Electricity & Electronics I

Aaron Chiu

March 5, 2019

Contents

1 Basic Circuit Analysis

1.1 Ohm's Law

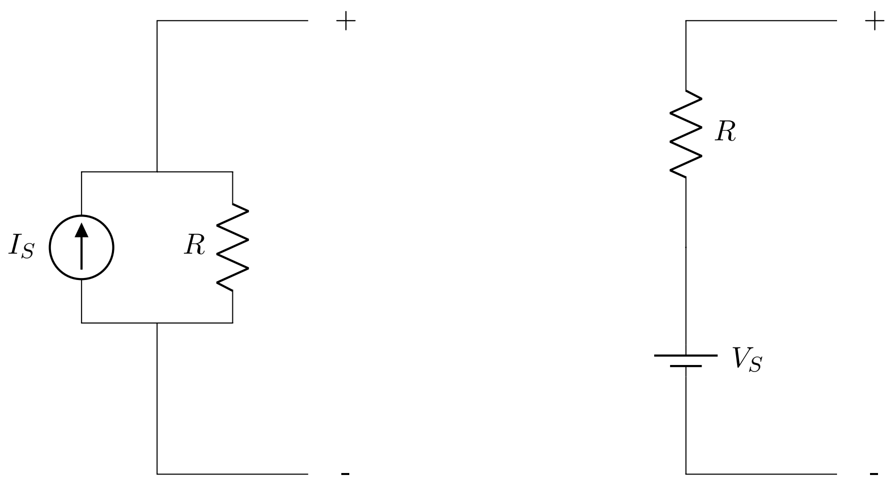
1.2 Kirchoff's Voltage & Current Laws

1.3 Series & Parallel Circuits

1.4 Open & Short Circuits

2 Advanced Circuit Analysis

2.1 Source Transformation (Conversion)



Current Sources in parallel with a resistor can be made into a voltage source with the same resistor in series. To convert between the two:

