

Math 135: Intermediate Algebra
Homework 8 Solving Quadratic Equations: Solutions

1. Difference of squares
2. $x^2 - 16 + 36 = (x - 6)^2$
3. $y^2 + 18y + 81 = (y + 9)^2$
4. $4a^2 + 25 - 20a = 4a^2 - 20a + 25 = (2a - 5)^2$
5. $x^2 - 16 = (x + 4)(x - 4)$
6. $121 - y^2 = (11 - y)(11 + y)$
7. $9x^2 - 4 = (3x - 2)(3x + 2)$
8. $64x^2 + 16xy + y^2 = (8x + y)^2$
9. $p^3 - 8 = (p - 2)(p^2 + 2p + 4)$
10. The area of the entire picture, including the border, is x^2 square inches. The area of just the picture inside the border is $8^2 = 64$ square inches. Therefore the area of the border is $x^2 - 64 = (x + 8)(x - 8)$ square inches.
11. The height is $324 - 16t^2 = 4(81 - 4t^2) = 4(9 - 2t)(9 + 2t)$ feet.
- 12.

$$\begin{aligned}(x + 3)(x - 4) &= 0 \\ x + 3 = 0 &\text{ or } x - 4 = 0 \\ x = -3 &\text{ or } x = 4\end{aligned}$$

13.

$$\begin{aligned}(4n - 3)(n - 2) &= 0 \\ 4n - 3 = 0 &\text{ or } n - 2 = 0 \\ 4n = 3 &\text{ or } n = 2 \\ n = \frac{3}{4} &\text{ or } n = 2\end{aligned}$$

14.

$$\begin{aligned}(2x + 1)(2x + 3) &= 0 \\ 2x + 1 = 0 &\text{ or } 2x + 3 = 0 \\ 2x = -1 &\text{ or } 2x = -3 \\ x = -\frac{1}{2} &\text{ or } x = -\frac{3}{2}\end{aligned}$$

15.

$$\begin{aligned}(3 - p)^2 &= 0 \\ 3 - p &= 0 \\ p &= 3\end{aligned}$$

16.

$$\begin{aligned}x^2 + 5x &= 0 \\x(x + 5) &= 0 \\x = 0 &\text{ or } x + 5 = 0 \\x = 0 &\text{ or } x = -5\end{aligned}$$

17.

$$\begin{aligned}6n^2 - 3n &= 0 \\3n(2n - 1) &= 0 \\3n = 0 &\text{ or } 2n - 1 = 0 \\n = 0 &\text{ or } 2n = 1 \\n = 0 &\text{ or } n = \frac{1}{2}\end{aligned}$$

18.

$$\begin{aligned}x^2 + 8x + 12 &= 0 \\(x + 2)(x + 6) &= 0 \\x + 2 = 0 &\text{ or } x + 6 = 0 \\x = -2 &\text{ or } x = -6\end{aligned}$$

19.

$$\begin{aligned}a^2 - 3a + 2 &= 0 \\(a - 2)(a - 1) &= 0 \\a - 2 = 0 &\text{ or } a - 1 = 0 \\a = 2 &\text{ or } a = 1\end{aligned}$$

20.

$$\begin{aligned}36 + 5t - t^2 &= 0 \\(9 - t)(4 + t) &= 0 \\9 - t = 0 &\text{ or } 4 + t = 0 \\t = 9 &\text{ or } t = -4\end{aligned}$$

21.

$$\begin{aligned}2x^2 + 5x - 7 &= 0 \\(2x + 7)(x - 1) &= 0 \\2x + 7 = 0 &\text{ or } x - 1 = 0 \\2x = -7 &\text{ or } x = 1 \\x = -\frac{7}{2} &\text{ or } x = 1\end{aligned}$$

22.

$$\begin{aligned}0 &= 4r^2 - 20r + 25 \\0 &= (2r - 5)^2 \\0 &= 2r - 5 \\2r &= 5 \\r &= \frac{5}{2}\end{aligned}$$

23.

$$\begin{aligned}
 20x^2 - 45 &= 0 \\
 4x^2 - 9 &= 0 \\
 (2x - 3)(2x + 3) &= 0 \\
 2x - 3 = 0 &\text{ or } 2x + 3 = 0 \\
 2x = 3 &\text{ or } 2x = -3 \\
 x = \frac{3}{2} &\text{ or } x = -\frac{3}{2}
 \end{aligned}$$

24.

$$\begin{aligned}
 p^2 - 32 &= -4p \\
 p^2 + 4p - 32 &= 0 \\
 (p + 8)(p - 4) &= 0 \\
 p + 8 = 0 &\text{ or } p - 4 = 0 \\
 p = -8 &\text{ or } p = 4
 \end{aligned}$$

25. We want to know when the height is zero, i.e. the sandbag is on the ground. Since the height $h = 900 - 16t^2$, this means we must solve for $h = 0$:

$$\begin{aligned}
 900 - 16t^2 &= 0 \\
 225 - 4t^2 &= 0 \\
 (15 - 2t)(15 + 2t) &= 0 \\
 15 - 2t = 0 &\text{ or } 15 + 2t = 0 \\
 2t = 15 &\text{ or } 2t = -15 \\
 t = 7.5 &\text{ or } t = -7.5.
 \end{aligned}$$

Since the sandbag must hit the ground after it is dropped, only the positive solution makes sense. The sandbag hits the ground 7.5 seconds after it is dropped.

26. The width is w and the length is $w + 50$, so the area is $w(w + 50)$. Since we are given that the area is 5000 square yards, we have an equation we can solve:

$$\begin{aligned}
 w(w + 50) &= 5000 \\
 w^2 + 50w &= 5000 \\
 w^2 + 50w - 5000 &= 0 \\
 (w + 100)(w - 50) &= 0 \\
 w + 100 = 0 &\text{ or } w - 50 = 0 \\
 w = -100 &\text{ or } w = 50.
 \end{aligned}$$

Since widths must be positive, only the positive solution makes sense. The width is 50 yards, and the length is $w + 50 = 100$ yards.

27. We are given the equation, so we just have to solve it:

$$\begin{aligned}
 8000[(1 + r)^2 - 1] &= 1680 \\
 8000(r^2 + 2r + 1 - 1) &= 1680 \\
 8000(r^2 + 2r) &= 1680 \\
 100(r^2 + 2r) &= 21 \\
 100r^2 + 200r - 21 &= 0
 \end{aligned}$$

$$\begin{aligned}
(10r - 1)(10r + 21) &= 0 \\
10r - 1 = 0 &\quad \text{or} \quad 10r + 21 = 0 \\
10r = 1 &\quad \text{or} \quad 10r = -21 \\
r = 0.1 &\quad \text{or} \quad r = -2.1.
\end{aligned}$$

Since the rate of return is positive, the solution that makes sense is $r = 0.1$. Since we are asked for the rate as a percentage, this is $r = 10\%$.

28. Using the Pythagorean Theorem, we have $x^2 + 12^2 = 15^2$. Solving:

$$\begin{aligned}
x^2 + 12^2 &= 15^2 \\
x^2 + 144 &= 225 \\
x^2 &= 81 \\
x &= \pm 9
\end{aligned}$$

Only the positive solution makes sense, since distances must be positive, so the ladder should be placed 9 feet from the house.