $h = 5 \times 5 = 25$

   - After ten years, the tree starts to make apples. If an apple falls from one half of the tree height, how far does it fall?

     $h = 10 \times 5 = 50$ meters

     The apple falls 25 meters.

   - While the tree is growing up, the roots are growing downward. The roots grow two meters per year. After ten years, how long is the tree from the top to the bottom?

     $d = -2 \times 10 = -20$ meters

     The tree is 20 meters deep and 50 meters high, so the entire tree is 70 meters.

4. Thomas earns \$2,000 per month after tax. His rent is \$500 per month, his car payment is \$100 per month, he puts \$100 per month into long-term savings, and his telephone and utility bills are \$200 per month. Thomas wants to save up for a vacation that will cost \$2400. Work out a budget for him, i.e. how much he can spend per month on food, clothes and entertainment that will let him take the vacation after one year.

   Thomas' expenditures per month are \$500 + \$100 + \$100 + \$200, or \$900 total. After spending this, he has \$2,000 - \$900 = \$1,100 left. If he wants to save up for a year for his vacation, which costs \$2,400, he will have to save \$2,400 ÷ 12 = \$200 per month. This means that he has \$1,100 - \$200 = \$900 per month for all his other expenditures - food, gasoline, clothes, entertainment. If he stays within this budget he'll be able to take his vacation.

Part b: Factors

1. Which of the following statements are true?
   (a) 3 is a factor of 18. (b) 3 is a multiple of 18. (c) 18 is a multiple of 3. (d) 27 has 7 as a factor. (e) 35 has 5 as a factor. (f) 12 has -3 and -2 as factors.

   Answer:
   (a) 3 is a factor of 18: 3×6 = 18

   (b) 3 is not a multiple of 18 because 3 divided by 18 is not a whole number with no remainder

   (c) 18 is a multiple of 3 because 18 is divisible by 3 with no remainder: 18 ÷ 3 = 6

   (d) 7 is not a factor of 27: 27 ÷ 7 = 3 with remainder 6

   (e) 5 is a factor of 35: 35 ÷ 5 = 7 with no remainder

   (f) -3 and -2 are both factors of 12: 12 ÷ -3 = -4, and 12 ÷ -2 = -6.

2. A *perfect number* is equal to the sum of all of its positive factors other than itself (all factors, not just prime factors). For example, 6 is perfect because its positive factors are $1, 2, 3, 6$, and $1+2+3=6$. The next perfect number after 6 is between 20 and 30. What is it?

   Answer:

   Let's look at the factors of the numbers from 20 to 30, excluding the number itself as a factor:

   20: factors 1,2,4,5,10: sum of factors 1+2+4+5+10 = 22

   21: factors 1,3,7: sum = 11

   22: factors 1,2,11: sum=14

   23: factors 1; sum = 1 (23 is a prime number)

   24: factors 1,2,3,4,6,8,12: sum = 35

   25: factors 1,5: sum = 6

   26: factors 1,2,13: sum = 16

27: factors 1,3,9: sum = 13

28: factors 1,2,4,7,14: sum = 28

29: factors 1: sum = 1 (29 is a prime number)

30: factors 1,2,3,5,6,10,15: sum = 42

It's clear from the above that 28 is a *perfect number*.

3. Find the prime factorizations of the following numbers:
   (a) 24 (b) 64 (c) 29 (d) 120 (e) 81 (f) 51

   Answer:

   $24 = 2\times2\times2\times3$

   $64 = 2\times2\times2\times2\times2\times2$

   29 is a prime number

   $120 = 2\times2\times2\times3\times5$

   $81 = 3\times3\times3\times3$

   $51 = 3\times17$ (check for divisibility by 3: 5+1 =6, so 51 is divisible by 3)

4. Find all of the primes between 1 and 30.

   Answer:

   First, let's eliminate all of the obvious numbers.

   (a) 1 is not a prime number: see notes

   (b) No even number greater than 2 is prime, because it is divisible by 2.
   This leaves 2,3,5,7,9,11,13,15,17,19,21,23,25,27,29 as candidates

   (c) no number with its last digit 5 (except 5 itself of course) is prime because it is divisible by 5. This removes 15 and 25 from consideration

(d) check for divisibility by 3 by adding the integers. 9 is divisible by 3; 15 is (1 + 5 = 6) and so on. Note that in the list under (b), every third number, starting at 3, is divisible by 3 (3,9,15,21,27) (and every fifth number is divisible by 5) - this is just the result of multiplication

(e) removing the numbers divisible by 3 and 5 from the list leaves us with 2,3,5,7,11,13,17,19,23,29

(f) So the first few prime numbers are 2,3,5,7. Try dividing the rest by 7 (nope), 11 (nope) and 13 (nope). Dividing by bigger primes would leave you with answers less than 1, so you're done. The prime numbers between 1 and 30 are 2,3,5,7,11,13,17,19,23 and 29.

5. Find the Greatest Common Factors of the following pairs of numbers, first using Method #1, and then using the Euclidean Algorithm: (a) (10,15) (b) (21,49)

Answer:

First, let's use Method #1 (prime factorization):

(a)
$10 = 2\times5$
$15 = 3\times5$

The largest subset of numbers common to both is 5. Thus the greatest common factor (GCF) of 10 and 15 is 5.

(b)
$21 = 3\times7$
$49 = 7\times7$

The GCF is 7.

Now let's use Euclid's method:

(a)
$15 \div 10 = 1$, remainder 5
$10 \div 5 = 2$, remainder 0

so the GCF of 15 and 10 is 5.

(b)
$49 \div 21 = 2$, remainder 7
$21 \div 7 = 3$, remainder 0

so the GCF of 21 and 49 is 7.

6. Find the Least Common Multiple of the number pairs in the previous problem

   Answer:

   Use the factors from Method #1:

   (a)
   10, 15: $2\times$ 5 and 3×5: $2\times3\times5 = 30$. The LCM of 10 and 15 is 30

   (b)
   21, 49: $3\times7\times7 = 147$. The LCM is 147

7. 48 boxes are to be stacked in a rectangular array a boxes wide, b boxes deep, and c boxes high. Find integers a, b, c that are as nearly equal to one another as possible. This will make a compact array.

   Answer:

   Let's look at the factors. If you pile the boxes in an array measuring $a \times b \times c$, the total number of boxes is $a \times b \times c$. The factors of 48 are: $2\times2\times2\times2\times3$. How do you combine these to make three numbers as close in size as possible? Make the factors 48 $= 4\times4\times3$. The array of boxes in 4 wide, 4 deep and 3 high (or 4 wide, 3 deep and 4 high).