Voltage → Current

$$I_S = \frac{V_S}{R}$$

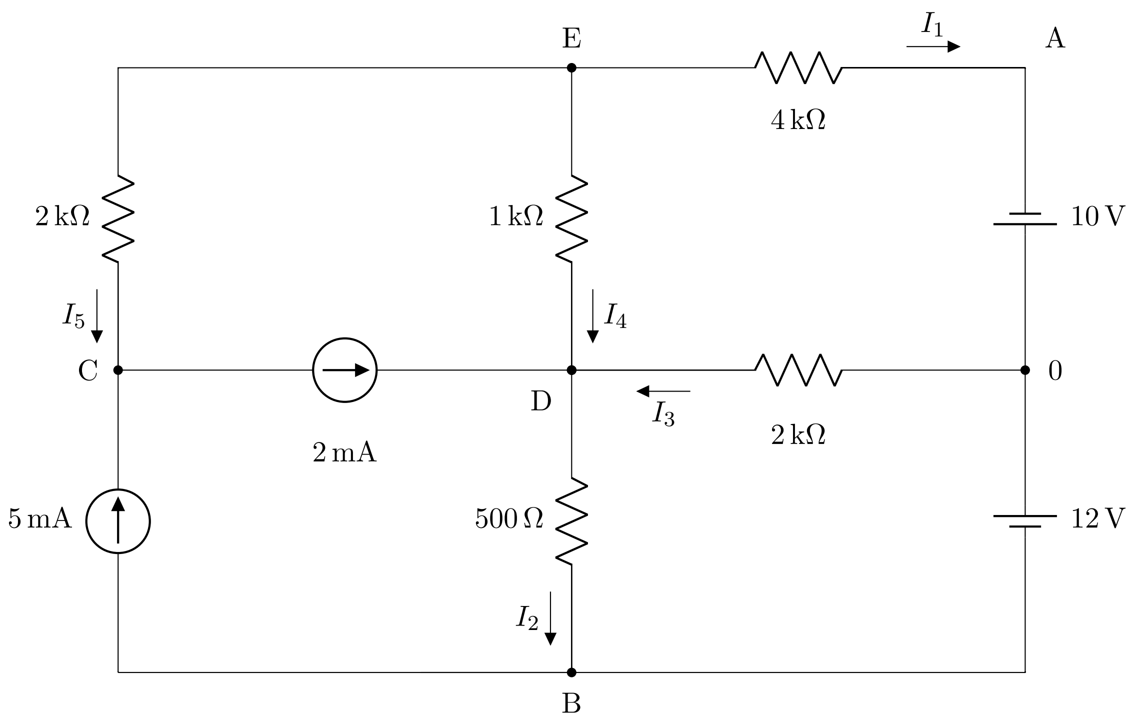
Current → Voltage

$$V_S = I_S R$$

Notes on Source Transformation

- A Current Source in parallel cannot be resolved into a Voltage Source or vice versa is out of the scope of the course
- The resulting Current/Battery Source should face the same way as the one before the transformation

2.2 Nodal Analysis



Step 1: Write the known voltages at the known nodes

$$\text{A: } A = -10\,V$$
$$\text{B: } B = -12\,V$$

Step 2: Write KCL equations for the other nodes

- Currents in equals currents out

$$\text{C: } I_5 + 5 = 2$$
$$\text{D: } I_4 + 2 + I_3 = I_2$$
$$\text{E: } 0 = I_5 + I_4 + I_1$$

Step 3: Rewrite in terms of voltages and resistances

- Note that Voltage is start minus end
- Write known voltages as their numerical value

$$\text{C: } \frac{V_E - V_C}{2} + 5 = 2$$
$$\text{D: } \frac{V_E - V_D}{1} + 2 + \frac{0 - V_D}{2} = \frac{V_D - (-12)}{0.5}$$
$$\text{E: } 0 = \frac{V_E - V_C}{2} + \frac{V_E - V_D}{1} + \frac{V_E - (-10)}{4}$$

Step 4: Isolate for voltages into a proper matrix form
(ie one voltage in one column and etc)

- It is recommended to reorganise voltages at the nodes so that the coefficients are positive since this will result in the main diagonal being positive
- This will also result in the coefficients being the sum of the conductances leading to the node in the main diagonal
- The coefficients of the negative will be the conductance leading out of the node to the other node

$$\begin{array}{lllll}
\text{C:} & +V_C\left[\frac{1}{2}\right] & -V_D[0] & -V_E\left[\frac{1}{2}\right] & =3 \\
\text{D:} & -V_C[0] & +V_D\left[\frac{1}{0.5}+\frac{1}{2}+\frac{1}{1}\right] & -V_E\left[\frac{1}{1}\right] & =-22 \\
\text{E:} & -V_C\left[\frac{1}{2}\right] & -V_D\left[\frac{1}{1}\right] & +V_E\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{1}\right] & =-2.5
\end{array}$$

