Math 135: Intermediate Algebra
Homework 9 Complex Numbers Solutions

Rewrite in terms of i:

1.

$$\sqrt{-4} = \pm 2i$$

2.

$$\sqrt{-\frac{1}{16}} = \pm\frac{1}{4}i$$

3.

$$\sqrt{-3} = \pm i\sqrt{3}$$

4.

$$\sqrt{-18} = \pm i\sqrt{18} = \pm 3i\sqrt{2}$$

5.

$$\sqrt{-500} = \pm i\sqrt{500} = \pm 10i\sqrt{5}$$

6.

$$-\sqrt{-9} = - \pm 3i = \mp 3i$$

7.

$$6\sqrt{-\frac{5}{16}} = \pm 6i\sqrt{\frac{5}{16}} = \pm\frac{3}{2}i\sqrt{5}$$

8.

$$-\frac{1}{4}\sqrt{-12} = - \pm \frac{1}{4}i\sqrt{12} = \mp\frac{1}{2}i\sqrt{3}$$

Please note that the *correct* answer for a square root should always include the $\pm$, since there are two square roots.

9. Ohm's Law for alternating-current circuits states that $V = IZ$, where V is the voltage (in volts), I is the current (in amperes) and Z the impedance (in ohms).

(a) Rewrite Ohm's law by solving for Z

Ohm's law states that $V = IZ$. We solve for Z by dividing both sides of the equation by I, giving us:

$$V = IZ; IZ = V; \frac{IZ}{I} = \frac{V}{I}; Z = \frac{V}{I}$$

(b) If V can be represented by $11 + 10i$ and I by $2 + 3i$, find Z

$$Z = \frac{V}{I} = \frac{11 + 10i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{22 - 33i + 20i + 30}{4 + 9} = \frac{52 = 13i}{13} = 4 - i$$

(c) Check your answer to part (c) by substituting into Ohm's Law

$V = IZ = (2 + 3i)(4 - i) = 8 - 2i + 12i + 3 = 11 + 10i$

The computed voltage, $11 + 10i$, agrees with the given information, confirming our answer in part (b)

(d) Rewrite Ohm's Law by solving for I

$V = IZ; IZ = V; I = \dfrac{V}{Z}$

(e) If V can be represented by $3 + 5i$ and Z by $i - 1$, find I

$I = \dfrac{V}{Z} = \dfrac{3 + 5i}{i - 1} = \dfrac{3 + 5i}{i - 1} \cdot \dfrac{i + 1}{i + 1} = \dfrac{(3 + 5i)(i + 1)}{i^2 - 1} = \dfrac{-2 + 8i}{-2} = 1 - 4i$

(f) Check your answer to part (e) by substituting into Ohm's Law

$V = IZ = (1 - 4i)(i - 1) = i - 1 + 4 + 4i = 5i + 3$, which checks

Evaluate

10. i^{18}

Since $i^4 = 1$, $i^{18} = i^{16} \cdot i^2 = -1$

11. i^{20}

$i^{20} = i^{4 \cdot 5} = 1$

12. i^{35}

$i^{35} = i^{32} \cdot i^3 = -i$

13. $\dfrac{i^{38}}{i^{19}}$

$\dfrac{i^38}{i^{19}} = i^{19} = i^{16} \cdot i^3 = -i$

14. $i^{15} \cdot i^{11}$

$i^{15} \cdot i^{11} = i^{26} = i^{24} \cdot i^2 = -1$

15. i^{41}

$i^{41} = i^{40} \cdot i = i$