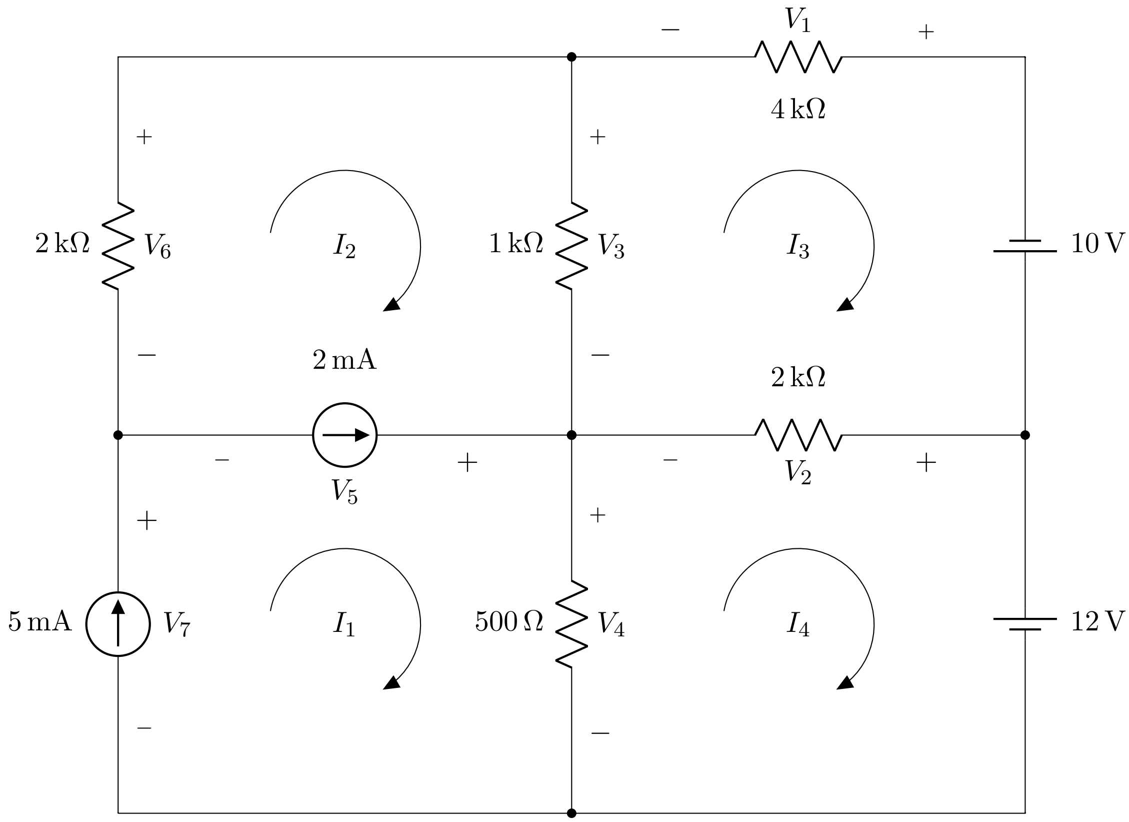
Step 5: Solve for the voltages

$$V_C = 0\,V \qquad V_D = -8\,V \qquad V_E = -6\,V$$

Notes on Nodal Analysis

- Always choose a reference node that has the most batteries connecting to it
- If possible use source conversion to convert the voltage sources to current sources
- To obtain the current through a battery, it is necessary to write KCL through a loop containing the battery

2.3 Mesh (Loop) Analysis



Step 1: Write the known currents at the known meshes

$$I_1 : \; I_1 = 5\,mA \qquad\qquad I_2 : \; 2 = I_1 - I_2$$
$$I_2 = 3\,mA$$

Step 2: Write KVL equations for the other meshes

- Travel in the direction of the loops and list down the voltages
- The sign you encounter first is the sign you write down

$$I_3 : \; -V_3 - V_1 - 10 + V_2 = 0$$
$$I_4 : \; -V_2 + 12 - V_4 = 0$$

Step 3: Rewrite in terms of currents and resistances

- Note that according to the direction of the voltage (+ → -), it is any current flowing with the voltage minus and current flowing against the voltage
- Write known currents as their numerical value and keep the signs of the variables

$$
\begin{aligned}
I_3 &: \; -(3 - I_3)(1) - (-I_3)(4) - 10 + (I_3 - I_4)(2) = 0 \\
I_4 &: \; -(I_3 - I_4)(2) + 12 - (5 - I_4)(0.5) = 0
\end{aligned}
$$

Step 4: Isolate for currents into a proper matrix form
(ie one current in one column and etc)

- It is recommended to reorganise currents at the meshes so that the coefficients are positive since this will result in the main diagonal being positive
- This will also result in the coefficients being the sum of the resistances around the mesh in the main diagonal
- The coefficients of the negative will be the resistances between the other mesh and the current mesh

$$
\begin{array}{llll}
I_3 : & + I_3(1+4+2) & - I_4(2) & = 13 \\
I_4 : & - I_3(2) & + I_4(2+0.5) & = -9.5
\end{array}
$$

Step 5: Solve for the currents

$$I_3 = 1\,mA \qquad\qquad I_4 = -3\,mA$$

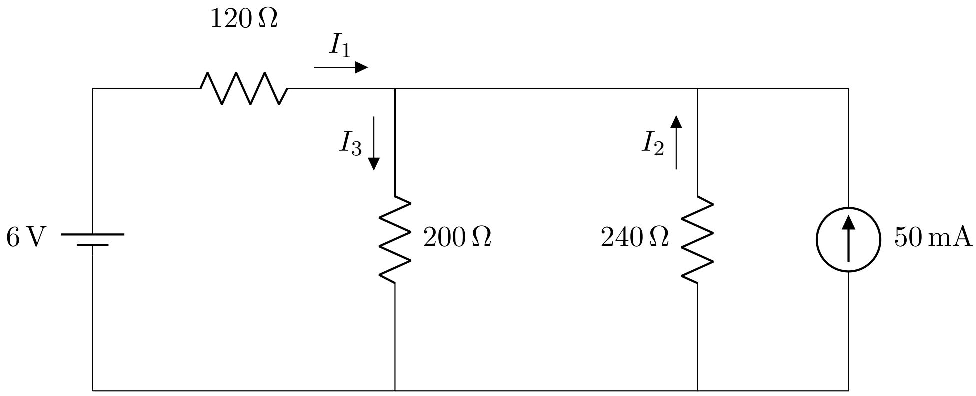
Notes on Mesh Analysis

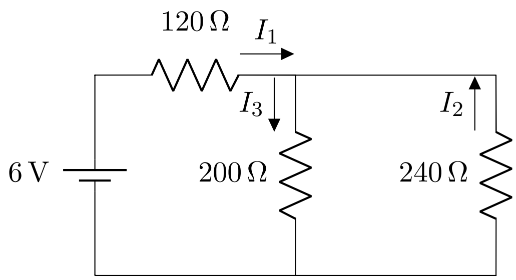
- It is recommended to make all your meshes turn clockwise
- If possible use source conversion to convert the current sources to voltage sources
- To obtain the voltage through a current source, it is necessary to write KVL through a loop containing the battery

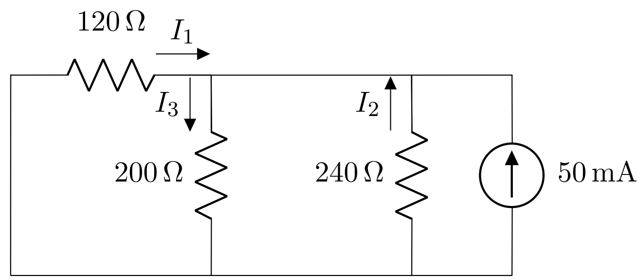
3 Network Theorems

3.1 Superposition Theorem

If a circuit has multiple sources, one can "kill" all sources but one. This results in the circuit only having one source making it easier to analyse. Current sources become open circuited and batteries become short circuited. The sum of both currents at the branch in question will be the actual current of the branch.







$$\begin{aligned} R_{eqA} &= 120\ \Omega + 200\ \Omega \| 240\ \Omega \\ &\approx 229.09\ \Omega \\ I_A &= \frac{6\ \mathrm{V}}{229.09\ \Omega} \\ &\approx 26.19\,\mathrm{mA} \end{aligned}$$

$$\begin{aligned} i_{3A} &= 26.19\,\mathrm{mA} \cdot \frac{240\ \Omega}{200\ \Omega + 240\ \Omega} \\ &\approx 14.29\,\mathrm{mA} \end{aligned}$$

$$\begin{aligned} i_{3B} &= 50\,\mathrm{mA} \ \cdot \frac{200\ \Omega}{120\ \Omega \| 200\ \Omega \| 240\ \Omega} \\ &\approx 14.29\,\mathrm{mA} \end{aligned}$$

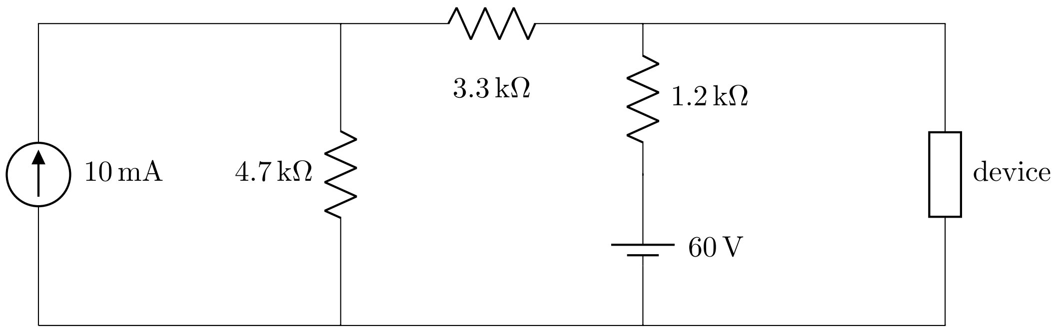
$$\begin{aligned} i_3 &= 14.29\,\mathrm{mA} + 14.29\,\mathrm{mA} \\ &\approx 28.58\,\mathrm{mA} \end{aligned}$$

3.2 Thévenin's Theorem

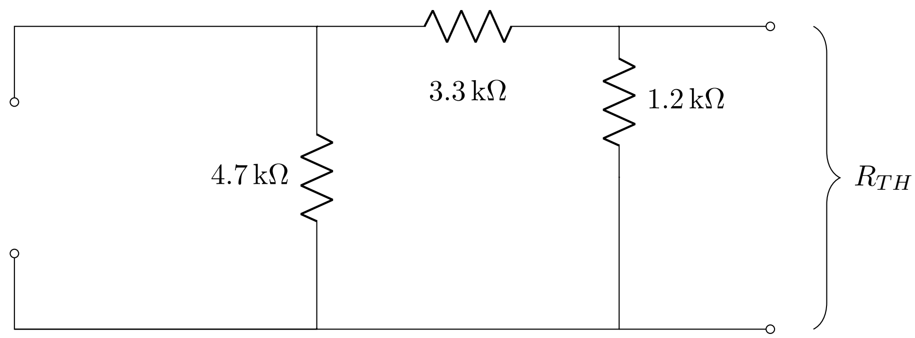
Any circuit that has two terminals (an open circuit) can be replaced with a Thévenin circuit is composed of a voltage source in series with a resistor named E_{TH} and R_{TH} respectively. E_{TH} is the voltage between the two open terminals. R_{TH} is the resistance when voltage sources are shorted and current sources are opened. An equation can be then derived.

$$V = E_{TH} + R_{TH}I$$

We are to find the relationship between voltage and current in the device.

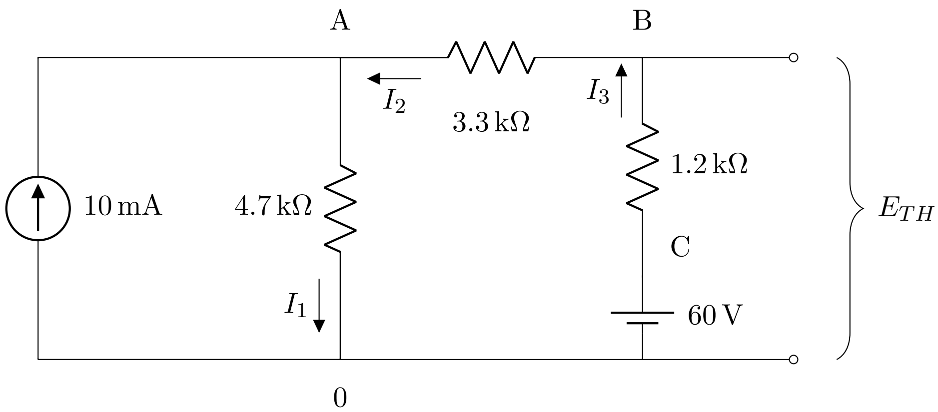


First thing is to label the network terminals, short all voltage sources and open all current sources.



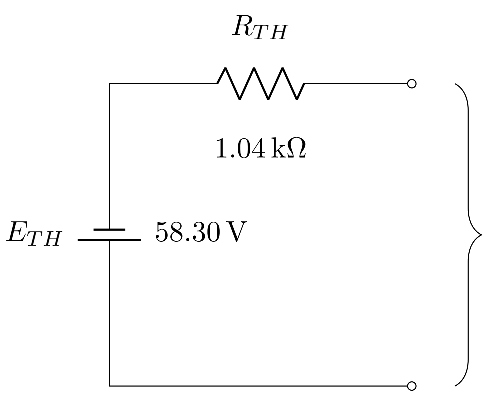
$$\begin{aligned} R_{TH} &= (4.7\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega})||1.2\,\mathrm{k\Omega} \\ &\approx 1.04\,\mathrm{k\Omega} \end{aligned}$$

Using known methods, we can calculate E_{TH} such as nodal analysis.



$$
\begin{aligned}
&A: 10 + I_2 = I_1 && 10 + \frac{V_B - V_A}{3.3} = \frac{V_A}{4.7} && V_A\left(\frac{1}{4.7} + \frac{1}{3.3}\right) - V_B\left(\frac{1}{3.3}\right) = 10 \\
&B: I_3 = I_2 && \frac{60 - V_B}{1.2} = \frac{V_B - V_A}{3.3} && -V_A\left(\frac{1}{3.3}\right) - V_B\left(\frac{1}{3.3} + \frac{1}{1.2}\right) = \frac{60}{1.2} \\
&C: V_C = 60\,\mathrm{V} && V_C = 60\,\mathrm{V} &&
\end{aligned}
$$

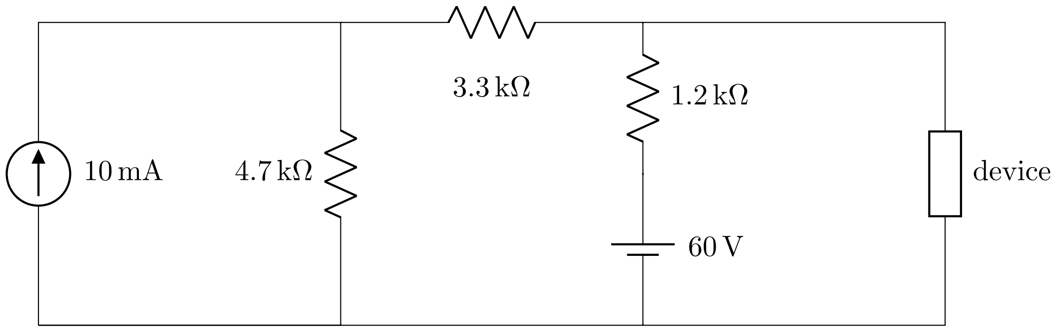
$$
\begin{aligned}
&A: V_A \approx 53.64\,\mathrm{V} \\
&B: V_B \approx 58.30\,\mathrm{V} \qquad\qquad \therefore E_{TH} \approx 58.30\,\mathrm{V} \\
&C: V_C = 60\,\mathrm{V}
\end{aligned}
$$



$$\boxed{V = 58.30 + 1.04I}$$

3.3 Norton's Theorem

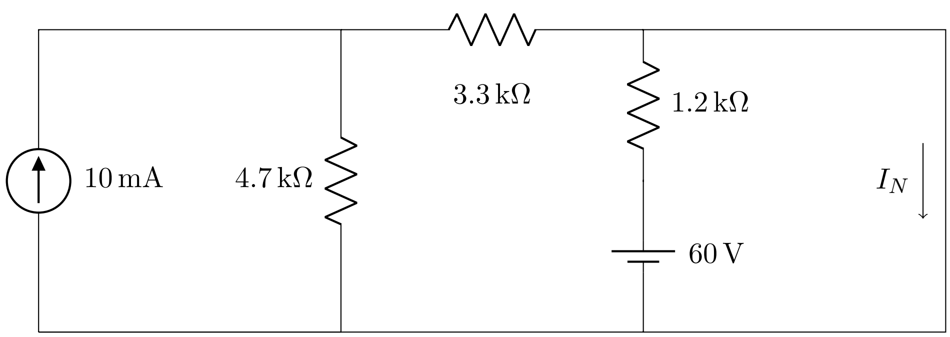
Similarly to Thévenin's theorem, this states a circuit can be replaced with a current source in parallel with a resistor named I_N and R_N respectively. Instead of opening the circuit where the device of question is, we short the device.
We are to find the relationship between voltage and current in the device.



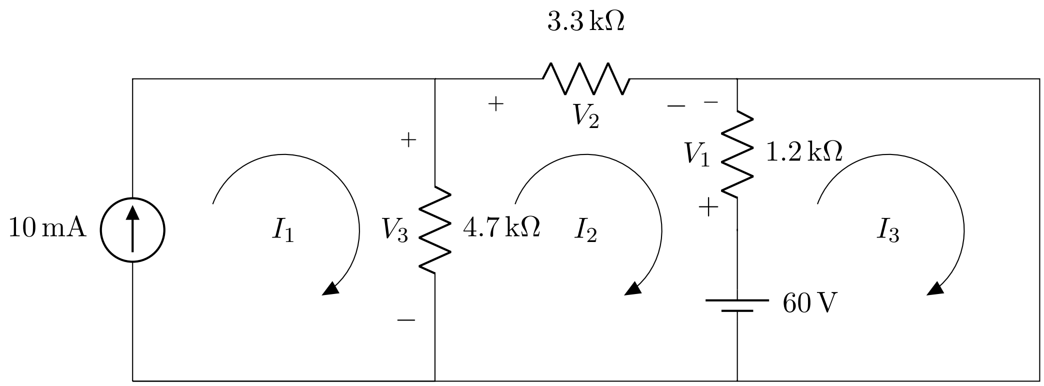
R_N will be the same as R_{TH} so we can use the same techniques to find it.

$$\begin{aligned} R_{TH} &= (4.7\,\mathrm{k\Omega} + 3.3\,\mathrm{k\Omega})||1.2\,\mathrm{k\Omega} \\ &\approx 1.04\,\mathrm{k\Omega} \end{aligned}$$

First thing is to short the device and label the current through it, short all voltage sources and open all current sources.



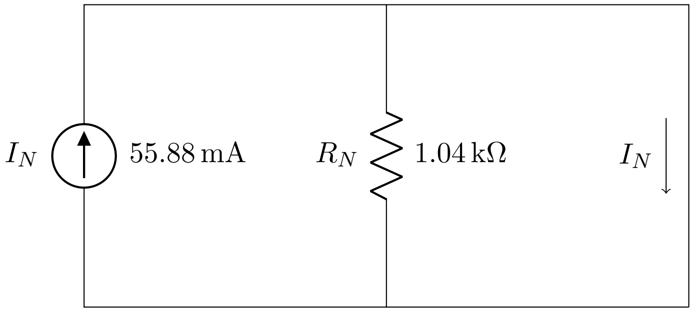
Using known methods, we can calculate I_N such as mesh analysis.



$$\begin{aligned} &I_1 : I_1 = 10\,\mathrm{mA} \\ &I_2 : 0 = 60 - V_3 + V_2 - V_1 && 0 = 60 - (10 - I_2)(4.7) + (I_2)(3.3) - (I_3 - I_2)(1.2) \\ &I_3 : 0 = V_1 + 60 && 0 = (I_3 - I_2)(1.2) + 60 \end{aligned}$$

$$\begin{aligned} &I_1 : && && I_1 = 10\,\mathrm{mA} \\ &I_2 : && I_2(4.7+3.3+1.2) - I_3(1.2) = -13 && I_2 = 5.88\,\mathrm{mA} \\ &I_3 : && -I_2(1.2) + I_3(1.2) = 60 && I_3 = 55.88\,\mathrm{mA} \end{aligned}$$

$$\therefore I_N = 55.88\,\mathrm{mA}$$



$$\boxed{I = 55.88 + \frac{V}{1.04}}$$

3.4 Relationships between Thévenin's and Norton's Theorem

3.5 Maximum Power Transfer Theorem